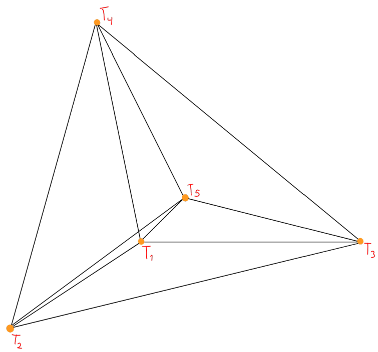
MEF 4D

Paso 1: Localización

Isoparametrización



$$N_1 = 1 - \epsilon - \eta - \phi - \gamma$$
$$N_2 = \epsilon$$
$$N_3 = \eta$$
$$N_4 = \phi$$
$$N_5 = \gamma$$

Partición de la unidad

$$N_1 + N_2 + N_3 + N_4 + N_5 = 1 - \cancel{\epsilon} - \cancel{\eta} - \cancel{\phi} - \cancel{\gamma} + \cancel{\epsilon} + \cancel{\eta} + \cancel{\phi} + \cancel{\gamma} = 1$$

$N_1 = (0,0,0,0) = 1$
$N_1 = (1,0,0,0) = 0$
$N_1 = (0,1,0,0) = 0$
$N_1 = (0,0,1,0) = 0$
$N_1 = (0,0,0,1) = 0$

$N_2 = (0,0,0,0) = 0$
$N_2 = (1,0,0,0) = 1$
$N_2 = (0,1,0,0) = 0$
$N_2 = (0,0,1,0) = 0$
$N_2 = (0,0,0,1) = 0$

$N_3 = (0,0,0,0) = 0$
$N_3 = (1,0,0,0) = 0$
$N_3 = (0,1,0,0) = 1$
$N_3 = (0,0,1,0) = 0$
$N_3 = (0,0,0,1) = 0$

$N_4 = (0,0,0,0) = 0$
$N_4 = (1,0,0,0) = 0$
$N_4 = (0,1,0,0) = 0$
$N_4 = (0,0,1,0) = 1$
$N_4 = (0,0,0,1) = 0$

$N_5 = (0,0,0,0) = 0$
$N_5 = (1,0,0,0) = 0$
$N_5 = (0,1,0,0) = 0$
$N_5 = (0,0,1,0) = 0$
$N_5 = (0,0,0,1) = 1$

Mantenimiento en la frontera

$$N_1(0,0,0,0)T_1 + N_2(0,0,0,0)T_2 + N_3(0,0,0,0)T_3 + N_4(0,0,0,0)T_4 + N_5(0,0,0,0)T_5 = T_1 + \cancel{0T_2} + \cancel{0T_3} + \cancel{0T_4} + \cancel{0T_5} = T_1$$

$$N_1(1,0,0,0)T_1 + N_2(1,0,0,0)T_2 + N_3(1,0,0,0)T_3 + N_4(1,0,0,0)T_4 + N_5(1,0,0,0)T_5 = \cancel{0T_1} + T_2 + \cancel{0T_3} + \cancel{0T_4} + \cancel{0T_5} = T_2$$

$$N_1(0,1,0,0)T_1 + N_2(0,1,0,0)T_2 + N_3(0,1,0,0)T_3 + N_4(0,1,0,0)T_4 + N_5(0,1,0,0)T_5 = \cancel{0T_1} + \cancel{0T_2} + T_3 + \cancel{0T_4} + \cancel{0T_5} = T_3$$

$$N_1(0,0,1,0)T_1 + N_2(0,0,1,0)T_2 + N_3(0,0,1,0)T_3 + N_4(0,0,1,0)T_4 + N_5(0,0,1,0)T_5 = \cancel{0T_1} + \cancel{0T_2} + \cancel{0T_3} + T_4 + \cancel{0T_5} = T_4$$

$$N_1(0,0,0,1)T_1 + N_2(0,0,0,1)T_2 + N_3(0,0,0,1)T_3 + N_4(0,0,0,1)T_4 + N_5(0,0,0,1)T_5 = \cancel{0T_1} + \cancel{0T_2} + \cancel{0T_3} + \cancel{0T_4} + T_5 = T_5$$

Paso 2: Interpolación

$$-\nabla(K\nabla T) = Q$$

$$T = N_1T_1 + N_2T_2 + N_3T_3 + N_4T_4 + N_5T_5$$

$$= \begin{bmatrix} N_1 & N_2 & N_3 & N_4 & N_5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = NT \qquad T \approx NT$$

Paso 3: Aproximación del modelo.

$$-\nabla(K\nabla T) = Q$$
$$-\nabla(K\nabla(NT) = Q$$

$$\mathcal{R} \approx Q + \nabla(K\nabla(NT))$$

Paso 4: M. de residuos ponderados

$$\int_{\Omega_4} W\mathcal{R}\, dV_4 = 0$$

$$\int_{\Omega_4} W(Q + \nabla(K\nabla(NT))\, dV_4 = 0$$

Paso 5: M. Galerkin.

$$W = N^{+}$$

$$\int_{\Omega_4} N^{+}\left(Q + \left(\nabla K \nabla (NT)\right)\right) dV_4 = 0$$

$$\int_{\Omega_4} N^{+}Q + N^{+}\nabla K\left(\nabla(NT)\right) dV_4 = 0$$

$$Q\int_{\Omega_4} N^{+} dV_4 + \int_{\Omega_4} N^{+}\nabla (K\nabla N)\, dV_4\, T = 0$$

$$-\int_{\Omega_4} N^{+}\nabla(K\nabla N)\, dV_4 = Q\int_{\Omega_4} N^{+} dV_4$$

Paso 6: Resolución de integrales

Lado derecho

$$Q\int_{\Omega_4} N^{+} dV_4 = Q\int_{\Omega_4} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix} dV_4 = Q\int \begin{bmatrix} 1-\epsilon-\eta-\phi-\gamma \\ \epsilon \\ \eta \\ \phi \\ \gamma \end{bmatrix} dV_4$$

$$Q\iiiint \begin{bmatrix} 1-\epsilon-\eta-\phi-\gamma \\ \epsilon \\ \eta \\ \phi \\ \gamma \end{bmatrix} dx\, dy\, dz\, dw$$

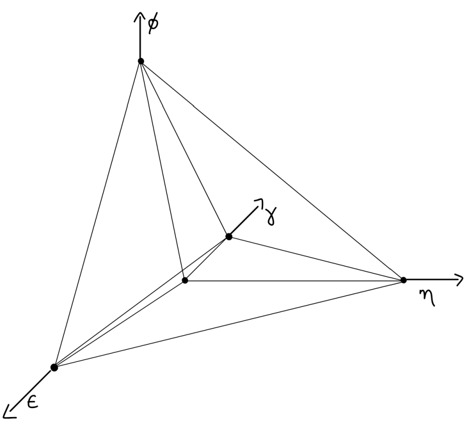
$dV_4 = dx\, dy\, dz\, dw$

$$J := |J| \rightarrow dx\,dy\,dz\,d\omega = J\,d\epsilon\,d\eta\,d\phi\,d\gamma$$

$$J = \begin{bmatrix} \frac{\delta x}{\delta \epsilon} & \frac{\delta x}{\delta \eta} & \frac{\delta x}{\delta \phi} & \frac{\delta x}{\delta \gamma} \\ \frac{\delta y}{\delta \epsilon} & \frac{\delta y}{\delta \eta} & \frac{\delta y}{\delta \phi} & \frac{\delta y}{\delta \gamma} \\ \frac{\delta z}{\delta \epsilon} & \frac{\delta z}{\delta \eta} & \frac{\delta z}{\delta \phi} & \frac{\delta z}{\delta \gamma} \\ \frac{\delta \omega}{\delta \epsilon} & \frac{\delta \omega}{\delta \eta} & \frac{\delta \omega}{\delta \phi} & \frac{\delta \omega}{\delta \gamma} \end{bmatrix} = \begin{bmatrix} x_2 - x_1 & x_3 - x_1 & x_4 - x_1 & x_5 - x_1 \\ y_2 - y_1 & y_3 - y_1 & y_4 - y_1 & y_5 - y_1 \\ z_2 - z_1 & z_3 - z_1 & z_4 - z_1 & z_5 - z_1 \\ \omega_2 - \omega_1 & \omega_2 - \omega_1 & \omega_4 - \omega_1 & \omega_5 - \omega_1 \end{bmatrix}$$

$$Q J \int_{\Omega_4} \begin{bmatrix} 1 - \epsilon - \eta - \phi - \gamma \\ \epsilon \\ \eta \\ \phi \\ \gamma \end{bmatrix} d\epsilon\,d\eta\,d\phi\,d\gamma$$

$$Q J \int\int\int\int \begin{bmatrix} 1 - \epsilon - \eta - \phi - \gamma \\ \epsilon \\ \eta \\ \phi \\ \gamma \end{bmatrix} d\epsilon\,d\eta\,d\phi\,d\gamma$$



$$\int_0^1 \int_0^{1-\gamma} \int_0^{1-\phi-\gamma} \int_0^{1-\phi-\gamma-\eta} \begin{bmatrix} 1-\epsilon-\eta-\phi-\gamma \\ \epsilon \\ \eta \\ \phi \\ \gamma \end{bmatrix} d\epsilon\, d\eta\, d\phi\, d\gamma$$

$$QJ \begin{bmatrix} \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{120} \\ \frac{1}{120} \end{bmatrix} = \frac{QJ}{120} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = b$$

Lado izquierdo

$$-\int_{\Omega_4} N^{+} \nabla (K \nabla N)\, dV_4 \qquad \begin{matrix} U = N^{+} \\ dU = \nabla N^{+} \end{matrix} \qquad \begin{matrix} dV = \nabla (K \nabla N) \\ V = K \nabla N \end{matrix}$$

$$-\left(\left[N^{+} K \nabla N \right] \Big|_{\Omega_4} - \int_{\Omega_4} \nabla N^{+} K \nabla N\, dV_4 \right) = -\left[N^{+} K \nabla N \right] + \underbrace{\int_{\Omega_4} \nabla N^{+} K \nabla N\, dV_4}$$

$$\int \nabla N^{+} K \nabla N\, dV_4 \qquad \nabla N = J^{-1} \begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \\ \frac{\delta}{\delta \phi} \\ \frac{\delta}{\delta \gamma} \end{bmatrix} \begin{bmatrix} 1-\epsilon-\eta-\phi-\gamma & \epsilon & \eta & \phi & \gamma \end{bmatrix}$$

$$-J^{-1}\begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} = J^{-1}B$$

$$= \frac{1}{J} AB \qquad \nabla N^{+} = \frac{1}{J} B^{+}A^{+}$$

$$= \int_{\Omega_4} \frac{1}{J} B^{+}A^{+} K \frac{1}{J} AB \, dV_4 = \frac{K}{J^2} B^{+}A^{+}AB \int_{\Omega_4} dV$$

$$= \frac{KV_4}{J^2} B^{+}A^{+}AB = K$$

$$- \left[N^{+} K \nabla N T\right]\Big|_{\Omega_4} + \left(\frac{kV_4}{J^2} B^{+}A^{+}AB\right)T = \frac{QJ}{120}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\left(\frac{kV_4}{J^2} B^{+}A^{+}AB\right)T = \frac{QJ}{120}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \left[N^{+} K \nabla N T\right]\Big|_{\Omega_4}$$

$$\left(\frac{kV_4}{J^2} B^{+}A^{+}AB\right)T = \frac{QJ}{120}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$K \quad T = b$$