UNIVERSIDAD CENTROAMERICANA “JOSÉ SIMEÓN CAÑAS”

Facultad de Ingeniería y Arquitectura Departamento de Electrónica e Informática

Técnicas de simulación en computadoras - Ciclo 01/2024

Tarea:

Segunda tarea práctica grupal

Docente:

Jorge Alfredo López Sorto

Integrantes:

Orellana Vividor, Gerardo André	00053520
Rodríguez Rodríguez, David Neftalí	00218621
Zacatales López, César Adonay	00223021
Alas Moscoso, Noé Bladimir	00262020

Sección: 01

Antiguo Cuscatlán, lunes 10 de junio de 2024

Grupo 10

Pentagono de lado 20

1) Localización: Mallado, tabla de conectividades y tabla de coordenadas de nodos.

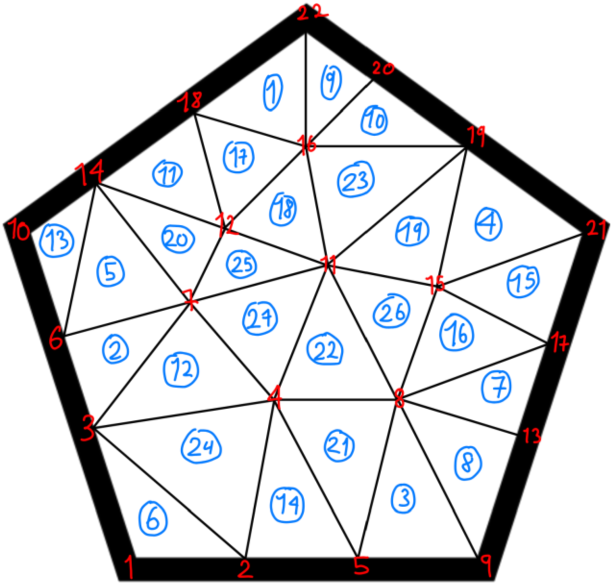


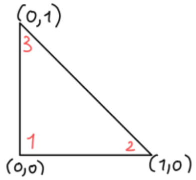
Tabla de coordenadas de nodos

Nodo	X	Y
1	-23.28	-13.66
2	-19.36	-13.66
3	-24.49	-9.13
4	-18.34	-9.21
5	-15.44	-13.66
6	-25.71	-6.21
7	-21.41	-5.29
8	-13.09	-8.81
9	-11.52	-13.66
10	-26.92	-2.48
11	-16.2	-3.89

Nodo	X	Y
12	-19.97	-2.04
13	-10.31	-9.94
14	-23.75	-0.18
15	-11.66	-4.34
16	-16.80	0.27
17	-9.10	-6.21
18	-20.58	2.13
19	-11.06	-0.18
20	-14.23	2.13
21	-7.89	-2.48
22	-17.40	4.43

Tabla de conectividades.

Elemento	1	2	3
1	22	18	16
2	6	3	7
3	5	9	8
4	21	19	15
5	14	6	7
6	1	2	3
7	13	17	8
8	13	8	9
9	20	22	16
10	20	16	19
11	18	14	12
12	3	4	7
13	10	6	14
14	2	5	4
15	17	21	15
16	17	15	8
17	16	18	12
18	16	12	11
19	15	19	11
20	12	14	7
21	4	5	8
22	4	8	11
23	16	11	19
24	2	4	3
25	11	12	7
26	15	11	8
27	7	4	11



$N_1 = 1 - \epsilon - \eta$

$N_2 = \epsilon$

$N_3 = \eta$

$A \to N_1(0,0) = 1-0-0 = 1$	$N_2(0,0) = 0$	$N_3(0,0) = 0$
$B \to N_1(1,0) = 1-1-0 = 0$	$N_2(1,0) = 1$	$N_3(1,0) = 0$
$C \to N_1(0,1) = 1-0-1 = 0$	$N_2(0,1) = 0$	$N_3(0,1) = 1$

$$NP = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$N_{1A}P_1 + N_{2A}P_2 + N_{3A}P_3 = (1)P_1 + \cancel{(0)P_2} + \cancel{(0)P_3} = P_1$

$N_{1B}P_1 + N_{2B}P_2 + N_{3B}P_3 = \cancel{(0)P_1} + (1)P_2 + \cancel{(0)P_3} = P_2$

$N_{1C}P_1 + N_{2C}P_2 + N_{3C}P_3 = \cancel{(0)P_1} + \cancel{(0)P_2} + (1)P_3 = P_3$

2) Interpolación.

$$P = NT$$

3) Aproximación del modelo.

$$-\eta^3 \nabla(\epsilon^2 \nabla P) \approx \eta + \epsilon^3$$

$$R = \eta + \epsilon^3 + \eta^3 \nabla(\epsilon^2 \nabla NP)$$

4) Método de residuos ponderados.

$$\int_A \omega R \, dA = 0$$

$$\int_A \omega(\eta + \epsilon^3 + \eta^3 \nabla(\epsilon^2 \nabla N) \, dA \, P = 0$$

5) Método de Galerkin

$$\omega = N^T$$

$$\int_A N^T(\eta + \epsilon^3 + \eta^3 \nabla(\epsilon^2 \nabla N) \, dA \, P = 0$$

$$\int_A N^T \eta \, dA + \int_A N^T \epsilon^3 + \int_A N^T(\eta^3 \nabla(\epsilon^2 \nabla N) \, dA \, P = 0$$

$$-\int_A N^T(\eta^3 \nabla(\epsilon^2 \nabla N) \, dA \, P = \int_A N^T \eta \, dA + \int_A N^T \epsilon^3$$

Continuación en siguiente página.

$$-\int_A N^{+}\left(\eta^3 \nabla\left(\epsilon^2 \nabla N\right)\right) dA\, P = \underset{A}{\int_A N^{+}\eta\, dA} + \underset{B}{\int_A N^{+}\epsilon^3}$$

Lado derecho

Resolución de A

$$\int_A N^{+}\eta\, dA \longrightarrow J\int_0^1\int_0^{1-\eta}\begin{bmatrix}1-\epsilon-\eta\\ \epsilon\\ \eta\end{bmatrix}\eta\, d\epsilon\, d\eta$$

Integral interna

$$\int_0^{1-\eta}\begin{bmatrix}\eta-\epsilon\eta-\eta^2\\ \epsilon\eta\\ \eta^2\end{bmatrix}d\epsilon = \begin{bmatrix}\epsilon\left(\eta-\eta^2\right)-\dfrac{\epsilon^2-\eta}{2}\\ \dfrac{\epsilon^2}{2}\\ \epsilon\,\eta^2\end{bmatrix}\Bigg|_0^{1-\eta} = \begin{bmatrix}\dfrac{\eta(\eta-1)^2}{2}\\ \dfrac{(\eta-1)^2}{2}\\ -\eta^2(\eta-1)\end{bmatrix}$$

Integral externa

$$\int_0^1\begin{bmatrix}\dfrac{\eta(\eta-1)^2}{2}\\ \dfrac{(\eta-1)^2}{2}\\ -\eta^2(\eta-1)\end{bmatrix}d\eta = \begin{bmatrix}\dfrac{\eta^2\left(3\eta^2-8\eta+6\right)}{24}\\ \dfrac{(\eta-1)^3}{6}\\ -\dfrac{\eta^3(3\eta-4)}{12}\end{bmatrix}\Bigg|_0^1 = \begin{bmatrix}\dfrac{1}{24}\\ \dfrac{1}{6}\\ \dfrac{1}{12}\end{bmatrix}$$

Resolución de B

$$\int_A N^{+}\epsilon^3\, dA = \int_0^1\int_0^{1-\eta}\begin{bmatrix}1-\epsilon-\eta\\ \epsilon\\ \eta\end{bmatrix}\epsilon^3\, d\epsilon\, d\eta$$

Continuación en siguiente página.

Integral interna

$$\int_0^{1-\eta}\begin{bmatrix}\epsilon^3-\epsilon^4-\epsilon^3\eta\\ \epsilon^4\\ \epsilon^3\eta\end{bmatrix}d\epsilon = \begin{bmatrix}-\dfrac{\epsilon^5}{5}-\dfrac{\epsilon^4(\eta-1)}{4}\\ \dfrac{\epsilon^5}{5}\\ \dfrac{\epsilon^4\eta}{4}\end{bmatrix}\Bigg|_0^{1-\eta} = \begin{bmatrix}-\dfrac{(\eta-1)^5}{20}\\ -\dfrac{(\eta-1)^5}{5}\\ \dfrac{\eta(\eta-1)^4}{4}\end{bmatrix}$$

Integral externa

$$\int_0^1 \begin{bmatrix} \frac{-(\eta-1)^5}{20} \\ \frac{-(\eta-1)^5}{5} \\ \frac{\eta(\eta-1)^4}{4} \end{bmatrix} = \begin{bmatrix} \frac{-(\eta-1)^6}{120} \\ \frac{-(\eta-1)^6}{30} \\ \frac{\eta^2(5\eta^4-24\eta^3+45\eta^2-40\eta+15)}{120} \end{bmatrix} = \begin{bmatrix} \frac{1}{120} \\ \frac{1}{30} \\ \frac{1}{120} \end{bmatrix}$$

$$b = J\left(\begin{bmatrix} \frac{1}{24} \\ \frac{1}{6} \\ \frac{1}{12} \end{bmatrix} + \begin{bmatrix} \frac{1}{120} \\ \frac{1}{30} \\ \frac{1}{120} \end{bmatrix} \right) = J \begin{bmatrix} \frac{1}{20} \\ \frac{3}{40} \\ \frac{1}{120} \end{bmatrix}$$

Lado izquierdo

$$\left(-\int N^t \nabla \eta^3 \epsilon^2 (\nabla N) \, dA \right) P$$

$U = N^t \qquad V = \nabla \eta^3 \epsilon^2 \nabla N$

$dU = \nabla N^t \qquad dv = \eta^3 \epsilon^2 \nabla N$

$$-\underbrace{\left(N^t \nabla \eta^3 \epsilon^2 \nabla N \right|_A}_{T_N} + \int_A \nabla N^t \eta^3 \epsilon^2 \nabla N \, dA$$

Resolución de integral de k

$$\int_A \nabla N^t \eta^3 \epsilon^2 \nabla N \, dA \qquad \nabla N = \begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \end{bmatrix} \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}$$

Continuación en siguiente página.

$$\nabla N = J^{-1} \begin{bmatrix} \frac{\delta}{\delta \epsilon} \\ \frac{\delta}{\delta \eta} \end{bmatrix} \begin{bmatrix} 1-\epsilon-\eta & \epsilon & \eta \end{bmatrix}$$

$$= J^{-1}\left[\begin{bmatrix}\frac{\delta}{\delta\epsilon}\\ \frac{\delta}{\delta\eta}\end{bmatrix}[1-\epsilon-\eta] \quad \begin{bmatrix}\frac{\delta}{\delta\epsilon}\\ \frac{\delta}{\delta\eta}\end{bmatrix}\epsilon \quad \begin{bmatrix}\frac{\delta}{\delta\epsilon}\\ \frac{\delta}{\delta\eta}\end{bmatrix}\eta\right] = \begin{bmatrix}\frac{\delta}{\delta\epsilon}(1-\epsilon-\eta) & \frac{\delta}{\delta\epsilon}(\epsilon) & \frac{\delta}{\delta\epsilon}(\eta)\\ \frac{\delta}{\delta\eta}(1-\epsilon-\eta) & \frac{\delta}{\delta\eta}(\epsilon) & \frac{\delta}{\delta\eta}(\eta)\end{bmatrix} = \underbrace{\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}}_{B}$$

$$\nabla N = J^{-1}B \qquad J^{-1} = \frac{1}{|J|}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix} = \frac{1}{|J|}\underbrace{\begin{bmatrix}(y_2-y_1) & (y_1-y_2)\\ (x_1-x_3) & (x_2-x_1)\end{bmatrix}}_{A} = \frac{1}{J}A$$

$$\nabla N = \frac{1}{J}AB \qquad \nabla N^{+} = \left(\frac{1}{J}AB\right)^{T} = \frac{1}{J}B^{T}A^{T}$$

$$= \int_A \eta^3\epsilon^2\, dA = \int_0^1\int_0^{1-\eta}\eta^3\epsilon^2\, d\epsilon\, d\eta = \frac{1}{420}$$

$$K = \frac{1}{420J^2}B^{+}A^{+}ABP$$

R11 $KT = b$

$$\downarrow$$

$$\frac{1}{420J^2}B^{+}A^{+}ABP = J\begin{bmatrix}\frac{1}{20}\\ \frac{3}{40}\\ \frac{1}{120}\end{bmatrix}$$