Computational Graphics: Lecture 11

Alberto Paoluzzi

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Outline: Geometria2

1. LAR-CC library
2. LAR representation
3. Facet extraction
4. Boundary computation
5. Extrusion
6. Cartesian product of complexes
7. Skeletons

LAR-CC library

download from github

```
$ git clone git@github.com:cvdlab/lar-cc.git
```

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```

In your python files:

```python
import sys
""" import modules from lar-cc/lib """
sys.path.insert(0, 'lar-cc/lib/py/')
from simplexn import *
from larcc import *
from lar2psm import *
from largrid import *
```

LAR representation

Input of a simplicial complex (brc2csr)

From BRC (Binary Row Compressed) to CSR (Compressed Sparse Row)

```
V = [[0, 0], [1, 0], [2, 0], [0, 1], [1, 1], [2, 1]]
FV = [[0, 1, 3], [1, 2, 4], [1, 3, 4], [2, 4, 5]]
EV = [[0,1],[0,3],[1,2],[1,3],[1,4],[2,4],[2,5],[3,4],[4,5]]

VIEW(STRUCT(MKPOLS((V,FV)))); VIEW(EXPLODE(1.2,1.2,1)(MKPOLS((V,FV))))
VIEW(STRUCT(MKPOLS((V,EV)))); VIEW(EXPLODE(1.2,1.2,1)(MKPOLS((V,EV))))

\pause

csrFV = csrCreate(FV)
csrEV = csrCreate(EV)

print "\ncsrCreate(FV) =\n", csrFV
print "\n>>> csr2DenseMatrix"
print "\nFV =\n", csr2DenseMatrix(csrFV)
print "\nEV =\n", csr2DenseMatrix(csrEV)
```

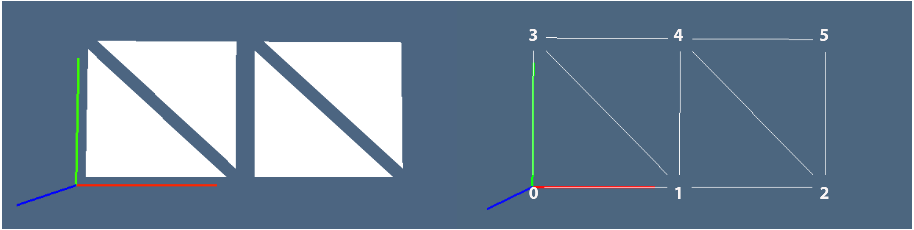
Input of a simplicial complex (brc2csr)

```
csrCreate(FV) =
  (0, 0)        1
  (0, 1)        1
  (0, 3)        1
  (1, 1)        1
  (1, 2)        1
  (1, 4)        1
  (2, 1)        1
  (2, 3)        1
  (2, 4)        1
  (3, 2)        1
  (3, 4)        1
  (3, 5)        1
```

```
>>> csr2DenseMatrix

FV =
[[1 1 0 1 0 0]
 [0 1 1 0 1 0]
 [0 1 0 1 1 0]
 [0 0 1 0 1 1]]

EV =
[[1 1 0 0 0 0]
 [1 0 0 1 0 0]
 [0 1 1 0 0 0]
 [0 1 0 1 0 0]
 [0 1 0 0 1 0]
 [0 0 1 0 1 0]
 [0 0 1 0 0 1]
 [0 0 0 1 1 0]
 [0 0 0 0 1 1]]
```



Facet extraction

Facet extraction from simplices

combinatorial approach

- A k-face of a d-simplex is defined as the convex hull of any subset of k vertices.

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$$\sigma^d = \langle v_0, v_1, \ldots, v_d \rangle$$

  is also called a facet.

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- Each of the $d+1$ facets of σ^d, obtained by removing a vertex from σ^d, is a $(d-1)$-simplex.

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- A simplex may be oriented in two different ways according to the permutation class of its vertices.

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  is also called a facet.
- Each of the $d+1$ facets of σ^d, obtained by removing a vertex from σ^d, is a $(d-1)$-simplex.
- A simplex may be oriented in two different ways according to the permutation class of its vertices.
- The simplex orientation is so changed by either multiplying the simplex by -1, or by executing an odd number of exchanges of its vertices.

Facet extraction from simplices

combinatorial approach

The chain of oriented boundary facets of σ^d, usually denoted as $\partial\sigma^d$, is generated combinatorially as follows:

$$\partial\,\sigma^d = \sum_{k=0}^{d} (-1)^d \langle v_0, \ldots, v_{k-1}, v_{k+1}, \ldots, v_d \rangle$$

Implementation

```
def larSimplexFacets(simplices):
    ''' To return the facets of a list of d-simplices '''
    out = []
    d = len(simplices[0])
    for simplex in simplices:
        out += [simplex[0:k]+simplex[k+1:d]
                for k in range(d)]
    out = sorted(out)
    return [facet for k,facet in enumerate(out[:-1])
                if out[k] != out[k+1]] + [out[-1]]
```

Test of implementation

```
>>>larSimplexFacets([[0]])
[[]]
>>>larSimplexFacets([[0,1]])
[[0],[1]]
>>>larSimplexFacets([[0,1,2,]])
[[0,1],[0,2],[1,2]]
>>>larSimplexFacets([[0,1,2,3]])
[[0,1,2],[0,1,3],[0,2,3],[1,2,3]]
>>>larSimplexFacets([[0,1,2,3,4]])
[[0,1,2,3],[0,1,2,4],[0,1,3,4],[0,2,3,4],[1,2,3,4]]
```

Test of implementation

```
>>>larSimplexFacets([[0]])
[[]]
>>>larSimplexFacets([[0,1]])
[[0],[1]]
>>>larSimplexFacets([[0,1,2,]])
[[0,1],[0,2],[1,2]]
>>>larSimplexFacets([[0,1,2,3]])
[[0,1,2],[0,1,3],[0,2,3],[1,2,3]]
>>>larSimplexFacets([[0,1,2,3,4]])
[[0,1,2,3],[0,1,2,4],[0,1,3,4],[0,2,3,4],[1,2,3,4]]
```

are such facets oriented?

Examples of facet extraction from 3D simplicial cube

```
V,CV = larSimplexGrid([1,1,1])

VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,CV))))
SK2 = (V,larSimplexFacets(CV))
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(SK2)))
SK1 = (V,larSimplexFacets(SK2[1]))
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(SK1)))
```

look also at

```
V,CV = larSimplexGrid([5,5,2])
```

Assignment

Change the larSimplexFacets so that the extracted facets are coherently oriented

Boundary computation

From cells and facets to boundary operator

```
def boundary(cells,facets):
    csrCV = csrCreate(cells)
    csrFV = csrCreate(facets)
    csrFC = matrixProduct(csrFV, csrTranspose(csrCV))
    facetLengths = [csrCell.getnnz() for csrCell in csrCV]
    return csrBoundaryFilter(csrFC,facetLengths)

def coboundary(cells,facets):
    Boundary = boundary(cells,facets)
    return csrTranspose(Boundary)
```

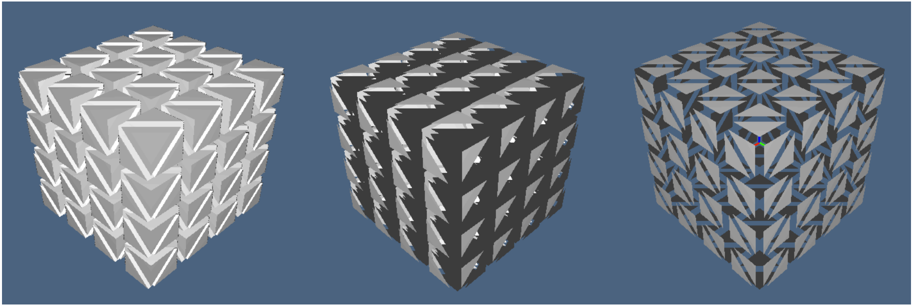
Oriented boundary example

```
V,CV = larSimplexGrid([4,4,4])
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,CV))))

FV = larSimplexFacets(CV)
EV = larSimplexFacets(FV)
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,FV))))

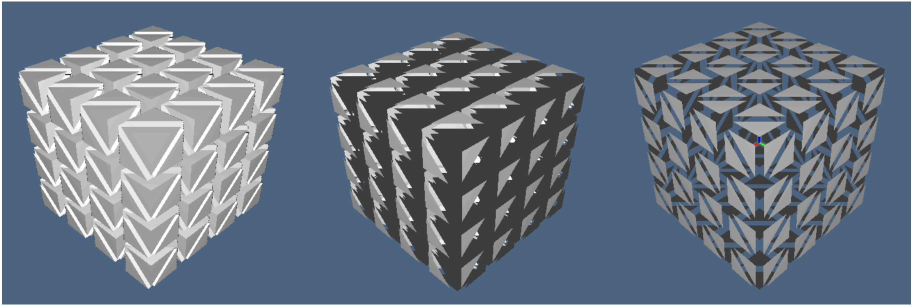
csrSignedBoundaryMat = signedBoundary (V,CV,FV)
boundaryCells_2 = signedBoundaryCells(V,CV,FV)
def swap(l): return [l[1],l[0],l[2]]
boundaryFV = [FV[-k] if k<0 else swap(FV[k]) for k in boundaryCells_2]
boundary = (V,boundaryFV)
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(boundary)))
```

Oriented boundary example



currently only the boundary of simplicial complexes can be oriented

Oriented boundary example



currently only the boundary of simplicial complexes can be oriented

Would you help?

Extrusion

Simplicial extrusion

Computation

Figure 1: Extrusion of (a) a point; (b) a straight line segment; (c) a triangle.

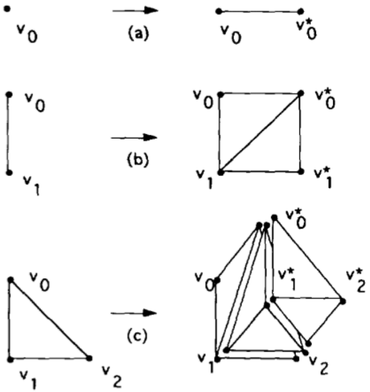


Figure : example caption

Simplicial extrusion

Computation

Let us concentrate on the generation of the simplex chain γ^{d+1} of dimension $d+1$ produced by combinatorial extrusion of a single simplex

$$\sigma^d = \langle v_0, v_1, \ldots, v_d, \rangle.$$

Then we have, with $|\gamma^{d+1}| = \sigma^d \times I$, and $I = [0,1]$:

$$\gamma^{d+1} = \sum_{k=0}^{d} (-1)^{kd} \langle v_k, \ldots v_d, v_0^*, \ldots v_k^* \rangle$$

with $v_k \in \sigma^d \times \{0\}$ and $v_k^* \in \sigma^d \times \{1\}$, and where the term $(-1)^{kd}$ is used to generate a chain of coherently-oriented extruded simplices.

Example of simplicial complex extrusion

```
V = [[0,0],[1,0],[2,0],[0,1],[1,1],[2,1],[0,2],[1,2],[2,2]]
FV = [[0,1,3],[1,2,4],[2,4,5],[3,4,6],[4,6,7],[5,7,8]]
model = larExtrude((V,FV),4*[1,2,-3])
VIEW(EXPLODE(1,1,1.2)(MKPOLS(model)))
```

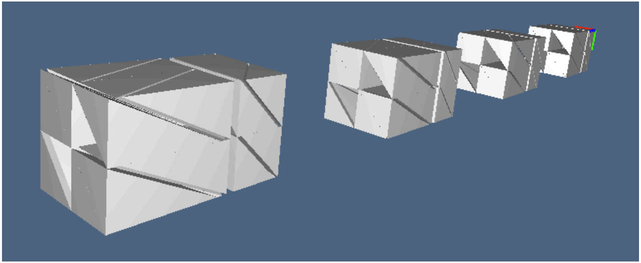


Figure : A simplicial complex providing a quite complex 3D assembly of tetrahedra.

Multidimensional simplicial grids

The generation of simplicial grids of any dimension and shape is amazingly simple

The input parameter `shape` is either a tuple or a list of integers used to specify the shape of the created array

```
VOID = V0,CV0 = [[]],[[0]]      # the empty simplicial model

def larSimplexGrid(shape):
    model = VOID
    for item in shape:
        model = larExtrude(model,item*[1])
    return model
```

The returned `model` has integer vertices, to be scaled and/or translated and/or mapped

Cartesian product of complexes

Cartesian product of two LAR models

```
def larModelProduct(twoModels):
    (V, cells1), (W, cells2) = twoModels
    @< Cartesian product of vertices @>
    @< Topological product of cells    @>
    model = [list(v) for v in vertices.keys()], cells
    return model
```

Cartesian product of two LAR models

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def larModelProduct(twoModels):
    (V, cells1), (W, cells2) = twoModels
    @< Cartesian product of vertices @>
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    model = [list(v) for v in vertices.keys()], cells
    return model
```

Cartesian product of vertices

```
vertices = collections.OrderedDict(); k = 0
for v in V:
    for w in W:
        id = tuple(v+w)
        if not vertices.has_key(id):
            vertices[id] = k
            k += 1  @}
```

Cartesian product of two LAR models

```
def larModelProduct(twoModels):
    (V, cells1), (W, cells2) = twoModels
    @< Cartesian product of vertices @>
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```

Cartesian product of vertices

```
vertices = collections.OrderedDict(); k = 0
for v in V:
    for w in W:
        id = tuple(v+w)
        if not vertices.has_key(id):
            vertices[id] = k
            k += 1  @}
```

Topological product of cells

```
cells = [ [vertices[tuple(V[v] + W[w])] for v in c1 for w in c2]
         for c1 in cells1 for c2 in cells2] @}
```

Cuboidal grids

```
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(larCuboids([3,2,1],True))))
```

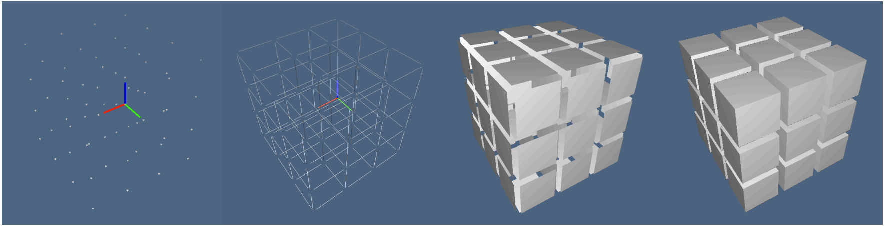

Figure : Exploded views of 0-, 1-, 2-, and 3-dimensional skeletons.

Cuboidal grids

```
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(larCuboids([3,2,1],True))))
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS(larCuboids([3,2,1],False))))
```

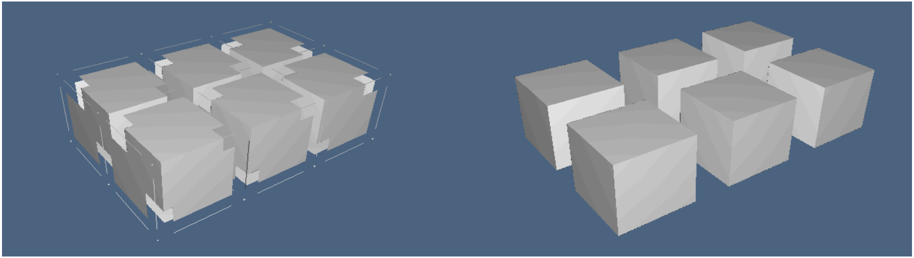

Figure : Exploded views of 0-, 1-, 2-, and 3-dimensional skeletons.

Skeletons

Cuboidal skeletons

A list of BRC characteristic matrices of cellular k-complexes ($0 \leq k \leq d$) with dimension d, where $d =$ `len(shape)`, is returned by the function `gridSkeletons` in the macro below, where the input is given by the shape of the grid, i.e.~by the list of cell items in each coordinate direction.

```
def gridSkeletons(shape):
    gridMap = larGridSkeleton(shape)
    skeletonIds = range(len(shape)+1)
    skeletons = [ gridMap(id) for id in skeletonIds ]
    return skeletons
```

Cuboidal skeletons

Just notice that the number of returned d-cells is equal to PROD(shape)

```
print "\ngridSkeletons([3]) =\n", gridSkeletons([3])
print "\ngridSkeletons([3,2]) =\n", gridSkeletons([3,2])
print "\ngridSkeletons([3,2,1]) =\n", gridSkeletons([3,2,1])
```

Generation of grid boundary complex

```
def gridBoundaryMatrices(shape):
    skeletons = gridSkeletons(shape)
    boundaryMatrices = [boundary(skeletons[k+1],faces)
                        for k,faces in enumerate(skeletons[:-
    return boundaryMatrices

for k in range(1):
    print "\ngridBoundaryMatrices([3]) =\n", \
          csr2DenseMatrix(gridBoundaryMatrices([3])[k])
for k in range(2):
    print "\ngridBoundaryMatrices([3,2]) =\n", \
          csr2DenseMatrix(gridBoundaryMatrices([3,2])[k])
for k in range(3):
    print "\ngridBoundaryMatrices([3,2,1]) =\n", \
          csr2DenseMatrix(gridBoundaryMatrices([3,2,1])[k])
```