Leveraging Phase Polynomials for Quantum Circuits Optimization

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Main Contribution:

PhasePoly: A **holistic** quantum **circuits optimization** framework via **Phase Polynomials.**

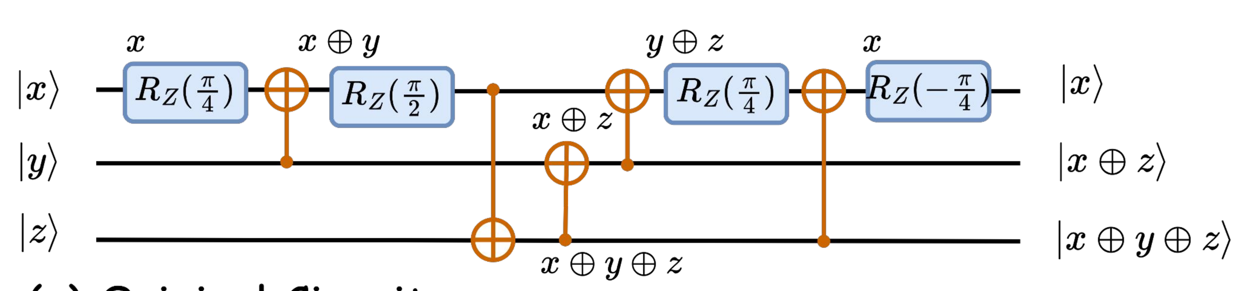
1. Background and Motivation

◇ Background

- Quantum circuits optimization is critical for reducing error and improving fidelity
- We can construct a **phase polynomials** circuit using $\{\mathbf{CNOT}, \mathbf{Rz}\}$.
- Phase polynomial circuits can be represented as ***sum-over-path***[2]:

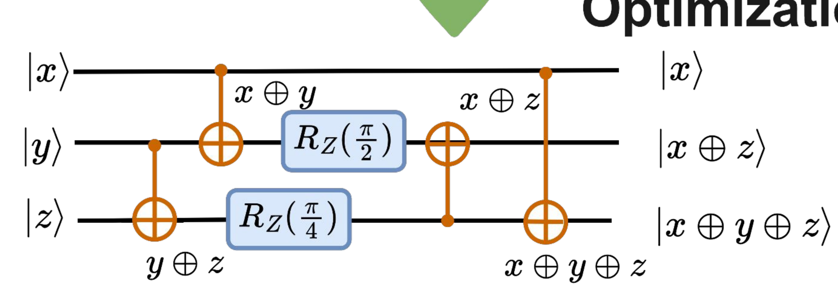
$$U\,|x_1,\ldots,x_n\rangle = e^{ip(x_1,\ldots,x_n)}\,|g(x_1,\ldots,x_n)\rangle$$

- Phase parity network: $p(x)$
- Output state CNOT network: $g(x)$



(a) Original Circuit
(**5** CNOT gates & **4** Rz gates)

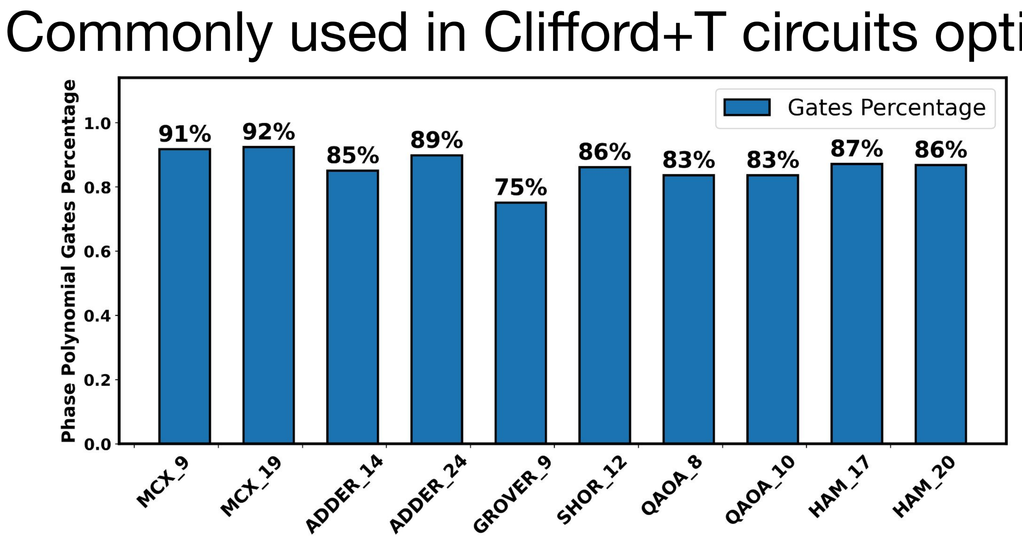
Phase Polynomials Optimization



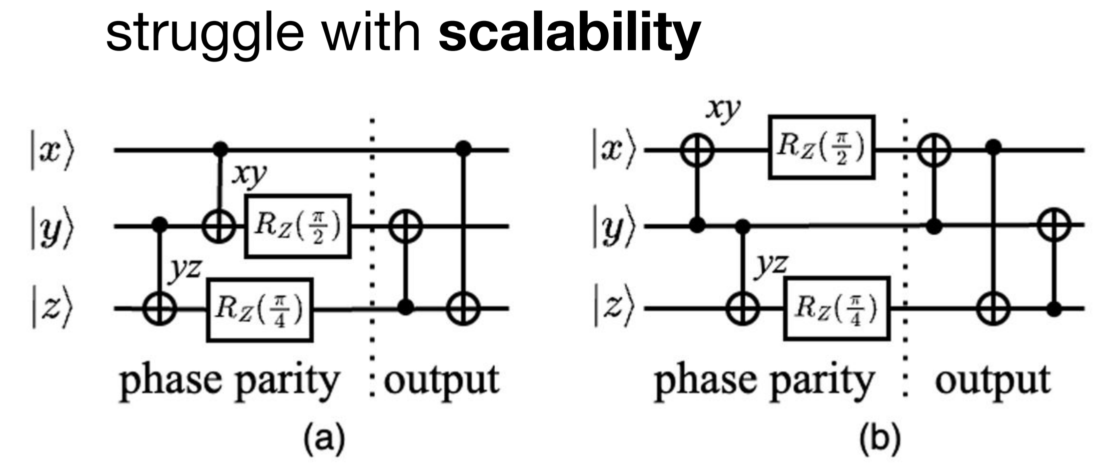
(b) Optimized Circuit
(**4** CNOT gates & **2** Rz gates)

◇ Motivation

- ◇ Phase polynomials are key **building block**
  - ◇ 75% of gates are $\{\mathbf{CNOT}, \mathbf{Rz}\}$ in selected circuits
  - ◇ Commonly used in Clifford+T circuits optimization



- ◇ Current phase polynomial approaches[3-5]
  - ◇ **Only** optimize phase parity network and single phase polynomial block **independently**
- ◇ Local equivalent subcircuit rewriting approaches[6] struggle with **scalability**

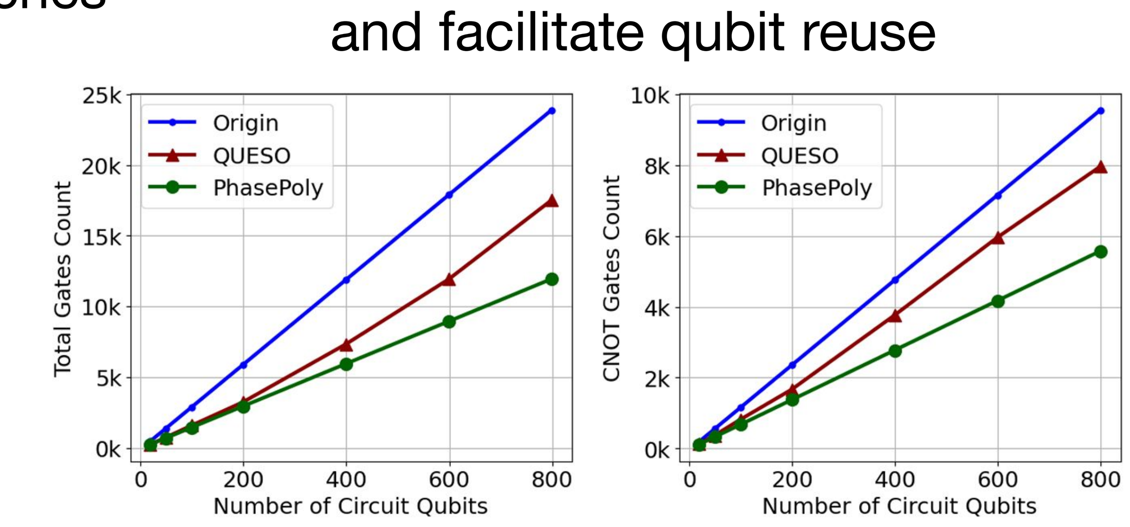


◇ Hardware-aware phase polynomials optimization

➔ Embed hardware constraints and qubit mapping cost into synthesis
➔ Maintain valid *sum-over-path* representation

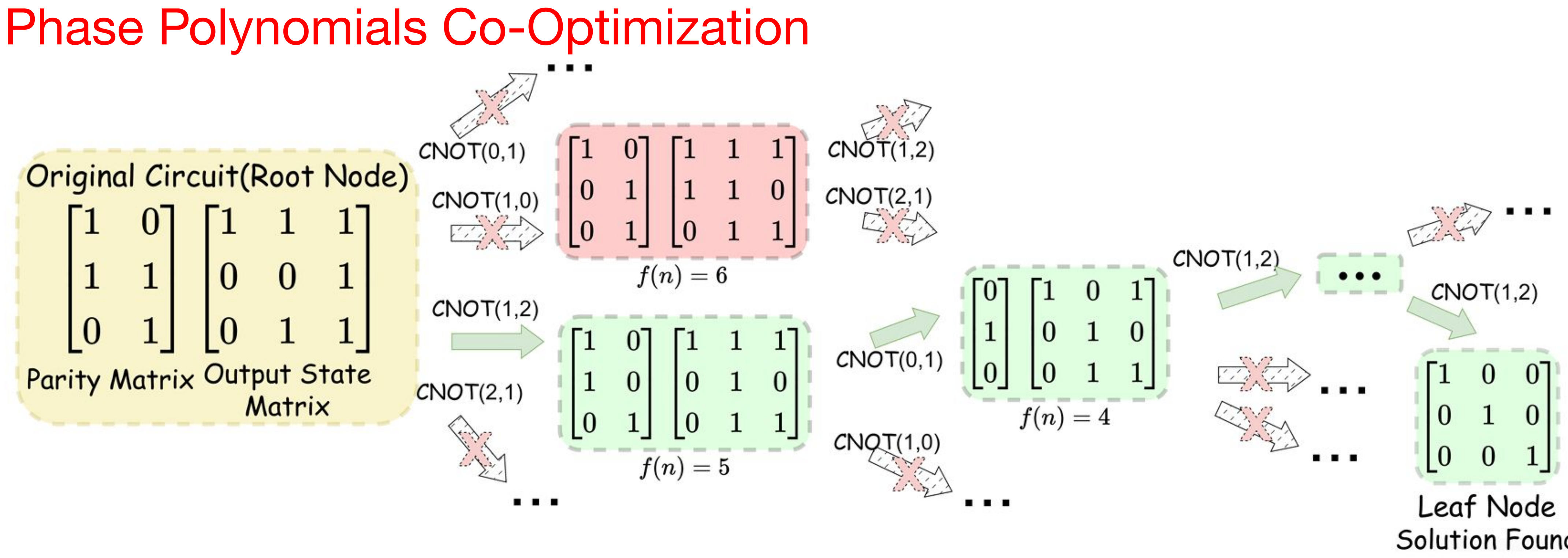
◇ Fault-tolerant friendly phase polynomials optimization

➔ Improve full-program FTQC performance via logical-level gains
➔ Optimize T gate placement via phase polynomials, potentially improve magic state scheduling, and facilitate qubit reuse



2. Our Approach: holistic phase polynomials optimization

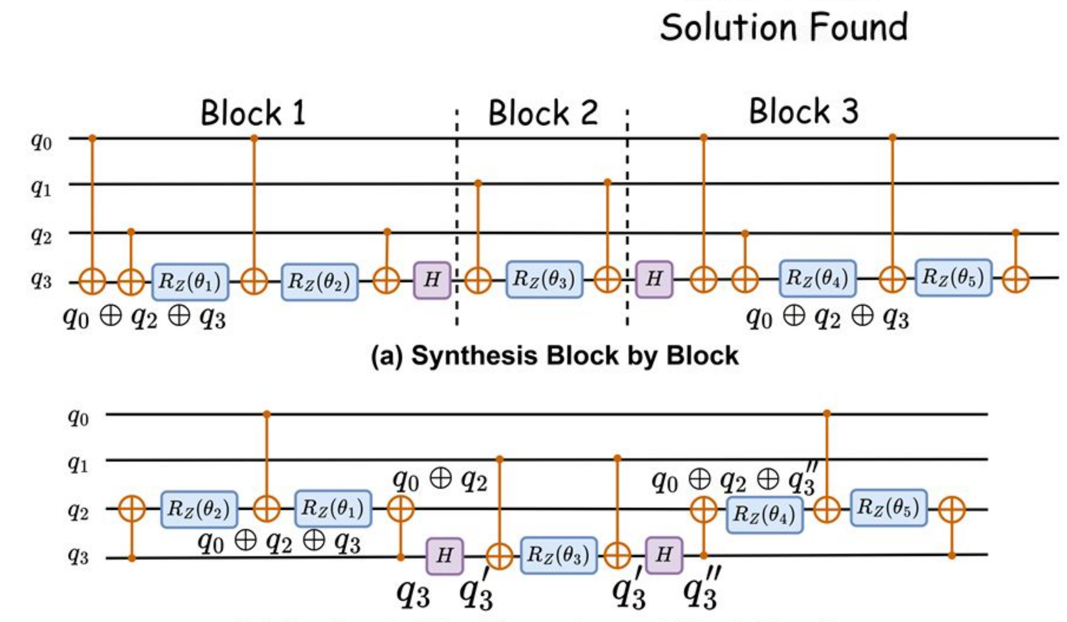
◇ Phase Polynomials Co-Optimization



◇ A* search in logical circuit optimization

- To evaluate each state after applying a CNOT from the **active row pair set**
  - Phase parity cost $h_1(n)$:
    Hamming weight of the phase parity matrix; estimates CNOTs needed for phase gates.
  - Output state parity cost $h_2(n)$:
    Estimated CNOTs for Gaussian elimination in output state recovery.
  - Actual cost $g(n)$:
    Cumulative CNOT count from root to current state.

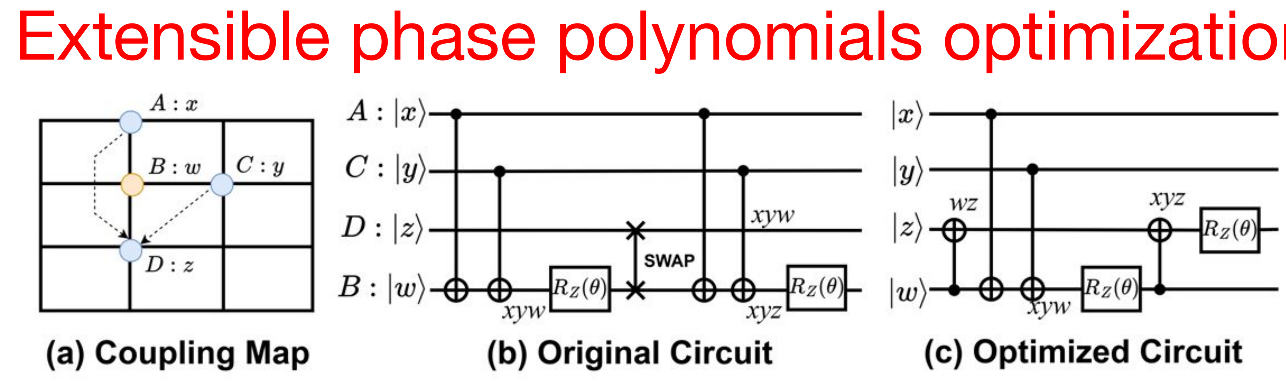
$$f(n) = g(n) + h_1(n) + h_2(n)$$



◇ Multi-block phase polynomials optimization

(a) Without SSA-style IR renaming. The term $q_0 \otimes q_2$, cannot be reused due to **block boundaries**
(b) After renaming, the blocks merge into a single phase polynomial block, reducing CNOT gates from 10 to 8 through the reuse of $q_0 \otimes q_2$

◇ Extensible phase polynomials optimization



◇ Results and Evaluation

- ◇ **Benchmarks**: Clifford+T benchmarks from prior work[3-6], as well as additional near-term and fault-tolerant quantum applications.
- ◇ **Metrics**: Total gate count and CNOT count.
- ◇ **Baselines**: GRAY-SYNTH[3] (with T gate optimizations) and QUESO[6], a state-of-the-art equivalent-subcircuit rewriting optimizer.
- ◇ **Verification**: Results passed equivalence checking by Qiskit and MQT QCEC
- ◇ **Results**: PhasePoly outperforms both prior frameworks individually, and achieves the best results when combined with QUESO, reducing up to **50%** total gate and **48.57%** CNOT reduction. (averaging 35.83% and 27.9%, respectively) ⬇️

Circuit	Org.	GRAY-SYNTH	QUESO	PhasePoly.	QUESO+PhasePoly.
	# Gates | CXs	# CXs	# Gates | CXs	# Gates | CXs	**# Gates | CXs**
rc_adder_6	200 | 93	71	176 | 79	152 | 71	**152 | 71**
tof_10	255 | 102	70	175 | 70	175 | 70	**175 | 70**
hwb6	259 | 116	110	218 | 103	200 | 96	**199 | 95**
mod_red_21	278 | 105	86	198 | 79	180 | 77	**179 | 76**
qaoa_n8_p4	440 | 96	-	244 | 92	240 | 88	**240 | 88**
ham15-low	443 | 236	208	343 | 200	336 | 198	**333 | 195**
qcla_com_7	443 | 186	136	295 | 127	256 | 120	**256 | 120**
barenco_tof_10	450 | 192	144	262 | 128	248 | 114	**248 | 114**
qcla_adder_10	521 | 233	214	443 | 207	391 | 176	**391 | 176**
grover_5	831 | 288	226	589 | 208	455 | 202	**455 | 202**
qcla_mod_7	884 | 382	360	778 | 356	628 | 289	**620 | 281**
adder_8	900 | 409	359	628 | 301	565 | 270	**561 | 270**
ham15-med	1272 | 534	357	773 | 374	658 | 327	**654 | 323**
mod_adder_1024	4285 | 1720	1390	2837 | 1237	2565 | 1217	**2550 | 1202**
ham15-high	5308 | 2149	1502	4653 | 2134	2792 | 1363	**2786 | 1357**
shor_15_7	36598 | 14858	-	34969 | 13308	31143 | 12814	**31143 | 12814**
Geo. Mean Reduction	-	16.70%	26.34% | 19.96%	35.61% | 27.46%	**35.83% | 27.90%**

Reference:

[1] Chen, Zihan, et al. PhasePoly: An Optimization Framework for Phase Polynomials in Quantum Circuits. (2025).
[2] Amy, Matthew, et al. Polynomial-time T-depth optimization of Clifford+T circuits via matroid partitioning. (2014).
[3] Amy, Matthew, et al. On the CNOT-complexity of CNOT-PHASE circuits. (2017).
[4] Nam, Yunseong, et al. Automated optimization of large quantum circuits with continuous parameters. (2018).
[5] Vandaele, Vivien, et al. Phase polynomials synthesis algorithms for NISQ architectures and beyond. (2022).
[6] Xu, Amanda, et al. Synthesizing quantum-circuit optimizers. (2023).