

# Quantitative methods and simulation in Finance

## Chapter 1 - Introduction



# P, Q quant and the course

- Syllabus
- Q quant : risk-neutral measure
- P quant : physical probability measure,  
time series analysis,  
statistical learning...

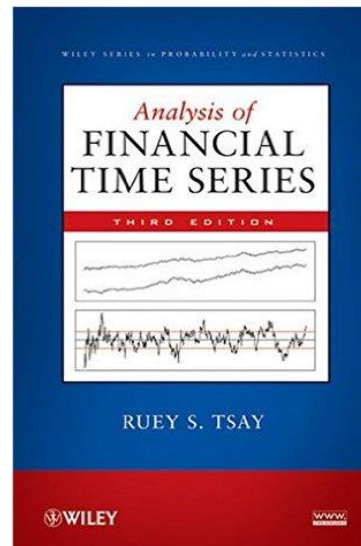
# Book and Lecture Notes

- Reference book :

Tsay, R.S. (2010) Analysis of Financial Time Series 3rd edition. Wiley.

John C. Hull (2014) Options, Futures and other derivatives 9<sup>th</sup> edition.

Robert I.Kabacoff (2016) R in action 2th edition.



# Book and Lecture Notes

- Emanuel Derman 《My Life as a Quant》
- An authentic autobiography by an experienced quant on Wall Street. Formerly a PhD in particle physics, Derman's combined quality of academia and business had suited him just fine. Listed and narrated, the glorious epic of Columbia's Physics Department, filled with Noble Prize receivers, gives us a glimpse of a half-lively half-miserable post-doc life surrounded by geniuses. After heading to Wall Street, hard mathematics and programming skills served him well to earn a place where people of different specialties and talents forced their entrance into the haven of investment and speculations. His experience as well as thinking about the area now known as Financial Engineering provides us with a general and basic scratch on quants, traders, salesman, or in summary, a overview on Street back in the glamorous old days, with some possibly subtle insights.

- <https://www.r-project.org/>
- <https://www.rstudio.com/>



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### Getting Started

R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To **download R**, please choose your preferred [CRAN mirror](#).

If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.

### News

- **R version 3.5.1 (Feather Spray)** has been released on 2018-07-02.
- The R Foundation has been awarded the Personality/Organization of the year 2018 award by the professional association of German market and social researchers.

# Grading

- 平时成绩 (20%)
- 期中考试 (30%)      10月30日
- 期末考试 (50%)

# ASSET RETURNS

- Most financial studies involve returns, instead of prices, of assets. Campbell, Lo, and MacKinlay (1997) give two main reasons for using returns.
- First, for average investors, return of an asset is a complete and scale-free summary of the investment opportunity.
- Second, return series are easier to handle than price series because the former have more attractive statistical properties.



# One-Period Simple Return

- Let  $P_t$  be the price of an asset at time index  $t$ . We discuss some definitions of returns that are used throughout the book. Assume for the moment that the asset pays no dividends.
- simple gross return, from date  $t - 1$  to date  $t$  :

$$1 + R_t = \frac{P_t}{P_{t-1}} \quad \text{or} \quad P_t = P_{t-1}(1 + R_t).$$

- The corresponding one-period simple net return or simple return is

$$R_t = \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}}.$$

# Multiperiod Simple Return

- $k$ -period simple gross return, from date  $t - k$  to  $t$  :

$$\begin{aligned} 1 + R_t[k] &= \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} \times \frac{P_{t-1}}{P_{t-2}} \times \cdots \times \frac{P_{t-k+1}}{P_{t-k}} \\ &= (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}) \\ &= \prod_{j=0}^{k-1} (1 + R_{t-j}). \end{aligned}$$

- Thus, the  $k$ -period simple gross return is just the product of the  $k$  one-period simple gross returns involved. This is called a compound return.
- annualized (average) return is

$$\text{Annualized } \{R_t[k]\} = \left[ \prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1.$$

# Multiperiod Simple Return

This is a geometric mean of the  $k$  one-period simple gross returns involved and can be computed by

$$\text{Annualized } \{R_t[k]\} = \exp \left[ \frac{1}{k} \sum_{j=0}^{k-1} \ln(1 + R_{t-j}) \right] - 1,$$

- Because it is easier to compute arithmetic average than geometric mean and the one-period returns tend to be small, one can use a first-order Taylor expansion to approximate the annualized return and obtain

$$\text{Annualized } \{R_t[k]\} \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}.$$

# Continuous Compounding

- Before introducing continuously compounded return, we discuss the effect of compounding. Assume that the interest rate of a bank deposit is 10% per annum and the initial deposit is \$1.00. If the bank pays interest once a year, then the net value of the deposit becomes
- $\$1(1 + 0.1) = \$1.1$
- $\$1(1 + 0.1/2)^2 = \$1.1025$
- $\$1(1 + 0.1/m)^m$

# Continuous Compounding

**TABLE 1.1 Illustration of Effects of Compounding: Time Interval Is 1 Year and Interest Rate Is 10% per Annum**

| Type         | Number of Payments | Interest Rate per Period | Net Value |
|--------------|--------------------|--------------------------|-----------|
| Annual       | 1                  | 0.1                      | \$1.10000 |
| Semiannual   | 2                  | 0.05                     | \$1.10250 |
| Quarterly    | 4                  | 0.025                    | \$1.10381 |
| Monthly      | 12                 | 0.0083                   | \$1.10471 |
| Weekly       | 52                 | 0.1/52                   | \$1.10506 |
| Daily        | 365                | 0.1/365                  | \$1.10516 |
| Continuously | $\infty$           |                          | \$1.10517 |

# Continuous Compounding

- continuous compounding:  $\exp(0.1)$
- In general, the net asset value  $A$  of continuous compounding is

$$A = C \exp(r \times n),$$

$$C = A \exp(-r \times n),$$

which is referred to as the present value of an asset.

# Continuously Compounded Return

- continuously compounded return or log return:

$$r_t = \ln(1 + R_t) = \ln \frac{P_t}{P_{t-1}} = p_t - p_{t-1},$$

- continuously compounded returns  $r_t$  enjoy some advantages over the simple net returns  $R_t$ .

$$\begin{aligned} r_t[k] &= \ln(1 + R_t[k]) = \ln[(1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1})] \\ &= \ln(1 + R_t) + \ln(1 + R_{t-1}) + \cdots + \ln(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}. \end{aligned}$$

- statistical properties of log returns are more tractable.

# Dividend Payment and Excess Return

- If an asset pays dividends periodically, we must modify the definitions of asset returns.
- Let  $D_t$  be the dividend payment of an asset, from date  $t-1$  to  $t$ .

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1, \quad r_t = \ln(P_t + D_t) - \ln(P_{t-1}).$$

- Excess return of an asset at time  $t$  is the difference between the asset's return and the return on some reference asset. e.g. short-term U.S. Treasury bill return.

$$Z_t = R_t - R_{0t}, \quad z_t = r_t - r_{0t},$$



# Summary of Relationship

- simple return  $R_t$  and continuously compounded (or log) return  $r_t$

$$r_t = \ln(1 + R_t), \quad R_t = e^{r_t} - 1.$$

- Temporal aggregation of the returns produces

$$1 + R_t[k] = (1 + R_t)(1 + R_{t-1}) \cdots (1 + R_{t-k+1}),$$

$$r_t[k] = r_t + r_{t-1} + \cdots + r_{t-k+1}.$$

- If the continuously compounded interest rate is  $r$  per annum, then the relationship between present and future values of an asset is

$$A = C \exp(r \times n), \quad C = A \exp(-r \times n).$$

# Summary of Relationship

- Example 1. If the monthly log return of an asset is 4.46%, then the corresponding monthly simple return is

$$r_t = \ln(1 + R_t)$$

$$100[\exp(4.46/100) - 1] = 4.56\%.$$

- Also, if the monthly log returns of the asset within a quarter are 4.46%, -7.34%, and 10.77%, respectively, then the quarterly log return of the asset is

$$(4.46 - 7.34 + 10.77)\% = 7.89\%.$$

- If the monthly simple returns of an asset are 4.46%, -7.34% and 10.77%, then what is the corresponding quarterly simple return?  
 $R = (1+0.0446)(1-0.0734)(1+0.1077)-1$

# DISTRIBUTIONAL PROPERTIES OF RETURNS

- To study asset returns, it is best to begin with their distributional properties.
- The objective here is to understand the behavior of the returns across assets and over time.
- Consider a collection of  $N$  assets held for  $T$  time periods, say,  $t = 1, \dots, T$ . For each asset  $i$ , let  $r_{it}$  be its log return at time  $t$ .

# Moments of a Random Variable

- the skewness and kurtosis of  $X$  are defined as

$$S(x) = E \left[ \frac{(X - \mu_x)^3}{\sigma_x^3} \right], \quad K(x) = E \left[ \frac{(X - \mu_x)^4}{\sigma_x^4} \right].$$

- The third central moment measures the symmetry of  $X$  with respect to its mean, whereas the fourth central moment measures the tail behavior of  $X$ .
- Under the normality assumption,  $\hat{S}(x)$  and  $\hat{K}(x) - 3$  are distributed asymptotically as normal with zero mean and variances  $6/T$  and  $24/T$ , respectively.

# Moments of a Random Variable

- Thus, the excess kurtosis of a normal random variable is zero. A distribution with positive excess kurtosis is said to have heavy tails, implying that the distribution puts more mass on the tails of its support than a normal distribution does.
- In practice, this means that a random sample from such a distribution tends to contain more extreme values.

$$\hat{S}(x) = \frac{1}{(T-1)\hat{\sigma}_x^3} \sum_{t=1}^T (x_t - \hat{\mu}_x)^3,$$

$$\hat{K}(x) = \frac{1}{(T-1)\hat{\sigma}_x^4} \sum_{t=1}^T (x_t - \hat{\mu}_x)^4$$

# Moments of a Random Variable

- Given an asset return series  $\{r_1, \dots, r_T\}$ , to test the skewness of the returns, we consider the null hypothesis
- $H_0 : S(r)=0$  versus the alternative hypothesis  $H_a : S(r) \neq 0$ .

$$t = \frac{\hat{S}(r)}{\sqrt{6/T}}.$$

# Moments of a Random Variable

- excess kurtosis of the return series using the hypotheses

$$H_0 : K(r) - 3 = 0 \text{ versus } H_a : K(r) - 3 \neq 0.$$

- The test statistic is

$$t = \frac{\hat{K}(r) - 3}{\sqrt{24/T}},$$

- Jarque and Bera (1987) (JB) combine the two prior tests and use the test statistic

$$JB = \frac{\hat{S}^2(r)}{6/T} + \frac{[\hat{K}(r) - 3]^2}{24/T},$$

which is asymptotically distributed as a chi-squared random variable with 2 degrees of freedom.

# Moments of a Random Variable

**TABLE 1.2 Descriptive Statistics for Daily and Monthly Simple and Log Returns of Selected Indexes and Stocks<sup>a</sup>**

| Security                        | Start     | Size | Mean  | Standard<br>Deviation | Skewness | Excess<br>Kurtosis | Minimum | Maximum |
|---------------------------------|-----------|------|-------|-----------------------|----------|--------------------|---------|---------|
| <i>Daily Simple Returns (%)</i> |           |      |       |                       |          |                    |         |         |
| SP                              | 70/01/02  | 9845 | 0.029 | 1.056                 | −0.73    | 22.81              | −20.47  | 11.58   |
| VW                              | 70/01/02  | 9845 | 0.040 | 1.004                 | −0.62    | 18.02              | −17.13  | 11.52   |
| EW                              | 70/01/02  | 9845 | 0.076 | 0.814                 | −0.77    | 17.08              | −10.39  | 10.74   |
| IBM                             | 70/01/02  | 9845 | 0.040 | 1.693                 | 0.06     | 9.92               | −22.96  | 13.16   |
| Intel                           | 72/12/15  | 9096 | 0.108 | 2.891                 | −0.15    | 6.13               | −29.57  | 26.38   |
| 3M                              | 670/01/02 | 9845 | 0.045 | 1.482                 | −0.36    | 13.34              | −25.98  | 11.54   |
| Microsoft                       | 86/03/14  | 5752 | 0.123 | 2.359                 | −0.13    | 9.92               | −30.12  | 19.57   |
| Citi-Grp                        | 86/10/30  | 5592 | 0.067 | 2.602                 | 1.80     | 55.25              | −26.41  | 57.82   |
| <i>Daily Log Returns (%)</i>    |           |      |       |                       |          |                    |         |         |
| SP                              | 70/01/02  | 9845 | 0.023 | 1.062                 | −1.17    | 30.20              | −22.90  | 10.96   |
| VW                              | 70/01/02  | 9845 | 0.035 | 1.008                 | −0.94    | 21.56              | −18.80  | 10.90   |
| EW                              | 70/01/02  | 9845 | 0.072 | 0.816                 | −1.00    | 17.76              | −10.97  | 10.20   |
| IBM                             | 70/01/02  | 9845 | 0.026 | 1.694                 | −0.27    | 12.17              | −26.09  | 12.37   |
| Intel                           | 72/12/15  | 9096 | 0.066 | 2.905                 | −0.54    | 7.81               | −35.06  | 23.41   |
| 3M                              | 70/01/02  | 9845 | 0.034 | 1.488                 | −0.78    | 20.57              | −30.08  | 10.92   |
| Microsoft                       | 86/03/14  | 5752 | 0.095 | 2.369                 | −0.63    | 14.23              | −35.83  | 17.87   |
| Citi-Grp                        | 86/10/30  | 5592 | 0.033 | 2.575                 | 0.22     | 33.19              | −30.66  | 45.63   |



# Moments of a Random Variable

- Example 1.2. Consider the daily simple returns of the International Business Machines (IBM) stock used in Table 1.2. The sample skewness and kurtosis of the returns are parts of the descriptive (or summary) statistics that can be obtained easily using various statistical software packages.
- To test the symmetry of return distribution, we use the test statistic

$$t = \frac{0.0614}{\sqrt{6/9845}} = \frac{0.0614}{0.0247} = 2.49,$$

- which gives a p value of about 0.013, indicating that the daily simple returns of IBM stock are significantly skewed to the right at the 5% level.

# R Demonstration

- R is a free software available from <http://www.r-project.org>. One can click CRAN on its Web page to select a nearby CRAN Mirror to download and install the software and selected packages.

```
**** Task: (a) Set the working directory
           (b) Load the library 'fBasics'.
           (c) Compute summary (or descriptive) statistics
           (d) Perform test for mean return being zero.
           (e) Perform normality test using the Jaque-Bera method.
           (f) Perform skewness and kurtosis tests.
```

```
> setwd("C:/Users/rst/teaching/bs41202/sp2016")  <== set working directory
> library(fBasics)  <== Load the library 'fBasics'.

> da=read.table("m-ibm-6815.txt",header=T)
> head(da)
  PERMNO    date    PRC ASKHI  BIDLO      RET    vwretd    ewretd    sprtrn
1  12490 19680131 594.50 623.0 588.75 -0.051834 -0.036330  0.023902 -0.043848
2  12490 19680229 580.00 599.5 571.00 -0.022204 -0.033624 -0.056118 -0.031223
3  12490 19680329 612.50 612.5 562.00  0.056034  0.005116 -0.011218  0.009400
4  12490 19680430 677.50 677.5 630.00  0.106122  0.094148  0.143031  0.081929
5  12490 19680531 357.00 696.0 329.50  0.055793  0.027041  0.091309  0.011169
6  12490 19680628 353.75 375.0 346.50 -0.009104  0.011527  0.016225  0.009120
> dim(da)
[1] 576  9
> ibm=da$RET % Simple IBM return
> lnIBM <- log(ibm+1) % compute log return
```

# R Demonstration

```
> ts.plot(ibm,main="Monthly IBM simple returns: 1968-2015") % Time plot
> mean(ibm)
[1] 0.008255663
> var(ibm)
[1] 0.004909968
> skewness(ibm)
[1] 0.2687105
attr(,"method")
[1] "moment"
> kurtosis(ibm)
[1] 2.058484
attr(,"method")
[1] "excess"
> basicStats(ibm)

                ibm
nobs           576.000000
NAs             0.000000
Minimum        -0.261905
Maximum         0.353799
1. Quartile    -0.034392
```

# R Demonstration

```
3. Quartile  0.048252
Mean         0.008256
Median       0.005600
Sum          4.755262
SE Mean      0.002920
LCL Mean     0.002521
UCL Mean     0.013990
Variance     0.004910
Stdev        0.070071
Skewness     0.268710
Kurtosis     2.058484
> basicStats(lnIBM) % log return
      lnIBM
nobs      576.000000
NAs        0.000000
Minimum    -0.303683
Maximum     0.302915
1. Quartile -0.034997
3. Quartile  0.047124
Mean        0.005813
Median      0.005585
Sum         3.348008
SE Mean     0.002898
LCL Mean    0.000120
UCL Mean    0.011505
Variance    0.004839
Stdev       0.069560
Skewness    -0.137286
Kurtosis    1.910438
> t.test(lnIBM) %% Test mean=0 vs mean .not. zero

      One Sample t-test

data:  lnIBM
t = 2.0055, df = 575, p-value = 0.04538
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.0001199015 0.0115051252
```

# R Demonstration

```
> t.test(lnIBM) %% Test mean=0 vs mean .not. zero
```

One Sample t-test

```
data: lnIBM
```

```
t = 2.0055, df = 575, p-value = 0.04538
```

```
alternative hypothesis: true mean is not equal to 0
```

```
95 percent confidence interval:
```

```
0.0001199015 0.0115051252
```

```
sample estimates:
```

```
mean of x
```

```
0.005812513
```

```
> normalTest(lnIBM,method='jb')
```

```
Title: Jarque - Bera Normalality Test
```

```
Test Results:
```

# R Demonstration

STATISTIC:

X-squared: 90.988

P VALUE:

Asymptotic p Value:  $< 2.2e-16$

```
> s3=skewness(lnIBM); T <- length(lnIBM)
> tst <- s3/sqrt(6/T) % test skewness
> tst
[1] -1.345125
> pv <- 2*pnorm(tst)
> pv
[1] 0.1785849
> k4 <- kurtosis(lnIBM)
> tst <- k4/sqrt(24/T) % test excess kurtosis
> tst
[1] 9.359197
>q() % quit R.
```

# Distributions of Returns

- Normal Distribution

Several statistical distributions have been proposed in the literature for the marginal distributions of asset returns.

financial study assumes the simple returns  $\{R_{it} | t = 1, \dots, T\}$  are independently and identically distributed as normal with fixed mean and variance.

First, the lower bound of a simple return is  $-1$ . Yet the normal distribution may assume any value in the real line and, hence, has no lower bound.

Second, if  $R_{it}$  is normally distributed, then the multiperiod simple return  $R_{it} [k]$  is not normally distributed. (product)

Third, the normality assumption is not supported by many empirical asset returns, which tend to have a positive excess kurtosis.

# Distributions of Returns

- Lognormal Distribution

Another commonly used assumption is that the log returns  $r_t$  of an asset are independent and identically distributed (iid) as normal with mean  $\mu$  and variance  $\sigma^2$ .

$$E(R_t) = \exp\left(\mu + \frac{\sigma^2}{2}\right) - 1, \quad \text{Var}(R_t) = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1].$$

Because the sum of a finite number of iid normal random variables is normal,  $r_t$  is also normally distributed under the normal assumption for  $\{r_t\}$ .

In addition, there is no lower bound for  $r_t$ , and the lower bound for  $R_t$  is satisfied using  $1 + R_t = \exp(r_t)$ .

In particular, many stock returns exhibit a positive excess kurtosis.



# Application:

- If the log return of an asset is normally distributed with mean 0.0119 and standard deviation 0.0663, then what is the mean and standard deviation of its simple return?

$$E(Y) = \exp \left[ 0.0119 + \frac{0.0663^2}{2} \right] = 1.014$$

$$V(Y) = \exp(2 \times 0.0119 + 0.0663^2) [\exp(0.0663^2) - 1] = 0.0045$$

- The simple return : gross return - 1

# Scale Mixture of Normal Distributions

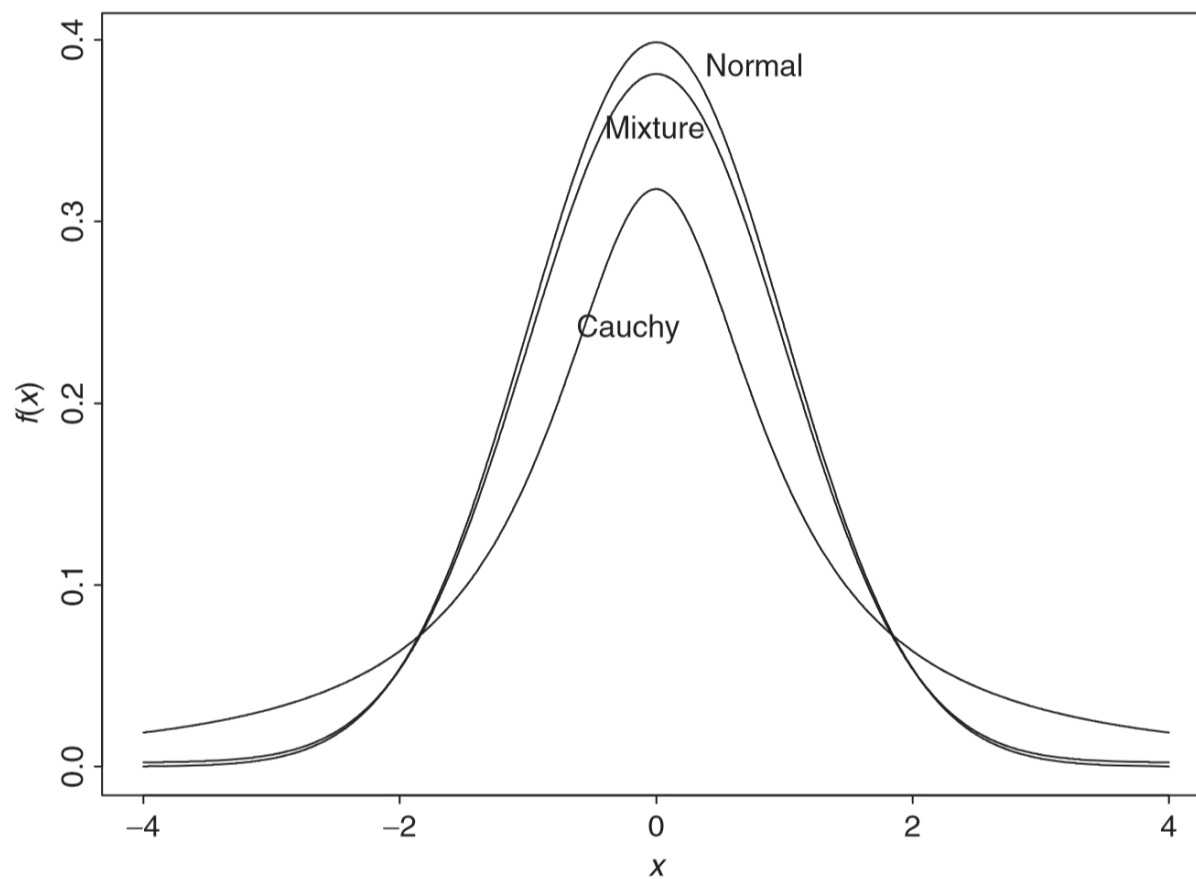
Under the assumption of scale mixture of normal distributions, the log return  $r_t$  is normally distributed with mean  $\mu$  and variance  $\sigma^2$  [i.e.,  $r_t \sim N(\mu, \sigma^2)$ ]. However,  $\sigma^2$  is a random variable that follows a positive distribution.

An example of finite mixture of normal distributions is

$$r_t \sim (1 - X)N(\mu, \sigma_1^2) + XN(\mu, \sigma_2^2),$$

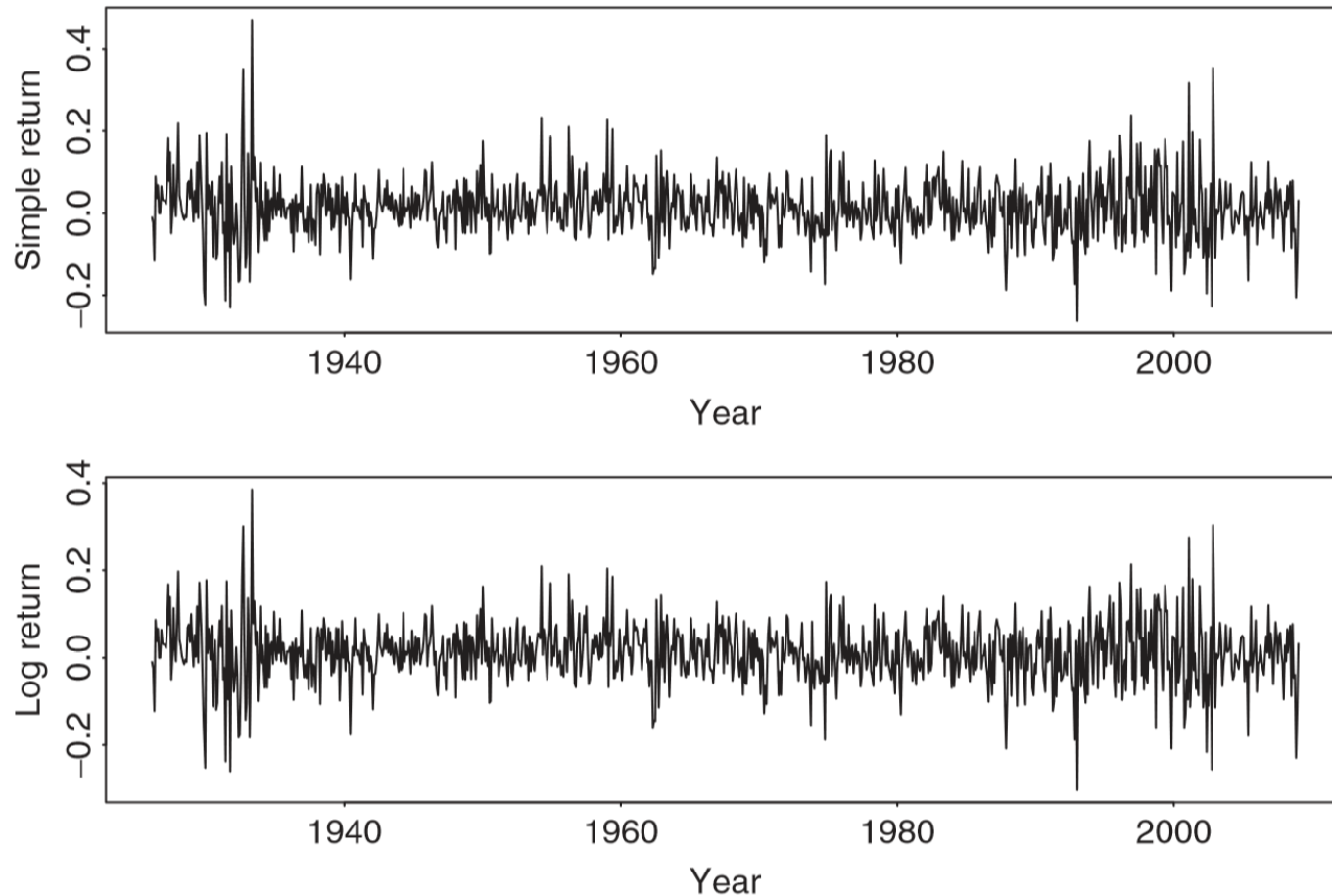
Advantages of mixtures of normal include that they maintain the tractability of normal, have finite higher order moments, and can capture the excess kurtosis. Yet it is hard to estimate the mixture parameters.

- the finite mixture says that 95% of the returns follow  $N(\mu, \sigma_1^2)$  and 5% follow  $N(\mu, \sigma_2^2)$ .
- The large value of  $\sigma_2^2$  enables the mixture to put more mass at the tails of its distribution. The low percentage of returns that are from  $N(\mu, \sigma_2^2)$  says that the majority of the returns follow a simple normal distribution.
- Advantages of mixtures of normal include that they maintain the tractability of normal, have finite higher order moments, and can capture the excess kurtosis.



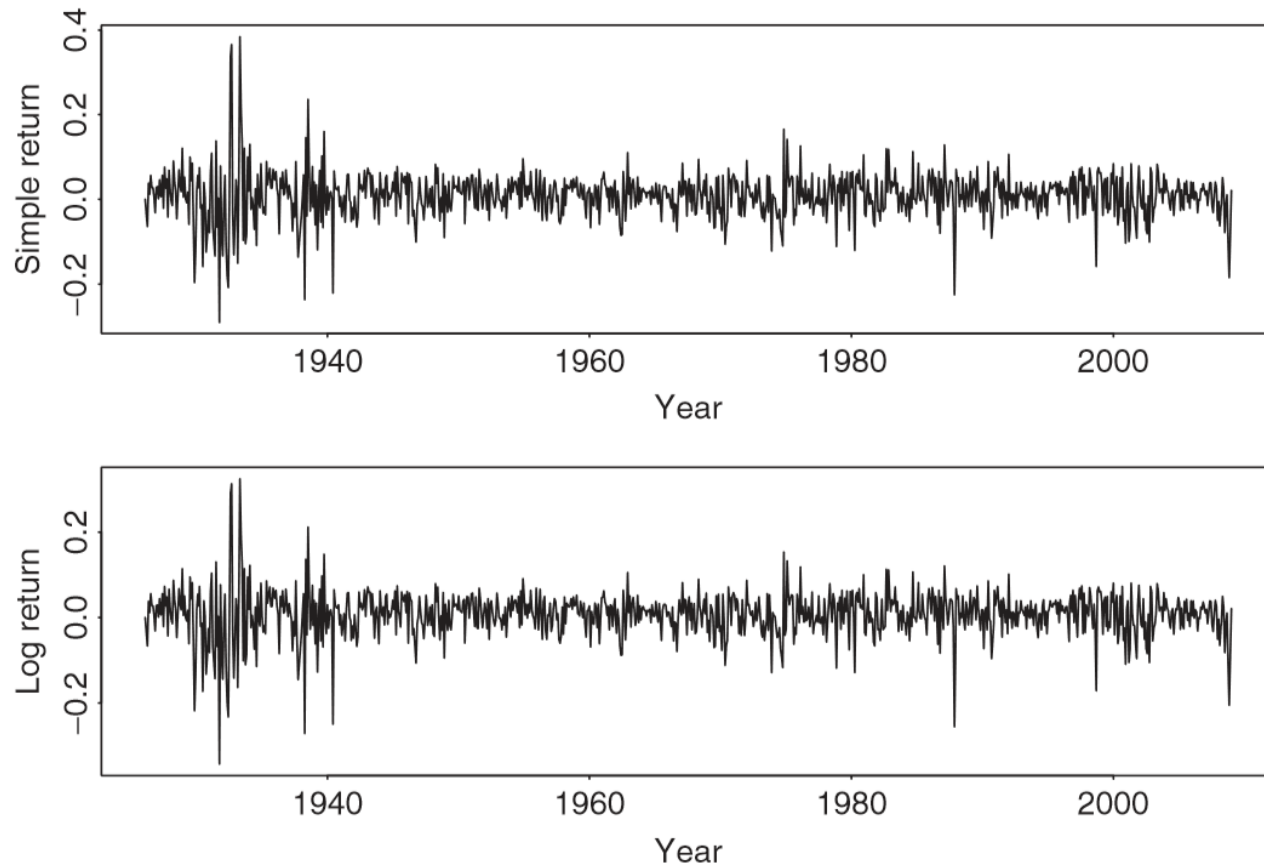
**Figure 1.1** Comparison of finite mixture, stable, and standard normal density functions.

# Empirical Properties of Returns



**Figure 1.2** Time plots of monthly returns of IBM stock from January 1926 to December 2008. Upper panel is for simple returns, and lower panel is for log returns.

# Empirical Properties of Returns



**Figure 1.3** Time plots of monthly returns of value-weighted index from January 1926 to December 2008. Upper panel is for simple returns, and lower panel is for log returns.

# Empirical Properties of Returns

**TABLE 1.2 Descriptive Statistics for Daily and Monthly Simple and Log Returns of Selected Indexes and Stocks<sup>a</sup>**

| Security                          | Start     | Size | Mean  | Standard<br>Deviation | Skewness | Excess<br>Kurtosis | Minimum | Maximum |
|-----------------------------------|-----------|------|-------|-----------------------|----------|--------------------|---------|---------|
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| VW                                | 70/01/02  | 9845 | 0.040 | 1.004                 | −0.62    | 18.02              | −17.13  | 11.52   |
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| Intel                             | 72/12/15  | 9096 | 0.108 | 2.891                 | −0.15    | 6.13               | −29.57  | 26.38   |
| 3M                                | 670/01/02 | 9845 | 0.045 | 1.482                 | −0.36    | 13.34              | −25.98  | 11.54   |
| Microsoft                         | 86/03/14  | 5752 | 0.123 | 2.359                 | −0.13    | 9.92               | −30.12  | 19.57   |
| Citi-Grp                          | 86/10/30  | 5592 | 0.067 | 2.602                 | 1.80     | 55.25              | −26.41  | 57.82   |
| <i>Daily Log Returns (%)</i>      |           |      |       |                       |          |                    |         |         |
| SP                                | 70/01/02  | 9845 | 0.023 | 1.062                 | −1.17    | 30.20              | −22.90  | 10.96   |
| VW                                | 70/01/02  | 9845 | 0.035 | 1.008                 | −0.94    | 21.56              | −18.80  | 10.90   |
| EW                                | 70/01/02  | 9845 | 0.072 | 0.816                 | −1.00    | 17.76              | −10.97  | 10.20   |
| IBM                               | 70/01/02  | 9845 | 0.026 | 1.694                 | −0.27    | 12.17              | −26.09  | 12.37   |
| Intel                             | 72/12/15  | 9096 | 0.066 | 2.905                 | −0.54    | 7.81               | −35.06  | 23.41   |
| 3M                                | 70/01/02  | 9845 | 0.034 | 1.488                 | −0.78    | 20.57              | −30.08  | 10.92   |
| Microsoft                         | 86/03/14  | 5752 | 0.095 | 2.369                 | −0.63    | 14.23              | −35.83  | 17.87   |
| Citi-Grp                          | 86/10/30  | 5592 | 0.033 | 2.575                 | 0.22     | 33.19              | −30.66  | 45.63   |
| <i>Monthly Simple Returns (%)</i> |           |      |       |                       |          |                    |         |         |
| SP                                | 26/01     | 996  | 0.58  | 5.53                  | 0.32     | 9.47               | −29.94  | 42.22   |
| VW                                | 26/01     | 996  | 0.89  | 5.43                  | 0.15     | 7.69               | −29.01  | 38.37   |
| EW                                | 26/01     | 996  | 1.22  | 7.40                  | 1.52     | 14.94              | −31.28  | 66.59   |
| IBM                               | 26/01     | 996  | 1.35  | 7.15                  | 0.44     | 3.43               | −26.19  | 47.06   |
| Intel                             | 73/01     | 432  | 2.21  | 12.85                 | 0.32     | 2.70               | −44.87  | 62.50   |
| 3M                                | 46/02     | 755  | 1.24  | 6.45                  | 0.22     | 0.98               | −27.83  | 25.80   |
| Microsoft                         | 86/04     | 273  | 2.62  | 11.08                 | 0.66     | 1.96               | −34.35  | 51.55   |
| Citi-Grp                          | 86/11     | 266  | 1.17  | 9.75                  | −0.47    | 1.77               | −39.27  | 26.08   |

# Empirical Properties of Returns

| <i>Monthly Log Returns (%)</i> |       |     |      |       |       |      |        |       |
|--------------------------------|-------|-----|------|-------|-------|------|--------|-------|
| SP                             | 26/01 | 996 | 0.43 | 5.54  | −0.52 | 7.93 | −35.58 | 35.22 |
| VW                             | 26/01 | 996 | 0.74 | 5.43  | −0.58 | 6.85 | −34.22 | 32.47 |
| EW                             | 26/01 | 996 | 0.96 | 7.14  | 0.25  | 8.55 | −37.51 | 51.04 |
| IBM                            | 26/01 | 996 | 1.09 | 7.03  | −0.07 | 2.62 | −30.37 | 38.57 |
| Intel                          | 73/01 | 432 | 1.39 | 12.80 | −0.55 | 3.06 | −59.54 | 48.55 |
| 3M                             | 46/02 | 755 | 1.03 | 6.37  | −0.08 | 1.25 | −32.61 | 22.95 |
| Microsoft                      | 86/04 | 273 | 2.01 | 10.66 | 0.10  | 1.59 | −42.09 | 41.58 |
| Citi-Grp                       | 86/11 | 266 | 0.68 | 10.09 | −1.09 | 3.76 | −49.87 | 23.18 |

<sup>a</sup>Returns are in percentages and the sample period ends on December 31, 2008. The statistics are defined in eqs. (1.10)–(1.13), and VW, EW and SP denote value-weighted, equal-weighted, and S&P composite index.

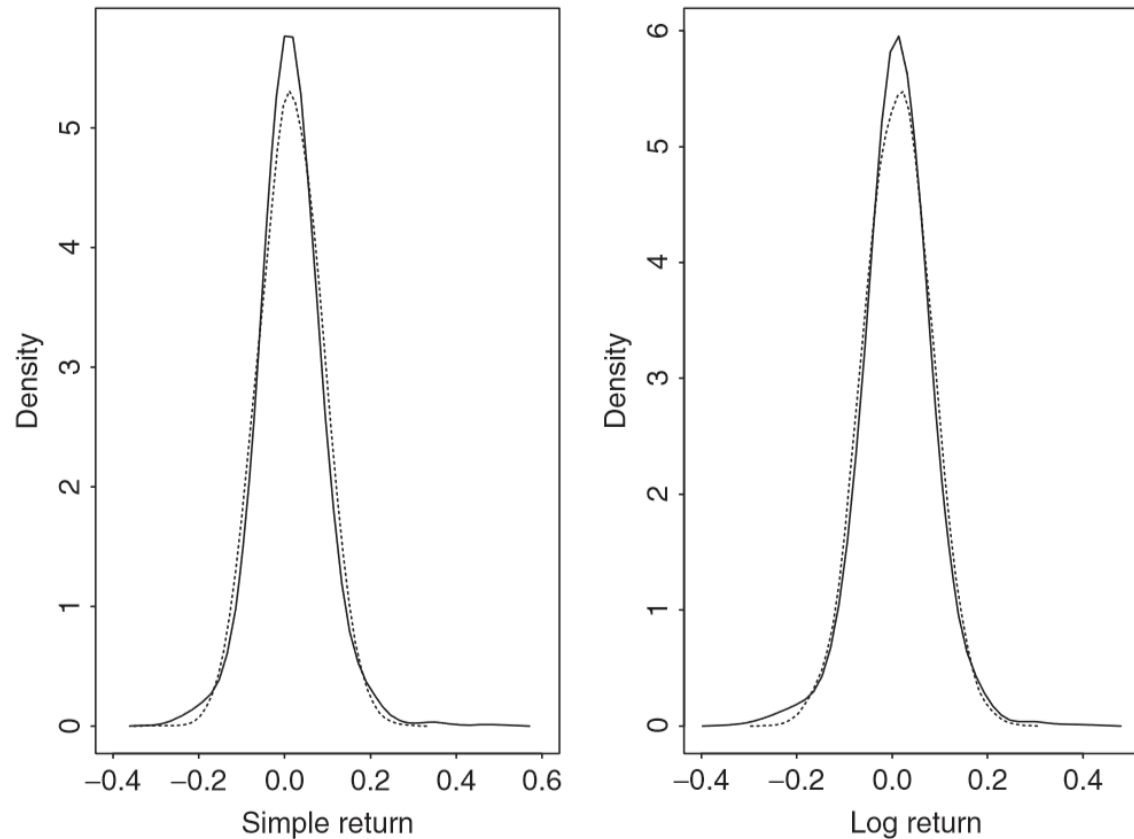
- The returns are for daily and monthly sample intervals and are in percentages. The data spans and sample sizes are also given in Table 1.2. From the table, we make the following observations. (a) Daily returns of the market indexes and individual stocks tend to have high excess kurtoses. For monthly series, the returns of market indexes have higher excess kurtoses than individual stocks.



# Empirical Properties of Returns

- (b) The mean of a daily return series is close to zero, whereas that of a monthly return series is slightly larger.
- (c) Monthly returns have higher standard deviations than daily returns.
- (d) Among the daily returns, market indexes have smaller standard deviations than individual stocks. This is in agreement with common sense.
- (e) The skewness is not a serious problem for both daily and monthly returns.
- (f) The descriptive statistics show that the difference between simple and log returns is not substantial.

# Empirical Properties of Returns



**Figure 1.4** Comparison of empirical and normal densities for monthly simple and log returns of IBM stock. Sample period is from January 1926 to December 2008. Left plot is for simple returns and right plot for log returns. Normal density, shown by the dashed line, uses sample mean and standard deviation given in Table 1.2.

# Empirical Properties of Returns

```
> da=read.table("m-gm3dx7508.txt",header=T) % Load data
> gm=da[,2] % Column 2 contains GM stock returns
> gm1=ts(gm,frequency=12,start=c(1975,1))
% Creates a ts object.
> par(mfcol=c(2,1)) % Put two plots on a page.

> plot(gm,type='l')
> plot(gm1,type='l')
> acf(gm,lag=24)
> acf(gm1,lag=24)
```

# R

```
> head(da)
  PERMNO      date    PRC ASKHI  BIDLO      RET    vwretd    ewretd    sprtrn
1  12490 19680131 594.50 623.0 588.75 -0.051834 -0.036330 0.023902 -0.043848
2  12490 19680229 580.00 599.5 571.00 -0.022204 -0.033624 -0.056118 -0.031223
3  12490 19680329 612.50 612.5 562.00 0.056034 0.005116 -0.011218 0.009400
4  12490 19680430 677.50 677.5 630.00 0.106122 0.094148 0.143031 0.081929
5  12490 19680531 357.00 696.0 329.50 0.055793 0.027041 0.091309 0.011169
6  12490 19680628 353.75 375.0 346.50 -0.009104 0.011527 0.016225 0.009120

> ibm <- da$RET
> sp <- da$sprtrn
> plot(sp,ibm)
> cor(sp,ibm)
[1] 0.5785249
> cor(sp,ibm,method="kendall")
[1] 0.4172056
> cor(sp,ibm,method="spearman")
[1] 0.58267
> cor(rank(ibm),rank(sp))
[1] 0.58267

> z=rnorm(1000) %% Genreate 1000 random variates from N(0,1)
> x=exp(z)
> y=exp(20*z)
> cor(x,y)
[1] 0.3187030
> cor(x,y,method='kendall')
[1] 1
> cor(x,y,method='spearman')
[1] 1
```

# R commands

```
> x=read.table("d-aapl0413.txt",header=T)  <== Load Apple stock returns
> dim(x)      <== check the size of the data file
[1] 2517 3
> x[1,]      <== show the first row of the data
  Permno      date      rtn
1  14593 20040102 -0.004212
> y=ts(x[,3],frequency=252,start=c(2004,1))  <== Create a time-series object in R
> plot(y,type='l',xlab='year',ylab='rtn')
> title(main='Daily returns of Apple stock: 2004 to 2013')

> par(mfcol=c(2,1))  <== To put two plots on a single page
> y=y*100             <== percentage returns
> hist(y,nclass=50)
> title(main='Percentage returns')
> d1=density(y)
> plot(d1$x,d1$y,xlab='returns',ylab='den',type='l')
```

```
> x=read.table("m-tb3ms.txt",header=T)  <== Load 3m-TB rates
> dim(x)
[1] 914  4

> y=read.table("m-tb6ms.txt",header=T)  <== Load 6m-TB rates
> dim(y)
[1] 615  4
> 914-615
[1] 299
> x[300,]  <== Check date of the 3m-TB
  year mon day value
300 1958 12  1  2.77
> y[1,]    <== Check date of the 1st observation of 6m-TB
  year mon day value
1 1958 12  1  3.01
```

# R commands

```
> int=cbind(x[300:914,4],y[,4])  <== Line up the two TB rates
> tdx=(c(1:615)+11)/12+1959
> par(mfcol=c(1,1))
> max(int)

[1] 16.3
> plot(tdx,int[,1],xlab='year',ylab='rate',type='l',ylim=c(0,16.5))
> lines(tdx,int[,2],lty=2)  <== Plot the 6m-TB rate on the same frame.

> plot(tdx,int[,2]-int[,1],xlab='year',ylab='spread',type='l')
> abline(h=c(0))  <== Draw a horizontal line to 'zero'.

> x=read.table("q-ko-earnings8309.txt",header=T)  <== Load KO data
> dim(x)
[1] 107  3
> x[1,]
      pends  anntime  value
1 19830331 19830426 0.0375
> tdx=c(1:107)/12+1983
> plot(tdx,x[,3],xlab='year',ylab='earnings',type='l')
> title(main='EPS of Coca Cola: 1983-2009')
> points(tdx,x[,3])
>
> y=read.table("d-exuseu.txt",header=T)  <== Load USEU exchange rates
> dim(y)
[1] 3567  4
> y[1,]
      year mon day  value
1 1999    1   4 1.1812
```

# R commands

```
> tdx=c(1:3567)/252+1999
> plot(tdx,y[,4],xlab='year',ylab='eu',type='l')
> title(main='Dollars per Euro')

> r=diff(log(y[,4])) <=== Compute log returns
> plot(tdx[2:3567],r,xlab='year',ylab='rtn',type='l')
> title(main='ln-rtn: US-EU')

> hist(r,nclass=50)
> title(main='useu: ln-rtn')
```

# Summary

- Return : definition
- Moment
- Distribution