Homework 1 Written Assignment

1)

a.

Still circular.

Let $(x_c + r_0\cos\theta, y_c + r_0\sin\theta, z_0)$ be a point on the edge of the disk. We have

$$\frac{x_i}{f} = \frac{x_o}{z_o}$$

$$\frac{y_i}{f} = \frac{y_o}{z_o}$$

where $x_0 = x_c + r_0\cos\theta, y_0 = y_c + r_0\sin\theta$.

Thus,

$$x_i = \frac{fx_0}{z_0}$$

$$y_i = \frac{fy_0}{z_0}$$

Then

$$(x_i - \frac{fx_c}{z_0})^2 + (y_i - \frac{fy_c}{z_0})^2 = \frac{f^2 r_0^2}{z_0^2}$$

The shape is still circular.

b.

(i)

Consider the simple case where A=C=D=0 and B=1.

The orientation vector can be expressed as $(l_x, 0, l_z)$.

We can write the two straight lines as

$$\{\ x = x_o + l_x t\ y = 0\ z = z_o + l_z t \qquad \{\ x = x'_o + l_x t\ y = 0\ z = z'_o + l_z t$$

The points projected on the image plane are

$$x_i = f\frac{x_o + l_x t}{z_o + l_z t} \quad x'_i = f\frac{x'_o + l_x t}{z'_o + l_z t}$$

$$y_i = 0 \quad y'_i = 0$$

Let $t \to \infty$, we have

$$x_{vp} = f\frac{l_x}{l_z} \quad y_{vp} = 0$$

Therefore, the vanishing points lie on the line $y = 0$.

(ii)

Consider the simple case where B=C=D=0 and A=1.

The orientation vector can be expressed as $(0, l_y, l_z)$.

We can write the two straight lines as

$$\{ x = 0\ y = y_o + l_y t\ z = z_o + l_z t \quad \{ x = 0\ y = y_o' + l_y t\ z = z_o' + l_z t$$

The points projected on the image plane are

$$x_i = 0 \quad x_i' = 0$$

$$y_i = f\frac{y_o + l_y t}{z_o + l_z t} \quad y_i' = f\frac{y_o' + l_y t}{z_o' + l_z t}$$

Let $t \to \infty$, we have

$$x_{vp} = 0 \quad y_{vp} = f\frac{l_y}{l_z}$$

Therefore, the vanishing points lie on the line $x = 0$.

c.

Now consider the general case. The orientation vector can be expressed as (l_x, l_y, l_z), which satisfies

$$Al_x + Bl_y + Cl_z = 0$$

Two parallel lines are

$$\{ x = x_o + l_x t\ y = y_o + l_y t\ z = z_o + l_z t \quad \{ x = x_o' + l_x t\ y = y_o' + l_y t\ z = z_o' + l_z t$$

And

$$x_i = f\frac{x_o + l_x t}{z_o + l_z t} \quad x_i' = f\frac{x_o' + l_x t}{z_o' + l_z t}$$

$$y_i = f\frac{y_o + l_y t}{z_o + l_z t} \quad y_i' = f\frac{y_o' + l_y t}{z_o' + l_z t}$$

Let $t \to \infty$, we have

$$x_{vp} = f\frac{l_x}{l_z} \quad y_{vp} = f\frac{l_y}{l_z}$$

We notice that x_{vp} and y_{vp} satisfies that

$$\frac{A}{f}x_{vp} + \frac{B}{f}y_{vp} + C = 0$$

Therefore, the vanishing points on the image plane lie on the line

$$\frac{A}{f}x + \frac{B}{f}y + C = 0$$