

# Matrix Multiplication Guide

## What is Matrix Multiplication?

A fundamental operation in linear algebra where two matrices are multiplied to produce a new matrix.

## How Matrix Multiplication Works

Given two matrices:

- A (size:  $m \times n$ )
- B (size:  $n \times p$ )

The resulting matrix  $C = A \times B$  has dimensions  $m \times p$ .

Each element  $C[i][j]$  is computed as the sum of products of the  $i$ -th row of A and the  $j$ -th column of B:

$C[i][j] = \sum(A[i][k] * B[k][j])$  for  $k$  from 1 to  $n$

## Example Calculation

Let's multiply two  $2 \times 2$  matrices:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Result:

$$C[0][0] = 1*5 + 2*7 = 19$$

$$C[0][1] = 1*6 + 2*8 = 22$$

$$C[1][0] = 3*5 + 4*7 = 43$$

$$C[1][1] = 3*6 + 4*8 = 50$$

Final matrix:

$\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$

## Applications in Computing

- Deep Learning: Neural network operations
- Computer Graphics: 3D transformations
- Scientific Computing: Solving linear systems
- Data Science: PCA, recommendation systems

## Python Implementations

NumPy (CPU):

```
>>> import numpy as np
>>> A = np.random.rand(1000, 1000)
>>> B = np.random.rand(1000, 1000)
>>> C = A @ B # Matrix multiplication
```

PyTorch (GPU):

```
>>> import torch
>>> A = torch.rand(1000, 1000).cuda()
>>> B = torch.rand(1000, 1000).cuda()
>>> C = torch.mm(A, B) # GPU-accelerated
```