Student Information

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Answer 1

a)

p	q	$\neg q$	$p \to q$	$p \wedge \neg q$	$(p \to q) \oplus (p \wedge \neg q)$
T	T	F	T	F	**T**
T	F	T	F	T	**T**
F	T	F	T	F	**T**
F	F	T	T	F	**T**

– Since the target column is full of True values, the statement is a tautology.

b)

$$
\begin{aligned}
p \to ((q \vee \neg p) \to r) &\equiv p \to (\neg(q \vee \neg p) \vee r) && \text{because "}p \to q \equiv \neg p \vee q\text{"} && \text{Table 7.1}\\
&\equiv p \to ((\neg q \wedge \neg\neg p) \vee r) && \text{by the second De Morgan's Law} && \text{Table 6}\\
&\equiv p \to ((\neg q \wedge p) \vee r) && \text{by the Double Negation Law} && \text{Table 6}\\
&\equiv \neg p \vee ((\neg q \wedge p) \vee r) && \text{because "}p \to q \equiv \neg p \vee q\text{"} && \text{Table 7.1}\\
&\equiv (\neg p \vee (\neg q \wedge p)) \vee r && \text{by the first Associative Law} && \text{Table 6}\\
&\equiv ((\neg p \vee \neg q) \wedge (\neg p \vee p)) \vee r && \text{by the first Distributive Law} && \text{Table 6}\\
&\equiv ((\neg p \vee \neg q) \wedge (p \vee \neg p)) \vee r && \text{by the first Commutative Law} && \text{Table 6}\\
&\equiv ((\neg p \vee \neg q) \wedge \mathrm{T}) \vee r && \text{by the first Negation Law} && \text{Table 6}\\
&\equiv (\neg p \vee \neg q) \vee r && \text{by the first Identity Law} && \text{Table 6}\\
&\equiv \neg(p \wedge q) \vee r && \text{by the first De Morgan's Law} && \text{Table 6}\\
&\equiv (p \wedge q) \to r && \text{because "}p \to q \equiv \neg p \vee q\text{"} && \text{Table 7.1}
\end{aligned}
$$

c)

1. F
2. F
3. F
4. T
5. T

Answer 2

a)

$$\exists y\,(P(\mathrm{Can}, y) \wedge T(y, \mathrm{L}))$$

b)

$$\forall x\,(T(x,\mathrm{S}) \to \exists y\,(P(y,x) \land N(y,\mathrm{Turkish}))$$

c)

$$\forall x\,(T(x,\mathrm{S}) \to \exists y \forall z\,(T(y,\mathrm{S}) \land R(x,y) \land (R(x,z) \to (\neg T(z,S) \lor y = z)) \\ \land\ (y = z \to R(x,z)) \land (y \neq x)))$$

d)

$$\forall y \forall x\,((P(x,y) \land N(x,\mathrm{English})) \to \neg W(\mathrm{M},y))$$

e)

$$\exists x \exists y \forall z \Big(\big(\big(P(y,\mathrm{G}) \land N(y,\mathrm{Turkish})\big) \land \big(P(x,\mathrm{G}) \land N(x,\mathrm{Turkish})\big) \big) \\ \land \big((y = z \lor x = z) \to \big(P(z,\mathrm{G}) \land N(z,\mathrm{Turkish})\big) \big) \\ \land \big(\big(P(z,\mathrm{G}) \land N(z,\mathrm{Turkish})\big) \to (y = z \lor x = z) \big) \\ \land (y \neq x) \Big)$$

f)

$$\exists x \exists y \exists z (T(x,y) \land T(x,z) \land y \neq z)$$

Answer 3

Table 1: Proof of $p \to q, (r \land s) \to p, (r \land \neg q) \vdash \neg s$

1	$p \to q$	*premise*
2	$(r \land s) \to p$	*premise*
3	$(r \land \neg q)$	*premise*
4	$\neg q$	$\land$e, 3
5	p	*assumption*
6	q	$\to$e, $1, 5$
7	$\bot$	$\neg$e, $4, 6$
8	$\neg p$	$\neg$i, $5 - 7$
9	r	$\land$e, 3
10	s	*assumption*
11	$r \land s$	$\land$i, $9, 10$
12	p	$\to$e, $2, 11$
13	$\bot$	$\neg$e, $8, 12$
14	$\neg s$	$\neg$i, $10 - 13$

Answer 4

a) $\exists x(P(x) \to S(x))$: Some students need to study for the exam in order to pass.

$\forall x(P(x))$: Every student passed the exam.

$\exists x(S(x))$: There is at least one student that studied for the exam.

Our claim is that $\exists x(P(x) \to S(x)), \forall x(P(x)) \vdash \exists x(S(x))$

b) Following natural deduction proves our claim:

Table 2: Proof of $\exists x(P(x) \to S(x)), \forall x(P(x)) \vdash \exists x(S(x))$

1	$\exists x(P(x) \to S(x)))$	*premise*
2	$\forall x(P(x))$	*premise*
3	$P(c) \to S(c)$	*assumption*
4	$P(c)$	$\forall$e, 2
5	$S(c)$	$\to$e, $3, 4$
6	$\exists x(S(x))$	$\exists$i, 5
7	$\exists x(S(x))$	$\exists$e, 1, $3 - 6$