Combination of Stress Sensitivities and the Heuristic Based Topology Optimization with Integrated Casting Simulation in LEOPARD|topo

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Abstract
The optimization process of the LEOPARD|topo suite relies on a binary material model in combination with bionic growth rules and additional heuristic procedures. Its common problems are weight or compliance minimizations under stress constraints. Therefore, the local von Mises stress is considered to be the indicator for material removal or addition. However, this procedure has proven to be insufficient when stress hotspots occur in notches at design space boundaries where no material can be added. Stress sensitivities which are crucial for density-based method optimizations can be used as a guidance to relax these situations. In this work, the von Mises stress values are augmented by the sensitivities of the von Mises stress norm. A new approach for the combination is proposed by a multiplication with a prior limitation of the order of magnitude for both quantities together with other manipulations. The influence of the sensitivities is penalized in higher iterations for a proper convergence. Two examples with several load situations are discussed where we also incorporate manufacturing restrictions and non-linear material behavior. The new augmented indicator offers the potential to reach superior local optima and a weight reduction of up to 45 % against the standard variant.
Keywords *Topology Optimization, Stress Sensitivities, Manufacturing Restrictions, Integrated Casting Simulation*

1. Introduction

Over the recent years topology optimization has become an important tool in part design aiding in the fulfillment of ever increasing lightweight ambitions. Due to this topic being at the link between the mathematical theory of optimization and the practical application in engineering, many different approaches exist. Their common aim is to determine an optimal material distribution with respect to application-specific objectives and constraints.

A common approach for continua optimization discretizes the given design space by Finite Elements (FE) that describe the part's geometry by their existence or non-existence. This is then taken to be the design variables of the optimization problem. On the one hand, there are approaches considering these variables to be continuous like the density method often combining the SIMP (Solid Isotropic Material with Penalization) approach [1,2,3], sensitivity analysis and gradient-based optimization algorithms like the MMA (Method of Moving Asymptotes) [4]. On the other hand, heuristic approaches with binary modelled design variables primarily use bionic growth rules [5] or genetically motivated searching techniques. Typical examples for this approach are the Bi-directional Evolutionary Structural Optimization (BESO) [6] and the Soft Kill Option (SKO) [7]. Other approaches make use of the Level Set method [8].

The topology optimization suite LEOPARD|topo [9] developed at Volkswagen uses a binary material model with a hard kill strategy in combination with a step size controller. This allows for the consideration of plasticity, contact and other nonlinearities in the FE Analysis as well as strict manufacturing restrictions with integrated manufacturing simulations [10]. The local von Mises stress is used as an indicator for material removal or addition. However, by this the relaxation of stress hotspots in notches is hard. It requires additional information for the indicator that can be acquired by the help of sensitivities. The development of stress sensitivities in order to solve stress constraint problems using the density method with SIMP is well described [11, 12]. In comparison to that the use of stress sensitivities in heuristic motivated topology optimization method is fairly new. There have been approaches to extend the BESO strategy [13, 14].

In this paper, a combination of LEOPARD|topo with stress sensitivities is presented. The focus lies on the development of a new way to combine values from the sensitivity analysis with the von Mises stress values in order to provide a stable optimization process and maintain the possibility of a combination with additional heuristic procedures. The positive impact is validated by two- and three-dimensional examples incorporating multiple load cases and plasticity.

2. Heuristic Based Topology Optimization in LEOPARD|topo

First introduced by [9], the topology optimization approach with LEOPARD|topo has become an important tool in the early development stages at Volkswagen and other brands belonging to the cooperation. Based on the idea of creating a fully stressed design, the local von Mises stress is the indicator of material removal or addition. This reduces weight in low strained regions and adds material around highly stressed elements. The main steps of every optimization iteration are schematically depicted in Fig. 1 (the step of calculating the sensitivities is new and forms the basis for the research presented here).

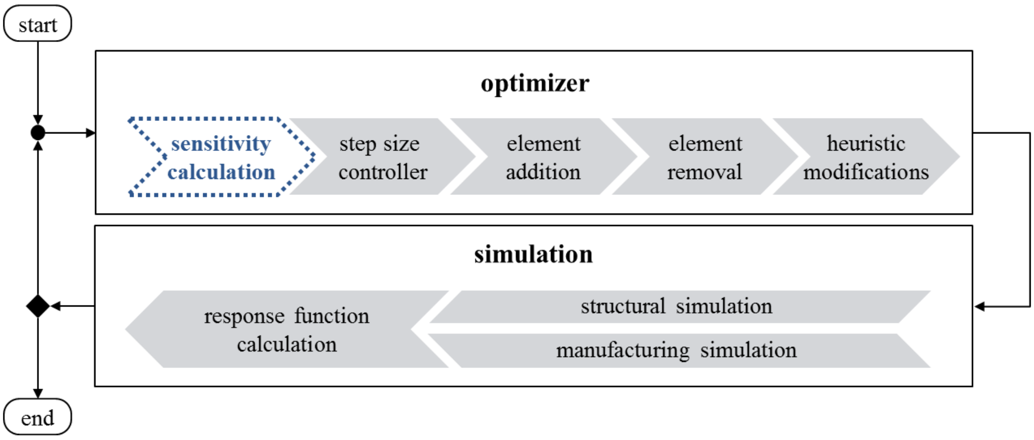

Figure 1. LEOPARD | topo optimization process

After every Finite Element Analysis the regular hexahedral elements of the design space are sorted according to their respective von Mises stress. Then, by the help of heuristic growth rules the status of the elements are updated. A step size controller examines the number of elements that are removed or added based on the current constraint violation. Constraints can be of various kinds, most notably in use are local von Mises stress maxima or maximum displacements in misuse cases [9]. If we consider multiple, the maximum violation is relevant for the step size controller.
The modification of the design variables is supported by additional heuristic functions that ensure connectivity or manufacturing restrictions. An integrated casting simulation [10] gives full insight into the manufacturability and allows for additional adjustments within the optimization process.
Unfortunately, this method experiences issues when stress hotspots occur at the design space boundary which happens at hard notches. The optimizer is unable to add material to relax the situation. A possible solution is to remove material instead resulting in a wider radius. Therefore, the ranking of the elements would have to be changed by a new criterion.

3. Stress Sensitivities for a binary material model

Generally speaking, sensitivities offer a measure of how the output of a potentially complex model changes under variation of the input quantities; hence, in topology optimization, they are the partial derivative of a response function w.r.t. a design variable. Their calculations are crucial for gradient-based optimization schemes but they can also be derived for a binary material model.
By the solution of the linear-elastic static state equation with the global stiffness matrix $\boldsymbol{K}$ and the global load vector $\boldsymbol{F}$

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{F}, \tag{1}$$

the global nodal displacement vector $\boldsymbol{u}$ is computed. The von Mises stress of the ith element can then be deduced by

$$\sigma_{vM,i} = x_i \sqrt{\boldsymbol{u}_i^T (\boldsymbol{E}\boldsymbol{B})^T \boldsymbol{V} (\boldsymbol{E}\boldsymbol{B}) \boldsymbol{u}_i} \tag{2}$$

with the local nodal displacement vector $\boldsymbol{u}_i$, the elasticity matrix $\boldsymbol{E}$, the strain-displacement matrix $\boldsymbol{B}$ and the von Mises matrix $\boldsymbol{V}$ [15]. The additional design variable x_i is used to ignore elements that are void due to the hard kill strategy within LEOPARD|topo. The von Mises stress field is evaluated at the element's center.

The dimension of the vector containing the von Mises stress of every element is collapsed by the general p-Norm

$$\sigma_k = \sqrt[p]{\sum_{i=1}^{N} \sigma_{vM,i}^{p}} \tag{3}$$

to form a quantity representing the global stress state. Contrary to evaluating the stress constraint for the step size controller with the infinity norm ($p \rightarrow \infty$), here $p = 6$ is used. The sensitivity of the global stress norm can then be derived as (see [14] or [15] for detailed derivation)

$$\frac{d\sigma_k}{dx_j} := \mu_j = \sigma_k^{1-p}[\sigma_{vM,j}^{p} - \kappa_j^T \boldsymbol{K}_j \boldsymbol{u}_j], \tag{4}$$

where $\boldsymbol{K}_j$ is the local stiffness matrix, $\boldsymbol{u}_j$ the local nodal displacement vector and κ_j the local pseudo nodal displacement vector. The latter one is calculated as the solution to the adjoint equation

$$\boldsymbol{K\kappa} = \sum_{i=1}^{N} x_i \sigma_i^{p-2} \boldsymbol{A}_i^T (\boldsymbol{EB})^T \boldsymbol{VEBu}_i. \tag{5}$$

The right-hand-side of this equation is called the pseudo force vector which is constructed by the help of the assembly matrix that is defined by $\boldsymbol{u}_i = \boldsymbol{A}_i \boldsymbol{u}$ which denotes the relation between the global degrees of freedom and the local degrees of freedom of element *i*. For every loadcase considered, a new adjoint equation has to be solved.

4. Combination of stresses and their sensitivities as a new optimization criterion

Now, for every element the local von Mises stress and the local sensitivity of the global stress norm are available. Since both quantities are not in the same order of magnitude, it is useful to apply a limitation to them.
Beforehand, both stresses ($\bar{\sigma}_{vM,i} \leftarrow \sigma_{vM,i}$) and sensitivities ($\bar{\mu}_i \leftarrow \mu_i$) are filtered by the well-known Sigmund-Filter [16]. To reduce the dimension for the results of multiple loadcases, the general p-norm similar to Eq. 3 is used. The maximum norm is applied to the stresses ($p \rightarrow \infty$) while the sensitivities are averaged ($p = 1$).
The following limiter is inspired by neural network activation functions [17]. Here, the hyperbolic tangent is used to smoothly map the stresses and sensitivities to the interval of $(1,\ 1.8)$ and $(0.2,\ 1.8)$, respectively. A negative sensitivity indicates that the element should be kept which in return means a higher ranking in the context of LEOPARD|topo; hence the sign of the sensitivities has to be flipped.

$$\tilde{\sigma}_{vM,i} = 1 + 0.8 * tanh(\frac{atanh(0.875)}{\eta_\sigma} * \bar{\sigma}_{vM,i}) \quad \text{and} \quad \tilde{\mu}_i = 1 + 0.8 * tanh(-\frac{atanh(0.875)}{\eta_\mu} * \bar{\mu}_i) \tag{6}$$

The *tanh* is further adjusted by the parameter η. The idea is to capture the majority of the elements within the steep gradient of the function. Therefore, we propose the idea of a generalized sign-sensitive median

$$\eta_\sigma = top(\sigma_{vM}) \quad \text{and} \quad \eta_\mu = max(top(|\mu_{neg}|),\ top(|\mu_{pos}|)). \tag{7}$$

The negative and the positive sensitivities are sorted, Then, \`top´ selects the element that is at a position within the vector defined by β. By this, a maximum of, e.g., 10 % ($\beta = 10$) of the elements are allowed to be above the threshold of 1.72 or below 0.28. This allows for a clear differentiation between regions that the sensitivities indicate to keep (value above 1) and those that should be removed (value below 1). Due to the monotonicity of the *tanh* function, the ordering of the elements within the vector stays unchanged.
Now, stresses and sensitivities share the same order of magnitude and can be combined. For the following, we use a multiplication of both with a penalization α of the sensitivities

$$\lambda_i = \tilde{\mu}_i{}^{\alpha} * \tilde{\sigma}_{vM,i}. \tag{8}$$

The value of α adapts the strength of the sensitivities against the stresses. The manipulations presented here are summarized in the flow chart of Fig. 2. The result of this manipulations, the indicator λ_i, is then used to rank the Finite Elements.

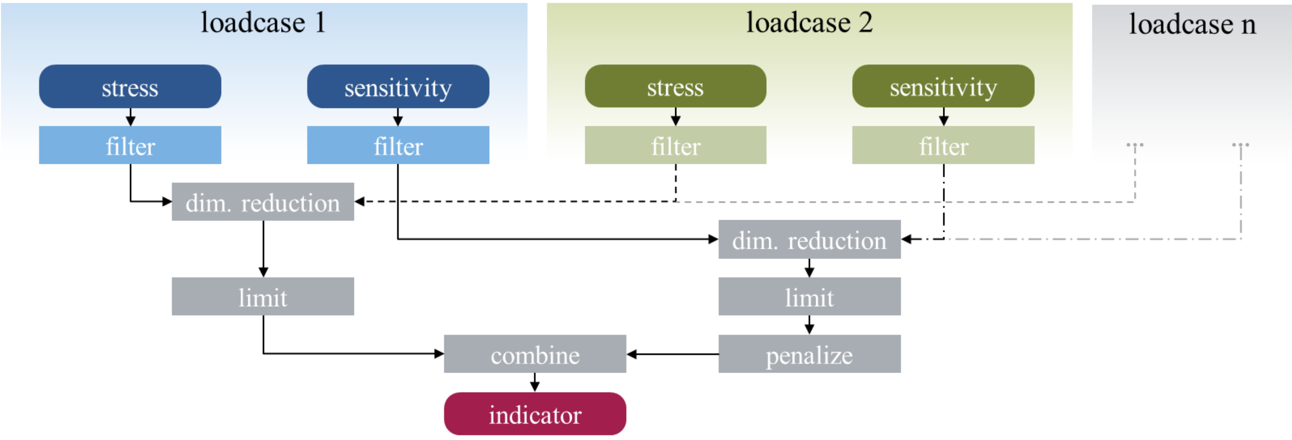

Figure 2. Flow chart of the sensitivity manipulation

5. Effects on the L-Bracket in various load case scenarios

The L-Bracket is a widely adopted example problem that consists of one 90 degrees hard notch (see, e.g., [12] and [14]). The geometry of the design space is displayed in Fig. 3. It is discretized by regular hexahedral elements with an edge length of 2 mm. The material is aluminum with a Young's modulus of 70000 MPa, a Poisson's ratio of 0.35 and a density of 2.65 t/m³. The stress constraint is set to be 250 MPa. Forces are applied within the right half-circular non-design areas by the help of a rigid body element connecting 8 nodes. The entire top of the bracket is clamped with an extending non-design area. The filter radius is set to 1.4 times the element width.

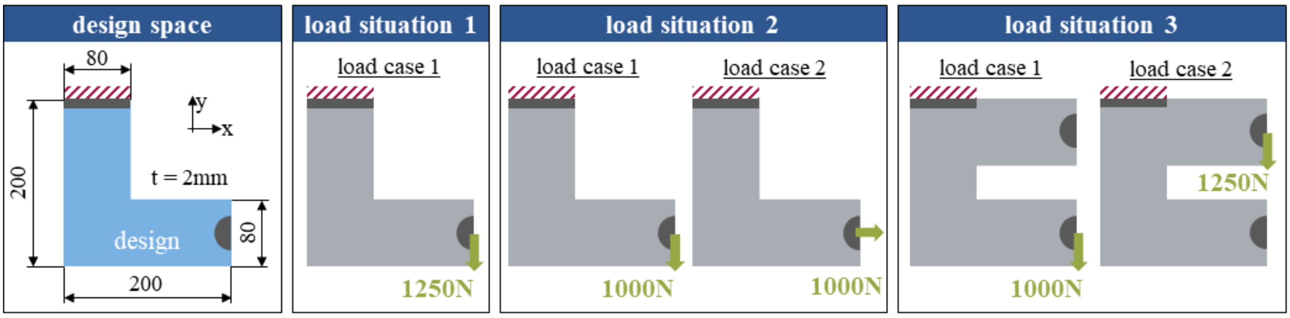

Figure 3. Design space and load situations of the L-Bracket

5.1 Load Situation 1: Bending load with linear-elastic material behavior

In this case, one bending load of 1250N is applied in the negative y direction. The initial value of the penalizing exponent $\alpha = 1$ is linearly decreased to 0.01 over the first two iterations. The top percent was set to $\beta = 10$.

The new approach results in a wide radius that precisely uses the available space which allows for a more fully stressed design that is 44.3% lighter. The initial impact of the sensitivities guide the optimizer into converging against a different local optimum. The results also show to be slightly lighter (by 3.3 %) than those of a standard implementation of the density method with SIMP of [15].

5.2 Load Situation 2: Bending load and pressure load with linear-elastic material behavior

The first load case of a bending load of 1000 N is augmented by a load of 1000 N in the negative x-direction. β is set to 3 and the initial α of 2.0 is linearly reduced to 0.01 over the first three iterations.

The influence of the additional force can be seen by the new struts of both L-Brackets (see Fig. 5). Due to the higher stress hotspot in the notch, the standard optimizer is incapable of decreasing the strut count and thickness. Here, the new approach presents a good way out resulting in a reduction of the weight by about 33.5 %.

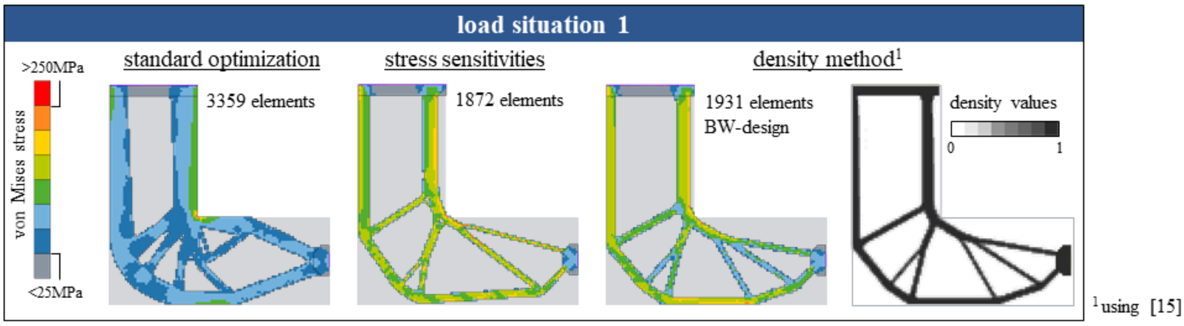

Figure 4. Optimization results of load situation 1

5.3 Load Situation 3: Bending load with linear-elastic material behavior and tension load with plasticity
Herein, the bending load of 1000 N, that is considered to be a regular use case, is extended by a misuse case of 1250 N at a different attack point incorporating a non-linear material behavior. In the first load case the constraint stays the maximum von Mises stress of 250 MPa while the residual plastic displacement at the attack point for the misuse case is limited to 0.1 mm.
By the initial influence of the sensitivities the optimizer is able to relax the stress notch even if there is no such wide radius like in the first load situation. The weight could be reduced by 39.9 %.

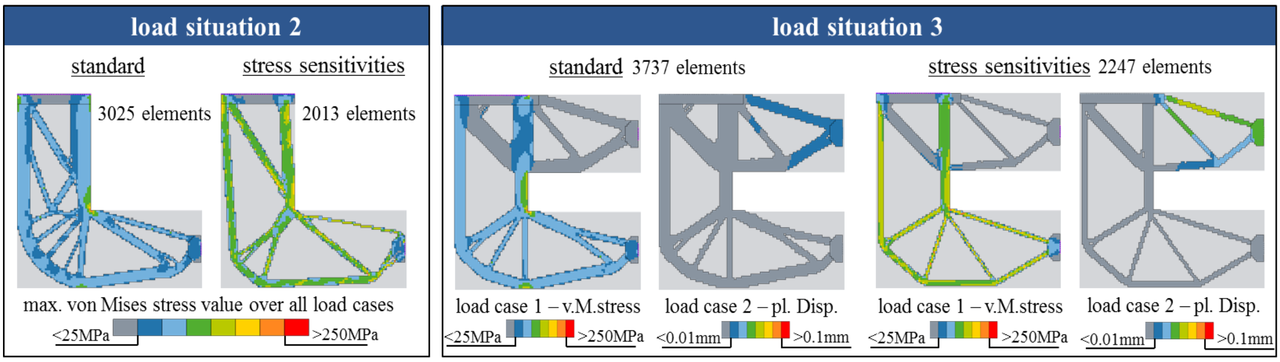

Figure 5. Optimization results of load situation 2 and 3

6. Automotive Steering Knuckle with Integrated Casting Simulation

As a final example, we present the effects of the sensitivities acting in the example of a three-dimensional steering knuckle based on real parts. Here, six different loadcases with linear-elastic material behavior are relevant. The material is aluminum with a Young's modulus of 70000 MPa, a Poisson's ratio of 0.35 and a density of 2.65 t/m³. The stress constraint is set to 180 MPa. Additionally, we employ the casting restrictions of LEOPARD|topo [10] that ensures the manufacturability. The initial value of $\alpha = 1.0$ is linearly decreased to 0.01 over the first two iterations.
The results can be seen in Fig. 6. The standard approach consists of multiple hard notches that the optimizer gets caught in whereas the most severe ones are marked by numbers. Notch 1 and 3 are partially relieved by the new approach and notch 2 is fully smeared. Due to the ongoing constraint violation the standard procedure is incapable of removing additional elements. In the end, the new approach reduces the weight of the part by about 18.7 %. Figure 7 shows the solidification behavior of the result with a directional solidification and no porosity occurring.

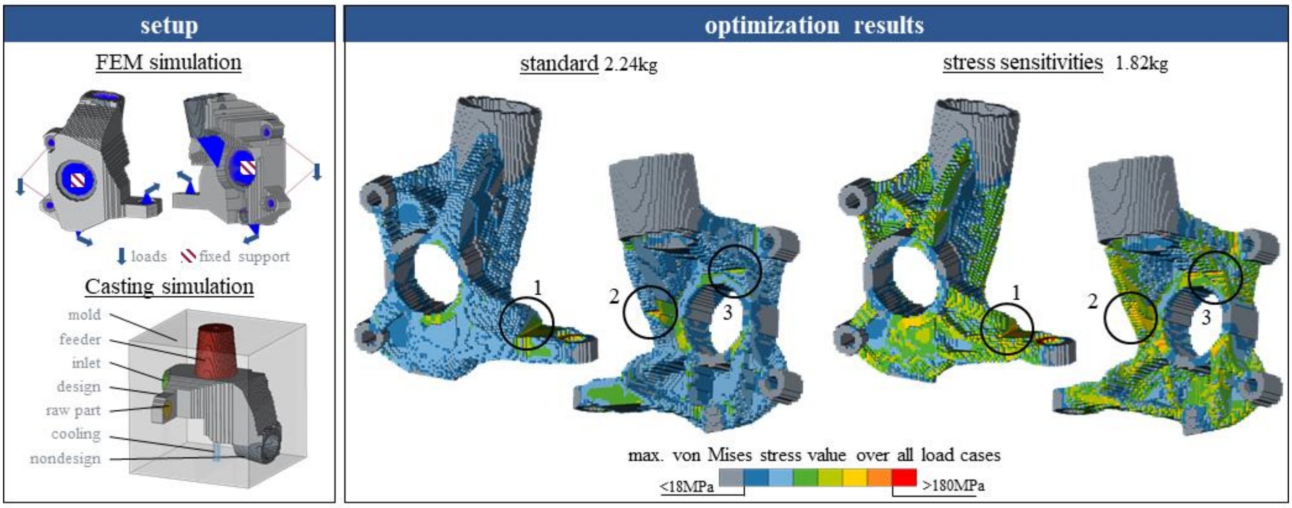

Figure 6. Comparison of the results of the 3D steering knuckle

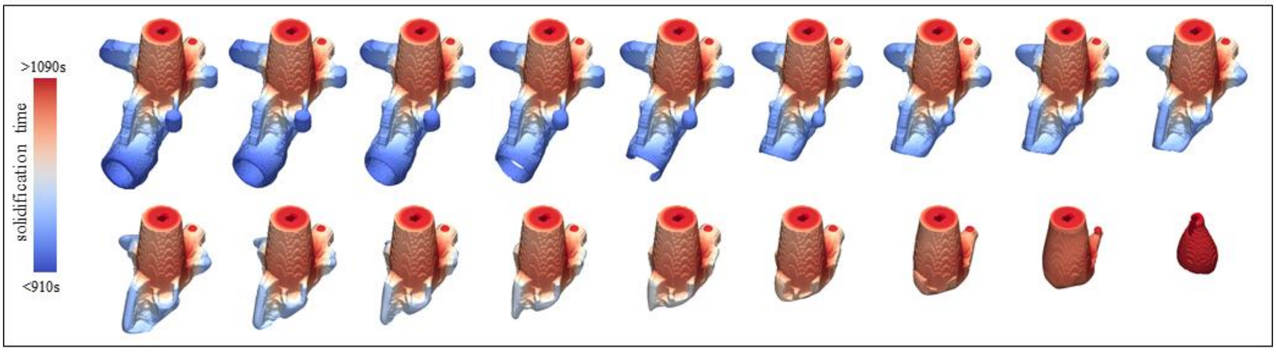

Figure 7 Solidification simulation of the optimization result

7. Conclusion and outlook

In this paper an approach of combining stress sensitivities with local von Mises stresses for ranking elements has been presented. Therefore, the concept of activation functions from artificial neural networks has been transferred to limit the order of magnitude for both quantities. The actual combination was done by a multiplication whereas the influence of the sensitivities has been penalized for higher iterations.
This new approach aided in relieving stress hotspots at design space boundaries by removing material instead of adding it. By this a weight reduction of up to 45% in the academic examples could be shown. Due to the validations presented, the new approach renders to be scalable to multiple loadcases even with some incorporating non-linear material behavior. Therefore, various load case scenarios for the well-known L-Bracket were optimized. Additionally, the new improvements have been further confirmed with the three-dimensional example of the steering knuckle where six loadcases have to be considered simultaneously. The integrated casting simulation of LEOPARD|topo was used to ensure the manufacturability. The influence of the sensitivities aided primarily in stress notch widening for small struts and structures.
Future research will consider the application of the sensitivities with the recently proposed Volume-of-Solid method for a combined topology and shape optimization as well as the impact of other sensitivities like eigenfrequencies.

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