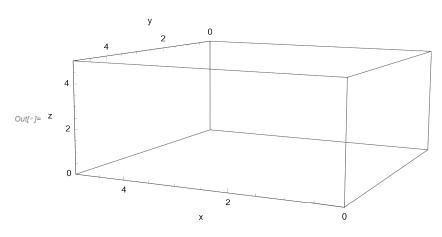
Solving the inverse kinematics

Deriving expressions for forward kinematics. Note that for simplifying the expressions, we subbed in arbitrary values for some of the padding constants.

$$ln[22]:=$$
 data = Flatten[Table[{x, y, z} /. fkinsub, {11, 0, 10, 0.05}, {12, 0, 10, 0.05}, {13, 0, 10, 0.05}], 1];

Out[ $\sigma$ ]= L - 13y - Py - y +  $\sqrt{-((13x - Px + Pz - x - z))(13x - Px - Pz - x + z))}$ 



Finding Jacobians related to forward and inverse kinematics

Out[@]//MatrixForm=

 $-\frac{19.46 \left(0.564+1.211\right)}{-11.3868+0.611-0.1812+0.7813} + \frac{\left(0.564+1.211\right)13}{-11.3868+0.611-0.1812+0.7813} + \frac{11.676 \left(-226.718+0.56411+0.611^2-7.0212+0.1812^2+30.357613-0.7813^2\right)}{\left(-11.3868+0.611-0.1812+0.7813\right)^2} + \frac{\left(0.564+1.211\right) \left(-226.718+0.56411+0.611^2-7.0212+0.1812^2+30.357613-0.7813^2\right)}{2 \left(-11.3868+0.611-0.1812+0.7813^2\right)} + \frac{11.676 \left(-226.718+0.56411+0.611^2-7.0212+0.1812^2+30.357613-0.7813^2\right)}{2 \left(-226.718+0.56411+0.611^2-7.0212+0.1812^2+30.357613-0.7813^2\right)} + \frac{11.676 \left(-226.718+0.56411+0.611^2-7.0212+0.1812^2+30.357613-0.7813^2\right)}{2 \left(-226.718+0.56411+0.6112-0.7813^2\right)} + \frac{11.676 \left(-226.718+0.56411+0.6112-7.0212+0.1812^2+30.357613-0.7813^2\right)}{2 \left(-226.718+0.56411+0.6112-0.7813^2\right)} + \frac{11.676 \left(-226.718+0.56411+0.6112-0.7813^2\right)}{2 \left(-226.718+0.56411+0.6112-0.7813^2\right)} + \frac{11.676 \left(-226.718+0.56411+0.6112-0.7813+0.6$ (-11.3868+0.611-0.1812+0.7813)2

Interplace | Jikin = FullSimplify[D[{11 /. l1soln, 12 /. l2soln, 13 /. l3soln}, {{x, y, z}}]]

$$\begin{array}{l} \text{Out[*]} = \left\{ \left\{ \frac{11x - Px - x}{\sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)}} \right\}, \\ \\ 1, \frac{-Pz + z}{\sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)}} \right\}, \\ \\ \left\{ -\frac{12x - Px + x}{\sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)}} \right\}, \\ \\ -Pz + z \\ \hline \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)}} \right\}, \\ \\ \left\{ \frac{13x - Px - x}{\sqrt{-\left(\left(13x - Px + Pz - x - z\right)\left(13x - Px - Pz - x + z\right)\right)}} \right\}, \\ -Pz + z \\ \hline -Pz + z \\ \hline \sqrt{-\left(\left(13x - Px + Pz - x - z\right)\left(13x - Px - Pz - x + z\right)\right)}} \right\} \right\} \\ \end{array}$$

## 

Out[@]//MatrixForm=

$$\begin{pmatrix} \frac{11x - Px - x}{\sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)}} & 1 & \frac{-Pz + z}{\sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)}} \\ - \frac{12x - Px + x}{\sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)}} & -1 & \frac{-Pz + z}{\sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)}} \\ \frac{13x - Px - x}{\sqrt{-\left(\left(13x - Px + Pz - x - z\right)\left(13x - Px - Pz - x + z\right)\right)}} & -1 & \frac{-Pz + z}{\sqrt{-\left(\left(13x - Px + Pz - x - z\right)\left(13x - Px - Pz - x + z\right)\right)}} \\ \end{pmatrix}$$

$$ln[*]:=$$
 Inverse[Jikin] /. {Px  $\rightarrow$  0.1, Py  $\rightarrow$  0.1, Pz  $\rightarrow$  0.1,  $l1x \rightarrow$  0.1,  $l1y \rightarrow$  0.1,  $l2x \rightarrow$  0.1,  $l2y \rightarrow$  0.1,  $l3x \rightarrow$  0.1,  $l3y \rightarrow$  0.2};

## In[\*]:= Inverse[Jikin] // MatrixForm

Out[@]//MatrixForm=

```
\sqrt{-\left(\left(11x-Px+Pz-x-z\right)\left(11x-Px-Pz-x+z\right)\right)} \sqrt{-\left(\left(12x-Px+Pz+x-z\right)\left(12x-Px-Pz+x+z\right)\right)}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \overline{\sqrt{-\left(\left(11x-Px+Pz-x-z\right)\,\left(11x-Px-Pz-x+z\right)\,\right)}} \quad \sqrt{-\left(\left(12x-Px+Pz+x-z\right)\,\left(12x-Px-Pz+x+z\right)\,\right)}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          √- ( (13x-Px+Pz-x-z ) (13x-P
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \sqrt{-((13x-Px+Pz-x-z)(13x-Px-Pz-x+z))}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       \sqrt{-(12x-Px+Pz+x-z)(12x-Px-Pz+x+z)}
\sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(11x - Px + Pz - x - z\right)\left(11x - Px - Pz - x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - Pz + x + z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - z\right)\right)} \sqrt{-\left(\left(12x - Px + Pz + x - z\right)\left(12x - Px - z\right)\right)} \sqrt{-\left(\left(12x - Px + z - z\right)\right)} \sqrt{-\left(\left(12x - Px + z - z\right)\right)} \sqrt{-\left(12x - Px + z - z\right)} \sqrt{-\left(\left(12x - Px + z - z\right)\right)} \sqrt{-\left(\left(12x - Px + z - z\right)\right)} \sqrt{-\left(\left(12x - Px + z - z\right)\right)} \sqrt{-\left(\left(12x - Px + z - 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 \sqrt{-((12x-Px+Pz+x-z))(12x-Pz+y-z)}
\sqrt{-\left(\left(11x - Px + Pz - x - z\right) \left(11x - Px - Pz - x + z\right)\right)} \ \sqrt{-\left(\left(12x - Px + Pz + x - z\right) \left(12x - Px - Pz + x + z\right)\right)}^{-} \ \sqrt{-\left(\left(11x - Px + Pz - x - z\right) \left(11x - Px - Pz - x + z\right)\right)} \ \sqrt{-\left(\left(12x - Px + Pz + x - z\right) \left(12x - Px - Pz + x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(11x - Px - Pz - x + z\right)\right)} \ \sqrt{-\left(\left(12x - Px + Pz + x - z\right) \left(12x - Px - Pz + x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px + Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x + z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - Pz - x - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - Pz - x - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - z - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - z - z\right) \left(12x - Px - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - z - z\right) \left(12x - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - Px - z - z\right) \left(12x - z - z\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z - z\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z - z\right)\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z - z\right)}^{-} \ \sqrt{-\left(\left(12x - z - z - z\right) \left(12x - z
```

Analyzing the internal angles of the linkages. This system should define the internal angles that the linkages should be given an x,y,z,  $l_1$ ,  $l_2$ ,  $l_3$ .

```
Solve [\{theta2 = theta1 + theta3, 11x + 1b * Cos[theta1] = x + px + 1t * Cos[theta3], \}
  11 + l1y + lb * Sin[theta1] == y - py - lt * Sin[theta3],
  (x - px)^2 + (y - py)^2 + (z - pz)^2 ==
    (Sqrt[2] * 1b)^2 + (Sqrt[2] * 1t)^2 + 2 * Sqrt[2]^2 * 1b * 1t * Cos[theta2]
 }, {theta1, theta3, theta2}]
```

- Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is ArcCos[Cos[theta1]] - ArcCos[Cos[theta2]] + ArcCos[Cos[theta3]] == 0.
- Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

```
Out[28]= \left\{ \left\{ \text{theta1} \rightarrow \right. \right. \right.
                                          -ArcCos[(-1t(-411^211x-411x^3-81111x11y-411x11y^2+411x1b^2-411x1t^2+411^2px+411x1t^2+411x^3-81111x11y^2+411x1t^2+411x1t^2+411^2px+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x1t^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2+411x^2
                                                                                           12 \, 11x^2 \, px + 8 \, 11 \, 11y \, px + 4 \, 11y^2 \, px - 4 \, 1b^2 \, px + 4 \, 1t^2 \, px - 12 \, 11x \, px^2 + 4 \, px^3 - 8 \, 11 \, 11x \, py -
                                                                                           8 11x 11y py + 8 11 px py + 8 11y px py - 4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 12 
                                                                                           8 11 11y x + 4 11y^2 x - 4 1b^2 x + 4 1t^2 x - 24 11x px x + 12 px^2 x + 8 11 py x + 8 11y py x +
                                                                                           4 \text{ py}^2 \text{ x} - 12 \text{ l1x } \text{x}^2 + 12 \text{ px } \text{x}^2 + 4 \text{ x}^3 + 8 \text{ l1 l1x } \text{y} + 8 \text{ l1x l1y } \text{y} - 8 \text{ l1 px } \text{y} - 8 \text{ l1y px } \text{y} +
                                                                                           \sqrt{\left(1t^{2} \left(-411^{2}11x-411x^{3}-81111x11y-411x11y^{2}+411x1b^{2}-411x1t^{2}+411x1t^{2}\right)}
                                                                                                                4 11^{2} px + 12 11x^{2} px + 8 11 11y px + 4 11y^{2} px - 4 1b^{2} px + 4 1t^{2} px -
                                                                                                                 12 11x px^{2} + 4 px^{3} - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                                                                                                 4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                                                                                                 4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                                                                 12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                                                                                 8 px py y - 8 l1 x y - 8 l1y x y - 8 py x y - 4 l1x y^2 + 4 px y^2 + 4 x y^2) -
                                                                                          4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y
                                                                                                                 py + 4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                                                                                  (11^4 + 211^211x^2 + 11x^4 + 411^311y + 41111x^211y + 611^211y^2 + 211x^211y^2 + 4
                                                                                                                 11\,11y^3 + 11y^4 - 2\,11^2\,1b^2 - 2\,11x^2\,1b^2 - 4\,11\,11y\,1b^2 - 2\,11y^2\,1b^2 + 1b^4 - 2\,11^2\,1t^2 + 2
```

```
11x^{2} 1t^{2} - 4 11 11y 1t^{2} - 2 11y^{2} 1t^{2} - 2 1b^{2} 1t^{2} + 1t^{4} - 4 11^{2} 11x px - 4 11x^{3} px - 8 11
                                                                                                                                      11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 + 4
                                                                                                                                      11\,11v\,px^2+2\,11y^2\,px^2-2\,1b^2\,px^2+2\,1t^2\,px^2-4\,11x\,px^3+px^4+4\,11^3\,py+4\,11\,11x^2
                                                                                                                                      py + 12 11^{2} 11y py + 4 11x^{2} 11y py + 12 11 11y^{2} py + 4 11y^{3} py - 4 11 1b^{2} py - 4 11y 1b^{2}
                                                                                                                                      py - 4111t^2 py - 411y1t^2 py - 81111x px py - 811x11y px py + 411 px^2 py + 411y
                                                                                                                                      px^2\;py\;+\;6\;11^2\;py^2\;+\;2\;11x^2\;py^2\;+\;12\;11\;11y\;py^2\;+\;6\;11y^2\;py^2\;-\;2\;1b^2\;py^2\;-\;2\;1t^2\;py^2\;-\;4
                                                                                                                                      11x px py^{2} + 2 px^{2} py^{2} + 4 11 py^{3} + 4 11y py^{3} + py^{4} - 4 11^{2} 11x x - 4 11x^{3} x - 8 11 11x
                                                                                                                                      11y \times -4 \times 11x \times 11y^2 \times +4 \times 11x \times 10^2 \times -4 \times 11x \times 11^2 \times 11x \times 11^2 \times 11x \times 11x^2 \times 11
                                                                                                                                      x + 4 \cdot 11y^2 px x - 4 lb<sup>2</sup> px x + 4 lt<sup>2</sup> px x - 12 l1x px<sup>2</sup> x + 4 px<sup>3</sup> x - 8 l1 l1x py x - 8 l1x
                                                                                                                                      11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x + 4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 +
                                                                                                                            4 11 11y x^{2} + 2 11y^{2} x^{2} - 2 1b^{2} x^{2} + 2 1t^{2} x^{2} - 12 11x px x^{2} + 6 px^{2} x^{2} + 4 11 py x^{2} + 4 11y
                                                                                                                                      py\ x^2 + 2\ py^2\ x^2 - 4\ l1x\ x^3 + 4\ px\ x^3 + x^4 - 4\ l1^3\ y - 4\ l1\ l1x^2\ y - 12\ l1^2\ l1y\ y - 4\ l1x^2
                                                                                                                                      11y y - 12 11 11y^2 y - 4 11y^3 y + 4 11 1b^2 y + 4 11y 1b^2 y + 4 11 1t^2 y + 4 11y 1t^2 y + 8
                                                                                                                                      11\ 11x\ px\ y + 8\ 11x\ 11y\ px\ y - 4\ 11\ px^2\ y - 4\ 11y\ px^2\ y - 12\ 11^2\ py\ y - 4\ 11x^2\ py\ y - 24
                                                                                                                                      11\ 11y\ py\ y - 12\ 11y^2\ py\ y + 4\ 1b^2\ py\ y + 4\ 1t^2\ py\ y + 8\ 11x\ px\ py\ y - 4\ px^2\ py\ y - 12\ 11
                                                                                                                                      py^2y - 12 lly py^2y - 4 py^3y + 8 ll llx xy + 8 llx lly xy - 8 ll px xy - 8 lly px xy +
                                                                                                                            8 11x py x y - 8 px py x y - 4 11 x^2 y - 4 11y x^2 y - 4 py x^2 y + 6 11^2 y^2 + 2 11x^2 y^2 + 12
                                                                                                                                    11\ 11y\ y^2 + 6\ 11y^2\ y^2 - 2\ 1b^2\ y^2 - 2\ 1t^2\ y^2 - 4\ 11x\ px\ y^2 + 2\ px^2\ y^2 + 12\ 11\ py\ y^2 + 12\ 11y
                                                                                                                                   py y^2 + 6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4)
                                                   (2 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                                                                     4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2))
                 ArcCos \left[\frac{1}{4 \ln 1t} \left(-2 \ln^2 - 2 \ln^2 + px^2 + py^2 + \frac{1}{2}\right)\right]
                                                pz<sup>2</sup> –
                                                          рx
                                                         x + x^2 - 2
                                                        y\,+\,y^2\,-\,2
                                                        z + z^2),
\texttt{theta3} \rightarrow -\texttt{ArcCos} \left[ \; \left( -\,\texttt{lt} \; \left( -\,4\,\,\texttt{l1}^2\,\,\texttt{l1x} \,-\,4\,\,\texttt{l1x}^3 \,-\,8\,\,\texttt{l1}\,\,\texttt{l1x}\,\,\texttt{l1y} \,-\,4\,\,\texttt{l1x}\,\,\texttt{l1y}^2 \,+\,4\,\,\texttt{l1x}\,\,\texttt{lb}^2 \,-\,4\,\,\texttt{l1x}\,\,\texttt{l1y}^2 \,+\,4\,\,\texttt{l1x}\,\,\texttt{l1y}^2 \,+\,4\,
                                                                                     4 \ln 1 \ln^2 + 4 \ln^2 px + 12 \ln^2 px + 8 \ln 1 \ln px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2
                                                                                      12 \ln x px^2 + 4 px^3 - 8 \ln 11x py - 8 \ln 11y py + 8 \ln px py + 8 \ln px py - 8 \ln 11y px py - 8 \ln
                                                                                      4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                                                                      4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                                      12\;px\;x^2+4\;x^3+8\;l1\;l1x\;y+8\;l1x\;l1y\;y-8\;l1\;px\;y-8\;l1y\;px\;y+8\;l1x\;py\;y-12\;px\;x^2+4\;x^3+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px\;y+12\;px
                                                                                     8\;px\;py\;y\;-\;8\;11\;x\;y\;-\;8\;11y\;x\;y\;-\;8\;py\;x\;y\;-\;4\;11x\;y^2\;+\;4\;px\;y^2\;+\;4\;x\;y^2\;\rangle\;\;-\;
                                                          \sqrt{\left(1t^2 \left(-4 11^2 11 x-4 11 x^3-8 11 11 x 11 y-4 11 x 11 y^2+4 11 x 1 b^2-4 11 x 1 t^2+1 x^3-8 11 11 x 11 y^2+4 11 x 1 b^2-4 11 x 1 t^2+1 x^3-8 11 11 x 11 x 11 y^2+4 11 x 1 b^2-4 11 x 1 t^2+1 x^3-8 11 11 x 11 x 11 y^2+4 11 x 1 b^2-4 11 x 1 t^2+1 t^2+1 x 1 t^2+1 t^2+1
                                                                                                                            4 \cdot 11^2 \text{ px} + 12 \cdot 11x^2 \text{ px} + 8 \cdot 11 \cdot 11y \text{ px} + 4 \cdot 11y^2 \text{ px} - 4 \cdot 1b^2 \text{ px} + 4 \cdot 1t^2 \text{ px} -
                                                                                                                            4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
```

```
4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                            12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                            8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2) -
                                           4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                                        4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                                (11^4 + 2 11^2 11x^2 + 11x^4 + 4 11^3 11y + 4 11 11x^2 11y + 6 11^2 11y^2 + 2 11x^2 11y^2 +
                                                        4\ 11\ 11y^3 + 11y^4 - 2\ 11^2\ 1b^2 - 2\ 11x^2\ 1b^2 - 4\ 11\ 11y\ 1b^2 - 2\ 11y^2\ 1b^2 + 1b^4 - 2\ 11^2\ 1t^2 +
                                                        211x^{2}1t^{2} - 41111y1t^{2} - 211y^{2}1t^{2} - 21b^{2}1t^{2} + 1t^{4} - 411^{2}11xpx - 411x^{3}px -
                                                       8 11 11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 +
                                                       4 11 11 y px^2 + 2 11 y^2 px^2 - 2 1 b^2 px^2 + 2 1 t^2 px^2 - 4 11 x px^3 + px^4 + 4 11^3 py +
                                                       4 11 11x^{2} py + 12 11^{2} 11y py + 4 11x^{2} 11y py + 12 11 11y^{2} py + 4 11y^{3} py - 4 11 1b^{2} py -
                                                       4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} + 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text
                                                       4 l1 px^2 py + 4 l1y px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 -
                                                       2 lb^2 pv^2 - 2 lt^2 pv^2 - 4 l1x px pv^2 + 2 px^2 pv^2 + 4 l1 pv^3 + 4 l1v pv^3 + pv^4 - 4 l1^2 l1x x -
                                                       4 11x^{3} x - 8 11 11x 11y x - 4 11x 11y^{2} x + 4 11x 1b^{2} x - 4 11x 1t^{2} x + 4 11^{2} px x +
                                                       12 \, 11x^2 \, px \, x + 8 \, 11 \, 11y \, px \, x + 4 \, 11y^2 \, px \, x - 4 \, 1b^2 \, px \, x + 4 \, 1t^2 \, px \, x - 12 \, 11x \, px^2 \, x +
                                                        4 px^3 x - 8 11 11x py x - 8 11x 11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x +
                                                       4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 + 4 11 11y x^2 + 2 11y^2 x^2 - 2 1b^2 x^2 + 2 1t^2 x^2 -
                                                        12 11x px x^2 + 6 px^2 x^2 + 4 11 py x^2 + 4 11y py x^2 + 2 py^2 x^2 - 4 11x x^3 + 4 px x^3 +
                                                       x^4 - 4 11^3 y - 4 11 11x^2 y - 12 11^2 11y y - 4 11x^2 11y y - 12 11 11y^2 y - 4 11y^3 y +
                                                       4 \, 11 \, 1b^2 \, y + 4 \, 11y \, 1b^2 \, y + 4 \, 11 \, 1t^2 \, y + 4 \, 11y \, 1t^2 \, y + 8 \, 11 \, 11x \, px \, y + 8 \, 11x \, 11y \, px \, y -
                                                       4 l1 px^2 y - 4 l1y px^2 y - 12 l1^2 py y - 4 l1x^2 py y - 24 l1 l1y py y - 12 l1y^2 py y +
                                                       4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1 py^2 y - 12 l1y py^2 y -
                                                       4 py^{3} y + 8 11 11x x y + 8 11x 11y x y - 8 11 px x y - 8 11y px x y + 8 11x py x y -
                                                       8 px py x y -411 x<sup>2</sup> y -411 y x<sup>2</sup> y -4 py x<sup>2</sup> y +611^2 y<sup>2</sup> +211x<sup>2</sup> y<sup>2</sup> +121111y y<sup>2</sup> +
                                                       6 \ln^2 y^2 - 2 \ln^2 y^2 - 2 \ln^2 y^2 - 4 \ln px y^2 + 2 px^2 y^2 + 12 \ln py 
                                                       6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4) ) \Big)
                      (2 1t^{2} (4 11^{2} + 4 11x^{2} + 8 11 11y + 4 11y^{2} - 8 11x px + 4 px^{2} + 8 11 py + 8 11y py +
                                     4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2))
   theta2 \rightarrow -ArcCos \left[\frac{1}{4 \ln 1 +} \left(-2 \ln^2 - 2 \ln^2 + \right)\right]
                         px^2 +
                         pv^2 +
                         pz^2 –
                              рх
                             x + x^2 - 2
                             ру
                             y + y^2 - 2
                             z + z^2) \}
\left\{ \text{theta1} \rightarrow -\text{ArcCos} \left[ \left( -\text{1t} \, \left( -4\, 11^2\, 11x - 4\, 11x^3 - 8\, 11\, 11x\, 11y - 4\, 11x\, 11y^2 + 4\, 11x\, 1b^2 - 4\, 11x\, 11y^2 \right) \right] \right\} \right\} = 0
```

```
4 11x 1t^2 + 4 11^2 px + 12 11x^2 px + 8 11 11y px + 4 11y^2 px - 4 1b^2 px + 4 1t^2 px -
                                  12 11x px^2 + 4 px^3 - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                  4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                  4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                  12 px x^{2} + 4 x^{3} + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                  8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2 -
                     \sqrt{\left(1t^{2} \left(-4 11^{2} 11x-4 11x^{3}-8 11 11x 11y-4 11x 11y^{2}+4 11x 1b^{2}-4 11x 1t^{2}+\right)}
                                                  4 11^{2} px + 12 11x^{2} px + 8 11 11y px + 4 11y^{2} px - 4 1b^{2} px + 4 1t^{2} px -
                                                  12 11x px^{2} + 4 px^{3} - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                                  4 11x py^{2} + 4 px py^{2} + 4 11^{2} x + 12 11x^{2} x + 8 11 11y x + 4 11y^{2} x - 4 1b^{2} x +
                                                  4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                  12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                  8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2) -
                                 4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y
                                                  py + 4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                      \left(11^4 + 2\,11^2\,11x^2 + 11x^4 + 4\,11^3\,11y + 4\,11\,11x^2\,11y + 6\,11^2\,11y^2 + 2\,11x^2\,11y^2 + 4\,11^3\,11y^2 + 4
                                                  11\,11y^3 + 11y^4 - 2\,11^2\,1b^2 - 2\,11x^2\,1b^2 - 4\,11\,11y\,1b^2 - 2\,11y^2\,1b^2 + 1b^4 - 2\,11^2\,1t^2 + 2
                                                  11x^{2} 1t^{2} - 4 11 11y 1t^{2} - 2 11y^{2} 1t^{2} - 2 1b^{2} 1t^{2} + 1t^{4} - 4 11^{2} 11x px - 4 11x^{3} px - 8 11
                                                  11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 + 4
                                                  11\ 11y\ px^2 + 2\ 11y^2\ px^2 - 2\ 1b^2\ px^2 + 2\ 1t^2\ px^2 - 4\ 11x\ px^3 + px^4 + 4\ 11^3\ py + 4\ 11\ 11x^2
                                                  py + 12 11^2 11y py + 4 11x^2 11y py + 12 11 11y^2 py + 4 11y^3 py - 4 11 1b^2 py - 4 11y 1b^2
                                                  py - 411 lt^2 py - 411y lt^2 py - 811 l1x px py - 811x l1y px py + 411 px^2 py + 411y
                                                  px^{2}py + 611^{2}py^{2} + 211x^{2}py^{2} + 121111ypy^{2} + 611y^{2}py^{2} - 21b^{2}py^{2} - 21t^{2}py^{2} - 4
                                                  11x px py^{2} + 2 px^{2} py^{2} + 4 11 py^{3} + 4 11y py^{3} + py^{4} - 4 11^{2} 11x x - 4 11x^{3} x - 8 11 11x
                                                  11y \times -4 \times 11x \times 11y^2 \times +4 \times 11x \times 10^2 \times -4 \times 11x \times 11^2 \times 11x \times 11x
                                                  x + 4 11y^2 px x - 4 1b^2 px x + 4 1t^2 px x - 12 11x px^2 x + 4 px^3 x - 8 11 11x py x - 8 11x
                                                  11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x + 4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 +
                                              4 11 11y x^2 + 2 11y^2 x^2 - 2 1b^2 x^2 + 2 1t^2 x^2 - 12 11x px x^2 + 6 px^2 x^2 + 4 11 py x^2 + 4 11y
                                                  py x^2 + 2 py^2 x^2 - 4 l1x x^3 + 4 px x^3 + x^4 - 4 l1^3 y - 4 l1 l1x^2 y - 12 l1^2 l1y y - 4 l1x^2
                                                  11yy - 121111y^2y - 411y^3y + 4111b^2y + 411y1b^2y + 4111t^2y + 411y1t^2y + 8
                                                  11\ 11x\ px\ y + 8\ 11x\ 11y\ px\ y - 4\ 11\ px^2\ y - 4\ 11y\ px^2\ y - 12\ 11^2\ py\ y - 4\ 11x^2\ py\ y - 24
                                                  11\ 11y\ py\ y - 12\ 11y^2\ py\ y + 4\ 1b^2\ py\ y + 4\ 1t^2\ py\ y + 8\ 11x\ px\ py\ y - 4\ px^2\ py\ y - 12\ 11
                                                  py^2y - 12 l1y py^2y - 4 py^3y + 8 l1 l1x x y + 8 l1x l1y x y - 8 l1 px x y - 8 l1y px x y +
                                              8 11x py x y - 8 px py x y - 4 11 x^2 y - 4 11y x^2 y - 4 py x^2 y + 6 11^2 y^2 + 2 11x^2 y^2 + 12
                                                  11 11y y^2 + 6 11y^2 y^2 - 2 1b^2 y^2 - 2 1t^2 y^2 - 4 11x px y^2 + 2 px^2 y^2 + 12 11 py y^2 + 12 11y
                                                  py y^2 + 6 py y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4)
              (2 1t^{2} (4 11^{2} + 4 11x^{2} + 8 11 11y + 4 11y^{2} - 8 11x px + 4 px^{2} + 8 11 py + 8 11y py +
                             4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2)) +
ArcCos \left[\frac{1}{4 \text{ lb lt}} \left(-2 \text{ lb}^2 - 2 \text{ lt}^2 + px^2 + py^2 + \right]
             pz<sup>2</sup> –
```

```
рх
                       x + x^2 - 2
                       ру
                       y + y^2 - 2
                       z + z^2
theta3 \rightarrow ArcCos \left[ \left( -1t \left( -411^2 11x - 411x^3 - 81111x11y - 411x11y^2 + 411x1b^2 - 411x1y^2 + 411x1b^2 - 411x1
                              4 l1x lt^{2} + 4 l1^{2} px + 12 l1x^{2} px + 8 l1 l1y px + 4 l1y^{2} px -
                              4 lb^2 px + 4 lt^2 px - 12 l1x px^2 + 4 px^3 - 8 l1 l1x py -
                              8 11x 11y py + 8 11 px py + 8 11y px py - 4 11x py^2 +
                              4 px py^{2} + 4 11^{2} x + 12 11x^{2} x + 8 11 11y x + 4 11y^{2} x -
                              4 lb^2 x + 4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x +
                              8 11y py x + 4 py^2 x - 12 11x x^2 + 12 px x^2 + 4 x^3 + 8 11 11x y +
                              8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y - 8 px py y -
                              8 11 \times y - 8 11y \times y - 8 py \times y - 4 11x y^2 + 4 px y^2 + 4 x y^2
                   \sqrt{\left(1t^{2}\left(-411^{2}11x-411x^{3}-81111x11y-411x11y^{2}+411x1b^{2}-411x1t^{2}+411^{2}px+411x1y^{2}\right)}
                                             12\ 11x^2\ px\ +\ 8\ 11\ 11y\ px\ +\ 4\ 11y^2\ px\ -\ 4\ 1b^2\ px\ +\ 4\ 1t^2\ px\ -\ 12\ 11x\ px^2\ +\ 4\ px^3\ -\ 8\ 11\ 11x
                                                 py - 8 11x 11y py + 8 11 px py + 8 11y px py - 4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x +
                                             8 11 11y x + 4 11y^2 x - 4 1b^2 x + 4 1t^2 x - 24 11x px x + 12 px^2 x + 8 11 py x + 8 11y py
                                                x + 4 py^{2} x - 12 11x x^{2} + 12 px x^{2} + 4 x^{3} + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px
                                                y + 8 11x py y - 8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2
                              4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                         4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                   (11^4 + 211^211x^2 + 11x^4 + 411^311y + 41111x^211y + 611^211y^2 + 211x^211y^2 +
                                         4\ 11\ 11v^3 + 11v^4 - 2\ 11^2\ 1b^2 - 2\ 11x^2\ 1b^2 - 4\ 11\ 11v\ 1b^2 - 2\ 11v^2\ 1b^2 + 1b^4 - 2\ 11^2\ 1t^2 +
                                         2 11x^{2} 1t^{2} - 4 11 11y 1t^{2} - 2 11y^{2} 1t^{2} - 2 1b^{2} 1t^{2} + 1t^{4} - 4 11^{2} 11x px - 4 11x^{3} px -
                                         8 11 11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 +
                                         4 11 11y px^{2} + 2 11y^{2} px^{2} - 2 1b^{2} px^{2} + 2 1t^{2} px^{2} - 4 11x px^{3} + px^{4} + 4 11^{3} py +
                                         4 11 11x^{2} py + 12 11^{2} 11y py + 4 11x^{2} 11y py + 12 11 11y^{2} py + 4 11y^{3} py - 4 11 1b^{2} py -
                                         4 \ln 1b^2 py - 4 \ln 1t^2 py - 4 \ln 1t^2 py - 8 \ln 11 px py - 8 \ln 11 px py + 8 \ln 11
                                         4 l1 px^2 py + 4 l1y px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 -
                                          2 \cdot 1b^2 \cdot py^2 - 2 \cdot 1t^2 \cdot py^2 - 4 \cdot 11x \cdot px \cdot py^2 + 2 \cdot px^2 \cdot py^2 + 4 \cdot 11 \cdot py^3 + 4 \cdot 11y \cdot py^3 + py^4 - 4 \cdot 11^2 \cdot 11x \cdot x - 11y \cdot py^3 + 11y \cdot py^
                                         4 11x^{3} x - 8 11 11x 11y x - 4 11x 11y^{2} x + 4 11x 1b^{2} x - 4 11x 1t^{2} x + 4 11^{2} px x +
                                         12 11x^2 px x + 8 11 11y px x + 4 11y^2 px x - 4 1b^2 px x + 4 1t^2 px x - 12 11x px^2 x +
                                         4 px^3 x - 8 11 11x py x - 8 11x 11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x +
                                         4 px py^{2} x + 2 l1^{2} x^{2} + 6 l1x^{2} x^{2} + 4 l1 l1y x^{2} + 2 l1y^{2} x^{2} - 2 lb^{2} x^{2} + 2 lt^{2} x^{2} -
                                         12 11x px x^2 + 6 px^2 x^2 + 4 11 py x^2 + 4 11y py x^2 + 2 py^2 x^2 - 4 11x x^3 + 4 px x^3 +
                                         x^4 - 4 \cdot 11^3 \cdot y - 4 \cdot 11 \cdot 11x^2 \cdot y - 12 \cdot 11^2 \cdot 11y \cdot y - 4 \cdot 11x^2 \cdot 11y \cdot y - 12 \cdot 11 \cdot 11y^2 \cdot y - 4 \cdot 11y^3 \cdot y +
                                         4 l1 lb^2 y + 4 l1y lb^2 y + 4 l1 lt^2 y + 4 l1y lt^2 y + 8 l1 l1x px y + 8 l1x l1y px y -
                                         4 l1 px^2 y - 4 l1y px^2 y - 12 l1^2 py y - 4 l1x^2 py y - 24 l1 l1y py y - 12 l1y^2 py y +
                                         4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1 py^2 y - 12 l1y py^2 y -
                                         4 py^{3} y + 8 11 11x x y + 8 11x 11y x y - 8 11 px x y - 8 11y px x y + 8 11x py x y -
                                          8 px py x y -411 x<sup>2</sup> y -411 y x<sup>2</sup> y -4 py x<sup>2</sup> y +611^2 y<sup>2</sup> +211x<sup>2</sup> y<sup>2</sup> +121111y y<sup>2</sup> +
```

```
6 \ln^2 y^2 - 2 \ln^2 y^2 - 2 \ln^2 y^2 - 4 \ln px y^2 + 2 px^2 y^2 + 12 \ln py y^2 + 12 \ln py y^2 + 2 \ln^2 y^2
                                                                                        6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4) \ \ \ \ \
                              (2 1t^{2} (4 11^{2} + 4 11x^{2} + 8 11 11y + 4 11y^{2} - 8 11x px + 4 px^{2} + 8 11 py + 8 11y py +
                                                          4 py^2 - 8 11x x + 8 px x + 4 x^2 -
                                                         8 11 y - 8 11 y y - 8 py y + 4 y^{2})
     theta2 \rightarrow ArcCos \left[\frac{1}{4 \ln 1 +} \left(-2 \ln^2 - 2 \ln^2 + px^2 + py^2 + \frac{1}{4 \ln 1 +} \ln 1 + \frac{1}{4 \ln 1 +} \ln 1
                                    pz^2 –
                                    2
                                            рх
                                            x + x^2 - 2
                                            y + y^2 - 2
                                           z + z^2
\left\{ \texttt{theta1} \to -\texttt{ArcCos} \left[ \, \left( -\,\texttt{1t} \, \left( -\,4\,\,\texttt{11}^2\,\,\texttt{11x} \,-\,4\,\,\texttt{11x}^3 \,-\,8\,\,\texttt{11}\,\,\texttt{11x}\,\,\texttt{11y} \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{1b}^2 \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{1b}^2 \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,
                                                                                 4 11x 1t^2 + 4 11^2 px + 12 11x^2 px + 8 11 11y px + 4 11y^2 px - 4 1b^2 px + 4 1t^2 px -
                                                                                  12 11x px^2 + 4 px^3 - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                                                                  4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                                                                  4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                                  12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                                                  \sqrt{\left(1t^{2} \left(-4 11^{2} 11x-4 11x^{3}-8 11 11x 11y-4 11x 11y^{2}+4 11x 1b^{2}-4 11x 1t^{2}+\right)}
                                                                                                                4 l1^{2} px + 12 l1x^{2} px + 8 l1 l1y px + 4 l1y^{2} px - 4 lb^{2} px + 4 lt^{2} px -
                                                                                                                12 11x px^2 + 4 px^3 - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                                                                                                4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                                                                                                4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                                                                12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                                                                                8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2) -
                                                                                  4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y
                                                                                                               py + 4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                                                                           (11^4 + 2 11^2 11x^2 + 11x^4 + 4 11^3 11y + 4 11 11x^2 11y + 6 11^2 11y^2 + 2 11x^2 11y^2 + 4 11^3 11y^2 + 4 
                                                                                                                11\,11y^3 + 11y^4 - 2\,11^2\,1b^2 - 2\,11x^2\,1b^2 - 4\,11\,11y\,1b^2 - 2\,11y^2\,1b^2 + 1b^4 - 2\,11^2\,1t^2 + 2
                                                                                                                11x^{2} 1t^{2} - 4 11 11y 1t^{2} - 2 11y^{2} 1t^{2} - 2 1b^{2} 1t^{2} + 1t^{4} - 4 11^{2} 11x px - 4 11x^{3} px - 8 11
                                                                                                                11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 + 4
                                                                                                                11\ 11y\ px^2 + 2\ 11y^2\ px^2 - 2\ 1b^2\ px^2 + 2\ 1t^2\ px^2 - 4\ 11x\ px^3 + px^4 + 4\ 11^3\ py + 4\ 11\ 11x^2
                                                                                                                py + 12 11^2 11y py + 4 11x^2 11y py + 12 11 11y^2 py + 4 11y^3 py - 4 11 1b^2 py - 4 11y 1b^2
                                                                                                                py - 411 lt^2 py - 411y lt^2 py - 811 l1x px py - 811x l1y px py + 411 px^2 py + 411y
                                                                                                                px^{2}py + 611^{2}py^{2} + 211x^{2}py^{2} + 121111ypy^{2} + 611y^{2}py^{2} - 21b^{2}py^{2} - 21t^{2}py^{2} - 4
                                                                                                                11x px py^2 + 2 px^2 py^2 + 4 11 py^3 + 4 11y py^3 + py^4 - 4 11^2 11x x - 4 11x^3 x - 8 11 11x
                                                                                                                11y \times -4 \times 11x \times 11y^2 \times +4 \times 11x \times 10^2 \times -4 \times 11x \times 11^2 \times 11^2 \times 11x \times 11x^2 \times 11
                                                                                                                x + 411y^2 px x - 4 lb<sup>2</sup> px x + 4 lt<sup>2</sup> px x - 12 l1x px<sup>2</sup> x + 4 px<sup>3</sup> x - 8 l1 l1x py x - 8 l1x
```

```
11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x + 4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 +
                                         4 11 11y x^{2} + 2 11y^{2} x^{2} - 2 1b^{2} x^{2} + 2 1t^{2} x^{2} - 12 11x px x^{2} + 6 px^{2} x^{2} + 4 11 py x^{2} + 4 11y
                                            py x^{2} + 2 py^{2} x^{2} - 4 11x x^{3} + 4 px x^{3} + x^{4} - 4 11^{3} y - 4 11 11x^{2} y - 12 11^{2} 11y y - 4 11x^{2}
                                            11yy - 121111y^2y - 411y^3y + 4111b^2y + 411y1b^2y + 4111t^2y + 411y1t^2y + 8
                                            11\ 11x\ px\ y + 8\ 11x\ 11y\ px\ y - 4\ 11\ px^2\ y - 4\ 11y\ px^2\ y - 12\ 11^2\ py\ y - 4\ 11x^2\ py\ y - 24
                                            11 \, 11y \, py \, y - 12 \, 11y^2 \, py \, y + 4 \, 1b^2 \, py \, y + 4 \, 1t^2 \, py \, y + 8 \, 11x \, px \, py \, y - 4 \, px^2 \, py \, y - 12 \, 11
                                            py^2y - 12 11y py^2y - 4 py^3y + 8 11 11x xy + 8 11x 11y xy - 8 11 px xy - 8 11y px xy +
                                         8 11x py x y - 8 px py x y - 4 11 x^2 y - 4 11y x^2 y - 4 py x^2 y + 6 11^2 y^2 + 2 11x^2 y^2 + 12
                                            11\ 11y\ y^2 + 6\ 11y^2\ y^2 - 2\ 1b^2\ y^2 - 2\ 1t^2\ y^2 - 4\ 11x\ px\ y^2 + 2\ px^2\ y^2 + 12\ 11\ py\ y^2 + 12\ 11y
                                           py y^2 + 6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4)
                 (2 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                            4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2))
      ArcCos \left[ \, \frac{1}{4 \; 1b \; 1t} \; \left( -2 \; 1b^2 - 2 \; 1t^2 + px^2 + py^2 \right. \right. + \\
                pz<sup>2</sup> –
                   рx
                   x + x^2 - 2
                   y + y^2 - 2
                   z + z^2
theta3 \rightarrow -ArcCos \Big[ \Big( -1t \Big( -411^2 11x - 411x^3 - 81111x 11y - 411x 11y^2 + 411x 1b^2 - 411x 11y^2 \Big) \Big] \Big]
                            4 l1x lt^{2} + 4 l1^{2} px + 12 l1x^{2} px + 8 l1 l1y px + 4 l1y^{2} px -
                            4 lb^2 px + 4 lt^2 px - 12 l1x px^2 + 4 px^3 - 8 l1 l1x py - 8 l1x l1y py +
                            8 11 px py + 8 11y px py - 4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x +
                            8 11 11y x + 4 11y^2 x - 4 1b^2 x + 4 1t^2 x - 24 11x px x + 12 px^2 x +
                            8 11 py x + 8 11y py x + 4 py^2 x - 12 11x x^2 + 12 px x^2 + 4 x^3 +
                            8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                            8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2 + 4 x y^2
                   \sqrt{\left(1t^{2} \left(-4 \cdot 11^{2} \cdot 11x - 4 \cdot 11x^{3} - 8 \cdot 11 \cdot 11x \cdot 11y - 4 \cdot 11x \cdot 11y^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1b^{2} -
                                         4 l1^2 px + 12 l1x^2 px + 8 l1 l1y px + 4 l1y^2 px - 4 lb^2 px + 4 lt^2 px -
                                         12 11x px^2 + 4 px^3 - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                         4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                         4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                         12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                         8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2) -
                            4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                      4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                (11^4 + 2 11^2 11x^2 + 11x^4 + 4 11^3 11y + 4 11 11x^2 11y + 6 11^2 11y^2 + 2 11x^2 11y^2 +
                                     4\ 11\ 11y^3 + 11y^4 - 2\ 11^2\ 1b^2 - 2\ 11x^2\ 1b^2 - 4\ 11\ 11y\ 1b^2 - 2\ 11y^2\ 1b^2 + 1b^4 - 2\ 11^2\ 1t^2 +
                                      2 11x^{2} 1t^{2} - 4 11 11y 1t^{2} - 2 11y^{2} 1t^{2} - 2 1b^{2} 1t^{2} + 1t^{4} - 4 11^{2} 11x px - 4 11x^{3} px - 4 11x^{2} px
```

```
8 11 11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 +
                                                                                                           4 11 11 y px^2 + 2 11 y^2 px^2 - 2 1 b^2 px^2 + 2 1 t^2 px^2 - 4 11 x px^3 + px^4 + 4 11^3 py +
                                                                                                           4\ 11\ 11x^2\ py + 12\ 11^2\ 11y\ py + 4\ 11x^2\ 11y\ py + 12\ 11\ 11y^2\ py + 4\ 11y^3\ py - 4\ 11\ 1b^2\ py -
                                                                                                           4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} + 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text
                                                                                                           4 11 px^2 py + 4 11y px^2 py + 6 11^2 py^2 + 2 11x^2 py^2 + 12 11 11y py^2 + 6 11y^2 py^2 -
                                                                                                           2 lb^2 py^2 - 2 lt^2 py^2 - 4 l1x px py^2 + 2 px^2 py^2 + 4 l1 py^3 + 4 l1y py^3 + py^4 - 4 l1^2 l1x x -
                                                                                                            4 11x^3 x - 8 11 11x 11y x - 4 11x 11y^2 x + 4 11x 1b^2 x - 4 11x 1t^2 x + 4 11^2 px x +
                                                                                                            12 11x^2 px x + 8 11 11y px x + 4 11y^2 px x - 4 1b^2 px x + 4 1t^2 px x - 12 11x px^2 x +
                                                                                                           4 px^3 x - 8 11 11x py x - 8 11x 11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x +
                                                                                                           4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 + 4 11 11y x^2 + 2 11y^2 x^2 - 2 1b^2 x^2 + 2 1t^2 x^2 -
                                                                                                            12 11x px x^2 + 6 px<sup>2</sup> x^2 + 4 11 py x^2 + 4 11y py x^2 + 2 py<sup>2</sup> x^2 - 4 11x x^3 + 4 px x^3 +
                                                                                                           x^4 - 411^3 y - 41111x^2 y - 1211^2 11y y - 411x^2 11y y - 121111y^2 y - 411y^3 y +
                                                                                                           4 11 1b^{2} y + 4 11y 1b^{2} y + 4 11 1t^{2} y + 4 11y 1t^{2} y + 8 11 11x px y + 8 11x 11y px y -
                                                                                                           4 11 px^{2} y - 4 11y px^{2} y - 12 11^{2} py y - 4 11x^{2} py y - 24 11 11y py y - 12 11y^{2} py y +
                                                                                                           4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1 py^2 y - 12 l1y py^2 y -
                                                                                                           4 py^{3} y + 8 11 11x x y + 8 11x 11y x y - 8 11 px x y - 8 11y px x y + 8 11x py x y -
                                                                                                           8 px py x y -411 x<sup>2</sup> y -411 y x<sup>2</sup> y -4 py x<sup>2</sup> y +611^2 y<sup>2</sup> +211x<sup>2</sup> y<sup>2</sup> +121111y y<sup>2</sup> +
                                                                                                            611y^2y^2 - 21b^2y^2 - 21t^2y^2 - 411x px y^2 + 2px^2y^2 + 1211 py y^2 + 1211y py y^2 +
                                                                                                          6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4) ) \Big)
                                          (2 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                                                        4\;py^2-8\;l1x\;x+8\;px\;x+4\;x^2-8\;l1\;y-8\;l1y\;y-8\;py\;y+4\;y^2\big)\;\big)\;\Big] ,
     theta2 \rightarrow -ArcCos \left[\frac{1}{4 \ln 1 +} \left(-2 \ln^2 - 2 \ln^2 + 1\right)\right]
                                                 px^2 +
                                                 py^2 +
                                                 pz^2 –
                                                          рх
                                                        x + x^2 - 2
                                                          ру
                                                         y + y^2 - 2
                                                        z + z^2) \}
\left\{ \texttt{theta1} \to -\texttt{ArcCos} \left[ \, \left( -\,\texttt{1t} \, \left( -\,4\,\,\texttt{11}^2\,\,\texttt{11x} \,-\,4\,\,\texttt{11x}^3 \,-\,8\,\,\texttt{11}\,\,\texttt{11x}\,\,\texttt{11y} \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{1b}^2 \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{1b}^2 \,-\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,4\,\,\texttt{11x}\,\,\texttt{11y}^2 \,+\,
                                                                                         4 \ln 1 + 4 \ln^2 px + 12 \ln^2 px + 8 \ln 1 \ln px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2 px - 4 \ln^2 px + 4 \ln^2 px 
                                                                                            12 \ln px^2 + 4 px^3 - 8 \ln \ln py - 8 \ln py + 8 \ln px py + 8 \ln px py - 8 \ln px py - 8 \ln px py + 8 \ln px py + 8 \ln px py - 9 \ln px py -
                                                                                            4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x + 8 11 11y x + 4 11y^2 x - 4 1b^2 x +
                                                                                            4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                                            12 px x^{2} + 4 x^{3} + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                                                            8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2 + 4 x y^2
                                                                   \sqrt{\left(1 t^2 \left(-4 11^2 11 x-4 11 x^3-8 11 11 x 11 y-4 11 x 11 y^2+4 11 x 1 b^2-4 11 x 1 t^2+1 x 1 x^2+4 11 x 1
                                                                                                                             4 l1^{2} px + 12 l1x^{2} px + 8 l1 l1y px + 4 l1y^{2} px - 4 lb^{2} px + 4 lt^{2} px -
```

```
12 11x px^{2} + 4 px^{3} - 8 11 11x py - 8 11x 11y py + 8 11 px py + 8 11y px py -
                                                                           4 11x py^{2} + 4 px py^{2} + 4 11^{2} x + 12 11x^{2} x + 8 11 11y x + 4 11y^{2} x - 4 1b^{2} x +
                                                                           4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                                                                           12 px x^2 + 4 x^3 + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y -
                                                                           8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2 - -
                                                      4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y
                                                                           py + 4 py^2 - 8 11x x + 8 px x + 4 x^2 - 8 11 y - 8 11y y - 8 py y + 4 y^2
                                                            (11^4 + 2 11^2 11x^2 + 11x^4 + 4 11^3 11y + 4 11 11x^2 11y + 6 11^2 11y^2 + 2 11x^2 11y^2 + 4
                                                                           11\ 11y^3 + 11y^4 - 2\ 11^2\ 1b^2 - 2\ 11x^2\ 1b^2 - 4\ 11\ 11y\ 1b^2 - 2\ 11y^2\ 1b^2 + 1b^4 - 2\ 11^2\ 1t^2 + 2
                                                                           11x^{2} + 1t^{2} - 41111y + 1t^{2} - 211y^{2} + 1t^{2} - 21b^{2} + 1t^{4} - 411^{2} + 11x + px - 411x^{3} + px - 811
                                                                           11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 + 4
                                                                           11\ 11y\ px^2 + 2\ 11y^2\ px^2 - 2\ 1b^2\ px^2 + 2\ 1t^2\ px^2 - 4\ 11x\ px^3 + px^4 + 4\ 11^3\ py + 4\ 11\ 11x^2
                                                                           py + 12 11^2 11y py + 4 11x^2 11y py + 12 11 11y^2 py + 4 11y^3 py - 4 11 1b^2 py - 4 11y 1b^2
                                                                           py - 4111t^2 py - 411y 1t^2 py - 81111x px py - 811x 11y px py + 411 px^2 py + 411y
                                                                           px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 - 2 lb^2 py^2 - 2 lt^2 py^2 - 4 l1y^2 py^2 + 6 l1y^2 py
                                                                           11x px py^{2} + 2 px^{2} py^{2} + 4 11 py^{3} + 4 11y py^{3} + py^{4} - 4 11^{2} 11x x - 4 11x^{3} x - 8 11 11x
                                                                           11y \times -4 \times 11x \times 11y^2 \times +4 \times 11x \times 10^2 \times -4 \times 11x \times 11^2 \times 11x \times 11x
                                                                           x + 411y^2 px x - 4 lb<sup>2</sup> px x + 4 lt<sup>2</sup> px x - 12 l1x px<sup>2</sup> x + 4 px<sup>3</sup> x - 8 l1 l1x py x - 8 l1x
                                                                           11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x + 4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 +
                                                                      4 11 11y x^2 + 2 11y^2 x^2 - 2 1b^2 x^2 + 2 1t^2 x^2 - 12 11x px x^2 + 6 px^2 x^2 + 4 11 py x^2 + 4 11y
                                                                           py x^2 + 2 py^2 x^2 - 4 l1x x^3 + 4 px x^3 + x^4 - 4 l1^3 y - 4 l1 l1x^2 y - 12 l1^2 l1y y - 4 l1x^2
                                                                           11y y - 12 11 11y^2 y - 4 11y^3 y + 4 11 1b^2 y + 4 11y 1b^2 y + 4 11 1t^2 y + 4 11y 1t^2 y + 8
                                                                           11\ 11x\ px\ y + 8\ 11x\ 11y\ px\ y - 4\ 11\ px^2\ y - 4\ 11y\ px^2\ y - 12\ 11^2\ py\ y - 4\ 11x^2\ py\ y - 24
                                                                           11\ 11y\ py\ y - 12\ 11y^2\ py\ y + 4\ 1b^2\ py\ y + 4\ 1t^2\ py\ y + 8\ 11x\ px\ py\ y - 4\ px^2\ py\ y - 12\ 11
                                                                           py^2y - 12 lly py^2y - 4 py^3y + 8 ll llx xy + 8 llx lly xy - 8 ll px xy - 8 lly px xy +
                                                                      8 11x py x y - 8 px py x y - 4 11 x^2 y - 4 11y x^2 y - 4 py x^2 y + 6 11^2 y^2 + 2 11x^2 y^2 + 12
                                                                           11 11y y^2 + 6 11y^2 y^2 - 2 1b^2 y^2 - 2 1t^2 y^2 - 4 11x px y^2 + 2 px^2 y^2 + 12 11 py y^2 + 12 11y
                                                                           py y^2 + 6 py y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4)
                             (2 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                                4\;py^2-8\;l1x\;x+8\;px\;x+4\;x^2-8\;l1\;y-8\;l1y\;y-8\;py\;y+4\;y^2\,\big)\;\Big)\;\Big]\;\;+
         ArcCos \left[\frac{1}{4 \ln 1+} \left(-2 \ln^2 - 2 \ln^2 + px^2 + py^2 + \frac{1}{2}\right)\right]
                          pz² –
                           2
                                 рх
                                x + x^2 - 2
                               y + y^2 - 2
                                z + z^2)
theta3 \rightarrow ArcCos \left[ \left( -1t \left( -411^2 11x - 411x^3 - 81111x11y - 411x11y^2 + 411x1b^2 - 411x1y^2 + 411x1b^2 - 411x1
                                           4 l1x lt^2 + 4 l1^2 px + 12 l1x^2 px + 8 l1 l1y px + 4 l1y^2 px -
```

```
4 lb^2 px + 4 lt^2 px - 12 l1x px^2 + 4 px^3 - 8 l1 l1x py -
                                                   8 11x 11y py + 8 11 px py + 8 11y px py - 4 11x py^2 +
                                                   4 px py^{2} + 4 11^{2} x + 12 11x^{2} x + 8 11 11y x + 4 11y^{2} x -
                                                   4 lb^2 x + 4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x +
                                                   8 11y py x + 4 py^2 x - 12 11x x^2 + 12 px x^2 + 4 x^3 + 8 11 11x y +
                                                   8 11x 11y y - 8 11 px y - 8 11y px y + 8 11x py y - 8 px py y -
                                                   8 11 \times y - 8 11y \times y - 8 py \times y - 4 11x y^2 + 4 px y^2 + 4 x y^2 +
                                \sqrt{\left(1t^{2} \left(-4 \cdot 11^{2} \cdot 11x - 4 \cdot 11x^{3} - 8 \cdot 11 \cdot 11x \cdot 11y - 4 \cdot 11x \cdot 11y^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11^{2} \cdot px + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11^{2} \cdot px + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4 \cdot 11x \cdot 1t^{2} + 4 \cdot 11x \cdot 1b^{2} - 4
                                                                            12\ 11x^2\ px\ +\ 8\ 11\ 11y\ px\ +\ 4\ 11y^2\ px\ -\ 4\ 1b^2\ px\ +\ 4\ 1t^2\ px\ -\ 12\ 11x\ px^2\ +\ 4\ px^3\ -\ 8\ 11\ 11x
                                                                                  py - 8 11x 11y py + 8 11 px py + 8 11y px py - 4 11x py^2 + 4 px py^2 + 4 11^2 x + 12 11x^2 x +
                                                                            8 11 11y x + 4 11y^2 x - 4 1b^2 x + 4 1t^2 x - 24 11x px x + 12 px^2 x + 8 11 py x + 8 11y py
                                                                                  x + 4 py^{2} x - 12 11x x^{2} + 12 px x^{2} + 4 x^{3} + 8 11 11x y + 8 11x 11y y - 8 11 px y - 8 11y px
                                                                                  y + 8 11x py y - 8 px py y - 8 11 x y - 8 11y x y - 8 py x y - 4 11x y^2 + 4 px y^2 + 4 x y^2
                                                  4 lt^{2} (4 l1^{2} + 4 l1x^{2} + 8 l1 l1y + 4 l1y^{2} - 8 l1x px + 4 px^{2} + 8 l1 py + 8 l1y py +
                                                                     4 \text{ py}^2 - 8 \text{ l1x x} + 8 \text{ px x} + 4 \text{ x}^2 - 8 \text{ l1 y} - 8 \text{ l1y y} - 8 \text{ py y} + 4 \text{ y}^2
                                                            (11^4 + 2 11^2 11x^2 + 11x^4 + 4 11^3 11y + 4 11 11x^2 11y + 6 11^2 11y^2 + 2 11x^2 11y^2 +
                                                                      4\ 11\ 11v^3 + 11v^4 - 2\ 11^2\ 1b^2 - 2\ 11x^2\ 1b^2 - 4\ 11\ 11v\ 1b^2 - 2\ 11v^2\ 1b^2 + 1b^4 - 2\ 11^2\ 1t^2 +
                                                                       211x^{2}1t^{2} - 41111y1t^{2} - 211y^{2}1t^{2} - 21b^{2}1t^{2} + 1t^{4} - 411^{2}11xpx - 411x^{3}px -
                                                                       8 11 11x 11y px - 4 11x 11y^2 px + 4 11x 1b^2 px - 4 11x 1t^2 px + 2 11^2 px^2 + 6 11x^2 px^2 +
                                                                      4 11 11y px^{2} + 2 11y^{2} px^{2} - 2 1b^{2} px^{2} + 2 1t^{2} px^{2} - 4 11x px^{3} + px^{4} + 4 11^{3} py +
                                                                     4 11 11x^2 py + 12 11^2 11y py + 4 11x^2 11y py + 12 11 11y^2 py + 4 11y^3 py - 4 11 1b^2 py -
                                                                     4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 4 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text{ py} + 8 \ln 10^2 \text{ py} - 8 \ln 10^2 \text
                                                                     4 l1 px^2 py + 4 l1y px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 -
                                                                       2 lb^2 py^2 - 2 lt^2 py^2 - 4 l1x px py^2 + 2 px^2 py^2 + 4 l1 py^3 + 4 l1y py^3 + py^4 - 4 l1^2 l1x x -
                                                                     4 11x^{3} x - 8 11 11x 11y x - 4 11x 11y^{2} x + 4 11x 1b^{2} x - 4 11x 1t^{2} x + 4 11^{2} px x +
                                                                      12 11x^2 px x + 8 11 11y px x + 4 11y^2 px x - 4 1b^2 px x + 4 1t^2 px x - 12 11x px^2 x +
                                                                      4 px^3 x - 8 11 11x py x - 8 11x 11y py x + 8 11 px py x + 8 11y px py x - 4 11x py^2 x +
                                                                      4 px py^2 x + 2 11^2 x^2 + 6 11x^2 x^2 + 4 11 11y x^2 + 2 11y^2 x^2 - 2 1b^2 x^2 + 2 1t^2 x^2 -
                                                                      12 11x px x^2 + 6 px^2 x^2 + 4 11 py x^2 + 4 11y py x^2 + 2 py^2 x^2 - 4 11x x^3 + 4 px x^3 +
                                                                      x^4 - 411^3 y - 41111x^2 y - 1211^211y y - 411x^211y y - 121111y^2 y - 411y^3 y +
                                                                      4 11 1b^2 y + 4 11y 1b^2 y + 4 11 1t^2 y + 4 11y 1t^2 y + 8 11 11x px y + 8 11x 11y px y -
                                                                     4 \ln px^2 y - 4 \ln px^2 y - 12 \ln^2 py y - 4 \ln^2 py y - 24 \ln \ln py y - 12 \ln^2 py y +
                                                                     4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1 py^2 y - 12 l1y py^2 y -
                                                                     4 py^{3} y + 8 11 11x x y + 8 11x 11y x y - 8 11 px x y - 8 11y px x y + 8 11x py x y -
                                                                      8 px py x y -411 x<sup>2</sup> y -411 y x<sup>2</sup> y -4 py x<sup>2</sup> y +611^2 y<sup>2</sup> +211x<sup>2</sup> y<sup>2</sup> +121111y y<sup>2</sup> +
                                                                     6 \, lly^2 \, y^2 - 2 \, lb^2 \, y^2 - 2 \, lt^2 \, y^2 - 4 \, llx \, px \, y^2 + 2 \, px^2 \, y^2 + 12 \, ll \, py \, y^2 + 12 \, lly \, py \, y^2 + 12 \, ly \, py \, y^2 + 
                                                                     6 py^2 y^2 - 4 11x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 11 y^3 - 4 11y y^3 - 4 py y^3 + y^4) ) \Big)
                    \left(2 \text{ lt}^2 \left(4 \text{ l1}^2 + 4 \text{ l1} \text{x}^2 + 8 \text{ l1} \text{ l1} \text{y} + 4 \text{ l1} \text{y}^2 - 8 \text{ l1} \text{x} \text{ px} + 4 \text{ px}^2 + 8 \text{ l1} \text{ py} + 8 \text{ l1} \text{y} \text{py} + 4 \text{ l1} \text{y}^2 + 8 \text{ l1} \text{ l1} \text{y} + 4 \text{ l1} \text{y}^2 + 8 \text{ l1} \text{ l2} \text{y} + 4 \text{ l2} \text{y}^2 + 8 \text{ l2} \text{ l2} \text{y} + 8 \text{ l2} \text{l2} \text{y}
                                             4 py^2 - 8 11x x + 8 px x + 4 x^2 -
                                            8 11 y - 8 11 y y - 8 py y + 4 y^2)
theta2 \rightarrow ArcCos \left[\frac{1}{4 \ln 1 +} \left(-2 \ln^2 - \frac{1}{4 \ln 1 +} \right)\right]
```

$$2 \\ 1t^{2} + \\ px^{2} + py^{2} + \\ pz^{2} - \\ 2 \\ px \\ x + x^{2} - 2 \\ py \\ y + y^{2} - 2 \\ pz \\ z + z^{2}) \, \Big] \Big\} \Big\}$$