Solving the inverse kinematics

In[1]:=
```
l1soln =
 FullSimplify[Solve[{Sqrt[(y - Py - l1 - l1y)^2 + (x + Px - l1x)^2] == z - Pz}, l1][[2]]]
```

Out[]=
$$\left\{l1 \to -l1y - Py + y + \sqrt{-((l1x - Px + Pz - x - z)(l1x - Px - Pz - x + z))}\right\}$$

In[2]:=
```
l2soln =
 FullSimplify[Solve[{Sqrt[(L - (y + Py) - l2 - l2y)^2 + (x - Px + l2x)^2] == z - Pz}, l2][[2]]]
```

Out[]=
$$\left\{l2 \to L - l2y - Py - y + \sqrt{-((l2x - Px + Pz + x - z)(l2x - Px - Pz + x + z))}\right\}$$

In[3]:=
```
l3soln =
 FullSimplify[Solve[{Sqrt[(L - (y + Py) - l3 - l3y)^2 + (x + Px - l3x)^2] == z - Pz}, l3][[2]]]
```

Out[]=
$$\left\{l3 \to L - l3y - Py - y + \sqrt{-((l3x - Px + Pz - x - z)(l3x - Px - Pz - x + z))}\right\}$$

In[]:=
```
l3 /. l3soln
```

Out[]=
$$L - l3y - Py - y + \sqrt{-((l3x - Px + Pz - x - z)(l3x - Px - Pz - x + z))}$$

Deriving expressions for forward kinematics. Note that for simplifying the expressions, we subbed in arbitrary values for some of the padding constants.

In[5]:=
```
fkin = Solve[{Sqrt[(y - Py - l1 - l1y)^2 + (x + Px - l1x)^2] == z - Pz,
    Sqrt[(L - (y + Py) - l2 - l2y)^2 + (x - Px + l2x)^2] == z - Pz,
    Sqrt[(L - (y + Py) - l3 - l3y)^2 + (x + Px - l3x)^2] == z - Pz
   }, {x, y, z}];

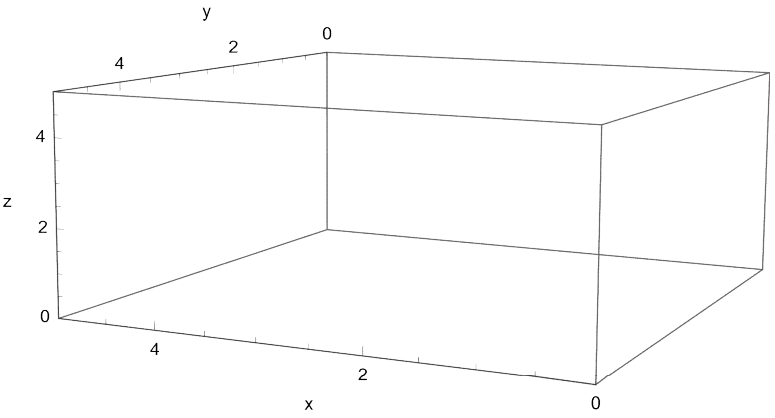
fkinsub = fkin[[1]] /. {Px → 0.5, Py → 0.34, Pz → 0.78, l1x → 0.12,
    l1y → 0.13, l2x → 0.1, l2y → 0.16, l3x → 0.3, l3y → 0.2, L → 5};
```

Initial attempt to plot the workspace

In[22]:=
```
data = Flatten[Table[{x, y, z} /. fkinsub,
    {l1, 0, 10, 0.05}, {l2, 0, 10, 0.05}, {l3, 0, 10, 0.05}], 1];
```

In[21]:=
```
ListPointPlot3D[data, PlotRange → {{0, 5}, {0, 5}, {0, 5}}, AxesLabel → {"x", "y", "z"}]
```

Out[]=



Finding Jacobians related to forward and inverse kinematics

In[ ]:=
```
Jfkin = D[{x, y, z} /. fkinsub , {{l1, l2, l3}}];
Jfkin // MatrixForm
```

Out[ ]//MatrixForm=

$$\left(\begin{array}{c} -\frac{19.46\,(0.564+1.2\,l1)}{-11.3868+0.6\,l1-0.18\,l2+0.78\,l3}+\frac{(0.564+1.2\,l1)\,l3}{-11.3868+0.6\,l1-0.18\,l2+0.78\,l3}+\frac{11.676\,(-226.718+0.564\,l1+0.6\,l1^2-7.02\,l2+0.18\,l2^2+30.3576\,l3-0.78\,l3^2)}{(-11.3868+0.6\,l1-0.18\,l2+0.78\,l3)^2}+\frac{(0.564+1.2\,l1)\,(-226.718+0.564\,l1+0.6\,l1^2-7.02\,l}{2\,(-11.3868+0.6\,l1-0.18\,l2} \end{array}\right)$$

In[ ]:=
```
Jikin = FullSimplify[D[{l1 /. l1soln, l2 /. l2soln, l3 /. l3soln} , {{x, y, z}}] ]
```

Out[ ]=

$$\left\{\left\{\frac{l1x-Px-x}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}},\ 1,\ \frac{-Pz+z}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}}\right\},\right.$$
$$\left\{-\frac{l2x-Px+x}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}},\ -1,\ \frac{-Pz+z}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}\right\},$$
$$\left.\left\{\frac{l3x-Px-x}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}},\ -1,\ \frac{-Pz+z}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}}\right\}\right\}$$

In[ ]:=
```
Jikin // MatrixForm
```

Out[ ]//MatrixForm=

$$\left(\begin{array}{ccc} \frac{l1x-Px-x}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}} & 1 & \frac{-Pz+z}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}} \\ -\frac{l2x-Px+x}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}} & -1 & \frac{-Pz+z}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}} \\ \frac{l3x-Px-x}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}} & -1 & \frac{-Pz+z}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}} \end{array}\right)$$

In[ ]:=
```
Inverse[Jikin] /. {Px → 0.1, Py → 0.1, Pz → 0.1,
   l1x → 0.1, l1y → 0.1, l2x → 0.1, l2y → 0.1, l3x → 0.1, l3y → 0.2 };
```

In[ ]:=
```
Inverse[Jikin] // MatrixForm
```

Out[ ]//MatrixForm=

$$
\left(\begin{array}{l}
\dfrac{-\frac{-Pz+z}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}}+\frac{-Pz+z}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-\cdots}}\cdots}{\frac{(l1x-Px-x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(l2x-Px+x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(-Pz+z)\left(-\frac{l1x\cdots}{\sqrt{-((l1x-Px+Pz-x\cdots}}\right.}{\sqrt{-((l\cdots}}\cdots} \\
\dfrac{\frac{(l3x-Px-x)\,(-Pz+z)}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{\cdots}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-P\cdots}}\cdots}{\frac{(l1x-Px-x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(l2x-Px+x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(-Pz+z)\left(-\frac{l1x\cdots}{\sqrt{-((l1x-Px+Pz-x\cdots}}\right.}{\sqrt{-((l\cdots}}\cdots} \\
\dfrac{\frac{l3x-Px-x}{\sqrt{-((l3x-Px+Pz-x-z)\,(l3x-Px-Pz-x+z))}}+\frac{l2x-Px+x}{\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-P\cdots}}\cdots}{\frac{(l1x-Px-x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(l2x-Px+x)\,(-Pz+z)}{\sqrt{-((l1x-Px+Pz-x-z)\,(l1x-Px-Pz-x+z))}\,\sqrt{-((l2x-Px+Pz+x-z)\,(l2x-Px-Pz+x+z))}}+\frac{(-Pz+z)\left(-\frac{l1x\cdots}{\sqrt{-((l1x-Px+Pz-x\cdots}}\right.}{\sqrt{-((l\cdots}}\cdots}
\end{array}\right.
$$

Analyzing the internal angles of the linkages. This system should define the internal angles that the linkages should be given an x,y,z, l_1, l_2, l_3.

```
Solve[{theta2 == theta1 + theta3, l1x + lb * Cos[theta1] == x + px + lt * Cos[theta3],
  l1 + l1y + lb * Sin[theta1] == y - py - lt * Sin[theta3],
  (x - px) ^2 + (y - py) ^2 + (z - pz) ^2 ==
   (Sqrt[2] * lb) ^2 + (Sqrt[2] * lt) ^2 + 2 * Sqrt[2] ^2 * lb * lt * Cos[theta2]
 }, {theta1, theta3, theta2}]
```

··· Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is ArcCos[Cos[theta1]] – ArcCos[Cos[theta2]] + ArcCos[Cos[theta3]] == 0.

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[28]=

$$
\begin{aligned}
&\{\{theta1 \to \\
&-\text{ArcCos}\Big[\Big(-lt\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 + 4\,l1^2\,px + \\
&\quad 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - \\
&\quad 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + \\
&\quad 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + \\
&\quad 4\,py^2\,x - 12\,l1x\,x^2 + 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + \\
&\quad 8\,l1x\,py\,y - 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big) - \\
&\sqrt{\Big(lt^2\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 +} \\
&\qquad 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&\qquad 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&\qquad 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&\qquad 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&\qquad 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&\qquad 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big)^2 - \\
&\quad 4\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y \\
&\qquad py + 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big) \\
&\quad \big(l1^4 + 2\,l1^2\,l1x^2 + l1x^4 + 4\,l1^3\,l1y + 4\,l1\,l1x^2\,l1y + 6\,l1^2\,l1y^2 + 2\,l1x^2\,l1y^2 + 4 \\
&\qquad l1\,l1y^3 + l1y^4 - 2\,l1^2\,lb^2 - 2\,l1x^2\,lb^2 - 4\,l1\,l1y\,lb^2 - 2\,l1y^2\,lb^2 + lb^4 - 2\,l1^2\,lt^2 + 2
\end{aligned}
$$

$$
\begin{aligned}
&\text{l1x}^2\,\text{lt}^2 - 4\,\text{l1}\,\text{l1y}\,\text{lt}^2 - 2\,\text{l1y}^2\,\text{lt}^2 - 2\,\text{lb}^2\,\text{lt}^2 + \text{lt}^4 - 4\,\text{l1}^2\,\text{l1x}\,\text{px} - 4\,\text{l1x}^3\,\text{px} - 8\,\text{l1}\,\text{l1x}\,\text{l1y}\,\text{px} - 4\,\text{l1x}\,\text{l1y}^2\,\text{px} + 4\,\text{l1x}\,\text{lb}^2\,\text{px} - 4\,\text{l1x}\,\text{lt}^2\,\text{px} + 2\,\text{l1}^2\,\text{px}^2 + 6\,\text{l1x}^2\,\text{px}^2 + 4\,\text{l1}\,\text{l1y}\,\text{px}^2 + 2\,\text{l1y}^2\,\text{px}^2 - 2\,\text{lb}^2\,\text{px}^2 + 2\,\text{lt}^2\,\text{px}^2 - 4\,\text{l1x}\,\text{px}^3 + \text{px}^4 + 4\,\text{l1}^3\,\text{py} + 4\,\text{l1}\,\text{l1x}^2\,\text{py} + 12\,\text{l1}^2\,\text{l1y}\,\text{py} + 4\,\text{l1x}^2\,\text{l1y}\,\text{py} + 12\,\text{l1}\,\text{l1y}^2\,\text{py} + 4\,\text{l1y}^3\,\text{py} - 4\,\text{l1}\,\text{lb}^2\,\text{py} - 4\,\text{l1y}\,\text{lb}^2\,\text{py} - 4\,\text{l1}\,\text{lt}^2\,\text{py} - 4\,\text{l1y}\,\text{lt}^2\,\text{py} - 8\,\text{l1}\,\text{l1x}\,\text{px}\,\text{py} - 8\,\text{l1x}\,\text{l1y}\,\text{px}\,\text{py} + 4\,\text{l1}\,\text{px}^2\,\text{py} + 4\,\text{l1y}\,\text{px}^2\,\text{py} + 6\,\text{l1}^2\,\text{py}^2 + 2\,\text{l1x}^2\,\text{py}^2 + 12\,\text{l1}\,\text{l1y}\,\text{py}^2 + 6\,\text{l1y}^2\,\text{py}^2 - 2\,\text{lb}^2\,\text{py}^2 - 2\,\text{lt}^2\,\text{py}^2 - 4\,\text{l1x}\,\text{px}\,\text{py}^2 + 2\,\text{px}^2\,\text{py}^2 + 4\,\text{l1}\,\text{py}^3 + 4\,\text{l1y}\,\text{py}^3 + \text{py}^4 - 4\,\text{l1}^2\,\text{l1x}\,\text{x} - 4\,\text{l1x}^3\,\text{x} - 8\,\text{l1}\,\text{l1x}\,\text{l1y}\,\text{x} - 4\,\text{l1x}\,\text{l1y}^2\,\text{x} + 4\,\text{l1x}\,\text{lb}^2\,\text{x} - 4\,\text{l1x}\,\text{lt}^2\,\text{x} + 4\,\text{l1}^2\,\text{px}\,\text{x} + 12\,\text{l1x}^2\,\text{px}\,\text{x} + 8\,\text{l1}\,\text{l1y}\,\text{px}\,\text{x} + 4\,\text{l1y}^2\,\text{px}\,\text{x} - 4\,\text{lb}^2\,\text{px}\,\text{x} + 4\,\text{lt}^2\,\text{px}\,\text{x} - 12\,\text{l1x}\,\text{px}^2\,\text{x} + 4\,\text{px}^3\,\text{x} - 8\,\text{l1}\,\text{l1x}\,\text{py}\,\text{x} - 8\,\text{l1x}\,\text{l1y}\,\text{py}\,\text{x} + 8\,\text{l1}\,\text{px}\,\text{py}\,\text{x} + 8\,\text{l1y}\,\text{px}\,\text{py}\,\text{x} - 4\,\text{l1x}\,\text{py}^2\,\text{x} + 4\,\text{px}\,\text{py}^2\,\text{x} + 2\,\text{l1}^2\,\text{x}^2 + 6\,\text{l1x}^2\,\text{x}^2 + 4\,\text{l1}\,\text{l1y}\,\text{x}^2 + 2\,\text{l1y}^2\,\text{x}^2 - 2\,\text{lb}^2\,\text{x}^2 + 2\,\text{lt}^2\,\text{x}^2 - 12\,\text{l1x}\,\text{px}\,\text{x}^2 + 6\,\text{px}^2\,\text{x}^2 + 4\,\text{l1}\,\text{py}\,\text{x}^2 + 4\,\text{l1y}\,\text{py}\,\text{x}^2 + 2\,\text{py}^2\,\text{x}^2 - 4\,\text{l1x}\,\text{x}^3 + 4\,\text{px}\,\text{x}^3 + \text{x}^4 - 4\,\text{l1}^3\,\text{y} - 4\,\text{l1}\,\text{l1x}^2\,\text{y} - 12\,\text{l1}^2\,\text{l1y}\,\text{y} - 4\,\text{l1x}^2\,\text{l1y}\,\text{y} - 12\,\text{l1}\,\text{l1y}^2\,\text{y} - 4\,\text{l1y}^3\,\text{y} + 4\,\text{l1}\,\text{lb}^2\,\text{y} + 4\,\text{l1y}\,\text{lb}^2\,\text{y} + 4\,\text{l1}\,\text{lt}^2\,\text{y} + 4\,\text{l1y}\,\text{lt}^2\,\text{y} + 8\,\text{l1}\,\text{l1x}\,\text{px}\,\text{y} + 8\,\text{l1x}\,\text{l1y}\,\text{px}\,\text{y} - 4\,\text{l1}\,\text{px}^2\,\text{y} - 4\,\text{l1y}\,\text{px}^2\,\text{y} - 12\,\text{l1}^2\,\text{py}\,\text{y} - 4\,\text{l1x}^2\,\text{py}\,\text{y} - 24\,\text{l1}\,\text{l1y}\,\text{py}\,\text{y} - 12\,\text{l1y}^2\,\text{py}\,\text{y} + 4\,\text{lb}^2\,\text{py}\,\text{y} + 4\,\text{lt}^2\,\text{py}\,\text{y} + 8\,\text{l1x}\,\text{px}\,\text{py}\,\text{y} - 4\,\text{px}^2\,\text{py}\,\text{y} - 12\,\text{l1}\,\text{py}^2\,\text{y} - 12\,\text{l1y}\,\text{py}^2\,\text{y} - 4\,\text{py}^3\,\text{y} + 8\,\text{l1}\,\text{l1x}\,\text{x}\,\text{y} + 8\,\text{l1x}\,\text{l1y}\,\text{x}\,\text{y} - 8\,\text{l1}\,\text{px}\,\text{x}\,\text{y} - 8\,\text{l1y}\,\text{px}\,\text{x}\,\text{y} + 8\,\text{l1x}\,\text{py}\,\text{x}\,\text{y} - 8\,\text{px}\,\text{py}\,\text{x}\,\text{y} - 4\,\text{l1}\,\text{x}^2\,\text{y} - 4\,\text{l1y}\,\text{x}^2\,\text{y} - 4\,\text{py}\,\text{x}^2\,\text{y} + 6\,\text{l1}^2\,\text{y}^2 + 2\,\text{l1x}^2\,\text{y}^2 + 12\,\text{l1}\,\text{l1y}\,\text{y}^2 + 6\,\text{l1y}^2\,\text{y}^2 - 2\,\text{lb}^2\,\text{y}^2 - 2\,\text{lt}^2\,\text{y}^2 - 4\,\text{l1x}\,\text{px}\,\text{y}^2 + 2\,\text{px}^2\,\text{y}^2 + 12\,\text{l1}\,\text{py}\,\text{y}^2 + 12\,\text{l1y}\,\text{py}\,\text{y}^2 + 6\,\text{py}^2\,\text{y}^2 - 4\,\text{l1x}\,\text{x}\,\text{y}^2 + 4\,\text{px}\,\text{x}\,\text{y}^2 + 2\,\text{x}^2\,\text{y}^2 - 4\,\text{l1}\,\text{y}^3 - 4\,\text{l1y}\,\text{y}^3 - 4\,\text{py}\,\text{y}^3 + \text{y}^4\big)\big)\big) \Big/ \\
&\big(2\,\text{lt}^2\,\big(4\,\text{l1}^2 + 4\,\text{l1x}^2 + 8\,\text{l1}\,\text{l1y} + 4\,\text{l1y}^2 - 8\,\text{l1x}\,\text{px} + 4\,\text{px}^2 + 8\,\text{l1}\,\text{py} + 8\,\text{l1y}\,\text{py} + 4\,\text{py}^2 - 8\,\text{l1x}\,\text{x} + 8\,\text{px}\,\text{x} + 4\,\text{x}^2 - 8\,\text{l1}\,\text{y} - 8\,\text{l1y}\,\text{y} - 8\,\text{py}\,\text{y} + 4\,\text{y}^2\big)\big)\Big] + \\
&\text{ArcCos}\Big[\frac{1}{4\,\text{lb}\,\text{lt}}\big(-2\,\text{lb}^2 - 2\,\text{lt}^2 + \text{px}^2 + \text{py}^2 + \text{pz}^2 - 2\,\text{px}\,\text{x} + \text{x}^2 - 2\,\text{py}\,\text{y} + \text{y}^2 - 2\,\text{pz}\,\text{z} + \text{z}^2\big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
\text{theta3} \to -\text{ArcCos}\Big[\Big(-\text{lt}\,\big(&-4\,\text{l1}^2\,\text{l1x} - 4\,\text{l1x}^3 - 8\,\text{l1}\,\text{l1x}\,\text{l1y} - 4\,\text{l1x}\,\text{l1y}^2 + 4\,\text{l1x}\,\text{lb}^2 - 4\,\text{l1x}\,\text{lt}^2 + 4\,\text{l1}^2\,\text{px} + 12\,\text{l1x}^2\,\text{px} + 8\,\text{l1}\,\text{l1y}\,\text{px} + 4\,\text{l1y}^2\,\text{px} - 4\,\text{lb}^2\,\text{px} + 4\,\text{lt}^2\,\text{px} - 12\,\text{l1x}\,\text{px}^2 + 4\,\text{px}^3 - 8\,\text{l1}\,\text{l1x}\,\text{py} - 8\,\text{l1x}\,\text{l1y}\,\text{py} + 8\,\text{l1}\,\text{px}\,\text{py} + 8\,\text{l1y}\,\text{px}\,\text{py} - 4\,\text{l1x}\,\text{py}^2 + 4\,\text{px}\,\text{py}^2 + 4\,\text{l1}^2\,\text{x} + 12\,\text{l1x}^2\,\text{x} + 8\,\text{l1}\,\text{l1y}\,\text{x} + 4\,\text{l1y}^2\,\text{x} - 4\,\text{lb}^2\,\text{x} + 4\,\text{lt}^2\,\text{x} - 24\,\text{l1x}\,\text{px}\,\text{x} + 12\,\text{px}^2\,\text{x} + 8\,\text{l1}\,\text{py}\,\text{x} + 8\,\text{l1y}\,\text{py}\,\text{x} + 4\,\text{py}^2\,\text{x} - 12\,\text{l1x}\,\text{x}^2 + 12\,\text{px}\,\text{x}^2 + 4\,\text{x}^3 + 8\,\text{l1}\,\text{l1x}\,\text{y} + 8\,\text{l1x}\,\text{l1y}\,\text{y} - 8\,\text{l1}\,\text{px}\,\text{y} - 8\,\text{l1y}\,\text{px}\,\text{y} + 8\,\text{l1x}\,\text{py}\,\text{y} - 8\,\text{px}\,\text{py}\,\text{y} - 8\,\text{l1}\,\text{x}\,\text{y} - 8\,\text{l1y}\,\text{x}\,\text{y} - 8\,\text{py}\,\text{x}\,\text{y} - 4\,\text{l1x}\,\text{y}^2 + 4\,\text{px}\,\text{y}^2 + 4\,\text{x}\,\text{y}^2\big) - \\
&\sqrt{\Big(\text{lt}^2\,\big(-4\,\text{l1}^2\,\text{l1x} - 4\,\text{l1x}^3 - 8\,\text{l1}\,\text{l1x}\,\text{l1y} - 4\,\text{l1x}\,\text{l1y}^2 + 4\,\text{l1x}\,\text{lb}^2 - 4\,\text{l1x}\,\text{lt}^2 + 4\,\text{l1}^2\,\text{px} + 12\,\text{l1x}^2\,\text{px} + 8\,\text{l1}\,\text{l1y}\,\text{px} + 4\,\text{l1y}^2\,\text{px} - 4\,\text{lb}^2\,\text{px} + 4\,\text{lt}^2\,\text{px} - 12\,\text{l1x}\,\text{px}^2 + 4\,\text{px}^3 - 8\,\text{l1}\,\text{l1x}\,\text{py} - 8\,\text{l1x}\,\text{l1y}\,\text{py} + 8\,\text{l1}\,\text{px}\,\text{py} + 8\,\text{l1y}\,\text{px}\,\text{py} - 4\,\text{l1x}\,\text{py}^2 + 4\,\text{px}\,\text{py}^2 + 4\,\text{l1}^2\,\text{x} + 12\,\text{l1x}^2\,\text{x} + 8\,\text{l1}\,\text{l1y}\,\text{x} + 4\,\text{l1y}^2\,\text{x} - 4\,\text{lb}^2\,\text{x} +
\end{aligned}
$$

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\begin{aligned}
&4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big)^2 - \\
&4\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big) \\
&\big(l1^4 + 2\,l1^2\,l1x^2 + l1x^4 + 4\,l1^3\,l1y + 4\,l1\,l1x^2\,l1y + 6\,l1^2\,l1y^2 + 2\,l1x^2\,l1y^2 + \\
&4\,l1\,l1y^3 + l1y^4 - 2\,l1^2\,lb^2 - 2\,l1x^2\,lb^2 - 4\,l1\,l1y\,lb^2 - 2\,l1y^2\,lb^2 + lb^4 - 2\,l1^2\,lt^2 + \\
&2\,l1x^2\,lt^2 - 4\,l1\,l1y\,lt^2 - 2\,l1y^2\,lt^2 - 2\,lb^2\,lt^2 + lt^4 - 4\,l1^2\,l1x\,px - 4\,l1x^3\,px - \\
&8\,l1\,l1x\,l1y\,px - 4\,l1x\,l1y^2\,px + 4\,l1x\,lb^2\,px - 4\,l1x\,lt^2\,px + 2\,l1^2\,px^2 + 6\,l1x^2\,px^2 + \\
&4\,l1\,l1y\,px^2 + 2\,l1y^2\,px^2 - 2\,lb^2\,px^2 + 2\,lt^2\,px^2 - 4\,l1x\,px^3 + px^4 + 4\,l1^3\,py + \\
&4\,l1\,l1x^2\,py + 12\,l1^2\,l1y\,py + 4\,l1x^2\,l1y\,py + 12\,l1\,l1y^2\,py + 4\,l1y^3\,py - 4\,l1\,lb^2\,py - \\
&4\,l1y\,lb^2\,py - 4\,l1\,lt^2\,py - 4\,l1y\,lt^2\,py - 8\,l1\,l1x\,px\,py - 8\,l1x\,l1y\,px\,py + \\
&4\,l1\,px^2\,py + 4\,l1y\,px^2\,py + 6\,l1^2\,py^2 + 2\,l1x^2\,py^2 + 12\,l1\,l1y\,py^2 + 6\,l1y^2\,py^2 - \\
&2\,lb^2\,py^2 - 2\,lt^2\,py^2 - 4\,l1x\,px\,py^2 + 2\,px^2\,py^2 + 4\,l1\,py^3 + 4\,l1y\,py^3 + py^4 - 4\,l1^2\,l1x\,x - \\
&4\,l1x^3\,x - 8\,l1\,l1x\,l1y\,x - 4\,l1x\,l1y^2\,x + 4\,l1x\,lb^2\,x - 4\,l1x\,lt^2\,x + 4\,l1^2\,px\,x + \\
&12\,l1x^2\,px\,x + 8\,l1\,l1y\,px\,x + 4\,l1y^2\,px\,x - 4\,lb^2\,px\,x + 4\,lt^2\,px\,x - 12\,l1x\,px^2\,x + \\
&4\,px^3\,x - 8\,l1\,l1x\,py\,x - 8\,l1x\,l1y\,py\,x + 8\,l1\,px\,py\,x + 8\,l1y\,px\,py\,x - 4\,l1x\,py^2\,x + \\
&4\,px\,py^2\,x + 2\,l1^2\,x^2 + 6\,l1x^2\,x^2 + 4\,l1\,l1y\,x^2 + 2\,l1y^2\,x^2 - 2\,lb^2\,x^2 + 2\,lt^2\,x^2 - \\
&12\,l1x\,px\,x^2 + 6\,px^2\,x^2 + 4\,l1\,py\,x^2 + 4\,l1y\,py\,x^2 + 2\,py^2\,x^2 - 4\,l1x\,x^3 + 4\,px\,x^3 + \\
&x^4 - 4\,l1^3\,y - 4\,l1\,l1x^2\,y - 12\,l1^2\,l1y\,y - 4\,l1x^2\,l1y\,y - 12\,l1\,l1y^2\,y - 4\,l1y^3\,y + \\
&4\,l1\,lb^2\,y + 4\,l1y\,lb^2\,y + 4\,l1\,lt^2\,y + 4\,l1y\,lt^2\,y + 8\,l1\,l1x\,px\,y + 8\,l1x\,l1y\,px\,y - \\
&4\,l1\,px^2\,y - 4\,l1y\,px^2\,y - 12\,l1^2\,py\,y - 4\,l1x^2\,py\,y - 24\,l1\,l1y\,py\,y - 12\,l1y^2\,py\,y + \\
&4\,lb^2\,py\,y + 4\,lt^2\,py\,y + 8\,l1x\,px\,py\,y - 4\,px^2\,py\,y - 12\,l1\,py^2\,y - 12\,l1y\,py^2\,y - \\
&4\,py^3\,y + 8\,l1\,l1x\,x\,y + 8\,l1x\,l1y\,x\,y - 8\,l1\,px\,x\,y - 8\,l1y\,px\,x\,y + 8\,l1x\,py\,x\,y - \\
&8\,px\,py\,x\,y - 4\,l1\,x^2\,y - 4\,l1y\,x^2\,y - 4\,py\,x^2\,y + 6\,l1^2\,y^2 + 2\,l1x^2\,y^2 + 12\,l1\,l1y\,y^2 + \\
&6\,l1y^2\,y^2 - 2\,lb^2\,y^2 - 2\,lt^2\,y^2 - 4\,l1x\,px\,y^2 + 2\,px^2\,y^2 + 12\,l1\,py\,y^2 + 12\,l1y\,py\,y^2 + \\
&6\,py^2\,y^2 - 4\,l1x\,x\,y^2 + 4\,px\,x\,y^2 + 2\,x^2\,y^2 - 4\,l1\,y^3 - 4\,l1y\,y^3 - 4\,py\,y^3 + y^4\big)\big)\big) \Big/ \\
&\big(2\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big)\big)\Big],
\end{aligned}
$$

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\begin{aligned}
\text{theta2} \to -\text{ArcCos}\Big[\frac{1}{4\,lb\,lt}\big(&-2\,lb^2 - 2\,lt^2 + \\
&px^2 + \\
&py^2 + \\
&pz^2 - \\
&2 \\
&\quad px \\
&\quad x + x^2 - 2 \\
&\quad py \\
&\quad y + y^2 - 2 \\
&\quad pz \\
&\quad z + z^2\big)\Big]\Big\},
\end{aligned}
$$

$$
\Big\{\text{theta1} \to -\text{ArcCos}\Big[\Big(-lt\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 -
$$

$$
\begin{aligned}
&4\,l1x\,lt^2 + 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big) - \\
&\sqrt{\Big(lt^2\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 + \\
&\quad 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&\quad 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&\quad 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&\quad 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&\quad 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&\quad 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big)^2 - \\
&4\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big) \\
&\quad \big(l1^4 + 2\,l1^2\,l1x^2 + l1x^4 + 4\,l1^3\,l1y + 4\,l1\,l1x^2\,l1y + 6\,l1^2\,l1y^2 + 2\,l1x^2\,l1y^2 + 4\,l1\,l1y^3 + l1y^4 - 2\,l1^2\,lb^2 - 2\,l1x^2\,lb^2 - 4\,l1\,l1y\,lb^2 - 2\,l1y^2\,lb^2 + lb^4 - 2\,l1^2\,lt^2 + 2\,l1x^2\,lt^2 - 4\,l1\,l1y\,lt^2 - 2\,l1y^2\,lt^2 - 2\,lb^2\,lt^2 + lt^4 - 4\,l1^2\,l1x\,px - 4\,l1x^3\,px - 8\,l1\,l1x\,l1y\,px - 4\,l1x\,l1y^2\,px + 4\,l1x\,lb^2\,px - 4\,l1x\,lt^2\,px + 2\,l1^2\,px^2 + 6\,l1x^2\,px^2 + 4\,l1\,l1y\,px^2 + 2\,l1y^2\,px^2 - 2\,lb^2\,px^2 + 2\,lt^2\,px^2 - 4\,l1x\,px^3 + px^4 + 4\,l1^3\,py + 4\,l1\,l1x^2\,py + 12\,l1^2\,l1y\,py + 4\,l1x^2\,l1y\,py + 12\,l1\,l1y^2\,py + 4\,l1y^3\,py - 4\,l1\,lb^2\,py - 4\,l1y\,lb^2\,py - 4\,l1\,lt^2\,py - 4\,l1y\,lt^2\,py - 8\,l1\,l1x\,px\,py - 8\,l1x\,l1y\,px\,py + 4\,l1\,px^2\,py + 4\,l1y\,px^2\,py + 6\,l1^2\,py^2 + 2\,l1x^2\,py^2 + 12\,l1\,l1y\,py^2 + 6\,l1y^2\,py^2 - 2\,lb^2\,py^2 - 2\,lt^2\,py^2 - 4\,l1x\,px\,py^2 + 2\,px^2\,py^2 + 4\,l1\,py^3 + 4\,l1y\,py^3 + py^4 - 4\,l1^2\,l1x\,x - 4\,l1x^3\,x - 8\,l1\,l1x\,l1y\,x - 4\,l1x\,l1y^2\,x + 4\,l1x\,lb^2\,x - 4\,l1x\,lt^2\,x + 4\,l1^2\,px\,x + 12\,l1x^2\,px\,x + 8\,l1\,l1y\,px\,x + 4\,l1y^2\,px\,x - 4\,lb^2\,px\,x + 4\,lt^2\,px\,x - 12\,l1x\,px^2\,x + 4\,px^3\,x - 8\,l1\,l1x\,py\,x - 8\,l1x\,l1y\,py\,x + 8\,l1\,px\,py\,x + 8\,l1y\,px\,py\,x - 4\,l1x\,py^2\,x + 4\,px\,py^2\,x + 2\,l1^2\,x^2 + 6\,l1x^2\,x^2 + 4\,l1\,l1y\,x^2 + 2\,l1y^2\,x^2 - 2\,lb^2\,x^2 + 2\,lt^2\,x^2 - 12\,l1x\,px\,x^2 + 6\,px^2\,x^2 + 4\,l1\,py\,x^2 + 4\,l1y\,py\,x^2 + 2\,py^2\,x^2 - 4\,l1x\,x^3 + 4\,px\,x^3 + x^4 - 4\,l1^3\,y - 4\,l1\,l1x^2\,y - 12\,l1^2\,l1y\,y - 4\,l1x^2\,l1y\,y - 12\,l1\,l1y^2\,y - 4\,l1y^3\,y + 4\,l1\,lb^2\,y + 4\,l1y\,lb^2\,y + 4\,l1\,lt^2\,y + 4\,l1y\,lt^2\,y + 8\,l1\,l1x\,px\,y + 8\,l1x\,l1y\,px\,y - 4\,l1\,px^2\,y - 4\,l1y\,px^2\,y - 12\,l1^2\,py\,y - 4\,l1x^2\,py\,y - 24\,l1\,l1y\,py\,y - 12\,l1y^2\,py\,y + 4\,lb^2\,py\,y + 4\,lt^2\,py\,y + 8\,l1x\,px\,py\,y - 4\,px^2\,py\,y - 12\,l1\,py^2\,y - 12\,l1y\,py^2\,y - 4\,py^3\,y + 8\,l1\,l1x\,x\,y + 8\,l1x\,l1y\,x\,y - 8\,l1\,px\,x\,y - 8\,l1y\,px\,x\,y + 8\,l1x\,py\,x\,y - 8\,px\,py\,x\,y - 4\,l1\,x^2\,y - 4\,l1y\,x^2\,y - 4\,py\,x^2\,y + 6\,l1^2\,y^2 + 2\,l1x^2\,y^2 + 12\,l1\,l1y\,y^2 + 6\,l1y^2\,y^2 - 2\,lb^2\,y^2 - 2\,lt^2\,y^2 - 4\,l1x\,px\,y^2 + 2\,px^2\,y^2 + 12\,l1\,py\,y^2 + 12\,l1y\,py\,y^2 + 6\,py^2\,y^2 - 4\,l1x\,x\,y^2 + 4\,px\,x\,y^2 + 2\,x^2\,y^2 - 4\,l1\,y^3 - 4\,l1y\,y^3 - 4\,py\,y^3 + y^4\big)\Big)\Big) \Big/ \\
&\big(2\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big)\big)\Big] + \\
&\text{ArcCos}\Big[\frac{1}{4\,lb\,lt}\big(-2\,lb^2 - 2\,lt^2 + px^2 + py^2 + \\
&\quad pz^2 - \\
&\quad 2
\end{aligned}
$$

```
      px
      x + x^2 - 2
      py
      y + y^2 - 2
      pz
      z + z^2) ],
theta3 → ArcCos[ (-lt (-4 l1^2 l1x - 4 l1x^3 - 8 l1 l1x l1y - 4 l1x l1y^2 + 4 l1x lb^2 -
        4 l1x lt^2 + 4 l1^2 px + 12 l1x^2 px + 8 l1 l1y px + 4 l1y^2 px -
        4 lb^2 px + 4 lt^2 px - 12 l1x px^2 + 4 px^3 - 8 l1 l1x py -
        8 l1x l1y py + 8 l1 px py + 8 l1y px py - 4 l1x py^2 +
        4 px py^2 + 4 l1^2 x + 12 l1x^2 x + 8 l1 l1y x + 4 l1y^2 x -
        4 lb^2 x + 4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x +
        8 l1y py x + 4 py^2 x - 12 l1x x^2 + 12 px x^2 + 4 x^3 + 8 l1 l1x y +
        8 l1x l1y y - 8 l1 px y - 8 l1y px y + 8 l1x py y - 8 px py y -
        8 l1 x y - 8 l1y x y - 8 py x y - 4 l1x y^2 + 4 px y^2 + 4 x y^2) -
    Sqrt[ (lt^2 (-4 l1^2 l1x - 4 l1x^3 - 8 l1 l1x l1y - 4 l1x l1y^2 + 4 l1x lb^2 - 4 l1x lt^2 + 4 l1^2 px +
          12 l1x^2 px + 8 l1 l1y px + 4 l1y^2 px - 4 lb^2 px + 4 lt^2 px - 12 l1x px^2 + 4 px^3 - 8 l1 l1x
          py - 8 l1x l1y py + 8 l1 px py + 8 l1y px py - 4 l1x py^2 + 4 px py^2 + 4 l1^2 x + 12 l1x^2 x +
          8 l1 l1y x + 4 l1y^2 x - 4 lb^2 x + 4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py
          x + 4 py^2 x - 12 l1x x^2 + 12 px x^2 + 4 x^3 + 8 l1 l1x y + 8 l1x l1y y - 8 l1 px y - 8 l1y px
          y + 8 l1x py y - 8 px py y - 8 l1 x y - 8 l1y x y - 8 py x y - 4 l1x y^2 + 4 px y^2 + 4 x y^2)^2 -
      4 lt^2 (4 l1^2 + 4 l1x^2 + 8 l1 l1y + 4 l1y^2 - 8 l1x px + 4 px^2 + 8 l1 py + 8 l1y py +
        4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2)
       (l1^4 + 2 l1^2 l1x^2 + l1x^4 + 4 l1^3 l1y + 4 l1 l1x^2 l1y + 6 l1^2 l1y^2 + 2 l1x^2 l1y^2 +
        4 l1 l1y^3 + l1y^4 - 2 l1^2 lb^2 - 2 l1x^2 lb^2 - 4 l1 l1y lb^2 - 2 l1y^2 lb^2 + lb^4 - 2 l1^2 lt^2 +
        2 l1x^2 lt^2 - 4 l1 l1y lt^2 - 2 l1y^2 lt^2 - 2 lb^2 lt^2 + lt^4 - 4 l1^2 l1x px - 4 l1x^3 px -
        8 l1 l1x l1y px - 4 l1x l1y^2 px + 4 l1x lb^2 px - 4 l1x lt^2 px + 2 l1^2 px^2 + 6 l1x^2 px^2 +
        4 l1 l1y px^2 + 2 l1y^2 px^2 - 2 lb^2 px^2 + 2 lt^2 px^2 - 4 l1x px^3 + px^4 + 4 l1^3 py +
        4 l1 l1x^2 py + 12 l1^2 l1y py + 4 l1x^2 l1y py + 12 l1 l1y^2 py + 4 l1y^3 py - 4 l1 lb^2 py -
        4 l1y lb^2 py - 4 l1 lt^2 py - 4 l1y lt^2 py - 8 l1 l1x px py - 8 l1x l1y px py +
        4 l1 px^2 py + 4 l1y px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 -
        2 lb^2 py^2 - 2 lt^2 py^2 - 4 l1x px py^2 + 2 px^2 py^2 + 4 l1 py^3 + 4 l1y py^3 + py^4 - 4 l1^2 l1x x -
        4 l1x^3 x - 8 l1 l1x l1y x - 4 l1x l1y^2 x + 4 l1x lb^2 x - 4 l1x lt^2 x + 4 l1^2 px x +
        12 l1x^2 px x + 8 l1 l1y px x + 4 l1y^2 px x - 4 lb^2 px x + 4 lt^2 px x - 12 l1x px^2 x +
        4 px^3 x - 8 l1 l1x py x - 8 l1x l1y py x + 8 l1 px py x + 8 l1y px py x - 4 l1x py^2 x +
        4 px py^2 x + 2 l1^2 x^2 + 6 l1x^2 x^2 + 4 l1 l1y x^2 + 2 l1y^2 x^2 - 2 lb^2 x^2 + 2 lt^2 x^2 -
        12 l1x px x^2 + 6 px^2 x^2 + 4 l1 py x^2 + 4 l1y py x^2 + 2 py^2 x^2 - 4 l1x x^3 + 4 px x^3 +
        x^4 - 4 l1^3 y - 4 l1 l1x^2 y - 12 l1^2 l1y y - 4 l1x^2 l1y y - 12 l1 l1y^2 y - 4 l1y^3 y +
        4 l1 lb^2 y + 4 l1y lb^2 y + 4 l1 lt^2 y + 4 l1y lt^2 y + 8 l1 l1x px y + 8 l1x l1y px y -
        4 l1 px^2 y - 4 l1y px^2 y - 12 l1^2 py y - 4 l1x^2 py y - 24 l1 l1y py y - 12 l1y^2 py y +
        4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1 py^2 y - 12 l1y py^2 y -
        4 py^3 y + 8 l1 l1x x y + 8 l1x l1y x y - 8 l1 px x y - 8 l1y px x y + 8 l1x py x y -
        8 px py x y - 4 l1 x^2 y - 4 l1y x^2 y - 4 py x^2 y + 6 l1^2 y^2 + 2 l1x^2 y^2 + 12 l1 l1y y^2 +
```

$$
\begin{aligned}
&6\,l1y^2\,y^2 - 2\,lb^2\,y^2 - 2\,lt^2\,y^2 - 4\,l1x\,px\,y^2 + 2\,px^2\,y^2 + 12\,l1\,py\,y^2 + 12\,l1y\,py\,y^2 + \\
&6\,py^2\,y^2 - 4\,l1x\,x\,y^2 + 4\,px\,x\,y^2 + 2\,x^2\,y^2 - 4\,l1\,y^3 - 4\,l1y\,y^3 - 4\,py\,y^3 + y^4\big)\big)\big) \Big/ \\
&\big(2\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&\quad 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - \\
&\quad 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big)\big)\Big],
\end{aligned}
$$

$$
\begin{aligned}
&\text{theta2} \to \text{ArcCos}\Big[\frac{1}{4\,lb\,lt}\big(-2\,lb^2 - 2\,lt^2 + px^2 + py^2 + \\
&\quad pz^2 - \\
&\quad 2 \\
&\quad\ px \\
&\quad\ x + x^2 - 2 \\
&\quad\ py \\
&\quad\ y + y^2 - 2 \\
&\quad\ pz \\
&\quad\ z + z^2\big)\Big]\Big\},
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{\text{theta1} \to -\text{ArcCos}\Big[\Big(-lt\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - \\
&\quad 4\,l1x\,lt^2 + 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&\quad 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&\quad 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&\quad 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&\quad 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&\quad 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big) + \\
&\quad \sqrt{\Big(lt^2\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 + \\
&\quad\quad 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&\quad\quad 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&\quad\quad 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&\quad\quad 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&\quad\quad 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&\quad\quad 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big)^2 - \\
&\quad\quad 4\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y \\
&\quad\quad\ py + 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big) \\
&\quad\quad \big(l1^4 + 2\,l1^2\,l1x^2 + l1x^4 + 4\,l1^3\,l1y + 4\,l1\,l1x^2\,l1y + 6\,l1^2\,l1y^2 + 2\,l1x^2\,l1y^2 + 4 \\
&\quad\quad\ l1\,l1y^3 + l1y^4 - 2\,l1^2\,lb^2 - 2\,l1x^2\,lb^2 - 4\,l1\,l1y\,lb^2 - 2\,l1y^2\,lb^2 + lb^4 - 2\,l1^2\,lt^2 + 2 \\
&\quad\quad\ l1x^2\,lt^2 - 4\,l1\,l1y\,lt^2 - 2\,l1y^2\,lt^2 - 2\,lb^2\,lt^2 + lt^4 - 4\,l1^2\,l1x\,px - 4\,l1x^3\,px - 8\,l1 \\
&\quad\quad\ l1x\,l1y\,px - 4\,l1x\,l1y^2\,px + 4\,l1x\,lb^2\,px - 4\,l1x\,lt^2\,px + 2\,l1^2\,px^2 + 6\,l1x^2\,px^2 + 4 \\
&\quad\quad\ l1\,l1y\,px^2 + 2\,l1y^2\,px^2 - 2\,lb^2\,px^2 + 2\,lt^2\,px^2 - 4\,l1x\,px^3 + px^4 + 4\,l1^3\,py + 4\,l1\,l1x^2 \\
&\quad\quad\ py + 12\,l1^2\,l1y\,py + 4\,l1x^2\,l1y\,py + 12\,l1\,l1y^2\,py + 4\,l1y^3\,py - 4\,l1\,lb^2\,py - 4\,l1y\,lb^2 \\
&\quad\quad\ py - 4\,l1\,lt^2\,py - 4\,l1y\,lt^2\,py - 8\,l1\,l1x\,px\,py - 8\,l1x\,l1y\,px\,py + 4\,l1\,px^2\,py + 4\,l1y \\
&\quad\quad\ px^2\,py + 6\,l1^2\,py^2 + 2\,l1x^2\,py^2 + 12\,l1\,l1y\,py^2 + 6\,l1y^2\,py^2 - 2\,lb^2\,py^2 - 2\,lt^2\,py^2 - 4 \\
&\quad\quad\ l1x\,px\,py^2 + 2\,px^2\,py^2 + 4\,l1\,py^3 + 4\,l1y\,py^3 + py^4 - 4\,l1^2\,l1x\,x - 4\,l1x^3\,x - 8\,l1\,l1x \\
&\quad\quad\ l1y\,x - 4\,l1x\,l1y^2\,x + 4\,l1x\,lb^2\,x - 4\,l1x\,lt^2\,x + 4\,l1^2\,px\,x + 12\,l1x^2\,px\,x + 8\,l1\,l1y\,px \\
&\quad\quad\ x + 4\,l1y^2\,px\,x - 4\,lb^2\,px\,x + 4\,lt^2\,px\,x - 12\,l1x\,px^2\,x + 4\,px^3\,x - 8\,l1\,l1x\,py\,x - 8\,l1x
\end{aligned}
$$

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\begin{aligned}
&\text{l1y py x} + 8\,\text{l1 px py x} + 8\,\text{l1y px py x} - 4\,\text{l1x py}^2\,\text{x} + 4\,\text{px py}^2\,\text{x} + 2\,\text{l1}^2\,\text{x}^2 + 6\,\text{l1x}^2\,\text{x}^2 + \\
&4\,\text{l1 l1y x}^2 + 2\,\text{l1y}^2\,\text{x}^2 - 2\,\text{lb}^2\,\text{x}^2 + 2\,\text{lt}^2\,\text{x}^2 - 12\,\text{l1x px x}^2 + 6\,\text{px}^2\,\text{x}^2 + 4\,\text{l1 py x}^2 + 4\,\text{l1y} \\
&\text{py x}^2 + 2\,\text{py}^2\,\text{x}^2 - 4\,\text{l1x x}^3 + 4\,\text{px x}^3 + \text{x}^4 - 4\,\text{l1}^3\,\text{y} - 4\,\text{l1 l1x}^2\,\text{y} - 12\,\text{l1}^2\,\text{l1y y} - 4\,\text{l1x}^2 \\
&\text{l1y y} - 12\,\text{l1 l1y}^2\,\text{y} - 4\,\text{l1y}^3\,\text{y} + 4\,\text{l1 lb}^2\,\text{y} + 4\,\text{l1y lb}^2\,\text{y} + 4\,\text{l1 lt}^2\,\text{y} + 4\,\text{l1y lt}^2\,\text{y} + 8 \\
&\text{l1 l1x px y} + 8\,\text{l1x l1y px y} - 4\,\text{l1 px}^2\,\text{y} - 4\,\text{l1y px}^2\,\text{y} - 12\,\text{l1}^2\,\text{py y} - 4\,\text{l1x}^2\,\text{py y} - 24 \\
&\text{l1 l1y py y} - 12\,\text{l1y}^2\,\text{py y} + 4\,\text{lb}^2\,\text{py y} + 4\,\text{lt}^2\,\text{py y} + 8\,\text{l1x px py y} - 4\,\text{px}^2\,\text{py y} - 12\,\text{l1} \\
&\text{py}^2\,\text{y} - 12\,\text{l1y py}^2\,\text{y} - 4\,\text{py}^3\,\text{y} + 8\,\text{l1 l1x x y} + 8\,\text{l1x l1y x y} - 8\,\text{l1 px x y} - 8\,\text{l1y px x y} + \\
&8\,\text{l1x py x y} - 8\,\text{px py x y} - 4\,\text{l1 x}^2\,\text{y} - 4\,\text{l1y x}^2\,\text{y} - 4\,\text{py x}^2\,\text{y} + 6\,\text{l1}^2\,\text{y}^2 + 2\,\text{l1x}^2\,\text{y}^2 + 12 \\
&\text{l1 l1y y}^2 + 6\,\text{l1y}^2\,\text{y}^2 - 2\,\text{lb}^2\,\text{y}^2 - 2\,\text{lt}^2\,\text{y}^2 - 4\,\text{l1x px y}^2 + 2\,\text{px}^2\,\text{y}^2 + 12\,\text{l1 py y}^2 + 12\,\text{l1y} \\
&\text{py y}^2 + 6\,\text{py}^2\,\text{y}^2 - 4\,\text{l1x x y}^2 + 4\,\text{px x y}^2 + 2\,\text{x}^2\,\text{y}^2 - 4\,\text{l1 y}^3 - 4\,\text{l1y y}^3 - 4\,\text{py y}^3 + \text{y}^4\big)\big)\big) \Big/ \\
&\big(2\,\text{lt}^2\,\big(4\,\text{l1}^2 + 4\,\text{l1x}^2 + 8\,\text{l1 l1y} + 4\,\text{l1y}^2 - 8\,\text{l1x px} + 4\,\text{px}^2 + 8\,\text{l1 py} + 8\,\text{l1y py} + \\
&4\,\text{py}^2 - 8\,\text{l1x x} + 8\,\text{px x} + 4\,\text{x}^2 - 8\,\text{l1 y} - 8\,\text{l1y y} - 8\,\text{py y} + 4\,\text{y}^2\big)\big)\Big] + \\
&\text{ArcCos}\Big[\frac{1}{4\,\text{lb lt}}\big(-2\,\text{lb}^2 - 2\,\text{lt}^2 + \text{px}^2 + \text{py}^2 + \\
&\text{pz}^2 - \\
&2 \\
&\text{px} \\
&\text{x} + \text{x}^2 - 2 \\
&\text{py} \\
&\text{y} + \text{y}^2 - 2 \\
&\text{pz} \\
&\text{z} + \text{z}^2\big)\Big], \\
&\text{theta3} \to -\text{ArcCos}\Big[\Big(-\text{lt}\,\big(-4\,\text{l1}^2\,\text{l1x} - 4\,\text{l1x}^3 - 8\,\text{l1 l1x l1y} - 4\,\text{l1x l1y}^2 + 4\,\text{l1x lb}^2 - \\
&4\,\text{l1x lt}^2 + 4\,\text{l1}^2\,\text{px} + 12\,\text{l1x}^2\,\text{px} + 8\,\text{l1 l1y px} + 4\,\text{l1y}^2\,\text{px} - \\
&4\,\text{lb}^2\,\text{px} + 4\,\text{lt}^2\,\text{px} - 12\,\text{l1x px}^2 + 4\,\text{px}^3 - 8\,\text{l1 l1x py} - 8\,\text{l1x l1y py} + \\
&8\,\text{l1 px py} + 8\,\text{l1y px py} - 4\,\text{l1x py}^2 + 4\,\text{px py}^2 + 4\,\text{l1}^2\,\text{x} + 12\,\text{l1x}^2\,\text{x} + \\
&8\,\text{l1 l1y x} + 4\,\text{l1y}^2\,\text{x} - 4\,\text{lb}^2\,\text{x} + 4\,\text{lt}^2\,\text{x} - 24\,\text{l1x px x} + 12\,\text{px}^2\,\text{x} + \\
&8\,\text{l1 py x} + 8\,\text{l1y py x} + 4\,\text{py}^2\,\text{x} - 12\,\text{l1x x}^2 + 12\,\text{px x}^2 + 4\,\text{x}^3 + \\
&8\,\text{l1 l1x y} + 8\,\text{l1x l1y y} - 8\,\text{l1 px y} - 8\,\text{l1y px y} + 8\,\text{l1x py y} - \\
&8\,\text{px py y} - 8\,\text{l1 x y} - 8\,\text{l1y x y} - 8\,\text{py x y} - 4\,\text{l1x y}^2 + 4\,\text{px y}^2 + 4\,\text{x y}^2\big) + \\
&\sqrt{\Big(\text{lt}^2\,\big(-4\,\text{l1}^2\,\text{l1x} - 4\,\text{l1x}^3 - 8\,\text{l1 l1x l1y} - 4\,\text{l1x l1y}^2 + 4\,\text{l1x lb}^2 - 4\,\text{l1x lt}^2 + \\
&4\,\text{l1}^2\,\text{px} + 12\,\text{l1x}^2\,\text{px} + 8\,\text{l1 l1y px} + 4\,\text{l1y}^2\,\text{px} - 4\,\text{lb}^2\,\text{px} + 4\,\text{lt}^2\,\text{px} - \\
&12\,\text{l1x px}^2 + 4\,\text{px}^3 - 8\,\text{l1 l1x py} - 8\,\text{l1x l1y py} + 8\,\text{l1 px py} + 8\,\text{l1y px py} - \\
&4\,\text{l1x py}^2 + 4\,\text{px py}^2 + 4\,\text{l1}^2\,\text{x} + 12\,\text{l1x}^2\,\text{x} + 8\,\text{l1 l1y x} + 4\,\text{l1y}^2\,\text{x} - 4\,\text{lb}^2\,\text{x} + \\
&4\,\text{lt}^2\,\text{x} - 24\,\text{l1x px x} + 12\,\text{px}^2\,\text{x} + 8\,\text{l1 py x} + 8\,\text{l1y py x} + 4\,\text{py}^2\,\text{x} - 12\,\text{l1x x}^2 + \\
&12\,\text{px x}^2 + 4\,\text{x}^3 + 8\,\text{l1 l1x y} + 8\,\text{l1x l1y y} - 8\,\text{l1 px y} - 8\,\text{l1y px y} + 8\,\text{l1x py y} - \\
&8\,\text{px py y} - 8\,\text{l1 x y} - 8\,\text{l1y x y} - 8\,\text{py x y} - 4\,\text{l1x y}^2 + 4\,\text{px y}^2 + 4\,\text{x y}^2\big)^2 - \\
&4\,\text{lt}^2\,\big(4\,\text{l1}^2 + 4\,\text{l1x}^2 + 8\,\text{l1 l1y} + 4\,\text{l1y}^2 - 8\,\text{l1x px} + 4\,\text{px}^2 + 8\,\text{l1 py} + 8\,\text{l1y py} + \\
&4\,\text{py}^2 - 8\,\text{l1x x} + 8\,\text{px x} + 4\,\text{x}^2 - 8\,\text{l1 y} - 8\,\text{l1y y} - 8\,\text{py y} + 4\,\text{y}^2\big) \\
&\big(\text{l1}^4 + 2\,\text{l1}^2\,\text{l1x}^2 + \text{l1x}^4 + 4\,\text{l1}^3\,\text{l1y} + 4\,\text{l1 l1x}^2\,\text{l1y} + 6\,\text{l1}^2\,\text{l1y}^2 + 2\,\text{l1x}^2\,\text{l1y}^2 + \\
&4\,\text{l1 l1y}^3 + \text{l1y}^4 - 2\,\text{l1}^2\,\text{lb}^2 - 2\,\text{l1x}^2\,\text{lb}^2 - 4\,\text{l1 l1y lb}^2 - 2\,\text{l1y}^2\,\text{lb}^2 + \text{lb}^4 - 2\,\text{l1}^2\,\text{lt}^2 + \\
&2\,\text{l1x}^2\,\text{lt}^2 - 4\,\text{l1 l1y lt}^2 - 2\,\text{l1y}^2\,\text{lt}^2 - 2\,\text{lb}^2\,\text{lt}^2 + \text{lt}^4 - 4\,\text{l1}^2\,\text{l1x px} - 4\,\text{l1x}^3\,\text{px} -
\end{aligned}
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\begin{aligned}
&8\,l1\,l1x\,l1y\,px - 4\,l1x\,l1y^2\,px + 4\,l1x\,lb^2\,px - 4\,l1x\,lt^2\,px + 2\,l1^2\,px^2 + 6\,l1x^2\,px^2 + \\
&4\,l1\,l1y\,px^2 + 2\,l1y^2\,px^2 - 2\,lb^2\,px^2 + 2\,lt^2\,px^2 - 4\,l1x\,px^3 + px^4 + 4\,l1^3\,py + \\
&4\,l1\,l1x^2\,py + 12\,l1^2\,l1y\,py + 4\,l1x^2\,l1y\,py + 12\,l1\,l1y^2\,py + 4\,l1y^3\,py - 4\,l1\,lb^2\,py - \\
&4\,l1y\,lb^2\,py - 4\,l1\,lt^2\,py - 4\,l1y\,lt^2\,py - 8\,l1\,l1x\,px\,py - 8\,l1x\,l1y\,px\,py + \\
&4\,l1\,px^2\,py + 4\,l1y\,px^2\,py + 6\,l1^2\,py^2 + 2\,l1x^2\,py^2 + 12\,l1\,l1y\,py^2 + 6\,l1y^2\,py^2 - \\
&2\,lb^2\,py^2 - 2\,lt^2\,py^2 - 4\,l1x\,px\,py^2 + 2\,px^2\,py^2 + 4\,l1\,py^3 + 4\,l1y\,py^3 + py^4 - 4\,l1^2\,l1x\,x - \\
&4\,l1x^3\,x - 8\,l1\,l1x\,l1y\,x - 4\,l1x\,l1y^2\,x + 4\,l1x\,lb^2\,x - 4\,l1x\,lt^2\,x + 4\,l1^2\,px\,x + \\
&12\,l1x^2\,px\,x + 8\,l1\,l1y\,px\,x + 4\,l1y^2\,px\,x - 4\,lb^2\,px\,x + 4\,lt^2\,px\,x - 12\,l1x\,px^2\,x + \\
&4\,px^3\,x - 8\,l1\,l1x\,py\,x - 8\,l1x\,l1y\,py\,x + 8\,l1\,px\,py\,x + 8\,l1y\,px\,py\,x - 4\,l1x\,py^2\,x + \\
&4\,px\,py^2\,x + 2\,l1^2\,x^2 + 6\,l1x^2\,x^2 + 4\,l1\,l1y\,x^2 + 2\,l1y^2\,x^2 - 2\,lb^2\,x^2 + 2\,lt^2\,x^2 - \\
&12\,l1x\,px\,x^2 + 6\,px^2\,x^2 + 4\,l1\,py\,x^2 + 4\,l1y\,py\,x^2 + 2\,py^2\,x^2 - 4\,l1x\,x^3 + 4\,px\,x^3 + \\
&x^4 - 4\,l1^3\,y - 4\,l1\,l1x^2\,y - 12\,l1^2\,l1y\,y - 4\,l1x^2\,l1y\,y - 12\,l1\,l1y^2\,y - 4\,l1y^3\,y + \\
&4\,l1\,lb^2\,y + 4\,l1y\,lb^2\,y + 4\,l1\,lt^2\,y + 4\,l1y\,lt^2\,y + 8\,l1\,l1x\,px\,y + 8\,l1x\,l1y\,px\,y - \\
&4\,l1\,px^2\,y - 4\,l1y\,px^2\,y - 12\,l1^2\,py\,y - 4\,l1x^2\,py\,y - 24\,l1\,l1y\,py\,y - 12\,l1y^2\,py\,y + \\
&4\,lb^2\,py\,y + 4\,lt^2\,py\,y + 8\,l1x\,px\,py\,y - 4\,px^2\,py\,y - 12\,l1\,py^2\,y - 12\,l1y\,py^2\,y - \\
&4\,py^3\,y + 8\,l1\,l1x\,x\,y + 8\,l1x\,l1y\,x\,y - 8\,l1\,px\,x\,y - 8\,l1y\,px\,x\,y + 8\,l1x\,py\,x\,y - \\
&8\,px\,py\,x\,y - 4\,l1\,x^2\,y - 4\,l1y\,x^2\,y - 4\,py\,x^2\,y + 6\,l1^2\,y^2 + 2\,l1x^2\,y^2 + 12\,l1\,l1y\,y^2 + \\
&6\,l1y^2\,y^2 - 2\,lb^2\,y^2 - 2\,lt^2\,y^2 - 4\,l1x\,px\,y^2 + 2\,px^2\,y^2 + 12\,l1\,py\,y^2 + 12\,l1y\,py\,y^2 + \\
&6\,py^2\,y^2 - 4\,l1x\,x\,y^2 + 4\,px\,x\,y^2 + 2\,x^2\,y^2 - 4\,l1\,y^3 - 4\,l1y\,y^3 - 4\,py\,y^3 + y^4\big)\big)\big) \Big/ \\
&\big(2\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&\quad 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big)\big)\Big],
\end{aligned}
$$

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\begin{aligned}
\text{theta2} \to -\text{ArcCos}\Big[\frac{1}{4\,lb\,lt}\big(&-2\,lb^2 - 2\,lt^2 + \\
&px^2 + \\
&py^2 + \\
&pz^2 - \\
&2 \\
&\quad px \\
&\quad x + x^2 - 2 \\
&\quad py \\
&\quad y + y^2 - 2 \\
&\quad pz \\
&\quad z + z^2\big)\Big]\Big\},
\end{aligned}
$$

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\begin{aligned}
\Big\{\text{theta1} \to -\text{ArcCos}\Big[\Big(&-lt\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - \\
&\quad 4\,l1x\,lt^2 + 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - \\
&\quad 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - \\
&\quad 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + \\
&\quad 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + \\
&\quad 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - \\
&\quad 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big) + \\
&\sqrt{\Big(lt^2\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 + \\
&\quad 4\,l1^2\,px + 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px -
\end{aligned}
$$

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                12 l1x px^2 + 4 px^3 - 8 l1 l1x py - 8 l1x l1y py + 8 l1 px py + 8 l1y px py -
                4 l1x py^2 + 4 px py^2 + 4 l1^2 x + 12 l1x^2 x + 8 l1 l1y x + 4 l1y^2 x - 4 lb^2 x +
                4 lt^2 x - 24 l1x px x + 12 px^2 x + 8 l1 py x + 8 l1y py x + 4 py^2 x - 12 l1x x^2 +
                12 px x^2 + 4 x^3 + 8 l1 l1x y + 8 l1x l1y y - 8 l1 px y - 8 l1y px y + 8 l1x py y -
                8 px py y - 8 l1 x y - 8 l1y x y - 8 py x y - 4 l1x y^2 + 4 px y^2 + 4 x y^2)^2 -
          4 lt^2 (4 l1^2 + 4 l1x^2 + 8 l1 l1y + 4 l1y^2 - 8 l1x px + 4 px^2 + 8 l1 py + 8 l1y
                py + 4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2)
           (l1^4 + 2 l1^2 l1x^2 + l1x^4 + 4 l1^3 l1y + 4 l1 l1x^2 l1y + 6 l1^2 l1y^2 + 2 l1x^2 l1y^2 + 4
                l1 l1y^3 + l1y^4 - 2 l1^2 lb^2 - 2 l1x^2 lb^2 - 4 l1 l1y lb^2 - 2 l1y^2 lb^2 + lb^4 - 2 l1^2 lt^2 + 2
                l1x^2 lt^2 - 4 l1 l1y lt^2 - 2 l1y^2 lt^2 - 2 lb^2 lt^2 + lt^4 - 4 l1^2 l1x px - 4 l1x^3 px - 8 l1
                l1x l1y px - 4 l1x l1y^2 px + 4 l1x lb^2 px - 4 l1x lt^2 px + 2 l1^2 px^2 + 6 l1x^2 px^2 + 4
                l1 l1y px^2 + 2 l1y^2 px^2 - 2 lb^2 px^2 + 2 lt^2 px^2 - 4 l1x px^3 + px^4 + 4 l1^3 py + 4 l1 l1x^2
                py + 12 l1^2 l1y py + 4 l1x^2 l1y py + 12 l1 l1y^2 py + 4 l1y^3 py - 4 l1 lb^2 py - 4 l1y lb^2
                py - 4 l1 lt^2 py - 4 l1y lt^2 py - 8 l1 l1x px py - 8 l1x l1y px py + 4 l1 px^2 py + 4 l1y
                px^2 py + 6 l1^2 py^2 + 2 l1x^2 py^2 + 12 l1 l1y py^2 + 6 l1y^2 py^2 - 2 lb^2 py^2 - 2 lt^2 py^2 - 4
                l1x px py^2 + 2 px^2 py^2 + 4 l1 py^3 + 4 l1y py^3 + py^4 - 4 l1^2 l1x x - 4 l1x^3 x - 8 l1 l1x
                l1y x - 4 l1x l1y^2 x + 4 l1x lb^2 x - 4 l1x lt^2 x + 4 l1^2 px x + 12 l1x^2 px x + 8 l1 l1y px
                x + 4 l1y^2 px x - 4 lb^2 px x + 4 lt^2 px x - 12 l1x px^2 x + 4 px^3 x - 8 l1 l1x py x - 8 l1x
                l1y py x + 8 l1 px py x + 8 l1y px py x - 4 l1x py^2 x + 4 px py^2 x + 2 l1^2 x^2 + 6 l1x^2 x^2 +
               4 l1 l1y x^2 + 2 l1y^2 x^2 - 2 lb^2 x^2 + 2 lt^2 x^2 - 12 l1x px x^2 + 6 px^2 x^2 + 4 l1 py x^2 + 4 l1y
                py x^2 + 2 py^2 x^2 - 4 l1x x^3 + 4 px x^3 + x^4 - 4 l1^3 y - 4 l1 l1x^2 y - 12 l1^2 l1y y - 4 l1x^2
                l1y y - 12 l1 l1y^2 y - 4 l1y^3 y + 4 l1 lb^2 y + 4 l1y lb^2 y + 4 l1 lt^2 y + 4 l1y lt^2 y + 8
                l1 l1x px y + 8 l1x l1y px y - 4 l1 px^2 y - 4 l1y px^2 y - 12 l1^2 py y - 4 l1x^2 py y - 24
                l1 l1y py y - 12 l1y^2 py y + 4 lb^2 py y + 4 lt^2 py y + 8 l1x px py y - 4 px^2 py y - 12 l1
                py^2 y - 12 l1y py^2 y - 4 py^3 y + 8 l1 l1x x y + 8 l1x l1y x y - 8 l1 px x y - 8 l1y px x y +
               8 l1x py x y - 8 px py x y - 4 l1 x^2 y - 4 l1y x^2 y - 4 py x^2 y + 6 l1^2 y^2 + 2 l1x^2 y^2 + 12
                l1 l1y y^2 + 6 l1y^2 y^2 - 2 lb^2 y^2 - 2 lt^2 y^2 - 4 l1x px y^2 + 2 px^2 y^2 + 12 l1 py y^2 + 12 l1y
                py y^2 + 6 py^2 y^2 - 4 l1x x y^2 + 4 px x y^2 + 2 x^2 y^2 - 4 l1 y^3 - 4 l1y y^3 - 4 py y^3 + y^4)))) /
      (2 lt^2 (4 l1^2 + 4 l1x^2 + 8 l1 l1y + 4 l1y^2 - 8 l1x px + 4 px^2 + 8 l1 py + 8 l1y py +
            4 py^2 - 8 l1x x + 8 px x + 4 x^2 - 8 l1 y - 8 l1y y - 8 py y + 4 y^2))) +
  ArcCos[ 1/(4 lb lt) (-2 lb^2 - 2 lt^2 + px^2 + py^2 +
    pz^2 -
    2
     px
     x + x^2 - 2
     py
     y + y^2 - 2
     pz
     z + z^2)],
theta3 -> ArcCos[ (-lt (-4 l1^2 l1x - 4 l1x^3 - 8 l1 l1x l1y - 4 l1x l1y^2 + 4 l1x lb^2 -
        4 l1x lt^2 + 4 l1^2 px + 12 l1x^2 px + 8 l1 l1y px + 4 l1y^2 px -
```

$$
\begin{aligned}
&4\,lb^2\,px + 4\,lt^2\,px - 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x\,py - \\
&8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - 4\,l1x\,py^2 + \\
&4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + 8\,l1\,l1y\,x + 4\,l1y^2\,x - \\
&4\,lb^2\,x + 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + \\
&8\,l1y\,py\,x + 4\,py^2\,x - 12\,l1x\,x^2 + 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + \\
&8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px\,y + 8\,l1x\,py\,y - 8\,px\,py\,y - \\
&8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big) + \\
\sqrt{\Big(}&lt^2\,\big(-4\,l1^2\,l1x - 4\,l1x^3 - 8\,l1\,l1x\,l1y - 4\,l1x\,l1y^2 + 4\,l1x\,lb^2 - 4\,l1x\,lt^2 + 4\,l1^2\,px + \\
&\quad 12\,l1x^2\,px + 8\,l1\,l1y\,px + 4\,l1y^2\,px - 4\,lb^2\,px + 4\,lt^2\,px - 12\,l1x\,px^2 + 4\,px^3 - 8\,l1\,l1x \\
&\quad py - 8\,l1x\,l1y\,py + 8\,l1\,px\,py + 8\,l1y\,px\,py - 4\,l1x\,py^2 + 4\,px\,py^2 + 4\,l1^2\,x + 12\,l1x^2\,x + \\
&\quad 8\,l1\,l1y\,x + 4\,l1y^2\,x - 4\,lb^2\,x + 4\,lt^2\,x - 24\,l1x\,px\,x + 12\,px^2\,x + 8\,l1\,py\,x + 8\,l1y\,py \\
&\quad x + 4\,py^2\,x - 12\,l1x\,x^2 + 12\,px\,x^2 + 4\,x^3 + 8\,l1\,l1x\,y + 8\,l1x\,l1y\,y - 8\,l1\,px\,y - 8\,l1y\,px \\
&\quad y + 8\,l1x\,py\,y - 8\,px\,py\,y - 8\,l1\,x\,y - 8\,l1y\,x\,y - 8\,py\,x\,y - 4\,l1x\,y^2 + 4\,px\,y^2 + 4\,x\,y^2\big)^2 - \\
&4\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&\quad 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big) \\
&\quad \big(l1^4 + 2\,l1^2\,l1x^2 + l1x^4 + 4\,l1^3\,l1y + 4\,l1\,l1x^2\,l1y + 6\,l1^2\,l1y^2 + 2\,l1x^2\,l1y^2 + \\
&\quad 4\,l1\,l1y^3 + l1y^4 - 2\,l1^2\,lb^2 - 2\,l1x^2\,lb^2 - 4\,l1\,l1y\,lb^2 - 2\,l1y^2\,lb^2 + lb^4 - 2\,l1^2\,lt^2 + \\
&\quad 2\,l1x^2\,lt^2 - 4\,l1\,l1y\,lt^2 - 2\,l1y^2\,lt^2 - 2\,lb^2\,lt^2 + lt^4 - 4\,l1^2\,l1x\,px - 4\,l1x^3\,px - \\
&\quad 8\,l1\,l1x\,l1y\,px - 4\,l1x\,l1y^2\,px + 4\,l1x\,lb^2\,px - 4\,l1x\,lt^2\,px + 2\,l1^2\,px^2 + 6\,l1x^2\,px^2 + \\
&\quad 4\,l1\,l1y\,px^2 + 2\,l1y^2\,px^2 - 2\,lb^2\,px^2 + 2\,lt^2\,px^2 - 4\,l1x\,px^3 + px^4 + 4\,l1^3\,py + \\
&\quad 4\,l1\,l1x^2\,py + 12\,l1^2\,l1y\,py + 4\,l1x^2\,l1y\,py + 12\,l1\,l1y^2\,py + 4\,l1y^3\,py - 4\,l1\,lb^2\,py - \\
&\quad 4\,l1y\,lb^2\,py - 4\,l1\,lt^2\,py - 4\,l1y\,lt^2\,py - 8\,l1\,l1x\,px\,py - 8\,l1x\,l1y\,px\,py + \\
&\quad 4\,l1\,px^2\,py + 4\,l1y\,px^2\,py + 6\,l1^2\,py^2 + 2\,l1x^2\,py^2 + 12\,l1\,l1y\,py^2 + 6\,l1y^2\,py^2 - \\
&\quad 2\,lb^2\,py^2 - 2\,lt^2\,py^2 - 4\,l1x\,px\,py^2 + 2\,px^2\,py^2 + 4\,l1\,py^3 + 4\,l1y\,py^3 + py^4 - 4\,l1^2\,l1x\,x - \\
&\quad 4\,l1x^3\,x - 8\,l1\,l1x\,l1y\,x - 4\,l1x\,l1y^2\,x + 4\,l1x\,lb^2\,x - 4\,l1x\,lt^2\,x + 4\,l1^2\,px\,x + \\
&\quad 12\,l1x^2\,px\,x + 8\,l1\,l1y\,px\,x + 4\,l1y^2\,px\,x - 4\,lb^2\,px\,x + 4\,lt^2\,px\,x - 12\,l1x\,px^2\,x + \\
&\quad 4\,px^3\,x - 8\,l1\,l1x\,py\,x - 8\,l1x\,l1y\,py\,x + 8\,l1\,px\,py\,x + 8\,l1y\,px\,py\,x - 4\,l1x\,py^2\,x + \\
&\quad 4\,px\,py^2\,x + 2\,l1^2\,x^2 + 6\,l1x^2\,x^2 + 4\,l1\,l1y\,x^2 + 2\,l1y^2\,x^2 - 2\,lb^2\,x^2 + 2\,lt^2\,x^2 - \\
&\quad 12\,l1x\,px\,x^2 + 6\,px^2\,x^2 + 4\,l1\,py\,x^2 + 4\,l1y\,py\,x^2 + 2\,py^2\,x^2 - 4\,l1x\,x^3 + 4\,px\,x^3 + \\
&\quad x^4 - 4\,l1^3\,y - 4\,l1\,l1x^2\,y - 12\,l1^2\,l1y\,y - 4\,l1x^2\,l1y\,y - 12\,l1\,l1y^2\,y - 4\,l1y^3\,y + \\
&\quad 4\,l1\,lb^2\,y + 4\,l1y\,lb^2\,y + 4\,l1\,lt^2\,y + 4\,l1y\,lt^2\,y + 8\,l1\,l1x\,px\,y + 8\,l1x\,l1y\,px\,y - \\
&\quad 4\,l1\,px^2\,y - 4\,l1y\,px^2\,y - 12\,l1^2\,py\,y - 4\,l1x^2\,py\,y - 24\,l1\,l1y\,py\,y - 12\,l1y^2\,py\,y + \\
&\quad 4\,lb^2\,py\,y + 4\,lt^2\,py\,y + 8\,l1x\,px\,py\,y - 4\,px^2\,py\,y - 12\,l1\,py^2\,y - 12\,l1y\,py^2\,y - \\
&\quad 4\,py^3\,y + 8\,l1\,l1x\,x\,y + 8\,l1x\,l1y\,x\,y - 8\,l1\,px\,x\,y - 8\,l1y\,px\,x\,y + 8\,l1x\,py\,x\,y - \\
&\quad 8\,px\,py\,x\,y - 4\,l1\,x^2\,y - 4\,l1y\,x^2\,y - 4\,py\,x^2\,y + 6\,l1^2\,y^2 + 2\,l1x^2\,y^2 + 12\,l1\,l1y\,y^2 + \\
&\quad 6\,l1y^2\,y^2 - 2\,lb^2\,y^2 - 2\,lt^2\,y^2 - 4\,l1x\,px\,y^2 + 2\,px^2\,y^2 + 12\,l1\,py\,y^2 + 12\,l1y\,py\,y^2 + \\
&\quad 6\,py^2\,y^2 - 4\,l1x\,x\,y^2 + 4\,px\,x\,y^2 + 2\,x^2\,y^2 - 4\,l1\,y^3 - 4\,l1y\,y^3 - 4\,py\,y^3 + y^4\big)\big)\Big) \Big/ \\
&\big(2\,lt^2\,\big(4\,l1^2 + 4\,l1x^2 + 8\,l1\,l1y + 4\,l1y^2 - 8\,l1x\,px + 4\,px^2 + 8\,l1\,py + 8\,l1y\,py + \\
&\quad 4\,py^2 - 8\,l1x\,x + 8\,px\,x + 4\,x^2 - \\
&\quad 8\,l1\,y - 8\,l1y\,y - 8\,py\,y + 4\,y^2\big)\big)\Big],
\end{aligned}
$$

$$
\text{theta2} \to \text{ArcCos}\Big[\frac{1}{4\,lb\,lt}\big(-2\,lb^2 -
$$

$$\begin{Bmatrix}\frac{1}{2}\left(-lt^2+px^2+py^2+pz^2-2\,px\,x+x^2-2\,py\,y+y^2-2\,pz\,z+z^2\right)\end{Bmatrix}$$