Métodos

Matrices de Pauli:

$$\sigma_0 = \mathbb{I} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

forman una base para el E.V de matrices 2x2 hermiticas

Matriz hermitica:

$$\mathbb{A} = \begin{pmatrix} z_1 & z_2 \\ z_3 & z_4 \end{pmatrix} = \operatorname{adj}(\mathbb{A}) = \begin{pmatrix} z_1^* & z_3^* \\ z_2^* & z_4^* \end{pmatrix}$$

$\hookrightarrow z_1^* = z_1 , \quad z_4^* = z_4 , \quad z_2^* = z_3$

* Tienen igual dimensión ✓
* Son 4, dim = 4 ✓
* ortogonales?

Si escribimos las matrices com vectores de $\mathbb{R}^4$, podemos formar la matriz de vectores columna:

$$(\sigma_1, \sigma_2, \sigma_3, \sigma_0) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & -i & 0 & 0 \\ 1 & i & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} = C$$

$$\det(C) = \det\begin{pmatrix} 1 & -i & 0 \\ 1 & i & 0 \\ 0 & 0 & 1 \end{pmatrix} - \det\begin{pmatrix} 1 & -i & 0 \\ 1 & i & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\det(C) = i + i + i + i = 4i \neq 0 \therefore$ son l.i y sí forman una base

Comprobamos si pueden generar matrices hermiticas.

$$\alpha\sigma_0 + \beta\sigma_1 + \gamma\sigma_2 + \lambda\sigma_3 = \begin{pmatrix} r_1 & z_2 \\ z_3 & r_4 \end{pmatrix}$$

$\hookrightarrow$

$$\begin{cases} \alpha + \lambda = r_1 \\ \beta - \gamma i = z_2 \\ \beta + \gamma i = z_3 \\ \alpha - \lambda = r_4 \end{cases} \rightarrow \begin{matrix} r_1 \in \mathbb{R}, \ \alpha + \lambda \in \mathbb{R} \ \checkmark \\ z_2^* = (\beta - \gamma i)^* = \beta + \gamma i = z_3 \ \checkmark \\ r_4 \in \mathbb{R}, \ \alpha - \lambda \in \mathbb{R} \ \checkmark \end{matrix}$$

Es ortogonal bajo la definición de producto interno:

$\langle a | b \rangle = T_r(\mathrm{adj}(\mathbb{A})\mathbb{B})$?

$\langle \sigma_0 | \sigma_1 \rangle = T_r(\mathbb{I}\,\sigma_1) = T_r(\sigma_1) = 0$ ✓

$\langle \sigma_0 | \sigma_2 \rangle = 0$ ✓ , $\langle \sigma_0 | \sigma_3 \rangle = 1 - 1 = 0$ ✓

σ_0 es ortogonal a $\sigma_1, \sigma_2, \sigma_3$

$\langle \sigma_1 | \sigma_2 \rangle = T_r(\sigma_1 \sigma_2) = T_r\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i - i = 0$ ✓

$\langle \sigma_1 | \sigma_3 \rangle = T_r\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = 0$ ✓

σ_1 es ortogonal a $\sigma_0, \sigma_2, \sigma_3$

$\langle \sigma_3 | \sigma_2 \rangle = T_r\begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = 0$ ✓

$\sigma_2 \perp \sigma_3$

$\therefore$ La base es ortogonal