

# Agents in Artificial Intelligence

An AI system can be defined as the study of the rational agent and its environment. The agents sense the environment through sensors and act on their environment through actuators. An AI agent can have mental properties such as knowledge, belief, intention, etc.

## What is an Agent?

An agent can be anything that perceives its environment through sensors and act upon that environment through actuators. An Agent runs in the cycle of **perceiving, thinking, and acting**. An agent can be:

- **Human-Agent:** A human agent has eyes, ears, and other organs which work for sensors and hand, legs, vocal tract work for actuators.
- **Robotic Agent:** A robotic agent can have cameras, infrared range finder, NLP for sensors and various motors for actuators.
- **Software Agent:** Software agent can have keystrokes, file contents as sensory input and act on those inputs and display output on the screen.

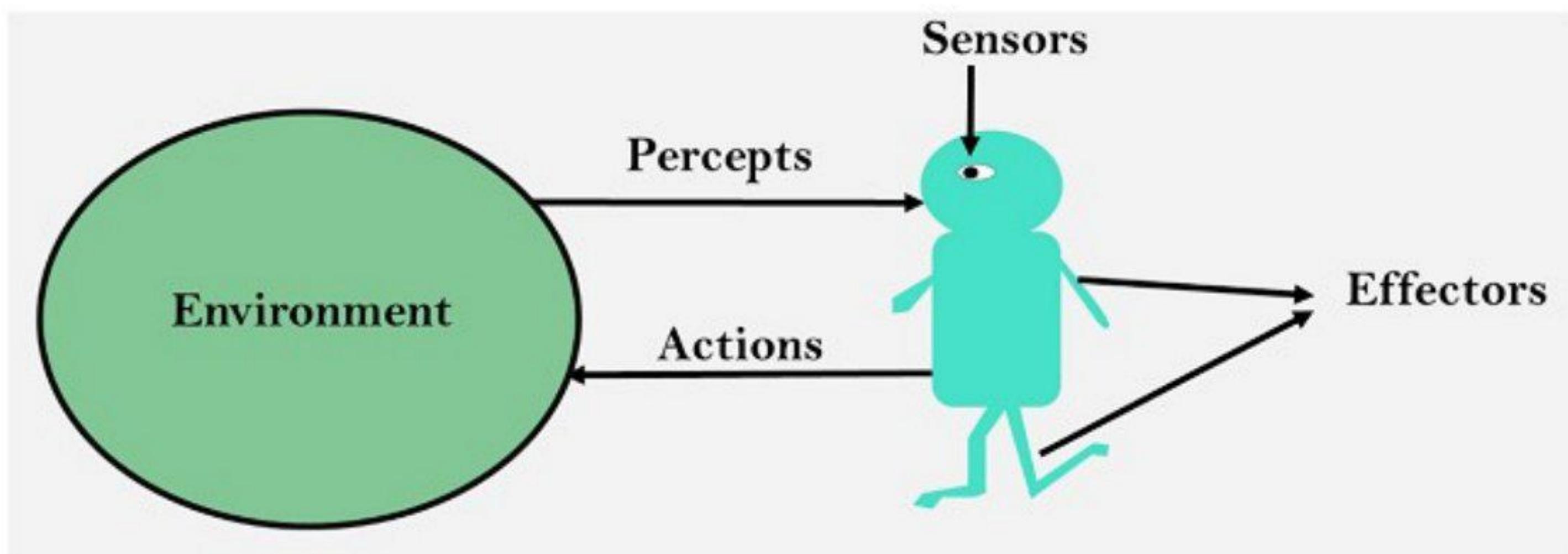
Hence the world around us is full of agents such as thermostat, cellphone, camera, and even we are also agents.

Before moving forward, we should first know about sensors, effectors, and actuators.

**Sensor:** Sensor is a device which detects the change in the environment and sends the information to other electronic devices. An agent observes its environment through sensors.

**Actuators:** Actuators are the component of machines that converts energy into motion. The actuators are only responsible for moving and controlling a system. An actuator can be an electric motor, gears, rails, etc.

**Effectors:** Effectors are the devices which affect the environment. Effectors can be legs, wheels, arms, fingers, wings, fins, and display screen.



## Intelligent Agents:

An intelligent agent is an autonomous entity which act upon an environment using sensors and actuators for achieving goals. An intelligent agent may learn from the environment to achieve their goals. A thermostat is an example of an intelligent agent.

Following are the main four rules for an AI agent:

- **Rule 1:** An AI agent must have the ability to perceive the environment.
- **Rule 2:** The observation must be used to make decisions.
- **Rule 3:** Decision should result in an action.
- **Rule 4:** The action taken by an AI agent must be a rational action.

## Rational Agent:

A rational agent is an agent which has clear preference, models uncertainty, and acts in a way to maximize its performance measure with all possible actions.

A rational agent is said to perform the right things. AI is about creating rational agents to use for game theory and decision theory for various real-world scenarios.

For an AI agent, the rational action is most important because in AI reinforcement learning algorithm, for each best possible action, agent gets the positive reward and for each wrong action, an agent gets a negative reward.

Note: Rational agents in AI are very similar to intelligent agents.

## Rationality:

The rationality of an agent is measured by its performance measure. Rationality can be judged on the basis of following points:

- Performance measure which defines the success criterion.
- Agent prior knowledge of its environment.
- Best possible actions that an agent can perform.
- The sequence of percepts.

Note: Rationality differs from Omniscience because an Omniscient agent knows the actual outcome of its action and act accordingly, which is not possible in reality.

## Structure of an AI Agent

The task of AI is to design an agent program which implements the agent function. The structure of an intelligent agent is a combination of architecture and agent program. It can be viewed as:

### 1. Agent = Architecture + Agent program

Following are the main three terms involved in the structure of an AI agent:

**Architecture:** Architecture is machinery that an AI agent executes on.

**Agent Function:** Agent function is used to map a percept to an action.

### 1. $f:P^* \rightarrow A$

**Agent program:** Agent program is an implementation of agent function. An agent program executes on the physical architecture to produce function f.

## PEAS Representation

PEAS is a type of model on which an AI agent works upon. When we define an AI agent or rational agent, then we can group its properties under PEAS representation model. It is made up of four words:

○ **P:** Performance measure

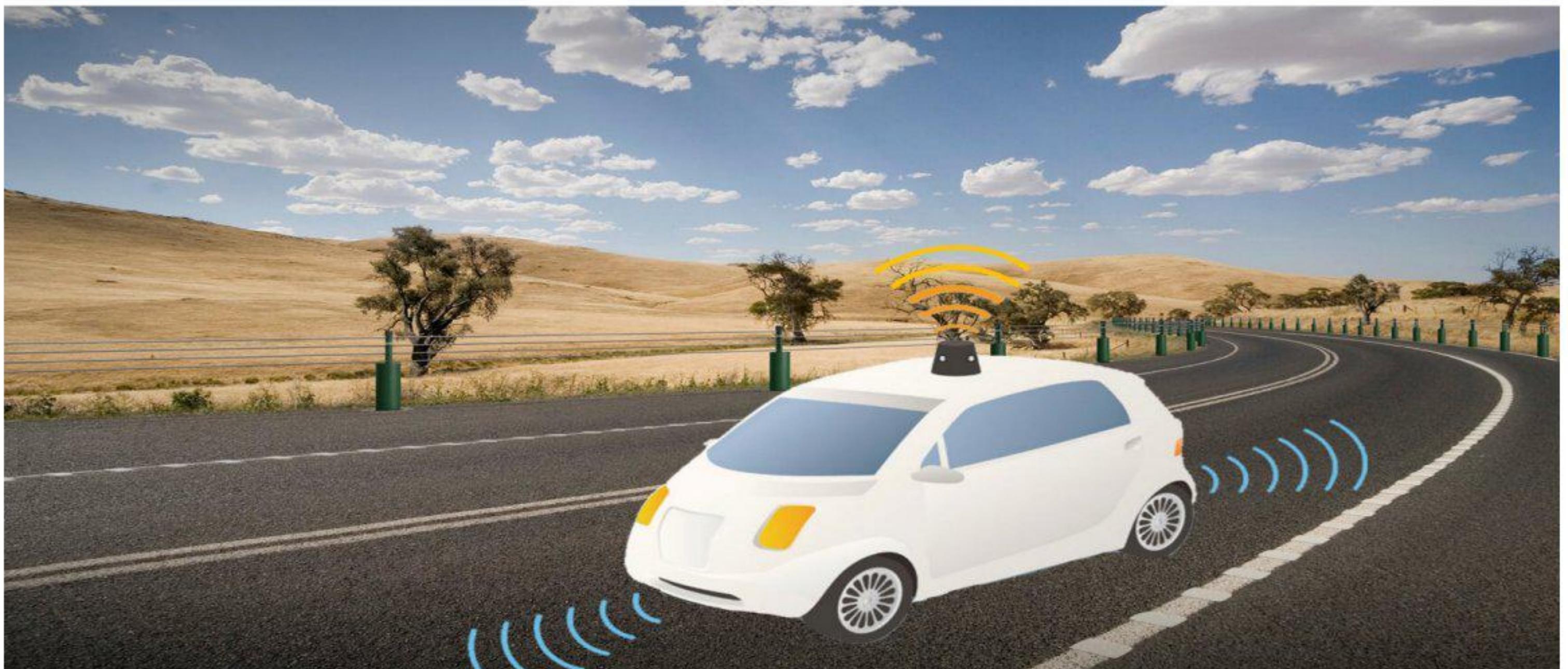
○ **E:** Environment

○ **A:** Actuators

## oS: Sensors

Here performance measure is the objective for the success of an agent's behavior.

## PEAS for self-driving cars:



Let's suppose a self-driving car then PEAS representation will be:

**Performance:** Safety, time, legal drive, comfort

**Environment:** Roads, other vehicles, road signs, pedestrian

**Actuators:** Steering, accelerator, brake, signal, horn

**Sensors:** Camera, GPS, speedometer, odometer, accelerometer, sonar.

## Example of Agents with their PEAS representation

Agent	Performance measure	Environment	Actuators	Sensors
1. Medical Diagnose	<ul style="list-style-type: none"><li>o Healthy patient</li><li>o Minimized cost</li></ul>	<ul style="list-style-type: none"><li>o Patient</li><li>o Hospital</li><li>o Staff</li></ul>	<ul style="list-style-type: none"><li>o Tests</li><li>o Treatment</li></ul>	Keyboard (Entry of symptoms)

				<ul style="list-style-type: none"> <li>○ Camera</li> <li>○ Dirt detection sensor</li> <li>○ Cliff sensor</li> <li>○ Bump Sensor</li> <li>○ Infrared Wall Sensor</li> </ul>
<b>2.</b> <b>Vacuum Cleaner</b>	<ul style="list-style-type: none"> <li>○ Cleanness</li> <li>○ Efficiency</li> <li>○ Battery life</li> <li>○ Security</li> </ul>	<ul style="list-style-type: none"> <li>○ Room</li> <li>○ Table</li> <li>○ Wood floor</li> <li>○ Carpet</li> <li>○ Various obstacles</li> </ul>	<ul style="list-style-type: none"> <li>○ Wheels</li> <li>○ Brushes</li> <li>○ Vacuum Extractor</li> </ul>	
<b>3. Part - picking Robot</b>	<ul style="list-style-type: none"> <li>○ Percentage of parts in correct bins.</li> </ul>	<ul style="list-style-type: none"> <li>○ Conveyor belt with parts,</li> <li>○ Bins</li> </ul>	<ul style="list-style-type: none"> <li>○ Jointed Arms</li> <li>○ Hand</li> </ul>	<ul style="list-style-type: none"> <li>○ Camera</li> <li>○ Joint angle sensors.</li> </ul>

## Types of Agents

Agents can be grouped into five classes based on their degree of perceived intelligence and capability :

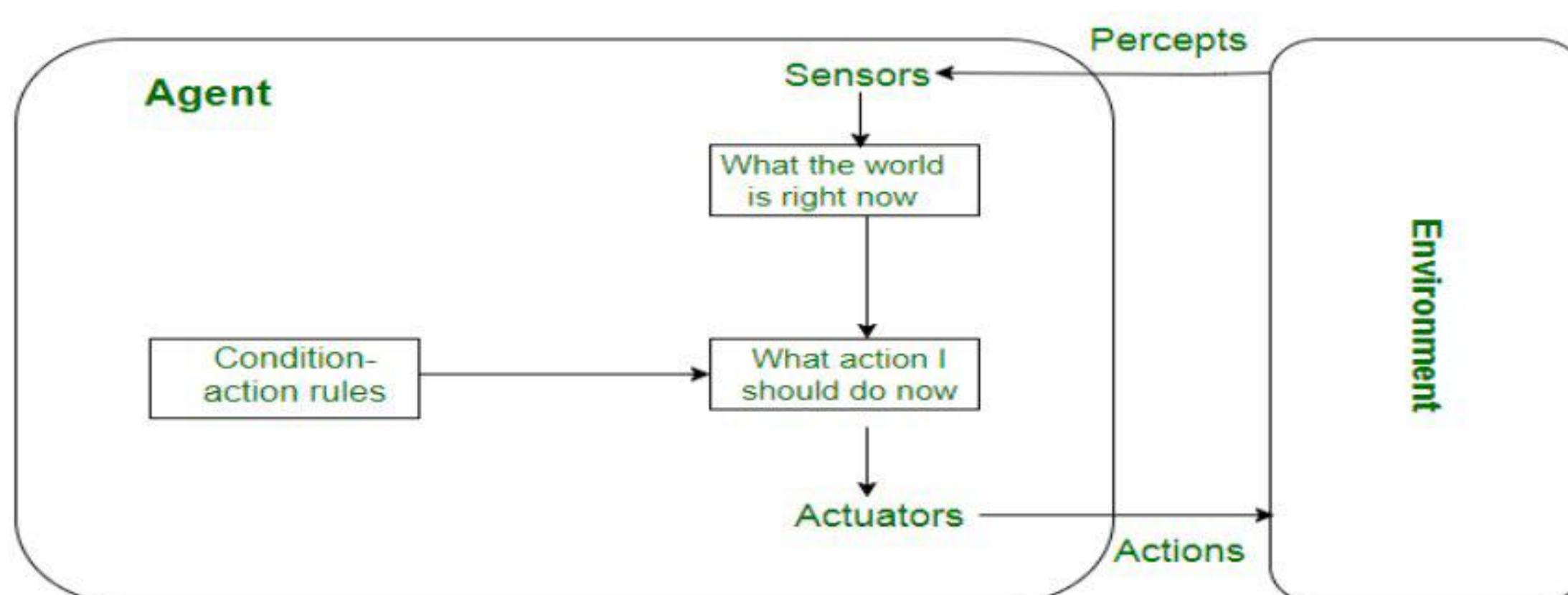
- Simple Reflex Agents
- Model-Based Reflex Agents
- Goal-Based Agents
- Utility-Based Agents
- Learning Agent

## Simple reflex agents

Simple reflex agents ignore the rest of the percept history and act only on the basis of the current percept. Percept history is the history of all that an agent has perceived to date. The agent function is based on the condition-action rule. A condition-action rule is a rule that maps a state i.e, condition to an action. If the condition is true, then the action is taken, else not. This agent function only succeeds when the environment is fully observable. For simple reflex agents operating in partially observable environments, infinite loops are often unavoidable. It may be possible to escape from infinite loops if the agent can randomize its actions.

Problems with Simple reflex agents are :

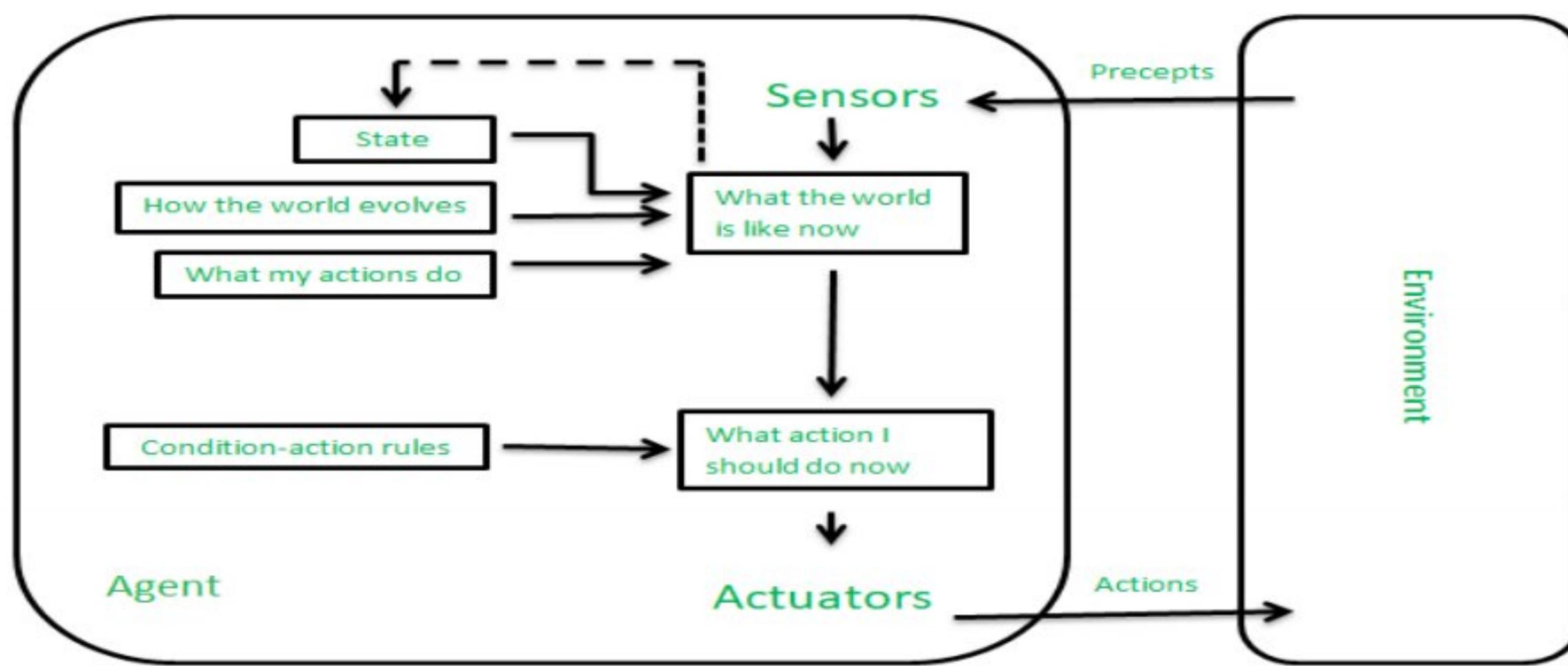
- Very limited intelligence.
- No knowledge of non-perceptual parts of the state.
- Usually too big to generate and store.
- If there occurs any change in the environment, then the collection of rules need to be updated.



## Model-based reflex agents

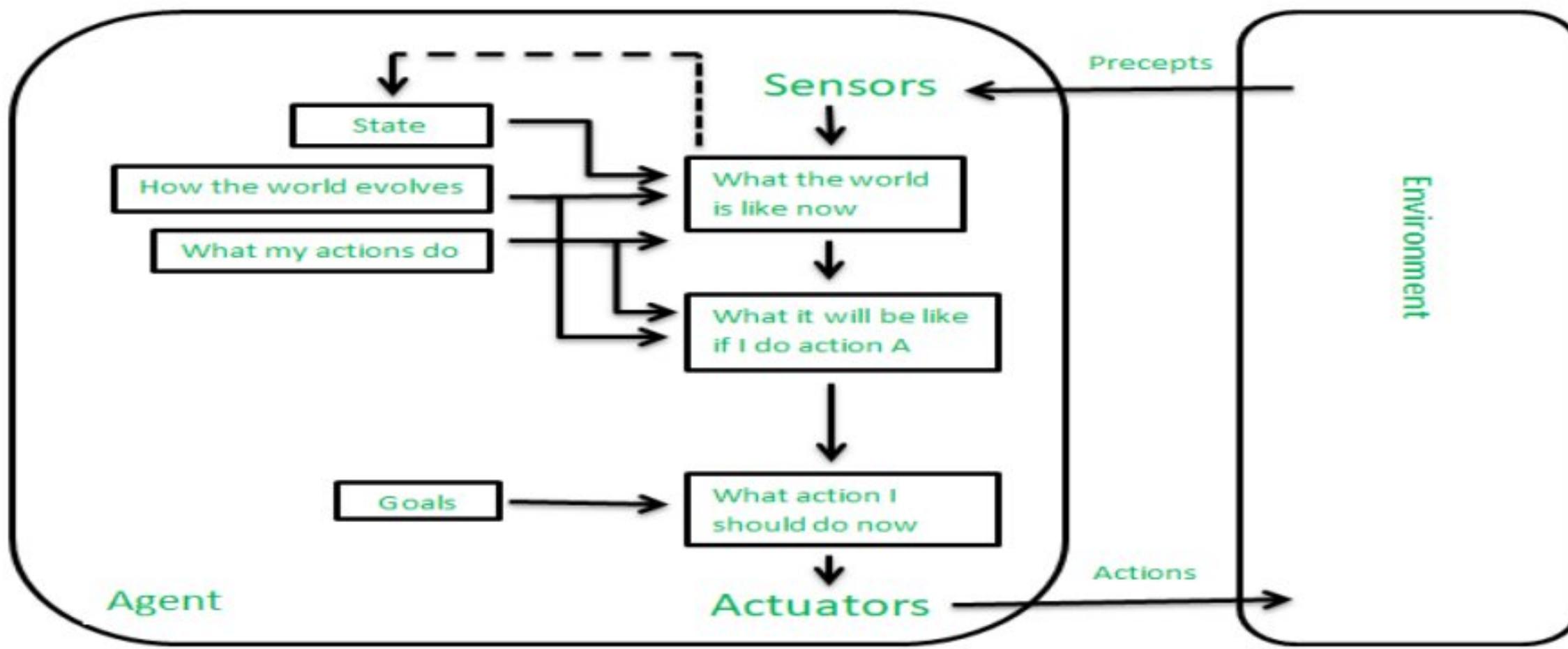
It works by finding a rule whose condition matches the current situation. A model-based agent can handle partially observable environments by the use of a model about the world. The agent has to keep track of the internal state which is adjusted by each percept and that depends on the percept history. The current state is stored inside the agent which maintains some kind of structure describing the part of the world which cannot be seen. Updating the state requires information about :

- how the world evolves independently from the agent, and
- how the agent's actions affect the world.



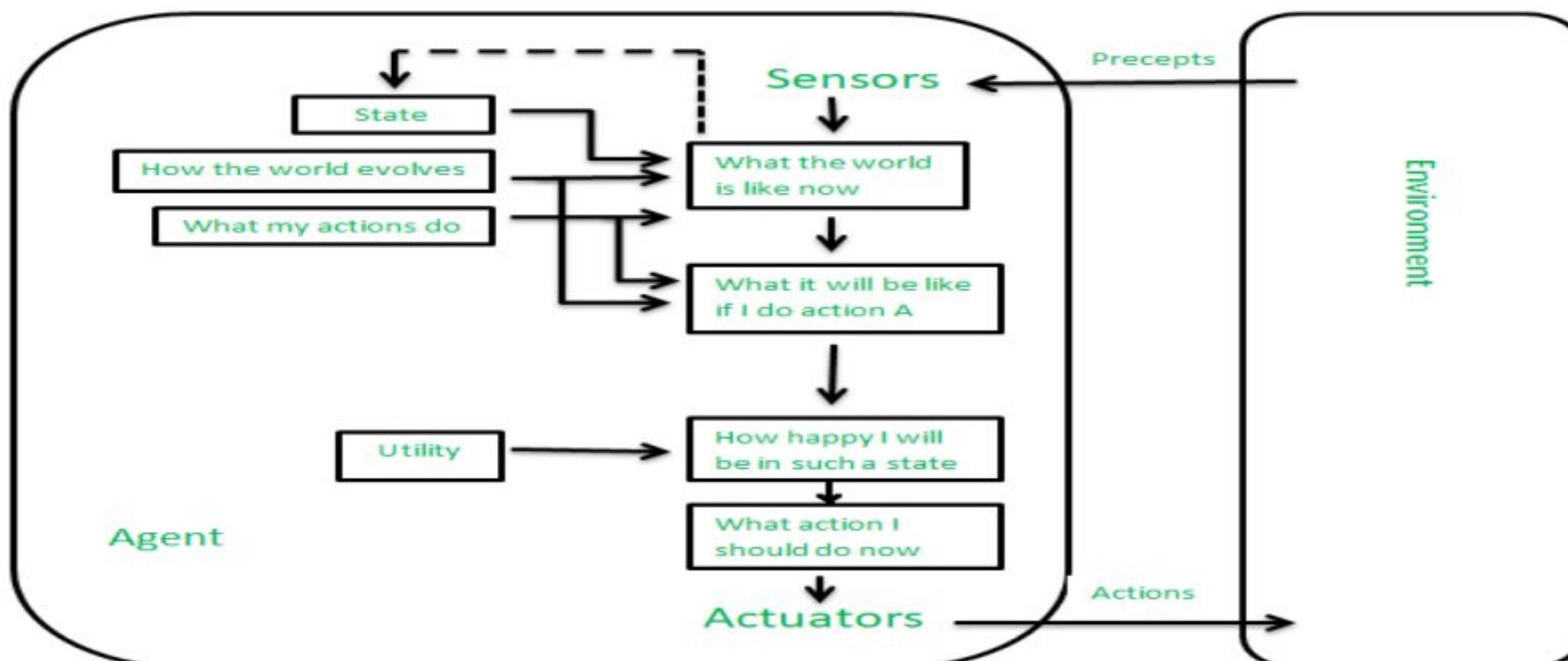
## Goal-based agents

These kinds of agents take decisions based on how far they are currently from their goal(description of desirable situations). Their every action is intended to reduce its distance from the goal. This allows the agent a way to choose among multiple possibilities, selecting the one which reaches a goal state. The knowledge that supports its decisions is represented explicitly and can be modified, which makes these agents more flexible. They usually require search and planning. The goal-based agent's behavior can easily be changed.



## Utility-based agents

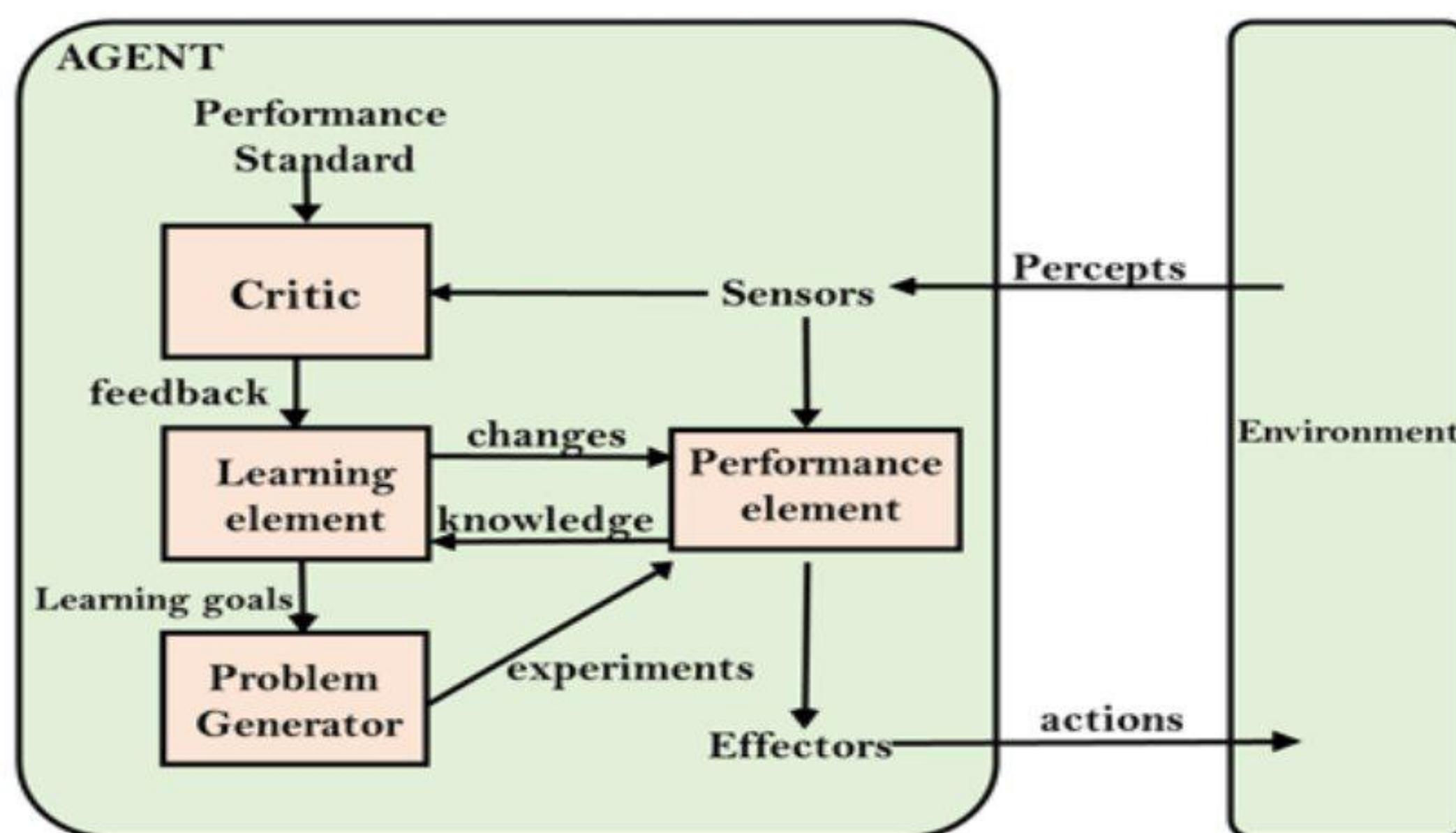
The agents which are developed having their end uses as building blocks are called utility-based agents. When there are multiple possible alternatives, then to decide which one is best, utility-based agents are used. They choose actions based on a preference (utility) for each state. Sometimes achieving the desired goal is not enough. We may look for a quicker, safer, cheaper trip to reach a destination. Agent happiness should be taken into consideration. Utility describes how “happy” the agent is. Because of the uncertainty in the world, a utility agent chooses the action that maximizes the expected utility. A utility function maps a state onto a real number which describes the associated degree of happiness.



## Learning Agent :

A learning agent in AI is the type of agent that can learn from its past experiences or it has learning capabilities. It starts to act with basic knowledge and then is able to act and adapt automatically through learning. A learning agent has mainly four conceptual components, which are:

- 1.Learning element: It is responsible for making improvements by learning from the environment
- 2.Critic: The learning element takes feedback from critics which describes how well the agent is doing with respect to a fixed performance standard.
- 3.Performance element: It is responsible for selecting external action
- 4.Problem Generator: This component is responsible for suggesting actions that will lead to new and informative experiences.



Artificial Intelligence tasks are divided into three groups, Mundane Tasks, Formal Tasks and Expert Tasks.

#### Mundane Tasks:

- Perception
- Vision
- Speech
- Natural Languages
  - Understanding
  - Generation
- Translation
- Common sense reasoning
- Robot Control

#### Formal Tasks

- Games: chess, checkers, etc
- Mathematics: Geometry, logic, Proving properties of programs

#### Expert Tasks:

- Engineering ( Design, Fault finding, Manufacturing planning)
- Scientific Analysis
- Medical Diagnosis
- Financial Analysis

## What is Intelligence?

The ability of a system to calculate, reason, perceive relationships and analogies, learn from experience, store and retrieve information from memory, solve problems, comprehend complex ideas, use natural language fluently, classify, generalize, and adapt new situations.

## Types of Intelligence

As described by Howard Gardner, an American developmental psychologist, the Intelligence comes in multifold –

Intelligence	Description	Example
Linguistic intelligence	The ability to speak, recognize, and use mechanisms of phonology (speech sounds), syntax (grammar), and semantics (meaning).	Narrators, Orators
Musical intelligence	The ability to create, communicate with, and understand meanings made of sound, understanding of pitch, rhythm.	Musicians, Singers, Composers
Logical-mathematical intelligence	The ability of use and understand relationships in the absence of action or objects. Understanding complex and abstract ideas.	Mathematicians, Scientists

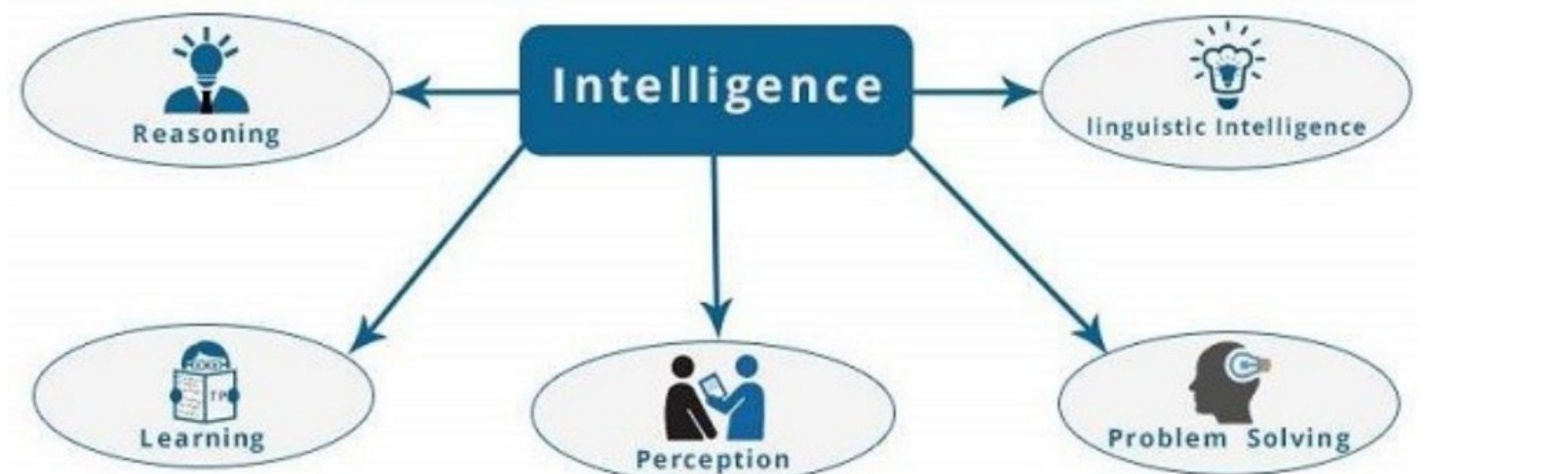
Spatial intelligence	The ability to perceive visual or spatial information, change it, and re-create visual images without reference to the objects, construct 3D images, and to move and rotate them.	Map readers, Astronauts, Physicists
Bodily-Kinesthetic intelligence	The ability to use complete or part of the body to solve problems or fashion products, control over fine and coarse motor skills, and manipulate the objects.	Players, Dancers
Intra-personal intelligence	The ability to distinguish among one's own feelings, intentions, and motivations.	Gautam Buddha
Interpersonal intelligence	The ability to recognize and make distinctions among other people's feelings, beliefs, and intentions.	Mass Communicators, Interviewers

You can say a machine or a system is **artificially intelligent** when it is equipped with at least one and at most all intelligences in it.

## What is Intelligence Composed of?

The intelligence is intangible. It is composed of –

- Reasoning
- Learning
- Problem Solving
- Perception
- Linguistic Intelligence



Let us go through all the components briefly –

- Reasoning** – It is the set of processes that enables us to provide basis for judgement, making decisions, and prediction. There are broadly two types –

Inductive Reasoning	Deductive Reasoning
It conducts specific observations to makes broad general statements.	It starts with a general statement and examines the possibilities to reach a specific, logical conclusion.
Even if all of the premises are true in a statement, inductive reasoning allows for the conclusion to be false.	If something is true of a class of things in general, it is also true for all members of that class.
Example – "Nita is a teacher. Nita is studious. Therefore, All teachers are studious."	Example – "All women of age above 60 years are grandmothers. Shalini is 65 years. Therefore, Shalini is a grandmother."

- Learning** – It is the activity of gaining knowledge or skill by studying, practising, being taught, or experiencing something. Learning enhances the awareness of the subjects of the study.

The ability of learning is possessed by humans, some animals, and AI-enabled systems. Learning is categorized as –

- Auditory Learning** – It is learning by listening and hearing. For example, students listening to recorded audio lectures.
- Episodic Learning** – To learn by remembering sequences of events that one has witnessed or experienced. This is linear and orderly.
- Motor Learning** – It is learning by precise movement of muscles. For example, picking objects, Writing, etc.
- Observational Learning** – To learn by watching and imitating others. For example, child tries to learn by mimicking her parent.
- Perceptual Learning** – It is learning to recognize stimuli that one has seen before. For example, identifying and classifying objects and situations.
- Relational Learning** – It involves learning to differentiate among various stimuli on the basis of relational properties, rather than absolute properties. For Example, Adding 'little less' salt at the time of cooking potatoes that came up salty last time, when cooked with adding say a tablespoon of salt.
- Spatial Learning** – It is learning through visual stimuli such as images, colors, maps, etc. For Example, A person can create roadmap in mind before actually following the road.

- **Stimulus-Response Learning** – It is learning to perform a particular behavior when a certain stimulus is present. For example, a dog raises its ear on hearing doorbell.
- **Problem Solving** – It is the process in which one perceives and tries to arrive at a desired solution from a present situation by taking some path, which is blocked by known or unknown hurdles.  
Problem solving also includes **decision making**, which is the process of selecting the best suitable alternative out of multiple alternatives to reach the desired goal are available.
- **Perception** – It is the process of acquiring, interpreting, selecting, and organizing sensory information.  
Perception presumes **sensing**. In humans, perception is aided by sensory organs. In the domain of AI, perception mechanism puts the data acquired by the sensors together in a meaningful manner.
- **Linguistic Intelligence** – It is one's ability to use, comprehend, speak, and write the verbal and written language. It is important in interpersonal communication.

## Difference between Human and Machine Intelligence

- Humans perceive by patterns whereas the machines perceive by set of rules and data.
- Humans store and recall information by patterns, machines do it by searching algorithms. For example, the number 40404040 is easy to remember, store, and recall as its pattern is simple.
- Humans can figure out the complete object even if some part of it is missing or distorted; whereas the machines cannot do it correctly.

## Applications of AI

AI has been dominant in various fields such as –

- **Gaming** – AI plays crucial role in strategic games such as chess, poker, tic-tac-toe, etc., where machine can think of large number of possible positions based on heuristic knowledge.
- **Natural Language Processing** – It is possible to interact with the computer that understands natural language spoken by humans.

**•Expert Systems** – There are some applications which integrate machine, software, and special information to impart reasoning and advising. They provide explanation and advice to the users.

**•Vision Systems** – These systems understand, interpret, and comprehend visual input on the computer. For example,

- A spying aeroplane takes photographs, which are used to figure out spatial information or map of the areas.
- Doctors use clinical expert system to diagnose the patient.
- Police use computer software that can recognize the face of criminal with the stored portrait made by forensic artist.

**•Speech Recognition** – Some intelligent systems are capable of hearing and comprehending the language in terms of sentences and their meanings while a human talks to it. It can handle different accents, slang words, noise in the background, change in human's noise due to cold, etc.

**•Handwriting Recognition** – The handwriting recognition software reads the text written on paper by a pen or on screen by a stylus. It can recognize the shapes of the letters and convert it into editable text.

**•Intelligent Robots** – Robots are able to perform the tasks given by a human. They have sensors to detect physical data from the real world such as light, heat, temperature, movement, sound, bump, and pressure. They have efficient processors, multiple sensors and huge memory, to exhibit intelligence. In addition, they are capable of learning from their mistakes and they can adapt to the new environment.

# Properties of Environment

The environment has multifold properties –

- **Discrete / Continuous** – If there are a limited number of distinct, clearly defined, states of the environment, the environment is discrete (For example, chess); otherwise it is continuous (For example, driving).
- **Observable / Partially Observable** – If it is possible to determine the complete state of the environment at each time point from the percepts it is observable; otherwise it is only partially observable.
- **Static / Dynamic** – If the environment does not change while an agent is acting, then it is static; otherwise it is dynamic.
- **Single agent / Multiple agents** – The environment may contain other agents which may be of the same or different kind as that of the agent.
- **Accessible / Inaccessible** – If the agent's sensory apparatus can have access to the complete state of the environment, then the environment is accessible to that agent.
- **Deterministic / Non-deterministic** – If the next state of the environment is completely determined by the current state and the actions of the agent, then the environment is deterministic; otherwise it is non-deterministic.
- **Episodic / Non-episodic** – In an episodic environment, each episode consists of the agent perceiving and then acting. The quality of its action depends just on the episode itself. Subsequent episodes do not depend on the actions in the previous episodes. Episodic environments are much simpler because the agent does not need to think ahead.

Steps problem-solving in AI: The problem of AI is directly associated with the nature of humans and their activities. So we need a number of finite steps to solve a problem which makes human easy works.

These are the following steps which require to solve a problem :

- Goal Formulation: This one is the first and simple step in problem-solving. It organizes finite steps to formulate a target/goals which require some action to achieve the goal. Today the formulation of the goal is based on AI agents.
- Problem formulation: It is one of the core steps of problem-solving which decides what action should be taken to achieve the formulated goal. In AI this core part is dependent upon software agent which consisted of the following components to formulate the associated problem.

Components to formulate the associated problem:

- Initial State: This state requires an initial state for the problem which starts the AI agent towards a specified goal. In this state new methods also initialize problem domain solving by a specific class.
- Action: This stage of problem formulation works with function with a specific class taken from the initial state and all possible actions done in this stage.
- Transition: This stage of problem formulation integrates the actual action done by the previous action stage and collects the final stage to forward it to their next stage.
- Goal test: This stage determines that the specified goal achieved by the integrated transition model or not, whenever the goal achieves stop the action and forward into the next stage to determines the cost to achieve the goal.
- Path costing: This component of problem-solving numerical assigned what will be the cost to achieve the goal. It requires all hardware software and human working cost.

## Search Algorithm Terminologies:

- **Search:** Searching is a step by step procedure to solve a search-problem in a given search space. A search problem can have three main factors:
  1. **Search Space:** Search space represents a set of possible solutions, which a system may have.
  2. **Start State:** It is a state from where agent begins **the search**.
  3. **Goal test:** It is a function which observe the current state and returns whether the goal state is achieved or not.
- **Search tree:** A tree representation of search problem is called Search tree. The root of the search tree is the root node which is corresponding to the initial state.
- **Actions:** It gives the description of all the available actions to the agent.
- **Transition model:** A description of what each action do, can be represented as a transition model.
- **Path Cost:** It is a function which assigns a numeric cost to each path.
- **Solution:** It is an action sequence which leads from the start node to the goal node.
- **Optimal Solution:** If a solution has the lowest cost among all solutions.

## Properties of Search Algorithms:

Following are the four essential properties of search algorithms to compare the efficiency of these algorithms:

**Completeness:** A search algorithm is said to be complete if it guarantees to return a solution if at least any solution exists for any random input.

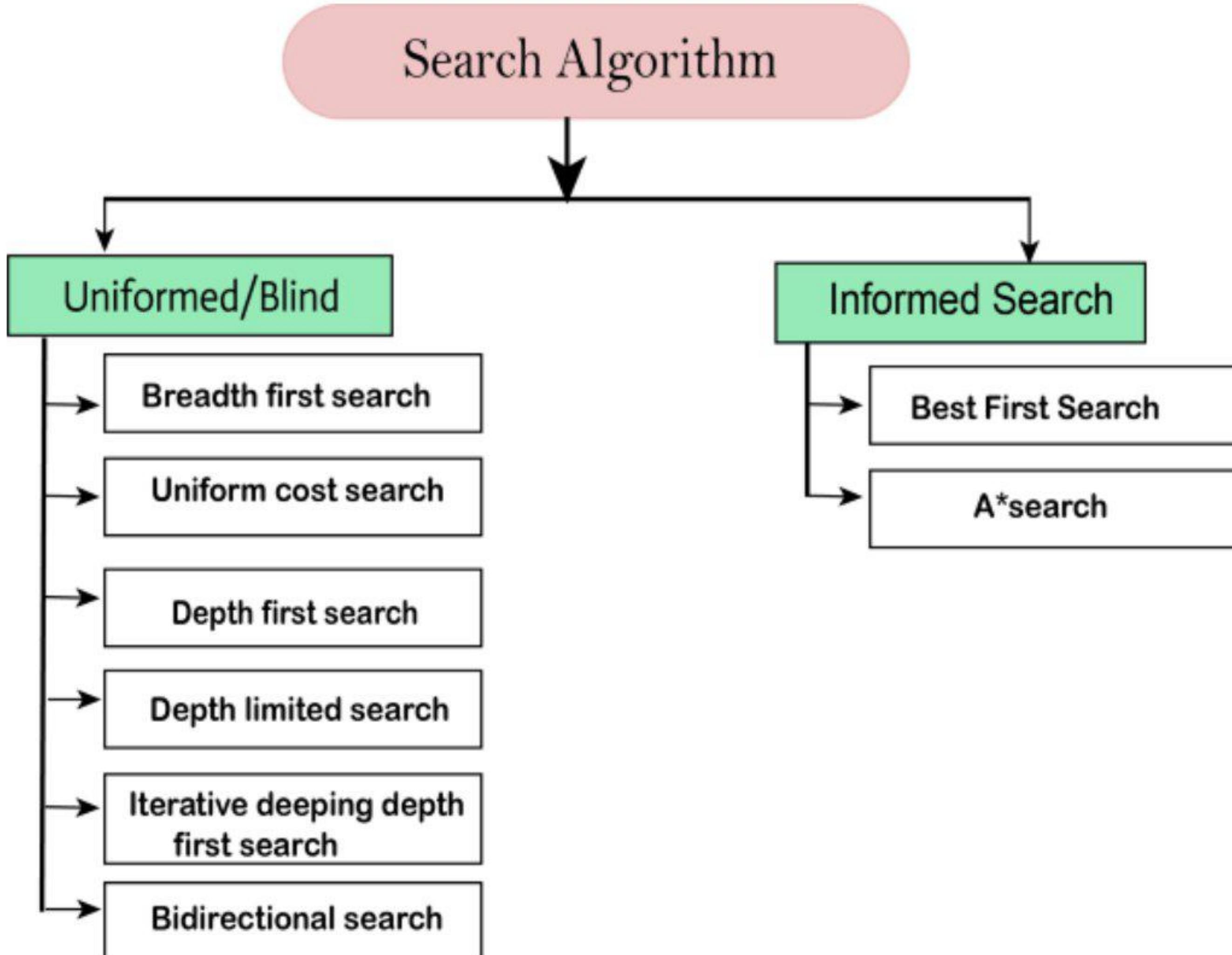
**Optimality:** If a solution found for an algorithm is guaranteed to be the best solution (lowest path cost) among all other solutions, then such a solution for is said to be an optimal solution.

**Time Complexity:** Time complexity is a measure of time for an algorithm to complete its task.

**Space Complexity:** It is the maximum storage space required at any point during the search, as the complexity of the problem.

## Types of search algorithms

Based on the search problems we can classify the search algorithms into uninformed (Blind search) search and informed search (Heuristic search) algorithms.



### Uninformed/Blind Search:

The uninformed search does not contain any domain knowledge such as closeness, the location of the goal. It operates in a brute-force way as it only includes information about how to traverse the tree and how to identify leaf and goal nodes. Uninformed search applies a way in which search tree is searched without any information about the search space like initial state operators and test for the goal, so it is also called blind search. It examines each node of the tree until it achieves the goal node.

**It can be divided into five main types:**

- Breadth-first search
- Uniform cost search
- Depth-first search
- Iterative deepening depth-first search
- Bidirectional Search

## Informed Search

Informed search algorithms use domain knowledge. In an informed search, problem information is available which can guide the search. Informed search strategies can find a solution more efficiently than an uninformed search strategy. Informed search is also called a Heuristic search.

A heuristic is a way which might not always be guaranteed for best solutions but guaranteed to find a good solution in reasonable time.

Informed search can solve much complex problem which could not be solved in another way.

An example of informed search algorithms is a traveling salesman problem.

1. Greedy Search
2. A\* Search

## 1. Breadth-first Search:

- Breadth-first search is the most common search strategy for traversing a tree or graph. This algorithm searches breadthwise in a tree or graph, so it is called breadth-first search.
- BFS algorithm starts searching from the root node of the tree and expands all successor node at the current level before moving to nodes of next level.
- The breadth-first search algorithm is an example of a general-graph search algorithm.
- Breadth-first search implemented using FIFO queue data structure.

### **Advantages:**

○BFS will provide a solution if any solution exists.

○If there are more than one solutions for a given problem, then BFS will provide the minimal solution which requires the least number of steps.

### Disadvantages:

○It requires lots of memory since each level of the tree must be saved into memory to expand the next level.

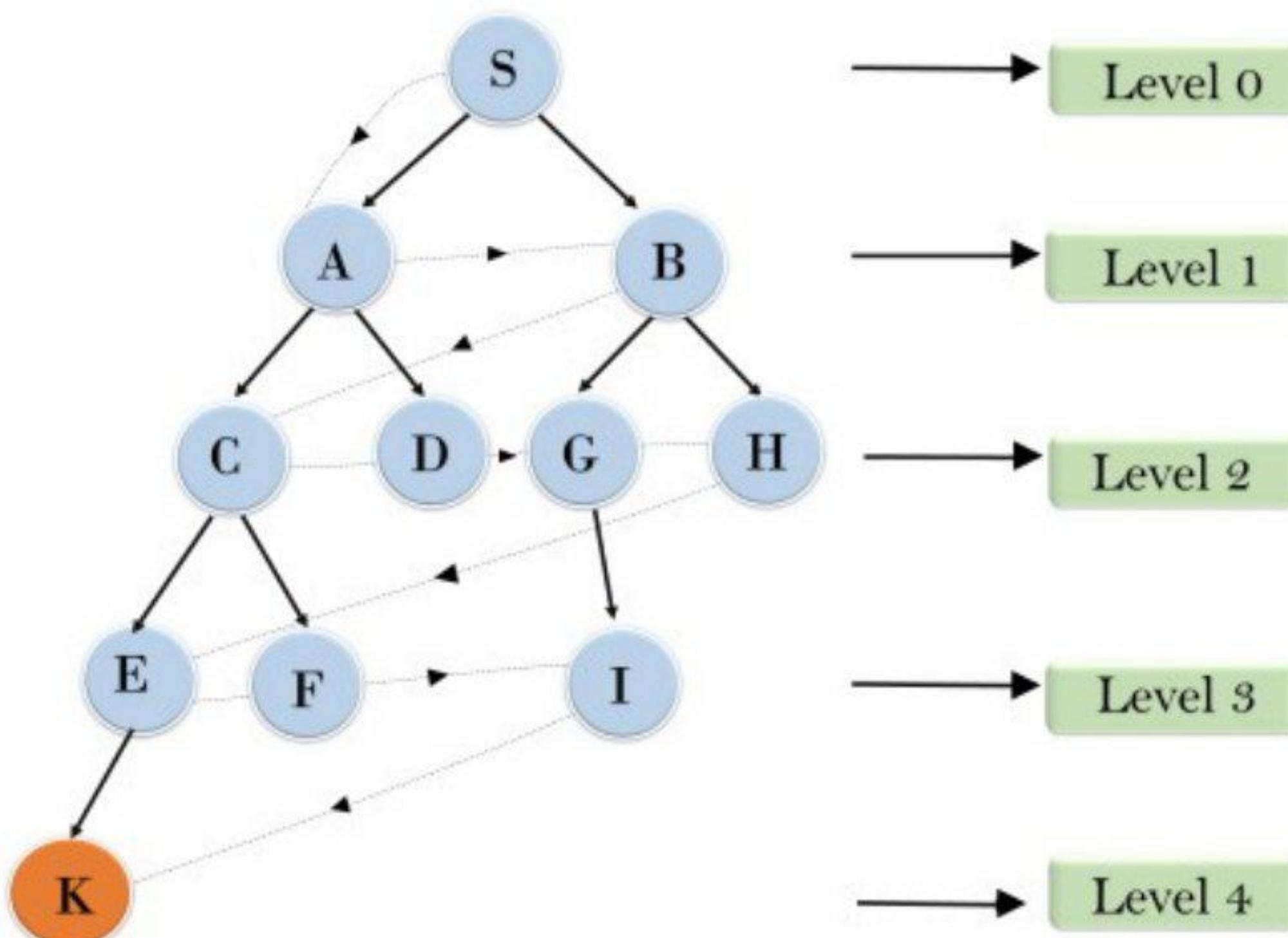
○BFS needs lots of time if the solution is far away from the root node.

### Example:

In the below tree structure, we have shown the traversing of the tree using BFS algorithm from the root node S to goal node K. BFS search algorithm traverse in layers, so it will follow the path which is shown by the dotted arrow, and the traversed path will be:

1. S---> A--->B---->C--->D---->G--->H--->E---->F---->I---->K

### Breadth First Search



**Time Complexity:** Time Complexity of BFS algorithm can be obtained by the number of nodes traversed in BFS until the shallowest Node. Where the d= depth of shallowest solution and b is a node at every state.

$$T(b) = 1+b^2+b^3+\dots+b^d = O(b^d)$$

**Space Complexity:** Space complexity of BFS algorithm is given by the Memory size of frontier which is  $O(b^d)$ .

**Completeness:** BFS is complete, which means if the shallowest goal node is at some finite depth, then BFS will find a solution.

**Optimality:** BFS is optimal if path cost is a non-decreasing function of the depth of the node.

## 2. Depth-first Search

- Depth-first search is a recursive algorithm for traversing a tree or graph data structure.
- It is called the depth-first search because it starts from the root node and follows each path to its greatest depth node before moving to the next path.
- DFS uses a stack data structure for its implementation.
- The process of the DFS algorithm is similar to the BFS algorithm.

**Note:** Backtracking is an algorithm technique for finding all possible solutions using recursion.

### Advantage:

- DFS requires very less memory as it only needs to store a stack of the nodes on the path from root node to the current node.
- It takes less time to reach to the goal node than BFS algorithm (if it traverses in the right path).

### Disadvantage:

- There is the possibility that many states keep re-occurring, and there is no guarantee of finding the solution.
- DFS algorithm goes for deep down searching and sometime it may go to the infinite loop.

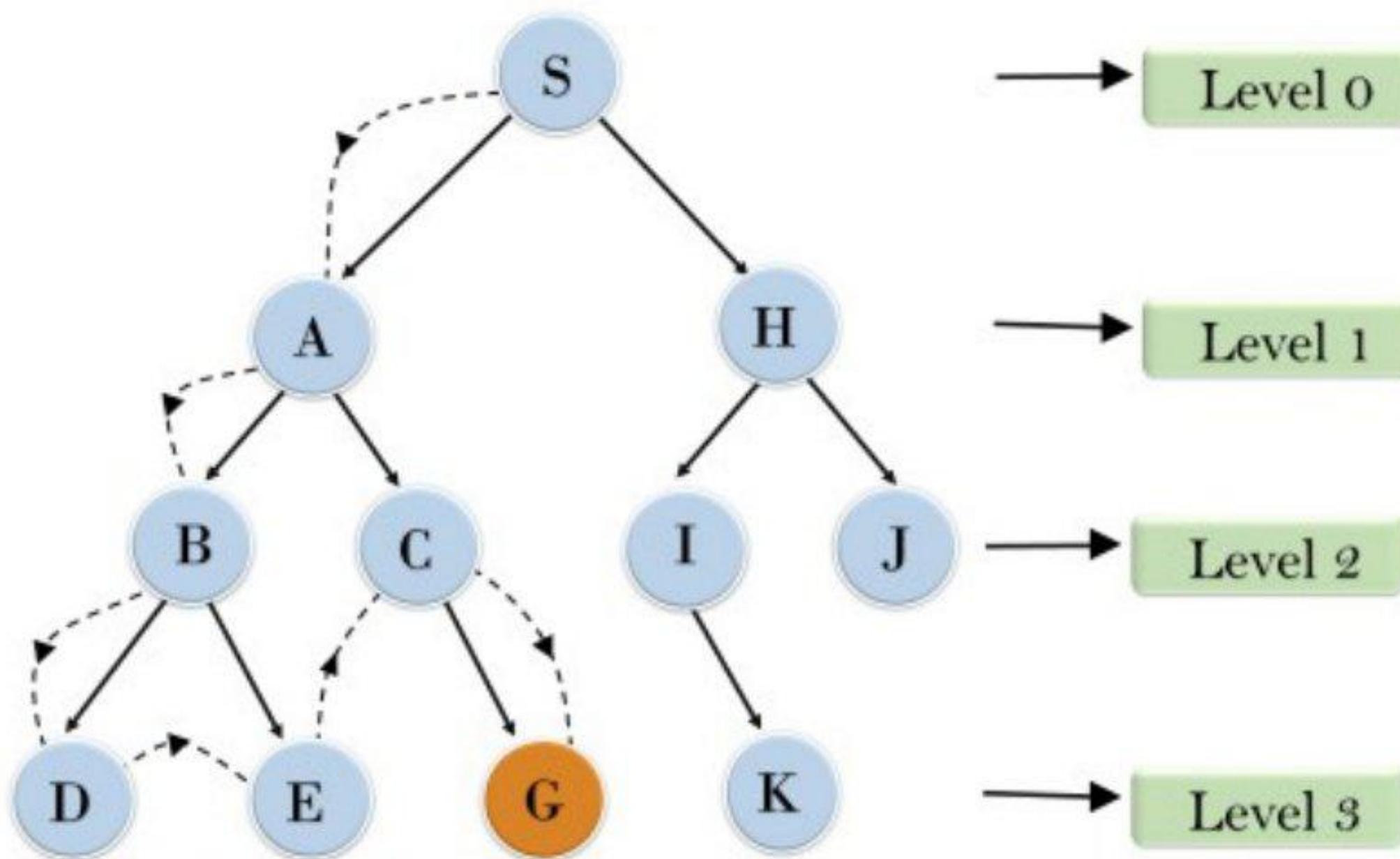
### Example:

In the below search tree, we have shown the flow of depth-first search, and it will follow the order as:

Root node-->Left node ----> right node.

It will start searching from root node S, and traverse A, then B, then D and E, after traversing E, it will backtrack the tree as E has no other successor and still goal node is not found. After backtracking it will traverse node C and then G, and here it will terminate as it found goal node.

## Depth First Search



**Completeness:** DFS search algorithm is complete within finite state space as it will expand every node within a limited search tree.

**Time Complexity:** Time complexity of DFS will be equivalent to the node traversed by the algorithm. It is given by:

$$T(n) = 1 + n^2 + n^3 + \dots + n^m = O(n^m)$$

**Where, m= maximum depth of any node and this can be much larger than d (Shallowest solution depth)**

**Space Complexity:** DFS algorithm needs to store only single path from the root node, hence space complexity of DFS is equivalent to the size of the fringe set, which is **O(bm)**.

**Optimal:** DFS search algorithm is non-optimal, as it may generate a large number of steps or high cost to reach to the goal node.

### 3. Depth-Limited Search Algorithm:

A depth-limited search algorithm is similar to depth-first search with a predetermined limit. Depth-limited search can solve the drawback of the infinite path in the Depth-first search. In this algorithm, the node at the depth limit will treat as it has no successor nodes further.

Depth-limited search can be terminated with two Conditions of failure:

- Standard failure value: It indicates that problem does not have any solution.
- Cutoff failure value: It defines no solution for the problem within a given depth limit.

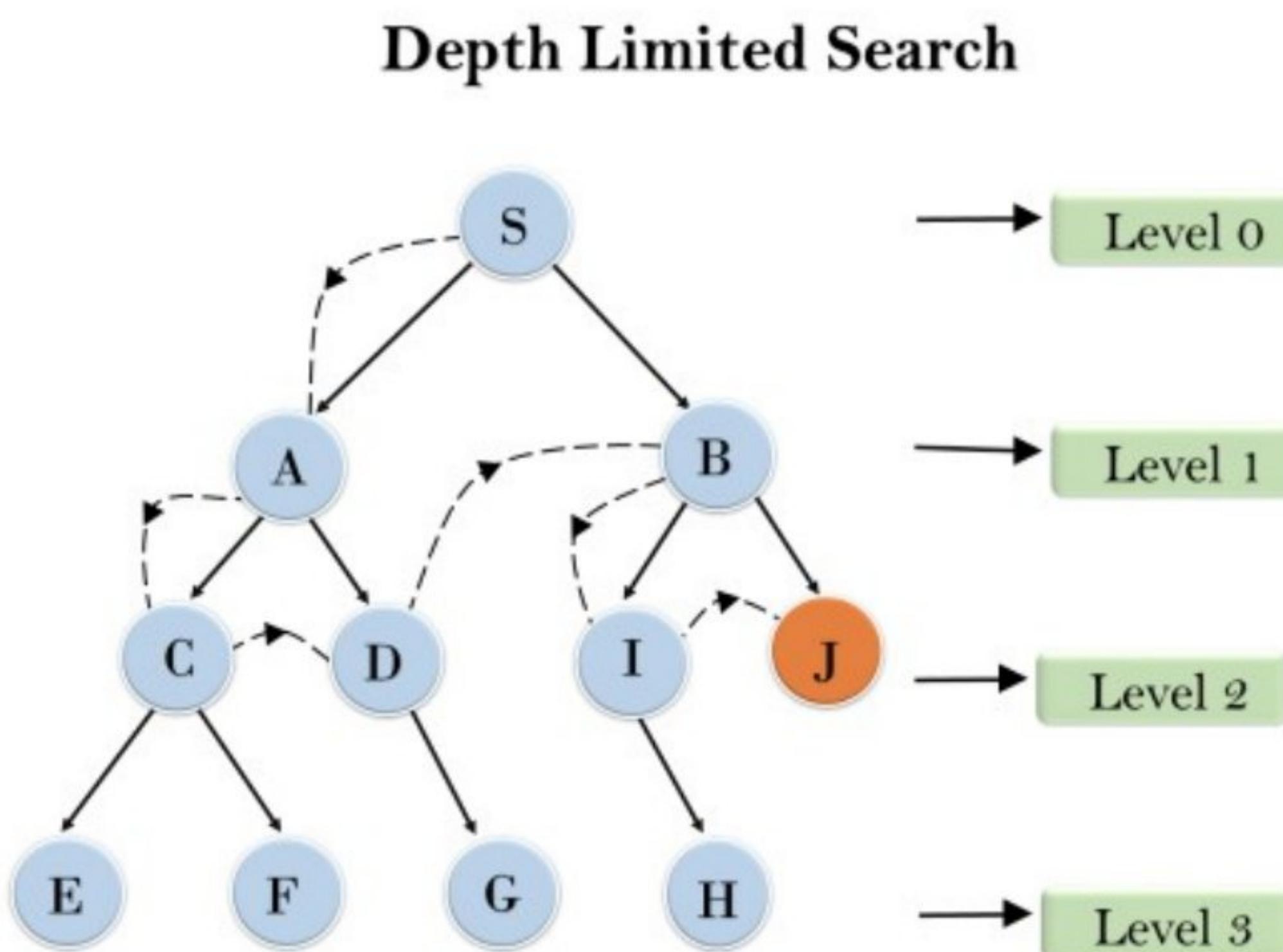
#### **Advantages:**

Depth-limited search is Memory efficient.

#### **Disadvantages:**

- Depth-limited search also has a disadvantage of incompleteness.
- It may not be optimal if the problem has more than one solution.

#### **Example:**



**Completeness:** DLS search algorithm is complete if the solution is above the depth-limit.

**Time Complexity:** Time complexity of DLS algorithm is  $O(b^l)$ .

**Space Complexity:** Space complexity of DLS algorithm is  $O(b \times l)$ .

**Optimal:** Depth-limited search can be viewed as a special case of DFS, and it is also not optimal even if  $l > d$ .

## 4. Uniform-cost Search Algorithm:

Uniform-cost search is a searching algorithm used for traversing a weighted tree or graph. This algorithm comes into play when a different cost is available for each edge. The primary goal of the uniform-cost search is to find a path to the goal node which has the lowest cumulative cost. Uniform-cost search expands nodes according to their path costs from the root node. It can be used to solve any graph/tree where the optimal cost is in demand. A uniform-cost search algorithm is implemented by the priority queue. It gives maximum priority to the lowest cumulative cost. Uniform cost search is equivalent to BFS algorithm if the path cost of all edges is the same.

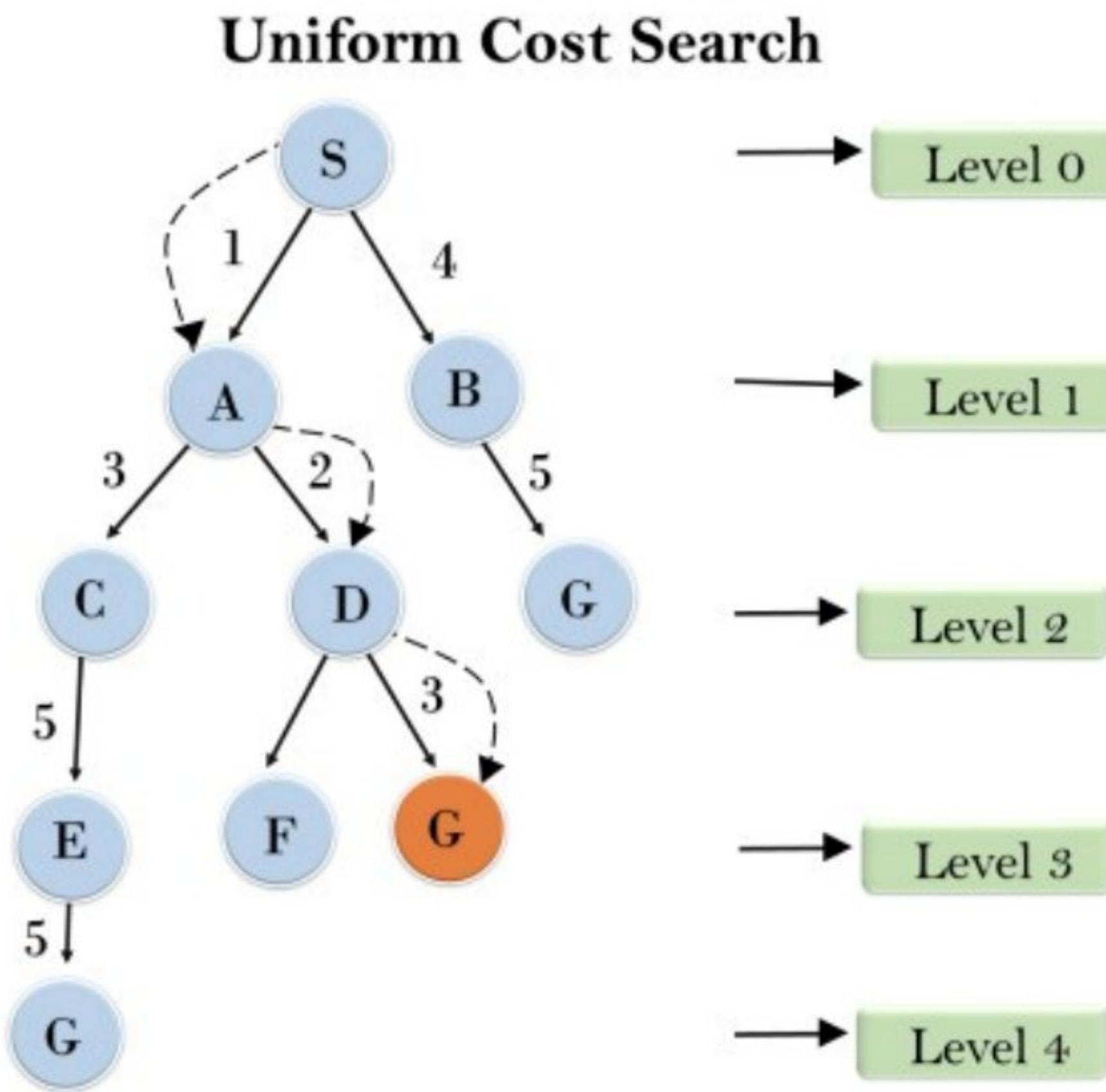
### **Advantages:**

- Uniform cost search is optimal because at every state the path with the least cost is chosen.

### **Disadvantages:**

- It does not care about the number of steps involved in searching and only concerned about path cost. Due to which this algorithm may be stuck in an infinite loop.

Example:



### Completeness:

Uniform-cost search is complete, such as if there is a solution, UCS will find it.

### Time Complexity:

Let  $C^*$  is **Cost of the optimal solution**, and  $\epsilon$  is each step to get closer to the goal node. Then the number of steps is  $= C^*/\epsilon + 1$ . Here we have taken  $+1$ , as we start from state 0 and end to  $C^*/\epsilon$ .

Hence, the worst-case time complexity of Uniform-cost search is  $O(b^{C^*/\epsilon})$ .

### Space Complexity:

The same logic is for space complexity so, the worst-case space complexity of Uniform-cost search is  $O(b^{C^*/\epsilon})$ .

### Optimal:

Uniform-cost search is always optimal as it only selects a path with the lowest path cost.

## 5. Iterative deepeningdepth-first Search:

The iterative deepening algorithm is a combination of DFS and BFS algorithms. This search algorithm finds out the best depth limit and does it by gradually increasing the limit until a goal is found.

This algorithm performs depth-first search up to a certain "depth limit", and it keeps increasing the depth limit after each iteration until the goal node is found.

This Search algorithm combines the benefits of Breadth-first search's fast search and depth-first search's memory efficiency.

The iterative search algorithm is useful uninformed search when search space is large, and depth of goal node is unknown.

### **Advantages:**

- It combines the benefits of BFS and DFS search algorithm in terms of fast search and memory efficiency.

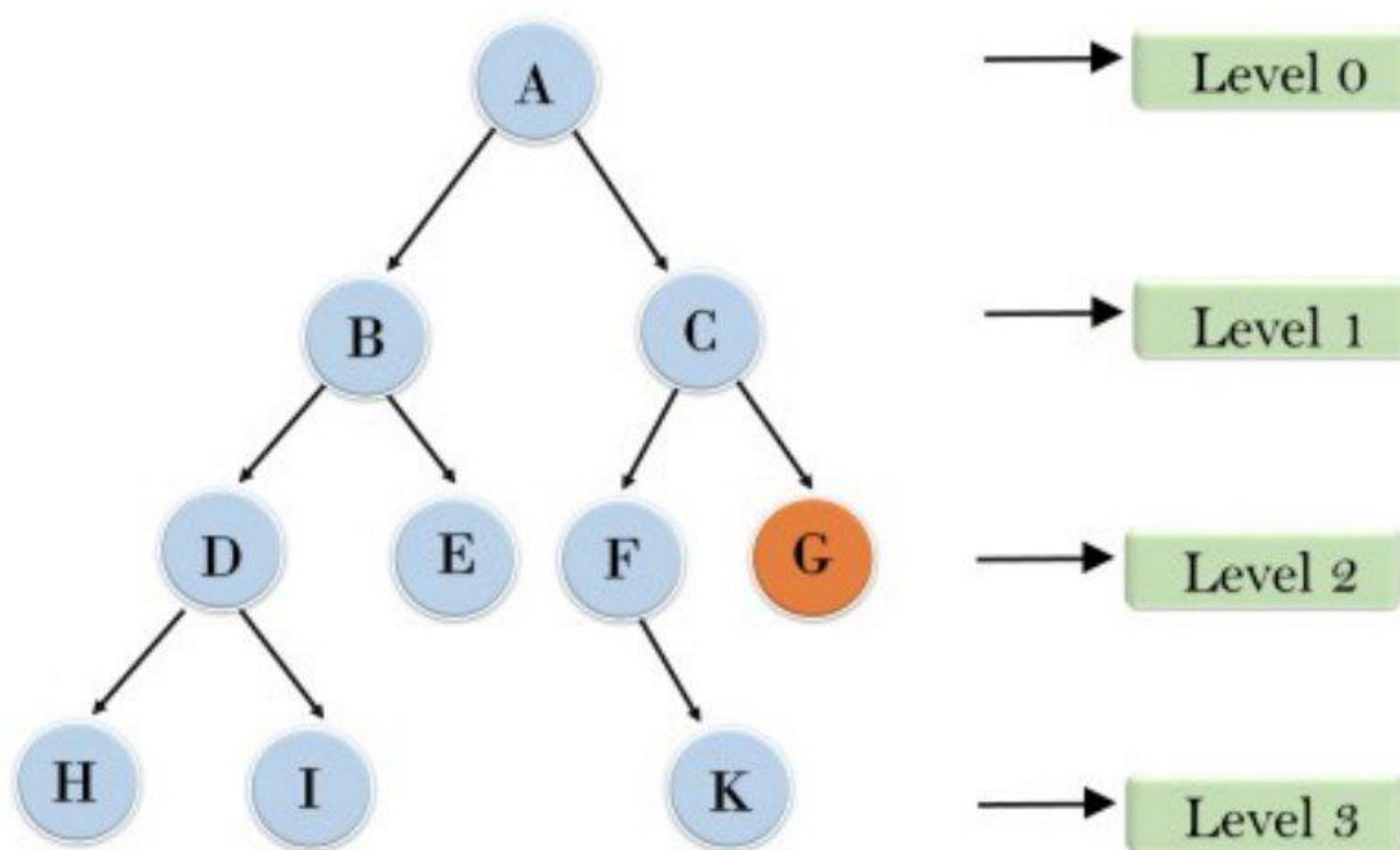
### **Disadvantages:**

- The main drawback of IDDFS is that it repeats all the work of the previous phase.

### **Example:**

Following tree structure is showing the iterative deepening depth-first search. IDDFS algorithm performs various iterations until it does not find the goal node. The iteration performed by the algorithm is given as:

## Iterative deepening depth first search



1'st	Iteration---->	A	C	G
2'nd	Iteration---->	B,	B,	C
3'rd	Iteration----->	D,	E,	F,
4'th	Iteration----->	H,	I,	K,
In the fourth iteration, the algorithm will find the goal node.				

### Completeness:

This algorithm is complete if the branching factor is finite.

### Time Complexity:

Let's suppose  $b$  is the branching factor and depth is  $d$  then the worst-case time complexity is  $O(b^d)$ .

### Space Complexity:

The space complexity of IDDFS will be  $O(bd)$ .

### Optimal:

IDDFS algorithm is optimal if path cost is a non-decreasing function of the depth of the node.

## 6. Bidirectional Search Algorithm:

**Bidirectional search algorithm runs two simultaneous searches, one from initial state called as forward-search and other from goal node called as backward-search, to find the goal node. Bidirectional search replaces one single search graph with two small subgraphs in which one starts the search from an initial vertex and other starts from goal vertex. The search stops when these two graphs intersect each other.**

**Bidirectional search can use search techniques such as BFS, DFS, DLS, etc.**

### **Advantages:**

- Bidirectional search is fast.
- Bidirectional search requires less memory

### **Disadvantages:**

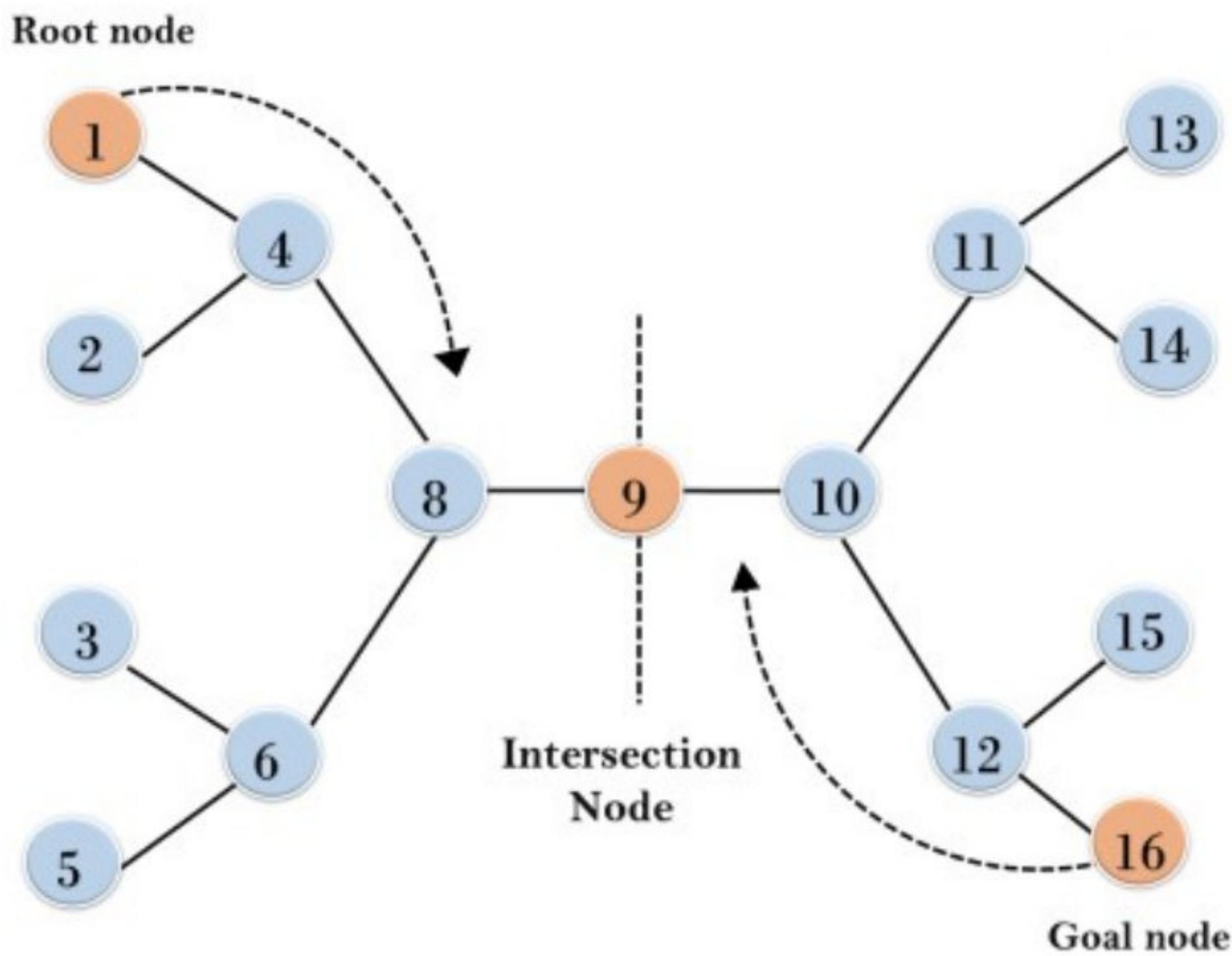
- Implementation of the bidirectional search tree is difficult.
- In bidirectional search, one should know the goal state in advance.

### **Example:**

In the below search tree, bidirectional search algorithm is applied. This algorithm divides one graph/tree into two sub-graphs. It starts traversing from node 1 in the forward direction and starts from goal node 16 in the backward direction.

The algorithm terminates at node 9 where two searches meet.

## Bidirectional Search



**Completeness:** Bidirectional Search is complete if we use BFS in both searches.

**Time Complexity:** Time complexity of bidirectional search using BFS is  $O(b^d)$ .

**Space Complexity:** Space complexity of bidirectional search is  $O(b^d)$ .

**Optimal:** Bidirectional search is Optimal.

informed search algorithm contains an array of knowledge such as how far we are from the goal, path cost, how to reach to goal node, etc. This knowledge help agents to explore less to the search space and find more efficiently the goal node.

The informed search algorithm is more useful for large search space. Informed search algorithm uses the idea of heuristic, so it is also called Heuristic search.

## What is Heuristics?

A heuristic is a technique that is used to solve a problem faster than the classic methods. These techniques are used to find the approximate solution of a problem when classical methods do not. Heuristics are said to be the problem-solving techniques that result in practical and quick solutions.

Heuristics are strategies that are derived from past experience with similar problems. Heuristics use practical methods and shortcuts used to produce the

solutions that may or may not be optimal, but those solutions are sufficient in a given limited timeframe.

**Heuristics function:** Heuristic is a function which is used in Informed Search, and it finds the most promising path. It takes the current state of the agent as its input and produces the estimation of how close agent is from the goal. The heuristic method, however, might not always give the best solution, but it guaranteed to find a good solution in reasonable time. Heuristic function estimates how close a state is to the goal. It is represented by  $h(n)$ , and it calculates the cost of an optimal path between the pair of states. The value of the heuristic function is always positive.

**Admissibility of the heuristic function is given as:**

$$1. h(n) \leq h^*(n)$$

Here  $h(n)$  is heuristic cost, and  $h^*(n)$  is the estimated cost. Hence heuristic cost should be less than or equal to the estimated cost.

## Pure Heuristic Search:

Pure heuristic search is the simplest form of heuristic search algorithms. It expands nodes based on their heuristic value  $h(n)$ . It maintains two lists, OPEN and CLOSED list. In the CLOSED list, it places those nodes which have already expanded and in the OPEN list, it places nodes which have yet not been expanded.

On each iteration, each node  $n$  with the lowest heuristic value is expanded and generates all its successors and  $n$  is placed to the closed list. The algorithm continues until a goal state is found.

In the informed search we will discuss two main algorithms which are given below:

- **Best First Search Algorithm(Greedy search)**

- **A\* Search Algorithm**

## 1.) Best-first Search Algorithm (Greedy Search):

Greedy best-first search algorithm always selects the path which appears best at that moment. It is the combination of depth-first search and breadth-first search algorithms. It uses the heuristic function and search. Best-first search allows us to take the advantages of both algorithms. With the help of best-first search, at each step, we can choose the most promising node. In the best first search algorithm,

we expand the node which is closest to the goal node and the closest cost is estimated by heuristic function, i.e.

1.  $f(n) = g(n)$ .

Where,  $h(n)$  = estimated cost from node  $n$  to the goal.

The greedy best first algorithm is implemented by the priority queue.

## Best first search algorithm:

- **Step 1:** Place the starting node into the OPEN list.
- **Step 2:** If the OPEN list is empty, Stop and return failure.
- **Step 3:** Remove the node  $n$ , from the OPEN list which has the lowest value of  $h(n)$ , and places it in the CLOSED list.
- **Step 4:** Expand the node  $n$ , and generate the successors of node  $n$ .
- **Step 5:** Check each successor of node  $n$ , and find whether any node is a goal node or not. If any successor node is goal node, then return success and terminate the search, else proceed to Step 6.
- **Step 6:** For each successor node, algorithm checks for evaluation function  $f(n)$ , and then check if the node has been in either OPEN or CLOSED list. If the node has not been in both list, then add it to the OPEN list.
- **Step 7:** Return to Step 2.

## Advantages:

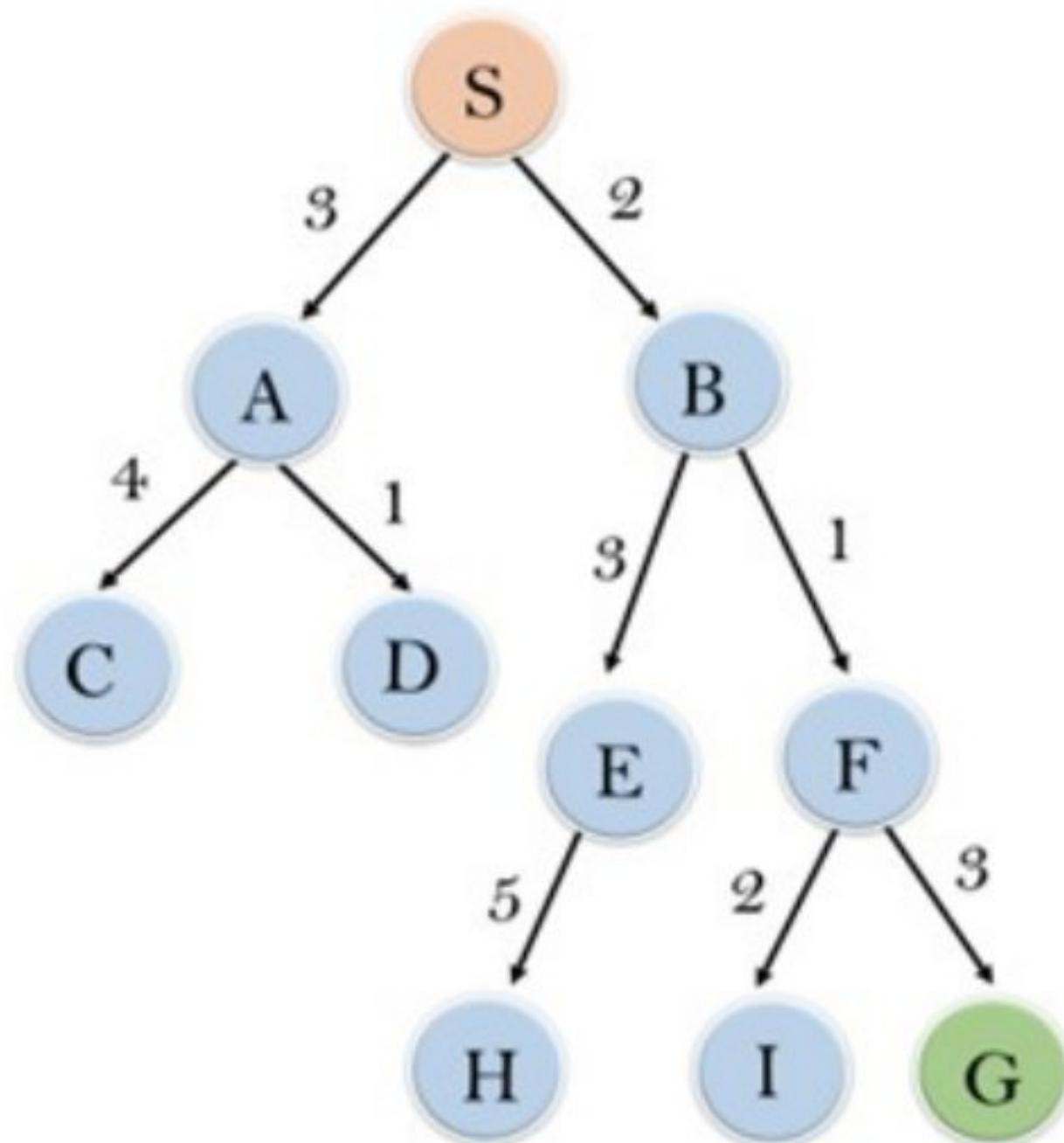
- Best first search can switch between BFS and DFS by gaining the advantages of both the algorithms.
- This algorithm is more efficient than BFS and DFS algorithms.

## Disadvantages:

- It can behave as an unguided depth-first search in the worst case scenario.
- It can get stuck in a loop as DFS.
- This algorithm is not optimal.

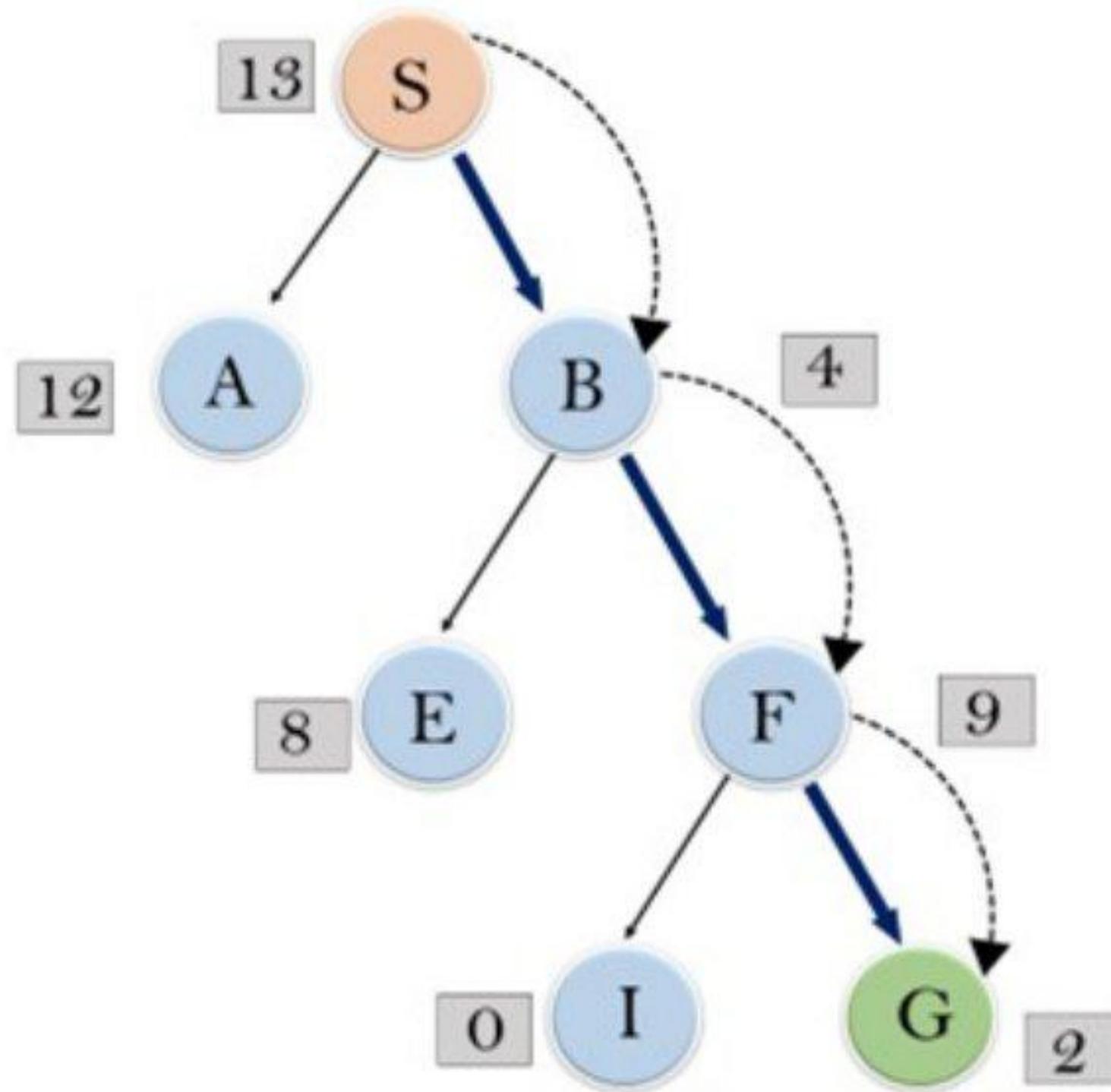
## Example:

Consider the below search problem, and we will traverse it using greedy best-first search. At each iteration, each node is expanded using evaluation function  $f(n)=h(n)$ , which is given in the below table.



node	H (n)
A	12
B	4
C	7
D	3
E	8
F	2
H	4
I	9
S	13
G	0

In this search example, we are using two lists which are **OPEN** and **CLOSED** Lists. Following are the iteration for traversing the above example.



**Expand the nodes of S and put in the CLOSED list**

**Initialization:** Open [A, B], Closed [S]

**Iteration 1:** Open [A], Closed [S, B]

**Iteration 2:** Open [E, F, A], Closed [S, B]  
 : Open [E, A], Closed [S, B, F]

**Iteration 3:** Open [I, G, E, A], Closed [S, B, F]  
 : Open [I, E, A], Closed [S, B, F, G]

Hence the final solution path will be: **S----> B---->F----> G**

**Time Complexity:** The worst case time complexity of Greedy best first search is  $O(b^m)$ .

**Space Complexity:** The worst case space complexity of Greedy best first search is  $O(b^m)$ . Where, m is the maximum depth of the search space.

**Complete:** Greedy best-first search is also incomplete, even if the given state space is finite.

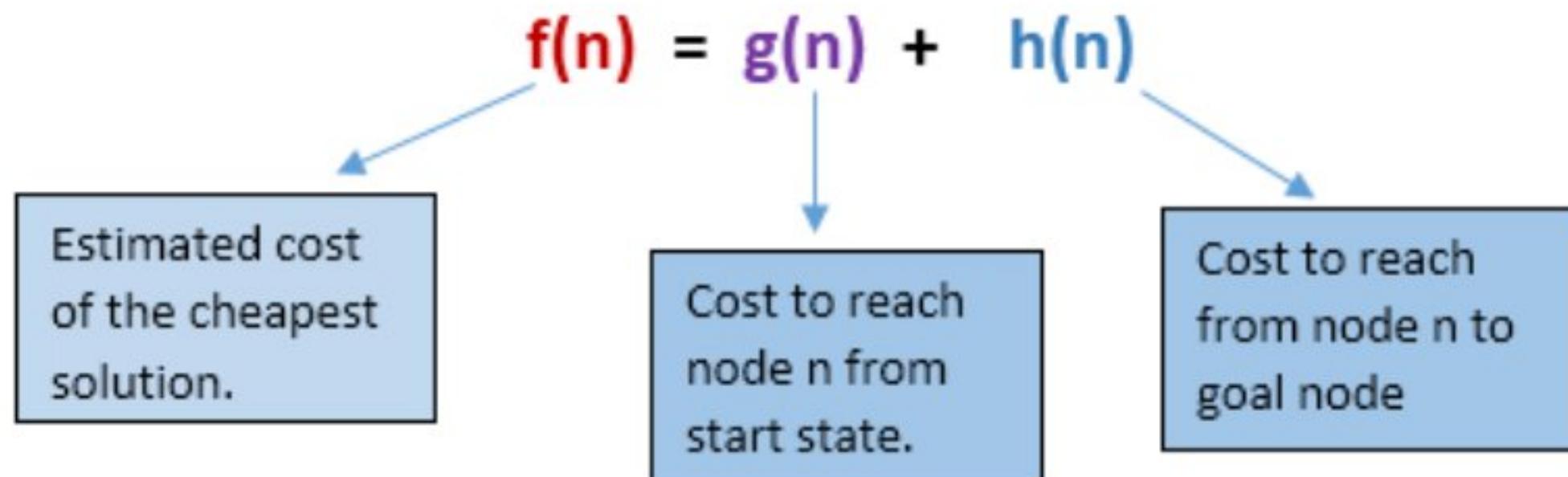
**Optimal:** Greedy best first search algorithm is not optimal.

## 2.) A\* Search Algorithm:

A\* search is the most commonly known form of best-first search. It uses heuristic function  $h(n)$ , and cost to reach the node n from the start state  $g(n)$ . It has combined features of UCS and greedy best-first search, by which it solve the

problem efficiently. A\* search algorithm finds the shortest path through the search space using the heuristic function. This search algorithm expands less search tree and provides optimal result faster. A\* algorithm is similar to UCS except that it uses  $g(n) + h(n)$  instead of  $g(n)$ .

In A\* search algorithm, we use search heuristic as well as the cost to reach the node. Hence we can combine both costs as following, and this sum is called as a **fitness number**.



At each point in the search space, only those node is expanded which have the lowest value of  $f(n)$ , and the algorithm terminates when the goal node is found.

## Algorithm of A\* search:

**Step 1:** Place the starting node in the OPEN list.

**Step 2:** Check if the OPEN list is empty or not, if the list is empty then return failure and stops.

**Step 3:** Select the node from the OPEN list which has the smallest value of evaluation function ( $g+h$ ), if node n is goal node then return success and stop, otherwise

**Step 4:** Expand node n and generate all of its successors, and put n into the closed list. For each successor  $n'$ , check whether  $n'$  is already in the OPEN or CLOSED list, if not then compute evaluation function for  $n'$  and place into Open list.

**Step 5:** Else if node  $n'$  is already in OPEN and CLOSED, then it should be attached to the back pointer which reflects the lowest  $g(n')$  value.

**Step 6:** Return to **Step 2**.

## Advantages:

- A\* search algorithm is the best algorithm than other search algorithms.
- A\* search algorithm is optimal and complete.
- This algorithm can solve very complex problems.

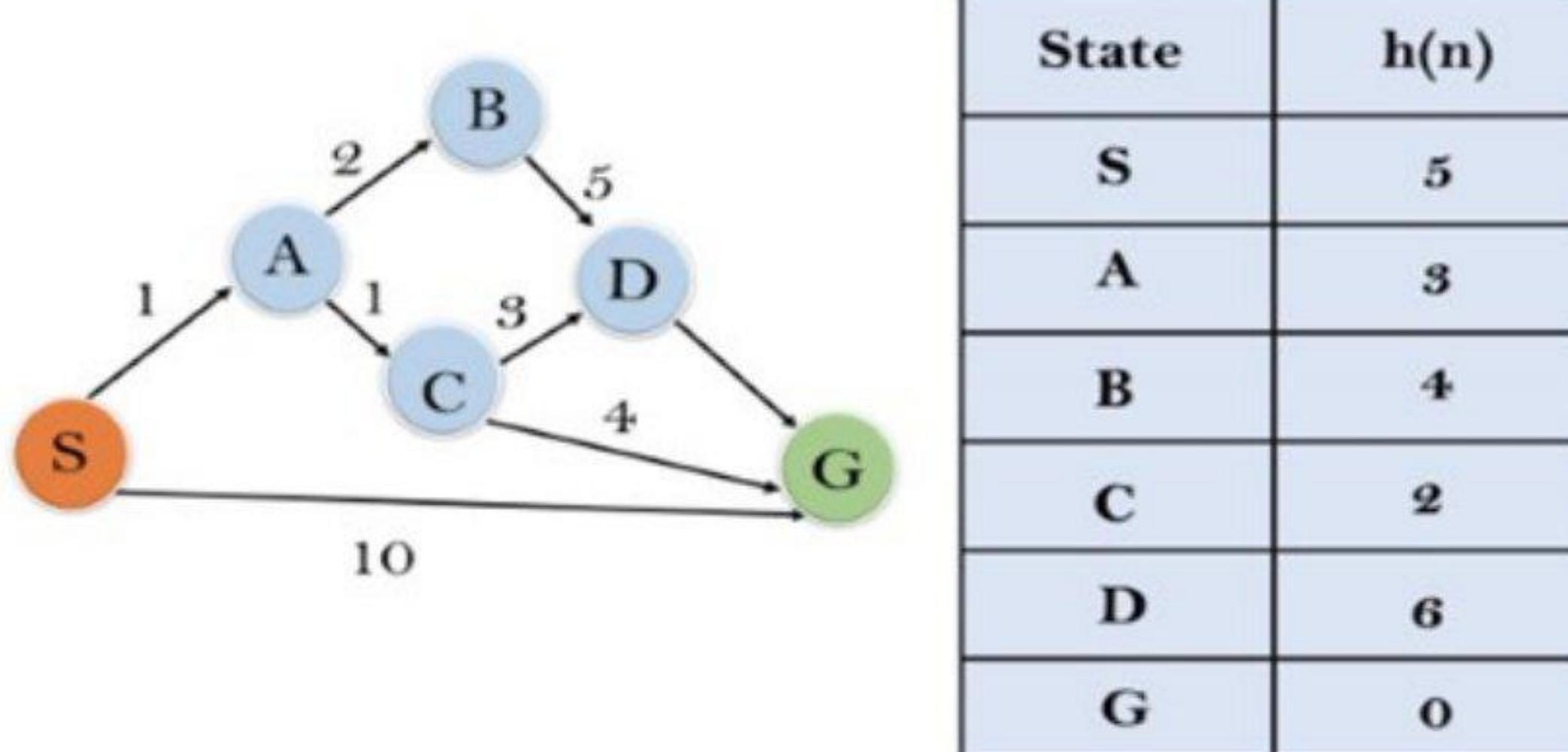
## Disadvantages:

- It does not always produce the shortest path as it mostly based on heuristics and approximation.
- A\* search algorithm has some complexity issues.
- The main drawback of A\* is memory requirement as it keeps all generated nodes in the memory, so it is not practical for various large-scale problems.

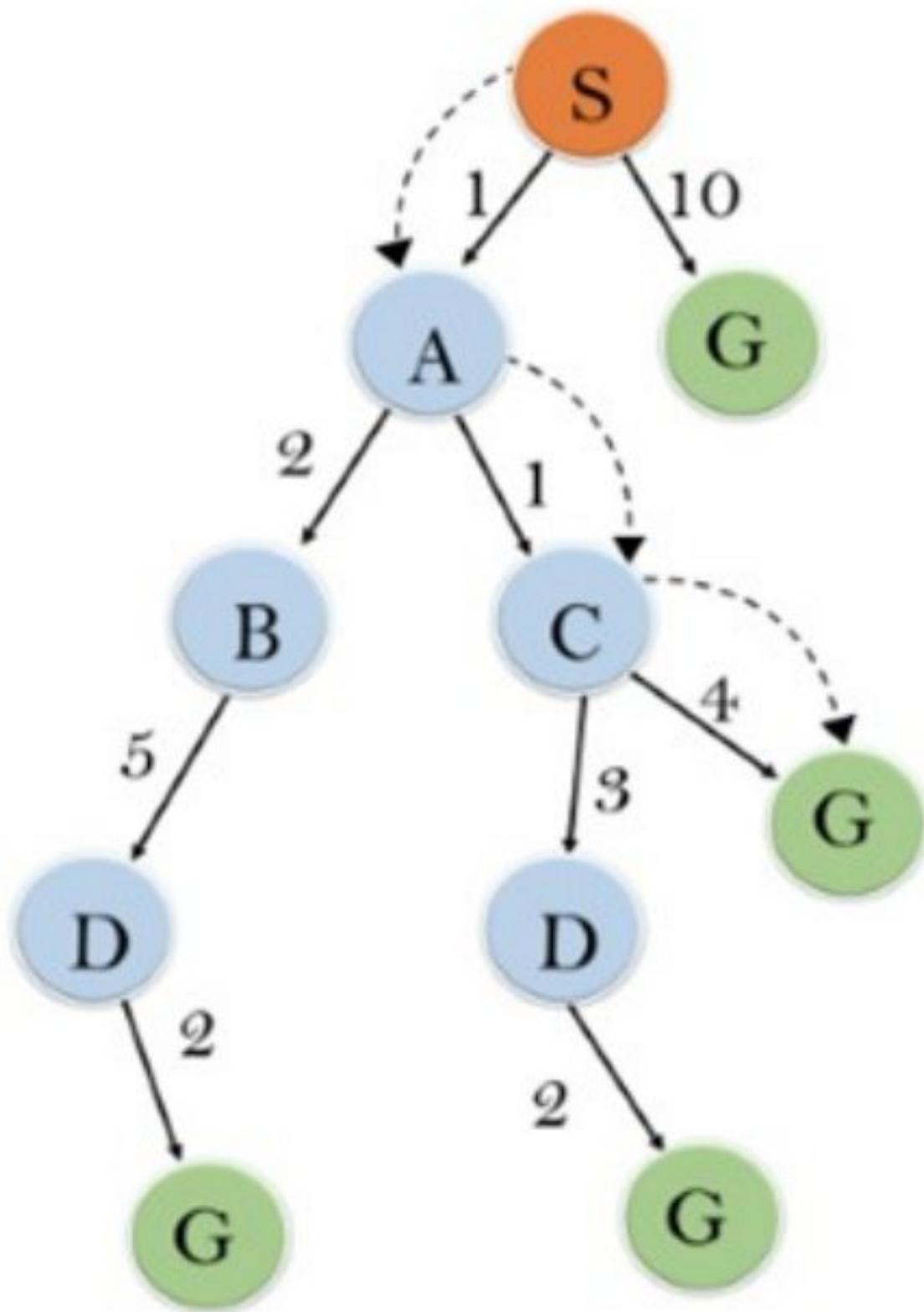
## Example:

In this example, we will traverse the given graph using the A\* algorithm. The heuristic value of all states is given in the below table so we will calculate the  $f(n)$  of each state using the formula  $f(n) = g(n) + h(n)$ , where  $g(n)$  is the cost to reach any node from start state.

Here we will use OPEN and CLOSED list.



## Solution:



**Initialization:**  $\{(S, 5)\}$

**Iteration1:**  $\{(S \rightarrow A, 4), (S \rightarrow G, 10)\}$

**Iteration2:**  $\{(S \rightarrow A \rightarrow C, 4), (S \rightarrow A \rightarrow B, 7), (S \rightarrow G, 10)\}$

**Iteration3:**  $\{(S \rightarrow A \rightarrow C \rightarrow G, 6), (S \rightarrow A \rightarrow C \rightarrow D, 11), (S \rightarrow A \rightarrow B, 7), (S \rightarrow G, 10)\}$

**Iteration 4** will give the final result, as **S--->A--->C--->G** it provides the optimal path with cost 6.

#### **Points to remember:**

- A\* algorithm returns the path which occurred first, and it does not search for all remaining paths.
- The efficiency of A\* algorithm depends on the quality of heuristic.
- A\* algorithm expands all nodes which satisfy the condition  $f(n)$

**Complete:** A\* algorithm is complete as long as:

- Branching factor is finite.
- Cost at every action is fixed.

**Optimal:** A\* search algorithm is optimal if it follows below two conditions:

○**Admissible:** the first condition requires for optimality is that  $h(n)$  should be an admissible heuristic for A\* tree search. An admissible heuristic is optimistic in nature.

○**Consistency:** Second required condition is consistency for only A\* graph-search.

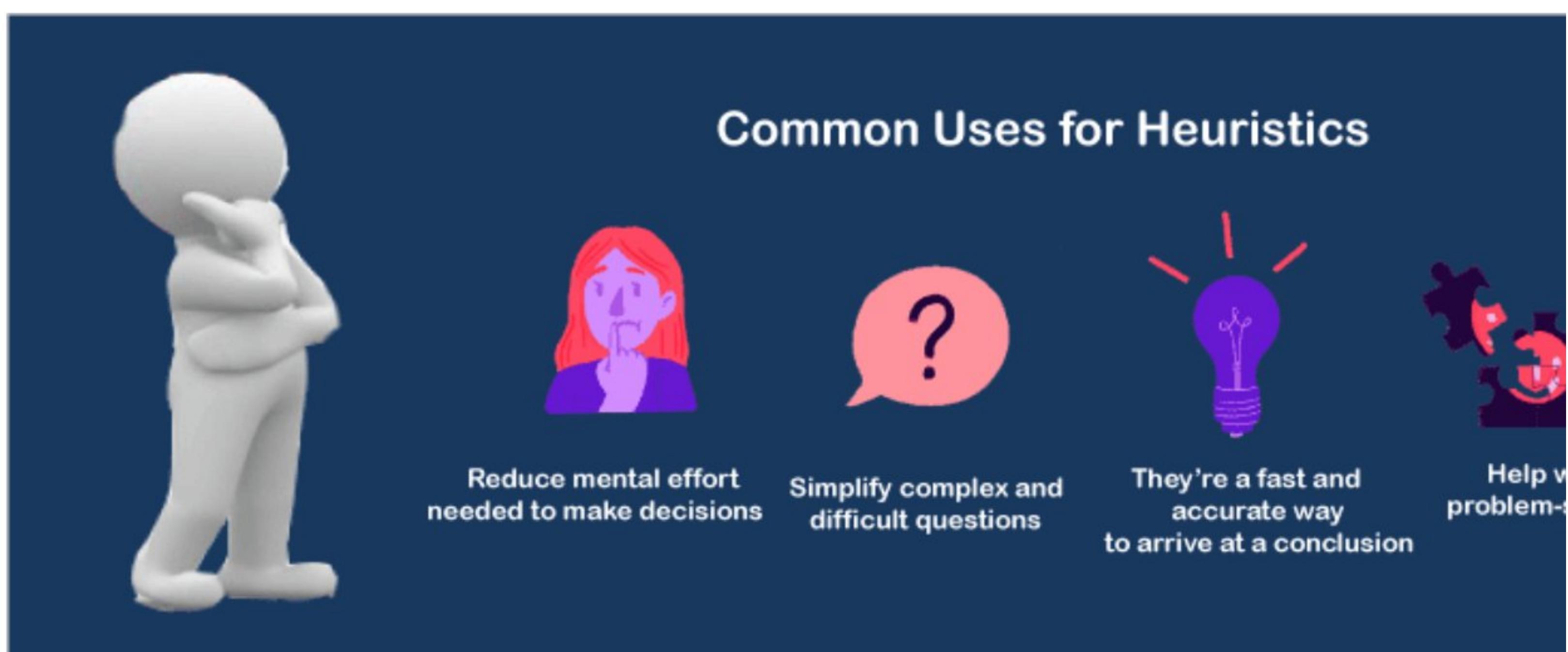
If the heuristic function is admissible, then A\* tree search will always find the least cost path.

**Time Complexity:** The time complexity of A\* search algorithm depends on heuristic function, and the number of nodes expanded is exponential to the depth of solution  $d$ . So the time complexity is  $O(b^d)$ , where  $b$  is the branching factor.

**Space Complexity:** The space complexity of A\* search algorithm is  **$O(b^d)$**

## Why do we need heuristics?

Heuristics are used in situations in which there is the requirement of a short-term solution. On facing complex situations with limited resources and time, Heuristics can help the companies to make quick decisions by shortcuts and approximated calculations. Most of the heuristic methods involve mental shortcuts to make decisions on past experiences.



The heuristic method might not always provide us the finest solution, but it is assured that it helps us find a good solution in a reasonable time.

Based on context, there can be different heuristic methods that correlate with the problem's scope. The most common heuristic methods are - trial and error, guesswork, the process of elimination, historical data analysis. These methods involve simply available information that is not particular to the problem but is most appropriate. They can include representative, affect, and availability heuristics.

## Heuristic search techniques in AI (Artificial Intelligence)



We can perform the Heuristic techniques into two categories:

### Direct Heuristic Search techniques in AI

It includes Blind Search, Uninformed Search, and Blind control strategy. These search techniques are not always possible as they require much memory and time. These techniques search the complete space for a solution and use the arbitrary ordering of operations.

The examples of Direct Heuristic search techniques include Breadth-First Search (BFS) and Depth First Search (DFS).

## Weak Heuristic Search techniques in AI

It includes Informed Search, Heuristic Search, and Heuristic control strategy. These techniques are helpful when they are applied properly to the right types of tasks. They usually require domain-specific information.

The examples of Weak Heuristic search techniques include Best First Search (BFS) and A\*.

Before describing certain heuristic techniques, let's see some of the techniques listed below:

- Bidirectional Search
- A\* search
- Simulated Annealing
- Hill Climbing
- Best First search
- Beam search

First, let's talk about the Hill climbing in Artificial intelligence.

## Hill Climbing Algorithm

It is a technique for optimizing the mathematical problems. Hill Climbing is widely used when a good heuristic is available.

It is a local search algorithm that continuously moves in the direction of increasing elevation/value to find the mountain's peak or the best solution to the problem. It terminates when it reaches a peak value where no neighbor has a higher value. Traveling-salesman Problem is one of the widely discussed examples of the Hill climbing algorithm, in which we need to minimize the distance traveled by the salesman.

It is also called greedy local search as it only looks to its good immediate neighbor state and not beyond that. The steps of a simple hill-climbing algorithm are listed below:

**Step 1:** Evaluate the initial state. If it is the goal state, then return success and Stop.

**Step 2:** Loop Until a solution is found or there is no new operator left to apply.

**Step 3:** Select and apply an operator to the current state.

#### **Step 4:** Check new state:

If it is a goal state, then return to success and quit.

Else if it is better than the current state, then assign a new state as a current state.

Else if not better than the current state, then return to step2.

#### **Step 5:** Exit.

## Best first search (BFS)

This algorithm always chooses the path which appears best at that moment. It is the combination of depth-first search and breadth-first search algorithms. It lets us to take the benefit of both algorithms. It uses the heuristic function and search. With the help of the best-first search, at each step, we can choose the most promising node.

#### **Best first search algorithm:**

**Step 1:** Place the starting node into the OPEN list.

**Step 2:** If the OPEN list is empty, Stop and return failure.

**Step 3:** Remove the node  $n$  from the OPEN list, which has the lowest value of  $h(n)$ , and places it in the CLOSED list.

**Step 4:** Expand the node  $n$ , and generate the successors of node  $n$ .

**Step 5:** Check each successor of node  $n$ , and find whether any node is a goal node or not. If any successor node is the goal node, then return success and stop the search, else continue to next step.

**Step 6:** For each successor node, the algorithm checks for evaluation function  $f(n)$  and then check if the node has been in either OPEN or CLOSED list. If the node has not been in both lists, then add it to the OPEN list.

**Step 7:** Return to Step 2.

## A\* Search Algorithm

A\* search is the most commonly known form of best-first search. It uses the heuristic function  $h(n)$  and cost to reach the node  $n$  from the start state  $g(n)$ . It has combined features of UCS and greedy best-first search, by which it solve the problem efficiently.

It finds the shortest path through the search space using the heuristic function. This search algorithm expands fewer search tree and gives optimal results faster.

#### **Algorithm of A\* search:**

**Step 1:** Place the starting node in the OPEN list.

**Step 2:** Check if the OPEN list is empty or not. If the list is empty, then return failure and stops.

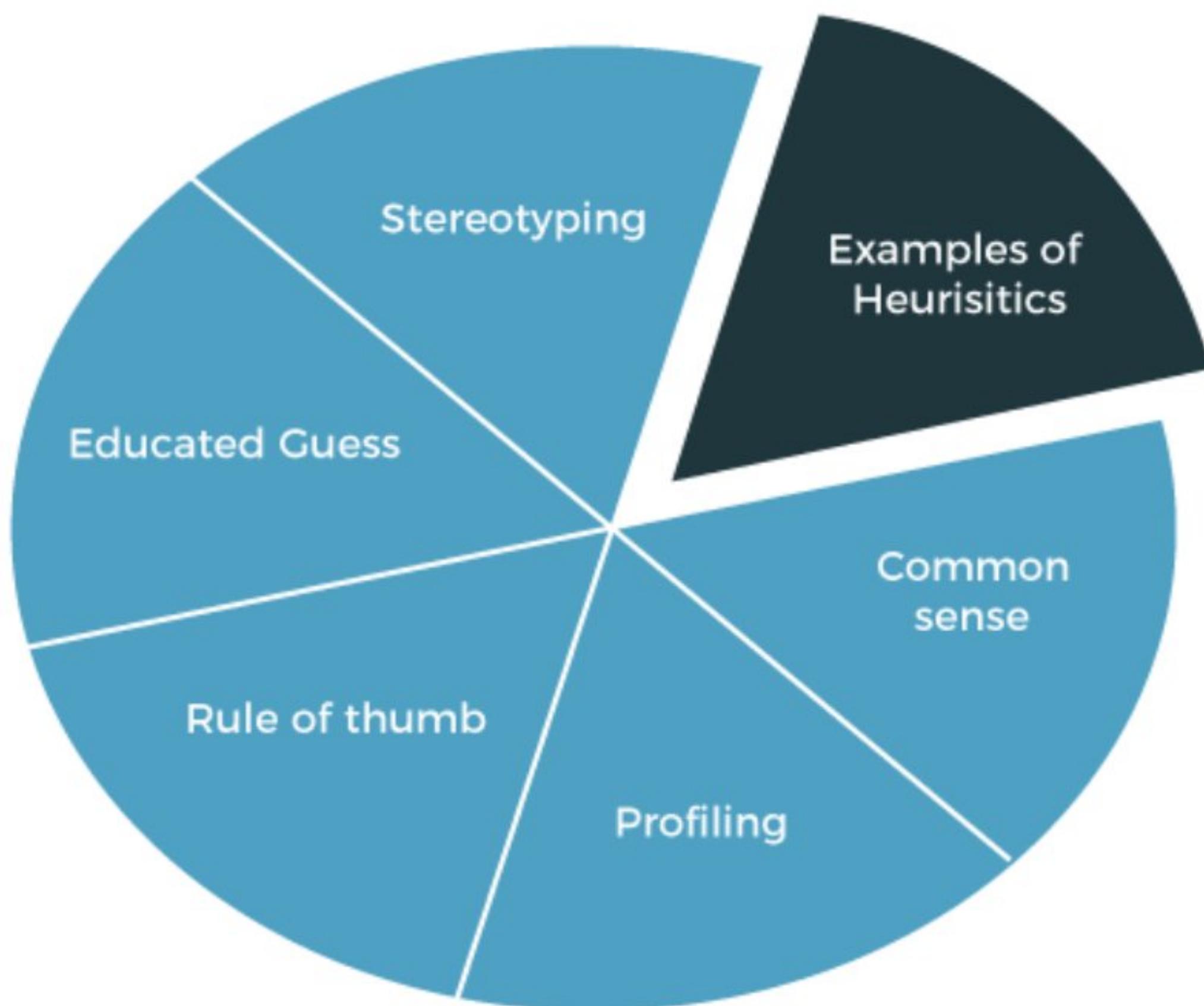
**Step 3:** Select the node from the OPEN list which has the smallest value of the evaluation function ( $g+h$ ). If node  $n$  is the goal node, then return success and stop, otherwise.

**Step 4:** Expand node  $n$  and generate all of its successors, and put  $n$  into the closed list. For each successor  $n'$ , check whether  $n'$  is already in the OPEN or CLOSED list. If not, then compute the evaluation function for  $n'$  and place it into the Open list.

**Step 5:** Else, if node  $n'$  is already in OPEN and CLOSED, then it should be attached to the back pointer which reflects the lowest  $g(n')$  value.

**Step 6:** Return to Step 2.

## Examples of heuristics in everyday life



Some of the real-life examples of heuristics that people use as a way to solve a problem:

- **Common sense:** It is a heuristic that is used to solve a problem based on the observation of an individual.

- **Rule of thumb:** In heuristics, we also use a term rule of thumb. This heuristic allows an individual to make an approximation without doing an exhaustive search.
- **Working backward:** It lets an individual solve a problem by assuming that the problem is already being solved by them and working backward in their minds to see how much a solution has been reached.
- **Availability heuristic:** It allows a person to judge a situation based on the examples of similar situations that come to mind.
- **Familiarity heuristic:** It allows a person to approach a problem on the fact that an individual is familiar with the same situation, so one should act similarly as he/she acted in the same situation before.
- **Educated guess:** It allows a person to reach a conclusion without doing an exhaustive search. Using it, a person considers what they have observed in the past and applies that history to the situation where there is not any definite answer has decided yet.

## Types of heuristics

There are various types of heuristics, including the availability heuristic, affect heuristic and representative heuristic. Each heuristic type plays a role in decision-making. Let's discuss about the Availability heuristic, affect heuristic, and Representative heuristic.

### Availability heuristic

Availability heuristic is said to be the judgment that people make regarding the likelihood of an event based on information that quickly comes into mind. On making decisions, people typically rely on the past knowledge or experience of an event. It allows a person to judge a situation based on the examples of similar situations that come to mind.

### Representative heuristic

It occurs when we evaluate an event's probability on the basis of its similarity with another event.

**Example:** We can understand the representative heuristic by the example of product packaging, as consumers tend to associate the products quality with the external packaging of a product. If a company packages its products that remind

you of a high quality and well-known product, then consumers will relate that product as having the same quality as the branded product.

So, instead of evaluating the product based on its quality, customers correlate the products quality based on the similarity in packaging.

## Affect heuristic

It is based on the negative and positive feelings that are linked with a certain stimulus. It includes quick feelings that are based on past beliefs. Its theory is one's emotional response to a stimulus that can affect the decisions taken by an individual.

When people take a little time to evaluate a situation carefully, they might base their decisions based on their emotional response.

**Example:** The affect heuristic can be understood by the example of advertisements. Advertisements can influence the emotions of consumers, so it affects the purchasing decision of a consumer. The most common examples of advertisements are the ads of fast food. When fast-food companies run the advertisement, they hope to obtain a positive emotional response that pushes you to positively view their products.

If someone carefully analyzes the benefits and risks of consuming fast food, they might decide that fast food is unhealthy. But people rarely take time to evaluate everything they see and generally make decisions based on their automatic emotional response. So, Fast food companies present advertisements that rely on such type of Affect heuristic for generating a positive emotional response which results in sales.

## Limitation of heuristics

Along with the benefits, heuristic also has some limitations.

- Although heuristics speed up our decision-making process and also help us to solve problems, they can also introduce errors just because something has worked accurately in the past, so it does not mean that it will work again.

- It will hard to find alternative solutions or ideas if we always rely on the existing solutions or heuristics.

# Conclusion

That's all about the article. Hence, in this article, we have discussed the heuristic techniques that are the problem-solving techniques that result in a quick and practical solution. We have also discussed some algorithms and examples, as well as the limitation of heuristics.

## Reasoning:

The reasoning is the mental process of deriving logical conclusion and making predictions from available knowledge, facts, and beliefs. Or we can say, "**Reasoning is a way to infer facts from existing data.**" It is a general process of thinking rationally, to find valid conclusions.

In artificial intelligence, the reasoning is essential so that the machine can also think rationally as a human brain, and can perform like a human.

## Types of Reasoning

In artificial intelligence, reasoning can be divided into the following categories:

- Deductive reasoning
- Inductive reasoning
- Abductive reasoning
- Common Sense Reasoning
- Monotonic Reasoning
- Non-monotonic Reasoning

Note: Inductive and deductive reasoning are the forms of propositional logic.

### 1. Deductive reasoning:

Deductive reasoning is deducing new information from logically related known information. It is the form of valid reasoning, which means the argument's conclusion must be true when the premises are true.

Deductive reasoning is a type of propositional logic in AI, and it requires various rules and facts. It is sometimes referred to as top-down reasoning, and contradictory to inductive reasoning.

In deductive reasoning, the truth of the premises guarantees the truth of the conclusion.

Deductive reasoning mostly starts from the general premises to the specific conclusion, which can be explained as below example.

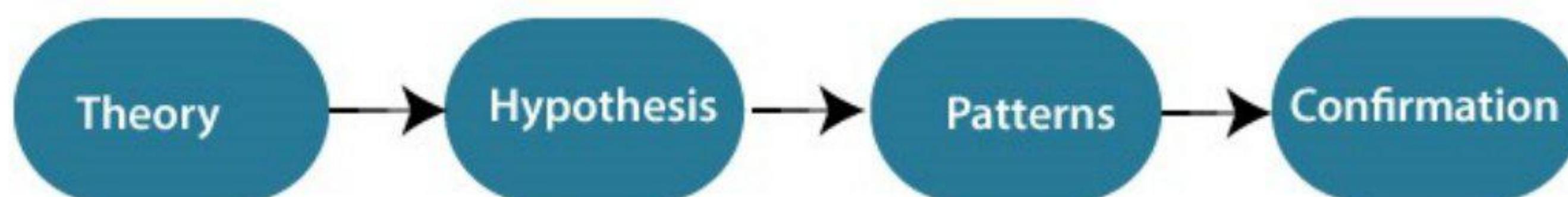
**Example:**

**Premise-1: All the human eats veggies**

**Premise-2: Suresh is human.**

**Conclusion: Suresh eats veggies.**

The general process of deductive reasoning is given below:



## 2. Inductive Reasoning:

Inductive reasoning is a form of reasoning to arrive at a conclusion using limited sets of facts by the process of generalization. It starts with the series of specific facts or data and reaches to a general statement or conclusion.

Inductive reasoning is a type of propositional logic, which is also known as cause-effect reasoning or bottom-up reasoning.

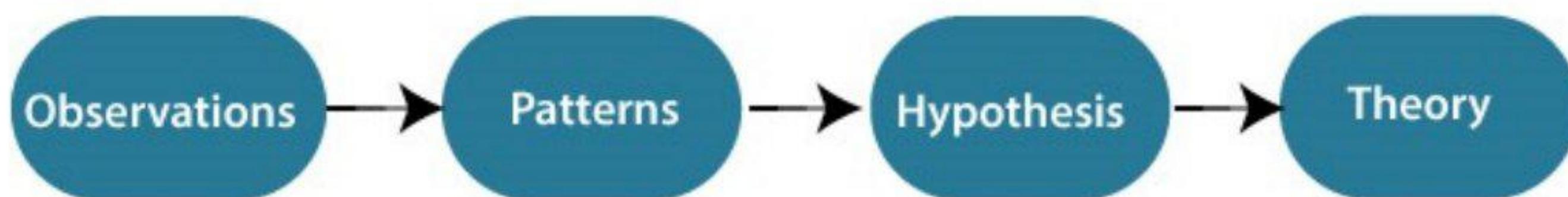
In inductive reasoning, we use historical data or various premises to generate a generic rule, for which premises support the conclusion.

In inductive reasoning, premises provide probable supports to the conclusion, so the truth of premises does not guarantee the truth of the conclusion.

**Example:**

**Premise: All of the pigeons we have seen in the zoo are white.**

**Conclusion: Therefore, we can expect all the pigeons to be white.**



### 3. Abductive reasoning:

Abductive reasoning is a form of logical reasoning which starts with single or multiple observations then seeks to find the most likely explanation or conclusion for the observation.

Abductive reasoning is an extension of deductive reasoning, but in abductive reasoning, the premises do not guarantee the conclusion.

**Example:**

**Implication:** Cricket ground is wet if it is raining

**Axiom:** Cricket ground is wet.

Conclusion It is raining.

### 4. Common Sense Reasoning

Common sense reasoning is an informal form of reasoning, which can be gained through experiences.

Common Sense reasoning simulates the human ability to make presumptions about events which occurs on every day.

It relies on good judgment rather than exact logic and operates on **heuristic knowledge** and **heuristic rules**.

**Example:**

1. **One person can be at one place at a time.**

2. **If I put my hand in a fire, then it will burn.**

The above two statements are the examples of common sense reasoning which a human mind can easily understand and assume.

### 5. Monotonic Reasoning:

In monotonic reasoning, once the conclusion is taken, then it will remain the same even if we add some other information to existing information in our knowledge base. In monotonic reasoning, adding knowledge does not decrease the set of prepositions that can be derived.

To solve monotonic problems, we can derive the valid conclusion from the available facts only, and it will not be affected by new facts.

Monotonic reasoning is not useful for the real-time systems, as in real time, facts get changed, so we cannot use monotonic reasoning.

Monotonic reasoning is used in conventional reasoning systems, and a logic-based system is monotonic.

Any theorem proving is an example of monotonic reasoning.

**Example:**

○**Earth revolves around the Sun.**

It is a true fact, and it cannot be changed even if we add another sentence in knowledge base like, "The moon revolves around the earth" Or "Earth is not round," etc.

**Advantages of Monotonic Reasoning:**

○In monotonic reasoning, each old proof will always remain valid.

○If we deduce some facts from available facts, then it will remain valid for always.

**Disadvantages of Monotonic Reasoning:**

○We cannot represent the real world scenarios using Monotonic reasoning.

○Hypothesis knowledge cannot be expressed with monotonic reasoning, which means facts should be true.

○Since we can only derive conclusions from the old proofs, so new knowledge from the real world cannot be added.

## 6. Non-monotonic Reasoning

In Non-monotonic reasoning, some conclusions may be invalidated if we add some more information to our knowledge base.

Logic will be said as non-monotonic if some conclusions can be invalidated by adding more knowledge into our knowledge base.

Non-monotonic reasoning deals with incomplete and uncertain models.

"Human perceptions for various things in daily life, "is a general example of non-monotonic reasoning.

**Example:** Let suppose the knowledge base contains the following knowledge:

○**Birds can fly**

○**Penguins cannot fly**

- **Pitty is a bird**

So from the above sentences, we can conclude that **Pitty can fly**.

However, if we add one another sentence into knowledge base "**Pitty is a penguin**", which concludes "**Pitty cannot fly**", so it invalidates the above conclusion.

### Advantages of Non-monotonic reasoning:

- For real-world systems such as Robot navigation, we can use non-monotonic reasoning.
- In Non-monotonic reasoning, we can choose probabilistic facts or can make assumptions.

### Disadvantages of Non-monotonic Reasoning:

- In non-monotonic reasoning, the old facts may be invalidated by adding new sentences.
- It cannot be used for theorem **proving**.

## Various levels of knowledge-based agent:

A knowledge-based agent can be viewed at different levels which are given below:

### 1. Knowledge level

Knowledge level is the first level of knowledge-based agent, and in this level, we need to specify what the agent knows, and what the agent goals are. With these specifications, we can fix its behavior. For example, suppose an automated taxi agent needs to go from a station A to station B, and he knows the way from A to B, so this comes at the knowledge level.

### 2. Logical level:

At this level, we understand that how the knowledge representation of knowledge is stored. At this level, sentences are encoded into different logics. At the logical level, an encoding of knowledge into logical sentences occurs. At the logical level we can expect to the automated taxi agent to reach to the destination B.

### 3. Implementation level:

This is the physical representation of logic and knowledge. At the implementation level agent perform actions as per logical and knowledge level. At this level, an automated taxi agent actually implement his knowledge and logic so that he can reach to the destination.

## Approaches to designing a knowledge-based agent:

There are mainly two approaches to build a knowledge-based agent:

- 1.1. **Declarative approach:** We can create a knowledge-based agent by initializing with an empty knowledge base and telling the agent all the sentences with which we want to start with. This approach is called Declarative approach.
- 2.2. **Procedural approach:** In the procedural approach, we directly encode desired behavior as a program code. Which means we just need to write a program that already encodes the desired behavior or agent.

However, in the real world, a successful agent can be built by combining both declarative and procedural approaches, and declarative knowledge can often be compiled into more efficient procedural code.

## Propositional logic in Artificial intelligence

Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions. A proposition is a declarative statement which is either true or false. It is a technique of knowledge representation in logical and mathematical form.

### Example:

1. a) It is Sunday.
2. b) The Sun rises from West (False proposition)
3. c)  $3+3=7$  (False proposition)
4. d) 5 is a prime number.

**Following are some basic facts about propositional logic:**

- Propositional logic is also called Boolean logic as it works on 0 and 1.

○In propositional logic, we use symbolic variables to represent the logic, and we can use any symbol for representing a proposition, such A, B, C, P, Q, R, etc.

○Propositions can be either true or false, but it cannot be both.

○Propositional logic consists of an object, relations or function, and **logical connectives**.

○These connectives are also called logical operators.

○The propositions and connectives are the basic elements of the propositional logic.

○Connectives can be said as a logical operator which connects two sentences.

○A proposition formula which is always true is called **tautology**, and it is also called a valid sentence.

○A proposition formula which is always false is called **Contradiction**.

○A proposition formula which has both true and false values is called

○Statements which are questions, commands, or opinions are not propositions such as "**Where is Rohini**", "**How are you**", "**What is your name**", are not propositions.

## Syntax of propositional logic:

The syntax of propositional logic defines the allowable sentences for the knowledge representation. There are two types of Propositions:

### 1. **Atomic Propositions**

### 2. **Compound propositions**

○**Atomic Proposition:** Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.

#### **Example:**

1. a) **2+2 is 4**, it is an atomic proposition as it is a **true** fact.

2. b) "The Sun is cold" is also a proposition as it is a **false** fact.

○**Compound proposition:** Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.

### Example:

1. a) "It is raining today, and street is wet."
2. b) "Ankit is a doctor, and his clinic is in Mumbai."

## Logical Connectives:

Logical connectives are used to connect two simpler propositions or representing a sentence logically. We can create compound propositions with the help of logical connectives. There are mainly five connectives, which are given as follows:

1. **Negation:** A sentence such as  $\neg P$  is called negation of P. A literal can be either Positive literal or negative literal.

2. **Conjunction:** A sentence which has  $\wedge$  connective such as,  $P \wedge Q$  is called a conjunction.

**Example:** Rohan is intelligent and hardworking. It can be written as,  
 $P = \text{Rohan}$        $is$        $\text{intelligent}$ ,  
 $Q = \text{Rohan is hardworking.} \rightarrow P \wedge Q$ .

3. **Disjunction:** A sentence which has  $\vee$  connective, such as  $P \vee Q$ . is called disjunction, where P and Q are the propositions.

**Example:** "Ritika is a doctor or Engineer",  
Here  $P = \text{Ritika is Doctor}$ .  $Q = \text{Ritika is Doctor}$ , so we can write it as  $P \vee Q$ .

4. **Implication:** A sentence such as  $P \rightarrow Q$ , is called an implication. Implications are also known as if-then rules. It can be represented as  
**If** it is raining, then the street is wet.

Let  $P = \text{It is raining}$ , and  $Q = \text{Street is wet}$ , so it is represented as  $P \rightarrow Q$

5. **Biconditional:** A sentence such as  $P \Leftrightarrow Q$  is a Biconditional sentence,  
**example If I am breathing, then I am alive**  
 $P = \text{I am breathing}$ ,  $Q = \text{I am alive}$ , it can be represented as  $P \Leftrightarrow Q$ .

Following is the summarized table for Propositional Logic Connectives:

Connective symbols	Word	Technical term	Example
$\wedge$	AND	Conjunction	$A \wedge B$
$\vee$	OR	Disjunction	$A \vee B$
$\rightarrow$	Implies	Implication	$A \rightarrow B$
$\Leftrightarrow$	If and only if	Biconditional	$A \Leftrightarrow B$
$\neg$ or $\sim$	Not	Negation	$\neg A$ or $\sim B$

## Truth Table:

In propositional logic, we need to know the truth values of propositions in all possible scenarios. We can combine all the possible combination with logical connectives, and the representation of these combinations in a tabular format is called **Truth table**. Following are the truth table for all logical connectives:

**For Negation:**

P	$\neg P$
True	False
False	True

**For Conjunction:**

P	Q	$P \wedge Q$
True	True	True
True	False	False
False	True	False
False	False	False

**For disjunction:**

P	Q	$P \vee Q$
True	True	True
False	True	True
True	False	True
False	False	False

**For Implication:**

P	Q	$P \rightarrow Q$
True	True	True
True	False	False
False	True	True
False	False	True

**For Biconditional:**

P	Q	$P \leftrightarrow Q$
True	True	True
True	False	False
False	True	False
False	False	True

## Truth table with three propositions:

We can build a proposition composing three propositions P, Q, and R. This truth table is made-up of  $8^n$  Tuples as we have taken three proposition symbols.

P	Q	R	$\neg R$	$P \vee Q$	$P \vee Q \rightarrow \neg R$
True	True	True	False	True	False
True	True	False	True	True	True
True	False	True	False	True	False
True	False	False	True	True	True
False	True	True	False	True	False
False	True	False	True	True	True
False	False	True	False	False	True
False	False	False	True	False	True

## Precedence of connectives:

Just like arithmetic operators, there is a precedence order for propositional connectors or logical operators. This order should be followed while evaluating a propositional problem. Following is the list of the precedence order for operators:

Precedence	Operators
First Precedence	Parenthesis
Second Precedence	Negation
Third Precedence	Conjunction(AN D)
Fourth Precedence	Disjunction(OR)
Fifth Precedence	Implication
Six Precedence	Biconditional

Note: For better understanding use parenthesis to make sure of the correct interpretations. Such as  $\neg R \vee Q$ , It can be interpreted as  $(\neg R) \vee Q$ .

## Logical equivalence:

Logical equivalence is one of the features of propositional logic. Two propositions are said to be logically equivalent if and only if the columns in the truth table are identical to each other.

Let's take two propositions A and B, so for logical equivalence, we can write it as  $A \Leftrightarrow B$ . In below truth table we can see that column for  $\neg A \vee B$  and  $A \rightarrow B$ , are identical hence A is Equivalent to B

A	B	$\neg A$	$\neg A \vee B$	$A \rightarrow B$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

## Properties of Operators:

### ○Commutativity:

$$\circ P \wedge Q = Q \wedge P, \text{ or}$$

$$\circ P \vee Q = Q \vee P.$$

### ○Associativity:

$$\circ (P \wedge Q) \wedge R = P \wedge (Q \wedge R),$$

$$\circ (P \vee Q) \vee R = P \vee (Q \vee R)$$

### ○Identity element:

$$\circ P \wedge \text{True} = P,$$

$$\circ P \vee \text{True} = \text{True}.$$

### ○Distributive:

$$\circ P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R).$$

$$\circ P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R).$$

### ○DE Morgan's Law:

$$\circ \neg (P \wedge Q) = (\neg P) \vee (\neg Q)$$

$$\circ \neg (P \vee Q) = (\neg P) \wedge (\neg Q).$$

### ○Double-negation elimination:

$$\circ \neg (\neg P) = P.$$

## Limitations of Propositional logic:

○ We cannot represent relations like ALL, some, or none with propositional logic. Example:

1. **All the girls are intelligent.**

2. **Some apples are sweet.**

○ Propositional logic has limited expressive power.

○ In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

In artificial intelligence, we need intelligent computers which can create new logic from old logic or by evidence, **so generating the conclusions from evidence and facts is termed as Inference.**

## Inference rules:

Inference rules are the templates for generating valid arguments. Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of the conclusion that leads to the desired goal.

In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:

○ **Implication:** It is one of the logical connectives which can be represented as  $P \rightarrow Q$ . It is a Boolean expression.

○ **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as  $Q \rightarrow P$ .

○ **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as  $\neg Q \rightarrow \neg P$ .

○ **Inverse:** The negation of implication is called inverse. It can be represented as  $\neg P \rightarrow \neg Q$ .

From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$\neg Q \rightarrow \neg P$	$\neg P \rightarrow \neg Q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Hence from the above truth table, we can prove that  $P \rightarrow Q$  is equivalent to  $\neg Q \rightarrow \neg P$ , and  $Q \rightarrow P$  is equivalent to  $\neg P \rightarrow \neg Q$ .

## Types of Inference rules:

### 1. Modus Ponens:

The Modus Ponens rule is one of the most important rules of inference, and it states that if  $P$  and  $P \rightarrow Q$  is true, then we can infer that  $Q$  will be true. It can be represented as:

$$\text{Notation for Modus ponens: } \frac{P \rightarrow Q, \quad P}{\therefore Q}$$

#### Example:

Statement-1: "If I am sleepy then I go to bed"  $\Rightarrow P \rightarrow Q$

Statement-2: "I am sleepy"  $\Rightarrow P$

Conclusion: "I go to bed."  $\Rightarrow Q$ .

Hence, we can say that, if  $P \rightarrow Q$  is true and  $P$  is true then  $Q$  will be true.

#### Proof by Truth table:

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

### 2. Modus Tollens:

The Modus Tollens rule state that if  $P \rightarrow Q$  is true and  $\neg Q$  is true, then  $\neg P$  will also true. It can be represented as:

$$\text{Notation for Modus Tollens: } \frac{P \rightarrow Q, \quad \neg Q}{\neg P}$$

**Statement-1:** "If I am sleepy then I go to bed"  $\Rightarrow P \rightarrow Q$

**Statement-2:** "I do not go to the bed."  $\Rightarrow \neg Q$

**Statement-3:** Which infers that "I am not sleepy"  $\Rightarrow \neg P$

**Proof by Truth table:**

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	
0	0	1	1	1	←
0	1	1	0	1	
1	0	0	1	0	
1	1	0	0	1	

### 3. Hypothetical Syllogism:

The Hypothetical Syllogism rule state that if  $P \rightarrow R$  is true whenever  $P \rightarrow Q$  is true, and  $Q \rightarrow R$  is true. It can be represented as the following notation:

**Example:**

**Statement-1:** If you have my home key then you can unlock my home.  $P \rightarrow Q$

**Statement-2:** If you can unlock my home then you can take my money.  $Q \rightarrow R$

**Conclusion:** If you have my home key then you can take my money.  $P \rightarrow R$

**Proof by truth table:**

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$	
0	0	0	1	1	1	←
0	0	1	1	1	1	←
0	1	0	1	0	1	
0	1	1	1	1	1	←
1	0	0	0	1	1	
1	0	1	0	1	1	
1	1	0	1	0	0	
1	1	1	1	1	1	←

### 4. Disjunctive Syllogism:

The Disjunctive syllogism rule state that if  $P \vee Q$  is true, and  $\neg P$  is true, then  $Q$  will be true. It can be represented as:

**Notation of Disjunctive syllogism:** 
$$\frac{P \vee Q, \neg P}{Q}$$

**Example:**

**Statement-1:** Today is Sunday or Monday.  $\Rightarrow P \vee Q$

**Statement-2:** Today is not Sunday.  $\Rightarrow \neg P$

**Conclusion:** Today is Monday.  $\Rightarrow Q$

**Proof by truth-table:**

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	1

## 5. Addition:

The Addition rule is one the common inference rule, and it states that If P is true, then  $P \vee Q$  will be true.

**Notation of Addition:** 
$$\frac{P}{P \vee Q}$$

**Example:**

**Statement:** I have a vanilla ice-cream.  $\Rightarrow P$

**Statement-2:** I have Chocolate ice-cream.

**Conclusion:** I have vanilla or chocolate ice-cream.  $\Rightarrow (P \vee Q)$

**Proof by Truth-Table:**

P	Q	$P \vee Q$
0	0	0
1	0	1
0	1	1
1	1	1

## 6. Simplification:

The simplification rule state that if  $P \wedge Q$  is true, then **Q or P** will also be true. It can be represented as:

**Notation of Simplification rule:** 
$$\frac{P \wedge Q}{Q}$$
 Or 
$$\frac{P \wedge Q}{P}$$

**Proof by Truth-Table:**

P	Q	$P \wedge Q$
0	0	0
1	0	0
0	1	0
1	1	1

## 7. Resolution:

The Resolution rule state that if  $P \vee Q$  and  $\neg P \wedge R$  is true, then  $Q \vee R$  will also be true. **It can be represented as**

$$\text{Notation of Resolution} \quad \frac{P \vee Q, \neg P \wedge R}{Q \vee R}$$

### Proof by Truth-Table:

P	$\neg P$	Q	R	$P \vee Q$	$\neg P \wedge R$	$Q \vee R$
0	1	0	0	0	0	0
0	1	0	1	0	0	1
0	1	1	0	1	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	0
1	0	0	1	1	0	1
1	0	1	0	1	0	1
1	0	1	1	1	0	1

## First-Order Logic in Artificial intelligence

In the topic of Propositional logic, we have seen that how to represent statements using propositional logic. But unfortunately, in propositional logic, we can only represent the facts, which are either true or false. PL is not sufficient to represent the complex sentences or natural language statements. The propositional logic has very limited expressive power. Consider the following sentence, which we cannot represent using PL logic.

- "Some humans are intelligent", or
- "Sachin likes cricket."

To represent the above statements, PL logic is not sufficient, so we required some more powerful logic, such as first-order logic.

## First-Order logic:

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as **Predicate logic or First-order predicate logic**. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.
- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:
  - **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, .....
  - **Relations:** It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
  - **Function:** Father of, best friend, third inning of, end of, .....
- As a natural language, first-order logic also has two main parts:
  - **Syntax**
  - **Semantics**

## Syntax of First-Order logic:

The syntax of FOL determines which collection of symbols is a logical expression in first-order logic. The basic syntactic elements of first-order logic are symbols. We write statements in short-hand notation in FOL.

## Basic Elements of First-order logic:

Following are the basic elements of FOL syntax:

<b>Constant</b>	1, 2, A, John, Mumbai,
-----------------	------------------------

	cat,....
<b>Variables</b>	x, y, z, a, b,....
<b>Predicates</b>	Brother, Father, >,....
<b>Function</b>	sqrt, LeftLegOf, ....
<b>Connective s</b>	$\wedge$ , $\vee$ , $\neg$ , $\Rightarrow$ , $\Leftrightarrow$
<b>Equality</b>	$=$
<b>Quantifier</b>	$\forall$ , $\exists$

## Atomic sentences:

- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as **Predicate (term1, term2, ....., term n)**.

**Example:** Ravi and Ajay are brothers:  $\Rightarrow$  Brothers(Ravi, Ajay).  
**Chinky is a cat:**  $\Rightarrow$  cat (Chinky).

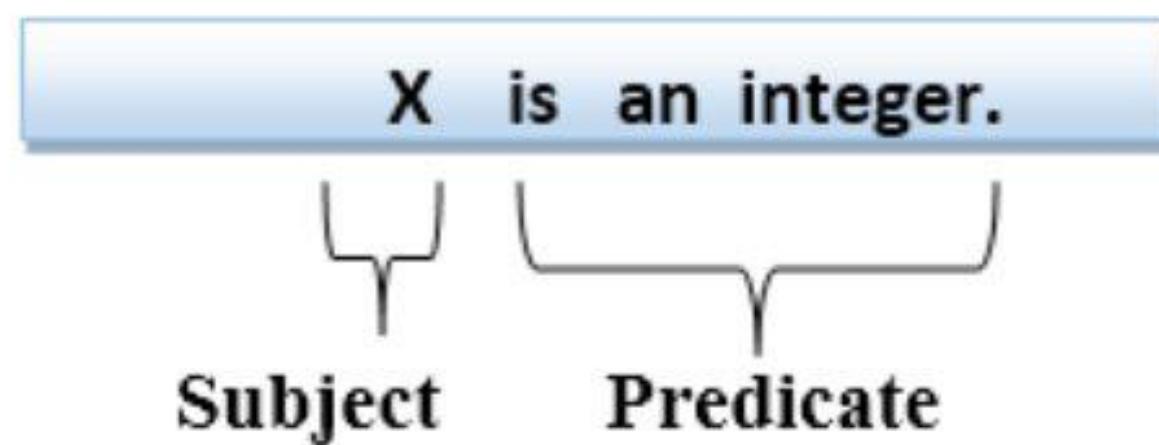
## Complex Sentences:

- Complex sentences are made by combining atomic sentences using connectives.

**First-order logic statements can be divided into two parts:**

- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.

**Consider the statement:** "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.



## Quantifiers in First-order logic:

- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression. There are two types of quantifier:

**1. Universal Quantifier, (for all, everyone, everything)**

**2. Existential quantifier, (for some, at least one).**

### Universal Quantifier:

Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.

The Universal quantifier is represented by a symbol  $\forall$ , which resembles an inverted A.

**Note:** In universal quantifier we use implication " $\rightarrow$ ".

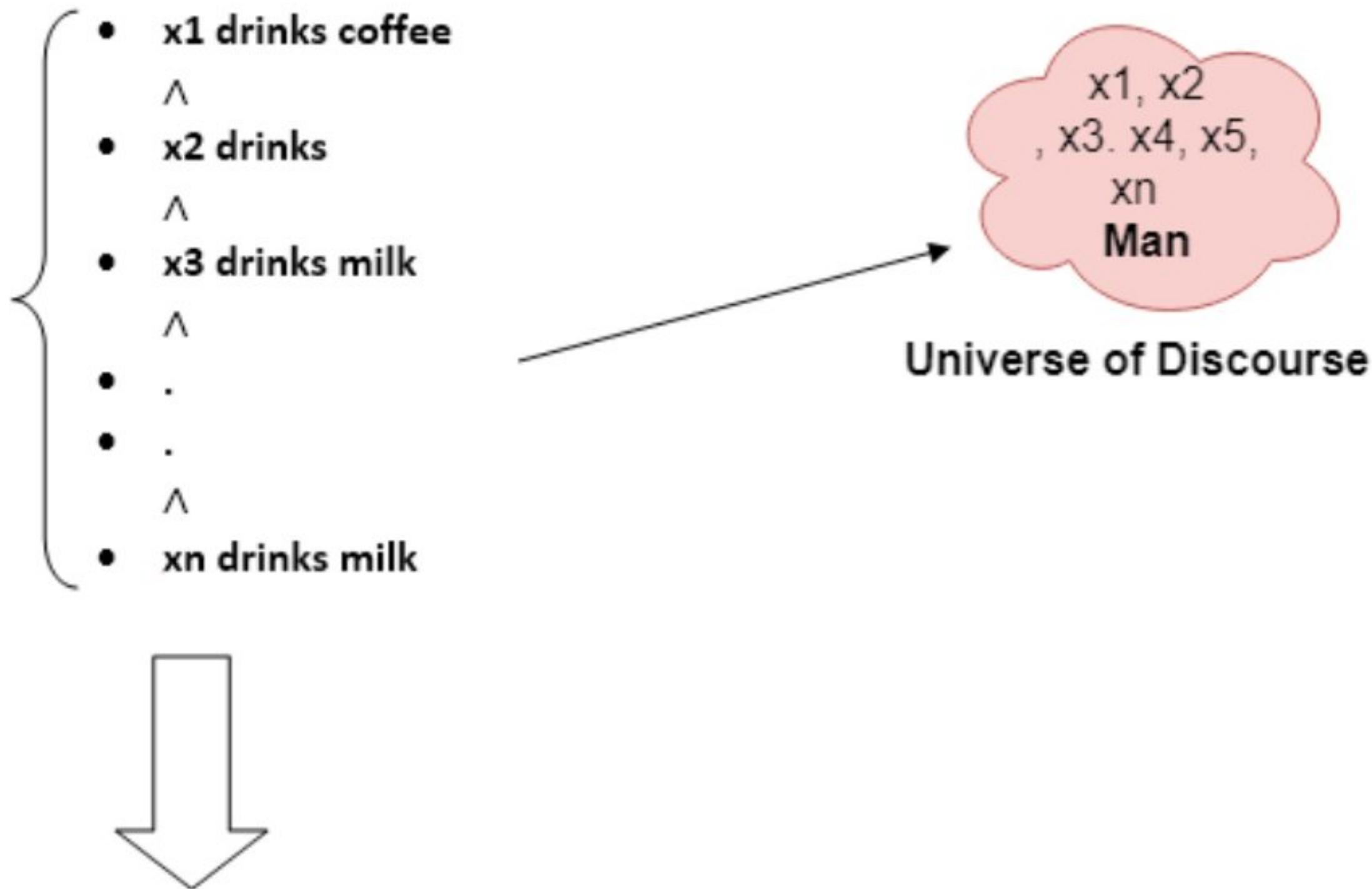
If x is a variable, then  $\forall x$  is read as:

- **For all x**
- **For each x**
- **For every x.**

## Example:

All man drink coffee.

Let a variable x which refers to a cat so all x can be represented in UOD as below:



So in shorthand notation, we can write it as :

$\forall x \text{ man}(x) \rightarrow \text{drink}(x, \text{coffee})$ .

It will be read as: There are all x where x is a man who drink coffee.

## Existential Quantifier:

Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.

It is denoted by the logical operator  $\exists$ , which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.

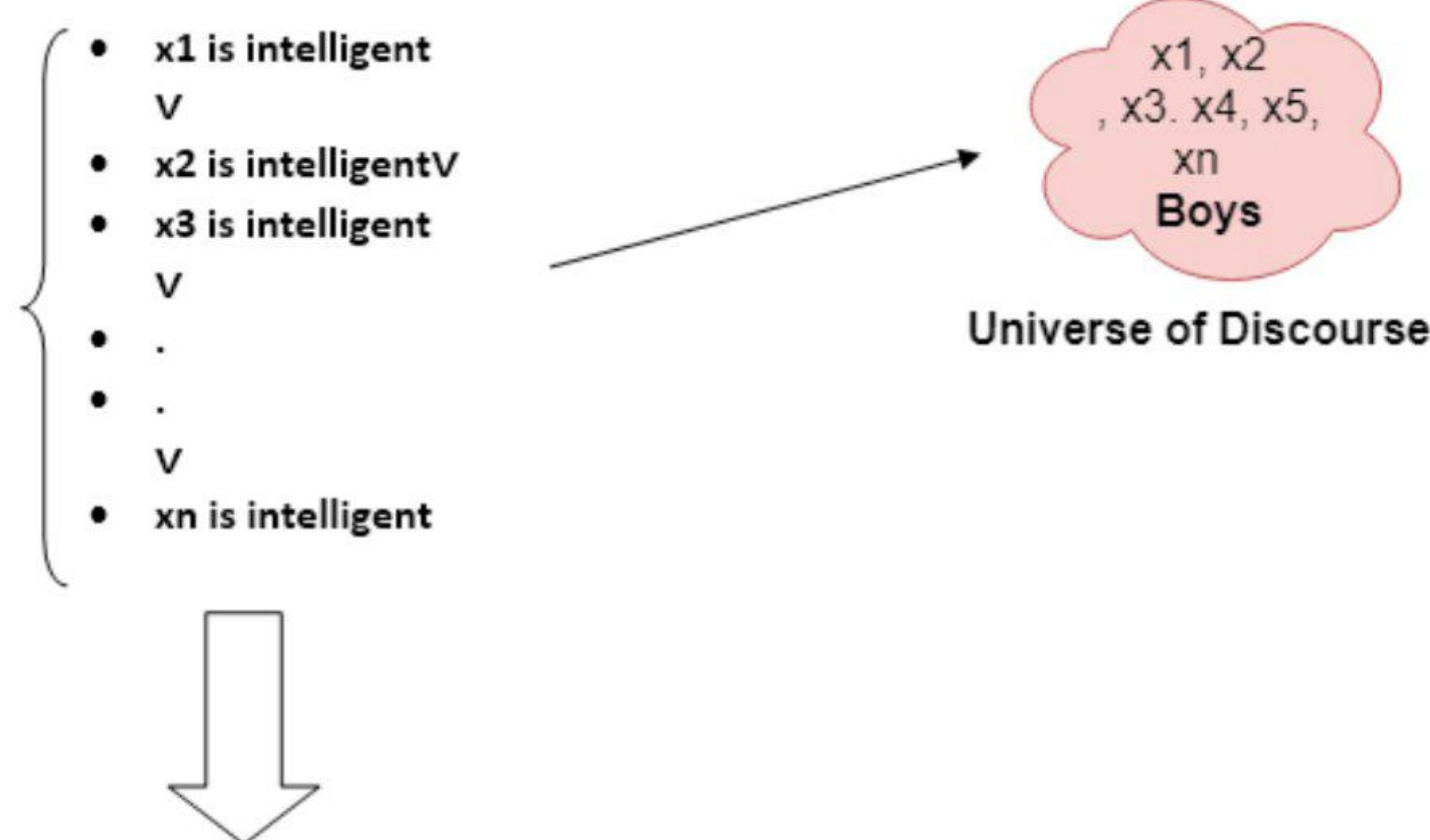
Note: In Existential quantifier we always use AND or Conjunction symbol ( $\wedge$ ).

If x is a variable, then existential quantifier will be  $\exists x$  or  $\exists(x)$ . And it will be read as:

- **There exists a 'x.'**
- **For some 'x.'**
- **For at least one 'x.'**

## Example:

Some boys are intelligent.



So in short-hand notation, we can write it as:

$\exists x: \text{boys}(x) \wedge \text{intelligent}(x)$

It will be read as: There are some x where x is a boy who is intelligent.

## Points to remember:

- The main connective for universal quantifier  $\forall$  is implication  $\rightarrow$ .
- The main connective for existential quantifier  $\exists$  is and  $\wedge$ .

## Properties of Quantifiers:

- In universal quantifier,  $\forall x \forall y$  is similar to  $\forall y \forall x$ .
- In Existential quantifier,  $\exists x \exists y$  is similar to  $\exists y \exists x$ .
- $\exists x \forall y$  is not similar to  $\forall y \exists x$ .

Some Examples of FOL using quantifier:

**1.**                   **All**                   **birds**                   **fly.**  
In           this       question       the       predicate       is       "fly(bird)."       
And since there are all birds who fly so it will be represented as follows.  
 $\forall x \text{ bird}(x) \rightarrow \text{fly}(x)$ .

**2. Every man respects his parent.**

In this question, the predicate is "**respect(x, y)**," where **x=man**, and **y=parent**.

Since there is every man so will use  $\forall$ , and it will be represented as follows:

$$\forall x \text{ man}(x) \rightarrow \text{respects}(x, \text{parent}).$$

**3. Some boys play cricket.**

In this question, the predicate is "**play(x, y)**," where **x= boys**, and **y= game**.

Since there are some boys so we will use  $\exists$ , **and it will be represented as:**

$$\exists x \text{ boys}(x) \rightarrow \text{play}(x, \text{cricket}).$$

**4. Not all students like both Mathematics and Science.**

In this question, the predicate is "**like(x, y)**," where **x= student**, and **y= subject**.

Since there are not all students, so we will use  $\forall$  **with negation, so** following representation for this:

$$\neg \forall (x) [\text{student}(x) \rightarrow \text{like}(x, \text{Mathematics}) \wedge \text{like}(x, \text{Science})].$$

**5. Only one student failed in Mathematics.**

In this question, the predicate is "**failed(x, y)**," where **x= student**, and **y= subject**.

Since there is only one student who failed in Mathematics, so we will use following representation for this:

$$\exists (x) [\text{student}(x) \rightarrow \text{failed}(x, \text{Mathematics}) \wedge \forall (y) [\neg(x==y) \wedge \text{student}(y) \rightarrow \neg\text{failed}(x, \text{Mathematics})]].$$

## Free and Bound Variables:

The quantifiers interact with variables which appear in a suitable way. There are two types of variables in First-order logic which are given below:

**Free Variable:** A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

**Example:**  $\forall x \exists (y) [P(x, y, z)]$ , where **z** is a free variable.

**Bound Variable:** A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

**Example:**  $\forall x [A(x) B(y)]$ , here **x** and **y** are the bound variables.

## Inference in First-Order Logic

Inference in First-Order Logic is used to deduce new facts or sentences from existing sentences. Before understanding the FOL inference rule, let's understand some basic terminologies used in FOL.

## **Substitution:**

Substitution is a fundamental operation performed on terms and formulas. It occurs in all inference systems in first-order logic. The substitution is complex in the presence of quantifiers in FOL. If we write **F[a/x]**, so it refers to substitute a constant "a" in place of variable "x".

Note: First-order logic is capable of expressing facts about some or all objects in the universe.

## **Equality:**

First-Order logic does not only use predicate and terms for making atomic sentences but also uses another way, which is equality in FOL. For this, we can use **equality symbols** which specify that the two terms refer to the same object.

### **Example: Brother (John) = Smith.**

As in the above example, the object referred by the **Brother (John)** is similar to the object referred by **Smith**. The equality symbol can also be used with negation to represent that two terms are not the same objects.

### **Example: $\neg(x=y)$ which is equivalent to $x \neq y$ .**

## **FOL inference rules for quantifier:**

As propositional logic we also have inference rules in first-order logic, so following are some basic inference rules in FOL:

- **Universal Generalization**
- **Universal Instantiation**
- **Existential Instantiation**
- **Existential introduction**

### **1. Universal Generalization:**

- Universal generalization is a valid inference rule which states that if premise  $P(c)$  is true for any arbitrary element  $c$  in the universe of discourse, then we can have a conclusion as  $\forall x P(x)$ .

$$\text{It can be represented as: } \frac{P(c)}{\forall x P(x)}$$

- This rule can be used if we want to show that every element has a similar property.

- In this rule, x must not appear as a free variable.

**Example:** Let's represent, P(c): "**A byte contains 8 bits**", so for  $\forall x P(x)$  "All bytes contain 8 bits.", it will also be true.

## 2. Universal Instantiation:

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.

- The new KB is logically equivalent to the previous KB.

- As per UI, **we can infer any sentence obtained by substituting a ground term for the variable.**

- The UI rule state that we can infer any sentence P(c) by substituting a ground term c (a constant within domain x) from  $\forall x P(x)$  **for any object in the universe of discourse.**

$$\frac{\forall x P(x)}{P(c)}$$

- It can be represented as:  $\frac{\forall x P(x)}{P(c)}$ .

### Example:1.

IF "Every person like ice-cream"  $\Rightarrow \forall x P(x)$  so we can infer that "John likes ice-cream"  $\Rightarrow P(c)$

### Example: 2.

Let's take a famous example,

"All kings who are greedy are Evil." So let our knowledge base contains this detail as in the form of FOL:

$\forall x \text{king}(x) \wedge \text{greedy}(x) \rightarrow \text{Evil}(x),$

So from this information, we can infer any of the following statements using Universal Instantiation:

- **King(John)  $\wedge$  Greedy (John)  $\rightarrow$  Evil (John),**

- **King(Richard)  $\wedge$  Greedy (Richard)  $\rightarrow$  Evil (Richard),**

- **King(Father(John))  $\wedge$  Greedy (Father(John))  $\rightarrow$  Evil (Father(John)),**

### 3. Existential Instantiation:

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.
- The new KB is not logically equivalent to old KB, but it will be satisfiable if old KB was satisfiable.
- This rule states that one can infer  $P(c)$  from the formula given in the form of  $\exists x P(x)$  for a new constant symbol  $c$ .
- The restriction with this rule is that  $c$  used in the rule must be a new term for which  $P(c)$  is true.

$$\frac{\exists x P(x)}{P(c)}$$

It can be represented as:

#### Example:

From the given sentence:  $\exists x \text{Crown}(x) \wedge \text{OnHead}(x, \text{John})$ ,

So we can infer:  $\text{Crown}(K) \wedge \text{OnHead}(K, \text{John})$ , as long as  $K$  does not appear in the knowledge base.

- The above used  $K$  is a constant symbol, which is called **Skolem constant**.
- The Existential instantiation is a special case of **Skolemization process**.

### 4. Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- This rule states that if there is some element  $c$  in the universe of discourse which has a property  $P$ , then we can infer that there exists something in the universe which has the property  $P$ .

$$\frac{P(c)}{\exists x P(x)}$$

It can be represented as:

○**Example:** Let's say that, "Priyanka got good marks in English." "Therefore, someone got good marks in English."

## Generalized Modus Ponens Rule:

For the inference process in FOL, we have a single inference rule which is called Generalized Modus Ponens. It is lifted version of Modus ponens.

Generalized Modus Ponens can be summarized as, " P implies Q and P is asserted to be true, therefore Q must be True."

According to Modus Ponens, for atomic sentences **pi**, **pi'**, **q**. Where there is a substitution  $\theta$  such that  $\text{SUBST}(\theta, \text{pi}') = \text{SUBST}(\theta, \text{pi})$ , it can be represented as:

$$\frac{\text{p1}', \text{p2}', \dots, \text{pn}', (\text{p1} \wedge \text{p2} \wedge \dots \wedge \text{pn} \Rightarrow \text{q})}{\text{SUBST}(\theta, \text{q})}$$

### Example:

**We will use this rule for Kings are evil, so we will find some x such that x is king, and x is greedy so we can infer that x is evil.**

1. Here let say, p1' is king(John)      p1 is king(x)
2. p2' is Greedy(y)                        p2 is Greedy(x)
3.  $\theta$  is {x/John, y/John}                q is evil(x)
4. SUBST( $\theta$ , q).

## Forward Chaining and backward chaining in AI

In artificial intelligence, forward and backward chaining is one of the important topics, but before understanding forward and backward chaining lets first understand that from where these two terms came.

### Inference engine:

The inference engine is the component of the intelligent system in artificial intelligence, which applies logical rules to the knowledge base to infer new

information from known facts. The first inference engine was part of the expert system. Inference engine commonly proceeds in two modes, which are:

### 1. **Forward chaining**

### 2. **Backward chaining**

#### **Horn Clause and Definite clause:**

Horn clause and definite clause are the forms of sentences, which enables knowledge base to use a more restricted and efficient inference algorithm. Logical inference algorithms use forward and backward chaining approaches, which require KB in the form of the **first-order definite clause**.

**Definite clause:** A clause which is a disjunction of literals with **exactly one positive literal** is known as a definite clause or strict horn clause.

**Horn clause:** A clause which is a disjunction of literals with **at most one positive literal** is known as horn clause. Hence all the definite clauses are horn clauses.

**Example:**  $(\neg p \vee \neg q \vee k)$ . It has only one positive literal k.

It is equivalent to  $p \wedge q \rightarrow k$ .

## A. Forward Chaining

Forward chaining is also known as a forward deduction or forward reasoning method when using an inference engine. Forward chaining is a form of reasoning which start with atomic sentences in the knowledge base and applies inference rules (Modus Ponens) in the forward direction to extract more data until a goal is reached.

The Forward-chaining algorithm starts from known facts, triggers all rules whose premises are satisfied, and add their conclusion to the known facts. This process repeats until the problem is solved.

#### **Properties of Forward-Chaining:**

○ It is a down-up approach, as it moves from bottom to top.

○ It is a process of making a conclusion based on known facts or data, by starting from the initial state and reaches the goal state.

○ Forward-chaining approach is also called as data-driven as we reach to the goal using available data.

- Forward-chaining approach is commonly used in the expert system, such as CLIPS, business, and production rule systems.

Consider the following famous example which we will use in both approaches:

### Example:

**"As per the law, it is a crime for an American to sell weapons to hostile nations. Country A, an enemy of America, has some missiles, and all the missiles were sold to it by Robert, who is an American citizen."**

Prove that "**Robert is criminal.**"

To solve the above problem, first, we will convert all the above facts into first-order definite clauses, and then we will use a forward-chaining algorithm to reach the goal.

### Facts Conversion into FOL:

- It is a crime for an American to sell weapons to hostile nations. (Let's say p, q, and r are variables)

**American(p) ∧ weapon(q) ∧ sells(p, q, r) ∧ hostile(r) → Criminal(p)**  
...(1)

- Country A has some missiles. **?p Owns(A, p) ∧ Missile(p).** It can be written in two definite clauses by using Existential Instantiation, introducing new Constant T1.

**Owns(A, T1) .....(2)**

**Missile(T1) .....(3)**

- All of the missiles were sold to country A by Robert.  
**?p Missiles(p) ∧ Owns(A, p) → Sells(Robert, p, A) .....(4)**

- Missiles are weapons.

**Missile(p) → Weapons(p) .....(5)**

- Enemy of America is known as hostile.

**Enemy(p, America) → Hostile(p) .....(6)**

- Country A is an enemy of America.

**Enemy(A, America) .....(7)**

o Robert is American  
**American(Robert).** .....(8)

## Forward chaining proof:

## **Step-1:**

In the first step we will start with the known facts and will choose the sentences which do not have implications, such as: **American(Robert)**, **Enemy(A, America)**, **Owns(A, T1)**, and **Missile(T1)**. All these facts will be represented as below.



## **Step-2:**

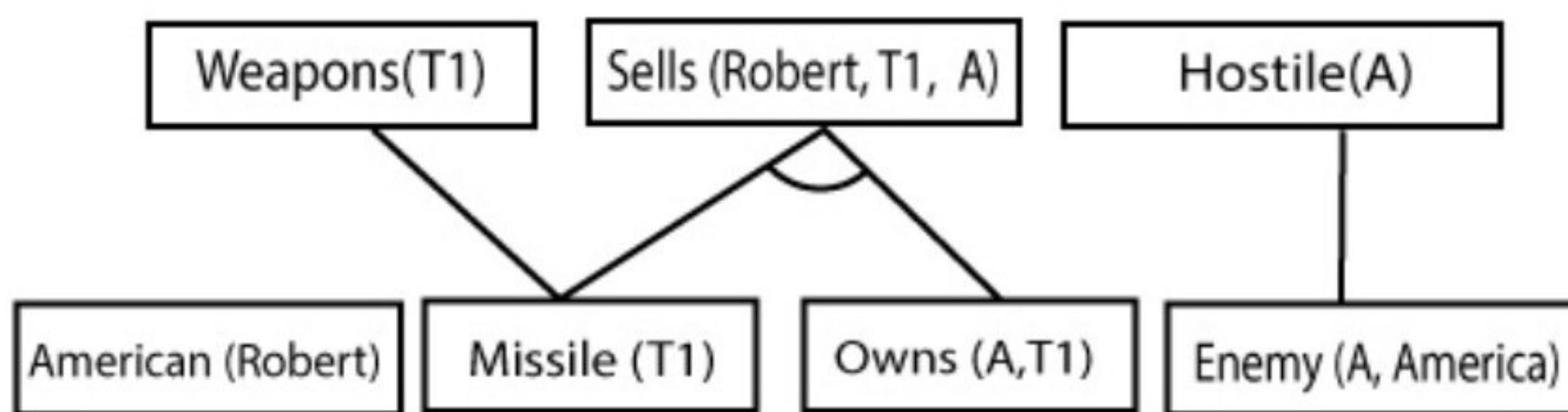
At the second step, we will see those facts which infer from available facts and with satisfied premises.

Rule-(1) does not satisfy premises, so it will not be added in the first iteration.

Rule-(2) and (3) are already added.

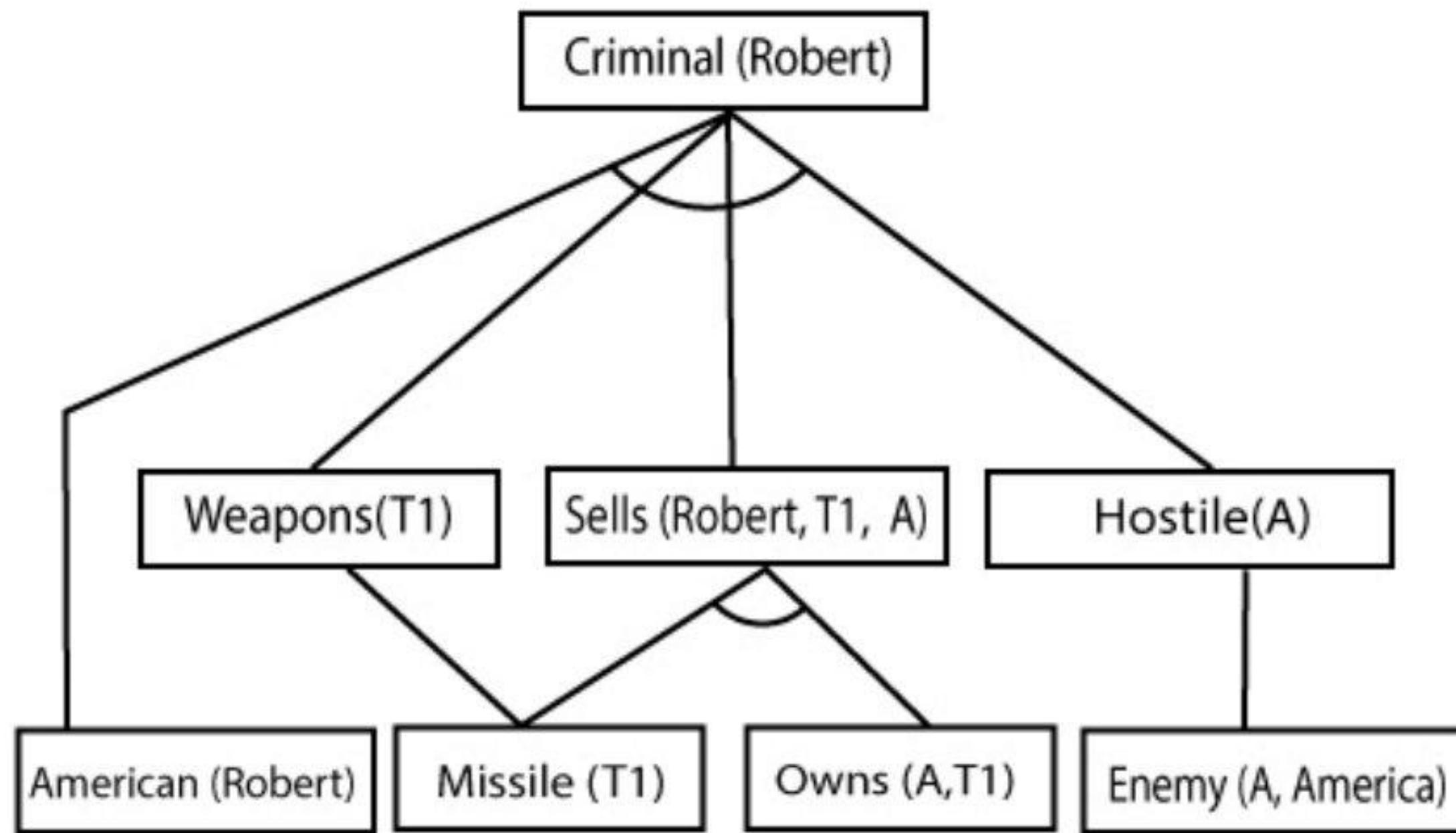
Rule-(4) satisfy with the substitution {p/T1}, so **Sells (Robert, T1, A)** is added, which infers from the conjunction of Rule (2) and (3).

Rule-(6) is satisfied with the substitution(p/A), so Hostile(A) is added and which infers from Rule-(7).



### **Step-3:**

At step-3, as we can check Rule-(1) is satisfied with the substitution **{p/Robert, q/T1, r/A}**, so we can add **Criminal(Robert)** which infers all the available facts. And hence we reached our goal statement.



**Hence it is proved that Robert is Criminal using forward chaining approach.**

## B. Backward Chaining:

Backward-chaining is also known as a backward deduction or backward reasoning method when using an inference engine. A backward chaining algorithm is a form of reasoning, which starts with the goal and works backward, chaining through rules to find known facts that support the goal.

### Properties of backward chaining:

- It is known as a top-down approach.
- Backward-chaining is based on modus ponens inference rule.
- In backward chaining, the goal is broken into sub-goal or sub-goals to prove the facts true.
- It is called a goal-driven approach, as a list of goals decides which rules are selected and used.
- Backward-chaining algorithm is used in game theory, automated theorem proving tools, inference engines, proof assistants, and various AI applications.
- The backward-chaining method mostly used a **depth-first search** strategy for proof.

## Example:

In backward-chaining, we will use the same above example, and will rewrite all the rules.

- **American (p)  $\wedge$  weapon(q)  $\wedge$  sells (p, q, r)  $\wedge$  hostile(r)  $\rightarrow$  Criminal(p)** .....(1)
- **Owns(A, T1)** .....(2)
- **Missile(T1)**
- **?p Missiles(p)  $\wedge$  Owns (A, p)  $\rightarrow$  Sells (Robert, p, A)** .....(4)
- **Missile(p)  $\rightarrow$  Weapons (p)** .....(5)
- **Enemy(p, America)  $\rightarrow$  Hostile(p)** .....(6)
- **Enemy (A, America)** .....(7)
- **American(Robert).** .....(8)

## Backward-Chaining proof:

In Backward chaining, we will start with our goal predicate, which is **Criminal(Robert)**, and then infer further rules.

### Step-1:

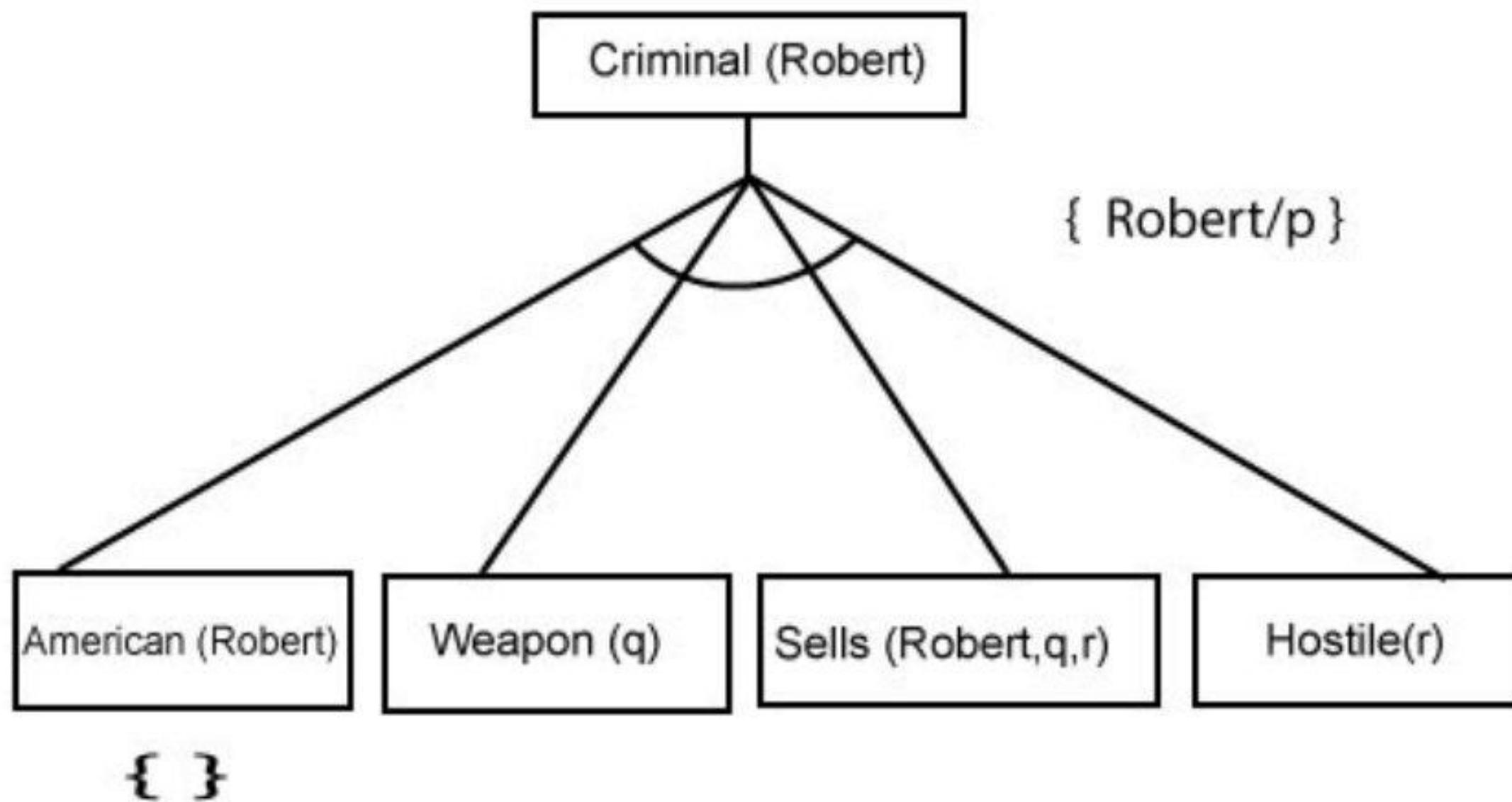
At the first step, we will take the goal fact. And from the goal fact, we will infer other facts, and at last, we will prove those facts true. So our goal fact is "Robert is Criminal," so following is the predicate of it.

Criminal (Robert)

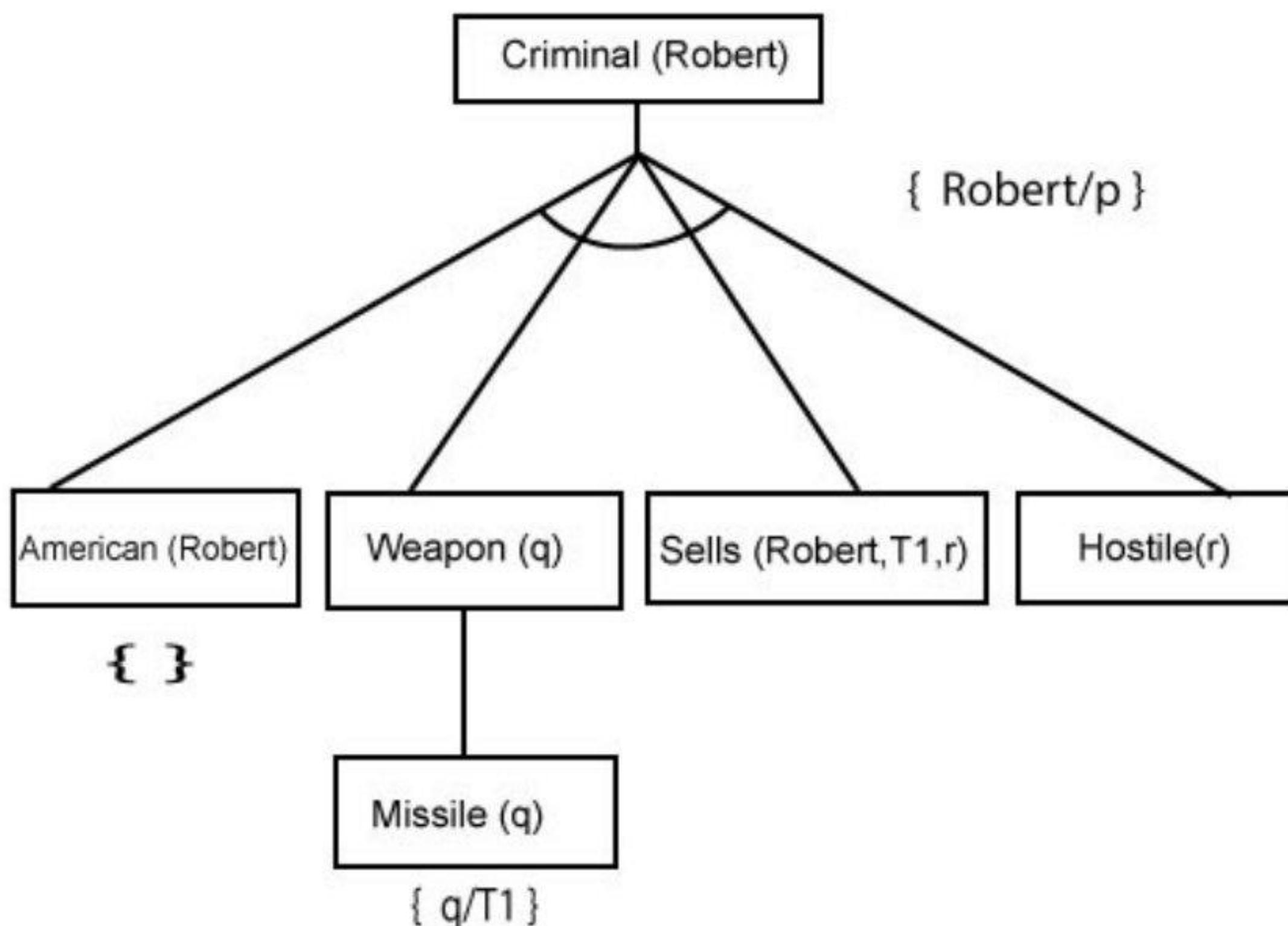
### Step-2:

At the second step, we will infer other facts form goal fact which satisfies the rules. So as we can see in Rule-1, the goal predicate Criminal (Robert) is present with substitution {Robert/P}. So we will add all the conjunctive facts below the first level and will replace p with Robert.

**Here we can see American (Robert) is a fact, so it is proved here.**

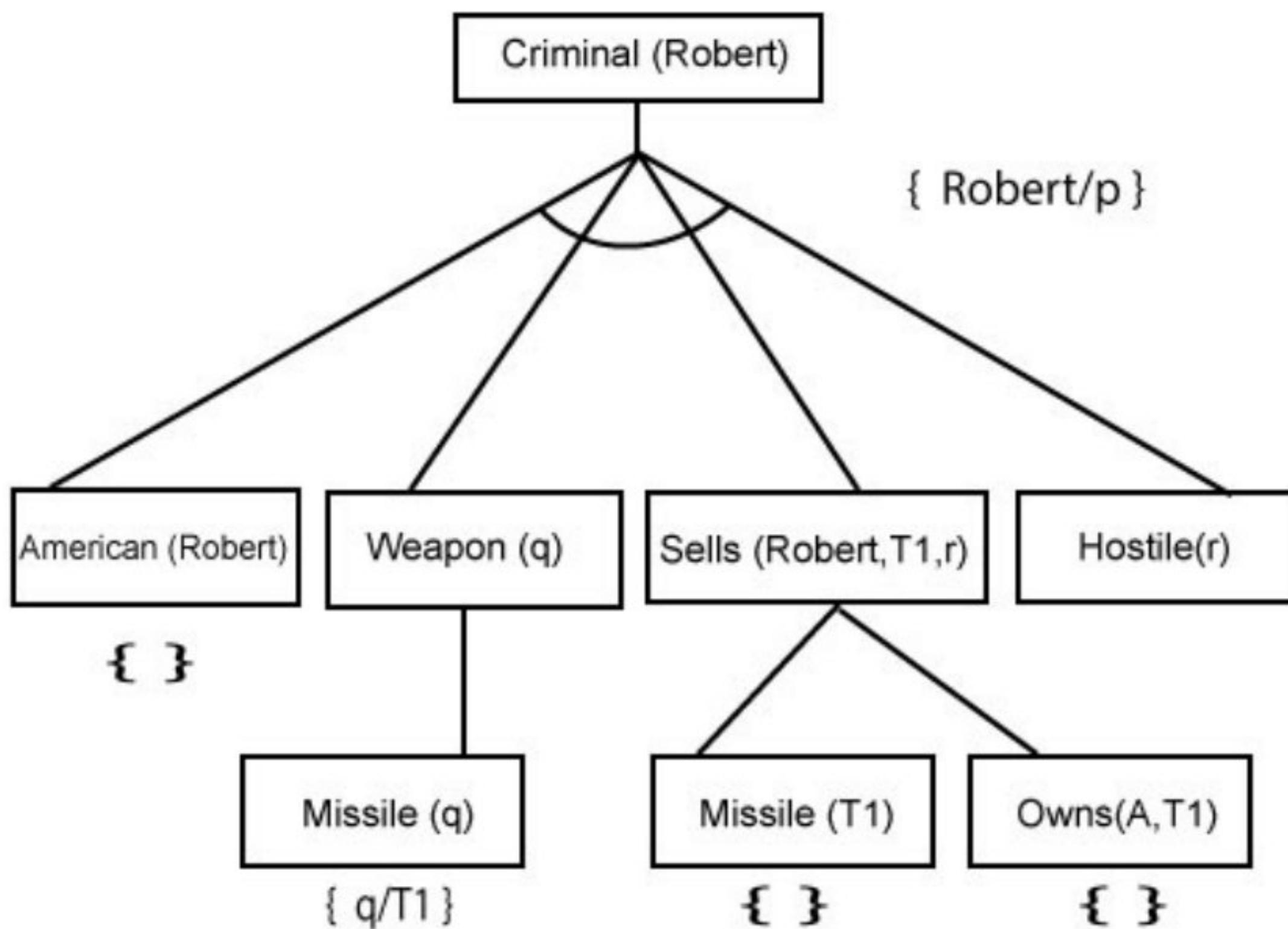


**Step-3:t** At step-3, we will extract further fact  $\text{Missile}(q)$  which infer from  $\text{Weapon}(q)$ , as it satisfies Rule-(5).  $\text{Weapon}(q)$  is also true with the substitution of a constant  $T1$  at  $q$ .



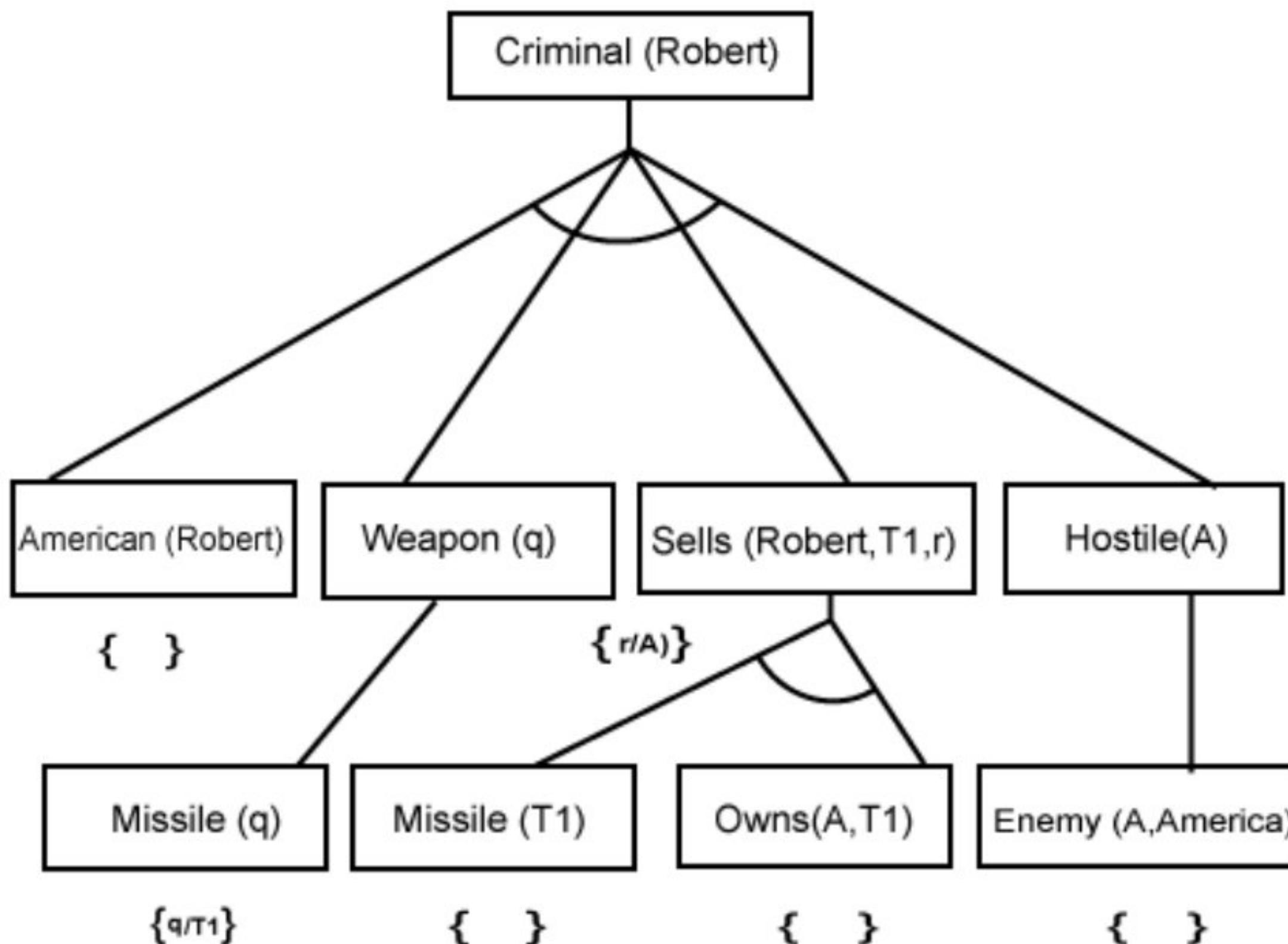
#### Step-4:

At step-4, we can infer facts  $\text{Missile}(T1)$  and  $\text{Owns}(A, T1)$  form  $\text{Sells}(\text{Robert}, T1, r)$  which satisfies the **Rule- 4**, with the substitution of  $A$  in place of  $r$ . So these two statements are proved here.



### Step-5:

At step-5, we can infer the fact **Enemy(A, America)** from **Hostile(A)** which satisfies Rule- 6. And hence all the statements are proved true using backward chaining.



<b>S. No.</b>	<b>Forward Chaining</b>	<b>Backward Chaining</b>
1.	Forward chaining starts from known facts and applies inference rule to extract more data until it reaches to the goal.	Backward chaining starts from the goal and works backward through inference rules to find the required facts that support the goal.
2.	It is a bottom-up approach	It is a top-down approach
3.	Forward chaining is known as data-driven inference technique as we reach to the goal using the available data.	Backward chaining is known as goal-driven technique as we start from the goal and divide into sub-goal to extract the facts.
4.	Forward chaining reasoning applies a breadth-first search strategy.	Backward chaining reasoning applies a depth-first search strategy.
5.	Forward chaining tests for all the available rules	Backward chaining only tests for few required rules.
6.	Forward chaining is suitable for the planning, monitoring, control, and interpretation application.	Backward chaining is suitable for diagnostic, prescription, and debugging application.
7.	Forward chaining can generate an infinite number of possible conclusions.	Backward chaining generates a finite number of possible conclusions.
8.	It operates in the forward direction.	It operates in the backward direction.
9.	Forward chaining is aimed for any	Backward chaining is only aimed for

conclusion.

the required data.

## What is Unification?

○ Unification is a process of making two different logical atomic expressions identical by finding a substitution. Unification depends on the substitution process.

○ It takes two literals as input and makes them identical using substitution.

○ Let  $\Psi_1$  and  $\Psi_2$  be two atomic sentences and  $\sigma$  be a unifier such that,  $\Psi_1\sigma = \Psi_2\sigma$ , then it can be expressed as **UNIFY( $\Psi_1$ ,  $\Psi_2$ )**.

○ **Example: Find the MGU for Unify{King(x), King(John)}**

Let  $\Psi_1 = \text{King}(x)$ ,  $\Psi_2 = \text{King}(\text{John})$ ,

**Substitution  $\theta = \{\text{John}/x\}$**  is a unifier for these atoms and applying this substitution, and both expressions will be identical.

○ The UNIFY algorithm is used for unification, which takes two atomic sentences and returns a unifier for those sentences (If any exist).

○ Unification is a key component of all first-order inference algorithms.

○ It returns fail if the expressions do not match with each other.

○ The substitution variables are called Most General Unifier or MGU.

**E.g.** Let's say there are two different expressions, **P(x, y)**, and **P(a, f(z))**.

In this example, we need to make both above statements identical to each other. For this, we will perform the substitution.

$$\begin{array}{lll} P(x, & y) \dots\dots\dots & (i) \\ P(a, f(z)) \dots\dots\dots & (ii) \end{array}$$

○ Substitute x with a, and y with f(z) in the first expression, and it will be represented as **a/x** and **f(z)/y**.

○ With both the substitutions, the first expression will be identical to the second expression and the substitution set will be: **[a/x, f(z)/y]**.

## Conditions for Unification:

Following are some basic conditions for unification:

- Predicate symbol must be same, atoms or expression with different predicate symbol can never be unified.
- Number of Arguments in both expressions must be identical.
- Unification will fail if there are two similar variables present in the same expression.

## Unification Algorithm:

**Algorithm: Unify( $\Psi_1$ ,  $\Psi_2$ )**

Step. 1: If  $\Psi_1$  or  $\Psi_2$  is a variable or constant, then:

a) If  $\Psi_1$  or  $\Psi_2$  are identical, then return NIL.

b) Else if  $\Psi_1$  is a variable,

a. then if  $\Psi_1$  occurs in  $\Psi_2$ , then return FAILURE

b. Else return  $\{(\Psi_2/\Psi_1)\}$ .

c) Else if  $\Psi_2$  is a variable,

a. If  $\Psi_2$  occurs in  $\Psi_1$  then return FAILURE,

b. Else return  $\{(\Psi_1/\Psi_2)\}$ .

d) Else return FAILURE.

Step.2: If the initial Predicate symbol in  $\Psi_1$  and  $\Psi_2$  are not same, then return FAILURE.

Step. 3: IF  $\Psi_1$  and  $\Psi_2$  have a different number of arguments, then return FAILURE.

Step. 4: Set Substitution set(SUBST) to NIL.

Step. 5: For i=1 to the number of elements in  $\Psi_1$ .

a) Call Unify function with the ith element of  $\Psi_1$  and ith element of  $\Psi_2$ , and put the result into S.

b) If S = failure then returns Failure

c) If S  $\neq$  NIL then do,

a. Apply S to the remainder of both L1 and L2.

b. SUBST= APPEND(S, SUBST).

Step.6: Return SUBST.

## Implementation of the Algorithm

**Step.1:** Initialize the substitution set to be empty.

**Step.2:** Recursively unify atomic sentences:

1. Check for Identical expression match.

2. If one expression is a variable  $v_i$ , and the other is a term  $t_i$  which does not contain variable  $v_i$ , then:

1. Substitute  $t_i / v_i$  in the existing substitutions

2. Add  $t_i / v_i$  to the substitution setlist.

3. If both the expressions are functions, then function name must be similar, and the number of arguments must be the same in both the expression.

**For each pair of the following atomic sentences find the most general unifier (If exist).**

**1. Find the MGU of { $p(f(a), g(Y))$  and  $p(X, X)$ }**

Sol:  $S_0 \Rightarrow$  Here,  $\Psi_1 = p(f(a), g(Y)),$  and  $\Psi_2 = p(X, X)$   
SUBST  $\theta = \{f(a) / X\}$

$S_1 \Rightarrow \Psi_1 = p(f(a), g(Y)),$  and  $\Psi_2 = p(f(a), f(a))$

SUBST  $\theta = \{f(a) / g(y)\},$  **Unification failed.**

Unification is not possible for these expressions.

**2. Find the MGU of { $p(b, X, f(g(Z)))$  and  $p(Z, f(Y), f(Y))$ }**

Here,  $\Psi_1 = p(b, X, f(g(Z)))$ , and  $\Psi_2 = p(Z, f(Y), f(Y))$   
 $S_0 \Rightarrow \{p(b, X, f(g(Z))), p(Z, f(Y), f(Y))\}$   
SUBST  $\theta = \{b/Z\}$

$S_1 \Rightarrow \{p(b, X, f(g(b))), p(b, f(Y), f(Y))\}$   
SUBST  $\theta = \{f(Y)/X\}$

$S_2 \Rightarrow \{p(b, f(Y), f(g(b))), p(b, f(Y), f(Y))\}$   
SUBST  $\theta = \{g(b)/Y\}$

$S_2 \Rightarrow \{p(b, f(g(b)), f(g(b))), p(b, f(g(b)), f(g(b)))\}$  **Unified Successfully.**  
**And Unifier = { b/Z, f(Y) /X , g(b) /Y }.**

### 3. Find the MGU of {p (X, X), and p (Z, f(Z))}

Here,  $\Psi_1 = \{p(X, X)$ , and  $\Psi_2 = p(Z, f(Z))$   
 $S_0 \Rightarrow \{p(X, X), p(Z, f(Z))\}$   
SUBST  $\theta = \{X/Z\}$   
 $S_1 \Rightarrow \{p(Z, Z), p(Z, f(Z))\}$   
SUBST  $\theta = \{f(Z)/Z\}$ , **Unification Failed.**

**Hence, unification is not possible for these expressions.**

### 4. Find the MGU of UNIFY(prime(11), prime(y))

Here,  $\Psi_1 = \{\text{prime}(11)$ , and  $\Psi_2 = \text{prime}(y)\}$   
 $S_0 \Rightarrow \{\text{prime}(11)$ ,  $\text{prime}(y)\}$   
SUBST  $\theta = \{11/y\}$   
 $S_1 \Rightarrow \{\text{prime}(11), \text{prime}(11)\}$ , **Successfully unified.**  
**Unifier: {11/y}.**

### 5. Find the MGU of Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)

Here,  $\Psi_1 = Q(a, g(x, a), f(y))$ , and  $\Psi_2 = Q(a, g(f(b), a), x)$   
 $S_0 \Rightarrow \{Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)\}$   
SUBST  $\theta = \{f(b)/x\}$   
 $S_1 \Rightarrow \{Q(a, g(f(b), a), f(y)), Q(a, g(f(b), a), f(b))\}$   
SUBST  $\theta = \{b/y\}$   
 $S_1 \Rightarrow \{Q(a, g(f(b), a), f(b)), Q(a, g(f(b), a), f(b))\}$ , **Successfully Unified.**

**Unifier: [a/a, f(b)/x, b/y].**

### 6. UNIFY(knows(Richard, x), knows(Richard, John))

Here,  $\Psi_1 = \text{knows}(\text{Richard}, x)$ , and  $\Psi_2 = \text{knows}(\text{Richard}, \text{John})$

$$S_0 \Rightarrow \{ \text{knows}(\text{Richard}, x); \text{knows}(\text{Richard}, \text{John}) \}$$

SUBST  $\theta = \{ \text{John}/x \}$

$$S_1 \Rightarrow \{ \text{knows}(\text{Richard}, \text{John}); \text{knows}(\text{Richard}, \text{John}) \}, \text{Successfully Unified.}$$

**Unifier: }{John/x}.**

## Resolution

Resolution is a theorem proving technique that proceeds by building refutation proofs, i.e., proofs by contradictions. It was invented by a Mathematician John Alan Robinson in the year 1965.

Resolution is used, if there are various statements are given, and we need to prove a conclusion of those statements. Unification is a key concept in proofs by resolutions. Resolution is a single inference rule which can efficiently operate on the **conjunctive normal form or clausal form**.

**Clause:** Disjunction of literals (an atomic sentence) is called a **clause**. It is also known as a unit clause.

**Conjunctive Normal Form:** A sentence represented as a conjunction of clauses is said to be **conjunctive normal form** or **CNF**.

Note: To better understand this topic, firstly learns the FOL in AI.

## The resolution inference rule:

The resolution rule for first-order logic is simply a lifted version of the propositional rule. Resolution can resolve two clauses if they contain complementary literals, which are assumed to be standardized apart so that they share no variables.

$$l_1 V \dots V l_k, \quad m_1 V \dots V m_n$$

---


$$\text{SUBST}(\theta, l_1 V \dots V l_{i-1} V l_{i+1} V \dots V l_k V m_1 V \dots V m_{j-1} V m_{j+1} V \dots V m_n)$$

Where  $l_i$  and  $m_j$  are complementary literals.

This rule is also called the **binary resolution rule** because it only resolves exactly two literals.

## Example:

We can resolve two clauses which are given below:

**[Animal (g(x) V Loves (f(x), x)] and [¬ Loves(a, b) V ¬ Kills(a, b)]**

Where two complimentary literals are: **Loves (f(x), x)** and **¬ Loves (a, b)**

These literals can be unified with unifier  $\theta = [a/f(x), \text{ and } b/x]$ , and it will generate a resolvent clause:

**[Animal (g(x) V ¬ Kills(f(x), x)].**

## Steps for Resolution:

1. Conversion of facts into first-order logic.
2. Convert FOL statements into CNF
3. Negate the statement which needs to prove (proof by contradiction)
4. Draw resolution graph (unification).

To better understand all the above steps, we will take an example in which we will apply resolution.

## Example:

1. **John likes all kind of food.**
2. **Apple and vegetable are food**
3. **Anything anyone eats and not killed is food.**
4. **Anil eats peanuts and still alive**
5. **Harry eats everything that Anil eats.**

**Prove by resolution that:**

6. **John likes peanuts.**

### **Step-1: Conversion of Facts into FOL**

In the first step we will convert all the given statements into its first order logic.

- a.  $\forall x: \text{food}(x) \rightarrow \text{likes}(\text{John}, x)$
- b.  $\text{food}(\text{Apple}) \wedge \text{food}(\text{vegetables})$
- c.  $\forall x \forall y: \text{eats}(x, y) \wedge \neg \text{killed}(x) \rightarrow \text{food}(y)$
- d.  $\text{eats}(\text{Anil}, \text{Peanuts}) \wedge \text{alive}(\text{Anil})$ .
- e.  $\forall x: \text{eats}(\text{Anil}, x) \rightarrow \text{eats}(\text{Harry}, x)$
- f.  $\forall x: \neg \text{killed}(x) \rightarrow \text{alive}(x)$
- g.  $\forall x: \text{alive}(x) \rightarrow \neg \text{killed}(x)$
- h.  $\text{likes}(\text{John}, \text{Peanuts})$

## **Step-2: Conversion of FOL into CNF**

In First order logic resolution, it is required to convert the FOL into CNF as CNF form makes easier for resolution proofs.

### **○ Eliminate all implication ( $\rightarrow$ ) and rewrite**

1.  $\forall x \neg \text{food}(x) \vee \text{likes}(\text{John}, x)$
2.  $\text{food}(\text{Apple}) \wedge \text{food}(\text{vegetables})$
3.  $\forall x \forall y \neg [\text{eats}(x, y) \wedge \neg \text{killed}(x)] \vee \text{food}(y)$
4.  $\text{eats}(\text{Anil}, \text{Peanuts}) \wedge \text{alive}(\text{Anil})$
5.  $\forall x \neg \text{eats}(\text{Anil}, x) \vee \text{eats}(\text{Harry}, x)$
6.  $\forall x \neg [\neg \text{killed}(x)] \vee \text{alive}(x)$
7.  $\forall x \neg \text{alive}(x) \vee \neg \text{killed}(x)$
8.  $\text{likes}(\text{John}, \text{Peanuts})$ .

### **○ Move negation ( $\neg$ ) inwards and rewrite**

1.  $\forall x \neg \text{food}(x) \vee \text{likes}(\text{John}, x)$
2.  $\text{food}(\text{Apple}) \wedge \text{food}(\text{vegetables})$
3.  $\forall x \forall y \neg \text{eats}(x, y) \vee \text{killed}(x) \vee \text{food}(y)$
4.  $\text{eats}(\text{Anil}, \text{Peanuts}) \wedge \text{alive}(\text{Anil})$
5.  $\forall x \neg \text{eats}(\text{Anil}, x) \vee \text{eats}(\text{Harry}, x)$

6. $\forall x \neg \text{killed}(x) \vee \text{alive}(x)$

7. $\forall x \neg \text{alive}(x) \vee \neg \text{killed}(x)$

8.likes(John, Peanuts).

○ **Rename variables or standardize variables**

1. $\forall x \neg \text{food}(x) \vee \text{likes}(\text{John}, x)$

2.food(Apple)  $\wedge$  food(vegetables)

3. $\forall y \forall z \neg \text{eats}(y, z) \vee \text{killed}(y) \vee \text{food}(z)$

4.eats (Anil, Peanuts)  $\wedge$  alive(Anil)

5. $\forall w \neg \text{eats}(\text{Anil}, w) \vee \text{eats}(\text{Harry}, w)$

6. $\forall g \neg \text{killed}(g) \vee \text{alive}(g)$

7. $\forall k \neg \text{alive}(k) \vee \neg \text{killed}(k)$

8.likes(John, Peanuts).

○ **Eliminate existential instantiation quantifier by elimination.**

In this step, we will eliminate existential quantifier  $\exists$ , and this process is known as **Skolemization**. But in this example problem since there is no existential quantifier so all the statements will remain same in this step.

○ **Drop Universal quantifiers.**

In this step we will drop all universal quantifier since all the statements are not implicitly quantified so we don't need it.

1. $\neg \text{food}(x) \vee \text{likes}(\text{John}, x)$

2.food(Apple)

3.food(vegetables)

4. $\neg \text{eats}(y, z) \vee \text{killed}(y) \vee \text{food}(z)$

5.eats (Anil, Peanuts)

6.alive(Anil)

7.  $\neg \text{eats}(\text{Anil}, w) \vee \text{eats}(\text{Harry}, w)$

8.  $\text{killed}(g) \vee \text{alive}(g)$

9.  $\neg \text{alive}(k) \vee \neg \text{killed}(k)$

10.  $\text{likes}(\text{John}, \text{Peanuts})$ .

Note: Statements "food(Apple)  $\wedge$  food(vegetables)" and "eats (Anil, Peanuts)  $\wedge$  alive(Anil)" can be written in two separate statements.

○ **Distribute conjunction  $\wedge$  over disjunction  $\neg$ .**

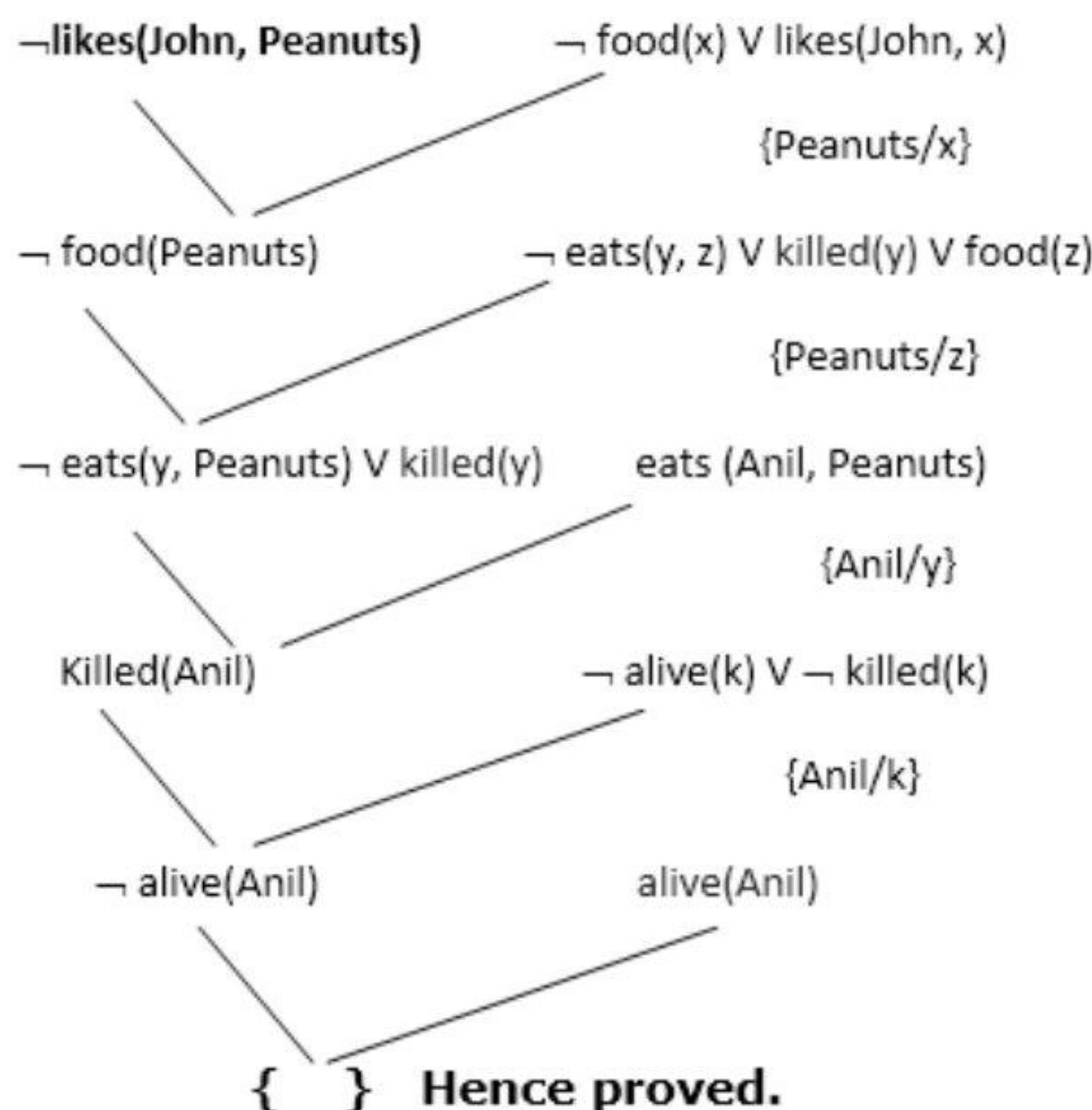
This step will not make any change in this problem.

### Step-3: Negate the statement to be proved

In this statement, we will apply negation to the conclusion statements, which will be written as  $\neg \text{likes}(\text{John}, \text{Peanuts})$

### Step-4: Draw Resolution graph:

Now in this step, we will solve the problem by resolution tree using substitution. For the above problem, it will be given as follows:



Hence the negation of the conclusion has been proved as a complete contradiction with the given set of statements.

## Explanation of Resolution graph:

In the first step of resolution graph,  $\neg \text{likes}(\text{John}, \text{Peanuts})$ , and  $\text{likes}(\text{John}, x)$  get resolved(canceled) by substitution of  $\{\text{Peanuts}/x\}$ , and we are left with  $\neg \text{food}(\text{Peanuts})$

In the second step of the resolution graph,  $\neg \text{food}(\text{Peanuts})$ , and  $\text{food}(z)$  get resolved (canceled) by substitution of  $\{\text{Peanuts}/z\}$ , and we are left with  $\neg \text{eats}(y, \text{Peanuts}) \vee \text{killed}(y)$ .

In the third step of the resolution graph,  $\neg \text{eats}(y, \text{Peanuts})$  and  $\text{eats}(\text{Anil}, \text{Peanuts})$  get resolved by substitution  $\{\text{Anil}/y\}$ , and we are left with  $\text{Killed}(\text{Anil})$ .

In the fourth step of the resolution graph,  $\text{Killed}(\text{Anil})$  and  $\neg \text{killed}(k)$  get resolved by substitution  $\{\text{Anil}/k\}$ , and we are left with  $\neg \text{alive}(\text{Anil})$ .

In the last step of the resolution graph  $\neg \text{alive}(\text{Anil})$  and  $\text{alive}(\text{Anil})$  get resolved.

## Approaches to knowledge representation

The only concept we are left with now of how we can store the information in the system. Of the different ways, there are 4 main approaches to knowledge representation in artificial intelligence, viz. simple relational knowledge, inheritable knowledge, inferential knowledge, and procedural knowledge—each of these ways corresponding to a technique of representing knowledge discussed above.

### •Simple Relational Knowledge

This is a relational method of storing facts which is among the simplest of the methods. This method helps in storing facts where each fact regarding an object is provided in columns. This approach is prevalent in DBMS (database management systems).

### •Inheritable Knowledge

Knowledge here is stored hierarchically. A well-structured hierarchy of classes is formed where data is stored, which provides the opportunity for inference. Here we can apply inheritance property, allowing us to

have inheritable knowledge. This way, the relations between instance and class (aka instance relation) can be identified. Unlike Simple Relations, here, the objects are represented as nodes.

#### **•Inferential Knowledge**

In this method, logics are used. Being a very formal approach, facts can be retrieved with a high level of accuracy.

#### **•Procedural Knowledge**

This method uses programs and codes that use simple if-then rules. This is the way many programming languages such as LISP, Prolog save information. We may not use this method to represent all forms of knowledge, but domain-specific knowledge can very efficiently be stored in this manner.

