

Data Mining

Classification III – Naïve Bayes (Part A)

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Bayes Theorem

- Conditional probability

$$P(A,B) = P(A|B)P(B) = P(B|A)P(A)$$

$$\rightarrow P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayes rule

Naïve Bayes Classifier

- Applying Bayes rule

$$\begin{aligned} P(Y_i|X_1 \dots X_n) &= \frac{P(Y_i) P(X_1 \dots X_n|Y_i)}{P(X_1 \dots X_n)} \\ &= \frac{P(Y_i) P(X_1|Y_i)P(X_2|Y_i) \dots P(X_n|Y_i)}{P(X_1 \dots X_n)} \end{aligned}$$

$$Y \leftarrow \arg \max P(Y_i) P(X_1|Y_i)P(X_2|Y_i) \dots P(X_n|Y_i)$$

Assumption: X_i 's are conditionally independent given Y

Terminology

$P(Y|\mathbf{X})$: posterior probability for Y

$P(Y)$: prior probability

$P(\mathbf{X}|Y)$: class-conditional probability

$P(\mathbf{X})$: evidence

Bayes theorem (Bayes rule) allows us to calculate the posterior probability $P(Y|\mathbf{X})$ using the prior probability $P(Y)$, the class-conditional probability $P(\mathbf{X}|Y)$ and the evidence $P(\mathbf{X})$ (which is constant and ignored).

Algorithmic Details (Training Phase)

- Naïve Bayes Algorithm

- Learning/training phase: given a training data set T ,

- For each label Y_i

- $P(Y_i) \leftarrow$ calculate $P(Y=Y_i)$ using training examples in T ;

- For each attribute value X_j

- calculate $P(X_j|Y_i)$ using training examples in T .

Training Data Set

Student	Assignment	Project	Exam	Label
1	Good	A	High	Pass
2	Good	B	High	Pass
3	Bad	B	Low	Fail
4	Bad	C	High	Fail
5	Good	C	Low	Fail
6	Good	C	High	Pass
7	Bad	B	High	Pass
8	Good	A	Low	Pass
9	Bad	A	Low	Fail
10	Good	B	Low	Pass

Training Phase

$$P(\text{Pass}) = 6/10; P(\text{Fail}) = 4/10$$

$$P(\text{Good}|\text{Pass}) = 5/6; P(\text{Bad}|\text{Pass}) = 1/6$$

$$P(\text{High}|\text{Fail}) = 1/4; P(\text{Low}|\text{Fail}) = 3/4$$

...

Algorithmic Details (Testing Phase)

- Naïve Bayes Algorithm

- Testing phase: given an unlabeled test record
 $X^* = \langle X_1 \dots X_n \rangle$

- Assign label Y to X^* based on
 $\max\{P(Y_i)P(X_1|Y_i)P(X_2|Y_i)\dots P(X_n|Y_i)\}$

End of Naïve Bayes Module (Part A)