2.) $A + 2B$

$$2B = \begin{bmatrix} 14 & -10 & 2 \\ 2 & -8 & -6 \end{bmatrix}$$

$$A+2B = \begin{bmatrix} 16 & -10 & 1 \\ 6 & -13 & -4 \end{bmatrix}$$

$3C - E$ = undefined. E is 2×1 and C is 2×2

$$CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$$

EB = undefined: E has 1 column but B has 2 rows

4)

$$A = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - 5_3 = \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 3 \\ -8 & 2 & -6 \\ -4 & 1 & 3 \end{bmatrix}$$

$$(5I_3)A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 9 & -1 & 3 \\ -8 & 7 & -6 \\ -4 & 1 & 8 \end{bmatrix} = \begin{bmatrix} 45 & -5 & 15 \\ -40 & 35 & -30 \\ -20 & 5 & 40 \end{bmatrix}$$

6)

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \qquad B = \begin{array}{c} \begin{array}{cc} b_1 & b_2 \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \end{array}$$

$$Ab_1 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix} \qquad A[b_1 \ \ b_2] = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

$$Ab_2 = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 14 \\ -9 \\ 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 4\cdot 1 + -2\cdot 2, & 4\cdot 3 + -2\cdot -1 \\ -3\cdot 1 + 0\cdot 2, & -3\cdot 3 + 0\cdot -1 \\ 3\cdot 1 + 5\cdot 2, & 3\cdot 3 + 5\cdot -1 \end{bmatrix} = \begin{bmatrix} 0 & 14 \\ -3 & -9 \\ 13 & 4 \end{bmatrix}$$

10) $A = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 8 & 4 \\ 5 & 5 \end{bmatrix} \qquad C = \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 2\cdot 8 + -3\cdot 5, & 2\cdot 4 + -3\cdot 5 \\ -4\cdot 8 + 6\cdot 5, & -4\cdot 4 + 6\cdot 5 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 2\cdot 5 + -3\cdot 3, & 2\cdot -2 + -3\cdot 1 \\ -4\cdot 5 + 6\cdot 3, & -4\cdot -2 + 6\cdot 1 \end{bmatrix}$$

$AB = AC$ ✓ but

$$= \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

12) $A = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$ ✓

$$AB = \begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3\cdot 2 + -6\cdot 1, & 3\cdot 4 + -6\cdot 2 \\ -1\cdot 2 + 2\cdot 1, & -1\cdot 4 + 2\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} ✓$$

13) $r_1, \ldots, r_p$ is in R^n so R is $n \times p$

Q is $m \times n$ so a matrix as the product would be QR a $m \times n$ matrix

So $QR = [Qr_1 \cdots Qr_p]$ since Rs n rows is the same as Qs n columns these matrices can be multiplied.

18) $$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 10 & 10 \end{bmatrix}$$

they would be the same because you are essentially multiplying matrix A by the same weights $b_1 = b_2$ twice

19) $\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}\begin{bmatrix}1 & 3 & 4\\ 2 & 4 & 6\end{bmatrix} = \begin{bmatrix}7 & 15 & 22\\ 10 & 22 & 32\end{bmatrix}$

The 3rd column of AB would be the sum of the first two columns because $A\cdot b_1 + A\cdot b_2$ is the same as $A(b_1 + b_2)$ according to Thrm 2.

20) that it will also be all zeros because every element in A will be multiplied by zero.

22) $x_1 b_1 + \dots + x_n b_n = 0$ with not all $x_i = 0$

B are lin dep.

So the coef of $AB = [Ab_1 \ Ab_2 \cdots Ab_n$

$A(x_1 b_1 + \dots + x_n b_n) = 0$

So if two columns add up to zero then just multiplying by A to both those columns would still mean that they could add together to be zero.

24) $\overset{m\times n}{A}\overset{n\times m}{D} = I_m$

$A\bar{x} = \bar{b}$ has a solution for any $\bar{b}$

$\bar{x}_{R^n}$ columns of A span R^m

$T(x) = A\bar{x}$ is onto.

$D\bar{b}$ = vector in $\mathbb{R}^n$

$= \bar{q}$

$A\bar{q} = A(D\bar{b}) = (AD)\bar{b} = I_m\bar{b} = \bar{b}$

so any solution works.

2.2)

3) $\begin{bmatrix} 8 & 5 \\ -7 & -5 \end{bmatrix}$ $\quad \frac{1}{ad-bc} = \frac{1}{8\cdot(-5)+35} = \frac{1}{-5}$

$A^{-1} = \frac{1}{-5}\begin{bmatrix} -5 & -5 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}$

6) $\bar{x} = A^{-1}b$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -7/5 & -8/5 \end{bmatrix}\begin{bmatrix} -9 \\ 11 \end{bmatrix} = \begin{bmatrix} -9 + 11 \\ 63/5 + -88/5 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

14) $(B-C)D = 0$ so B must equal C.

so $BD - CD = 0$

$BD = CD$

$BDD^{-1} = CDD^{-1}$

$BI_n = CI_n$

$B = C$ ✓

18) Suppose P is invertable and $A = PBP^{-1}$

So $AP = PBP^{-1}$

$AP = PBI_n$

$AP = PB$

$A = PBP^{-1}$

$P^{-1}A = P^{-1}(PBP^{-1})$

$\quad = (P^{-1}P)BP^{-1}$

$P^{-1}A = BP^{-1}$

$(P^{-1}A)P = (BP^{-1})P$

$P^{-1}AP = B$ ✓

24) b is in $\mathbb{R}^m$ and $AD = I_m$ $Ax = b$

The identity matrix solves $I_m b = b$

So $ADb = I_m b = b$

so ... ...

AD is a matrix and b is a vector
so $x = Db$ and the equation has a
solution.

If $Ax = b$ has a solution then there
must be a pivot position in every row
if A has more rows than columns then
there is a row with all zeros.

22)
consider columns b_1 b_2 in B that are
are linearly dependent. this means that one
must be a multiple of the other.

so $A(b_1 \ b_2) = Ab_1, Ab_2$. A is a multiple
of A and b_1 is a multiple of b_2 consider C_1
to be some multiple on b_1
C is some multiple.

so $b_1 = b_1 c_1 = b_2$

$Ab_1 = Ab_1 c_1$

$Ab_1 = c_1(Ab_1)$ by thrm 2

—

since Ab_1 is just some multiple on
b_2 shown as $b_1 = Cb_2$ Ab_2 is
a multiple C of Ab_1 and so

they must be literally delusional.