LAB-04

Question 1:

1. $1^2 + 2^2 + 3^2 + \ldots \ N^2 \qquad \Theta(N^3)$.

$\quad N(N^2) = N^3 \ \rightarrow \ O(N^3) \quad - \text{ upper bound}$

$\quad \Omega(N^3) \Rightarrow \text{ lower bound}.$

$\Rightarrow \Omega(N^3) \leq f(n) \leq O(N^3)$.

2. $1 + 2 + 3 + \ldots + N^2 \qquad \Theta(N^4)$.

$\quad N^2(N^2) \rightarrow N^4 \rightarrow O(N^4)$.

So,

$\quad \Omega(N^4) \leq f(n) \leq O(N^4)$.

3. $1 + 3 + 5 + 7 + 9 + \ldots + (2N+1) \qquad \Theta(N^2)$.

$\quad (2N+1)(2N+1) = 4N^2 + 2N + 2N + 1$

$\quad \quad \quad \quad \quad = 4N^2 + 4N + 1 \ \rightarrow \ O(N^2)$.

$\Omega(N^2) \leq f(n) \leq O(N^2)$.

4. $2 + 4 + 6 + 8 + \ldots + 2N \qquad \Theta(N^2)$

$\quad 2N(2N) = 4N^2 \rightarrow O(N^2)$.

$\Omega(N^2) \leq f(n) \leq O(N^2)$.

5. $1 + 2 + 3 + 4 + \ldots + N/2 \ldots \quad \Theta(N^2)$

$\quad N/2(N/2) = N^2/4 \rightarrow O(N^2)$

So,

$\quad \Omega(N^2) \leq f(n) \leq O(N^2)$

6. $1 + 2 + 4 + 6 + \ldots + N^2 \qquad \Theta(N^2)$.
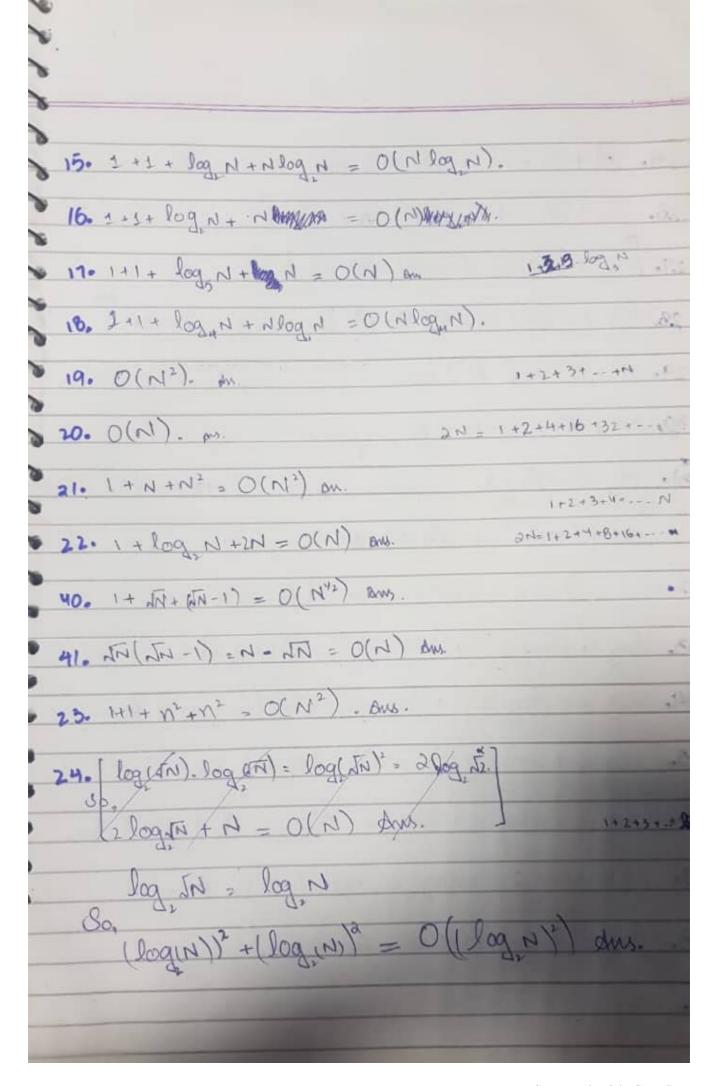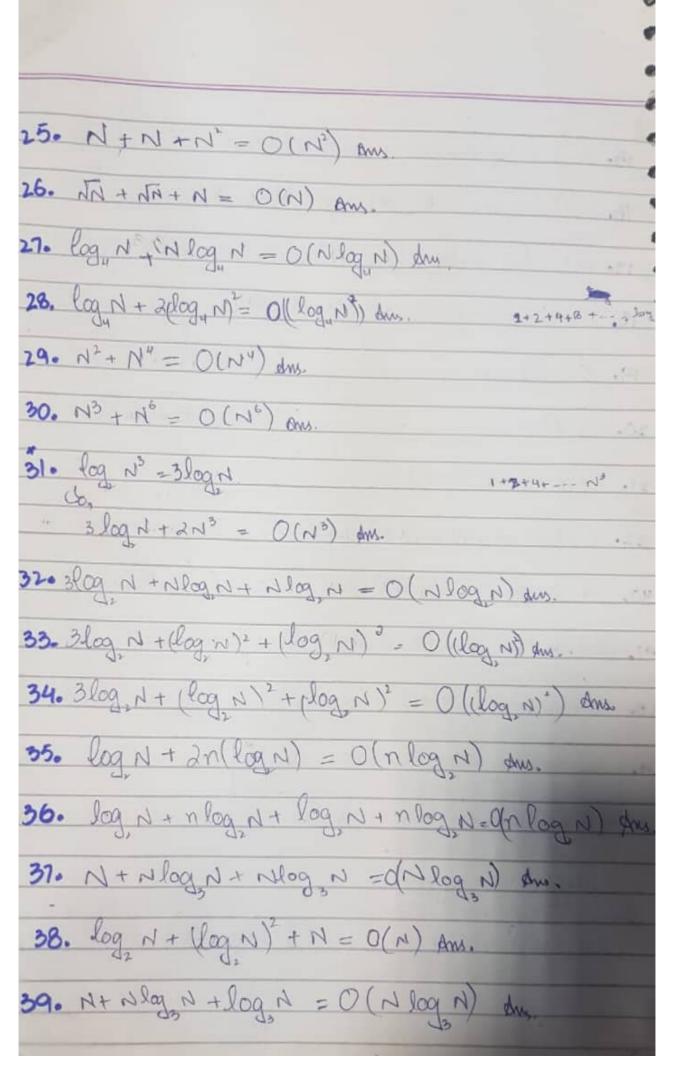
$\quad 2N^2 \rightarrow O(N^2)$

$\Omega(N^2) \leq f(n) \leq O(N^2)$.

Question 2:-

1. $O(1) + O(N) + O(N^2) + O(N^2) = O(N^2)$. Ans.

2. $O(1) + O(N) + O(N) + O(N) + O(N) = O(N)$. Ans.

3. $O(1) + O(N) + O(N^2) + O(N^3) + O(N^3) = O(N^3)$
   $O(N) + O(N^2) + O(N^3) + O(N^3) = O(N^3)$.
   $= O(N^3)$. Ans

4. $O(1) + O(N) + O(N) + O(N) + O(N) \cdots$
   $= O(N)$ Ans.

5. $O(1) + O(N) + O(N^2) + O(N^3) + O(N^3) = O(N^3)$ Ans.

6. $O(1) + O(N) + O(N^2) + O(N^3) + O(N^3) = O(N^3)$ Ans.

7. $O(1) + O(\log_2 N) + O(N) + O(N) = O(N)$. Ans
   $O(\log_2 N^2) + O(\log_2 N^2) = O((\log(N))^2)$.

8. $O(1) + O(\log_2 N) + O(\log_2 N) + O(\log_2 N) + O(\log_2 N) = O(\log_2 N)$

9. $O(N^2) + O(N^2 \cdot \log_2 (N^2)) + " "$
   $= O(N^2 \log_2 N)$ Ans.

10. $O(N^2) + O(\log_2 N) = O(\log_2 N)$. Ans. $\dfrac{\log N^2}{2 \log_2 N}$

11. $O(\log_2 N) + O((\log_2 N)^2) = O((\log_2 N)^2)$ Ans.

12. $O(\log_2 N) + O(\log_2 N) = O(\log_2 N)$ Ans.

13. $O(1) + O(\log_2 N) + O((\log_2 N)^2) + O(\log_2 N^3) = O((\log_2 N)^3)$

14. $O(1) + O(\log N) + O(\log N) + O(\log N) = O(\log N)$ Ans.

15. $1 + 1 + \log_2 N + N\log_2 N = O(N \log_2 N)$.

16. $1 + 1 + \log_2 N + N \log_2 N = O(N) \log_2 N$.

17. $1 + 1 + \log_3 N + \log_3 N = O(N)$ Ans.

    $1 + 2 \cdot 9 \cdot \log_3 N$

18. $1 + 1 + \log_4 N + N\log_4 N = O(N\log_4 N)$.

19. $O(N^2)$. Ans.

    $1 + 2 + 3 + \cdots + N$

20. $O(N)$. Ans.

    $2N = 1 + 2 + 4 + 16 + 32 + \cdots$

21. $1 + N + N^2 = O(N^2)$ Ans.

    $1 + 2 + 3 + 4 + \cdots N$

22. $1 + \log_2 N + 2N = O(N)$ Ans.

    $2N = 1 + 2 + 4 + 8 + 16 + \cdots N$

40. $1 + \sqrt{N} + (\sqrt{N} - 1) = O(N^{1/2})$ Ans.

41. $\sqrt{N}(\sqrt{N} - 1) = N - \sqrt{N} = O(N)$ Ans.

23. $1 + 1 + n^2 + n^2 = O(N^2)$. Ans.

24. $\left[\log_2(\sqrt{N}) \cdot \log_2(\sqrt{N}) = \log(\sqrt{N})^2 = 2\log_2 \sqrt{2} \right]$

    So,
    $2\log_2\sqrt{N} + N = O(N)$ Ans.

    $1 + 2 + 3 + \cdots N$

    $\log_2 \sqrt{N} = \log_2 N$

    So,
    $(\log(N))^2 + (\log_2(N))^2 = O((\log_2 N)^2)$ Ans.

25. $N + N + N^2 = O(N^2)$ Ans.

26. $\sqrt{N} + \sqrt{N} + N = O(N)$ Ans.

27. $\log_4 N + N \log_4 N = O(N \log_4 N)$ Ans.

28. $\log_4 N + 2(\log_4 N)^2 = O((\log_4 N)^2)$ Ans.

$1 + 2 + 4 + 8 + \cdots = \log_2$

29. $N^2 + N^4 = O(N^4)$ Ans.

30. $N^3 + N^6 = O(N^6)$ Ans.

* 31. $\log_2 N^3 = 3 \log_2 N$

$1 + 2 + 4 + \cdots N^3$

So,

$3 \log_2 N + 2 N^3 = O(N^3)$ Ans.

32. $3 \log_2 N + N \log_2 N + N \log_2 N = O(N \log_2 N)$ Ans.

33. $3 \log_2 N + (\log_2 N)^2 + (\log_2 N)^3 = O((\log_2 N)^3)$ Ans.

34. $3 \log_2 N + (\log_2 N)^2 + (\log_2 N)^2 = O((\log_2 N)^2)$ Ans.

35. $\log_2 N + 2n(\log_2 N) = O(n \log_2 N)$ Ans.

36. $\log_2 N + n \log_2 N + \log_2 N + n \log_2 N = O(n \log_2 N)$ Ans.

37. $N + N \log_3 N + N \log_3 N = O(N \log_3 N)$ Ans.

38. $\log_2 N + (\log_2 N)^2 + N = O(N)$ Ans.

39. $N + N \log_3 N + \log_3 N = O(N \log_3 N)$ Ans.

Question 3:

f_1 :-

$$1 + \log_2 N + N = O(N).$$

f_2 :-

$$1 + N + N = O(N).$$

f_3 :-

$$1 + N + \log_2 \sqrt{N}$$
$$= 1 + N + \frac{1}{2}\log_2 N = O(N).$$

f_4 :-

$$1 + N + N + N^3 = O(N^3).$$

f_5 :-

$$1 + N + N + N^2 = O(N^2).$$

f_6 :-

$$1 + O(N) + O(N) + O(N^1) + O(N^3) + O(N^2) = O(N^3).$$