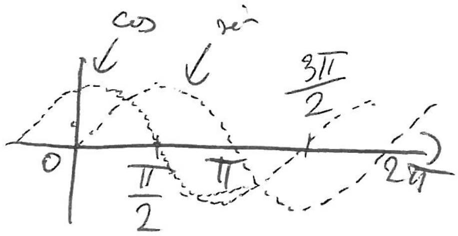
1) $\Omega = \{0, 1, 2, 3\}$, $P(\{k\}) = \frac{1}{4}$ $k = 0, 1, 2, 3$,

$X(\omega) = \sin \frac{\pi\omega}{2}$ $Y(\omega) = \cos \frac{\pi\omega}{2}$




a)

| $\omega$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(\{\omega\})$ | 1/4 | 1/4 | 1/4 | 1/4 |
| $x = X(\omega)$ | 0 | 1 | 0 | -1 |
| $y = Y(\omega)$ | 1 | 0 | -1 | 0 |

ROZKŁAD P-STWA X:

| $x_i$ | -1 | 0 | 1 |
|---|---|---|---|
| | $\frac{1}{4}$ | $\frac{1}{2}$ $\left(\frac{1}{4} + \frac{1}{4}\right)$ | $\frac{1}{4}$ |

ROZKŁAD P-STWA Y =

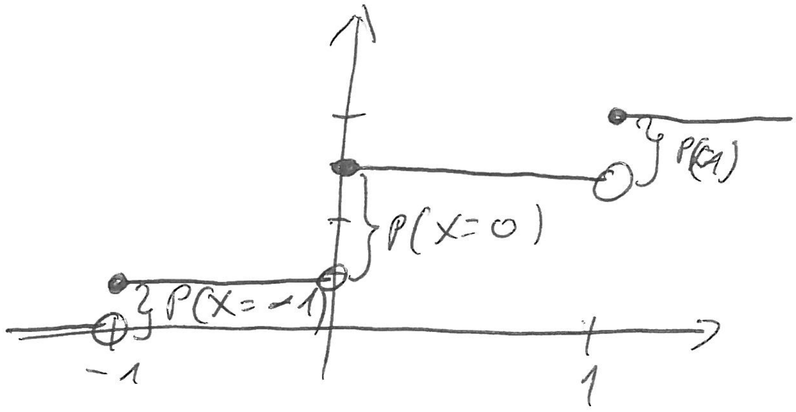
| $x_i$ | -1 | 0 | 1 |
|---|---|---|---|
| | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

b) $F(t) = P(X \le t) = P(Y \le t) = \sum_{i: x_i \le t} p_i =$

$= \begin{cases} 0 \\ \frac{1}{4} \\ \frac{3}{4} \\ 1 \end{cases}$

1

$$= \begin{cases} 0 = & t < -1 \\ \frac{1}{4} = & -1 < t < 0 \\ \frac{3}{3} = & 0 < t < 1 \\ 1 = & t \geq 1 \end{cases}$$




② $\Omega = \{1, 2, 3, 4, 5, 6\}$ $P(\{k\}) = \frac{1}{6}$ dla $k = 1, 2, 3, 4, 5, 6$

| $\omega$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(\{k\})$ | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |
| $X$ | 0 | 1 | -2 | 1 | -2 | 1 |

ROZKŁAD

| $x_i$ | -2 | 0 | 1 |
|---|---|---|---|
| ~~P(X)~~ pi $P(X=x_i)$ | $\frac{1}{3}$ $\left(\frac{1}{6}+\frac{1}{6}\right)$ | $\frac{1}{6}$ | $\frac{1}{2}$ $\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right)$ |

ŚREDNIA WARTOŚĆ

$$E(X) = -2 \cdot \frac{1}{3} + 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{2} =$$

$$= -\frac{1}{6}$$

(3)

X - LICZBA ORŁÓW

| $\omega$ | OOO | OOR | ORO | ROO | RRO | ROR | ORR | RRR |
|---|---|---|---|---|---|---|---|---|
| $P(\{\omega\})$ | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 | 1/8 |
| $x = X(\omega)$ | 3 | 2 | 2 | 2 | 1 | 1 | 1 | 0 |

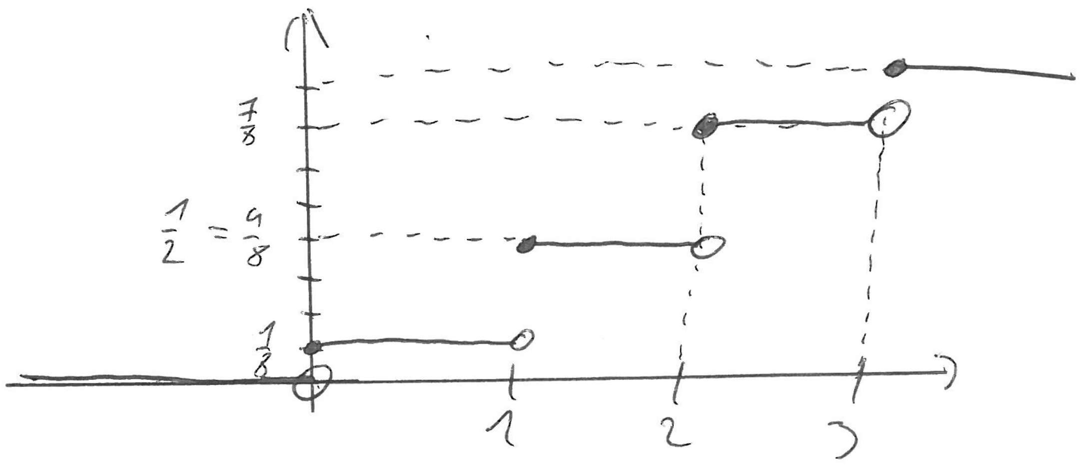
a)
ROZKŁAD

| $x_i$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $p_i = P(X = x_i)$ | 1/8 | 3/8 | 3/8 | 1/8 |

$3/8 = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $\quad 3/8 = \frac{1}{8}+\frac{1}{8}+\frac{1}{8}$

DYST

$$F(t) = P(X \leq t) = \begin{cases} 0, & t < 0 \\ \frac{1}{8} & 0 \leq t < 1 \\ \frac{1}{2} & 1 \leq t < 2 \\ \frac{7}{8} & 2 \leq t < 3 \\ 1 & t \geq 3 \end{cases}$$



a) $P(X \leq 2) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

$P(X > 1) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

$P(X = 2) = \frac{3}{8}$

$P(2 \leq X < 4) = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

c) $E(X) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8} =$

$= \frac{12}{8} = \frac{3}{2}$ ← WARTOŚĆ OCZEKIWANA

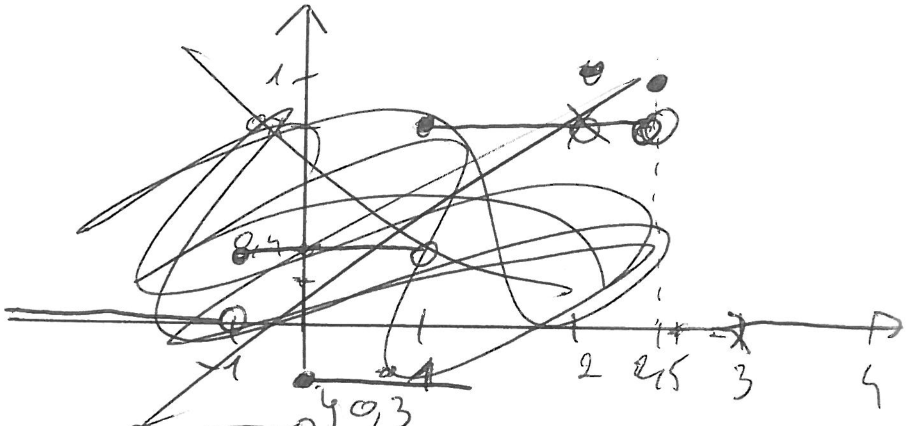
$E(X^2) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{3}{8} + 3^2 \cdot \frac{1}{8} =$

$= 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = 3$

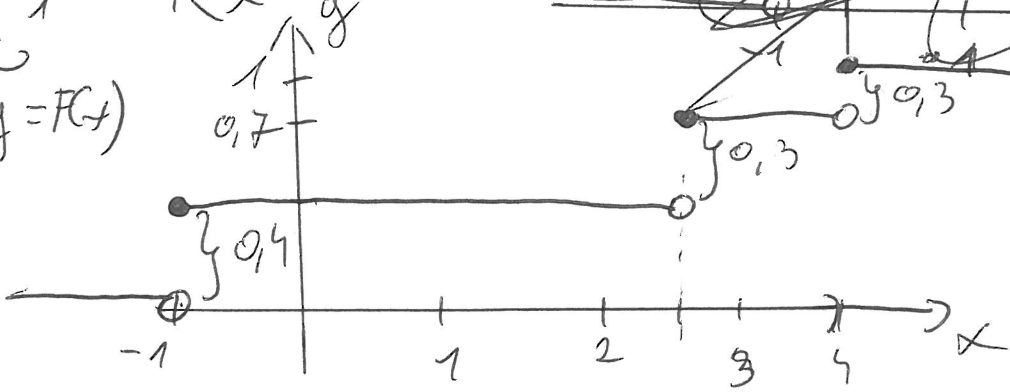
$VAR(X) = 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4} = \frac{3}{4}$ ← WARIANCJA

$\sigma_X = \sqrt{VAR(X)} = \sqrt{\frac{3}{4}} \approx 0{,}87$ ← ODCHYLENIE STND.

4) $F(x) = \begin{cases} 0 & x < -1 \\ 0{,}4 & -1 \leq x < 2{,}5 \\ 0{,}7 & 2{,}5 \leq x < 4 \\ 1 & 4 \leq x \end{cases}$




$y = F(x)$




$0 \leq F(x) \leq 1 \quad \forall\ x \in R$

F(x) JEST NIE MAL.

F(x) JEST PRAWO-STRONNIE CIĄG.

$\lim_{x \to -\infty} F(x) = 0 \qquad \lim_{x \to +\infty} F(x) = 1$

ROZKŁAD P-STWA ZL. X

| $x_i$ | $-1$ | $2{,}5$ | $4$ |
|---|---|---|---|
| $p_i = P(X = x_i)$ | $0{,}4$ | $0{,}3$ | $0{,}3$ |

b) $P(X > 0) = 0{,}3 + 0{,}3 = 0{,}6$

$P(1 \leq X \leq 3) = 0{,}3$

$P(-2 \leq X \leq 3) = 0{,}4 + 0{,}3 = 0{,}7$

c) $E(x) = -1 \cdot 0,4 + 2,5 \cdot 0,3 + 4 \cdot 0,3$

$E(x) = -0,4 + 0,75 + 1,2 = 1,55$ ← OCZEKIWANA

$E(x^2) = \cancel{(\ldots)} \; (-1)^2 \cdot 0,4 + (2,5)^2 \cdot 0,3 + (4)^2 \cdot 0,3 =$

$= 0,4 + 1,875 + 4,8 = 7,075$

$VAR(x) = 7,075 - (1,55)^2 = 4,6725$ ← WARIANCJA

$\sigma_x = \sqrt{VAR(x)} = \sqrt{4,6725} \approx 2,16$

6)

| $x_i$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $p_i$ | $a$ 0,35 | 0,4 | 0,2 | $b$ 0,05 |

$E(x) = 2,95$

$$\begin{cases} a + 0,4 + 0,2 + b = 1 \\ 2a + 3 \cdot 0,4 + 4 \cdot 0,2 + 5b = 2,95 \end{cases}$$

$$\begin{cases} a + b = 0,4 \\ 2a + 5b = 0,95 \end{cases}$$

$$\begin{cases} a = 0,4 - b \\ 2(0,4 - b) + 5b = 0,95 \end{cases}$$

$0,8 - 2b + 5b = 0,95$

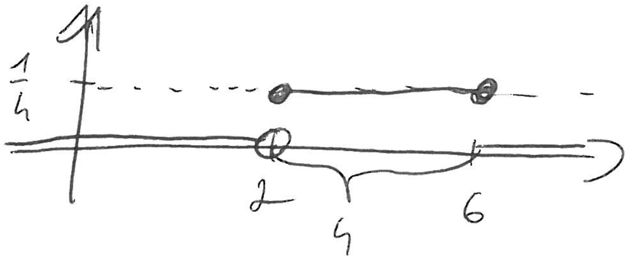
$3b = 0,15 \quad /:3$

$b = 0,05 \quad a = 0,35$

$$F(t) = \begin{cases} 0 & t < 2 \\ 0{,}35 & 2 \leq t < 3 \\ 0{,}75 & 3 \leq t < 4 \\ 0{,}95 & 4 \leq t < 5 \\ 1 & 5 \leq t \end{cases}$$

c) $P(\cancel{x}\, t \geq 4) = 0{,}2 + 0{,}05 = 0{,}25$

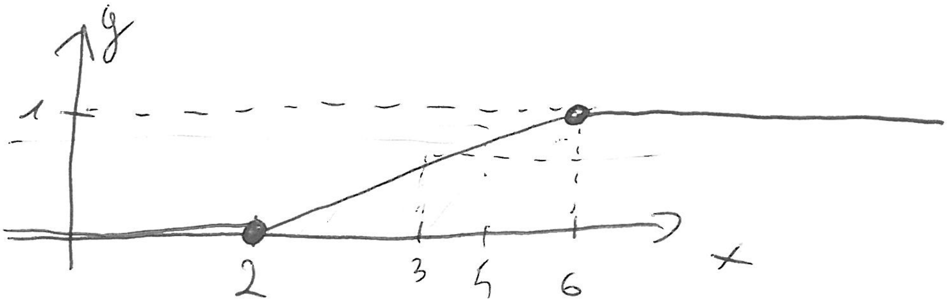
g) $f(t) = \begin{cases} A & \rightarrow x \in [2;6] \\ 0 & \rightarrow x \notin [2;6] \end{cases}$




$6 - 2 = 4$

$A = \frac{1}{4}$

G) $F(t) = \begin{cases} \int_{-\infty}^{t} 0\,dx = 0 & \rightarrow t < 2 \\ 0 + \int_{2}^{t} \frac{1}{4}\,dx = \frac{t-2}{4} & 2 \leq t < 6 \\ 0 + \frac{t-2}{4} + \int_{6}^{t} 0\,dx = 1 & 6 \leq t \end{cases}$




$$P(X \le 5) = \int_{-\infty}^{5} f(x)\,dx = \frac{1}{4}(5-2) = \frac{3}{4}$$

$$P(x > 3) = \int_{3}^{+\infty} f(x)\,dx = \frac{1}{4}(6-3) = \frac{3}{4}$$

$$P(x = 3) = 0$$

$$P(1 \le X \le 4) = \int_{1}^{4} f(x)\,dx = \frac{1}{4}(4-2) = \frac{1}{2}$$

BO 2>1

c) $$E(X) = \underbrace{\int_{-\infty}^{2} x \cdot 0\,dx}_{0} + \underbrace{\int_{2}^{6} x \tfrac{1}{4}\,dx}_{2} + \underbrace{\int_{6}^{+\infty} x\, 0\,dx}_{0} =$$

d)

$$= \frac{1}{4}\int_{2}^{6} x\,dx = \frac{1}{4}\left(\frac{1}{2}x^2\right)_{2}^{6} = \frac{1}{4}\cdot\frac{1}{2}\left(x^2\right)_{2}^{6} =$$

$$= \frac{1}{8}\left(6^2 - 2^2\right) = 4$$ ← OCZEKIWANA

$$E(X^2) = \int_{-\infty}^{2} x^2 0\,dx + \int_{2}^{6} x^2 \tfrac{1}{4}\,dx + \int_{6}^{+\infty} x\, 0\,dx =$$

$$= 0 + \frac{1}{4}\left(\frac{x^3}{3}\right)_{2}^{6} + 0 =$$

$$= \frac{1}{4}\left(\frac{1}{3}x^3\right)_2^6 = \frac{1}{4}\cdot\frac{1}{3}\left(x^3\right)_2^6 = \frac{1}{12}\left(6^3 - 2^3\right) =$$

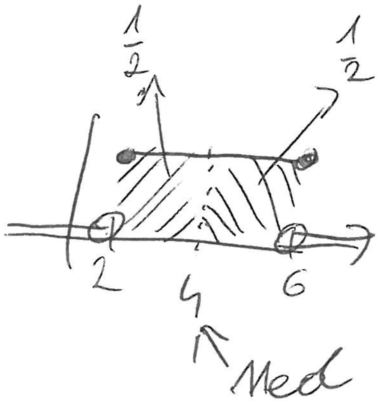
$$= \frac{216-8}{12} = \frac{52}{3}$$

$$VAR(X) = E(X^2) - (E(X))^2 = \frac{52}{3} - 4^2 = \frac{52}{3} - \frac{48}{3} =$$

$$= \frac{4}{3} \leftarrow \text{VARIANCJA}$$

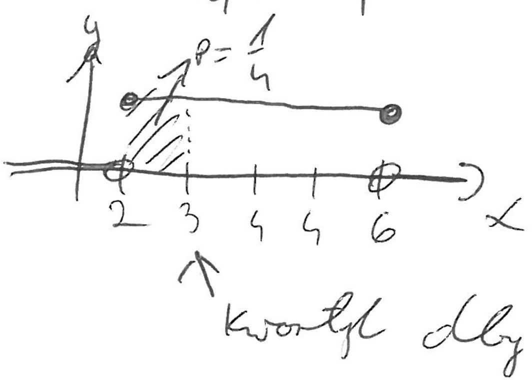
$$\sigma_X = \sqrt{VAR(X)} = \sqrt{\frac{4}{3}} \approx 1,15$$

$F(Med) = \frac{1}{2}$



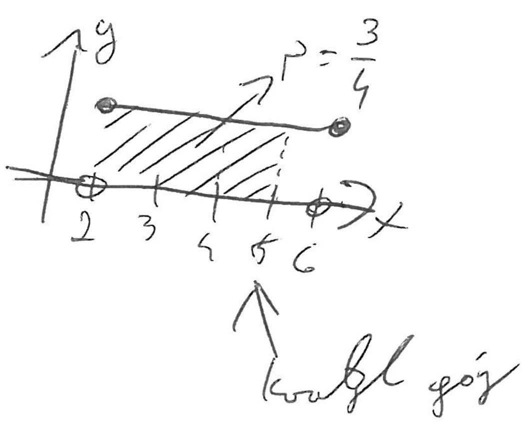

kwantyl dolny $Q_1 = q_{1/4}$

$F(q_{1/4}) = \frac{1}{4}$



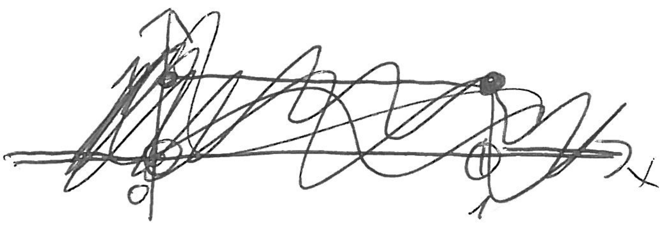

kwantyl górny $Q_3 = q_{3/4}$

$F(q_{3/4}) = \frac{3}{4}$



10)

$$f(x) = \begin{cases} A(x-1) & \to x \in [0;1] \\ 0 & \to x \notin [0;1] \end{cases}$$




$\cancel{A(0-1)=1}$

$\cancel{2A \quad A \quad /:2}$

$\cancel{A = \frac{1}{2}}$

$\cancel{2(x-1)}$

$\cancel{2x+2}$

$\cancel{x-A}$

$\cancel{0 \quad 1 \quad 0}$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\underbrace{\int_{-\infty}^{0} 0\,dx}_{0} + \int_{0}^{1} A(x-1)\,dx + \underbrace{\int_{1}^{\infty} 0\,dx}_{0} = 1$$

$\cancel{A\left[\frac{x^2}{2} - x\right]_0^1}$

$$A\left[\frac{x^2}{2} - x\right]_0^1 = A\cdot\left(-\frac{1}{2}\right) = 1 \quad /:\left(-\frac{1}{2}\right)$$

$$A = -2$$

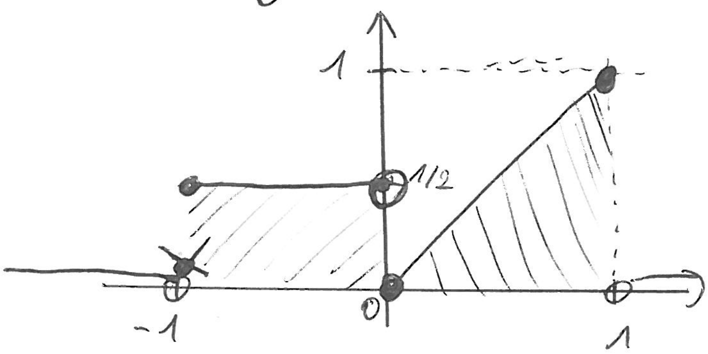
$-2(x-1)$

$-2x+2$

$$F(t) = \begin{cases} \int_{-\infty}^{t} 0\,dx = 0 & \rightarrow t < 0 \\ 0 + \int_{0}^{t} -2x + 2\,dx = & \Rightarrow 0 \le t < 1 \\ \cancel{0 + \int_0^t} = 0 - t^2 + 2t & \rightarrow 0 \le t < 1 \\ 1 & \rightarrow t \geq 1 \end{cases}$$

12)

$$f(x) = \begin{cases} 1/2 & \rightarrow x \in [1; 0) \\ x & \rightarrow x \in [0; 1] \\ 0 & \rightarrow x \notin [-1; 1] \end{cases}$$

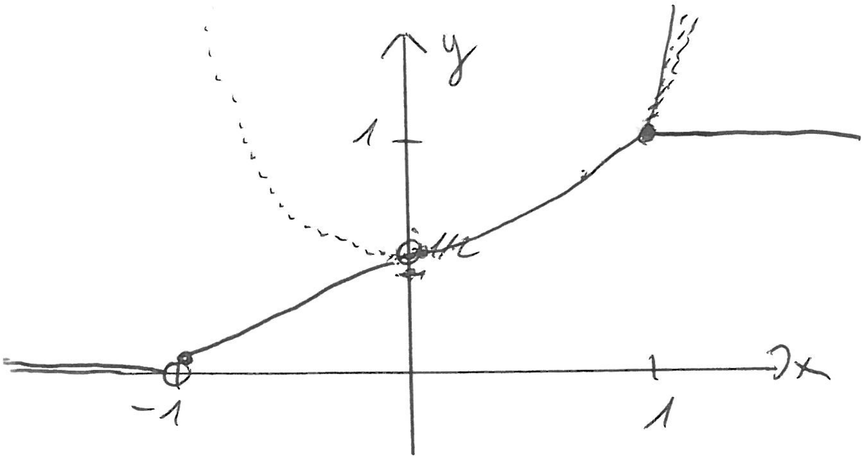



DYSTRYBUANTA

$$F(t) = \int_{-\infty}^{t} f(x)\,dx = \begin{cases} \int_{-\infty}^{t} 0\,dx = 0 & \rightarrow t < -1 \\ \frac{t+1}{2} & \rightarrow -1 \le t < 0 \\ \frac{t^2+1}{2} & \rightarrow 0 \le t < 1 \\ 1 & \rightarrow t \geq 1 \end{cases}$$

① $\int_{-1}^{t} \frac{1}{2} dx = \frac{1}{2} \int_{-1}^{t} dx = \frac{1}{2} [x]_{-1}^{t} = \frac{t+1}{2}$

② $\int_{-1}^{0} \frac{1}{2} dx + \int_{0}^{t} x\, dx = \frac{1}{2} + \frac{t^2}{2} = \frac{t^2+1}{2}$




$P(|X - 1/8| < 5/8)$

$|X - \frac{1}{8}| < \frac{5}{8} \iff -\frac{5}{8} < X - \frac{1}{8} < \frac{5}{8}$

$-\frac{5}{8} + \frac{1}{8} < X < \frac{5}{8} + \frac{1}{8}$

$-\frac{4}{8} = -\frac{1}{2} < X < \frac{6}{8} = \frac{3}{4}$

$P(-\frac{1}{2} < X < \frac{3}{4}) = F(\frac{3}{4}) - F(-\frac{1}{2}) =$

$= \frac{(\frac{3}{4})^2 + 1}{2} - \frac{(-\frac{1}{2}) + 1}{2} \approx 0{,}78 - 0{,}25 = 0{,}53$

c) $EX = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{-\infty}^{-1} x \cdot 0\,dx + \int_{-1}^{0} x \cdot \frac{1}{2}\,dx + \int_{0}^{1} x \cdot x\,dx +$

$+ \int_{1}^{+\infty} x \cdot 0\,dx = \frac{1}{2}\left[\frac{1}{2}x^2\right]_{-1}^{0} + \left[\boxed{\frac{1}{3}}x^3\right]_{0}^{1} =$

$= -\frac{1}{4} + \frac{1}{3} = \frac{1}{12}$ ← OCZEKIWANA

$VAR\,X = E(X^2) - (EX)^2.$

$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \int_{-\infty}^{-1} x^2 \cdot 0\,dx + \int_{-1}^{0} x^2 \cdot \frac{1}{2}\,dx +$

$+ \int_{0}^{1} x^2 \cdot x\,dx + \int_{1}^{+\infty} x^2\, 0\,dx =$

$= +\frac{1}{6} + \left(+\frac{1}{4}\right) = \frac{5}{12}$

$VAR\,X = \cancel{\frac{1}{12} - \frac{1}{144}}\ \frac{5}{12} - \frac{1}{144} = \frac{59}{144}$

$\sigma_X = \sqrt{VAR\,X} \approx 0{,}64$

18) X – LICZBA POPRAWNYCH ODPOWIEDZI

$X \sim B(n; p)$, $n = 10$ $p = \frac{1}{2}$

$P(X=8) = \binom{10}{8}\left(\frac{1}{2}\right)^8\left(1-\frac{1}{2}\right)^{10-8} \approx 0,044$

19) X – LICZBA PACJENTÓW Z UBOCZNYMI

$X \sim B(n; p)$ $n = 10$ $p = \frac{1}{3}$

$P(X<2) = P(X=0) + P(X=1) =$

$= \binom{10}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{10} + \binom{10}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^9 \approx 0,1$

20) X – NIE LUBIĄCY RPS

$X \sim B(n; p)$ $n = 6$ $p = 0,15$

~~$P(X=6) = \binom{6}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$~~

$P(X=6) = \binom{6}{6}(0,15)^6(0,85)^0 \approx 0,0001$

$EX = 6 \cdot 0,15 = 0,9$ ← OCZEKIWANA

$VAR\,X = 6 \cdot 0,15(1-0,15) = 0,765$ ← WARIANCJA

$\sigma_X = \sqrt{VAR\,X} = \sqrt{0,765} \approx 0,87$