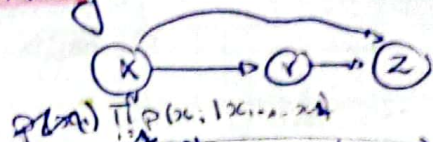


Q1) Considering multiple RV, explain the chain rule decomposition for the joint probability. Provide examples of different statistical relationship between RV. Explain the Markov chain of the first order and second order.

• Chain rule decomposition for the joint probability: study dependencies

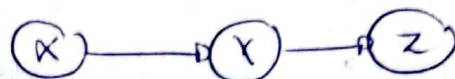
$$P(X, Y, Z) = P(X) \times P(Y|X) \times P(Z|X, Y)$$



• Markovianity first order

$$P(x_1, \dots, x_n) \Rightarrow P(x_1) \prod_{i=2}^n P(x_i | x_{i-1})$$

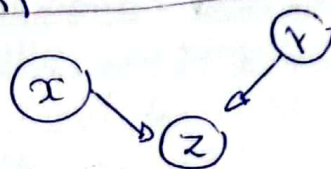
$$P(X, Y, Z) = P(X) P(Y|X) P(Z|Y)$$



Take only one element of the past into account. (dependence height)

• Conditional independence. 1

$$P(X, Y, Z) = P(Z|X, Y) P(X) P(Y)$$



• Conditional dependence

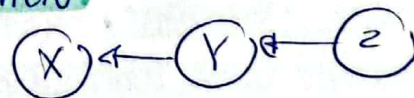
$$P(X, Y, Z) = P(Z) P(X|Z) P(Y|Z)$$



Generally X and Y are independent but knowing Z makes them dependent

• Markovianity first order in both directions

$$P(X, Y, Z) = P(Z) P(Y|Z) P(X|Y)$$



• chain rule for joint probability

independencies

$$P(X, Y, Z) = P(X) P(Y) P(Z)$$



Markov 2nd order

$$P(x_i | x_{i-1}, \dots, x_1) \Rightarrow P(x_i | x_{i-1}, x_{i-2})$$

$$P(x_1, x_2, \dots, x_n) \Rightarrow P(x_1) \prod_{i=2}^n P(x_i | x_{i-1}, x_{i-2})$$

