Session 4 SMV: Rewriting and strategies

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1 Rewriting & Strategies

In this section, the purpose is to understand how rewriting works.
We give the following ADT:

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ADT Natural;
Interface Booleans
  Sorts natural;
  Operations
  Generator:
    0: -> natural;
    s _ : natural -> natural
  Modifier:
    _ + _: natural, natural -> natural
    _ - _: natural, natural -> natural
  Axioms:
    (1) x + 0 = x
    (2) x + s(y) = s(x + y)
    (3) x - 0 = x
    (4) 0 - x = 0
    (5) s(x) - s(y) = x - y

  Corresponding Rewriting rules:
    (1) x + 0 ⇝ x
    (2) x + s(y) ⇝ s(x + y)
    (3) x - 0 ⇝ x
    (4) 0 - x ⇝ 0
    (5) s(x) - s(y) ⇝ x - y

  Where x,y: natural
```

1.1 Syntax of a strategy

A strategy S is inductively defined as follow:

- $Rew_{Ax} \in S$
- $Identity \in S$
- $Fail \in S$
- $Sequence(s_1, s_2) \in S, s_1, s_2 \in S$
- $Choice(s_1, s_2) \in S, s_1, s_2 \in S$
- $All(s_1, \ldots, s_n) \in S, s_1, \ldots, s_n \in S$
- $One(s_1, \ldots, s_n) \in S, s_1, \ldots, s_n \in S$
- $Try(s) \in S, s \in S$
- $Repeat(s) \in S, s \in S$
- $OnceBottomUp(s) \in S, s \in S$
- $BottomUp(s) \in S, s \in S$
- $TopDown(s) \in S, s \in S$
- $Innermost(s) \in S, s \in S$
- $Outermost(s) \in S, s \in S$

The semantic of strategies can be directly found in the course.
Examples: Let suppose the term $t = (0+0)-(0+0)$. Let's try to apply different strategies to understands how it works:

- $(Rew_{Ax})[t] = fail$
- $(Identity)[t] = t$
- $(Fail)[t] = fail$
- $(Try(Rew_{Ax}))[t] = Choice(Rew_{Ax}, Identity)[t] = t$
- $All(Rew_{Ax})[t] = 0 - 0$
- $Sequence(All(Rew_{Ax}), Rew_{Ax})[t] = 0$

1. Using the examples above and strategy semantics in the course, show how we get each of these results.
2. Let suppose the term: $t_1 = (0+0)+0$. Redo the same example as before by changing t by t_1 and observe the new results.
3. Keeping the same value for t_1, apply the strategy $Repeat(Rew_{Ax})$. Explain each step of your computation ! (Hint: Use a lazy procedure)
4. Now, apply the strategy $BottomUp(Rew_{Ax})$ and $TopDown(Rew_{Ax})$. What do we get ? Is there a difference ?