TP 9 : Introduction to the Lattie Boltzmann Method (LBM)

Cours de modélisation numérique

5 May 2023

Introduction

The objective of this tutorial is the introduction of a first simplified model of Boltzmann on lattice (*LBM - Lattice Boltzmann Method*) in two dimensions, simulating a tornado.

Model

We consider here the LBM to simulate a simplified case of a tornado seen from space, whose center (the "eye") is fixed. To do this, we give ourselves a grid of dimensions n_x, n_y, and a LBM D2Q9 model, i.e. with 2 spatial dimensions, and 9 possible speeds. We initialize the system at equilibrium with a zero velocity everywhere, and a density $\rho = 1$. We then apply at any time a perturbation at the center of the domain, of the form :

$$\tilde{\mathbf{u}}(t) = u_{LB} \begin{bmatrix} \cos(\omega_p t) \\ \sin(\omega_p t) \end{bmatrix} \tag{1}$$

where $u_{LB} < 1$ is the propagation speed of the populations in the network, and ω_p is the tornado pulse frequency. On the boundaries of the domain, we apply the *outflow* conditions, which are equivalent to Von Neumann conditions on the populations, i.e. for the right edge of the domain :

$$f^{in}(n_x, t+1) = f^{in}(n_x - 1, t). \tag{2}$$

We do the same for the other 3 edges.

Work to do

It is provided on Moodle the beginning of a code simulating the above situation. You are asked to complete it with the help of the course. In particular, you must implement :

- The table $\mathbf{v}$ containing the possible speeds in the D2Q9 model.
- The perturbation function $\tilde{\mathbf{u}}(t)$.
- The outflow boundary conditions.
- The calculation of the macroscopic quantities u, rho, P (pressure), from the populations.

Once the code is complete, test it, and try to answer the following questions :

- How does the system vary according to the parameters Re, u_{LB}, ω_p, $n_{x,y}$?
- The fact that the eye of the tornado is static is obviously not realistic. How would you make it move ?