Session 7 SMV: Encoding - Decoding / SFDD to Kripke structure

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9 December 2021

1 Encoding - Decoding

Definition (Canonical form)

Let T be a set of terms, and $< \in T \times T$ a total ordering on T. A SFDD $S \in \mathbb{S}$ is canonical if and only if

- S is the rejecting terminal $\bot$
- S is the accepting terminal $\top$
- $S = \langle t, \tau, \sigma \rangle$ where
  - $\tau = \langle t_\tau, \tau_\tau, \sigma_\tau \rangle \implies t < t_\tau$ and $\tau \neq \bot$
  - $\sigma = \langle t_\sigma, \tau_\sigma, \sigma_\sigma \rangle \implies t < t_\sigma$
  - τ and σ are canonical

τ is the subtree of the take node. (full arrow)
σ is the subtree of the skip node. (dash arrow)

1.1 Encoding a set

The encoding of a set into a SFDD is given by:

$$
\begin{aligned}
enc(\varnothing) &= \bot \\
enc(\{\varnothing\}) &= \top \\
enc(S \cup \{s\}) &= enc(S) \cup enc(\{s\}) \\
t < \min(s) \implies enc(\{s \cup \{t\}\}) &= \langle t, enc(\{s\}), \bot \rangle
\end{aligned}
$$

Let assume we have the following ordering: $a < b < c < \ldots$

1. Encode the following family of sets: $\{\{a, b\}$.
2. Encode the following family of sets: $\{\{a, b\}, \{b, c\}, \{c\}\}$.

1.2 Decoding a SFDD

The decoding of one SFDD is given by:

$$
\begin{aligned}
dec(\bot) &= \varnothing \\
dec(\top) &= \{\varnothing\} \\
dec(\langle t, \tau, \sigma \rangle) &= (dec(\tau) \oplus t) \cup dec(\sigma)
\end{aligned}
$$

Where $\oplus$ is defined as follows:

$$\bigcup_{s \in S} \{s\} \oplus t = \bigcup_{s \in S} \{s \cup \{t\}\}$$

1. Decode the following SFDD: $\langle a, \top, \top \rangle$
2. Decode the following SFDD: $\langle a, \langle c, \top, \bot \rangle, \top \rangle$

2 Microwave: From a Kripke structure to a SFDD

In this exercise, the goal is to represent a microwave by a Kripke structure, then to transform this Kripke structure into a SFDD. Let's define some information about the microwave !

The possible atomic propositions are:

- $open, close$
- on, off

Using negations, we get the following representation:

- $p = open$
- $\bar{p} = close$
- $q = on$
- $\bar{q} = off$

We describe formally the microwave as the following Kripke structure $K = \langle S, S_0, R, L \rangle$ where:

- $S = \{s_0, s_1, s_2\}$
- $S_0 = \{s_1\}$
- $R = \{(s_0, s_1), (s_1, s_0), (s_1, s_2), (s_2, s_1)\}$
- $L(s_0) = \{p, \bar{q}\},\ L(s_1) = \{\bar{p}, \bar{q}\},\ L(s_2) = \{\bar{p}, q\}$

2.1 Representation of the Kripke structure

1. Draw the Kripke structure that is depicted above.
2. Explain in a few words in what states a microwave can be or not.
3. Let assume we want to add another property to manage the ventilation which can be on or off. How could you do it ? Adapt the microwave Kripke structure to contain your new property.

2.2 From Kripke structure to SFDD

In this exercise, we use the original microwave without the ventilation. To help us in this exercise, we need to introduce the siblings of $\{p, q\}$ which we call $\{p', q'\}$. The key order is the following: $p < p' < q < q'$

1. Explain why we need to introduce siblings ? What do they bring to us ? Look at the examples in the course to help you.
2. Reusing what you did in the previous exercises, write the possible states using the family of sets. For instance, the relation (s_0, s_1) is represented by the set: $\{p, \bar{q}, \bar{p}', \bar{q}'\}$
3. How can you obtain the SFDD ? Do it !
4. How can you verify the CTL formula $\neg EG(p, q)$? Do it !