



Q) 10] Explain the difference between the characteristic function and moment generating function.

• Moments : mean, variance, skewness, kurtosis  
 1<sup>st</sup> 2<sup>nd</sup>

↳ For continuous random variable (pdf) +  
 ↳ For discrete RV (pmf) + (distributions)

• For some distributions, only two first moments are sufficient (eg Gaussian dist) to completely define the distribution.  ⇒ moments

• For some other distribution, we can compute it moments but from the moment we can not find the pdf.  → moment from only few moments. (not enough to do an approximation).

• CF of a pdf: similar in Fourier T.  $j = \sqrt{-1}$  complex

$$\Phi(t) = \int_{-\infty}^{\infty} f(x) e^{jtx} dx = \mathbb{E}_{f(x)} [e^{jtx}]$$

• pdf of a CF (to recover the pdf)

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(t) e^{-jtx} dt$$

• Moment generating function  $\Pi(t) = \mathbb{E}_{f(x)} [e^{tx}]$   
 (real function) no  $j$

• Those exponents can be extended with the Taylor series expansion

$$e(tx) = \sum_{n=0}^{\infty} \frac{t^n x^n}{n!} \quad ; \quad e(jtx) = \sum_{n=0}^{\infty} \frac{(jt)^n x^n}{n!}$$

• Then we can rewrite  $\Phi(t)$  and  $\Pi(t)$  :  
 $\Phi(t) = 1 + \frac{jt \mathbb{E}_{f(x)} [x]}{1} - \frac{t^2 \mathbb{E}_{f(x)} [x^2]}{2!} + \dots + \frac{(jt)^n \mathbb{E}_{f(x)} [x^n]}{n!}$   
 (+/-)

$$\Pi(t) = 1 + \frac{t \mathbb{E}_{f(x)} [x]}{1} + \frac{t^2 \mathbb{E}_{f(x)} [x^2]}{2!} + \dots + \frac{t^n \mathbb{E}_{f(x)} [x^n]}{n!}$$

mean                  variance  
 (if we subtract the mean)

⇒ We can get the characteristic fct from the distribution  $f(x)$ .

⇒ Sometimes we can cut the previous formula and use only few moment to approximate the distribution (other moments might be small or 0)

! Sometimes we don't have the distribution actually most of the time we have a lot of data but not the distribution. ( $\Delta$  i.i.d data)

⇒ we derive from the data an estimation of our moments :

• With moment estimation from data:

$$\hat{m}_n = \frac{1}{N} \sum_{i=1}^N x_i^n ; n \geq 1$$

When  $n$  is very large (lot of data)  $\xrightarrow{\text{avg}}$  true mean.

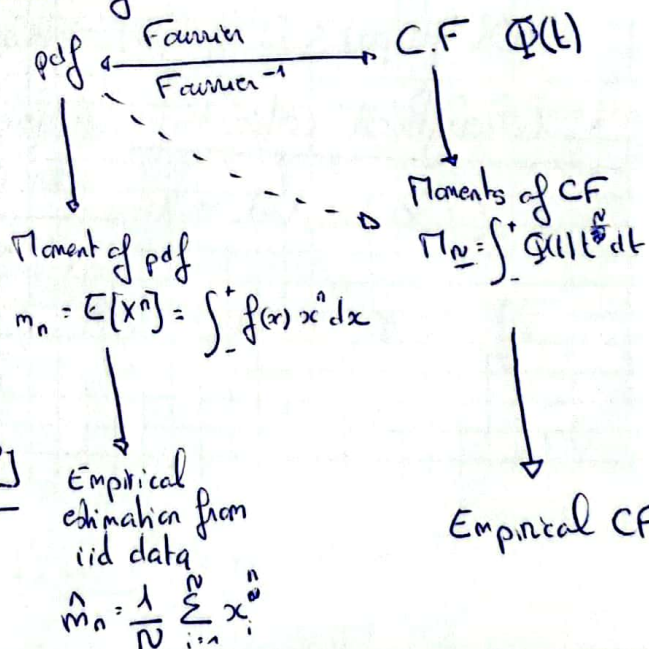
⇒ From data we can compute moments directly w/o knowing the true dist

⇒ From moment, i can approximate the CF (or moment gen fct)

⇒ From CF i can compute the pdf with the inverse Fourier T. (idea)

Apply: Signal's Fake detection

Summary



• need DWT of PCA to have I.I.d data.

• Use moments for discrimination of data to distinguish btw two things for example (pdf will be the same)