Q1) Considering multiple RV, explain the chain rule decomposition for the joint probability. Provide examples of different statistical relationship between RV. Explain the markov chain of the first order and second order.

- Chain rule decomposition for the joint probability: study dependencies

$$p(X,Y,Z) = p(X) * p(Y|X) * p(Z|X,Y)$$

$$\begin{tikzcd} X \arrow[r] \arrow[rr, bend left] & Y \arrow[r] & Z \end{tikzcd}$$

$p(x_1) \prod p(x_i | x_{i-1} \dots x_1)$

- Markovianity first order

$p(x_1, \dots x_N) \Rightarrow p(x_1) \prod_{i=2} p(x_i | x_{i-1})$

$$p(x,y,z) = p(x)\, p(y|x)\, p(z|y)$$

$$\begin{tikzcd} X \arrow[r] & Y \arrow[r] & Z \end{tikzcd}$$

Take only one element of the past (neigh) into account. (dependencie)

- Conditional independence. ⊥

$$p(x,y,z) = p(z|x,y)\, p(x)\, p(y)$$

$$\begin{tikzcd} x \arrow[r] & z & y \arrow[l] \end{tikzcd}$$

- Conditional dependence

$$p(x,y,z) = p(z)\, p(x|z)\, p(y|z)$$

$$\begin{tikzcd} X & z \arrow[l] \arrow[r] & Y \end{tikzcd}$$

Generally X and Y are independend given ~~x~~ z ~~but knowing z makes thed dependent~~

- Markovianity first order in both direction

$$p(x,y,z) = p(z)\, p(y|z)\, p(x|y)$$

$$\begin{tikzcd} X & Y \arrow[l] & z \arrow[l] \end{tikzcd}$$

- chaine rule for joint probability   independencies

$$p(x,y,z) = p(x)\, p(y)\, p(z)$$

$X \quad Y \quad z$

Markov 2nd order

$$p(x_i | x_{i-1}, \dots x_1) \Longrightarrow p(x_i | x_{i-1}, x_{i-2})$$

$$p(x_1, x_2, \dots x_N) \Rightarrow p(x_1) \prod_{i=2}^{N} p(x_i | x_{i-1}, x_{i-2})$$