06.04.23

Types of modulation

- Robust DWM
- Steganography
- Tamper detection

$\underline{m} = (0101\ldots)^T$ $\begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$

① $\underline{x} \rightarrow$ Enc $f(\cdot)$ $\rightarrow \underline{y}$; $\underline{K} = (100101)^T$

$D_{xy} = \|\underline{x} - \underline{y}\|_2^2 \leq D_{emb}$

$= (\underline{x}-\underline{y})^T(\underline{x}-\underline{y})$

min

- Note: l_2-norm $\neq$ HVS;

φ-mask

$\varphi = H$: 1 → wm visible; 0 → wm is invisible

$\|\underline{x} - \underline{y}\|_\varphi^2 = (\underline{x}-\underline{y})^T \varphi^{-1} (\underline{x}-\underline{y}) = \sum_{i=1}^{H}\sum_{j=1}^{W} \varphi(i,j) \cdot (x(i,j) - y(i,j))^2$

$\underline{y} = f_{\underline{K}}(\underline{x}, \underline{m})$ — modulation or WM embedding function.

② Types of $f_K(\cdot)$ —
- (1) Direct domain $\underline{y} = f_{\underline{K}}(\underline{x}, \underline{m})$
- (2) Transform domain: T, T^{-1}, s.t: $T\,T^{-1} = I$
  - (a) $\tilde{\underline{x}} = T(\underline{x}) = T\underline{x}$ (direct)
  - (b) $\tilde{\underline{y}} = f_K(\tilde{\underline{x}}, m)$ (embedding)
  - (c) $\underline{y} = T^{-1}(\tilde{\underline{y}}) = T^{-1}\underline{y}$ (inverse)

Hand-crafted transform (defined by humans)

- non-learnable (not ML)
- data independent (security)

(a) T → $\tilde{x}$ → (b) $f_K(\tilde{x}, m)$ → $\tilde{y}$ → (c) T^{-1}

- DCT
- DFT
- DWT
- RP (key → ...) DP DP ...

public / key-based

min D_{xy}

T: $f_\phi(\cdot)$

T^{-1}: $g_\theta(\cdot)$

▷ fixed transforms

- data agnostic → $\perp\!\!\!\perp \{\underline{x}\}$; indep.

Machine-learning based transform domain modulations

$\underline{x}$ → Neural Network $f_\phi(\underline{x}) = \tilde{\underline{x}}$ → $\tilde{x} \in C$ → $f_K(\tilde{x}, m)$ → $\tilde{y}$ → $g_\theta(\tilde{y})$ → $\underline{y}$

data $\{\underline{x}\}$

min D_{xy}

$\{\underline{x}_i\}_{i=1}^N$ → $f_\phi(x)$, $g_\theta(\tilde{y})$

Training data

- domain specific:
  - medical
  - astro
  - biometrics
  - ⋮

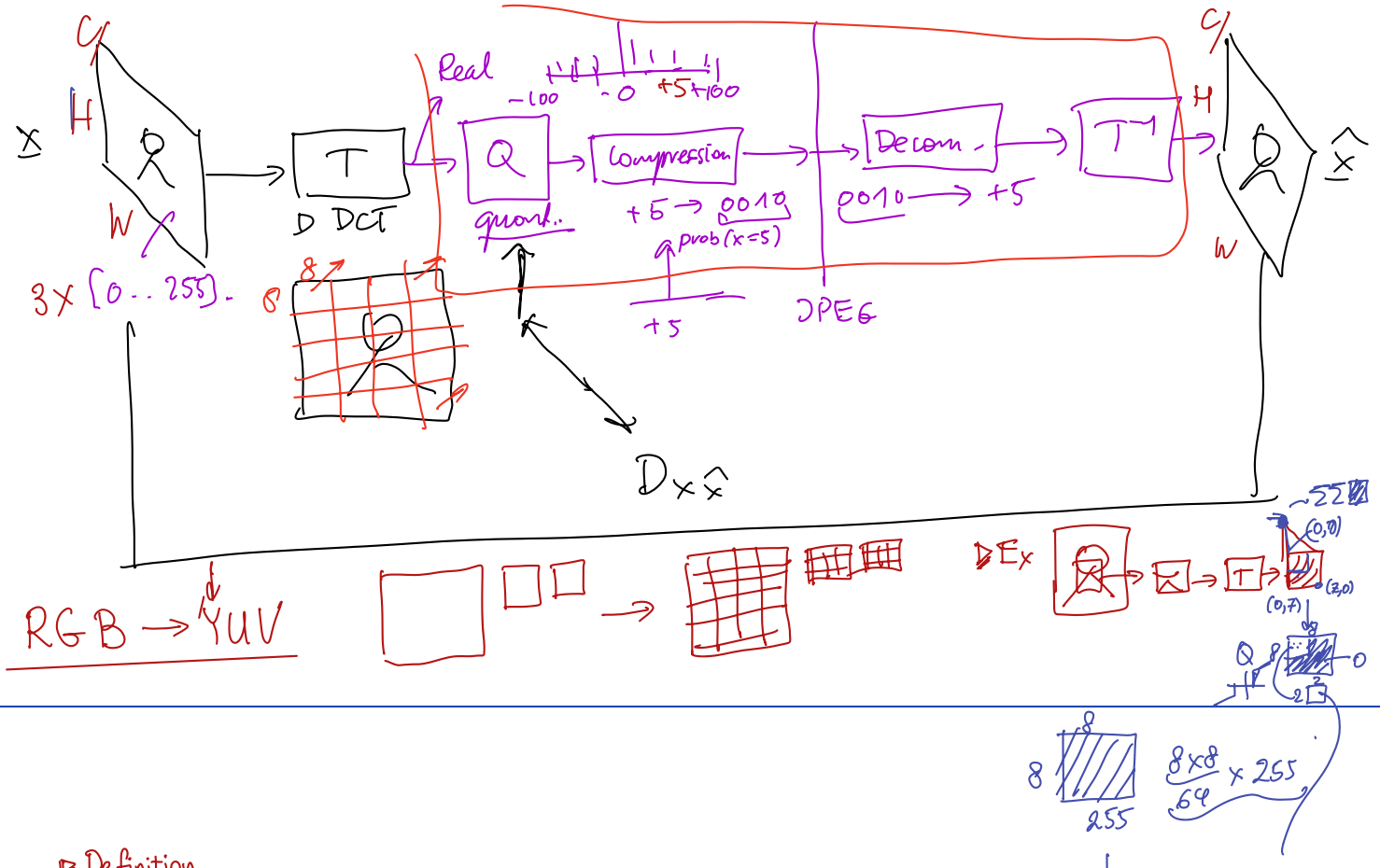
→ ML(T) ↑

- generic

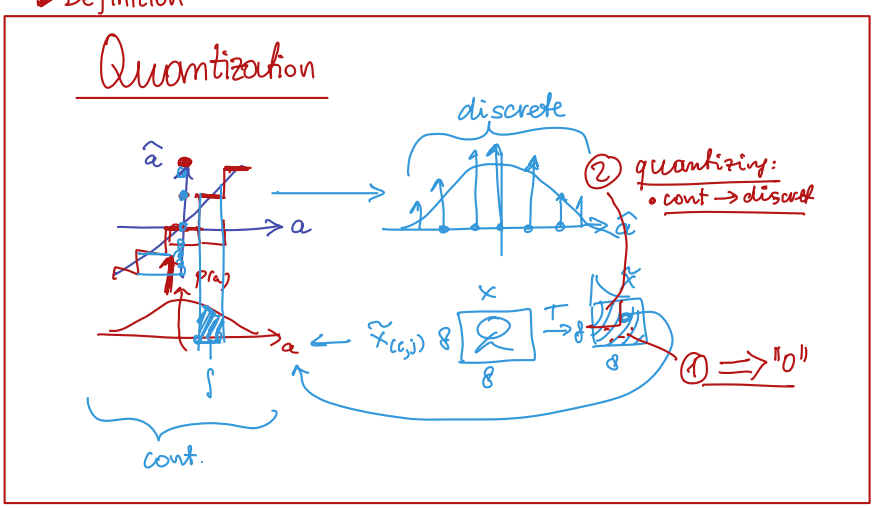
$T_{HC} \approx T_{ML}$

▷ This transform is learned in a function of data $\{\underline{x}\}$.

Link to lossy (JPEG) image compression



▷ Definition



▷ $f_K(\tilde{x}, m) = Q_K(\tilde{x}, m)$ - quantization based embedding.

• Dither quantizer
$Q(\tilde{x} + d)$
$\Pi \leftarrow (K, m)$

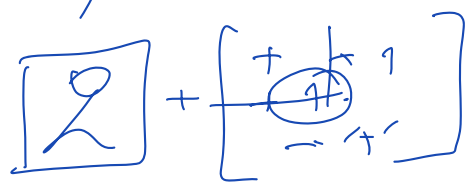
~~+0|-0|0|-0~~

Recall:

$\underline{y} = \underline{x} + \underline{w}$ $\quad \underline{m}, \underline{K}$

in the direct /image/ domain.

- direct domain (T=I)
- additive modulation
- quantization modulation



$\underline{m} = [0\ 1\ 0\ 1\ \dots\ 1]$

$\underline{K} \rightarrow (x_1, y_1)$ $\quad 0 \rightarrow -1$, $1 \rightarrow +1$

Table: general classification of DH modulation techniques.

$\underline{y} = f_{\underline{K}}(\underline{x}, \underline{m})$

- Direct domain (I) = T
  - Additive modulation: $\underline{y} = \underline{x} + \underline{w}$ ($\underline{m}, \underline{K}$)
  - Quantization modulation
- Transform domain (T)
  - HC ($T \parallel X$)
    - Additive modulation
    - Quantization modulation
  - ML ($T \not\parallel X$)
    - Additive modulation
    - Quantization modulation

$f_K(\tilde{x}, m) \leftarrow \tilde{x} = T(x) \leftarrow x$

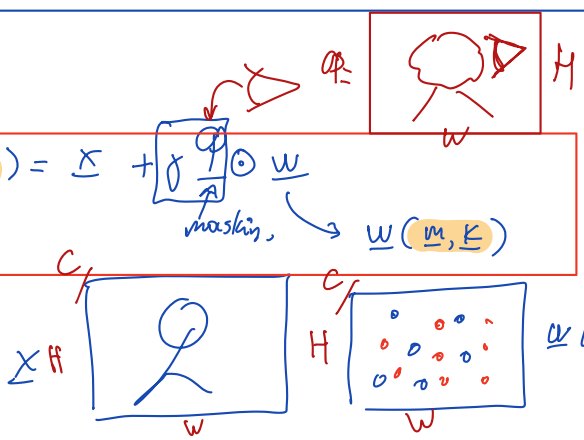
$\rightarrow T^{-1}(\cdot) \rightarrow \underline{y}$

1) Additive modulation.

1.1. Direct domain:

$\underline{y} = f_{\underline{K}}(\underline{x}, \underline{m}) = \underline{x} + \underline{\gamma} \odot \underline{w}$

masking, $\rightarrow \underline{w}(\underline{m}, \underline{K})$



$\underline{w}(\underline{m}, \underline{K})$

• $\rightarrow +1$

• $\rightarrow -1$

$\underline{y} = T^{-1}[T(x)] = \underline{x} + T^{-1}(\tilde{\gamma} \odot \tilde{w})$

$\tilde{x}_{T^{-1}}$ $\quad \tilde{x}$

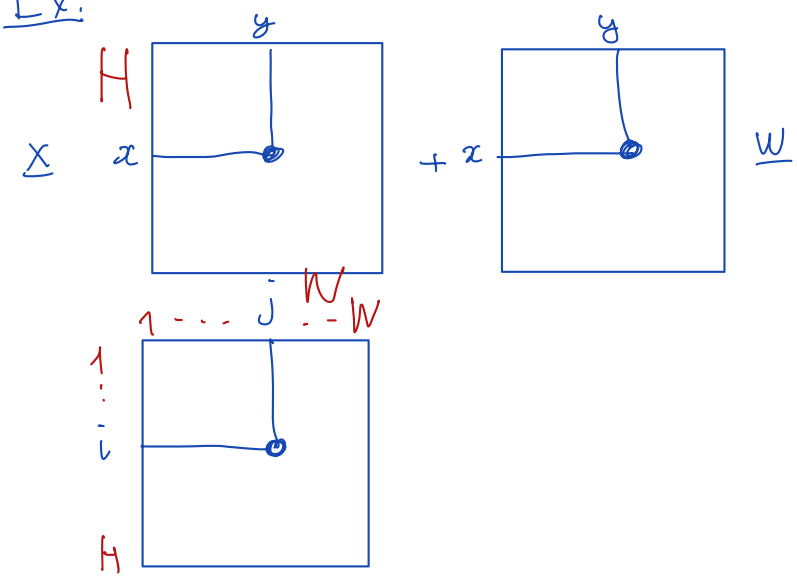
1.2. Transform domain.

$\underline{y} = f_{\underline{K}}(\underline{x}, \underline{m}) = T^{-1}\left(T(x) + \tilde{\gamma} \odot \tilde{w}\right)$

HC $\quad$ ML $\quad$ $\underline{w}(\underline{m}, \underline{K})$

- $T(aX + bY) = aT(X) + bT(Y)$

▷ Ex:



$$\underline{y} = \underline{x} + \underline{w}$$

$$y[i,j] = x[i,j] + w[i,j]$$

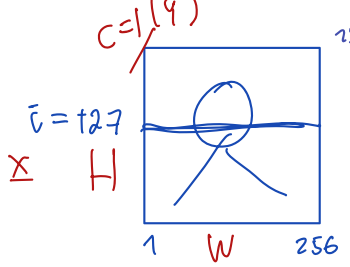
$x[i,j] \in [0 \dots 255]$; $w[i,j] \in \{\pm 1\}$

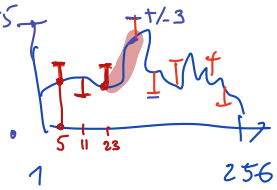
$x \cdot \gamma \cdot \varphi[i,j]$; $\varphi[i,j] \in [0 \dots 1)$

$\gamma > 1$; ± 3 ; ± 7

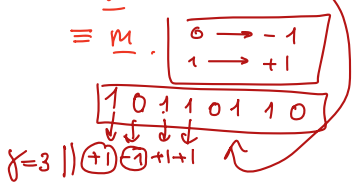
---

① Embedding (encoding) $f_K(\underline{x}, \underline{m})$



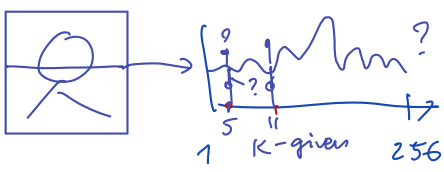


- position $\equiv \underline{K} \Rightarrow (5, 11, 23 \dots)$
- sign $\equiv \underline{m}$. 0 → −1, 1 → +1



② Extraction (decoding)

⚠ The extraction of WM is NOT a trivial problem.

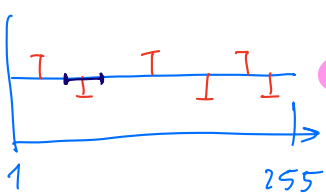


Defider (defender) — Attacker.

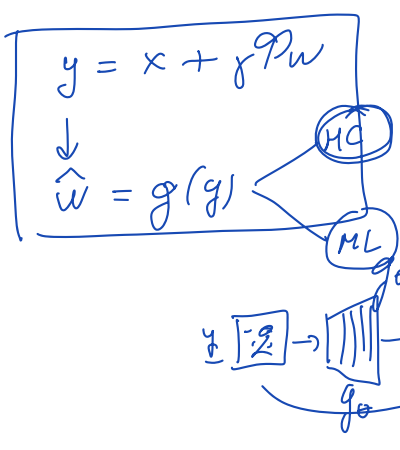
Why?

▷ Intuition.

>▷ Suppose:

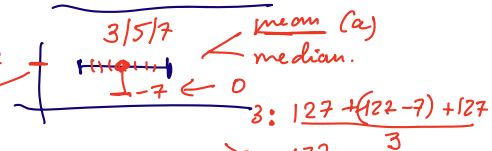


0 → −1
1 → +1



• extraction on the window → "local mean".

Local mean (a)



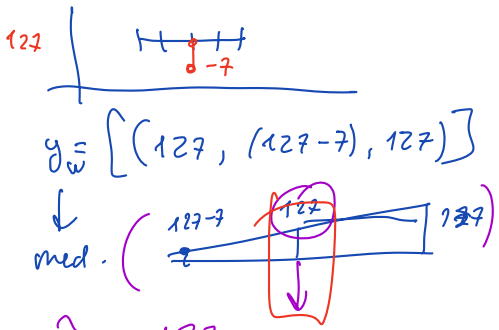
$y = x + \underset{\{\pm 1\}}{w}$

$\hat{x} = \frac{1}{3}\sum_{i=1}^{3} x_i$, $\frac{3 \times 127}{3} - \frac{7}{3} = \hat{x}$

$\hat{w} = y - \hat{x} = (127 - 7) - \left(127 - \frac{7}{3}\right) = -7 + \frac{7}{3} = -5 \rightarrow -1$

Local media. (5)

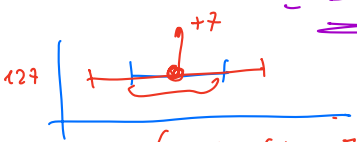
Watermark w faces a strong interference from the side of host image x.



|| local mean → local median

$\hat{x}_{med} = 127.$

$\hat{w} = y - \hat{x} = (127-7) - 127 = -7.$



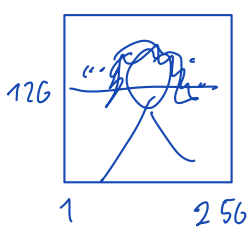
$y_w = (127, (127+7), 127)$

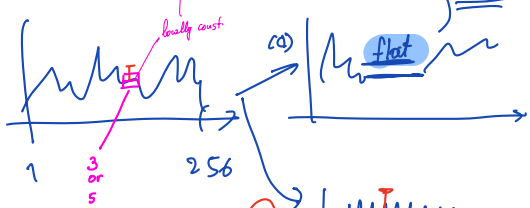
$\hat{x}_{med}$ 127 127 127+7

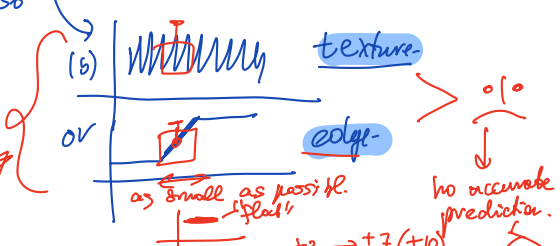
127.

$\hat{w} = y - \hat{x}_{med} = (127+7) - 127 = +7.$

▷ Real signals







attacker cannot do the accurate prediction.

Security

*

▷ In case of image:

$y = x + w = x + \epsilon$ (noise.)

O[JP]: denoiser: $w \equiv \epsilon$ as noise.

(a) $\hat{x} = \text{denoiser}(y)$ ~~$\hat{x} = y$~~

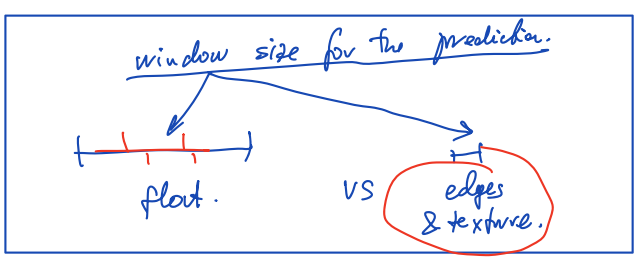
Matlab: wiener2(y, [3 3], σ_ϵ^2) (5 5, 5×3, auto.)

(b) $\hat{w} = y - \hat{x} = x + w - \hat{x}$

5.1. if $\hat{x} = x$: $\hat{w} = w$.

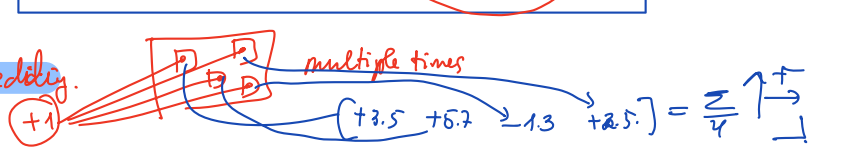
6.2: if $\hat{x} = x + z_{est}$:

$\hat{w} = y - \hat{x} = x + w - x - z_{est} = w - z_{est}$.



→ repetive embedding.

one bit

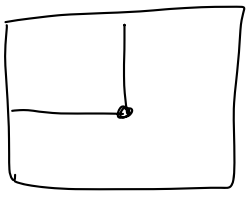


"Distributed" watermark embedding into multiple pixels: the redundancy again

$\underline{M} = [0\ 1\ 0\ \ldots\ 1]$ bits. (1, 2, L)

① $0 \to -1$, $1 \to +1$ → $[-1\ +1\ -1\ \ldots\ +1]$ bits.

②



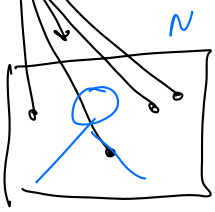
Key $\underline{K}$ →

(a) $(x_1, y_1)\ (x_2, y_2)\ \ldots\ (x_L, y_L)$.

(no redundancy).

1 bit ≡ 1 location. (L)

(b). $[-1\ +1\ +1\ \ldots\ +1]$ bits (1, 2, L)



$(x_1^{(1)}, y_1^{(1)})\ (x_2^{(1)}, y_1^{(1)})\ \ldots$

R = # of repetitions.

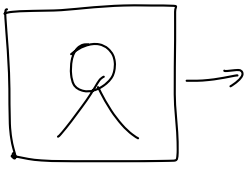
R × L

(label A)

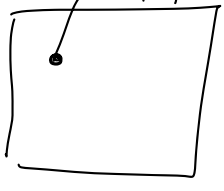
RL positions

Attacker

$\underline{y}$ →

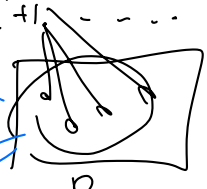


+/−1 − ? or ∅ (not modified)



- has no info where the same bit was placed.

[+1 − − −]



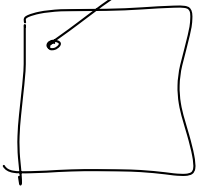
? → Σ → +/− → CLT → −1 / +1

$P_r = 0.5$ → +1 / −1

Defender

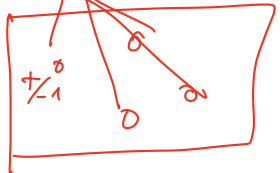
$\underline{y}$ →

+/−1 − ? or ∅



▷ has an info about "R times" location.

[+1 − −]

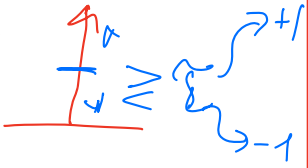


① ∅ → is not active.

(+/−1)

② R → + → Σ

1 +0.2
2 +1.3
+0.75
+0.5
−0.1
R ⋮



Robustness to geometrical attacks.

CIP: Theme 5

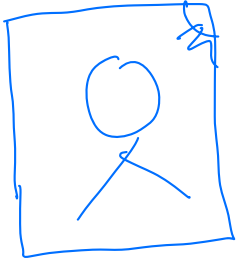
- given

$\underline{y} = \underline{x} + \underline{w}$

stego

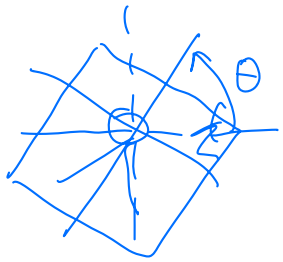
(a) affine
(b) projective

rescale

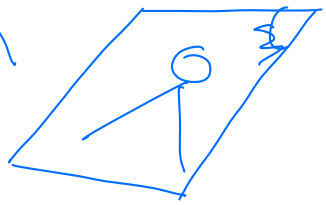


or

rotation

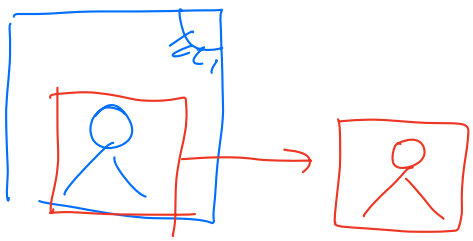


Skewing

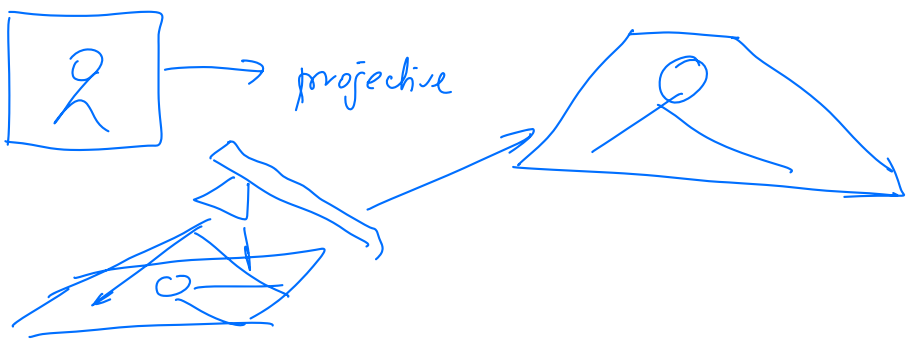


Slipping

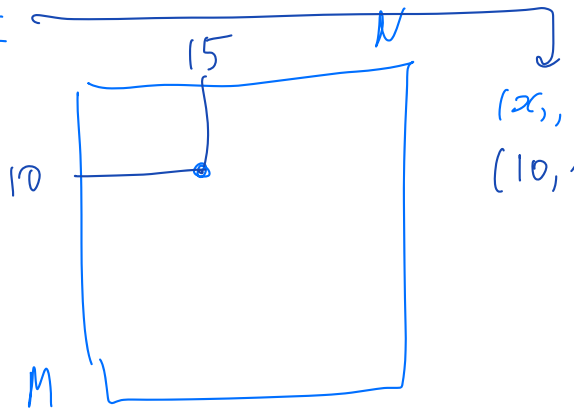
Cropping



Practical use case:

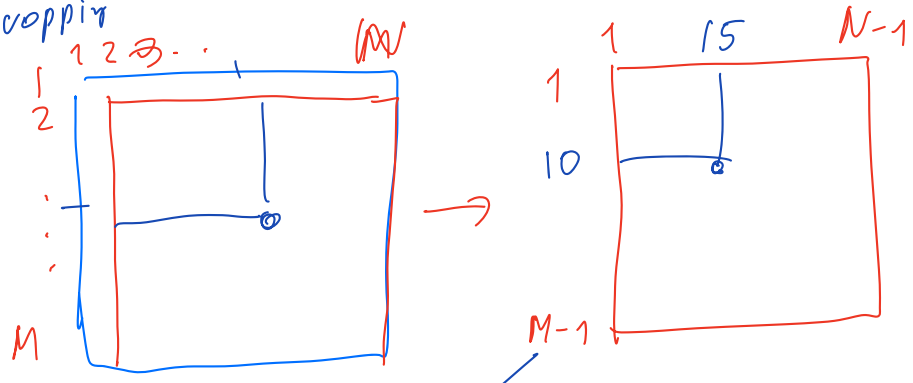


① K:



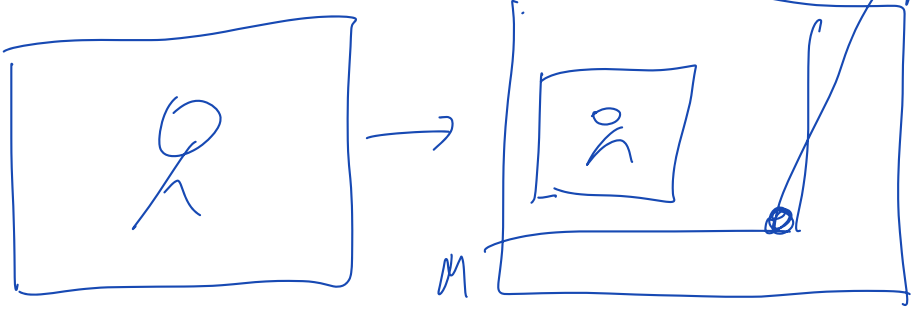
② Extraction

(a) Cropping

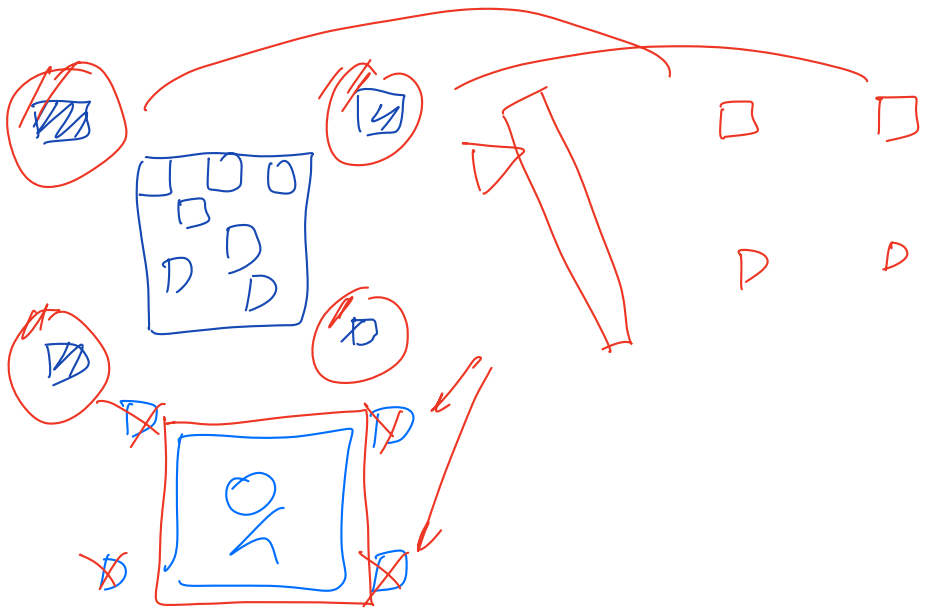


K ⟶ (x_1, y_1) (x_2, y_2) - - - -

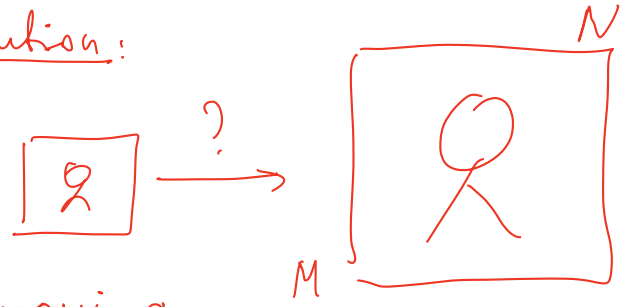
(b)



QR codes



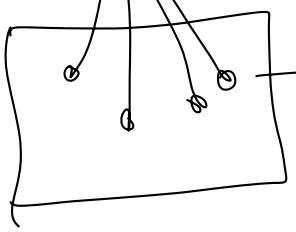
▷ Solution:



Not knowing what was done with this image?!

1) Label A

$\underline{m} = [+1 \; +1 \; -1 \; \ldots \;]$ (1, 2, L)



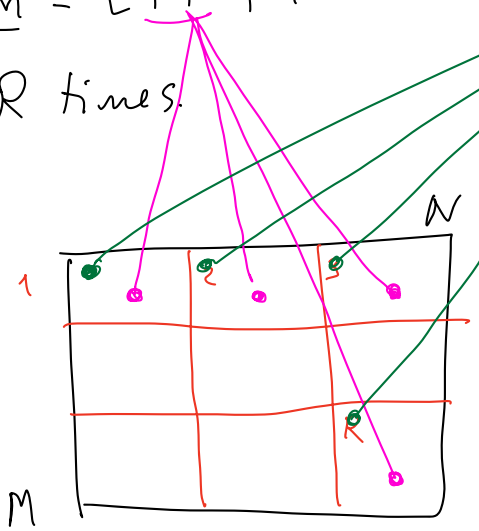
R times $\equiv$ For redundancy. (errors of prediction in view of interference with $\underline{x}$).

Embedding

2).
- $\underline{m} = [+1 \; +1 \; -1 \; \ldots \;]$ (L, −1)
- R times.

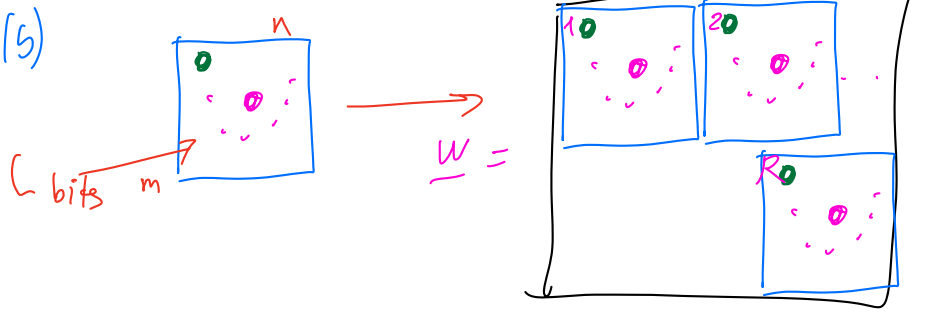
(Ordered placement).

(a)



$\underline{w} =$ R-blocks.

(b)



$\underline{w} =$ – repeated R times

$$\underline{y} = \underline{x} + \underline{w}$$

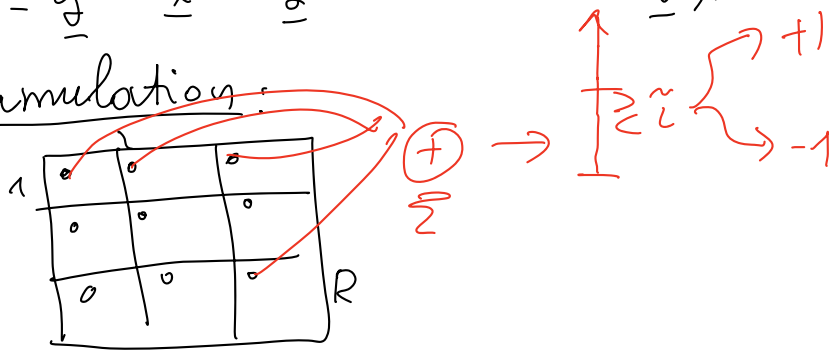
Extraction — No geom. attacks

1) ▷ $\underline{y} = \underline{x} + \underline{w}$

2) Prediction of $\hat{\underline{w}}$:

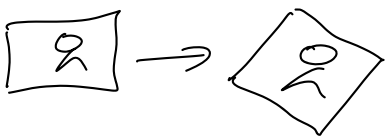
$\hat{\underline{w}} = \underline{y} - \hat{\underline{x}} = \underline{y} - \text{denoiser}(\underline{y})$ | wiener2(.)

3) Accumulation:



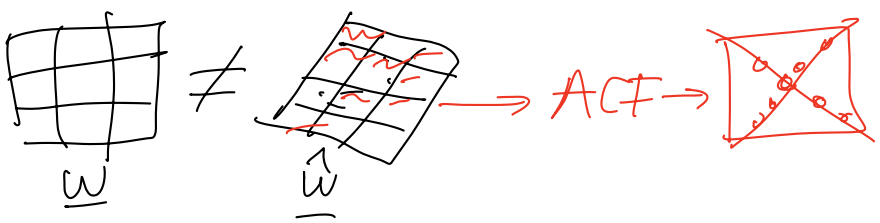
With geom. attacks-

1) $y = G(\underline{x} + \underline{w})$

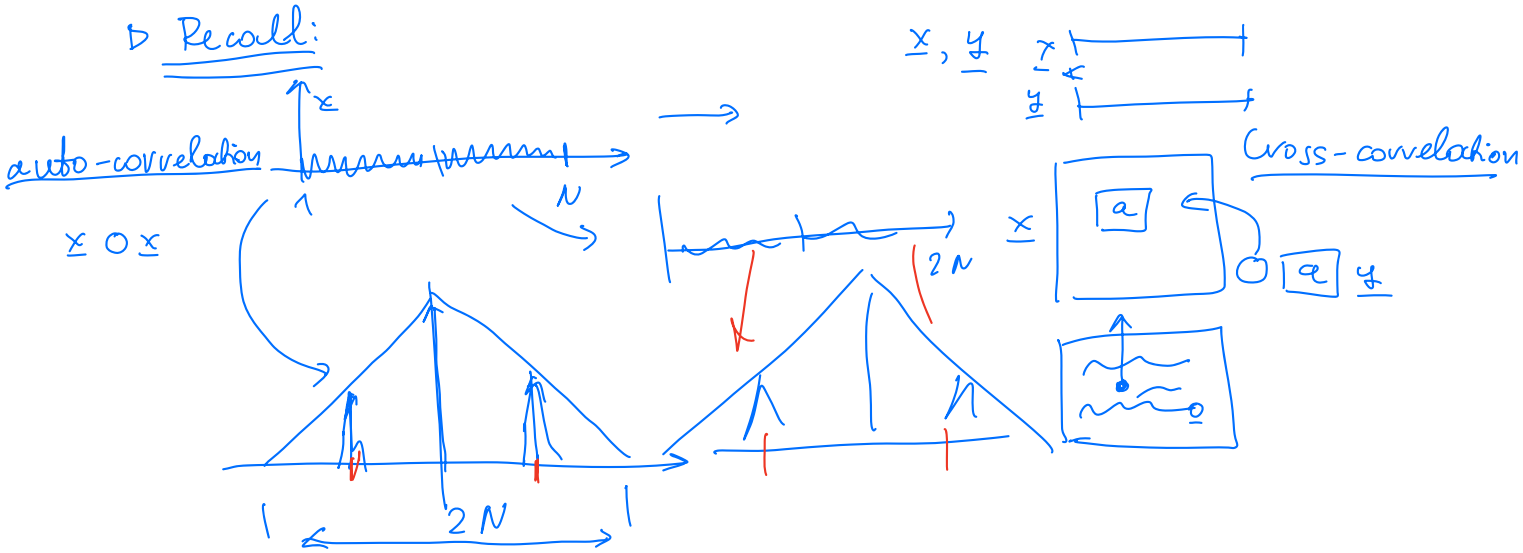


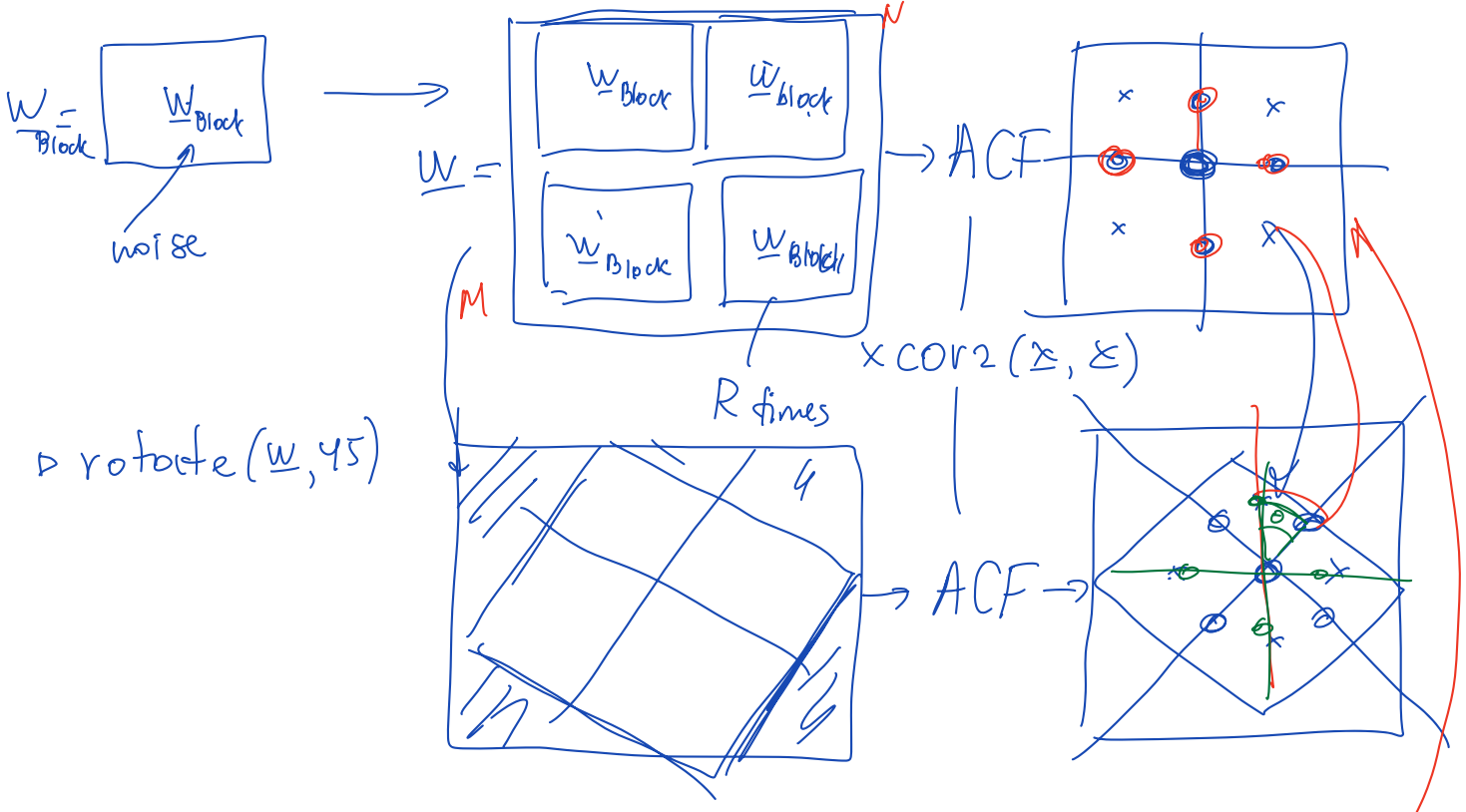
2) Prediction

$\hat{\underline{w}} = \underline{y} - \hat{\underline{x}} = \underline{y} - \text{denoiser}(\underline{y})$ | wiener2(.)



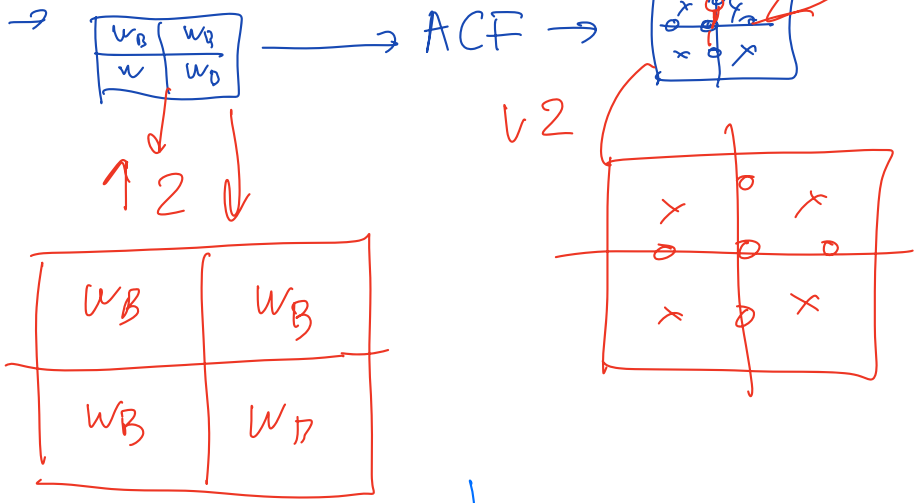
▷ Recall:

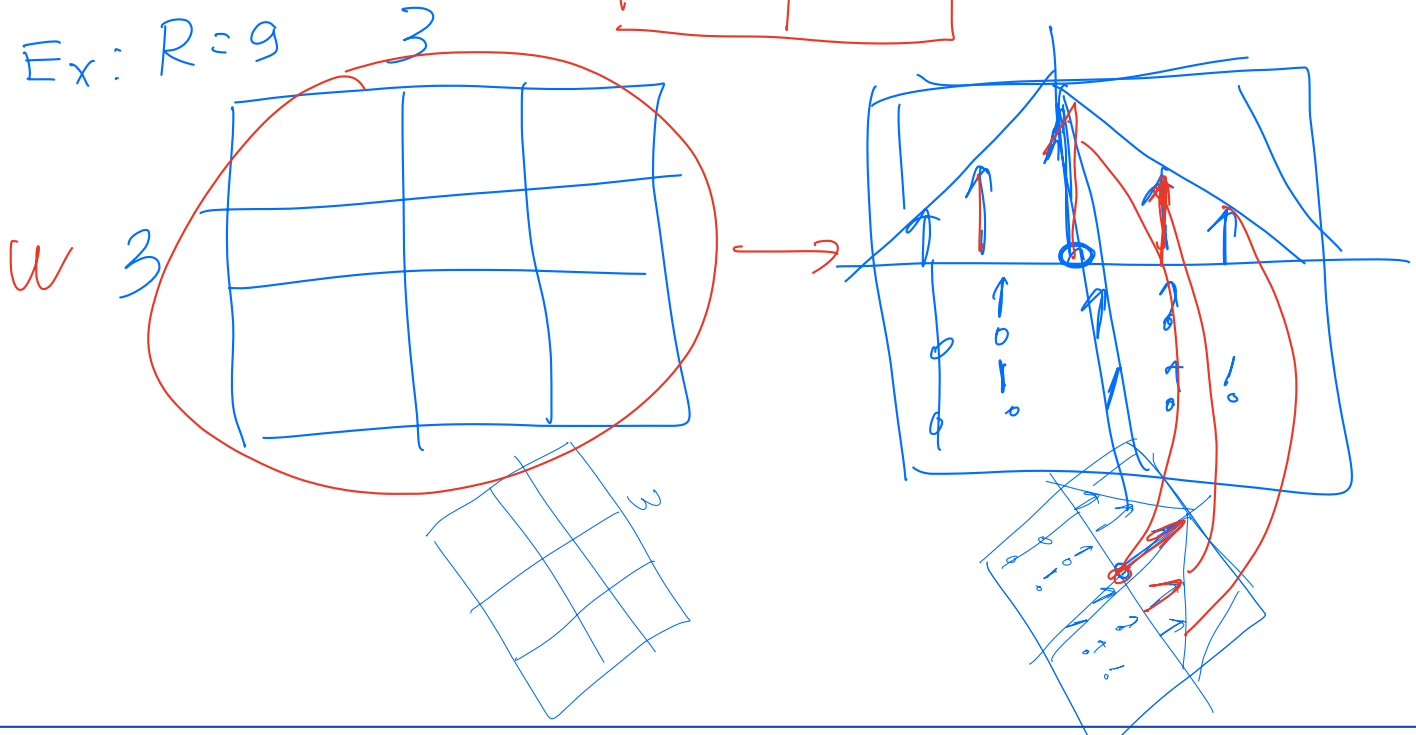




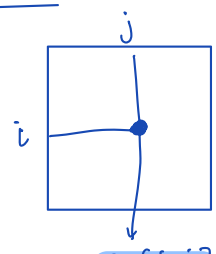
▷ scale

imresize(W, 0.5^2) →





2) Quantization modulation.



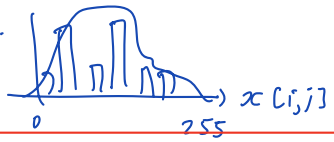
▷ additive modulation

• Direct domain:

$$y[i,j] = x[i,j] + w[i,j] \quad \hookrightarrow (M, K).$$

▷ quantization modulation

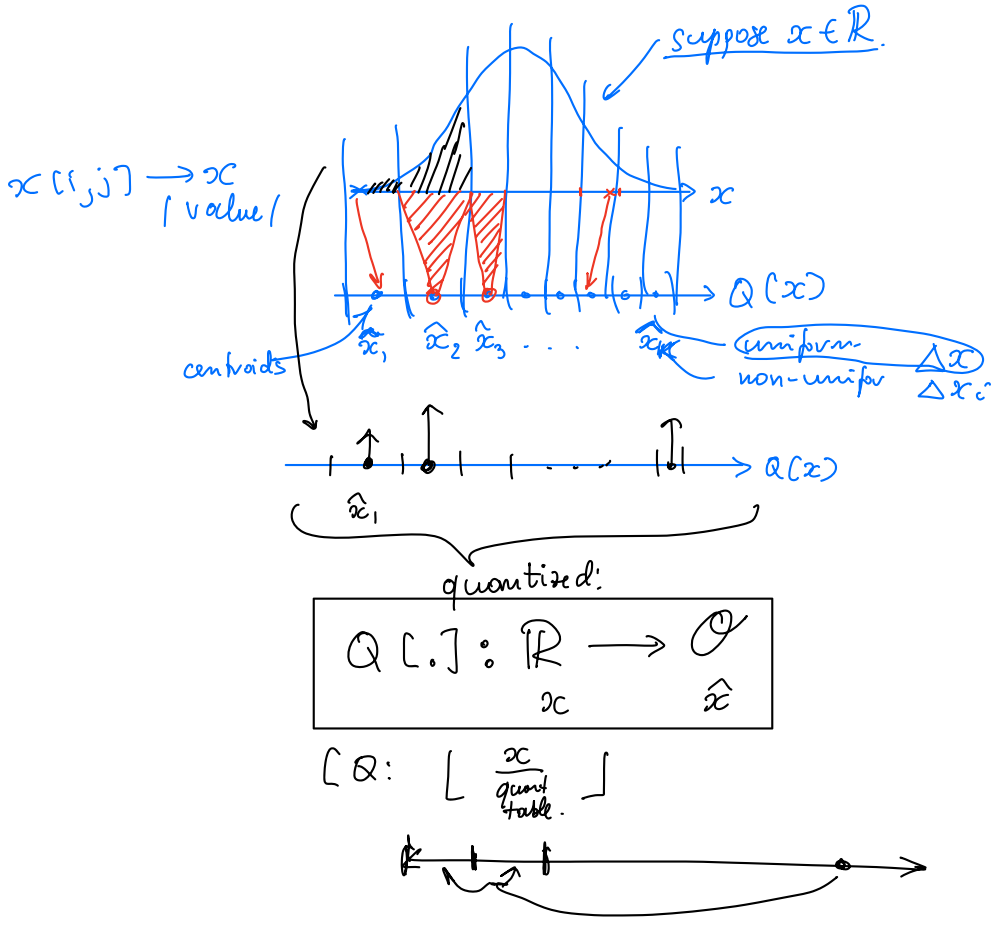
• histogram.



$$y[i,j] = \begin{cases} Q_0[x[i,j]], & \text{for } m=0, \\ Q_1[x[i,j]], & \text{for } m=1 \end{cases}$$

▷ where $Q_0[\cdot]$ for the quantizer for bit 0
$Q_1[\cdot]$ for bit 1, respectively

▷ Recall: quantization



Modulation

$m = [0\ (1)\ 0 \ldots]$

$\underline{x} \rightarrow [f_{\underline{K}}(\underline{x}, m)] \rightarrow \underline{y}$

$\underline{K}$

$m = 0$: $Q_0[x(i,j)]$

$m = 1$: $Q_1[x(i,j)]$

0/1

i, j

$x(i,j) \longrightarrow = 125$

127

x^* – value (magn.)

$Q_0[x]$ 5 10 15 20 125

$y(i,j) = Q_0[x(i,j)] = 125$

$Q_1[x]$ 5 127

$y(i,j) = Q_1[x(i,j)] = 127$

J

i

Selection of interval Δx

Robustness to distortions

visibility of modulation distortion.

Extraction

- given: $\underline{K}$

$\underline{y}$ i j

value $\equiv y = 125 \rightarrow \hat{m}$ (bit) → 0 / 1

(a) (no noise)

$j = 1, \ldots$

$|y - Q_0^j|$

$|y - Q_1^j|$

centroid y

Q_0 1 2 - - - J $m=0$

Q_1 $m=1$

$\hat{m} = 0$.

▷ Decision rule: $|y - Q_0^j| \gtrless |y - Q_1^j|$ → 0 / 1

(L.SB).

Additive:

float: → $w \Rightarrow \hat{m}$ → accurate.

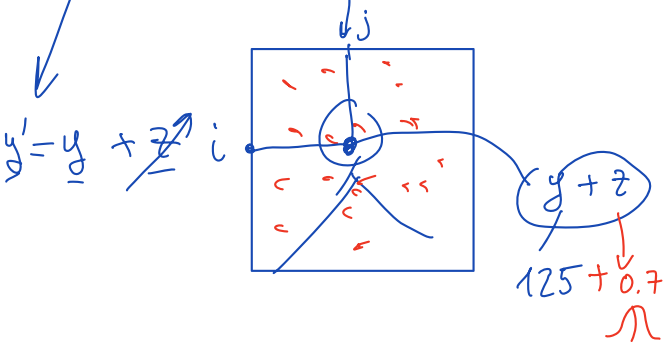
e/t → $\hat{m}$ → not very accurate

Q: is it important x is → float ? e & t. ?

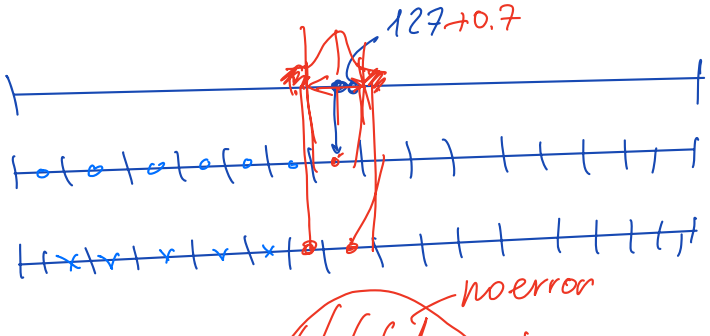
it is → $\hat{x} \rightarrow y - \hat{x} = \hat{a}$

↳ Q → 0 / 1

- given K

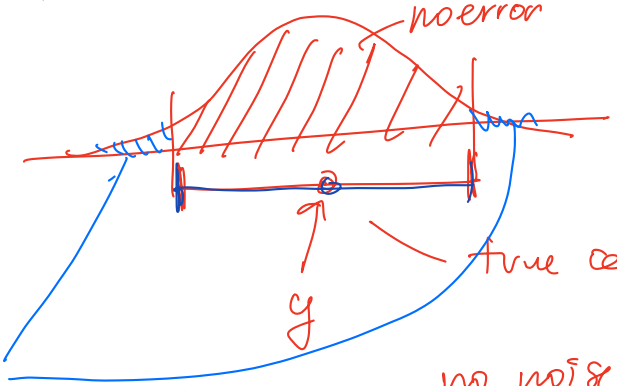


(b) with noise.

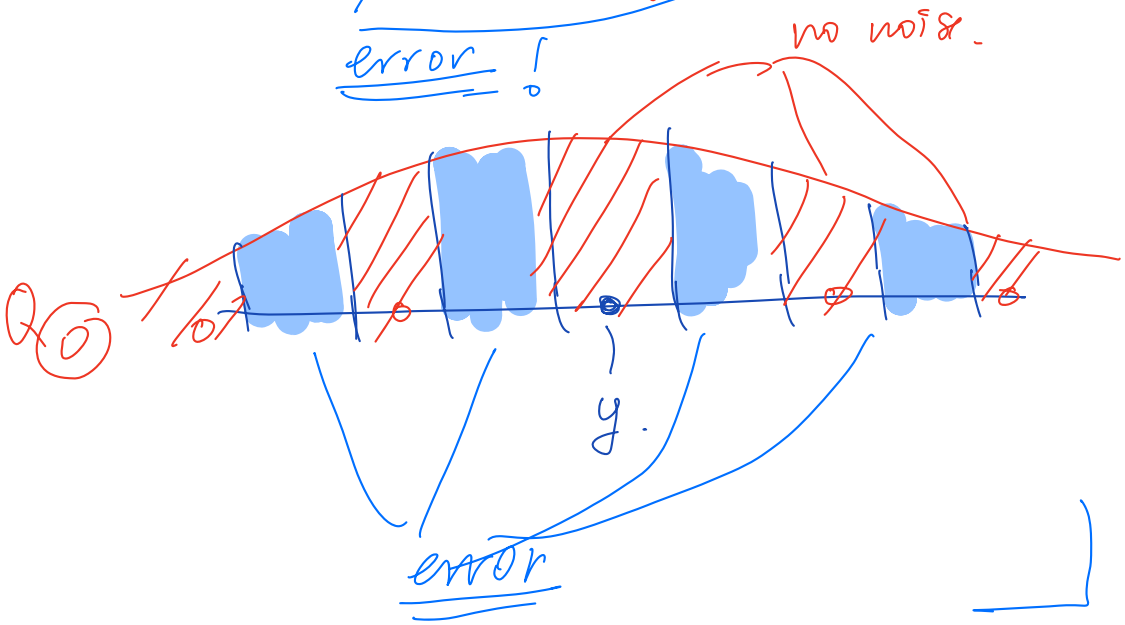


▷ Robustness.

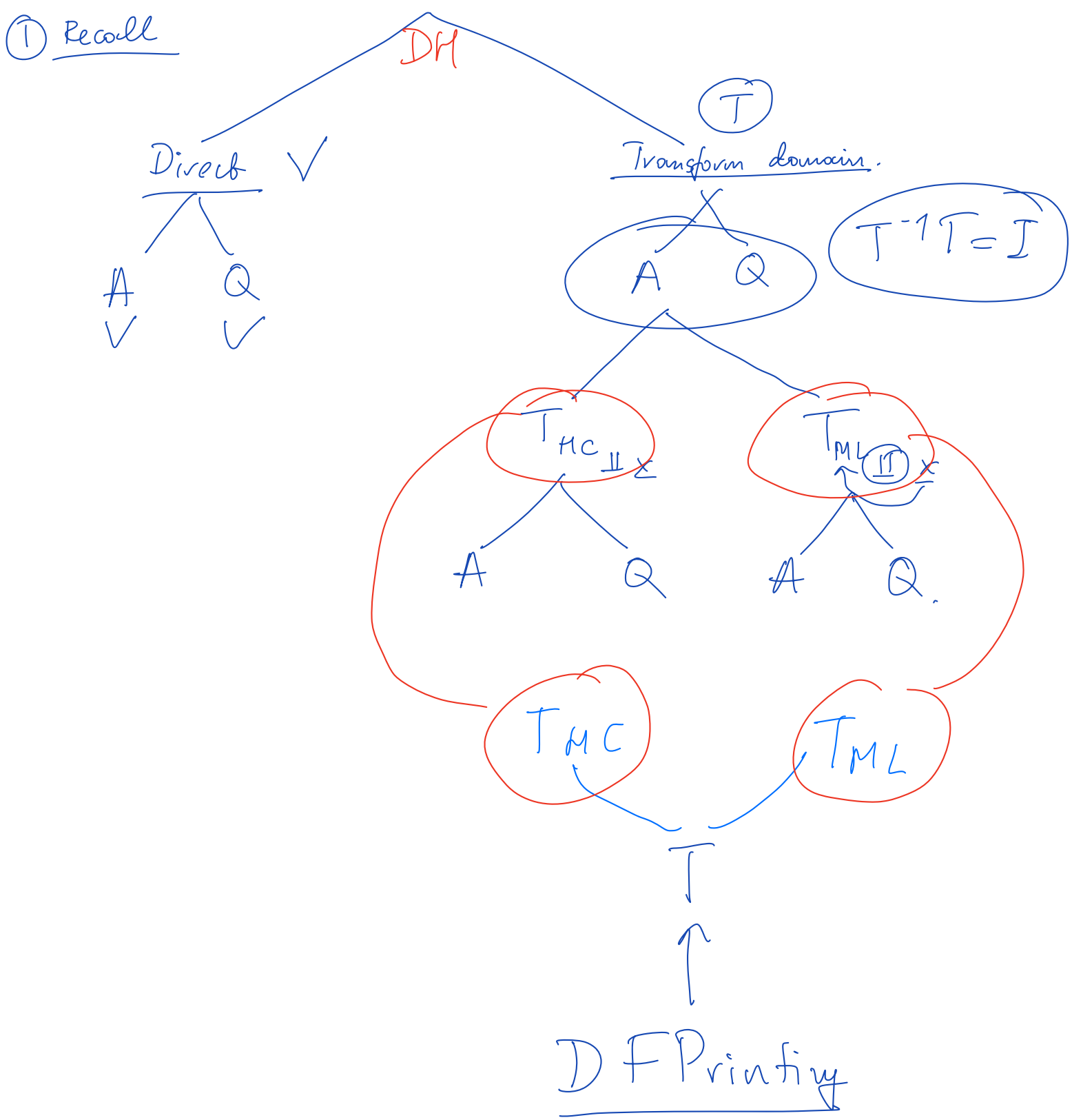
(1)



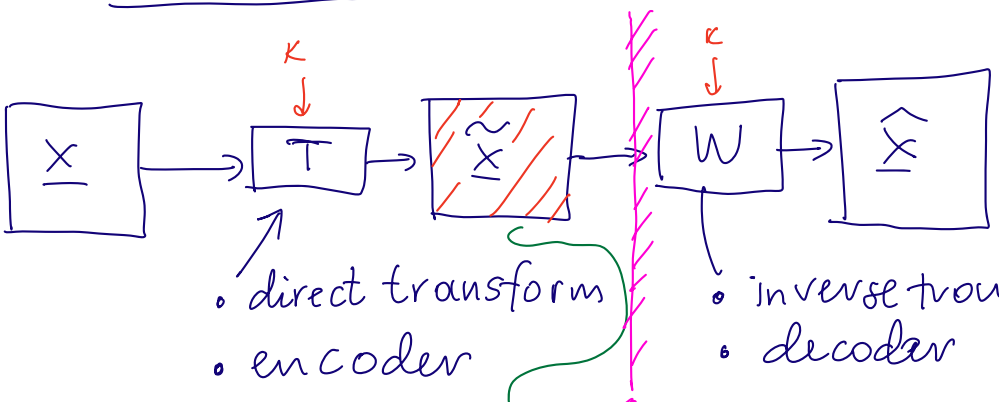
error !



▷ LSB: a part. case of quant. modulation.


① Recall
DH
Direct
A
Q
V
V
T
Transform domain.
A
Q
$T^{-1}T = I$
T_{MC}
T_{ML}
A
Q
A
Q
T_{MC}
T_{ML}
T
DFPrinting

1) Transforms (Defender)

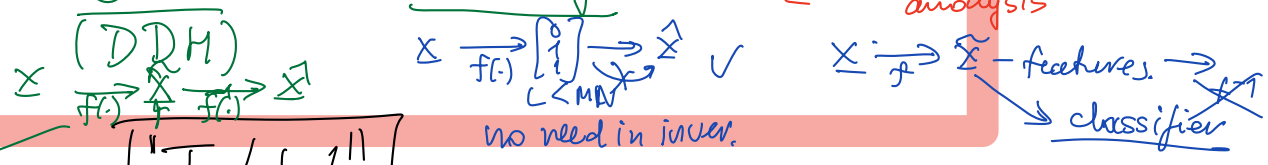


stop

▷ DWM (DRM)

▷ DFPting

[T.H. forensic analysis]



"T / W"

Yes, we need to choose T ⇒ W = T⁻¹

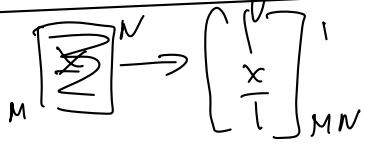
HC

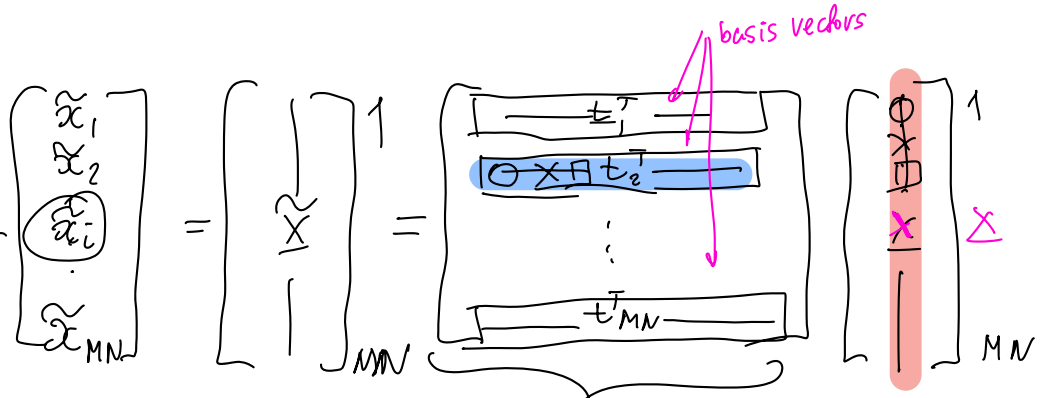
ML

Sect. A → see below

1) HC (hand-crafter ≡ engineered) transform

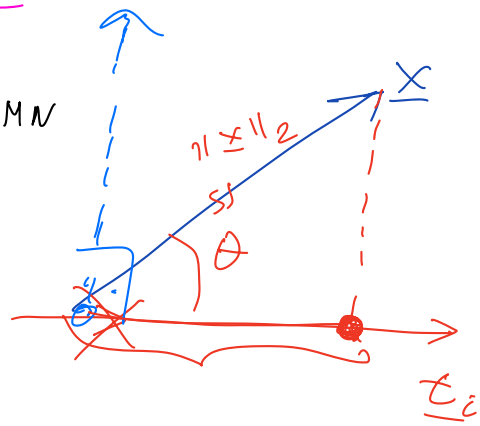
▷ Recall.





$\tilde{x} = T x$

$$\tilde{x}_i = t_i^T x = x^T t_i = \sum_{j=1}^{M \cdot N} x_j \cdot t_{ji}$$



▷ HC Transform : a selection of $\hat{T}$ (resp. W).

▷ "manually" → expertise of person.

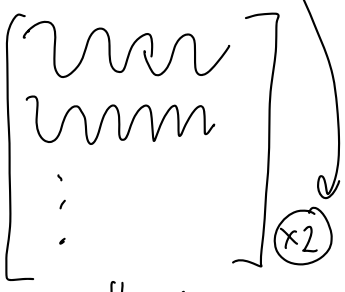
▷ selection is $\perp\!\!\!\perp$ to data X. (no ML)

- in practice:

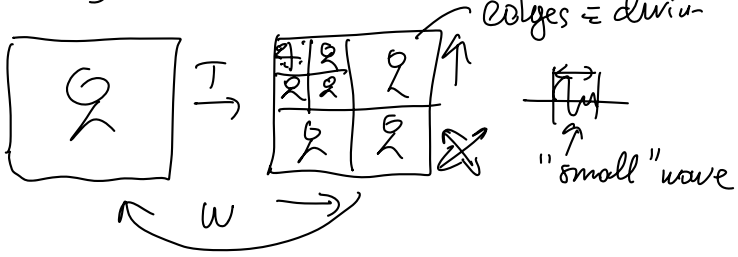
T to be:

- DFT

- DCT



- DWT ↑ wavelet



- Macdonnan.

invertable

Invertability

- it depends on the construction of matrix T. $[K \times (MN)]$

- Random projection

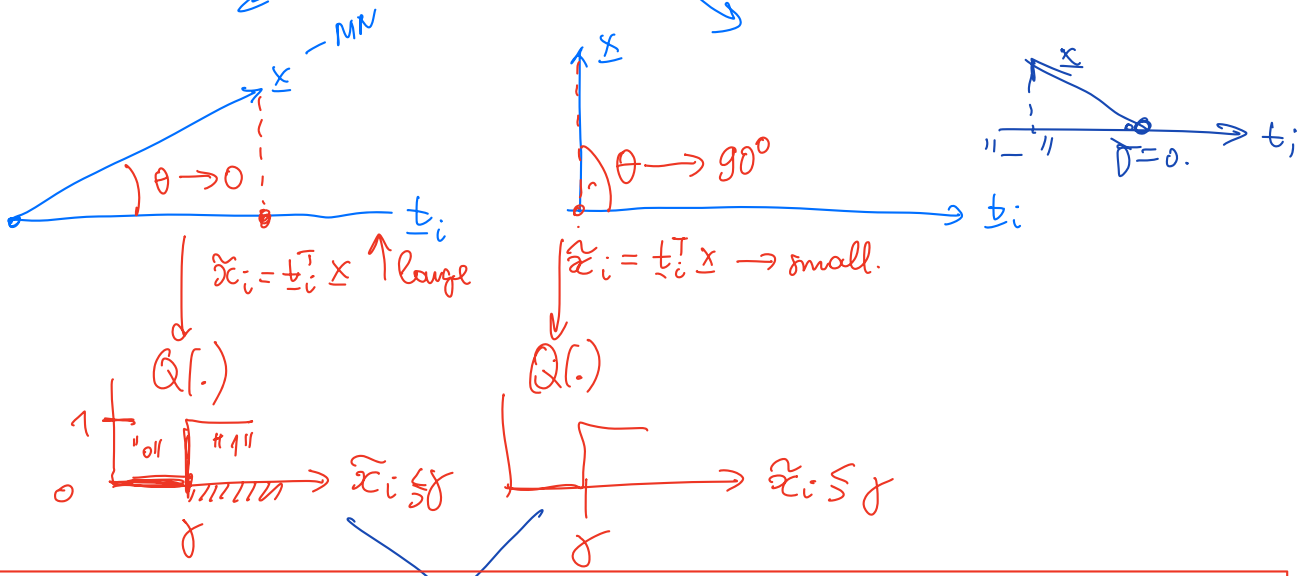
$\mathcal{N}(0, \underline{\underline{I}}_{MN})$

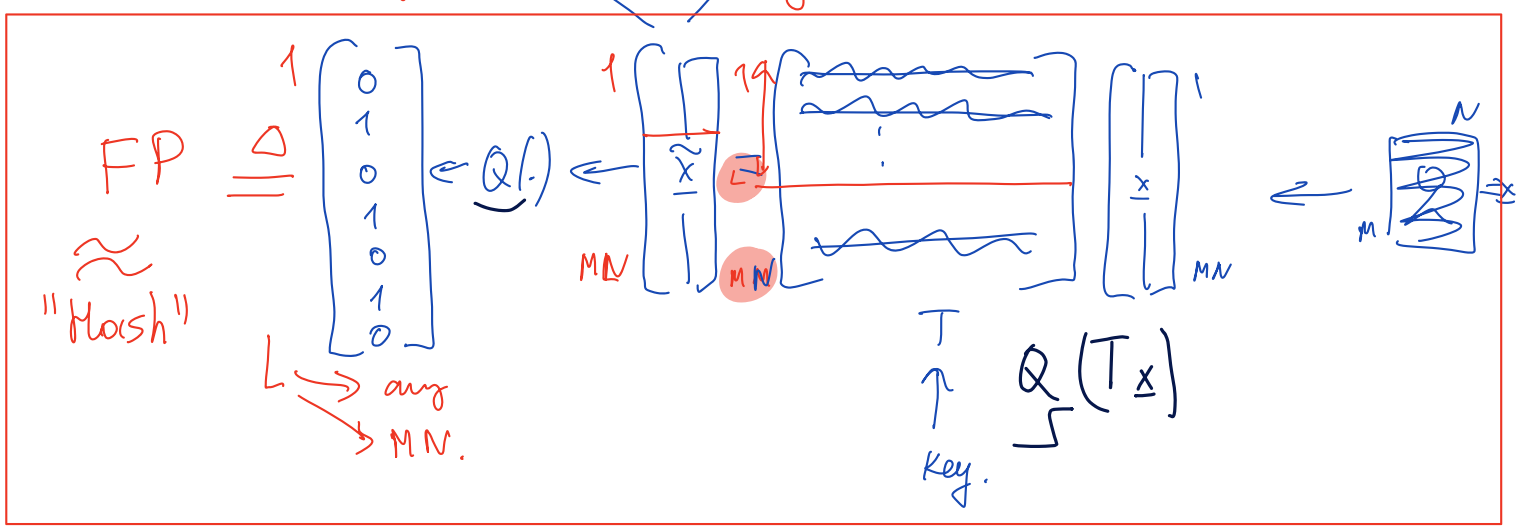
$$T = \begin{bmatrix} t_1^T \\ t_2^T \\ \vdots \\ t_{MN}^T \end{bmatrix} \begin{bmatrix} | \\ x \\ | \end{bmatrix}$$

① $T \perp\!\!\!\perp X$

② $T \leftarrow$ keep K · seed (K). (security).

random $\underline{b} \perp\!\!\!\perp \underline{x}$.



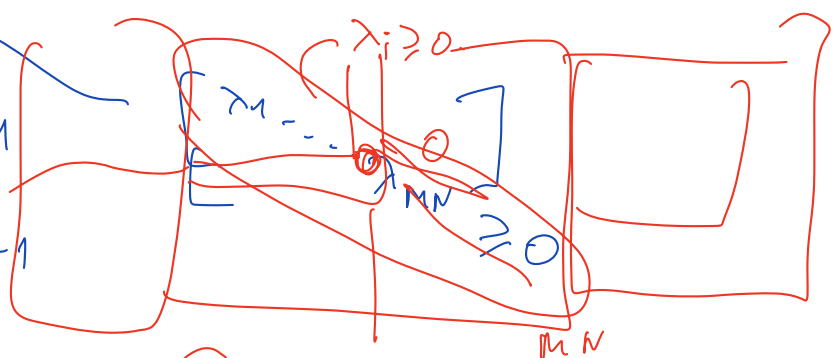


- $L < ML \to T \longrightarrow T^{-1}$ – ?

SVD: $T = U S V^{-1}$

$T^{-1} = (\quad)^{-1}$

$= V S^{-1} U^{-1}$



① $L < MN$ to avoid a perfect inversability

② $\tilde{\underline{x}} \to Q(\tilde{\underline{x}}) \to 1$ >0.

Defender (FP)

▷ **Attacker** : $Q(\tilde{x}) \longmapsto \hat{x} \longleftrightarrow x$.

• **Invertability**

MC (math) :(

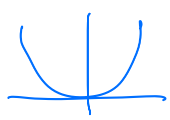
"SVD":

→ approximate solution (exact).

$\tilde{x} = Tx$ — linear equations.

$\varphi(x)$ or $\mathcal{L}(x) = \frac{1}{2}\|\tilde{x} - Tx\|_2^2$

$$\hat{x} = \underset{x \in \mathbb{R}^{MN}}{\operatorname{argmin}} \mathcal{L}(x) = \underset{x}{\operatorname{argmin}} \underbrace{\frac{1}{2}\|\tilde{x} - Tx\|_2^2}_{\ell_2}$$



$\nabla_x \mathcal{L}(x) = 0$. W_θ

$$\hat{x} = (T^T T)^{-1} T^T \tilde{x}$$

T^{-1} → $(T^T T)^{-1}$

① T not invertable $L \ll MN$.

② $Q(\tilde{x})$ → loss of inform.

→ stoch. (M.A.P.)

$$\mathcal{L}(x) = \frac{1}{2}\|\tilde{x} - Tx\|_2^2 + \lambda \underbrace{\Omega(x)}_{\text{prior}}$$

$\Omega(x)$

ℓ_2: $\|x\|_2$ → ridge

ℓ_1: $\|x\|_1$ → lasso.

$\nabla_x \mathcal{L}(x) = 0$..

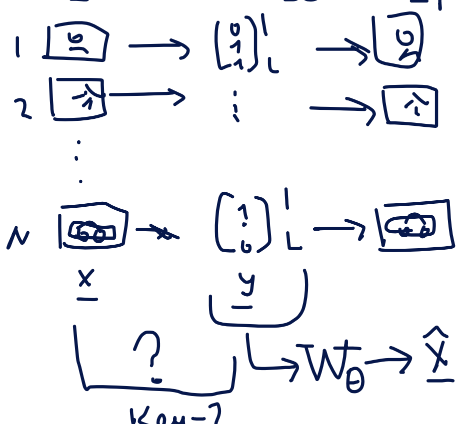
$Q(\tilde{x})$ → No

ML ← 15:13

• $\{x_i, Q(\tilde{x}_i)\}_{i=1}^N$

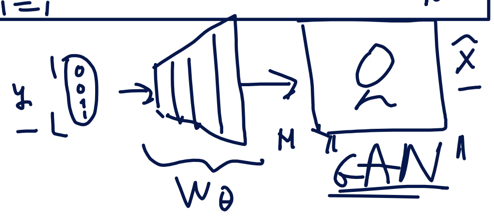
y_i; $L = Tx_i$

$x_i \to T \to \tilde{x}_i \to Q \to y_i$



$\to W_\theta \to \hat{x}$

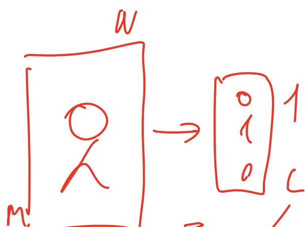
$$\mathcal{L}(\theta) = \sum_{i=1}^{N} \|x_i - W_\theta(y_i)\|_2^2$$



$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \mathcal{L}(\theta)$$

$L \sim MN$

:(



• Not feasible.

:(for the attacker.

▷ Sec A. T/W based on ML. (Defender)

DWM /DMI

DFP

$\hat{\underline{X}} = \underline{X}$

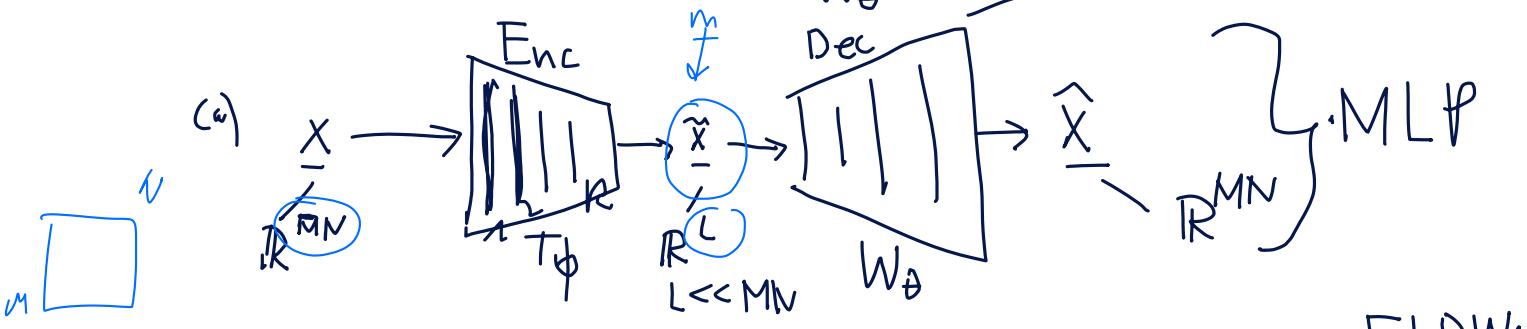
$\underline{X} \rightarrow [T] \rightarrow \underline{\tilde{X}} \rightarrow [W] \rightarrow \underline{\hat{X}}$

▷ ML: the transf: $T \longrightarrow T_\phi$

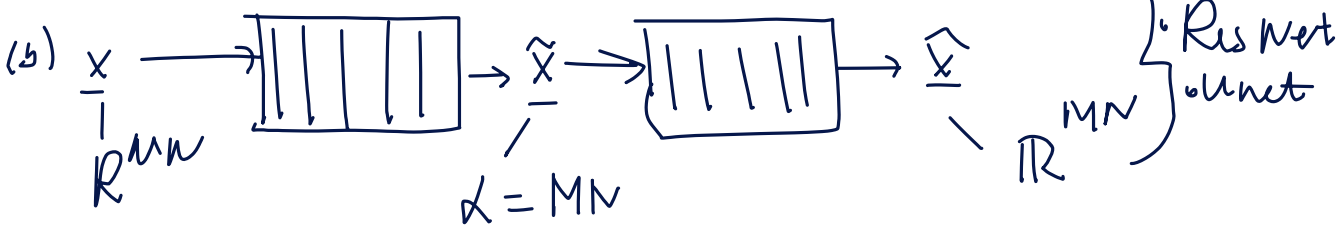
$W \longrightarrow W_\theta$

Auto-encoder (AE)

(a)



(b)



Ex: Enc:

$$\tilde{\underline{X}} = T_\phi(\underline{X}) = \sigma_R\left(T_R \cdots \sigma_2\left(T_2 \cdot \sigma_1\left(T_1 \underline{X} + \underline{b}_1\right) + \underline{b}_2\right) + \cdots \underline{b}_R\right)$$

(Q)

$\underline{X}_1$

$\phi = \{T_1, T_2, \ldots, T_k, \underline{b}_1, \ldots \underline{b}_R\}$

Dec:

$$\hat{\underline{X}} = W_\theta(\tilde{\underline{X}}) = \sigma_R(W_R - \cdots - \sigma_1(W_1 \tilde{\underline{X}} + \underline{a}_1) - \cdots + \underline{b}_R)$$

▷ Training

$$\mathcal{L}(\phi, \theta) = \sum_{i=1}^{N} \| \underline{x}_i - \underline{\hat{x}}_i \|_2 = \sum_{i=1}^{N} \| \underline{x}_i - W_\theta (T_\phi(\underline{x}_i)) \|_2^2$$

$\underline{\hat{x}}_i = W_\theta T_\phi(\underline{x})$

$\underline{\tilde{x}}$

1) $\nabla_{\phi,\theta} = 0$

2) GD: $(\hat{\phi}, \hat{\theta}) = (\phi, \theta)^K - \beta \nabla_{\phi,\theta} \mathcal{L}(\phi, \theta)$

Remarks:

① # in $\phi, \theta \longrightarrow N \uparrow$

② $\underline{x} \mapsto \triangleright \mapsto \underline{\tilde{x}} \mapsto \triangleleft \mapsto \underline{\hat{x}}$

T_ϕ^* $\quad \tilde{x}$ $\quad W_\theta^*$

DWM

$(T_\phi^* \& W_\theta^*)$

DFP

$\underline{x} \mapsto T_\phi^* \mapsto \underline{\tilde{x}} \mapsto Q(\cdot) \mapsto \underline{y}$