TP2 : Probabilities and Statistics. High-dimensional Data.

1 Probabilities and Statistics

1. We have the following joint probability :

X	Y	$p_{X,Y}(x,y)$
0	0	1/2
0	1	1/8
1	0	1/4
1	1	1/8

- $p_X(x)$:
  $p_X(0) = p_{X,Y}(x=0,y=0) + p_{X,Y}(x=0,y=1) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$
  $p_X(1) = p_{X,Y}(x=1,y=0) + p_{X,Y}(x=1,y=1) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

- $p_Y(y)$:
  $p_Y(0) = p_{X,Y}(x=0,y=0) + p_{X,Y}(x=1,y=0) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
  $p_Y(1) = p_{X,Y}(x=0,y=1) + p_{X,Y}(x=1,y=1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

- $p_{X|Y}(x|y=0)$:
  $p_{X|Y}(0|y=0) = \frac{p_{X,Y}(0,0)}{p_Y(0)} = \frac{1/2}{3/4} = \frac{2}{3}$
  $p_{X|Y}(1|y=0) = \frac{p_{X,Y}(1,0)}{p_Y(0)} = \frac{1/4}{3/4} = \frac{1}{3}$

- $p_{Y|X}(y|x=1)$:
  $p_{Y|X}(0|x=1) = \frac{p_{X,Y}(1,0)}{p_X(1)} = \frac{1/4}{3/8} = \frac{2}{3}$
  $p_{Y|X}(1|x=1) = \frac{p_{X,Y}(1,1)}{p_X(1)} = \frac{1/8}{3/8} = \frac{1}{3}$

2.

2 Simulations by using acceptance-rejection method

3 Two Lines

4 Distribution of Pair-wise Distances

5 Distribution of angles