Session 6 SMV: SFDD

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1 SFDD homomorphisms

1.1 SFDD Canonical Definition

Definition (Canonical form)

Let T be a set of terms, and $< \in T \times T$ a total ordering on T. A SFDD $S \in \mathbb{S}$ is canonical if and only if

- S is the rejecting terminal $\bot$
- S is the accepting terminal $\top$
- $S = \langle t, \tau, \sigma \rangle$ where
  - $\tau = \langle t_\tau, \tau_\tau, \sigma_\tau \rangle \implies t < t_\tau$ and $\tau \neq \bot$
  - $\sigma = \langle t_\sigma, \tau_\sigma, \sigma_\sigma \rangle \implies t < t_\sigma$
  - τ and σ are canonical

1.2 Clean

For the rest of the exercises, we suppose that we use the lexicographic order for the keys, i.e. $a < b < c < \ldots$
The definition of clean operation is as follows:

clean : $\mathbb{S} \rightarrow \mathbb{S}$ removes a negative node from all sets that contain it:

$$clean(\bot) = \bot$$
$$clean(\top) = \top$$
$$clean(\langle t, \tau, \sigma \rangle) = \begin{cases} clean(\sigma) & \text{if } \tau = \bot \\ \langle t, clean(\tau), clean(\sigma) \rangle & \text{if } \textit{otherwise} \end{cases}$$

An example of the application of the clean operation:
$S = \langle a, \top, \langle b, \bot, \top \rangle \rangle$
$clean(S) = clean(\langle a, \top, \langle b, \bot, \top \rangle \rangle)$
$clean(S) = \langle a, clean(\top), clean(\langle b, \bot, \top \rangle) \rangle$
$clean(\top) = \top$
$clean(\langle b, \bot, \top \rangle) = \top$
$clean(S) = \langle a, \top, \top \rangle$

1. Apply the *clean* homomorphism on the following SFDD: $\langle a, \bot, \langle b, \bot, \langle c, \top, \bot \rangle \rangle \rangle$
2. Draw the two SFDDs from the last question.
3. What is the purpose of the clean operation ?

1.3 Union

The union of two SFDDs is given by:

$$
\begin{aligned}
A \cup B &= B \cup A \\
A \cup A &= A \\
\bot \cup A &= A \\
\top \cup \langle t, \tau, \sigma \rangle &= \langle t, \tau, \top \cup \sigma \rangle \\
\langle t, \tau, \sigma \rangle \cup \langle t', \tau', \sigma' \rangle &= \begin{cases} \langle t, \tau, \sigma \cup \langle t', \tau', \sigma' \rangle \rangle & \text{if } t < t' \\ \langle t, \tau \cup \tau', \sigma \cup \sigma' \rangle & \text{if } t = t' \\ \langle t', \tau', \sigma' \cup \langle t, \tau, \sigma \rangle \rangle & \text{if } t > t' \end{cases}
\end{aligned}
$$

1. Apply the *union* homomorphism on the following SFDD: $\langle a, \langle b, \top, \bot \rangle, \bot \rangle \cup \langle b, \top, \bot \rangle$. Then draw the final result of the union.
2. Apply the *union* homomorphism on the following SFDD: $\langle a, \langle b, \top, \bot \rangle, \bot \rangle \cup \langle a, \top, \bot \rangle$. Then draw the final result of the union.
3. Apply the *union* homomorphism on the following SFDD: $\langle c, \top, \bot \rangle \cup \langle a, \top, \bot \rangle$. Then draw the final result of the union.

1.4 Intersection

The intersection of two SFDDs is given by:

$$
\begin{aligned}
A \cap B &= B \cap A \\
A \cap A &= A \\
\bot \cap A &= \bot \\
\top \cap \langle t, \tau, \sigma \rangle &= \top \cap \sigma \\
\langle t, \tau, \sigma \rangle \cap \left\langle t', \tau', \sigma' \right\rangle &= \begin{cases} \sigma \cap \langle t', \tau', \sigma' \rangle & \text{if } t < t' \\ \langle t, \tau \cap \tau', \sigma \cap \sigma' \rangle & \text{if } t = t' \\ \langle t, \tau, \sigma \rangle \cap \sigma' & \text{if } t > t' \end{cases}
\end{aligned}
$$

1. Apply the *intersection* homomorphism on the following SFDD: $\langle a, \langle b, \top, \top \rangle, \bot \rangle \wedge \langle a, \top, \bot \rangle$. Then draw the final result of the intersection.
2. Apply the *intersection* homomorphism on the following SFDD: $\langle a, \langle b, \bot, \top \rangle, \bot \rangle \wedge \langle a, \bot, \langle b, \top, \bot \rangle \rangle$ Then draw the final result of the intersection.
3. Apply the *intersection* homomorphism on the following SFDD: $\langle c, \top, \bot \rangle \wedge \langle a, \top, \langle c, \top, \bot \rangle \rangle$. Then draw the final result of the intersection.

1.5 Insertion

$\oplus : \mathbb{S}, T \rightarrow \mathbb{S}$ inserts a term $a \in T$ into all sets of a SFDD:

$$
\begin{aligned}
\bot \oplus a &= \bot \\
\top \oplus a &= \langle a, \top, \bot \rangle \\
\langle t, \tau, \sigma \rangle \oplus a &= \begin{cases} \langle t, \tau \oplus a, \sigma \oplus a \rangle & \text{if } t < a \\ \langle t, \tau \cup \sigma, \bot \rangle & \text{if } t = a \\ \langle a, \langle t, \tau, \sigma \rangle, \bot \rangle & \text{if } t > a \end{cases}
\end{aligned}
$$

1. Draw the following SFDD before the insertion: $\langle a, \langle b, \top, \bot \rangle, \bot \rangle \oplus a$. Then compute the result of the insertion, and draw the new result.
2. Draw the following SFDD before the insertion: $\langle a, \langle b, \top, \bot \rangle, \langle b, \top, \bot \rangle \rangle \oplus a$ Then compute the result of the insertion, and draw the new result.