TP 3 : Hopfield Network

Cours de modélisation numérique

10 March 2023

Introduction

During the lessons dedicated to dynamic systems, it was shown a simple model called *Hopfield network*[1], able to model certain elementary cerebral functions. It consists of a network of N interconnected binary nodes (neurons) $s_i = \pm 1$ with weights w_{ij}. The state of a neuron at a future time is determined at each instant by the state of all other neurons following the rule :

$$s_i(t+1) = sgn\left(\sum_j w_{ij} s_j(t)\right) = sgn\left(\frac{1}{N}\sum_j s_i^1 s_j^1 s_j(t)\right) \tag{1}$$

The key property of the Hopfield model is that the network is able to memorise a certain state $S^1 = \{s_1^1, ..., s_N^1\}$ provided that the weights are well chosen, as defined above (this choice is not unique, in this case we used the *Hebb rule*). By this we mean that S^1 is a fixed point, better an attractor, of the dynamic rule, this means that if we start from a state reasonably different from S^1, we will find (in a single iteration) the state S^1.

It is also possible to memorise multiple patterns $S^1, ..., S^P$, adapting the weights as :

$$w_{ij} = \frac{1}{N}\sum_{K=1}^{p} s_i^K s_j^K, \tag{2}$$

as long as p does not exceed a certain fraction of N.

Work to do

This TP consists in verifying experimentally the essential aspects of the Hopfield model mentioned above. To do so, you'll have to complete the following steps :

1. Implement a Hopfield network able to process the provided images (see below).
2. Provide a script using your implementation of the Hopfield network. Th script should :
   — Learn to recognise the 4 given images

1. Introduced in 1982 by the physicist J. J. Hopfield

— Roughen one of the images and display the result
— Reconstruct the image and display it

In addition, answer the following questions :

1. With a few tries, can you determine the number of changed bits at which the network loses its recognition capability ? Is it a lot ?
2. How does the weight matrix evolve as a function of the number of images learned ? Show the matrix and support your explanations with pictures.
3. Although we are not going to test here the limitations of the network in terms of the number of states that can be memorised, discuss the number and size of images that can be stored for a given population of neurons, or, vice versa, the capacity required to store an arbitrary number of images of arbitrary size. For information, a standard human brain has about 10^{11} neurons (but not all interconnected).
4. Propose a way around the limitations outlined above.

Given code

You will find on Moodle an archive containing :

1. A library (*utils.py*) allowing you to read a binary image. If an error message tells you that PIL is not installed on your machine follow the instructions in `installation_PIL.txt`.
2. Four beautiful binary images of size 20x20 pixels.

The tutorial file `example.py` shows you how to read, roughen and write and image. You can use it as a basis for your own program.