TP 12 : Complex Networks

Cours de modélisation numérique

26 May 2023

Introduction

You saw in class that graphs are powerful tools for building simple models of a wide variety of phenomena. Examples include electrical distribution systems, road transport networks and biological processes such as metabolism.

These graph-based models describe how structures emerge and how they evolve. Ideally, the graph generator used should be capable of producing a graph that mimics the properties of the phenomenon studied. In this series of exercises, you will implement and study a *scale-free* graph generator.

The Barabási Albert model

The Barabási Albert model, proposed by Albert-László Barabási, is a model generating *scale-free* graphs that reproduce the evolution of the growth of many real networks, such as the Internet (Fig. 1) and the WWW, or social networks such as the one social network underlying Facebook.

Construction can be broken down into two stages : growth and preferential attachment. Growth involves adding new vertices to the graph. Preferential attachment means that new vertices have a higher probability of establishing links with vertices with a higher degree. The term *the Matthew effect* refers to the Gospel of Matthew.

« *For to him who has shall be given, and he shall have abundance ; but to him who has not shall be taken away even that which he has.* »

– Matthew 25 : 29

The steps in the algorithm follow one another as follows, with an **initial situation** which is a graph $G = (V, E)$ composed of n_0 vertices ($n_0 \geq 2$) each with a degree of at least 1. Then the following steps are repeated for t time steps :

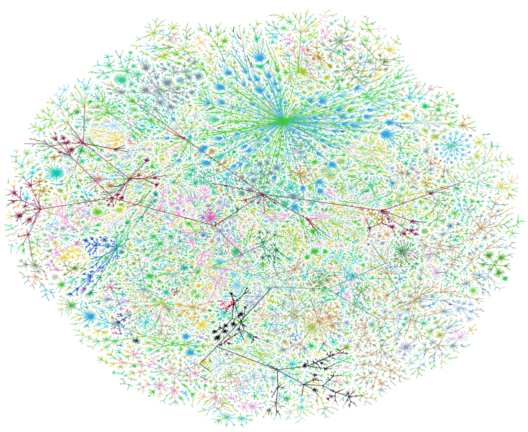

Figure 1 – The internet (29 June 1999)

Growth : at each time step, a new vertex is added to the graph with n arcs that connect it to n different vertices already present in the graph.

Preferential attachment : the choice of vertices to connect to the new depends on their degree. The new vertex is connected to one of the existing i vertices with the following probability P_i :

$$P_i = \frac{k_i}{\sum_j k_j} = \frac{k_i}{2|E|}$$

where k_i is the degree of vertex i.

After repeating these steps for t iterations, the resulting graph is composed of $t + n_0$ vertices and nt arcs.

The graphs generated by this model have the following properties, among others :

— the distribution P_k of degrees k follows a power-law distribution :

$$P_k \propto k^{-\gamma} \tag{1}$$

— the *average path length* l_G is proportional to :

$$l_G \propto \frac{\ln(n)}{\ln\big(\ln(n)\big)}, \tag{2}$$

where n is the number of vertices in the graph G. This measure defines the average number of steps it takes to connect all pairs of vertices in the graph along the shortest paths.

Work to do

On Moodle, you'll find the beginnings of a `Python` script designed to implement Barabási Albert's model, and producing attractive output graphs. To produce the graphs, you'll need the `networkx` package from `Python`, which you can install by typing the command `pip install networkx` in your `Python` environment.

We ask you to complete the code in order to generate the graphs. To help you understand the formalism used in the script, the Erdős-Rényi method has been implemented.

Once you can generate graphs with Barabási Albert's method, you can check that the generated graphs do indeed possess the (1) degree distribution property. The (2) property is left as a bonus. To do this, you'll need to build fairly large graphs, measure each magnitude *numerically*, and compare it with the value calculated *analytically* by the formulas above.