Bachelor's Thesis

Bachelor's Degree in Industrial Technologies and Economic Analysis

Offshore Wind Park Optimization

June 2024

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Call: Spring 2023-2024

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Escola Tècnica Superior
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Abstract

Short and must include results.

Keywords: offshore wind power plant, power flow, renewable energy, HVAC, transmission system, optimization, mixed-integer programming, genetic algorithms.

MSC codes: 90C11, 90C15, 90C29, 90C30, 90C59

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Resum

Short and must include results.

Paraules clau: parc eòlic marí, flux de potència, energia renovable, HVAC, sistema de transmissió, optimització, programació enter mixta, algoritmes genètics.

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Resumen

Short and must include results.

Palabras clave: parque eólico marino, flujo de potencia, energía renovable, HVAC, sistema de transmisión, optimización, programación entera mixta, algoritmos genéticos.

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Nomenclature

The next list describes several abbreviations and symbols that will be later used within the body of the thesis.

AC   Alternating Current

DC   Direct Current

HVAC High Voltage Alternating Curent

HVDC High Voltage Direct Current

MVRSM Mixed-Variable ReLU-based Surrogate Modelling

N-R   Newton-Raphson Method

OPF   Optimal Power Flow

OSS   Offshore Substation

OWF   Offshore Wind Farm

OWPP Offshore Wind Power Plant

PF   Power Flow

SCR   Short Circuit Ratio

XLPE Cross-Linked Polyethylene

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1 Preface

Arguably, climate change is one of the most pressing challenges we are facing today as humanity. That's why I wanted to develop a project revolving around sustainable solutions for the energy system of the future. As an engineering student I wanted to explore how renewable energy sources can by integrated into the grid and what challenges it poses, that's why I contacted Oriol Gomis to explore thesis topics within this field.

He introduced me diverse research areas and also eRoots, a spin-off from the UPC-CITCEA that develops software solutions for modern grid modelling, analyis and optimization. Then Josep Fanals, my supervisor and eRoots CEO, presented to my various topics that they would be potenitially interested to develop a thesis on. This is when he introduced me to the topic of design, sizing and optimization of the transmission system of offshore wind power plants. The research group CITCEA-UPC has been working in this area [2] and further developement on this field was the breeding ground for this proposal. The topic inmediatly caught my attention and that is how I ended up as an intern at eRoots developing software solutions for the optimization of offshore wind power plants.

Figure 1: Una imatge del logo de l'ETSEIB

Per referir-se a la Fig.1. Per no repetir informació és millor referir-se a altres apartats 6.

I recorda, sempre és important citar a la bibliografia [2].
La bibliografia ha d'estar ordenada, en teniu un exemple a la pàgina 36

2   Introduction

2.1   Motivation

During the industrial engineering studies you get in touch with a wide range of topics that can be applied to different fields. During the last years to get introduced to electrical engineering fundamentals and its applications. I discovered a deep interests for those topics and realized is a key tool for ensuring a future towards energy systems that can inegrate renwable energy sources.

The main driving force behind choosing this topic is the need to develop a sustainable energy system that can ensure a future for the next generations. The energy system is a key player in the fight against climate change. Moreover, the last report on global sustainable development [1] highlights how *Goal 7: Affordable and clean energy* is failing to meet its targets. In fact, it actually notices a backward trend in the 2020-2023 period when it comes to this goal targets, which signals that it is an area where efforts have to be put in.

This thesis is my modest and passionate contribution to provide sustainable solutions for our future.

2.2   Scope

This work will limit its study to the optimal design of HVAC transmission systems, without considering the cost comparision with HVDC. Moreover, it will limit its study to the steady-state of balanced three phase load systems, without considering unbalanced or transient states.

2.3   Objectives

The main objectives of the thesis are the following:

- Model all the elements of the transmission system of an offshore wind power plant and find its equivalent circuit.

- Implement a power flow solver with Python.

- Formulate the optimization problem to minimize all types of costs taking into account the technical constraints of the system.

- Bencharmark different optimization algorithms thar find solutions faster than state of the art methods.

- Study some specific cases applicable to real offshore wind power plants.

2.4 Outline

The thesis is structured as follows:

- Chapter 3 introduces offshore wind power plants, the problem we want to tackle and models the elements that we find in a HVAC transmission system.

- Chapter 5 formulates the minimization problem, including objective functions and constraints. In this section we also build the power flow solver that deals with the equality contraints and the algorithm for computing objective function values.

- Chapter 6 explains state of the art methods to solve the problem, its limitations, and our new approaches, involving genetic algorithms and an optimal power flow approach.

- Chapter 7 showcases the results for a specific case study and benchmarks the performance of the optimization algorithms. and computational time.

- Chapter 8 collects the main outcomes of the thesis and proposes future lines of work.

- Chapter 9 presents the planning and viability studies for the project.

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3   Technical background: grid to study

3.1   Offshore wind power plants

As global energy demands surge and the pressing need for sustainable development becomes ever more urgent, the quest for renewable energy sources has intensified. Among these, wind power has emerged as a frontrunner due to its potential to generate substantial amounts of clean electricity. While onshore wind farms have been widely implemented, their offshore counterparts are gaining increasing attention for their ability to harness the stronger and more consistent winds found over the oceans.

Offshore wind power, defined as the use of wind turbines located in bodies of water to generate electricity, presents several advantages over onshore installations. The primary benefit lies in the higher wind speeds and lower turbulence experienced offshore, which contribute to greater energy yields. Additionally, offshore wind farms can be situated closer to urban centers located along coastlines, thereby reducing transmission losses and enhancing energy efficiency. The development of offshore wind technology has seen rapid advancements in recent years. From the deployment of the first offshore wind farm in Denmark in 1991 to the establishment of massive installations such as the Hornsea Project in the UK, the scale and capacity of these projects have grown significantly. Technological innovations, including larger turbines, floating foundations, and improved grid integration techniques, have further propelled the industry forward.

3.2   Transmission system: design and reactive power compensation problem

When designing the transmission system of an OWPP, several factors must be considered to ensure optimal performance and efficiency. The system must be capable of transmitting the generated power from the wind turbines to the onshore grid while trying to be as energy and cost-efficient as possible. There are two main types of technologies that can bu used, HVDC and HVAC. In this thesis we will focus our study to the HVAC technology.

To put in context the relevance of this topic, the work and software developed in this thesis will be used as the breeding-ground of a parternship between eRoots and Acciona, a leading infrastructre company in Spain that has shown interest in developing a tool for the optimal design of OWPP's transmission systems.

One drawback of using HVAC cables is the high shunt capcitance they have, which is even larger in underground cables, which are the ones used for OWPP.

	Overhead Lines	Underground Lines
Capacitance per unit length (μF/km)	0.01 - 0.02	0.3 - 0.6

Table 1: Comparison of Capacitance per unit length of Overhead and Underground Lines

The charging current of this capacitance limits the active power tranfer capacity of the line and in-

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creases power losses and voltage across the line due to the Ferranti effect[3]. This effect can be described by the votage difference between the sending and reciving end of transmission line under no-load conditions:

$$\frac{V_o - V_i}{V_o} = \omega^2 CL \frac{l^2}{2} \tag{1}$$

where V_o and V_i are the receiving and sending end voltages respectively, ω is the frequency, C is the capacitance per unit length, L is the inductance per unit length and l is the length of the line. Note that the voltage difference is proportional to the square of the length of the line which leads to overvoltages for long transmission lines.

However, the possbility to include reactive power compensation elements helps reduce the reactive power generation. Figure 2 shows how including this compensation reduces power losses, especially when we approach the no-load condition, which is equavialent to not having any type of active power injection from the OWPP, i.e. the wind speed is very low.
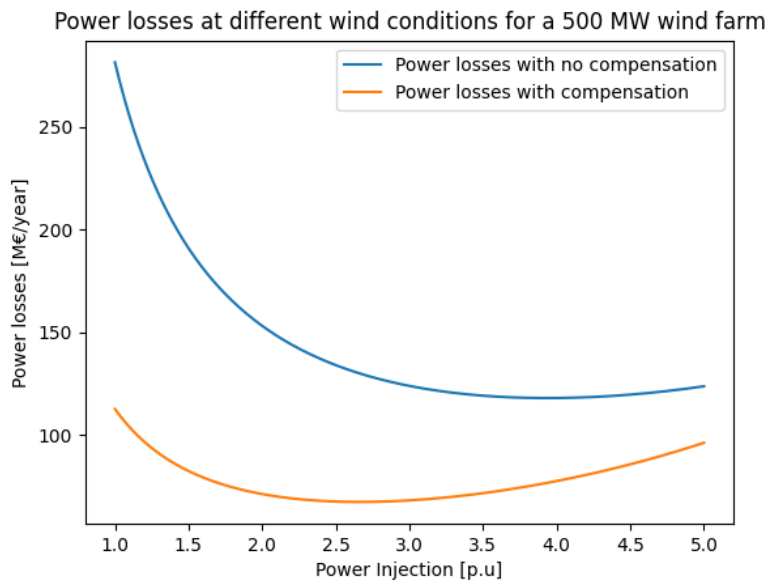
Figure 2: Power losses comparision when including reactive power compensation

The goal of the project is to determine where this compensation has to be placed and how to size it. But this is only part of the design carachteristics we want to optimize. A full descrption of the optimitization variables will be presented in Chapter 5.

Taking all this into consderation, a HVAC transmission system layout for an OWPP with reactive power compensation looks like the one in Figure 3.
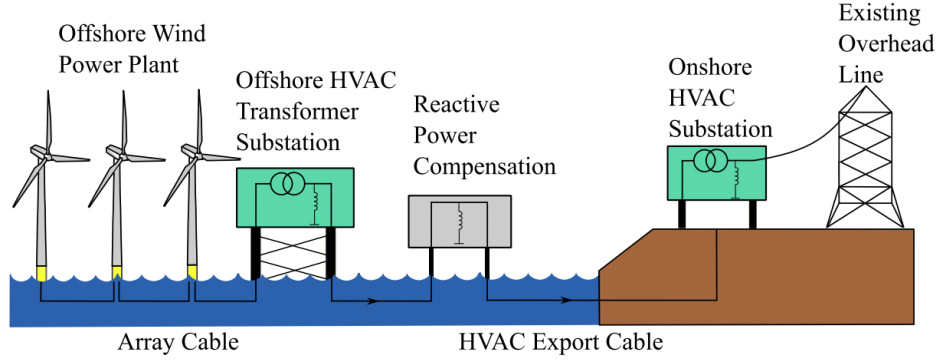
Figure 3: HVAC transmission system layout for an OWPP with reactive power compensation[2]

From this general layout we will be able to create the network model using the equivalent circuits of the elements involved.

3.3   Grid elements

To be able to do a steady-state analysis of the transmission system we need to model all the elements in the grid. This section models this elements and presents some important concepts to understand the grid.

3.3.1   Cables

Cables are an essential part of HVAC transmission systems. We will consider three-core cross-linked Polyethylene (XLPE) cables, which are the most common type of cables used in OWPP's.

The equivalent circuit of a cable is shown in Figure 4. We will use a model
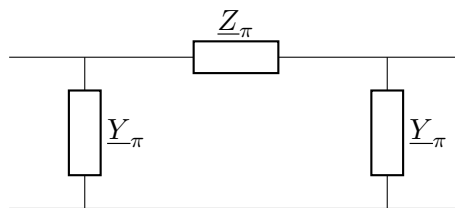
Figure 4: Pi model transmission line

$$\begin{bmatrix} \underline{V}_s \\ \underline{I}_s \end{bmatrix} = \begin{bmatrix} \cosh(\underline{\theta}) & \underline{Z}_c \sinh(\underline{\theta}) \\ \frac{1}{\underline{Z}_c} \sinh(\underline{\theta}) & \cosh(\underline{\theta}) \end{bmatrix} \begin{bmatrix} \underline{V}_r \\ \underline{I}_r \end{bmatrix} \tag{2}$$

$$\underline{Z}_c = \sqrt{\frac{\underline{Z}}{\underline{Y}}}; \quad \underline{\theta} = l\sqrt{\underline{ZY}} \tag{3}$$

$$\underline{Z} = R + j\omega L; \quad \underline{Y} = j\omega \frac{C}{2} \tag{4}$$

Therefore, the equivalent π model is given by:

$$\underline{Z}_\pi = \underline{Z}_c \sinh(\theta) = \underline{Z}l\frac{\sinh(\theta)}{\theta} \tag{5}$$

$$\underline{Y}_\pi = \frac{\tanh(\theta/2)}{\underline{Z}_c} = \frac{\underline{Y}l}{2}\frac{\tanh(\theta/2)}{\theta/2} \tag{6}$$

3.3.2 Transformers
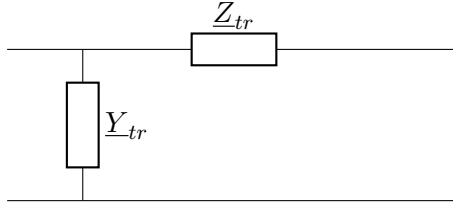
Figure 5: Transformer model

$$\underline{Z}_{tr} = R_{tr} + jX_{tr}; \quad \underline{Y}_{tr} = G_{tr} + jB_{tr} \tag{7a}$$

$$R_{tr} = P_{Cu}^{loss}\left(\frac{U_{r-tr}}{S_{r-tr}}\right)^2 \quad X_{tr} = \sqrt{\left(u_k\frac{U_{r-tr}}{S_{r-tr}}\right)^2 - R_{tr}^2} \tag{7b}$$

$$G_{tr} = \frac{P_{Fe}^{loss}}{U_{r-tr}^2} \quad B_{tr} = i_o\frac{S_{r-tr}}{U_{r-tr}^2} \tag{7c}$$

3.3.3 Shunt reactor: reactive power compensation

To compensate reactive power generation we will use shunt reactors. They are reactive power absorvers and in our case will be connected from the line directly to the ground. The equivalent circuit is shown in Figure 6.

$$\underline{y}_{sh} = \frac{1}{j\omega L} \tag{8}$$

where L is the inductance of the reactor.
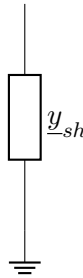
Figure 6: Shunt reactor model

We will consider the possibility to include 5 different shunt reactors in the system. Further development on the reasoning of this scheme on 4

3.3.4 Main grid

It will be the slack bus.

We will model the grid we are connected to using a Thévenin equivalent circuit where $\underline{u}_g$ is the Thévenin voltage and $\underline{z}_g$ is the Thévenin impedance:

$$\underline{z}_g = R_g + jX_g \tag{9a}$$

$$R_g = \sqrt{\frac{\frac{U_g^2}{SCR} p_{owf} S_{base}}{(\frac{X_g}{R_g})^2 + 1}} \tag{9b}$$

Note that we need the short circuit ratio (SCR) and the $\frac{X_g}{R_g}$ ratio as parameters to the equivalent circuit.

- The SCR is the ratio of the short circuit apparent power in the case of a line-line-line-ground fault at the location in the grid where some generator is connected to the power rating of the generator itself. It is somehow a measure of the grid strength to small disturbances.
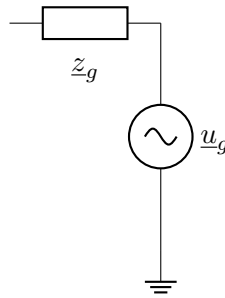
Figure 7: Main grid model

3.3.5 Power plant

We will model our OWPP as a simple power injection at one end of the transmission line. For our analysis we will consider that there is no reactive power generation, therefore $q_{owf} = 0$. p_{owf} will be the active power generation that will depend on the wind conditions of the plant. The reach of the work will limit its analysis to a fix power generation. Explicar aquí que differents vents, diferents potènices injectadees i referenciar a future work.

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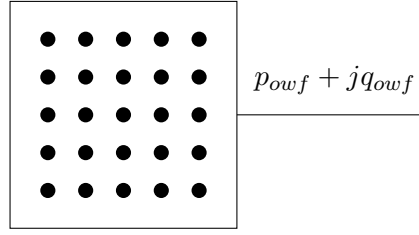
Figure 8: Power plant

3.4 Costs modelling

In this section we will derive the cost equations for the different elements of our transmission system. This costs function will be essential for our optimization problem since the will be the objective function to minimize and the parameters on which thwey depend will be the vector of decision variables we want to get.

3.4.1 Cables

Equations from [8]:

$$C_{cables} = n_{cables}(A + B\exp(\frac{CS_n}{10^8})) \cdot E_{\frac{eu}{sek}} \cdot \frac{1}{10^6} \quad \left[\frac{M\text{\EUR}}{\text{km}}\right] \tag{10}$$

$$S_n = \sqrt{3}V_{rated}I_{rated} \quad [\text{VA}] \tag{11}$$

where S_n is the rated power of the cable, n_{cables} is the number of cables, A, B and C are parameters that depend on the rated voltage of the cable, $E_{\frac{eu}{sek}=0.087}$ is the exchange rate between euros and sek and V_{rated} and I_{rated} are the rated voltage and current of the cable respectively. The parameters A, B and C are shown in Table 2.

Rated Voltage (kV)	A [10^6]	B [10^6]	C
132	1.971	0.209	1.66
220	3.181	0.11	1.16

Table 2: Parameters A, B, C for different rated voltages [8]

3.4.2 Transformers

$$C_{tr} = 0.0427 \cdot S_{r-tr}^{0.7513} \quad [M\text{\EUR}] \tag{12}$$

where S_{r-tr} is the rated power of the transformer.

3.4.3 Shunt reactor

$$C_{sh} = K \cdot Q_l \cdot + P = K \cdot Y_l \cdot U_{AC-N}^2 + P \quad [M\text{\EUR}] \tag{13}$$

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where K and P are constants that depends on the location of the reactor, Q_l is the reactive power absorbed by the reactor, Y_l is the admittance of the reactor and U_{AC-N} is the nominal transmission voltage in kV where the compensation is placed.

Location	K	P
Onshore	0.01049	0.8312
Offshore	0.01576	1.244
Mid cable	0.01576	12.44

Table 3: Parameters K and P for different positions of the reactive power compensation [2]

3.4.4 Switchgears

$$C_{gis-AC} = 0.0117 \cdot U_{AC-N} + 0.0231 \quad [M\text{€}] \tag{14}$$

where U_{AC-N} is the nominal transmission voltage in kV.

3.4.5 Substation platform

$$C_{ss-AC} = 2.534 + 0.0887 \cdot P_{owf} \quad [M\text{€}] \tag{15}$$

where P_{owf} is the nominal power generated by the OWPP in MW.

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4 Power flow analysis

Now we can fully define the power flow analysis. This approach will allow as to model the transmission system as a set of buses (or nodes) interconnected by transmission links. This will allow as to solve for the steady-state powers and voltages of the system. This step is crucial to latter on formulate our optimization since power flow equations will be the equality constraint of the optimization problem, **??**, and the objective function will also depend on the solution of the power flow (PF).

4.1 Types of buses

In this section we will breifly describe what is a bus and what types we have.

Buses are points of the grid which either supplied by generators, *generator buses*, or those without generators, *load buses*. More formally, a n-bus system, $N = \{B_1, ..., B_i, ..., B_n\}$ where N is the set of n-nodes, is defined as:

$$\forall B_i \in N, \begin{cases} S_i = P_i + jQ_i, & \text{where } S_i \text{ is the apparent power at bus } i \\ V_i = |V_i|e^{j\theta_i}, & \text{where } V_i \text{ is the complex voltage at bus } i \end{cases} \tag{16}$$

As we can see, for each bus i we have 4 variables:

- P_i and Q_i are the active and reactive power at bus i respectively.

- $|V_i|$ and θ_i are the voltage magnitude and angle at bus i respectively.

It is important to note that in general we cannot specify all the P_i's independently since there is a constraint imposed by the need to balance active power. In our case, a transmission system with losses, which are unknown before the PF, the sum of P_i's must be equal to losses. To tackle this we will define one bus as the $slack$ bus, where power injection is left free. Taking all of this account, depeding on the variables are known and unknown for a cerain bus we can classify them as:

- **Slack bus:** The slack bus is the reference bus of the system. It is the bus where the voltage magnitude and angle are known, typically $|V_i| = 1, \theta_i = 0$. All other buses angles will be referenced to the $slack$. It is used to balance the active and reactive power in the system.

- **Generator bus (PV):** The generator bus is the bus where the active power and voltage are known

- **Load bus (PQ):** The load bus is the bus where the active and reactive power are known. The voltage magnitude and angle are unknown.

In summary:

	Slack Bus	PQ	PV
Voltage Magnitude ($|V_i|$)	Yes	No	Yes
Voltage Angle (θ_i)	Yes	No	No
Active Power (P_i)	No	Yes	Yes
Reactive Power (Q_i)	No	Yes	No

Table 4: Known and unknown variables for each type of bus

4.2   Per unit system (p.u.)

In power systems analysis, it is common to use the per unit system to normalize the magnitudes of the variables. This is very useful when we are dealing with several transformers and voltage levels. The per unit system is defined as:

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \tag{17}$$

In our case we will use $S_{base} = 100$ MVA and $V_{base} =$ transmission voltage in kV as the base values. This means that the per unit system will be defined as:

$$\begin{cases} I_{base} = S_{base} V_{base} \\ Y_{base} = \frac{S_{base}}{V_{base}^2} \end{cases} \tag{18}$$

Using the per unit system will be particularly useful for analyzing the results and for dealing with the inequality constraints 5.2.2.

4.3   Grid model

Now we have all the information neeeded to build our full model. In [2] they consider five possible postions for the shunt reactors, as seen in Figure 9.

Figure 9: Possible locations for the shunt reactors [2]
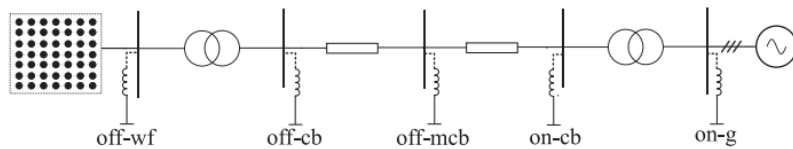
They consider when optimizing the cost function only the combinations where: if at a given transformer you place a reactor before it, you won't consider placing another one after the same transformer. For sake of generality, we will consider that any combination within the five possible positions is valid. This will lead to a total of $2^5 = 32$ possible combinations.
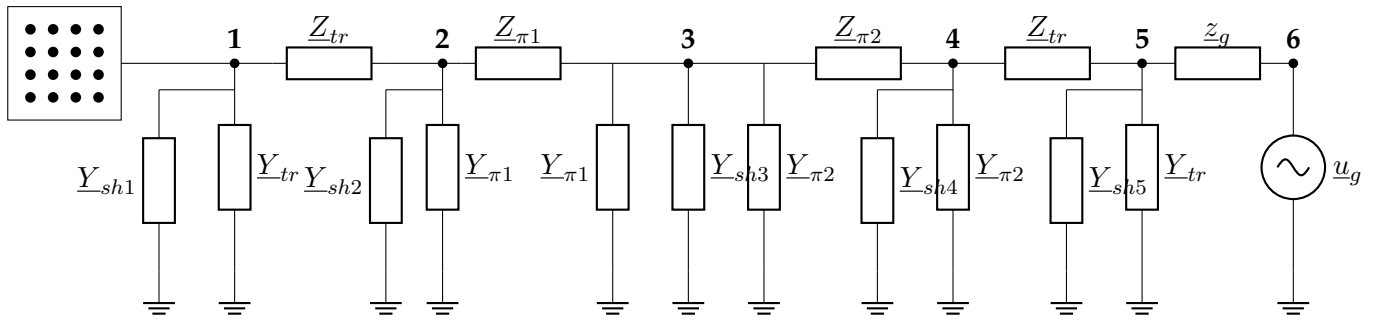
Figure 10: Transmission system model and buses

Now we choose where we want to define our buses and which type they are. We classify them as in Table 5.

Bus Number	Bus Type
1	PQ
2	PQ
3	PQ
4	PQ
5	PQ
6	Slack

Table 5: Transmission system bus types

A brief explanation of the bus types is exposed:

- **Bus 1:** It is the bus where the WPP is injecting power therefore active and reactive power are specified as $P_1 = p_{owf}$ and $Q_1 = q_{owf}$.

- **Bus 2, 3, 4, 5:** Those buses are plain PQ load buses, where $P_i = 0, Q_i = 0$. We want to consider them as we will be interested in the voltage levels in the lines in order to compute possible over and undervoltages in the cables, as we will see in Chapter 5.

- **Bus 6:** We will use this one as the slack bus. Note that this bus power flow solution for P_6 will allow us to compute the losses in the system and will be essential part of the optimization problem. Also, since we define it as the slack bus, it will be the reference for the voltage angles and in charge of balacinf the power in the transmission system.

4.4 Admittance matrix

The admittance matrix encodes all the information of the elements of the grid and the relationship between them and allows us to perform the steady-state analysis of the grid. Now we can build the full HVAC trasnmission system model and fins its admittance matrix $\underline{Y}_{bus}$, which will be essential for the power flow solver.

Using the Kirchhoff's Current Law (KCL):

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V} \tag{19}$$

where $\mathbf{I}$ is the vector of injected currents, $\mathbf{Y}_{bus}$ is the admittance matrix and $\mathbf{V}$ is the vector of bus voltages.

Since we will be using the per unit system:

$$y_i = \frac{Y_i}{Y_{base}} \tag{20}$$

Note that when will build it, we take the series impedances in 10, compute their inverse and add the subindex s to identify them. For example:

$$\underline{y}_{\pi 1s} = \frac{1}{\underline{z}_{\pi 1}} \tag{21}$$

$$\mathbf{Y_{bus}} = \begin{bmatrix} (\underline{y}_{tr} + \underline{y}_{trs} + \underline{y}_{sh1}) & -\underline{y}_{trs} & 0 & 0 & 0 & 0 \\ -\underline{y}_{trs} & (\underline{y}_{\pi 1} + \underline{y}_{\pi 1s} + \underline{y}_{sh2} + \underline{y}_{trs}) & -\underline{y}_{\pi 1s} & 0 & 0 & 0 \\ 0 & -\underline{y}_{\pi 1s} & (2\underline{y}_{\pi 1} + 2\underline{y}_{\pi 1s} + \underline{y}_{sh3}) & -\underline{y}_{\pi 2s} & 0 & 0 \\ 0 & 0 & -\underline{y}_{\pi 2s} & (\underline{y}_{\pi 2} + \underline{y}_{\pi 2s} + \underline{y}_{sh4} + \underline{y}_{trs}) & -\underline{y}_{trs} & 0 \\ 0 & 0 & 0 & -\underline{y}_{trs} & (\underline{y}_{tr} + \underline{y}_{tr} + \underline{y}_{sh5} + \underline{y}_{g}) & -\underline{y}_{g} \\ 0 & 0 & 0 & 0 & -\underline{y}_{g} & \underline{y}_{g} \end{bmatrix} \tag{22}$$

Some properties of the admittance matrix are:

- It is a square matrix of size $n \times n$, where n is the number of buses in the system.

- It is symmetric.

- The diagonal elements, Y_{ii}, are self-admittance, equal to the sum of the admittances of elements connected to bus i.

- Y_{ij} is the negative of the admittance between buses i and j

- For our system, the sparcity is $\frac{20}{36} = 56\%$. Nevertheless, for large networks, the matrix es very sparse. The level of sparsity, which is the percentage of zero elements in a matrix, increases with the size of the network. For instance, in a 1000-bus system, the matrix approximately is 99% sparse You can take advantage (and it will be essential to do it for very big networks!) of this sparsity using computational techniques [5].

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4.5   Grid parameters

We will use the following set of grid elements parameters for the transformers and the main grid:

Parameter	Value
SCR	5
$\frac{X_g}{R_g}$	10
P_{Cu}^{loss}	60 kW
P_{Fe}^{loss}	40 kW
u_k	18 %
i_o	1.2 %

Table 6: Grid parameters [2]

4.6   Power flow

4.6.1   Power flow equations

We first going to derive an expression for the PF equations. From 19 we can get the injected current for the i-th component:

$$I_i = \sum_{j=1}^{n} Y_{ik} V_k \quad i = 1, 2, ..., n \tag{23}$$

Now we can compute the i-th bus power:

$$S_i = V_i I_i^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^* \quad i = 1, 2, ..., n \tag{24}$$

Now if fe let $V_i = |V_i|e^{j\theta_i}$, and $Y_{ik} = G_{ik} + jB_{ik}$, we can write the power at bus i as:

$$S_i = \sum_{k=1}^{n} |V_i||V_k|e^{j\theta_i}(G_{ik} - jB_{ik}) = \sum_{k=1}^{n} |V_i||V_k|(cos(\theta_{ik}) + jsin(\theta_{ik}))(G_{ik} - jB_{ik}) \quad i = 1, 2, ..., n \tag{25}$$

Note that the real part of the admittance, G_{ik}, is the conductance and the imaginary part, B_{ik}, is the susceptance. Now if we separate the real and imaginary parts of the power we get:

$$P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}cos(\theta_{ik}) + B_{ik}sin(\theta_{ik})) \quad i = 1, 2, ..., n \tag{26}$$

$$Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}sin(\theta_{ik}) - B_{ik}cos(\theta_{ik})) \quad i = 1, 2, ..., n \tag{27}$$

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4.6.2 Newton-Raphson Solver

Problem Solvability   First, we have to make sure that our set of equations is solvable. First, if we strip away the $slack$-bus, we have 5 PQ-buses remaing. Each PQ-bus introduces 2 unknows, $|V_i|$ and θ_i, and 2 equations, P_i and Q_i. This means that we have 10 equations and 10 unknowns. After solving the system, from 26 and 27 we can solve for the power injections of the slack bus, P_6 and Q_6. Hence, the condition for solvability is fullfiled.

To solve a set of non-linear equations, numerical methods are the standalone way to go. For the PF, on of the most used methods is the Newton-Raphson (N-R) method [6]. A brief description of this method will be presented for the n-dimentional case, but further descreiption can be found in [4].

We will descibe the process considering an n-bus system with all PQ buses except one slack bus. Note that in our specific case this conditions are met and $n = 6$. Once we strip away the slack bus, it remains to finnd the unknowns in the right side of equations 26 and 27. Therefore it will be convinient to define the following vectors of unknowns of PQ-buses:

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n-1} \end{bmatrix} \quad |\mathbf{V}| = \begin{bmatrix} |V_1| \\ |V_2| \\ \vdots \\ |V_{n-1}| \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \theta \\ |\mathbf{V}| \end{bmatrix} \tag{28}$$

Now the dependency between the power flow equations and the vector of unknowns $\mathbf{x}$ can be easily shown:

$$P_i = P_i(\mathbf{x}) \quad i = 1, 2, ..., n-1 \tag{29a}$$
$$Q_i = Q_i(\mathbf{x}) \quad i = 1, 2, ..., n-1 \tag{29b}$$

The N-R method is an iterative method that starts with an initial guess of the unknowns, $\mathbf{x}_0$, and then iterates the vector $\mathbf{x}$ until the right side matches the left side of the equations. Therefore we can define the power mismatch vector $\mathbf{f}(\mathbf{x})$ as:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_1(\mathbf{x}) - P_1 \\ \vdots \\ P_{n-1}(\mathbf{x}) - P_{n-1} \\ Q_1(\mathbf{x}) - Q_1 \\ \vdots \\ Q_{n-1}(\mathbf{x}) - Q_{n-1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Delta P}(\mathbf{x}) \\ \mathbf{\Delta Q}(\mathbf{x}) \end{bmatrix} = \mathbf{0} \tag{30}$$

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Now we have to consider $\mathbf{J}$, the Jacobian matrix of $\mathbf{f}(\mathbf{x})$. For clarity, is convininet to parition $\mathbf{J}$ as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J_{11}} & \mathbf{J_{12}} \\ \mathbf{J_{21}} & \mathbf{J_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{\Delta P}}{\partial \theta} & \frac{\partial \mathbf{\Delta P}}{\partial |\mathbf{V}|} \\ \frac{\partial \mathbf{\Delta Q}}{\partial \theta} & \frac{\partial \mathbf{\Delta Q}}{\partial |\mathbf{V}|} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \theta} & \frac{\partial \mathbf{P}}{\partial |\mathbf{V}|} \\ \frac{\partial \mathbf{Q}}{\partial \theta} & \frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|} \end{bmatrix} \tag{31}$$

if interest check derivation of the partial derivatives of the power flow equations with respect to the unknowns check the Appendix.

Now it just remain to find the step $\Delta \mathbf{x^v}$ to update $\mathbf{x}$ solving the following linear system by Gauss elimination:

$$\mathbf{J^k} \Delta \mathbf{x^k} = -\mathbf{f}(\mathbf{x^k}) \tag{32}$$

Now we update $\mathbf{x^{k+1}} = \mathbf{x^k} + \Delta \mathbf{x^k}$ and iterate until it converges. The convergence criteria is usually set as the maximum of the absolute value of the power mismatch vector, $\max(\mathbf{f}(\mathbf{x}))$, is less than a tolerance, tol.

The N-R solver has been implemented in Python(check the Appendix) and the procedure can be found in Algorithm 1.

Algorithm 1 Newton-Raphson Method
---
procedure RUN_PF
    Initialize V with ones and θ with zeros
    Set $V[slack]$ to 1, $\theta[slack]$ to 0
    Set P, Q based on OWPP data
    Set $iter = 0$, tol and $k = 0$
    while $iter < max\_iter$ and $\Delta PQ > tol$ **do**
        Compute $P\_present$, $Q\_present$ using V, θ, Y_{bus}
        Compute mismatch $\Delta PQ = [dP, dQ]$ as difference between calculated and given P, Q
        Compute the Jacobian matrix J.
        Solve the linear system $J \cdot \Delta x_k = -\Delta PQ$.
        Compute the updated $x^{k+1} = x^k + \Delta x^k$.
        Update V, θ using dP, dQ, Y_bus
        if $\max(\Delta PQ) < tol$ **then**
            break
        end if
    end while
    return V, θ
    Compute P_{slack}, Q_{slack}
end procedure

Finally note that close to the solution vector $\mathbf{x}^*$ it can be proven that N-R normally presents quadratic convergence [7].

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5 Optimization problem formulation

Generally an optimization problem that involves equality and inequality constraints can be formulated as:

$$
\begin{aligned}
\min \quad & f(\mathbf{x}) \\
\text{subject to} \quad & \mathbf{H}(\mathbf{x}) = 0 \\
& \mathbf{G}(\mathbf{x}) \leq 0
\end{aligned}
\tag{33}
$$

where $\mathbf{x} \in \mathbb{R}^n$ is the n decision variables vector, $f(\mathbf{x})$ is the objective function to be minimized, $\mathbf{H}(\mathbf{x})$ are the equality constraints and $\mathbf{G}(\mathbf{x})$ are the inequality constraints.

Now we have to decide which decision variables we include in the optimization problem. It is easy to see that we can include all the design varaibles in which the transmission system model depends, which are all included in the cost function and therefore have a direct impact in the cost of the system. By checking the equivalent circuits of the elements in the grid we can define the vector of unknowns as:

$$
\mathbf{x} = \begin{bmatrix} vol_{tr}, & n_{cables}, & react_{bi_1}, & ,... & ,react_{bi5} & ,react_{cont1} & ,... & ,react_{cont5} & ,react_{bi5} & ,S_{trafo} \end{bmatrix}
\tag{34}
$$

where:

- vol_{tr}: Is the voltage level of the transmission cables. Note that this will be an integer variable, where you can choose one voltage level given by manufacturers with its associated parameters and cost. Check table 2 to see available options considered.

- n_{cables}: Is an integer number that that represents the number of cables placed in parallel.

- $react_{bi_i} \quad i = 1, ..., 5$: Is a binary variable (i.e. either 0 or 1) that tells if we place or not a reactor at position i.

- $react_{cont_i} \quad i = 1, ..., 5$: Sizing of reactor at position i, therefore its Y_{shi} in p.u.

- S_{trafo}: Rated power of the transformer in MVA.

A very important remark with respect to the type of variables we have involves in the problem. Considering $\mathbf{x}$ we see that we have binary, integer and continous variables. This kind of problems are labeled as mixed-integer optimization and constraints the optimization methods we can use as we will see in 6.

5.1 Objective function: multiobjective and mixed-integer

Explicar pq separem en multiobjective pq aixi traiem PARETTO SET, ventages respsecvte solució tradicional

5.2 Constraints

5.2.1 Equality: Power Flow

The equality constraints $\mathbf{H}(\mathbf{x}) = 0$ in the optimization problem in 33 are the imposed by the power flow equations. In fact, the power mismatch vector 30 is the one that take this form

5.2.2 Inequality: Technical constraints

The operational constraints are the ones that ensure the system operates within the limits of the equipment. This constraints of inequaility nature, setting upper and lower bounds for certain varaibles. This constraints will be the the inequality constraints $\mathbf{G}(\mathbf{x}) \leq 0$ in the optimization problem in 33. We will denote as $\mathbf{L_i^X}$ each different limit at bus i and for the sake of coherence we will express the constraints in the $\mathbf{G}(\mathbf{x}) \leq 0$ form.

Note that all the following limits can be based on grid code requirements, but we will use some standard values.

Bus voltage limits
We set the nodal bus-i voltage, V_i, limits as:

$$
\begin{aligned}
L_i^{Vu} = V_i - V_{max} \leq 0, \quad i \in N \quad \text{is the upper limit} \\
L_i^{Vl} = V_{min} - V_i \leq 0, \quad i \in N \quad \text{is the lower limit}
\end{aligned}
\tag{35}
$$

where $\mathbf{V_{max}}$ and $\mathbf{V_{min}}$ are the maximum and minimum voltages. These values have been taken to be ± 0.1 p.u. of the nominal voltage respectively.

Maximum current limits for the lines
The current in the lines should not exceed the maximum current that the cables can handle. We will set $I_{max} = 1.1$ p.u. of the rated current of the cable I_{rated}. This rated current is given by manufacturers [9]. Therefore the constraint will be:

$$
L_i^I = I_i - I_{max} \leq 0, \quad i \in N_{lines}
\tag{36}
$$

Reactive power delivered to the grid
In general reactive power delivered to the grid, Q_6 can be defined by grid code requirements. Nevertheless, in this case we will aim to keep it as close as possible to zero. Therefore we will set the limits

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as:

$$L_6^{Qu} = Q_6 - Q_{max} \leq 0$$
$$L_6^{Ql} = Q_{min} - Q_6 \leq 0$$

(37)

where $Q_{max} = Q_{min} = 0$.

5.3   Algorithm overview
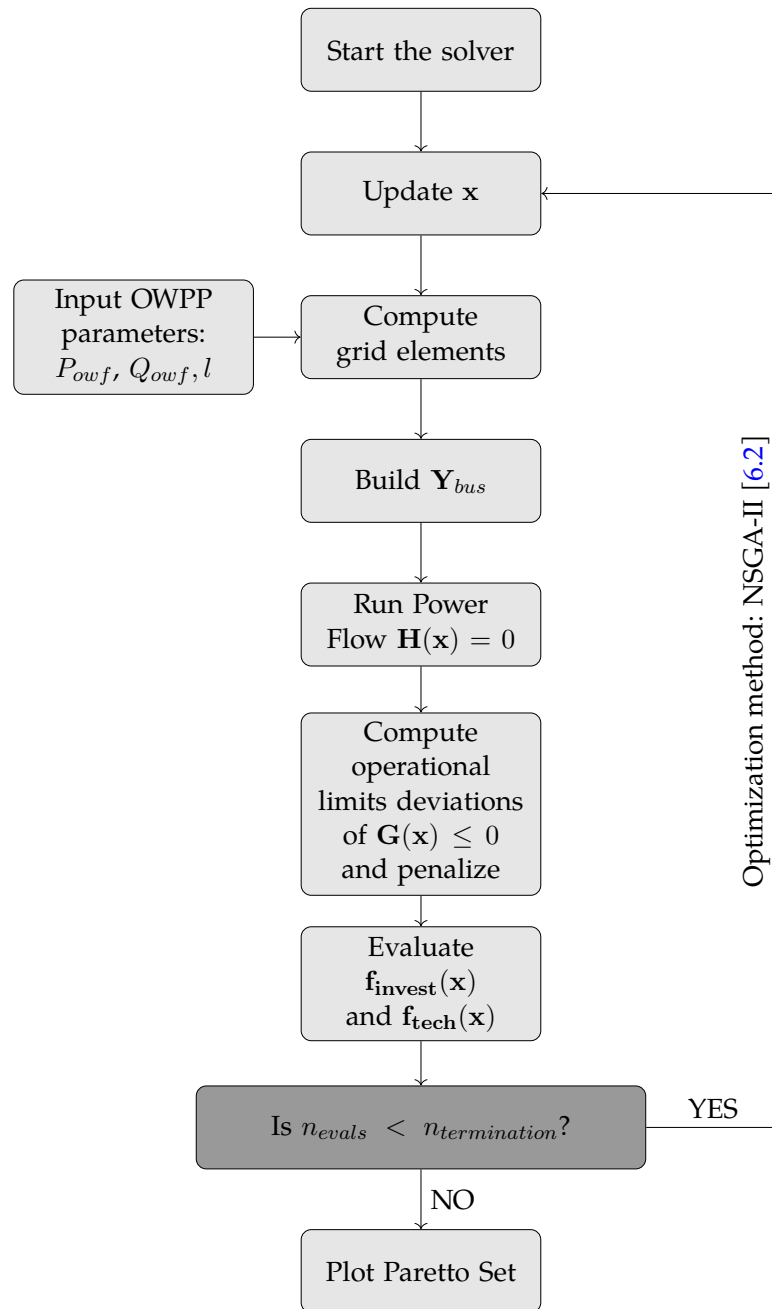
Figure 11: Block diagram of the optimization algorithm

6   Optimization methods

6.1   State of the art limitations: interior point method

6.2   NSGA-II: Genetic algorithm

Parameter	Value
Population size	300
Survival	Rank and Crowding "cd"
Termination	300 evaluations
Seed	1

Table 7: Tuning parameters for the NSGA-II

6.3   Optimal Power Flow approach for compensation sizing

h

7 Results: Case study

7.1 500 MW OWPP

h

8 Conclusions

8.1 Outcome

8.2 Future work

h

9 Planning and viability studies

9.1 Time Planning

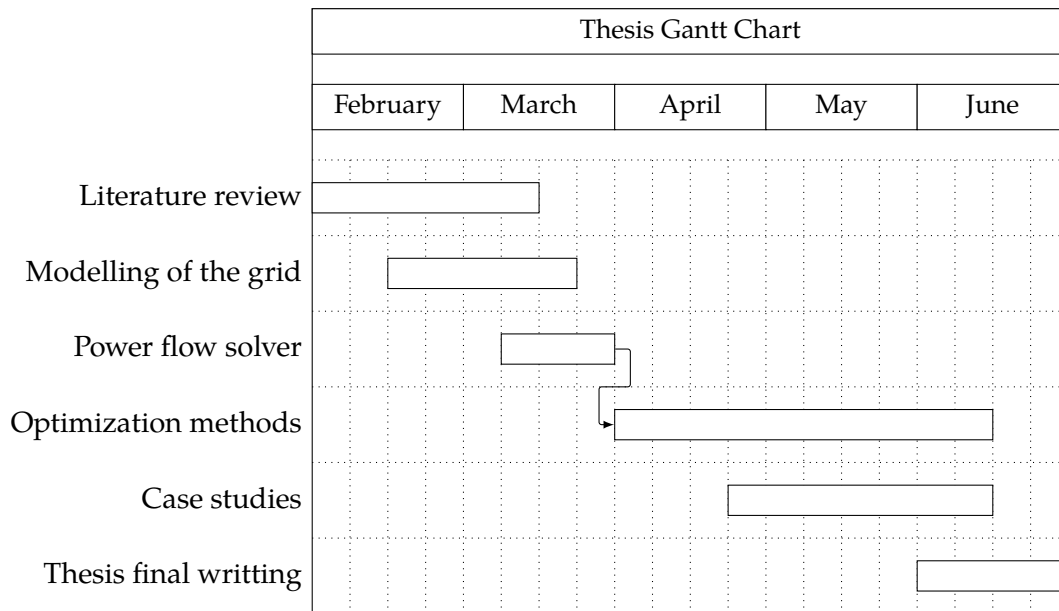Figure 12 shows the time distribution for the tasks carried out in the thesis.

Thesis Gantt Chart				
February	March	April	May	June

Literature review

Modelling of the grid

Power flow solver

Optimization methods

Case studies

Thesis final writting

Figure 12: Thesis Gantt Chart

9.2 Economic assessment

Concept	Cost
Computer	$XXX
Working hours	$XXX
Tutor supervision	$XXX
Total without IVA	$600
Total with IVA (21%)	$720

Table 8: Thesis Costs

9.3 Environmental assessment

This environmental assessment evaluates the energy costs incurred during the development of a thesis over five months, using a computer as the primary tool. The primary energy consumption arises from the computer's usage, which includes writing, research, data analysis, and communication.

Assuming an average laptop with a power consumption of 60 watts, used for approximately 6 hours daily, the total energy consumption over five months is around 54 kWh. This consumption translates to roughly 30 kg of CO_2 emissions, assuming the average emission factor for electricity generation.

To reduce these energy costs and associated environmental impacts in future thesis projects, several

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strategies can be employed. Utilizing energy-efficient computers, enabling power-saving modes, and limiting usage time can significantly lower consumption. Additionally, adopting renewable energy sources, such as solar panels for charging devices, further reduces the carbon footprint, contributing to a more sustainable academic practice.

9.4   Social and gender equality assessment

This assessment evaluates the social and gender equality aspects of a bachelor's thesis focused on developing a tool for optimizing renewable energy system design, authored by a 22-year-old white engineering student. While the thesis itself addresses a critical area in sustainable development, examining its social dimensions is essential to ensure inclusivity and equality.

The demographic profile of the author reflects broader trends in STEM fields, where women and minority groups remain underrepresented. This lack of diversity can influence the perspectives and priorities embedded in the research. Ensuring diverse representation in such projects is crucial for incorporating a wide range of insights and addressing the needs of various communities.

To promote social and gender equality, the research should consider the differential impacts of renewable energy systems on diverse populations. Inclusive design processes involve consulting with and incorporating feedback from women, marginalized communities, and other underrepresented groups. This approach ensures that the developed tools and technologies are accessible and beneficial to all segments of society.

Moreover, educational institutions should encourage and support participation from diverse backgrounds in engineering and renewable energy fields. Mentorship programs, scholarships, and targeted recruitment can help bridge the gender and social gap, fostering an environment where innovative solutions for renewable energy are developed through diverse and inclusive contributions.

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Acknowledgements

Primer de tot vull agraïr profundament el suport constant del Josep, el meu tutor, que m'ha donat consell i ajuda quan l'he necessitada cada dia des de que vaig entrar a eRoots. Per extensió, agraïr la rebuda i suport de tot l'equip d'eRoots durant aquests mesos, especialment al meu tutot acadèmic,l'Oriol, que ha proporcionat noves idees i enfocs al desenvolupament del treball.

Aquest TFG tanca un capítol de quatre anys de la meva vida que han sigut molt feliços gràcies als meus amics del grau; gràcies a tots nou.

Finalment, agraïr a la meva família i a la Mire el seu suport incondicional cada dia i a ensenyar-me a estimar.

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Appendix

In the following link you can find the Git repository with all the code used for the development of this work: https://github.com/Ch4rlieStone/tfg_eroots

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