

# Bachelor's Thesis

Bachelor's Degree in Industrial Technologies and Economic Analysis

## Offshore Wind Park Optimization

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# ETSEIB



## Abstract

Short and must include results.

**Keywords:** offshore wind power plant, power flow, renewable energy, HVAC, transmission system, optimization, mixed-integer programming, genetic algorithms.

**MSC codes:** 90C11, 90C15, 90C29, 90C30, 90C59

## Resum

Short and must include results.

**Paraules clau:** parc eòlic marí, flux de potència, energia renovable, HVAC, sistema de transmissió, optimització, programació enter mixta, algoritmes genètics.

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## Resumen

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## Nomenclature

The next list describes several abbreviations and symbols that will be later used within the body of the thesis.

AC Alternating Current

DC Direct Current

HVAC High Voltage Alternating Current

HVDC High Voltage Direct Current

MVRSM Mixed-Variable ReLU-based Surrogate Modelling

N-R Newton-Raphson Method

OPF Optimal Power Flow

OSS Offshore Substation

OWF Offshore Wind Farm

OWPP Offshore Wind Power Plant

PF Power Flow

SCR Short Circuit Ratio

XLPE Cross-Linked Polyethylene

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## 1 Preface

Arguably, climate change is one of the most pressing challenges we are facing today as humanity. That's why I wanted to develop a project revolving around sustainable solutions for the energy system of the future. As an engineering student I wanted to explore how renewable energy sources can be integrated into the grid and what challenges it poses, that's why I contacted Oriol Gomis to explore thesis topics within this field.

He introduced me diverse research areas and also eRoots, a spin-off from the UPC-CITCEA that develops software solutions for modern grid modelling, analysis and optimization. Then Josep Fanals, my supervisor and eRoots CEO, presented to me various topics that they would be potentially interested in developing a thesis on. This is when he introduced me to the topic of design, sizing and optimization of the transmission system of offshore wind power plants. The research group CITCEA-UPC has been working in this area [2] and further development on this field was the breeding ground for this proposal. The topic immediately caught my attention and that is how I ended up as an intern at eRoots developing software solutions for the optimization of offshore wind power plants.

## 2 Introduction

### 2.1 Motivation

During the industrial engineering studies you get in touch with a wide range of topics that can be applied to different fields. During the last years to get introduced to electrical engineering fundamentals and its applications. I discovered a deep interests for those topics and realized is a key tool for ensuring a future towards energy systems that can inegrate renewable energy sources.

The main driving force behind choosing this topic is the need to develop a sustainable energy system that can ensure a future for the next generations. The energy system is a key player in the fight against climate change. Moreover, the last report on global sustainable development [1] highlights how *Goal 7: Affordable and clean energy* is failing to meet its targets. In fact, it actually notices a backward trend in the 2020-2023 period when it comes to this goal targets, which signals that it is an area where efforts have to be put in.

This thesis is my modest and passionate contribution to provide sustainable solutions for our future.

### 2.2 Scope

This work will limit its study to :

- The optimal design of HVAC transmission systems, without considering the comparison with HVDC.
- Study the steady-state of balanced three phase load systems, without considering unbalanced or transient states.
- We will consider constant power injection form the OWPP at its nominal value. In this sense, we are not including the wind speeds distribution that yields different power injections.
- We will assume that the main grid we are supplying power to is completely stable, in the sense that it does not suffer voltage level perturbations.

### 2.3 Objectives

The main objectives of the thesis are the following:

- Model all the elements of the transmission system of an offshore wind power plant and find its equivalent circuit.
- Implement a power flow solver with Python.
- Formulate the optimization problem of the reactive power compensation and the transmission system design.

- Design and implement an optimization algorithm in Python that allows to find the Pareto Frontier of optimal solutions and compare its advantages with respect to the state-of-the-art.
- Study a specific case study applicable to real offshore wind power plants.

## 2.4 Outline

The thesis is structured as follows:

- Chapter 3 introduces offshore wind power plants, the problem we want to tackle and models the elements that we find in a HVAC transmission system.
- Chapter 4 presents the power flow analysis and builds the full transmission system model.
- Chapter 5 formulates the minimization problem, including objective functions and constraints. In this section we describe the algorithm structure that deals with the optimization formulation.
- Chapter 6 explains state of the art methods to solve the problem, its limitations, and our new approaches, involving genetic algorithms and an optimal power flow approach.
- Chapter 7 showcases the results for a specific case study and analyses the performance and results of the optimization algorithm. and computational time.
- Chapter 8 collects the main outcomes of the thesis and proposes future lines of work.
- Chapter 9 presents the planning and viability studies for the project.

### 3 Technical background: grid to study

#### 3.1 Offshore wind power plants

As global energy demands surge and the pressing need for sustainable development becomes ever more urgent, the quest for renewable energy sources has intensified. Among these, wind power has emerged as a frontrunner due to its potential to generate substantial amounts of clean electricity. While onshore wind farms have been widely implemented, their offshore counterparts are gaining increasing attention for their ability to harness the stronger and more consistent winds found over the oceans.

Offshore wind power, defined as the use of wind turbines located in bodies of water to generate electricity, presents several advantages over onshore installations. The primary benefit lies in the higher wind speeds and lower turbulence experienced offshore, which contribute to greater energy yields. Additionally, offshore wind farms can be situated closer to urban centers located along coastlines, thereby reducing transmission losses and enhancing energy efficiency. The development of offshore wind technology has seen rapid advancements in recent years. From the deployment of the first offshore wind farm in Denmark in 1991 to the establishment of massive installations such as the Hornsea Project in the UK, the scale and capacity of these projects have grown significantly. Technological innovations, including larger turbines, floating foundations, and improved grid integration techniques, have further propelled the industry forward.

#### 3.2 Transmission system: design and reactive power compensation problem

When designing the transmission system of an OWPP, several factors must be considered to ensure optimal performance and efficiency. The system must be capable of transmitting the generated power from the wind turbines to the onshore grid while trying to be as energy and cost-efficient as possible. There are two main types of technologies that can be used, HVDC and HVAC. In this thesis we will focus our study to the HVAC technology.

To put in context the relevance of this topic, the work and software developed in this thesis will be used as the breeding-ground of a partnership between eRoots and Acciona, a leading infrastructure company in Spain that has shown interest in developing a tool for the optimal design of OWPP's transmission systems.

One drawback of using HVAC cables is the high shunt capacitance they have, which is even larger in underground cables, which are the ones used for OWPP.

	Overhead Lines	Underground Lines
Capacitance per unit length ( $\mu\text{F}/\text{km}$ )	0.01 - 0.02	0.3 - 0.6

Table 1: Comparison of Capacitance per unit length of Overhead and Underground Lines

The charging current of this capacitance limits the active power transfer capacity of the line and in-

creases power losses and voltage across the line due to the Ferranti effect[3]. This effect can be described by the voltage difference between the sending and receiving end of transmission line under no-load conditions:

$$\frac{V_o - V_i}{V_o} = \omega^2 CL \frac{l^2}{2} \quad (1)$$

where  $V_o$  and  $V_i$  are the receiving and sending end voltages respectively,  $\omega$  is the frequency,  $C$  is the capacitance per unit length,  $L$  is the inductance per unit length and  $l$  is the length of the line. Note that the voltage difference is proportional to the square of the length of the line which leads to overvoltages for long transmission lines.

However, the possibility to include reactive power compensation elements helps reduce the reactive power generation. Figure 1 shows how including this compensation reduces power losses, especially when we approach the no-load condition, which is equivalent to not having any type of active power injection from the OWPP, i.e. the wind speed is very low.

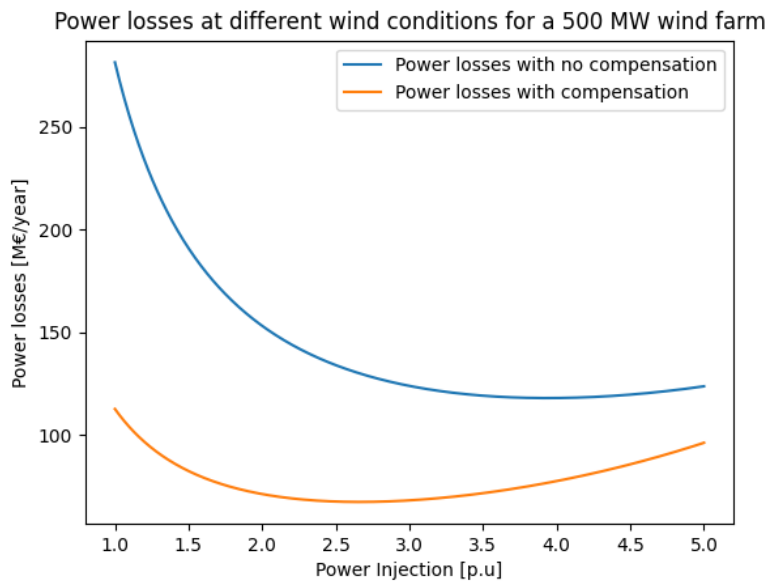


Figure 1: Power losses comparison when including mid-cable reactive power compensation

The goal of the project is to determine where this compensation has to be placed and how to size it. But this is only part of the design characteristics we want to optimize. A full description of the optimization variables will be presented in Chapter 5.

Taking all this into consideration, a HVAC transmission system layout for an OWPP with reactive power compensation looks like the one in Figure 2.



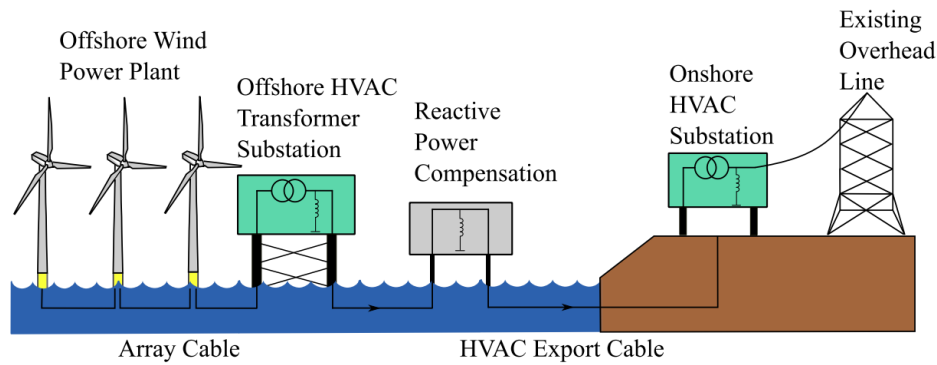


Figure 2: HVAC transmission system layout for an OWPP with reactive power compensation[2]

From this general layout we will be able to create the network model using the equivalent circuits of the elements involved.

### 3.3 Grid elements

To be able to do a steady-state analysis of the transmission system we need to model all the elements in the grid. This section models these elements and presents some important concepts to understand the grid.

#### 3.3.1 Cables

Cables are an essential part of HVAC transmission systems, since they are in charge of transmitting the power from the OWPP to shore and are the main source of active power losses and reactive power generation. We will consider three-core cross-linked Polyethylene (XLPE) cables, which are the most common type of cables used in AC OWPP's.

Three-core XLPE cables (see Figure 3) have a steel wire armour and can have either copper or aluminum as conductor. We will consider the copper ones.



Figure 3: three-core XLPE cable cross-section [10]

Recall, as seen in 1, that the capacitance of underground cables is quite large, which leads to high reactive power generation. Reactive power compensation will help us mitigate this effect.

For the equivalent circuit of a long transmission cable, we can use the following  $\pi$ -model, shown in Figure 4.

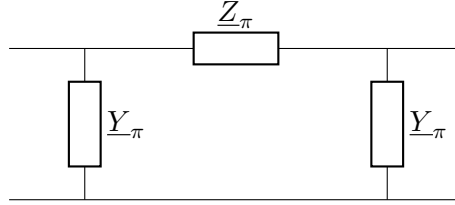


Figure 4:  $\pi$ -model transmission line

We can use the following equations [2] to get the parameters:

$$\begin{bmatrix} \underline{V}_s \\ \underline{I}_s \end{bmatrix} = \begin{bmatrix} \cosh(\underline{\theta}) & \underline{Z}_c \sinh(\underline{\theta}) \\ \frac{1}{\underline{Z}_c} \sinh(\underline{\theta}) & \cosh(\underline{\theta}) \end{bmatrix} \begin{bmatrix} \underline{V}_r \\ \underline{I}_r \end{bmatrix} \quad (2)$$

$$\underline{Z}_c = \sqrt{\frac{\underline{Z}}{\underline{Y}}}; \quad \underline{\theta} = l\sqrt{\underline{Z}\underline{Y}} \quad (3)$$

$$\underline{Z} = R + j\omega L; \quad \underline{Y} = j\omega \frac{C}{2} \quad (4)$$

where  $\underline{V}_s$  and  $\underline{I}_s$  are the sending end voltage and current and  $\underline{V}_r$  and  $\underline{I}_r$  are the receiving end voltage and current,  $\underline{Z}_c$  is the characteristic impedance and  $\underline{\theta}$  is the characteristic angle. The parameters in Table 2 are obtained from manufacturer data [10].

Afegir lo de R effects.

Symbol		Description
$R$	$\frac{\Omega}{\text{km}}$	Resistance per unit length
$L$	$\frac{H}{\text{km}}$	Inductance per unit length
$C$	$\frac{F}{\text{km}}$	Capacitance per unit length

Table 2: Electrical parameters of AC cables

Therefore, the equivalent  $\pi$  model is given by:

$$\underline{Z}_\pi = \underline{Z}_c \sinh(\underline{\theta}) = \underline{Z}l \frac{\sinh(\underline{\theta})}{\underline{\theta}} \quad (5)$$

$$\underline{Y}_\pi = \frac{\tanh(\underline{\theta}/2)}{\underline{Z}_c} = \frac{\underline{Y}l \tanh(\underline{\theta}/2)}{2 \underline{\theta}/2} \quad (6)$$

### 3.3.2 Transformers

Transformers are essential devices in power transmission systems that transfer electrical energy between circuits through electromagnetic induction. They enable voltage levels to be increased (stepped up) for efficient, long-distance transmission and decreased (stepped down) for safe distribution to homes and businesses. This voltage transformation minimizes energy losses and enhances the stability of the power grid. Without transformers, the high currents required for low-voltage transmission would lead to excessive heat generation and energy waste, reducing the efficiency and reliability of the electrical supply. Thus, transformers are crucial for optimizing power delivery and ensuring the safety and efficiency of electrical systems.

In our OWPP transmission system we will need to employ transformers to step up the voltage of the power generated by the wind turbines to the nominal voltage of the transmission system and another one at onshore to set the voltage required by the main grid we are supplying to.

We will use the the model in Figure 5 to represent the transformer:

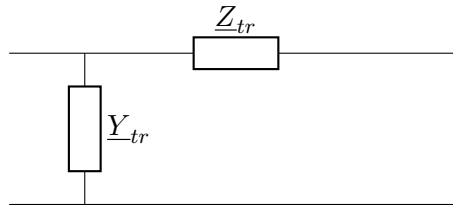


Figure 5: Transformer model

The equivalent impedance  $\underline{Z}_{tr}$  and admittance  $\underline{Y}_{tr}$  are given by:

$$\underline{Z}_{tr} = R_{tr} + jX_{tr}; \quad \underline{Y}_{tr} = G_{tr} + jB_{tr} \quad (7a)$$

$$R_{tr} = P_{Cu}^{loss} \left( \frac{U_{r-tr}}{S_{r-tr}} \right)^2 \quad X_{tr} = \sqrt{\left( u_k \frac{U_{r-tr}}{S_{r-tr}} \right)^2 - R_{tr}^2} \quad (7b)$$

$$G_{tr} = \frac{P_{Fe}^{loss}}{U_{r-tr}^2} \quad B_{tr} = i_o \frac{S_{r-tr}}{U_{r-tr}^2} \quad (7c)$$

where  $P_{Cu}^{loss}$  are the copper losses,  $U_{r-tr}$  the rated voltage of the transformer at the transmission system side,  $S_{r-tr}$  the rated power of the transformer,  $u_k$  the short-circuit voltage,  $P_{Fe}^{loss}$  the iron losses and  $i_o$  the open circuit current.

### 3.3.3 Shunt reactor: reactive power compensation

To compensate reactive power generation we will use shunt reactors. They are reactive power absorbers and in our case will be connected from the line directly to the ground. The equivalent circuit

is shown in Figure 6.

$$\underline{y}_{sh} = \frac{1}{j\omega L} \quad (8)$$

where  $L$  is the inductance of the reactor.

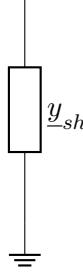


Figure 6: Shunt reactor model

We will consider the possibility to include 5 different shunt reactors in the system. Further development on the reasoning of this scheme on [4](#)

### 3.3.4 Main grid

The main grid is the distribution grid we are supplying power to fit the OWPP. We will model the grid we are connected to using a Thévenin equivalent circuit where  $\underline{U}_g$  is the Thévenin voltage and  $\underline{Z}_g$  is the Thévenin impedance:

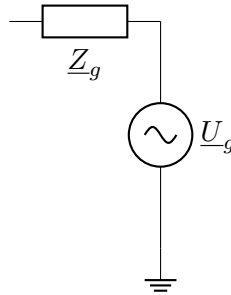


Figure 7: Main grid model

$$\underline{Z}_g = R_g + jX_g \quad (9a)$$

$$R_g = \sqrt{\frac{U_g^2}{\frac{SCR}{Powf} S_{base}}} X_g = R_g \frac{X_g}{R_g} \quad (9b)$$

where  $U_g$  is the rated voltage of the grid,  $SCR$  the short-circuit ratio and  $\frac{X_g}{R_g}$  the ratio of the system reactance.

- The  $SCR$  is the ratio of the short circuit apparent power in the case of a line-line-line-ground

(3LG) fault at the location in the grid where some generator is connected to the power rating of the generator itself. It is somehow a measure of the grid strength to changes in active and reactive power injections.

### 3.3.5 Power plant

We will model our OWPP as a simple power injection at one end of the transmission line. For our analysis we will consider that there is no reactive power generation, therefore  $q_{owf} = 0$ .  $p_{owf}$  will be the active power generation that will depend on the wind conditions of the plant. The reach of the work will limit its analysis to a fix power generation.

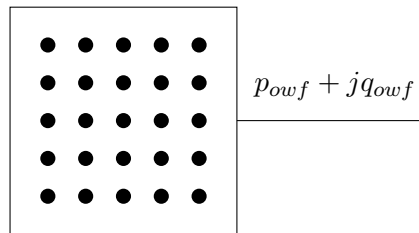


Figure 8: Power plant

Further development on how considering different wind conditions could be implemented in our work, check [8.2](#).

### 3.4 Costs modelling

In this section we will derive the cost equations for the different elements of our transmission system. This costs function will be essential for our optimization problem since it will be the objective function to minimize and the parameters on which they depend will be the vector of decision variables we want to get.

#### 3.4.1 Cables

Equations to describe AC cables costs are presented in [8] as:

$$C_{cables} = n_{cables} (A + B \exp(\frac{C S_n}{10^8})) \cdot E_{\frac{eu}{sek}} \cdot \frac{1}{10^6} \left[ \frac{M\text{€}}{\text{km}} \right] \quad (10)$$

$$S_n = \sqrt{3} V_{rated} I_{rated} \quad [\text{VA}] \quad (11)$$

where  $S_n$  is the rated power of the cable,  $n_{cables}$  is the number of cables,  $A$ ,  $B$  and  $C$  are parameters that depend on the rated voltage of the cable,  $E_{\frac{eu}{sek}=0.087}$  is the exchange rate between euros and sek and  $V_{rated}$  and  $I_{rated}$  are the rated voltage and current of the cable respectively. The parameters  $A$ ,  $B$  and  $C$  are shown in Table 3:

Rated Voltage (kV)	A [ $10^6$ ]	B [ $10^6$ ]	C
132	1.971	0.209	1.66
220	3.181	0.11	1.16

Table 3: Parameters A, B, C for different rated voltages [8]

We will consider the transmission voltage levels presented in Table 3 as decision variables in our optimization.

#### 3.4.2 Transformers

For the transformers, costs are defined in [20] as:

$$C_{tr} = 0.0427 \cdot S_{r-tr}^{0.7513} \quad [M\text{€}] \quad (12)$$

where  $S_{r-tr}$  is the rated power of the transformer.

#### 3.4.3 Shunt reactor

Cost functions for shunt reactors are presented in [2] as:

$$C_{sh} = K \cdot Q_l \cdot + P = K \cdot Y_l \cdot U_{AC-N}^2 + P \quad [M\text{€}] \quad (13)$$

where  $K$  and  $P$  are constants that depend on the location of the reactor and can be found in Figure

4,  $Q_l$  is the reactive power absorbed by the reactor,  $Y_l$  is the admittance of the reactor and  $U_{AC-N}$  is the nominal transmission voltage in kV where the compensation is placed.

Location	K	P
Onshore	0.01049	0.8312
Offshore	0.01576	1.244
Mid cable	0.01576	12.44

Table 4: Parameters K and P for different positions of the reactive power compensation [2]

### 3.4.4 Switchgears

For switchgears, costs are defined in [9] as:

$$C_{gis-AC} = 0.0117 \cdot U_{AC-N} + 0.0231 \quad [M\text{€}] \quad (14)$$

where  $U_{AC-N}$  is the nominal transmission voltage in kV.

### 3.4.5 Substation platform

For the substation platform, costs are presented in [8] as:

$$C_{ss-AC} = 2.534 + 0.0887 \cdot P_{owf} \quad [M\text{€}] \quad (15)$$

where  $P_{owf}$  is the nominal power generated by the OWPP in MW.

### 3.4.6 AC power losses

Lastly, but probably one of the most important costs to consider, are the power losses in the transmission system. To able to compare this term with other investment costs related to the electric elements, it would be useful to express this term in monetary units. We will use the following equation to estimate the cost of power losses in the system:

$$C_{loss-AC} = 8760 \cdot t_{owf} \cdot C_{energy} \cdot P_{loss} \cdot 10^{-6} \quad [M\text{€}] \quad (16)$$

where 8760 is the number of operating hours in a year,  $t_{owf}$  is the expected lifetime of the OWPP in years,  $C_{energy}$  is the cost of energy in €/MWh and  $P_{loss}$  is the power losses in the system in MW. We will be using the following estimated values from [2]:

Parameter	Value
$t_{owf}$	30 years
$C_{energy}$	100 €/MWh

Table 5: Parameters for computing power losses cost [2]

Now it remains to compute the power losses  $P_{losses}$  in the system. This will be derived in 4 as it will be trivial to get the power losses once we have the power flow solution.



## 4 Power flow analysis

Now we can fully define the power flow analysis. This approach will allow us to model the transmission system as a set of buses (or nodes) interconnected by transmission links. This will allow us to solve for the steady-state powers and voltages of the system. This step is crucial to latter on formulate our optimization since power flow equations will be the equality constraint of the optimization problem, and the objective function will also depend on the solution of the power flow (PF).

### 4.1 Types of buses

In this section we will briefly describe what is a bus and what types we have.

Buses are points of the grid which either supplied by generators, *generator buses*, or those without generators, *load buses*. More formally, a  $n$ -bus system,  $N = \{B_1, \dots, B_i, \dots, B_n\}$  where  $N$  is the set of  $n$ -nodes, is defined as:

$$\forall B_i \in N, \begin{cases} S_i = P_i + jQ_i, & \text{where } S_i \text{ is the apparent power at bus } i \\ V_i = |V_i|e^{j\theta_i}, & \text{where } V_i \text{ is the complex voltage at bus } i \end{cases} \quad (17)$$

As we can see, for each bus  $i$  we have 4 variables:

- $P_i$  and  $Q_i$  are the active and reactive power at bus  $i$  respectively.
- $|V_i|$  and  $\theta_i$  are the voltage magnitude and angle at bus  $i$  respectively.

It is important to note that in general we cannot specify all the  $P_i$ 's independently since there is a constraint imposed by the need to balance active power. In our case, a transmission system with losses, which are unknown before the PF, the sum of  $P_i$ 's must be equal to losses. To tackle this we will define one bus as the *slack* bus, where power injection is left free. Taking all of this account, depending on the variables are known and unknown for a certain bus we can classify them as:

- **Slack bus:** The slack bus is the reference bus of the system. It is the bus where the voltage magnitude and angle are known, typically  $|V_i| = 1, \theta_i = 0$ . All other buses angles will be referenced to the *slack*. It is used to balance the active and reactive power in the system.
- **Generator bus (PV):** The generator bus is the bus where the active power and voltage are known
- **Load bus (PQ):** The load bus is the bus where the active and reactive power are known. The voltage magnitude and angle are unknown.

In summary:

	Slack Bus	PQ	PV
Voltage Magnitude ( $ V_i $ )	Yes	No	Yes
Voltage Angle ( $\theta_i$ )	Yes	No	No
Active Power ( $P_i$ )	No	Yes	Yes
Reactive Power ( $Q_i$ )	No	Yes	No

Table 6: Known and unknown variables for each type of bus

## 4.2 Per unit system (p.u.)

In power systems analysis, it is common to use the per unit system to normalize the magnitudes of the variables. This is very useful when we are dealing with several transformers and voltage levels. The per unit system is defined as:

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \quad (18)$$

In our case we will use  $S_{base} = 100$  MVA and  $V_{base} =$  transmission voltage in kV as the base values. This means that the per unit system will be defined as:

$$\begin{cases} I_{base} = S_{base} / V_{base} \\ Y_{base} = \frac{S_{base}}{V_{base}^2} \end{cases} \quad (19)$$

Using the per unit system will be particularly useful for analyzing the results and for dealing with the inequality constraints 5.1.2.

## 4.3 Grid model

Now we have all the information needed to build our full model. In [2] they consider five possible postions for the shunt reactors, as seen in Figure 9.

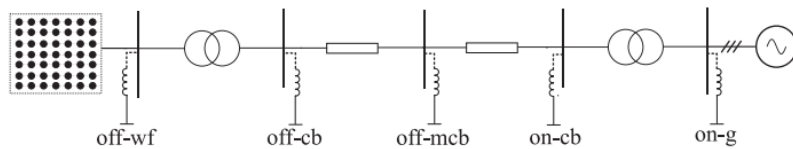


Figure 9: Possible locations for the shunt reactors [2]

They consider when optimizing the cost function only the combinations where: if at a given transformer you place a reactor before it, you won't consider placing another one after the same transformer. For sake of generality, we will consider that any combination within the five possible positions is valid. This will lead to a total of  $2^5 = 32$  possible combinations.

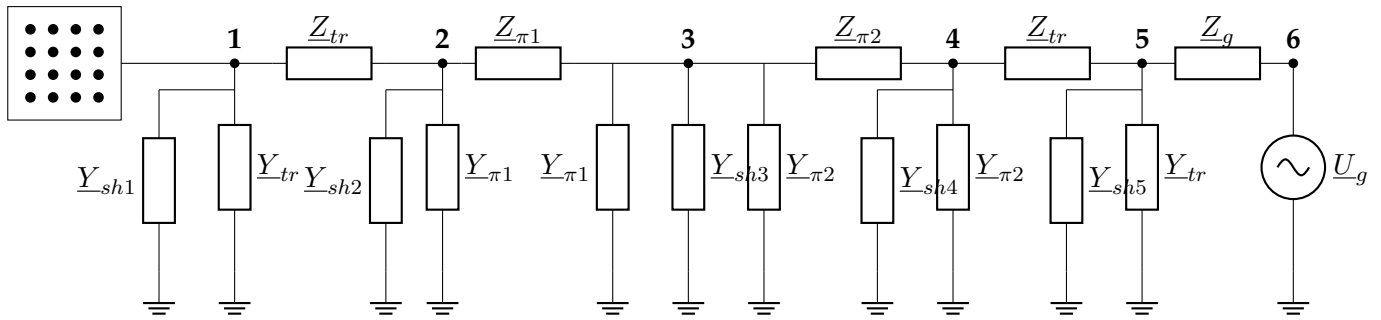


Figure 10: Transmission system model and buses

Now we choose where we want to define our buses and which type they are. We classify them as in Table 7.

Bus Number	Bus Type
1	PQ
2	PQ
3	PQ
4	PQ
5	PQ
6	Slack

Table 7: Transmission system bus types

A brief explanation of the bus types is exposed:

- **Bus 1:** It is the bus where the OWPP is injecting power therefore active and reactive power are specified as  $P_1 = p_{owf}$  and  $Q_1 = q_{owf}$ .
- **Bus 2, 3, 4, 5:** Those buses are plain PQ load buses, where  $P_i = 0, Q_i = 0$ . We want to consider them as we will be interested in the voltage levels in the lines in order to compute possible over and undervoltages in the cables, as we will see in Chapter 5.
- **Bus 6:** We will use this one as the slack bus. Note that this bus power flow solution for  $P_6$  will allow us to compute the losses in the system and will be essential part of the optimization problem. Also, since we define it as the slack bus, it will be the reference for the voltage angles and in charge of balancing the power in the transmission system.

#### 4.4 Admittance matrix

The admittance matrix encodes all the information of the elements of the grid and the relationship between them and allows us to perform the steady-state analysis of the grid. Now we can build the full HVAC transmission system model and find its admittance matrix  $\underline{Y}_{bus}$ , which will be essential for the power flow solver.

Inspecting the Kirchhoff's Current Law (KCL) we can observe how the admittance matrix is the fundamental relationship between voltages and currents :

$$\mathbf{I} = \mathbf{Y}_{bus} \mathbf{V} \quad (20)$$

where  $\mathbf{I}$  is the vector of injected currents,  $\mathbf{Y}_{bus}$  is the admittance matrix and  $\mathbf{V}$  is the vector of bus voltages.

Since we will be using the per unit system:

$$y_i = \frac{Y_i}{Y_{base}} \quad (21)$$

Note that when will build  $\mathbf{Y}_{bus}$  , we take the series impedances in 10, compute their inverse and add the subindex  $s$  to identify them. For example:

$$\underline{y}_{\pi 1s} = \frac{1}{\underline{z}_{\pi 1}} \quad (22)$$

$$\mathbf{Y}_{bus} = \begin{bmatrix} (\underline{y}_{tr} + \underline{y}_{trs} + \underline{y}_{sh1}) & -\underline{y}_{trs} & 0 & 0 & 0 & 0 \\ -\underline{y}_{trs} & (\underline{y}_{\pi 1} + \underline{y}_{\pi 1s} + \underline{y}_{sh2} + \underline{y}_{trs}) & -\underline{y}_{\pi 1s} & 0 & 0 & 0 \\ 0 & -\underline{y}_{\pi 1s} & (2\underline{y}_{\pi 1} + 2\underline{y}_{\pi 1s} + \underline{y}_{sh3}) & -\underline{y}_{\pi 2s} & 0 & 0 \\ 0 & 0 & -\underline{y}_{\pi 2s} & (\underline{y}_{\pi 2} + \underline{y}_{\pi 2s} + \underline{y}_{sh4} + \underline{y}_{trs}) & -\underline{y}_{trs} & 0 \\ 0 & 0 & 0 & -\underline{y}_{trs} & (\underline{y}_{tr} + \underline{y}_{tr} + \underline{y}_{sh5} + \underline{y}_g) & -\underline{y}_g \\ 0 & 0 & 0 & 0 & -\underline{y}_g & \underline{y}_g \end{bmatrix} \quad (23)$$

Some properties of the admittance matrix are:

- It is a square matrix of size  $n \times n$ , where  $n$  is the number of buses in the system.
- It is symmetric.
- The diagonal elements,  $Y_{ii}$ , are self-admittance, equal to the sum of the admittances of elements connected to bus  $i$ .
- $Y_{ij}$  is the negative of the admittance between buses  $i$  and  $j$
- For our system, the sparsity is  $\frac{20}{36} = 56\%$ . Nevertheless, for large networks, the matrix es very sparse. The level of sparsity, which is the percentage of zero elements in a matrix, increases with the size of the network. For instance, in a 1000-bus system, the matrix approximately is 99% sparse You can take advantage (and it will be essential to do it for very big networks!) of this sparsity using computational techniques [5].

## 4.5 Grid parameters

We will use the following set of grid elements parameters for the transformers and the main grid:

Parameter	Value
$SCR$	5
$\frac{X_g}{R_g}$	10
$P_{Cu}^{loss}$	60 kW
$P_{Fe}^{loss}$	40 kW
$u_k$	18 %
$i_o$	1.2 %

Table 8: Grid parameters [2]

## 4.6 Power flow

### 4.6.1 Power flow equations

We first going to derive an expression for the PF equations. From 20 we can get the injected current for the  $i$ -th component:

$$I_i = \sum_{j=1}^n Y_{ik} V_k \quad i = 1, 2, \dots, n \quad (24)$$

Now we can compute the  $i$ -th bus power:

$$S_i = V_i I_i^* = V_i \sum_{k=1}^n Y_{ik}^* V_k^* \quad i = 1, 2, \dots, n \quad (25)$$

Now if fe let  $V_i = |V_i|e^{j\theta_i}$ , and  $Y_{ik} = G_{ik} + jB_{ik}$ , we can write the power at bus  $i$  as:

$$S_i = \sum_{k=1}^n |V_i||V_k|e^{j\theta_i}(G_{ik} - jB_{ik}) = \sum_{k=1}^n |V_i||V_k|(\cos(\theta_{ik}) + j\sin(\theta_{ik}))(G_{ik} - jB_{ik}) \quad i = 1, 2, \dots, n \quad (26)$$

Note that the real part of the admittance,  $G_{ik}$ , is the conductance and the imaginary part,  $B_{ik}$ , is the susceptance. Now if we separate the real and imaginary parts of the power we get:

$$P_i = \sum_{k=1}^n |V_i||V_k|(G_{ik}\cos(\theta_{ik}) + B_{ik}\sin(\theta_{ik})) \quad i = 1, 2, \dots, n \quad (27)$$

$$Q_i = \sum_{k=1}^n |V_i||V_k|(G_{ik}\sin(\theta_{ik}) - B_{ik}\cos(\theta_{ik})) \quad i = 1, 2, \dots, n \quad (28)$$

#### 4.6.2 Newton-Raphson Solver

**Problem Solvability** First, we have to make sure that our set of equations is solvable. First, if we strip away the *slack*-bus, we have 5 *PQ*-buses remaining. Each *PQ*-bus introduces 2 unknowns,  $|V_i|$  and  $\theta_i$ , and 2 equations,  $P_i$  and  $Q_i$ . This means that we have 10 equations and 10 unknowns. After solving the system, from 27 and 28 we can solve for the power injections of the slack bus,  $P_6$  and  $Q_6$ . Hence, the condition for solvability is fulfilled.

To solve a set of non-linear equations, numerical methods are the standalone way to go. For the PF, one of the most used methods is the Newton-Raphson (N-R) method [6]. A brief description of this method will be presented for the  $n$ -dimensional case, but further description can be found in [4].

We will describe the process considering an  $n$ -bus system with all *PQ* buses except one slack bus. Note that in our specific case these conditions are met and  $n = 6$ . Once we strip away the slack bus, it remains to find the unknowns in the right side of equations 27 and 28. Therefore it will be convenient to define the following vectors of unknowns of *PQ*-buses:

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{n-1} \end{bmatrix} \quad |\mathbf{V}| = \begin{bmatrix} |V_1| \\ |V_2| \\ \vdots \\ |V_{n-1}| \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \theta \\ |\mathbf{V}| \end{bmatrix} \quad (29)$$

Now the dependency between the power flow equations and the vector of unknowns  $\mathbf{x}$  can be easily shown:

$$P_i = P_i(\mathbf{x}) \quad i = 1, 2, \dots, n-1 \quad (30a)$$

$$Q_i = Q_i(\mathbf{x}) \quad i = 1, 2, \dots, n-1 \quad (30b)$$

The N-R method is an iterative method that starts with an initial guess of the unknowns,  $\mathbf{x}_0$ , and then iterates the vector  $\mathbf{x}$  until the right side matches the left side of the equations. Therefore we can define the power mismatch vector  $\mathbf{f}(\mathbf{x})$  as:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} P_1(\mathbf{x}) - P_1 \\ \vdots \\ P_{n-1}(\mathbf{x}) - P_{n-1} \\ Q_1(\mathbf{x}) - Q_1 \\ \vdots \\ Q_{n-1}(\mathbf{x}) - Q_{n-1} \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}) \\ \Delta \mathbf{Q}(\mathbf{x}) \end{bmatrix} = \mathbf{0} \quad (31)$$

Now we have to consider  $\mathbf{J}$ , the Jacobian matrix of  $\mathbf{f}(\mathbf{x})$ . For clarity, is convinient to parition  $\mathbf{J}$  as:

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_{11} & \mathbf{J}_{12} \\ \mathbf{J}_{21} & \mathbf{J}_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial \Delta \mathbf{P}}{\partial \theta} & \frac{\partial \Delta \mathbf{P}}{\partial |\mathbf{V}|} \\ \frac{\partial \Delta \mathbf{Q}}{\partial \theta} & \frac{\partial \Delta \mathbf{Q}}{\partial |\mathbf{V}|} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \theta} & \frac{\partial \mathbf{P}}{\partial |\mathbf{V}|} \\ \frac{\partial \mathbf{Q}}{\partial \theta} & \frac{\partial \mathbf{Q}}{\partial |\mathbf{V}|} \end{bmatrix} \quad (32)$$

if interest check derivation of the partial derivatives of the power flow equations with respect to the unknowns check the [Appendix](#).

Now it just remain to find the step  $\Delta \mathbf{x}^v$  to update  $\mathbf{x}$  solving the following linear system by Gauss elimination:

$$\mathbf{J}^k \Delta \mathbf{x}^k = -\mathbf{f}(\mathbf{x}^k) \quad (33)$$

Now we update  $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta \mathbf{x}^k$  and iterate until it converges. The convergence criteria is usually set as the maximum of the absolute value of the power mismatch vector,  $\max(\mathbf{f}(\mathbf{x}))$ , is less than a tolerance,  $tol$ , which we have set to  $10^{-6}$ .

The N-R solver has been implemented in Python(check the [Appendix](#)) and the procedure can be found in Algorithm 1.

---

#### Algorithm 1 Newton-Raphson Method

---

```

procedure RUN_PF
  Initialize  $V$  with ones and  $\theta$  with zeros
  Set  $V[slack]$  to 1,  $\theta[slack]$  to 0
  Set  $P, Q$  based on OWPP data
  Set  $iter = 0, tol$  and  $k = 0$ 
  while  $iter < max\_iter$  and  $\Delta PQ > tol$  do
    Compute  $P\_present, Q\_present$  using  $V, \theta, Y_{bus}$ 
    Compute mismatch  $\Delta PQ = [dP, dQ]$  as difference between calculated and given  $P, Q$ 
    Compute the Jacobian matrix  $J$ .
    Solve the linear system  $J \cdot \Delta x_k = -\Delta PQ$ .
    Compute the updated  $x^{k+1} = x^k + \Delta x^k$ .
    Update  $V, \theta$  using  $dP, dQ, Y_{bus}$ 
    if  $\max(\Delta PQ) < tol$  then
      break
    end if
  end while
  return  $V, \theta$ 
  Compute  $P_{slack}, Q_{slack}$ 
end procedure

```

---

Finally note that close to the solution vector  $\mathbf{x}^*$  it can be proven that N-R normally presents quadratic convergence [7]. Moreover, in terms of computational time, Alogorithm 1 takes about 0.06 s in average to solve the PF.

Now we can observe that the PF is essential for computing the power losses (see 16) in the system, which will be in the objective function of the optimization problem. It is clear to see that  $P_{losses}$  will be the difference between the injected active power from the OWPP and the active power delivered to the slack bus, which is the main grid:

$$P_{losses} = (P_{owf} - P_6) \cdot S_{base} \cdot 10^{-6} \quad \text{MW} \quad (34)$$



## 5 Optimization problem formulation

Generally an optimization problem that involves equality and inequality constraints can be formulated as:

$$\begin{aligned} \min \quad & \mathbf{f}(\mathbf{x}) = (f_1(x), f_2(x), \dots, f_k(x)) \\ \text{subject to} \quad & \mathbf{H}(\mathbf{x}) = 0 \\ & \mathbf{G}(\mathbf{x}) \leq 0 \end{aligned} \quad (35)$$

where  $\mathbf{x} \in \mathbb{R}^n$  is the  $n$  decision variables vector,  $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^k$  is set of objective functions to be minimized. Note we have presented the problem in its more general form, where the objective function can be a set of  $n$ -objectives to have to be optimized simultaneously and will be proved to be useful in 5.2.  $\mathbf{H}(\mathbf{x}) = \mathbf{0}$  are the equality constraints and  $\mathbf{G}(\mathbf{x}) \leq \mathbf{0}$  are the inequality constraints.

Now we have to decide which decision variables we include in the optimization problem. It is easy to see that we can include all the design variables in which the transmission system model depends, which are all included in the cost function and therefore have a direct impact in the cost of the system. By checking the parameters on which the admittance matrix depends, which can be found in ??, we can define the vector of unknowns as:

$$\mathbf{x} = [vol_{tr}, n_{cables}, react_{bi1}, \dots, react_{bi5}, react_{cont1}, \dots, react_{cont5}, react_{bi5}, S_{trafo}] \quad (36)$$

where:

- $vol_{tr}$ : Is the voltage level of the transmission cables. Note that this will be an integer variable, where you can choose one voltage level given by manufacturers with its associated parameters and cost. Check table 3 to see available options considered.
- $n_{cables}$ : Is an integer number that represents the number of cables placed in parallel.
- $react_{bi_i}$   $i = 1, \dots, 5$ : Is a binary variable (i.e. either 0 or 1) that tells if we place or not a reactor at position  $i$ .
- $react_{cont_i}$   $i = 1, \dots, 5$ : Sizing of reactor at position  $i$ , therefore its  $Y_{shi}$  in p.u.
- $S_{trafo}$ : Rated power of the transformer in VA.

A very important remark with respect to the type of variables we have involves in the problem. Considering  $\mathbf{x}$  we see that we have binary, integer and continuous variables. This kind of problems are labeled as mixed-integer optimization and constraints the optimization methods we can use as we will see in 6.

## 5.1 Constraints

### 5.1.1 Equality: Power Flow

The equality constraints  $\mathbf{H}(\mathbf{x}) = 0$  in the optimization problem in 35 are the imposed by the power flow equations. In fact, the power mismatch vector 31 is the one that take this form. therefore:

$$\mathbf{H}(\mathbf{x}) = \begin{bmatrix} \Delta \mathbf{P}(\mathbf{x}) \\ \Delta \mathbf{Q}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} P_1(\mathbf{x}) - P_1 \\ \vdots \\ P_{n-1}(\mathbf{x}) - P_{n-1} \\ Q_1(\mathbf{x}) - Q_1 \\ \vdots \\ Q_{n-1}(\mathbf{x}) - Q_{n-1} \end{bmatrix} = \mathbf{0} \quad (37)$$

### 5.1.2 Inequality: Technical constraints

The operational constraints are the ones that ensure the system operates within the limits of the equipment. This constraints of inequaility nature, setting upper and lower bounds for certain variables. This constraints will be the the inequality constraints  $\mathbf{G}(\mathbf{x}) \leq 0$  in the optimization problem in 35. We will denote as  $\mathbf{L}_i^{\mathbf{X}}$  each different limit at bus  $i$  and for the sake of coherence we will express the constraints in the  $\mathbf{G}(\mathbf{x}) \leq 0$  form.

Note that all the following limits can be based on grid code requirements, but we will use some standard values.

#### Bus voltage limits

We set the nodal bus- $i$  voltage,  $V_i$ , limits as:

$$\begin{aligned} L_i^{Vu} &= V_i - V_{max} \leq 0, \quad i \in N \quad \text{is the upper limit} \\ L_i^{Vl} &= V_{min} - V_i \leq 0, \quad i \in N \quad \text{is the lower limit} \end{aligned} \quad (38)$$

where  $V_{max}$  and  $V_{min}$  are the maximum and minimum voltages. These values have been taken to be  $\pm 0.1$  p.u. of the nominal voltage respectively.

#### Maximum current limits for the lines

The current in the lines should not exceed the maximum current that the cables can handle. We will set  $I_{max} = 1.1$  p.u. of the rated current of the cable  $I_{rated}$ . This rated current is given by manufacturers [10]. Therefore the constraint will be:

$$L_i^I = I_i - I_{max} \leq 0, \quad i \in N_{lines} \quad (39)$$

### Reactive power delivered to the grid

In general reactive power delivered to the grid,  $Q_6$  can be defined by grid code requirements. Nevertheless, in this case we will aim to keep it as close as possible to zero. Therefore we will set the limits as:

$$\begin{aligned} L_6^{Qu} &= Q_6 - Q_{max} \leq 0 \\ L_6^{Ql} &= Q_{min} - Q_6 \leq 0 \end{aligned} \quad (40)$$

where  $Q_{max} = Q_{min} = 0$ . Also note that potentially the wind turbines could be used to absorb reactive power, but we will not consider this in this case.

## 5.2 Objective function: multiobjective

Probably one of the critical parts is to identify the objective function we want to minimize and decide how we treat it. Now, we will describe the terms that we will include in the objective function and why we have decided to work with a multiobjective approach.

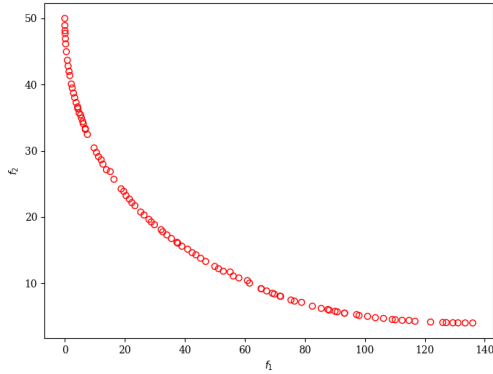
### 5.2.1 Why bi-objective?

Let us imagine you are an engineer of a firm that is planning to build an OWPP. You have to ensure both that the transmission system is reliable and robust in terms of operational constraints but also to make the investment as profitable as possible, therefore you want to minimize its investment cost. Given this scenario, you can expect that it exists a tradeoff between the two objectives, investing more money can make your transmission system more robust and yield smaller power losses and viceversa. In fact, in eRoots conversations with Acciona, they have shown interest on getting insights on how this two objectives interact with them and decide which option to go for given the relationship.

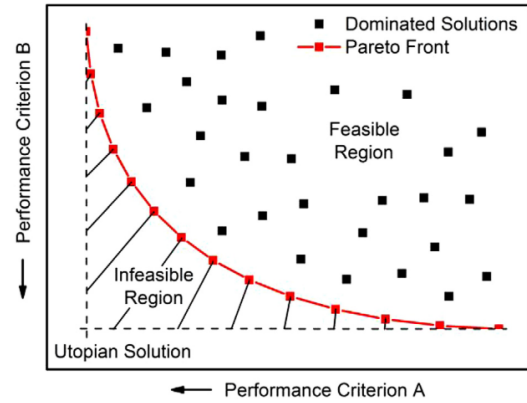
Literature can offer solutions for this kind of scenarios. Multi-objective optimization, also known as Pareto optimization, deals with situations where optimal decision making needs to be taken in presence of various conflicting objectives. Within this context, there is no guarantee that there exist a unique solution that minimizes both objectives. The main idea is to find the set of solutions that are not dominated by any other solution in the set; that is solution where no objective can be further minimized without worsening another. In this sense, any point found in this set can be considered optimal and be regarded as a potential solution. This set is called the Pareto Frontier and the solutions are called Pareto optimal solutions.

Using the formulation in 35, let  $\mathbf{x}^* \in \mathbf{X}$ , where  $\mathbf{X}$  is the feasible set of solutions that satisfy the constraints, have its associated outcome vector  $\mathbf{f}(\mathbf{x}^*)$ . Then a feasible solution  $\mathbf{x}_1$  is said to (Pareto) dominate another solution  $\mathbf{x}_2$  if:

$$\begin{aligned} \forall i \in \{1, 2, \dots, k\}, f_i(\mathbf{x}_1) &\leq f_i(\mathbf{x}_2) \quad (\text{Condition 1}) \\ \exists i \in \{1, 2, \dots, k\}, f_i(\mathbf{x}_1) &< f_i(\mathbf{x}_2) \quad (\text{Condition 2}) \end{aligned} \quad (41)$$



(a) Pareto Front for Binh and Korn function



(b) Explanation of domination [15]

In Fig. 11a you can see the Pareto Front of the Binh and Korn function [14], a usual test function for multiobjective optimization.

Now we can go back to our problem and decide how to define both objectives. Therefore we define the objective function as:

$$\mathbf{f}(\mathbf{x}) = (f_{invest}(\mathbf{x}), f_{tech}(\mathbf{x})) \quad (42)$$

where the definition of  $f_{invest}(\mathbf{x})$  is quite straight-forward; the addition of the cost of the elements in the transmission system:

$$f_{invest}(\mathbf{x}) = C_{cables}(\mathbf{x}) + C_{tr}(\mathbf{x}) + C_{sh}(\mathbf{x}) + C_{gis-AC}(\mathbf{x}) + C_{ss-AC}(\mathbf{x}) \quad (43)$$

When it comes to how we handle  $f_{tech}(\mathbf{x})$  it becomes a little more tricky. We wanted this part to capture how optimal is the transmission system, in terms of steady-state operational conditions and power losses. In other words, this term has to measure if the inequality constraints, the ones related to operational limits, are being satisfied and try to minimize  $C_{loss-AC}$ . To do so we will use a penalty method to deal with the inequality constraints.

Exhaustive development of this method can be found in [16], but the main idea is to expand the original feasible set of solutions,  $\mathbf{X}$ , to all of  $\mathbb{R}^n$ , but a large cost or "penalty" is added to the objective function when the solution is not satisfying  $\mathbf{G}(\mathbf{x}) \leq 0$ . Therefore:

$$f_{tech}(\mathbf{x}) = C_{loss-AC}(\mathbf{x}) + c \cdot \mathbf{p}(\mathbf{x}) \quad (44)$$

where  $c$  is the penalty factor and  $\mathbf{p}(\mathbf{x})$  is the penalty function:

$$\mathbf{p}(\mathbf{x}) = \sum_{L_i^X \in G(x)} \max(0, \mathbf{L}_i(\mathbf{x})) \quad (45)$$

In practice, setting a big value for  $c$  will make the optimization method to look for solutions that satisfy the constraints.

### 5.3 Algorithm overview

Now we have setup all the elements that define the sequential process for finding the desired Pareto Front. Now it just remains to find the recursive optimization method that suits best our problem. Note that here we advance that we will use the genetic algorithm NSGA-II to solve the optimization problem (see section 6.2). Figure 12 shows the block diagram of the solver implemented in Python (check the [Appendix](#)).

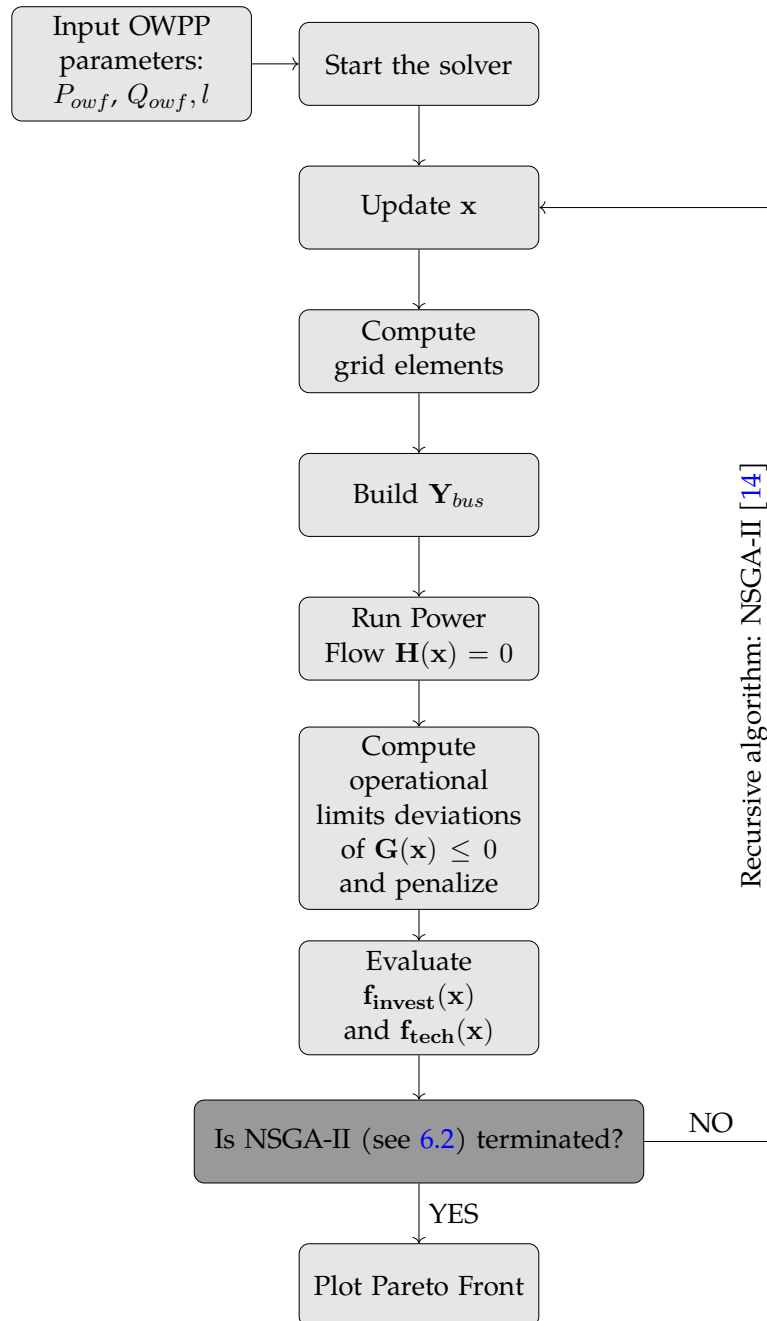


Figure 12: Block diagram of the optimization algorithm

## 6 Optimization methods

After what we have summarized in the previous section we want an optimization method that:

- Can handle mixed-integer optimization problems, meaning that it can handle binary, integer and continuous variables as decision vector  $\mathbf{x}$ .
- Can handle bi-objective optimization problem with equality and inequality constraints and can deal with a penalty function.
- Can find the whole Pareto front to analyse the tradeoff between the two objectives, investment and technical.

### 6.1 State of the art limitations: interior point method

In [2] they propose an iterative method for solving the reactive power compensation problem. This methodology solves the optimization for continuous variables using an interior point method solver [17] for every possible combination of integer variables. Then, they compare the total cost for all options and pick the one with smaller cost. The algorithm is summarized in 13:

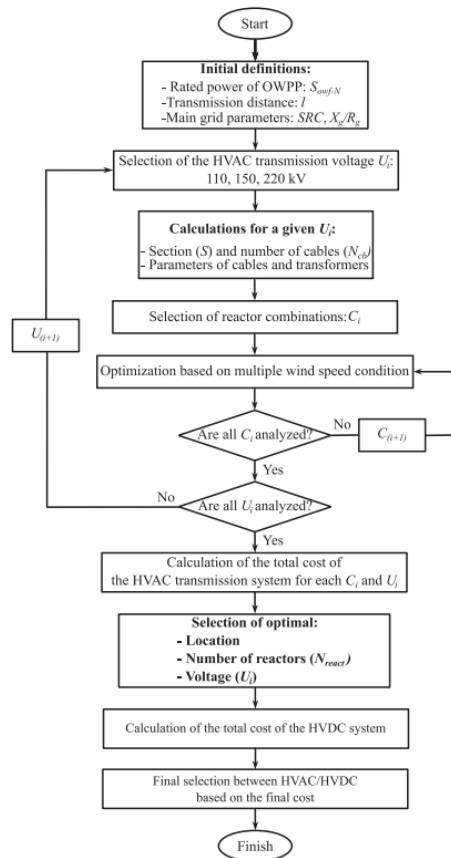


Figure 13: State-of-the-art algorithm for reactive power compensation optimization [2]

This method is not suitable for our case, as we want to find the whole Pareto Front and not just one optimal solution. Moreover, the number of possible combinations of integer variables can increase quickly in further expansions of the problem (for instance if you also consider the electrical collection system between turbines in the OWPP), making the method computationally expensive.

Another traditional method to solve multiobjective problems is to basically transform the problem into a single objective problem by using a weighted sum of the objectives [11]:

$$f(\mathbf{x}) = \sum_{i=1}^k w_i \cdot f_i(\mathbf{x}) \quad \text{where} \quad \mathbf{x} \in \mathbf{X} \text{ and } \sum_{i=1}^k w_i = 1 \quad (46)$$

From 46 is clear that modifying the weights  $w_i$  we change the relative importance we give to each objective. The main drawback of this approach, even though that we can deal with more than one objectives at the same time, we still get just one solution for each combinations of weights, not a set of optimal solutions, i.e. the Pareto Front. Moreover, how do we choose this  $w_i$ ? Optimal point is sensible to this weights, therefore you must have a clear understanding of you priorities between the objectives which is something we do not want to impose in our approach.

## 6.2 NSGA-II: Genetic algorithm

This is where genetic algorithms (GA's) come into play. GA's are a class of optimization algorithms that are inspired by the process of natural selection that uses the principles of evolution to solve optimization problems. The main idea is that they start from a random population of candidate solutions and, in one single simulation run, they are able to find multiple Pareto optimal solutions.

That is why we will use the NSGA-II algorithm to solve our problem. It accomplishes the requirements in 6 and it has a time complexity of  $\mathcal{O}(MN^2)$ , where  $M$  is the number of objectives and  $N$  the population size. Moreover, it allows us to treat inequality constraints with penalty functions, as proposed in 5.2.1.

An exhaustive theoretical development of how NSGA-II works is out of the scope of this work, but a complete and formal description of the algorithm can be found in [12]. Just for the sake of completeness, one can simplify how NSGA-II works in the steps on Figure 14:



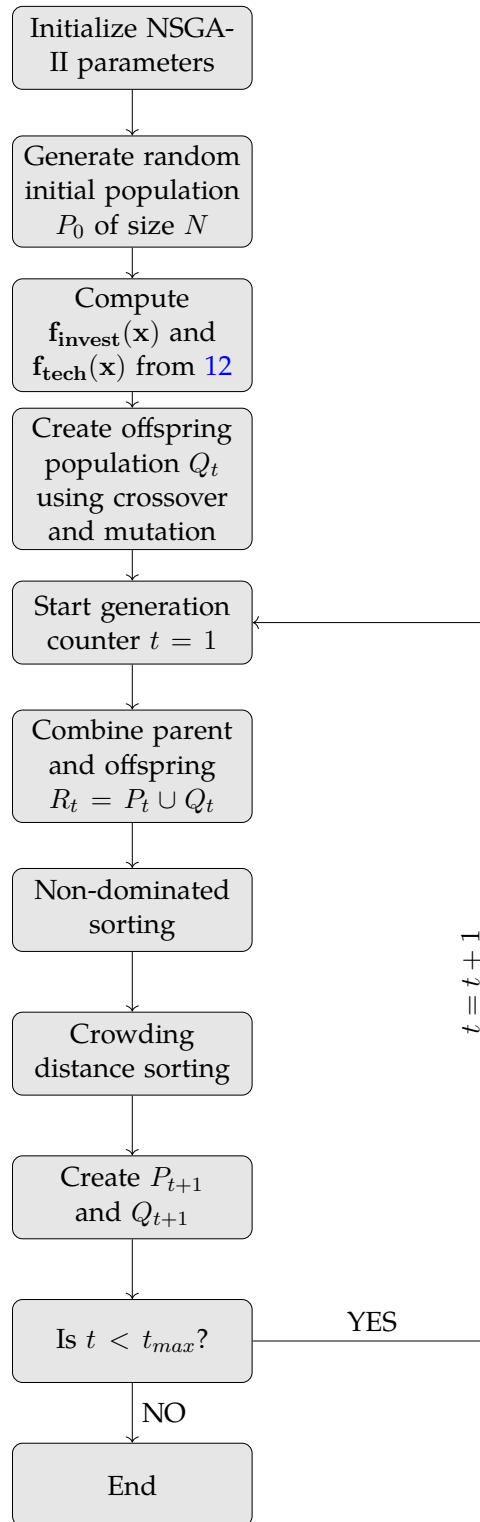


Figure 14: Block diagram of NSGA-II

The algorithm 12 has been implemented in pymoo [13], a Python library for multiobjective optimization that includes the NSGA-II algorithm 14 and allows to formulate mixed-variable problems. It is worth mentioning that the proposed method 12 is performed on a whole set of candidate solutions  $\mathbf{x}$

from the population set  $P$ .

### 6.3 Optimal Power Flow approach for compensation sizing

In this section we propose a different approach based on the Optimal Power Flow (OPF) to validate the results obtained with NSGA-II, specifically the ones relates to the sizing of the reactive power compensation.

The idea is to take one optimal solution from NSGA-II, i.e. all the decision variables in  $\mathbf{x}$ , and recompute just the values of sizing of the shunt reactors  $Y_{sh_i}$ . The main idea is to compare how close is the stochastic approach solutions of NSGA-II to the solutions given by a standard deterministic approach as the OPF.

The traditional AC-OPF objective function [18] minimizes the cost of generating active power:

$$\min_{P_G} \mathbf{c}^T \mathbf{P}_G \quad \text{where} \quad \mathbf{c} = \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} \quad (47)$$

where  $\mathbf{P}_G$  is the vector of active power generated by the generators and  $c_0$ ,  $c_1$  and  $c_2$  are the independent, linear and quadratic power generations costs respectively.

As we saw in 13, shunt reactors compensate a reactive power equivalent to  $Q_{sh} = Y_{sh} \cdot U^2$ . Therefore, finding the optimal size of the shunt reactors is equivalent to minimize the cost of the reactive power compensation. Therefore we can modify the objective function of the OPF to:

$$\min_{Q_{sh}} \mathbf{c}_{sh}^T \mathbf{Q}_{sh} \quad \text{where} \quad \mathbf{c}_{sh} = \begin{bmatrix} K \\ P \\ 0 \end{bmatrix} \quad (48)$$

where  $K$  and  $P$  are from 4.

We have made this modifications implemented in the AC-OPF solver in Grical [19], an open-source Python library for power systems analysis. Note that the derivatives of the objective functions, Jacobian and Hessian, have been modified according to the new objective function 48.

## 7 Case study

In this section we will study in depth the results that the algorithm yields for a specific design parameters of an OWPP. We will benchmark the results obtained with NSGA-II with a "brute" force approach using random search and compare the results of reactors sizing comparing with the OPF approach. Moreover, we will do an in-depth analysis of the optimal solution cost breakdown and compare it to optimal points without considering reactive power compensation to understand the need to include this in the optimization problem.

The results obtained have been computed in a laptop with the following specifications:

- Processor: Processor: Intel(R) Core(TM) i5-8365U CPU @ 1.60GHz
- RAM: 8 GB
- OS: Windows 11
- IDE: Visual Studio Code and Pycharm
- Python version: 3.11.8
- Pymoo version: 0.6.1
- GridCal version: 5.1.11

The tuning parameters used for the NSGA-II algorithm are shown in Table 9.

Parameter	Value
Population size	150
Survival sorting	Rank and Crowding " <i>pcd</i> "
Termination	15 generations
" <i>c</i> ", constraint penalty	10e4

Table 9: Tuning parameters for the NSGA-II

### 7.1 500 MW, 100 km OWPP

We pick as case study a 500 MW OWPP located 100 km from shore. Results have been separated in different subsections to study specific results.

### 7.1.1 Random search compared to NSGA-II

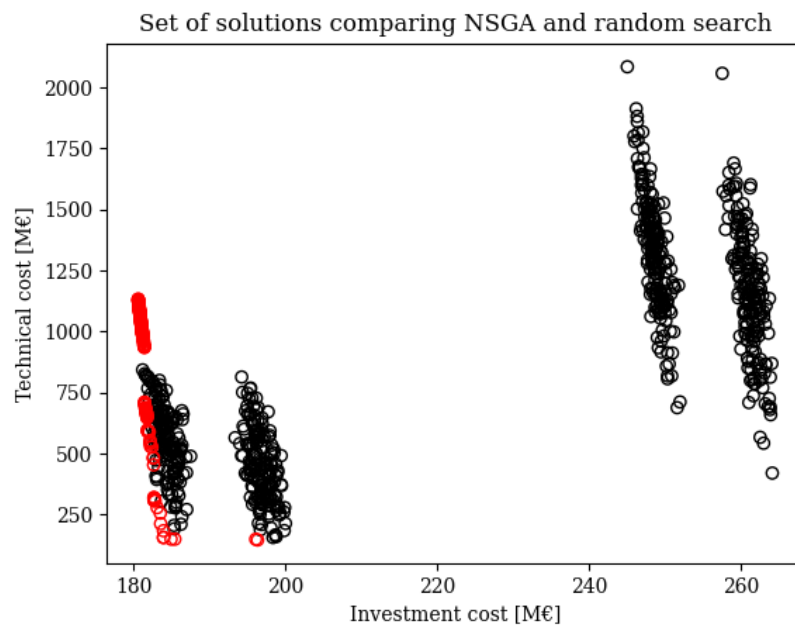


Figure 15: Random search (black) compared to NSGA-II (red)

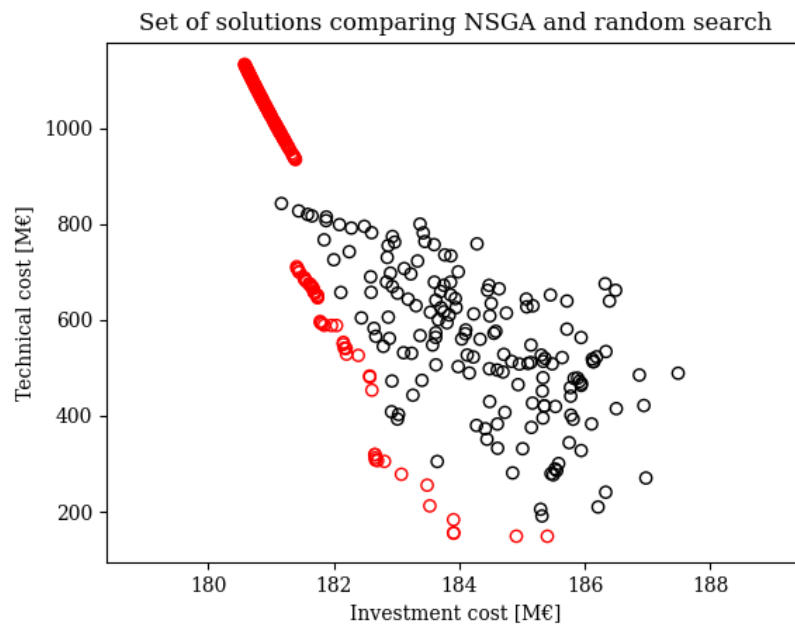


Figure 16: Zoom on the Pareto Front area

### 7.1.2 OPF validation

Now we showcase how the OPF can help to validate the results obtained with NSGA-II. We pick one point from the Pareto Set and take its solution for all the variables in  $\mathbf{x}$  except for the continuous variables of the sizing of the reactors,  $Y_{sh_i}$ . We run the OPF proposed in 48 and compare the results.

First, we check the convergence and running time of the OPF:

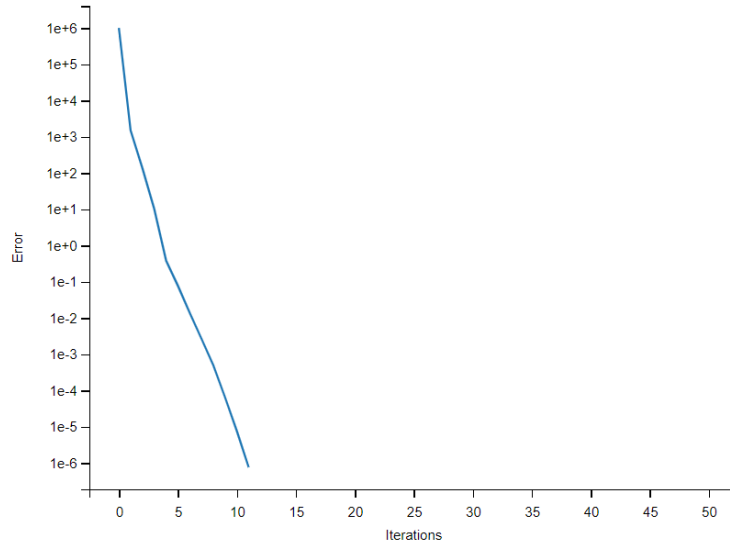


Figure 17: OPF convergence

Figure 17 shows how the OPF converges to the optimal solution in about 12 iterations and it takes about 1.5 seconds. For comparison, running the traditional PF in 4.6.2 takes around 0.06 s. That makes OPF around 25 times slower than the traditional PF.

To do so, from the Pareto Front, we take a random point, chosen to be the one of minimizing both objectives with equal weights. The vector of solutions we get is:

Parameter	Value
Transmission voltage level	220 kV
Number of cables	2
Reactors	[1, 1, 0, 1, 0]
Rated power of the transformer	509.72 MVA

Table 10: Specifications (except reactors sizing) of the optimal solution

Now we pass these values to the OPF and see how the new values of the compensation affect the objective function:

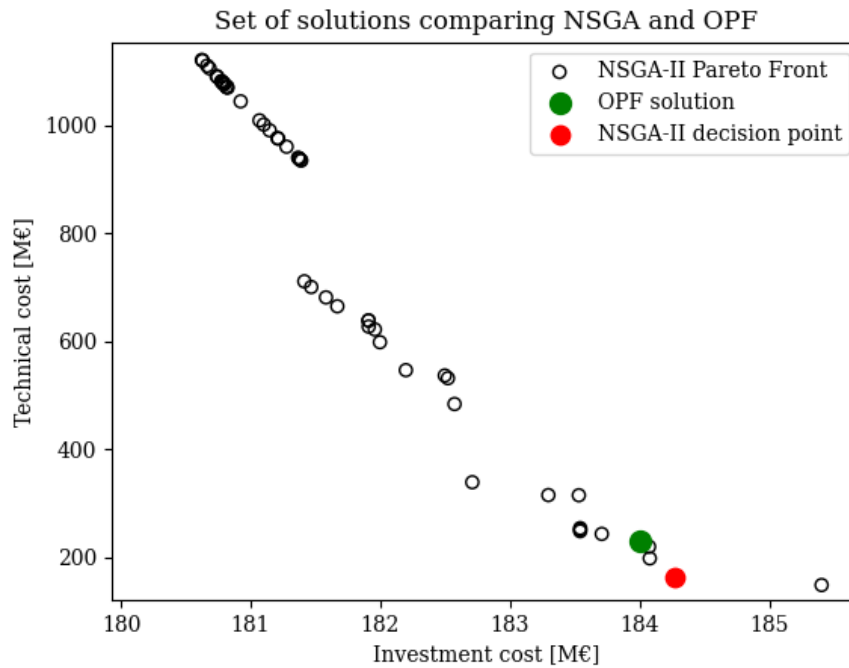


Figure 18: NSGA-II vs OPF for reactors sizing

As we see in Fig. 18, the OPF gives a solution that is very close to the one obtained with NSGA-II. This validates the results obtained with NSGA-II, since OPF, which is a strictly traditional optimization method, does fall on the same Pareto Set, meaning that it has not found a combination of reactor sizing that Pareto dominates the solutions from NSGA-II.

7.1.3 Cost breakdown

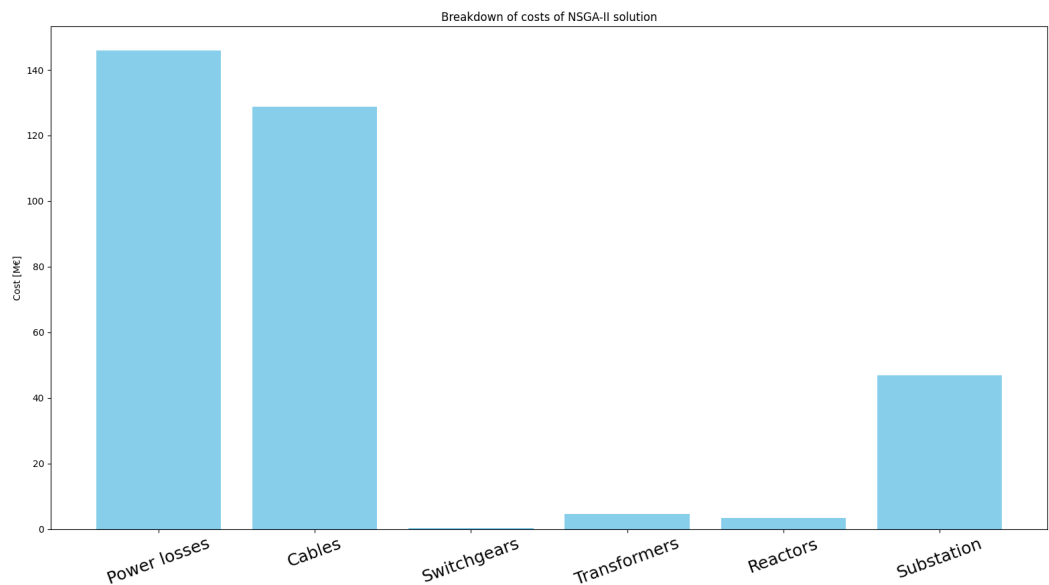


Figure 19: NSGAI optimal solution cost breakdown

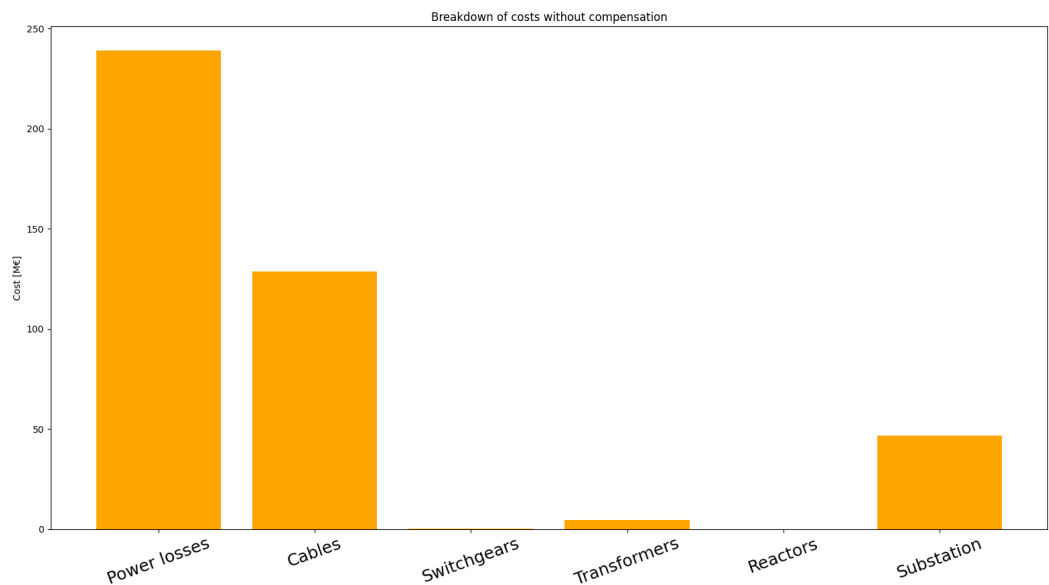


Figure 20: No compensation solution cost breakdown

Now we can compare the cumulative total costs of NSGA-II optimal solution with the one of no compensation:

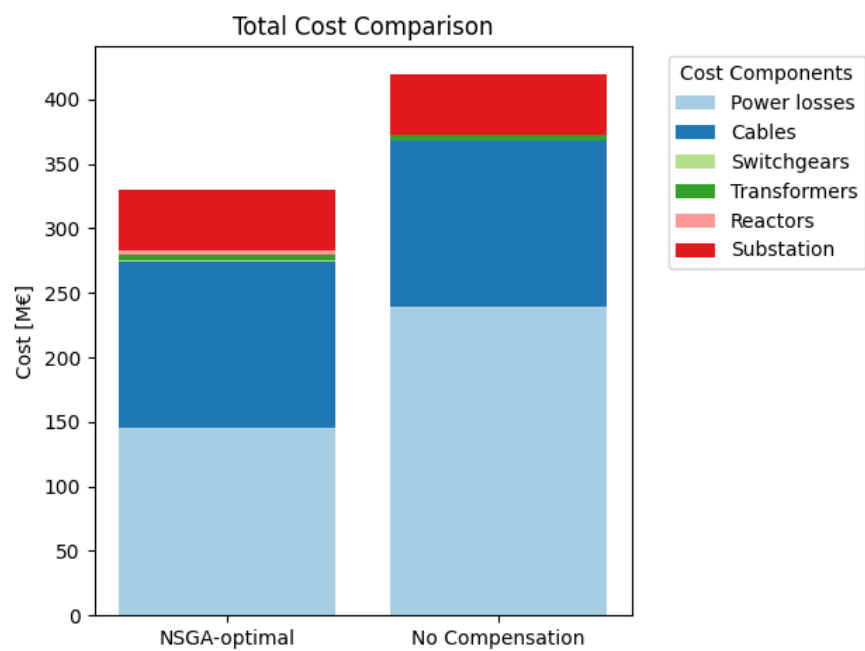


Figure 21: NSGA-II optimal solution and no-compensation total cost comparison



## 8 Conclusions

All the objectives proposed at the beginning at 2.3 have been accomplished. This chapter summarizes the main conclusions of the work and presents some proposals to take into consideration for future related work.

### 8.1 Outcome

The main outcomes derived from the objectives met from each chapter of interest are:

- Chapter 5 covers the mathematical foundations of the problem formulation and optimization approach. The analysis shows how the multiobjective approach helps us achieve the desire to obtain the full Pareto Front and how this approximation becomes useful to understand the underlying behaviour of the power flow analysis of the transmission system model.
- Chapter 6 presents the state-of-the-art optimization algorithm and presents the different techniques that have been considered to be used. It establishes the limitations that have been found and how the applications of a genetic algorithm such as NSGA-II can help to overcome them and meet the requirements of the optimization problem. It presents some theoretical foundations of the algorithm used and the schematic of the recursive solver that has been implemented.
- Chapter 7 applies the optimization algorithm to a specific case study of an OWPP. First, the algorithm implementation has been demonstrated to be fully functional and meets the requirements in 6. The results show how we obtain a coherent Pareto Front and how the results obtained outperform any brute force approach, revealing a set of non-dominated optimal solutions. Moreover, the comparison with the OPF approach validates our results and shows that NSGA-II is able to reach the "best" points using a stochastic approach given that OPF is unable to explore any dominating area below the Pareto Front. The cost breakdown and evolution of costs for different power injections give an intuitive explanation of the need for reactive power compensation.

All in all, it can be concluded that the innovative approach of using a GA for the optimization of the transmission system of an OWPP has been successful and it has promising properties as a potential powerful tool to deal with this kind of optimization with reduced computational time.

### 8.2 Future work

The implementation of NSGA-II for the optimization of OWPP transmission system design has potential to become a powerful tool. In this section we will propose a set of possible improvements that could enhance the overall quality of the proposed method and give new insights.

The main simplification and assumption through the development of the work has been assuming a constant power supplied by the OWPP at its nominal value, in other words, we are assuming a constant wind speed at the desired value. As shown in 1 and in (), the need of reactive power com-

pensation increases as the power injections of our plant decreases, i.e. low wind or no wind. The next and logical step is to let the proposed algorithm to consider different wind conditions. To be more accurate, we should consider a wind speed distribution, normally assumed to follow a Weibull distribution shown in 22.

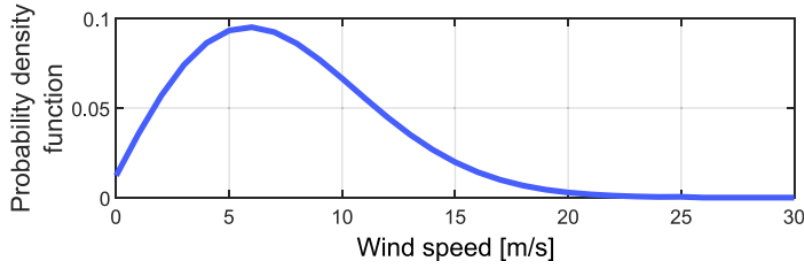


Figure 22: Weibull distribution of wind speeds [2]

To consider this distribution, we propose to create a set of  $N$  wind speeds  $u_j$ , and associate a probability  $p_j$  and power generation  $P_{owf_j}$  to each of them. Now it would just remain that the step on which we solve the power flow in 12 to compute a weighted average of the power injections of the OWPP,  $P_{owf}$  with the probabilities  $p_j$ . Doing so, the objective function will be a weighted sum of the costs of the different wind conditions:

$$\begin{aligned}
 f_{invest}(\mathbf{x}) &= \sum_{j=1}^N p_j \cdot [C_{cables}(\mathbf{x}) + C_{tr}(\mathbf{x}) + C_{sh}(\mathbf{x}) + C_{gis-AC}(\mathbf{x}) + C_{ss-AC}(\mathbf{x})] \\
 f_{tech}(\mathbf{x}) &= \sum_{j=1}^N p_j [C_{loss-AC}(\mathbf{x}) + c \cdot \mathbf{p}(\mathbf{x})]
 \end{aligned} \tag{49}$$

This approach will give a more realistic view of the system and allow to do a fair comparison between the state-of-the-art results in terms of reactive power compensation sizing for every different combination of  $P_{owf}$  and  $l$  for OWPP.

Moreover, recall that we have limit our work to the HVAC technology, but HVDC technology should also be considered as an option. Literature [2] shows that after considering shunt reactors for reactive power compensation purposes, after the break-even distance of around 150 km, using HVDC becomes cheaper than HVAC, for which power losses increase notably for high distances. Computing costs for HVDC would allow to extend the decision variables of the optimization to its full domain of possibilities.

Moreover, further work could be to consider how disturbances of the voltage of the grid we are supplying to or the layout of the electrical collection system (see Figure 23) inside the OWPP could affect the optimization problem. Also, this work could be implemented in a software like Gridcal. Implementing this algorithm in an open-source software would allow to make it accesible to a wider

audience, with a more user oriented interface and to be used in real-life projects.

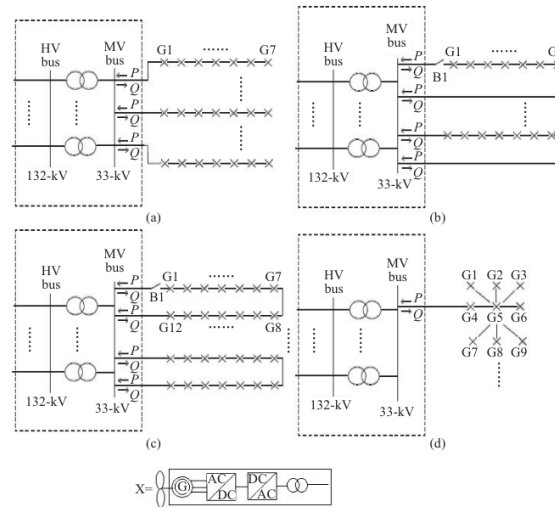


Figure 23: Collection system for OWPP possible layouts: Radial(a), Single-sided ring(b), Double sided ring(c) and star(d) [21]

Finally, one of the main criticism to this work would be the validity of the cost functions used for the objective function. Finding and approximating this functions is no easy work, since costs associated to each element vary from manufacturer to manufacturer and cost analysis can be extended not just initial investment ones, but to other concepts such as maintenance, operation and depreciation costs. Studying more complex and accurate versions of costs functions is a another potential line of work.

All in all, the work developed in this thesis provides a good theoretical and practical basis for further research in the optimization of transmission systems in OWPP. In fact, this thesis will be used as the the starting point for eRoots and Acciona to implement an operative and complete version of the algorithm in GridCal.

## 9 Planning and viability studies

### 9.1 Time Planning

Figure 24 shows the time distribution for the tasks carried out in the thesis.

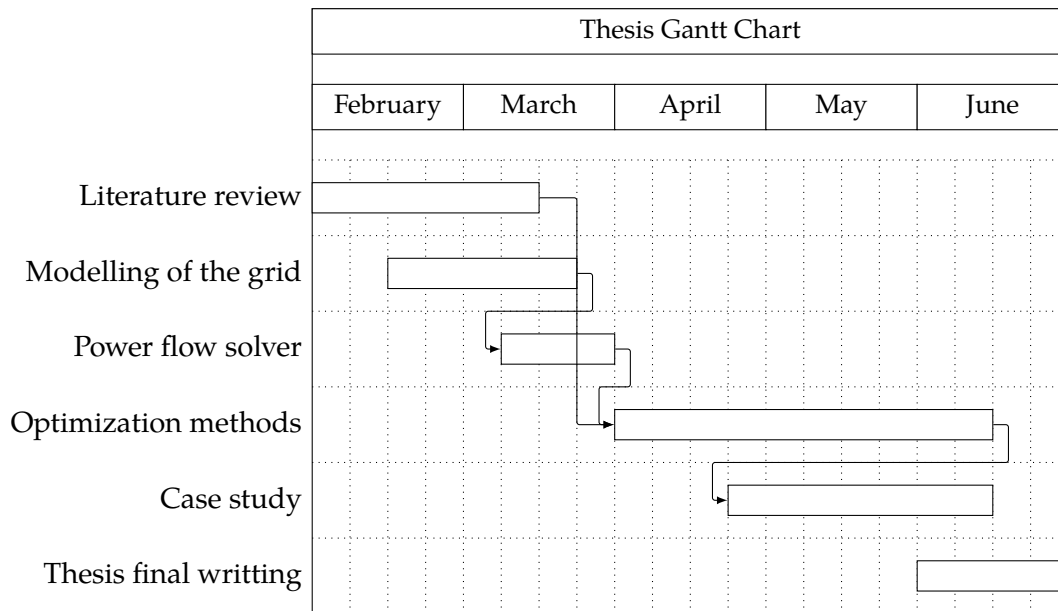


Figure 24: Thesis Gantt Chart

### 9.2 Economic assessment

The budgeting for this work includes the equipment and human resources used during the development of the thesis. The total cost has been broken down with and without the Value Added Tax (VAT). The complete budgeting is shown in Table 11.

Concept	Unit Cost	Quantity	Total (€)
Computer	1000 €	1	1000
Working hours	8 €/h	450 h	3600
Tutor supervision	25 €/h	50 h	1250
<b>Total without VAT</b>			5850
<b>Total with VAT (21%)</b>			7078.5

Table 11: Thesis Costs

### 9.3 Environmental assessment

#### 9.3.1 Energy consumption

This environmental assessment evaluates the energy consumption costs incurred during the development of a thesis over five months, using a computer as the primary tool. The primary energy

consumption arises from the computer's usage, which includes writing, research, data analysis, and communication.

Assuming an average laptop with a power consumption of 60 watts, used for approximately 6 hours daily, the total energy consumption over five months is around 54 kWh. This consumption translates to roughly 30 kg of CO<sub>2</sub> emissions, assuming the average emission factor for electricity generation.

To reduce these energy costs and associated environmental impacts in future thesis projects, several strategies can be employed. Utilizing energy-efficient computers, enabling power-saving modes, and limiting usage time can significantly lower consumption. Additionally, adopting renewable energy sources, such as solar panels for charging devices, further reduces the carbon footprint, contributing to a more sustainable academic practice.

### 9.3.2 Potential impact

Optimizing transmission systems in OWPP can significantly enhance environmental impact by maximizing energy efficiency and reducing carbon emissions. Improved transmission reduces energy losses, ensuring that more clean energy reaches the grid, thereby displacing fossil fuel-based power generation. Additionally, optimization can lead to less intrusive infrastructure, minimizing harm to marine ecosystems and reducing the physical footprint of OWPP. Efficient transmission systems also facilitate the integration of larger amounts of renewable energy, supporting the transition to a sustainable energy future and helping to combat climate change by lowering greenhouse gas emissions.

## 9.4 Social and gender equality assessment

This assessment evaluates the social and gender equality aspects of a bachelor's thesis focused on developing a tool for optimizing renewable energy system design, authored by a 22-year-old white engineering student. While the thesis itself addresses a critical area in sustainable development, examining its social dimensions is essential to ensure inclusivity and equality.

The demographic profile of the author reflects broader trends in STEM fields, where women and minority groups remain underrepresented. This lack of diversity can influence the perspectives and priorities embedded in the research. Ensuring diverse representation in such projects is crucial for incorporating a wide range of insights and addressing the needs of various communities.

To promote social and gender equality, the research should consider the differential impacts of renewable energy systems on diverse populations. This approach ensures that the developed tools and technologies are accessible and beneficial to all segments of society.

Moreover, educational institutions should encourage and support participation from diverse backgrounds in engineering and renewable energy fields. Mentorship programs, scholarships, and targeted recruitment can help bridge the gender and social gap, fostering an environment where innovative solutions for renewable energy are developed through diverse and inclusive contributions.

## Acknowledgements

Primer de tot vull agrair profundament el suport constant del Josep, el meu tutor, que m'ha donat consell i ajuda quan l'he necessitada cada dia des de que vaig entrar a eRoots. Per extensió, agrair la rebuda i suport de tot l'equip d'eRoots durant aquests mesos, especialment al meu tutor acadèmic, l'Oriol, que ha proporcionat noves idees i enfoc al desenvolupament del treball.

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Finalment, agrair a la meva família i a la Mire el seu suport incondicional cada dia i a ensenyar-me a estimar.

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## Appendix

In the following link you can find the Git repository with all the code used for the development of this work: [https://github.com/Ch4rlieStone/tfg\\_eroots](https://github.com/Ch4rlieStone/tfg_eroots)