Chapter 17 Waves II

5. If d is the distance from the location of the earthquake to the seismograph and v_s is the speed of the S waves, then the time for these waves to reach the seismograph is $t_s = d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

so

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(3.0\,\text{min})(60\,\text{s}/\text{min})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9\times 10^3\ \text{km}.$$

We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

6. Let ℓ be the length of the rod. Then the time of travel for sound in air (speed v_s) will be $t_s = \ell / v_s$. And the time of travel for compressional waves in the rod (speed v_r) will be $t_r = \ell / v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12\,\text{s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right).$$

Thus, with v_s = 343 m/s and $v_r = 15v_s$ = 5145 m/s, we find $\ell = 44\,\text{m}$.

7. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives

$$d = \frac{1}{2} g t_f^2 \quad \Rightarrow \quad t_f = \sqrt{2d/g}.$$

The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d. Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

13. The problem says "At one instant..." and we choose that instant (without loss of generality) to be $t = 0$. Thus, the displacement of "air molecule A" at that instant is

$$s_A = +s_m = s_m\cos(kx_A - \omega t + \phi)|_{t=0} = s_m\cos(kx_A + \phi),$$

where $x_A = 2.00$ m. Regarding "air molecule B" we have

$$s_B = +\tfrac{1}{3}s_m = s_m\cos(kx_B - \omega t + \phi)|_{t=0} = s_m\cos(kx_B + \phi).$$

These statements lead to the following conditions:

$$kx_A + \phi = 0$$
$$kx_B + \phi = \cos^{-1}(1/3) = 1.231$$

where $x_B = 2.07$ m. Subtracting these equations leads to

$$k(x_B - x_A) = 1.231 \quad \Rightarrow \quad k = 17.6 \text{ rad/m}.$$

Using the fact that $k = 2\pi/\lambda$ we find $\lambda = 0.357$ m, which means

$$f = v/\lambda = 343/0.357 = 960 \text{ Hz}.$$

Another way to complete this problem (once k is found) is to use $kv = \omega$ and then the fact that $\omega = 2\pi f$.

14. (a) The period is $T = 2.0$ ms (or 0.0020 s) and the amplitude is $\Delta p_m = 8.0$ mPa (which is equivalent to 0.0080 N/m^2). From Eq. 17-15 we get

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi/T)} = 6.1\times10^{-9} \text{ m}$$

where $\rho = 1.21$ kg/m^3 and $v = 343$ m/s.

(b) The angular wave number is $k = \omega/v = 2\pi/vT = 9.2$ rad/m.

(c) The angular frequency is $\omega = 2\pi/T = 3142$ rad/s $\approx 3.1\times10^3$ rad/s.

The results may be summarized as $s(x, t) = (6.1 \text{ nm}) \cos[(9.2 \text{ m}^{-1})x - (3.1 \times 10^3 \text{ s}^{-1})t]$.

(d) Using similar reasoning, but with the new values for density ($\rho' = 1.35$ kg/m^3) and speed ($v' = 320$ m/s), we obtain

(b) The discussion about the "quiet" directions was started in part (a). The number of values in the list: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5 along with –2.5, –1.5, –0.5, +0.5, +1.5, +2.5 (for the lower-half plane) is 14. There are 14 "quiet" directions.

20. (a) The problem indicates that we should ignore the decrease in sound amplitude, which means that all waves passing through point P have equal amplitude. Their superposition at P if $d = \lambda/4$ results in a net effect of zero there since there are four sources (so the first and third are $\lambda/2$ apart and thus interfere destructively; similarly for the second and fourth sources).

(b) Their superposition at P if $d = \lambda/2$ also results in a net effect of zero there since there are an even number of sources (so the first and second being $\lambda/2$ apart will interfere destructively; similarly for the waves from the third and fourth sources).

(c) If $d = \lambda$ then the waves from the first and second sources will arrive at P in phase; similar observations apply to the second and third, and to the third and fourth sources. Thus, four waves interfere constructively there with net amplitude equal to $4s_m$.

21. Let L_1 be the distance from the closer speaker to the listener. The distance from the other speaker to the listener is $L_2 = \sqrt{L_1^2 + d^2}$, where d is the distance between the speakers. The phase difference at the listener is $\phi = 2\pi(L_2 - L_1)/\lambda$, where λ is the wavelength.

For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where n is an integer. Thus,

$$\lambda = 2(L_2 - L_1)/(2n + 1).$$

The frequency is

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{2\left(\sqrt{L_1^2+d^2}-L_1\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(3.75\,\text{m})^2+(2.00\,\text{m})^2}-3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range. This means n ranges from 0 to 28.

(a) The lowest frequency that gives minimum signal is ($n = 0$) $f_{\min,1} = 343$ Hz.

(b) The second lowest frequency is ($n = 1$) $f_{\min,2} = [2(1)+1]343\text{ Hz} = 1029\text{ Hz} = 3f_{\min,1}$. Thus, the factor is 3.

(c) The third lowest frequency is ($n = 2$) $f_{\min,3} = [2(2)+1]343\text{ Hz} = 1715\text{ Hz} = 5f_{\min,1}$. Thus, the factor is 5.

For a maximum in intensity at the listener, $\phi = 2n\pi$, where n is any positive integer. Thus $\lambda = (1/n)\left(\sqrt{L_1^2 + d^2} - L_1\right)$ and

$$f = \frac{v}{\lambda} = \frac{nv}{\sqrt{L_1^2 + d^2} - L_1} = \frac{n(343\,\text{m/s})}{\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, n must be in the range from 1 to 29 for the frequency to be audible.

(d) The lowest frequency that gives maximum signal is (n = 1) $f_{\text{max},1} = 686$ Hz.

(e) The second lowest frequency is (n = 2) $f_{\text{max},2} = 2(686\text{ Hz}) = 1372\text{ Hz} = 2 f_{\text{max},1}$. Thus, the factor is 2.

(f) The third lowest frequency is (n = 3) $f_{\text{max},3} = 3(686\text{ Hz}) = 2058\text{ Hz} = 3 f_{\text{max},1}$. Thus, the factor is 3.

22. At the location of the detector, the phase difference between the wave that traveled straight down the tube and the other one, which took the semi-circular detour, is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r).$$

For $r = r_{\text{min}}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus,

$$r_{\text{min}} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0\,\text{cm}}{2(\pi - 2)} = 17.5\,\text{cm}.$$

23. (a) If point P is infinitely far away, then the small distance d between the two sources is of no consequence (they seem effectively to be the same distance away from P). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces fully constructive interference.

(c) For finite values of x, the difference in source positions becomes significant. The path lengths for waves to travel from S_1 and S_2 become now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

The path length difference for waves traveling from S_1 and S_2 is

$$\beta = (10\ \text{dB})\log\frac{I}{I_0}$$

where $I_0 = 10^{-12}\ \text{W/m}^2$ is the standard reference intensity. In this problem, let the two intensities be I_1 and I_2 such that $I_2 > I_1$. The sound levels are $\beta_1 = (10\ \text{dB})\log(I_1/I_0)$ and $\beta_2 = (10\ \text{dB})\log(I_2/I_0)$. With $\beta_2 = \beta_1 + 1.0\ \text{dB}$, we have

$$(10\ \text{dB})\log(I_2/I_0) = (10\ \text{dB})\log(I_1/I_0) + 1.0\ \text{dB},$$

or

$$(10\ \text{dB})\log(I_2/I_0) - (10\ \text{dB})\log(I_1/I_0) = 1.0\ \text{dB}.$$

Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 0.1$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is

$$\frac{I_2}{I_1} = 10^{0.1} = 1.26\,.$$

29. The intensity is the rate of energy flow per unit area perpendicular to the flow. The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where A $(= 4\pi r^2)$ is the surface area of a sphere of radius r. Thus

$$P = 4\pi(2.50\ \text{m})^2\,(1.91\times10^{-4}\ \text{W/m}^2) = 1.50\times10^{-2}\ \text{W}.$$

30. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r = 3.00$ m,

$$I = \frac{1.00\times10^{-6}\ \text{W}}{4\pi(3.00\ \text{m})^2} = 8.84\times10^{-9}\ \text{W/m}^2.$$

(b) The sound level there is

$$\beta = 10\log\left(\frac{8.84\times10^{-9}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 39.5\,\text{dB}.$$

31. We use $\beta = 10\log(I/I_o)$ with $I_o = 1\times10^{-12}\ \text{W/m}^2$ and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\,\text{m})^2\left(1\times10^{-12}\ \text{W/m}^2\right)10^{6.2} = 2\times10^{-6}\ \text{W} = 2\ \mu\text{W}.$$

48. (a) Since the difference between consecutive harmonics is equal to the fundamental frequency (see section 17-6) then f_1 = (390 – 325) Hz = 65 Hz. The next harmonic after 195 Hz is therefore (195 + 65) Hz = 260 Hz.

(b) Since $f_n = nf_1$, then n = 260/65 = 4.

(c) Only *odd* harmonics are present in tube B, so the difference between consecutive harmonics is equal to *twice* the fundamental frequency in this case (consider taking differences of Eq. 17-41 for various values of n). Therefore,

$$f_1 = \tfrac{1}{2}(1320 - 1080)\ \text{Hz} = 120\ \text{Hz}.$$

The next harmonic after 600 Hz is consequently [600 + 2(120)] Hz = 840 Hz.

(d) Since $f_n = nf_1$ (for n odd), then n = 840/120 = 7.

49. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f = (n/2L)\sqrt{\tau/\mu}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between, so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_2 = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_2 - f_1 = (1/2L)\sqrt{\tau/\mu}$ and

$$\tau = 4L^2\mu(f_2 - f_1)^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\ \text{kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 = 45.3\,\text{N}.$$

(a) When the wind is blowing from the source to the observer with a speed w, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f\left(\frac{v+u'_D}{v+u'_S}\right) = f\left(\frac{v+w}{v+w}\right) = 2.0\times10^3\text{ Hz.}$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both u'_D and u'_S. The result is that there is still no Doppler shift:

$$f' = f\left(\frac{v-u'_D}{v-u'_S}\right) = f\left(\frac{v-w}{v-w}\right) = 2.0\times10^3\text{ Hz.}$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

66. We use Eq. 17-47 with f = 500 Hz and v = 343 m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 30.5\text{ m/s}}{343\text{ m/s} - 30.5\text{ m/s}}\right) = 598\text{ Hz.}$$

(b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 0}{343\text{ m/s} - 2(30.5\text{ m/s})}\right) = 608\text{ Hz.}$$

(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\text{ Hz})\left(\frac{343\text{ m/s} + 2(30.5\text{ m/s})}{343\text{ m/s} - 0}\right) = 589\text{ Hz.}$$