Chapter 7 Kinetic Energy and Work

30. Hooke's law and the work done by a spring is discussed in the chapter. We apply the work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in Figure 7-35 at x = 1.0 m and x = 2.0 m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$. should be F, not P

$$4\text{ J} = P(1.0\text{ m}) - \frac{1}{2}k(1.0\text{ m})^2$$

$$0 = P(2.0\text{ m}) - \frac{1}{2}k(2.0\text{ m})^2.$$

(a) Simultaneous solution leads to P = 8.0 N.

(b) Similarly, we find k = 8.0 N/m.

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular “areas” in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the “areas” under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at $x = 7.0$ m.

(c) And at $x = 9.0$ m, the work is 12 J.

(d) Equation 7-10 (along with Eq. 7-1) leads to speed $v = 6.5$ m/s at $x = 4.0$ m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at $x = 4.0$ m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at $x = 7.0$ m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed $v = 3.5$ m/s at $x = 9.0$ m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

50. (a) Using Eq. 7-48 and Eq. 3-23, we obtain

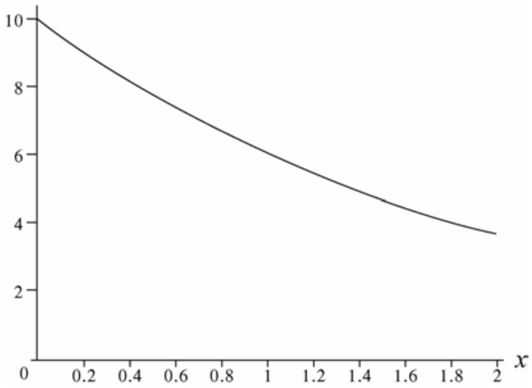
$$P = \vec{F} \cdot \vec{v} = (4.0\ \text{N})(-2.0\ \text{m/s}) + (9.0\ \text{N})(4.0\ \text{m/s}) = 28\ \text{W}.$$

(b) We again use Eq. 7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\text{j}}$.

$$P = \vec{F} \cdot \vec{v} \quad \Rightarrow -12\ \text{W} = (-2.0\ \text{N})v.$$

which yields $v = 6$ m/s.

73. (a) The plot of the function (with SI units understood) is shown below.



Estimating the area under the curve allows for a range of answers. Estimates from 11 J to 14 J are typical. 在这个范围内都可以

(b) Evaluating the work analytically (using Eq. 7-32), we have

$$W = \int_0^2 10e^{-x/2}dx = -20e^{-x/2}\Big|_0^2 = 12.6 \text{ J} \approx 13 \text{ J}.$$

Chapter 8 Potential Energy and Conservation of Energy

26. (a) With energy in joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)\,dx' \;.$$

Therefore, with $U(0) = 27$ J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $dU/dx = 0$ condition, or we can treat this as a force equilibrium situation — which is the approach we show.

$$F = 0 \Rightarrow 6x_{eq} - 12 = 0$$

Thus, $x_{eq} = 2.0$ m, and the above expression for the potential energy becomes $U = 39$ J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the negative value of x for which $U = 0$ to be $x = -1.6$ m.

(d) Similarly, we find the positive value of x for which $U = 0$ to be $x = 5.6$ m.

31. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so k = 1960 N/m and x_0 = –0.200 m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2\text{ J}$.

(b) Since initially U_g = 0, the change in U_g is the same as its final value mgh where m = 2.00 kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g$ = 39.2 J.

(c) The principle of mechanical energy conservation leads to

$$K_0 + U_0 = K_f + U_f$$

$$0 + \frac{1}{2}kx_0^2 = 0 + mgh$$

which yields h = 2.00 m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30°$ = 4.00 m.

39. From the figure, we see that at x = 4.5 m, the potential energy is U_1 = 15 J. If the speed is v = 7.0 m/s, then the kinetic energy is

$$K_1 = mv^2/2 = (0.90 \text{ kg})(7.0 \text{ m/s})^2/2 = 22 \text{ J}.$$

The total energy is $E_1 = U_1 + K_1 = (15 + 22)$ J = 37 J.

(a) At x = 1.0 m, the potential energy is U_2 = 35 J. By energy conservation, we have K_2 = 2.0 J > 0. This means that the particle can reach there with a corresponding speed

$$v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(2.0 \text{ J})}{0.90 \text{ kg}}} = 2.1 \text{ m/s}.$$

(b) The force acting on the particle is related to the potential energy by the negative of the slope:

$$F_x = -\frac{\Delta U}{\Delta x}$$

From the figure we have $F_x = -\dfrac{35\text{ J}-15\text{ J}}{2\text{ m}-4\text{ m}} = +10\text{ N}$.

(c) Since the magnitude $F_x > 0$, the force points in the $+x$ direction.

(d) At x = 7.0 m, the potential energy is U_3 = 45 J, which exceeds the initial total energy E_1. Thus, the particle can never reach there. At the turning point, the kinetic energy is zero. Between x = 5 and 6 m, the potential energy is given by

$$U(x) = 15 + 30(x-5), \qquad 5 \le x \le 6.$$

Thus, the turning point is found by solving $37 = 15 + 30(x-5)$, which yields x = 5.7 m.

(e) At x = 5.0 m, the force acting on the particle is

$$F_x = -\frac{\Delta U}{\Delta x} = -\frac{(45-15)\text{ J}}{(6-5)\text{ m}} = -30\text{ N}.$$

The magnitude is $|F_x| = 30\text{ N}$.

(f) The fact that $F_x < 0$ indicated that the force points in the $-x$ direction.

65. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-33 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this

occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{th} = \mu_k mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{th} = \mu_k mg[(n - 1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\left((n-1)L + d\right)$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n.$$

The first two terms give $1 + 1/2\mu_k = 3.5$, so that the requirement $0 \le d/L \le 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \frac{1}{2}$, or

$$d = \frac{1}{2}L = \frac{1}{2}(40 \text{ cm}) = 20 \text{ cm}$$

and that this occurs on its third pass through the flat region.