

Name _____ Student ID _____ School _____ Major/Class _____ Seat No. _____

Probability and Statistics 2023-2024-(2) Exam Paper A

Question No.	I	II	III	IV	V	VI	VII	Sum
Score								

Score

- Probability and Statistics Midterm Exam Page 1 of 7

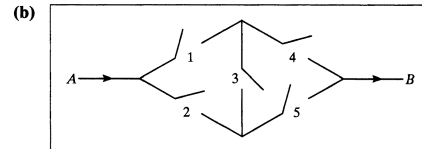
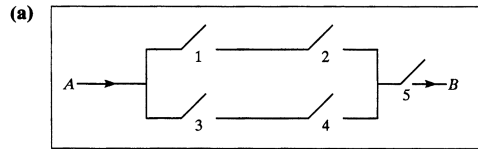
II. (15 points) A red die, a blue die, and a yellow die (all six sided) are rolled. We are interested in the probability that the number appearing on the blue die is less than that appearing on the yellow die, which is less than that appearing on the red die. That is, with B , Y , and R denoting, respectively, the number appearing on the blue, yellow, and red die, we are interested in $P(B < Y < R)$.

Score

1. What is the probability that no two of the dice land on the same number?
2. Given that no two of the dice land on the same number, what is the conditional probability that $B < Y < R$?
3. What is $P(B < Y < R)$?

III. (15 points) The probability of the closing of the i th relay in the circuits shown in the following figure is given by p_i , $i = 1, 2, 3, 4, 5$. If all relays function independently, what is the probability that a current flows between A and B for the respective circuits? In the circuit (a), find also the conditional probability that relays 1 and 2 are both closed given that a current flows from A to B .

Score



IV. (10 points) Ten balls are to be distributed among 5 urns, with each ball going into urn i with probability p_i , $\sum_{i=1}^5 p_i = 1$. Let X_i denote the number of balls that go into urn i . Assume that events corresponding to the locations of different balls are independent.

Score

1. What type of random variable is X_i ? Be as specific as possible.
2. For $i \neq j$, what type of random variable is $X_i + X_j$?
3. Find $P(X_1 + X_2 + X_3 = 7)$.

V. (20 points) If Y is uniformly distributed over $(-5, 5)$, find

Score

1. the probability that $|Y| > 2$;
2. the density function of $2024|Y|$;
3. the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real.
4. What random variable, having a linear relation with Y , is uniformly distributed over $(0, 1)$?

VI. (15 points) Suppose that X_i ($i = 1, 2, 3$) are independent Poisson random variables with respective means λ_i ($i = 1, 2, 3$). Let $X = X_1 + X_2$ and $Y = X_2 + X_3$.

Score

1. Find the pmf of X .
2. Find the joint pmf of X and Y .
3. Find $E(XY)$.

VII. (15 points) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} e^{-y}, & 0 \leq x \leq 1, y \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Score

1. Are X and Y independent?
2. Find the pdf of $Z = 2X + Y$.
3. Compute $P(Z > 3)$.