Chapter 9 Center of Mass and Linear Momentum

14. (a) The phrase (in the problem statement) "such that it [particle 2] always stays directly above particle 1 during the flight" means that the shadow (as if a light were directly above the particles shining down on them) of particle 2 coincides with the position of particle 1, at each moment. We say, in this case, that they are vertically aligned. Because of that alignment, $v_{2x} = v_1 = 10.0$ m/s. Because the initial value of v_2 is given as 20.0 m/s, then (using the Pythagorean theorem) we must have

$$v_{2y} = \sqrt{v_2^2 - v_{2x}^2} = \sqrt{300} \text{ m/s}$$

for the initial value of the y component of particle 2's velocity. Equation 2-16 (or conservation of energy) readily yields $y_{max} = 300/19.6 = 15.3$ m. Thus, we obtain

$$H_{max} = m_2 y_{max} / m_{total} = (3.00 \text{ g})(15.3 \text{ m})/(8.00 \text{ g}) = 5.74 \text{ m}.$$

(b) Since both particles have the same horizontal velocity, and particle 2's vertical component of velocity vanishes at that highest point, then the center of mass velocity then is simply $(10.0 \text{ m/s})\hat{\text{i}}$ (as one can verify using Eq. 9-17).

(c) Only particle 2 experiences any acceleration (the free fall acceleration downward), so Eq. 9-18 (or Eq. 9-19) leads to

$$a_{com} = m_2 g / m_{total} = (3.00 \text{ g})(9.8 \text{ m/s}^2)/(8.00 \text{ g}) = 3.68 \text{ m/s}^2$$

for the magnitude of the downward acceleration of the center of mass of this system. Thus, $\vec{a}_{\text{com}} = (-3.68 \text{ m/s}^2)\,\hat{\text{j}}$.

15. (a) The net force on the *system* (of total mass $m_1 + m_2$) is $m_2 g$. Thus, Newton's second law leads to $a = g(m_2/(m_1 + m_2)) = 0.4g$. For block 1, this acceleration is to the right (the $\hat{\text{i}}$ direction), and for block 2 this is an acceleration downward (the $-\hat{\text{j}}$ direction). Therefore, Eq. 9-18 gives

$$\vec{a}_{\text{com}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} = \frac{(0.6)(0.4g\hat{\text{i}}) + (0.4)(-0.4g\hat{\text{j}})}{0.6 + 0.4} = (2.35\,\hat{\text{i}} - 1.57\,\hat{\text{j}})\text{ m/s}^2 .$$

(b) Integrating Eq. 4-16, we obtain

$$\vec{v}_{\text{com}} = (2.35\,\hat{\text{i}} - 1.57\hat{\text{j}})\,t$$

(with SI units understood), since it started at rest. We note that the *ratio* of the *y*-component to the *x*-component (for the velocity vector) does not change with time, and it is that ratio which determines the angle of the velocity vector (by Eq. 3-6), and thus the direction of motion for the center of mass of the system.

(c) The last sentence of our answer for part (b) implies that the path of the center-of-mass is a straight line.

(d) Equation 3-6 leads to $\theta = -34°$. The path of the center of mass is therefore straight, at downward angle 34°.

16. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then

$$M_R(L/2 - x) = mx + M_C(L/2 + x).$$

Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, $x = 40$ cm/2 $= 0.20$ m, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)\left(\frac{3.0}{2} - 0.20\right) - (30)(0.20)}{(3.0/2) + 0.20} = 58 \text{ kg}.$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16, $M\Delta x_{\text{com}} = 0 = m_b\Delta x_b + m_d\Delta x_d$, which implies

$$|\Delta x_b| = \frac{m_d}{m_b}|\Delta x_d|.$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance d = 2.4 m:

$$|\Delta x_b| + |\Delta x_d| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}|(\Delta x_d)| + |\Delta x_d| = d$$

which leads to $|\Delta x_d| = \dfrac{d}{1 + m_d/m_b} = \dfrac{2.4\text{ m}}{1 + (4.5/18)} = 1.92\text{ m.}$

The dog is therefore 1.9 m closer to the shore than initially (where it was D = 6.1 m from it). Thus, it is now $D - |\Delta x_d|$ = 4.2 m from the shore.

18. The magnitude of the ball's momentum change is

$$\Delta p = m|v_i - v_f| = (0.70\text{ kg})|(5.0\text{ m/s}) - (-2.0\text{ m/s})| = 4.9\text{ kg}\cdot\text{m/s}.$$

19. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(2100\text{ kg})\left((51\text{ km/h})^2 - (41\text{ km/h})^2\right)\\ &= 9.66\times10^4\text{ kg}\cdot(\text{km/h})^2\left((10^3\text{ m/km})(1\text{ h/3600 s})\right)^2\\ &= 7.5\times10^4\text{ J.}\end{aligned}$$

(b) The magnitude of the change in velocity is

$$|\Delta\vec{v}| = \sqrt{(-v_i)^2 + (v_f)^2} = \sqrt{(-41\text{ km/h})^2 + (51\text{ km/h})^2} = 65.4\text{ km/h}$$

so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m|\Delta\vec{v}| = (2100\text{ kg})(65.4\text{ km/h})\left(\frac{1000\text{ m/km}}{3600\text{ s/h}}\right) = 3.8\times10^4\text{ kg}\cdot\text{m/s.}$$

(c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\text{ km/h}}{51\text{ km/h}}\right) = 39°.$$

(b) The negative sign in v_f indicates that the velocity is in the $-x$ direction, which is opposite to the initial direction of travel.

(c) From the above, the average magnitude of the force is $F_{\text{avg}} = 1.20 \times 10^3$ N.

(d) The direction of the impulse on the ball is $-x$, same as the applied force.

Note: In vector notation, $\vec{F}_{\text{avg}}\Delta t = \vec{J} = \Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$, which gives

$$\vec{v}_f = \vec{v}_i + \frac{\vec{J}}{m} = \vec{v}_i + \frac{\vec{F}_{\text{avg}}\Delta t}{m}.$$

Since $\vec{J}$ or $\vec{F}_{\text{avg}}$ is in the opposite direction of $\vec{v}_i$, the velocity of the ball decreases. The ball first moves in the $+x$-direction, but then slows down and comes to a stop under the influence of the applied force, and reverses its direction of travel.

28. (a) The magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (0.70 \text{ kg})(13 \text{ m/s}) \approx 9.1 \text{ kg} \cdot \text{m/s} = 9.1 \text{ N} \cdot \text{s}.$$

(b) With duration of $\Delta t = 5.0 \times 10^{-3}$ s for the collision, the average force is

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.1 \text{ N} \cdot \text{s}}{5.0 \times 10^{-3} \text{ s}} \approx 1.8 \times 10^3 \text{ N}.$$

29. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3 \times 10^{-3} \text{kg})(500 \text{ m/s})}{(1 \text{min})(60 \text{ s/min})} \approx 5 \text{ N}.$$

30. (a) By Eq. 9-30, impulse can be determined from the "area" under the $F(t)$ curve. Keeping in mind that the area of a triangle is $\frac{1}{2}$(base)(height), we find the impulse in this case is $1.00 \text{ N} \cdot \text{s}$.

(b) By definition (of the average of function, in the calculus sense) the average force must be the result of part (a) divided by the time (0.010 s). Thus, the average force is found to be 100 N.

(c) Consider ten hits. Thinking of ten hits as 10 $F(t)$ triangles, our total time interval is 10(0.050 s) = 0.50 s, and the total area is 10(1.0 $\mathrm{N\cdot s}$). We thus obtain an average force of 10/0.50 = 20.0 N. One could consider 15 hits, 17 hits, and so on, and still arrive at this same answer.

31. (a) By energy conservation, the speed of the passenger when the elevator hits the floor is

$$\frac{1}{2}mv^2 = mgh \quad\Rightarrow\quad v = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(36\ \mathrm{m})} = 26.6\ \mathrm{m/s}.$$

Thus, the magnitude of the impulse is

$$J = |\Delta p| = m|\Delta v| = mv = (90\ \mathrm{kg})(26.6\ \mathrm{m/s}) \approx 2.39\times10^3\ \mathrm{N\cdot s}.$$

(b) With duration of $\Delta t = 5.0\times10^{-3}$ s for the collision, the average force is

$$F_{\mathrm{avg}} = \frac{J}{\Delta t} = \frac{2.39\times10^3\ \mathrm{N\cdot s}}{5.0\times10^{-3}\ \mathrm{s}} \approx 4.78\times10^5\ \mathrm{N}.$$

(c) If the passenger were to jump upward with a speed of $v' = 7.0\ \mathrm{m/s}$, then the resulting downward velocity would be

$$v'' = v - v' = 26.6\ \mathrm{m/s} - 7.0\ \mathrm{m/s} = 19.6\ \mathrm{m/s},$$

and the magnitude of the impulse becomes

$$J'' = |\Delta p''| = m|\Delta v''| = mv'' = (90\ \mathrm{kg})(19.6\ \mathrm{m/s}) \approx 1.76\times10^3\ \mathrm{N\cdot s}.$$

(d) The corresponding average force would be

$$F''_{\mathrm{avg}} = \frac{J''}{\Delta t} = \frac{1.76\times10^3\ \mathrm{N\cdot s}}{5.0\times10^{-3}\ \mathrm{s}} \approx 3.52\times10^5\ \mathrm{N}.$$

32. (a) By the impulse-momentum theorem (Eq. 9-31) the change in momentum must equal the "area" under the $F(t)$ curve. Using the facts that the area of a triangle is $\frac{1}{2}$ (base)(height), and that of a rectangle is (height)(width), we find the momentum at t = 4 s to be (30 kg·m/s)$\hat{\mathrm{i}}$.

(b) Similarly (but keeping in mind that areas beneath the axis are counted negatively) we find the momentum at t = 7 s is (38 kg·m/s)$\hat{\mathrm{i}}$.

(c) At t = 9 s, we obtain $\vec{v}$ = (6.0 m/s)$\hat{\mathrm{i}}$.

40. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires

$$(M + m)v_0 = mv + M(v - v_r).$$

Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300 \text{ km/h} + \frac{(4m)(82 \text{ km/h})}{4m+m} = 4.4 \times 10^3 \text{ km/h}.$$

41. (a) With SI units understood, the velocity of block L (in the frame of reference indicated in the figure that goes with the problem) is $(v_1 - 3)\hat{\text{i}}$. Thus, momentum conservation (for the explosion at $t = 0$) gives

$$m_L(v_1 - 3) + (m_C + m_R)v_1 = 0$$

which leads to

$$v_1 = \frac{3\, m_L}{m_L + m_C + m_R} = \frac{3(2 \text{ kg})}{10 \text{ kg}} = 0.60 \text{ m/s}.$$

Next, at $t = 0.80$ s, momentum conservation (for the second explosion) gives

$$m_C v_2 + m_R(v_2 + 3) = (m_C + m_R)v_1 = (8 \text{ kg})(0.60 \text{ m/s}) = 4.8 \text{ kg} \cdot \text{m/s}.$$

This yields $v_2 = -0.15$. Thus, the velocity of block C after the second explosion is

$$v_2 = -(0.15 \text{ m/s})\hat{\text{i}}.$$

(b) Between $t = 0$ and $t = 0.80$ s, the block moves $v_1\Delta t = (0.60 \text{ m/s})(0.80 \text{ s}) = 0.48$ m. Between $t = 0.80$ s and $t = 2.80$ s, it moves an additional

$$v_2\Delta t = (-0.15 \text{ m/s})(2.00 \text{ s}) = -0.30 \text{ m}.$$

Its net displacement since $t = 0$ is therefore 0.48 m – 0.30 m = 0.18 m.

42. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\text{i}}$; the mass of the less massive piece is m_1; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}m \text{ and } m_2 = \frac{3}{4}m.$$

Conservation of linear momentum requires

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80\,.$$

58. We think of this as having two parts: the first is the collision itself, where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet, and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_m. The first part involves momentum conservation (with $+x$ rightward):

$$m_1v_1 = (m_1+m_2)v \quad \Rightarrow \quad (2.0\text{ kg})(4.0\text{ m/s}) = (3.0\text{ kg})\vec{v}$$

which yields $\vec{v} = 2.7\text{ m/s}$. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\text{ kg})\,(2.7\text{ m/s})^2 = \frac{1}{2}\,(200\text{ N/m})x_{\text{m}}^2$$

which gives the result $x_m = 0.33$ m.

59. As hinted in the problem statement, the velocity v of the system as a whole, when the spring reaches the maximum compression x_m, satisfies

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 = \frac{(m_1v_{1i}+m_2v_{2i})^2}{2(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)\, v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_{\text{m}}^2 = -\Delta K \Rightarrow x_{\text{m}} = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35\text{ J})}{1120\text{ N/m}}} = 0.25\text{ m}.$$

60. (a) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_Av_{Ai} + m_Bv_{Bi} = m_Av_{Af} + m_Bv_{Bf}$$

and

$$v_1' = \frac{v_1 \sin\theta}{\sin(\phi - \theta)} = \frac{(500\text{ m/s})\sin(-30°)}{\sin(-90°)} = 250\text{ m/s}.$$

76. We use Eq. 9-88. Then

$$v_f = v_i + v_{\text{rel}} \ln\left(\frac{M_i}{M_f}\right) = 105\text{ m/s} + (253\text{ m/s}) \ln\left(\frac{6090\text{ kg}}{6010\text{ kg}}\right) = 108\text{ m/s}.$$

77. We consider what must happen to the coal that lands on the faster barge during a time interval Δt. In that time, a total of Δm of coal must experience a change of velocity (from slow to fast) $\Delta v = v_{\text{fast}} - v_{\text{slow}}$, where rightward is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta p}{\Delta t} = \frac{(\Delta m)}{\Delta t}\Delta v = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}})$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$. Note that we ignore the transverse speed of the coal as it is shoveled from the slower barge to the faster one.

(a) Given that $(\Delta m/\Delta t) = 1000\text{ kg/min} = (16.67\text{ kg/s})$, $v_{\text{fast}} = 20\text{ km/h} = 5.56\text{ m/s}$ and $v_{\text{slow}} = 10\text{ km/h} = 2.78\text{ m/s}$, the force that must be applied to the faster barge is

$$F_{\text{fast}} = \left(\frac{\Delta m}{\Delta t}\right)(v_{\text{fast}} - v_{\text{slow}}) = (16.67\text{ kg/s})(5.56\text{ m/s} - 2.78\text{ m/s}) = 46\text{ N}$$

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

78. We use Eq. 9-88 and simplify with $v_i = 0$, $v_f = v$, and $v_{\text{rel}} = u$.

$$v_f - v_i = v_{\text{rel}} \ln\frac{M_i}{M_f} \Rightarrow \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$ we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7$.

(b) If $v = 2u$ we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4$.