Seat No.______ Major/Class______ School______ Student ID ______ Name ______

(DON NOT WRITE YOUR ANSWER IN THIS AREA)

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WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

Notice:
1. Make sure that you have filled the form on the left side of the seal line.
2. Write your answers on the exam paper.
3. This is a close-book exam.
4. The exam with full score of 100 points lasts 120 minutes.

Question No.	I	II	III	IV	V	VI	Sum
Score							

Score

I. (20 points) Let

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 5 & 4 \end{bmatrix}$$

1. Find the determinant of matrix A.
2. Give the inverse of matrix A.
3. Let I be a 3×3 Identity matrix and $B = \begin{bmatrix} A & 0 \\ I & -A \end{bmatrix}$, which is a partitioned matrix with four 3×3 blocks. Find the inverse of matrix B.

II. (15 points) For the vector space

Score

$$H = \left\{ \begin{bmatrix} a+b+2c+5d \\ a+2b+3c+8d \\ b+2c+5d \\ a+2b+4c+10d \end{bmatrix} : a,b,c,d \in \mathbb{R} \right\},$$

1. If H is a subspace of $\mathbb{R}^k$, what is the number k?
2. Find a set of basis for H and the dimension of H
3. Find a set of basis for the orthogonal compliment $H^{\perp}$ of H.

III (20 points) Let $\mathscr{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis in $\mathbb{R}^3$,

$$P = \begin{pmatrix} 1 & 2 & -1 \\ -3 & -5 & 0 \\ 4 & 6 & 1 \end{pmatrix},$$

and

Score

$$\mathbf{v}_1 = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -8 \\ 5 \\ 2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -7 \\ 2 \\ 6 \end{pmatrix}.$$

1. Show that $\mathscr{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis of $\mathbb{R}^3$.
2. Show the change-of-coordinates matrix $P_{\mathscr{E} \leftarrow \mathscr{B}}$ from basis $\mathscr{B}$ to the standard basis $\mathscr{E}$.
3. Find a basis $\mathscr{D} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for $\mathbb{R}^3$ such that P is the change-of-coordinates matrix from $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ to the basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.[Hint: One can use the fact $P_{\mathscr{B} \leftarrow \mathscr{D}} = P_{\mathscr{B} \leftarrow \mathscr{E}} P_{\mathscr{E} \leftarrow \mathscr{D}}$.]

Score

IV. (15 points) Consider

$$A = \begin{pmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{pmatrix}.$$

1. Find the eigenvalues of matrix A.
2. Diagonalize the matrix A, if possible and if not, explain the reason.

Score

V(15 points) Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ be two eigenvectors of a 2×2 matrix A related to eigenvalues $\lambda_1 = 3$ and $\lambda_2 = -2$ respectively.

1. Compute $A^2 \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ without using the exact formula of A.

2. Find the exact formula of A

Score

VI(15 points) Let $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ are vectors in $\mathbb{R}^n$. Suppose that vectors $\mathbf{u}_1, \mathbf{u}_2$ are orthogonal and the norm of $\mathbf{u}_2$ is 4 and $\mathbf{u}_2^{\mathrm{T}}\mathbf{u}_3 = 7$. Find the value of the real number a in $\mathbf{u}_1 = \mathbf{u}_2 + a\mathbf{u}_3$.