# Chapter 35 Interference

11. (a) Equation 35-11 (in absolute value) yields

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\,\text{m})}{500\times10^{-9}\,\text{m}}(1.60-1.50) = 1.70.$$

(b) Similarly,

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\,\text{m})}{500\times10^{-9}\,\text{m}}(1.72-1.62) = 1.70.$$

(c) In this case, we obtain

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(3.25\times10^{-6}\,\text{m})}{500\times10^{-9}\,\text{m}}(1.79-1.59) = 1.30.$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

12. (a) We note that ray 1 travels an extra distance $4L$ more than ray 2. To get the least possible $L$ that will result in destructive interference, we set this extra distance equal to half of a wavelength:

$$4L = \frac{\lambda}{2} \quad\Rightarrow\quad L = \frac{\lambda}{8} = \frac{420.0\text{ nm}}{8} = 52.50\text{ nm}.$$

(b) The next case occurs when that extra distance is set equal to $\frac{3}{2}\lambda$. The result is

$$L = \frac{3\lambda}{8} = \frac{3(420.0\text{ nm})}{8} = 157.5\text{ nm}.$$

13. (a) We choose a horizontal $x$ axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 35-11 (with $L$ in that equation replaced with $L_2$). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 35-11 but with $L$ replaced with $L_1 - L_2$ and $n_2$ replaced with 1 (since the top ray in Fig. 35-35 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences with $\lambda = 0.600\ \mu$m, we have

$$\phi = d\sin\theta/\lambda = (4.24\ \mu\text{m})\sin(20°)/(0.500\ \mu\text{m}) = 2.90\,.$$

(b) Multiplying this by $2\pi$ gives $\phi = 18.2$ rad.

(c) The result from part (a) is greater than $\frac{5}{2}$ (which would indicate the third minimum) and is less than 3 (which would correspond to the third side maximum).

19. The condition for a maximum in the two-slit interference pattern is $d\sin\theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, $m$ is an integer, and $\theta$ is the angle made by the interfering rays with the forward direction. If $\theta$ is small, $\sin\theta$ may be approximated by $\theta$ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer $m$ and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance $D$ away is given by

$$\Delta y = D\,\Delta\theta = \lambda D/d.$$

Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\ \text{mm}.$$

20. (a) We use Eq. 35-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2(550\times10^{-9}\,\text{m})}{7.70\times10^{-6}\,\text{m}}\right] = 0.216\,\text{rad}.$$

(b) $\theta = (0.216)\,(180°/\pi) = 12.4°$.

21. The maxima of a two-slit interference pattern are at angles $\theta$ given by $d\sin\theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, and $m$ is an integer. If $\theta$ is small, $\sin\theta$ may be replaced by $\theta$ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of $m$ is

$$\Delta\theta = (m/d)(\lambda_2 - \lambda_1),$$

and their separation on a screen a distance $D$ away is

$$\Delta y = D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)$$
$$= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right](600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}) = 7.2\times10^{-5}\,\text{m}.$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

where we have plugged in $D = 20\lambda$, $d = 3\lambda$ and $x = 6\lambda$. Thus, the phase difference at that point is 1.71 wavelengths.

(f) We note that the answer to part (e) is closer to $\frac{3}{2}$ (destructive interference) than to 2 (constructive interference), so that the point is "intermediate" but closer to a minimum than to a maximum.

25. Let the distance in question be $x$. The path difference (between rays originating from $S_1$ and $S_2$ and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \ldots$. After some algebraic steps, we solve for the distance in terms of $m$:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

To obtain the largest value of $x$, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900\text{ nm}) = 7.88\times10^3\text{ nm} = 7.88\,\mu\text{m}.$$

26. (a) We use Eq. 35-14 to find $d$:

$$d\sin\theta = m\lambda \quad \Rightarrow \quad d = (4)(450\text{ nm})/\sin(90°) = 1800\text{ nm}.$$

For the third-order spectrum, the wavelength that corresponds to $\theta = 90°$ is

$$\lambda = d\sin(90°)/3 = 600\text{ nm}.$$

Any wavelength greater than this will not be seen. Thus, 600 nm < $\theta$ ≤ 700 nm are absent.

(b) The slit separation $d$ needs to be decreased.

(c) In this case, the 400 nm wavelength in the $m = 4$ diffraction is to occur at 90°. Thus

$$d_{\text{new}}\sin\theta = m\lambda \quad \Rightarrow \quad d_{\text{new}} = (4)(400\text{ nm})/\sin(90°) = 1600\text{ nm}.$$

This represents a change of

$$|\Delta d| = d - d_{\text{new}} = 200\text{ nm} = 0.20\,\mu\text{m}.$$

27. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness $x$ is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where $\lambda_m$ is the wavelength in the mica and $n$ is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for $\lambda_m$. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\,\text{m})}{1.58-1} = 6.64\times10^{-6}\,\text{m}.$$

28. The problem asks for "the greatest value of $x$… exactly out of phase," which is to be interpreted as the value of $x$ where the curve shown in the figure passes through a phase value of $\pi$ radians. This happens as some point $P$ on the $x$ axis, which is, of course, a distance $x$ from the top source and (using Pythagoras' theorem) a distance $\sqrt{d^2+x^2}$ from the bottom source. The difference (in normal length units) is therefore $\sqrt{d^2+x^2} - x$, or (expressed in radians) is $\frac{2\pi}{\lambda}(\sqrt{d^2+x^2} - x)$. We note (looking at the leftmost point in the graph) that at $x = 0$, this latter quantity equals $6\pi$, which means $d = 3\lambda$. Using this value for $d$, we now must solve the condition

$$\frac{2\pi}{\lambda}\left(\sqrt{d^2+x^2}-x\right) = \pi\,.$$

Straightforward algebra then leads to $x = (35/4)\lambda$, and using $\lambda = 400$ nm we find $x = 3500$ nm, or 3.5 $\mu$m.

29. The intensity is proportional to the square of the resultant field amplitude. Let the electric field components of the two waves be written as

$$E_1 = E_{10}\sin\omega t$$
$$E_2 = E_{20}\sin(\omega t+\phi),$$

where $E_{10} = 1.00$, $E_{20} = 2.00$, and $\phi = 60°$. The resultant field is $E = E_1 + E_2$. We use the phasor diagram to calculate the amplitude of $E$.

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2 L}{2m+1}$$

which yields $\lambda$ = 2208 nm, 736 nm, 442 nm … for the different values of $m$. We note that only the 442-nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

Note: A light ray reflected by a material changes phase by π rad (or 180°) if the refractive index of the material is greater than that of the medium in which the light is traveling. Otherwise, there is no phase change. Note that refraction at an interface does not cause a phase shift.

56. For constructive interference (which is obtained for λ = 600 nm) in this circumstance, we require

$$2L = \frac{k}{2}\lambda_n = \frac{k\lambda}{2n}$$

where $k$ = some positive <u>odd</u> integer and $n$ is the index of refraction of the thin film. Rearranging and plugging in $L$ = 272.7 nm and the wavelength value, this gives

$$n = \frac{k\lambda}{4L} = \frac{k(600 \text{ nm})}{4(272.7 \text{ nm})} = \frac{k}{1.818} = 0.55k .$$

Since we expect $n$ > 1, then $k$ = 1 is ruled out. However, $k$ = 3 seems reasonable, since it leads to $n$ = 1.65, which is close to the "typical" values found in Table 34-1. Taking this to be the correct index of refraction for the thin film, we now consider the destructive interference part of the question. Now we have 2$L$ = (integer)$\lambda_{\text{dest}}/n$. Thus,

$$\lambda_{\text{dest}} = (900 \text{ nm})/(\text{integer}).$$

We note that setting the integer equal to 1 yields a $\lambda_{\text{dest}}$ value outside the range of the visible spectrum. A similar remark holds for setting the integer equal to 3. Thus, we set it equal to 2 and obtain $\lambda_{\text{dest}}$ = 450 nm.

57. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \quad \lambda = \frac{4Ln_2}{2m+1} \ , \qquad m = 0, 1, 2, \ldots$$

Therefore,

Converting this angle into degrees, we arrive at $\theta = 0.012°$.

72. We apply Eq. 35-27 to both scenarios: $m$ = 4001 and $n_2 = n_{air}$, and $m$ = 4000 and $n_2 = n_{vacuum}$ = 1.00000:

$$2L = (4001)\frac{\lambda}{n_{air}} \quad \text{and} \quad 2L = (4000)\frac{\lambda}{1.00000}.$$

Since the 2$L$ factor is the same in both cases, we set the right-hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{air} = (1.00000)\frac{4001}{4000} = 1.00025.$$

We remark that this same result can be obtained starting with Eq. 35-43 (which is developed in the textbook for a somewhat different situation) and using Eq. 35-42 to eliminate the $2L/\lambda$ term.

73. Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by $\pi$ rad. At a place where the thickness of the air film is $L$, the condition for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$ where $\lambda$ (= 683 nm) is the wavelength and $m$ is an integer. This is satisfied for $m$ = 140:

$$L = \frac{(m + \frac{1}{2})\lambda}{2} = \frac{(140.5)(683 \times 10^{-9}\ \text{m})}{2} = 4.80 \times 10^{-5}\ \text{m} = 0.048\ \text{mm}.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m$ = 0. There are, therefore, 141 bright fringes in all.

74. By the condition $m\lambda = 2y$ where $y$ is the thickness of the air film between the plates directly underneath the middle of a dark band), the edges of the plates (the edges where they are not touching) are $y = 8\lambda/2$ = 2400 nm apart (where we have assumed that the *middle* of the ninth dark band is at the edge). Increasing that to $y'$ = 3000 nm would correspond to $m' = 2y'/\lambda$ = 10 (counted as the eleventh dark band, since the first one corresponds to $m$ = 0). There are thus 11 dark fringes along the top plate.

75. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a $\pi$ rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is $d$, the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda,$ where $\lambda$ is the wavelength in air and $m$ is an integer. Therefore,

$$d = (2m + 1)\lambda/4.$$