# Chapter 16 Waves I

9. (a) The amplitude $y_m$ is half of the 6.00 mm vertical range shown in the figure, that is, $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = kv = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times 10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between $kx$ and $\omega t$) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction, see Section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times 10^2\, t) .$$

10. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find $\lambda$ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times 10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times 10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of $y$) is

$$u_{\max} = 2\pi f y_m = \left(4.0\pi \text{ s}^{-1}\right)\left(6.0 \text{ cm}\right) = 75 \text{ cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

32. (a) Let the phase difference be $\phi$. Then from Eq. 16-52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to $2\pi$ rad), this is equivalent to $1.45 \text{ rad}/2\pi = 0.230$ wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00 \text{ mm}$, as stated in the problem.

(b) The figure indicates that $\lambda = 40$ cm $= 0.40$ m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t = (56.0 \text{ cm})/(8.0 \text{ ms}) = 70$ m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude $y_m$. The amplitude of their resultant wave, as shown in the figure, is $y'_m = 4.00$ mm. Equation 16-52 applies:

$$y'_m = 2y_m \cos(\tfrac{1}{2}\phi_2) \quad \Rightarrow \quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it was shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, $\phi$ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100t) \quad \text{and} \quad y_2 = (0.00900)\sin(16x + 1100t + 2.7).$$

52. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16-20(b), where (see Eq. 16-65)

$$\lambda = L \quad \text{and} \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$, then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \Rightarrow \lambda = 4.0\,\text{m} \Rightarrow L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$, then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \Rightarrow v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16-26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\ \text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

the period is $T = 1/f = 0.11$ s.

53. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^2\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for $\lambda$.

(d) The string speed is given by $u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\,(0.50\,\text{cm})\,\sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$