Chapter 10 Rotation

6. If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad/s})t - (3.0 \text{ rad/s}^2)t^2 + (1.0 \text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Equation 10-6 leads to

$$\omega = \frac{d}{dt}(4t - 3t^2 + t^3) = 4 - 6t + 3t^2.$$

Evaluating this at t = 2 s yields ω_2 = 4.0 rad/s.

(b) Evaluating the expression in part (a) at t = 4 s gives ω_4 = 28 rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2) = -6 + 6t.$$

Evaluating this at t = 2 s produces α_2 = 6.0 rad/s^2.

(e) Evaluating the expression in part (d) at t = 4 s yields α_4 = 18 rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

7. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5 \text{ rev/s}} = 0.050 \text{ s}.$$

The minimum speed of the arrow is then $v_{\min} = \dfrac{20 \text{ cm}}{0.050 \text{ s}} = 400 \text{ cm/s} = 4.0 \text{ m/s}.$

(b) No—there is no dependence on radial position in the above computation.

8. (a) We integrate (with respect to time) the $\alpha = 6.0t^4 - 4.0t^2$ expression, taking into account that the initial angular velocity is 2.0 rad/s. The result is

$$\omega = 1.2\, t^5 - 1.33\, t^3 + 2.0.$$

(b) Integrating again (and keeping in mind that $\theta_o = 1$) we get

$$\theta = 0.20t^6 - 0.33\, t^4 + 2.0\, t + 1.0\ .$$

9. (a) With $\omega = 0$ and $\alpha = -\,4.2$ rad/s^2, Eq. 10-12 yields $t = -\omega_o/\alpha = 3.00$ s.

(b) Eq. 10-4 gives $\theta - \theta_o = -\,\omega_o^{\,2} / 2\alpha = 18.9$ rad.

10. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25 \text{ rad} = \frac{1}{2}\alpha(5.0 \text{ s})^2 \;\;\Rightarrow\;\; \alpha = 2.0 \text{ rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25 \text{ rad}}{5.0 \text{ s}} = 5.0 \text{ rad/s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0 \text{ rad/s}^2\right)(5.0 \text{ s}) = 10 \text{ rad/s}\ .$$

(d) According to Eq. 10-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0 \text{ rad/s}^2)(10 \text{ s})^2 = 100 \text{ rad}.$$

Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100$ rad – 25 rad = 75 rad.

$$\theta_1 = \frac{1}{2}\alpha t_1^2, \quad \theta_2 = \frac{1}{2}\alpha t_2^2$$

Given $\Delta\theta = \theta_2 - \theta_1$, we can solve for t_1, which tells us how long the wheel has been in motion up to the beginning of the 4.0 s-interval. The above expressions can be combined to give

$$\Delta\theta = \theta_2 - \theta_1 = \frac{1}{2}\alpha\left(t_2^2 - t_1^2\right) = \frac{1}{2}\alpha(t_2 + t_1)(t_2 - t_1)$$

With $\Delta\theta = 120\ \text{rad}$, $\alpha = 3.0\ \text{rad/s}^2$, and $t_2 - t_1 = 4.0\ \text{s}$, we obtain

$$t_2 + t_1 = \frac{2(\Delta\theta)}{\alpha(t_2 - t_1)} = \frac{2(120\ \text{rad})}{(3.0\ \text{rad/s}^2)(4.0\ \text{s})} = 20\ \text{s},$$

which can be further solved to give $t_2 = 12.0\ \text{s}$ and $t_1 = 8.0\ \text{s}$. So, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

Note: We can readily verify the results by calculating θ_1 and θ_2 explicitly:

$$\theta_1 = \frac{1}{2}\alpha t_1^2 = \frac{1}{2}(3.0\ \text{rad/s}^2)(8.0\ \text{s})^2 = 96\ \text{rad}$$
$$\theta_2 = \frac{1}{2}\alpha t_2^2 = \frac{1}{2}(3.0\ \text{rad/s}^2)(12.0\ \text{s})^2 = 216\ \text{rad}.$$

Indeed the difference is $\Delta\theta = \theta_2 - \theta_1 = 120\ \text{rad}$.

16. (a) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(1.5\ \text{rad/s}^2)\,t_1^2$$

where $\theta - \theta_o$ = (2 rev)(2π rad/rev). Therefore, t_1 = 4.09 s.

(b) We can find the time to go through a full 4 rev (using the same equation to solve for a new time t_2) and then subtract the result of part (a) for t_1 in order to find this answer.

$$(4\ \text{rev})(2\pi\ \text{rad/rev}) = 0 + \tfrac{1}{2}(1.5\ \text{rad/s}^2)\,t_2^2 \quad \Rightarrow \quad t_2 = 5.789\ \text{s}.$$

Thus, the answer is 5.789 s – 4.093 s ≈ 1.70 s.

35. Since the rotational inertia of a cylinder is $I = \frac{1}{2}MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2.$$

(a) For the smaller cylinder, we have

$$K_1 = \frac{1}{4}(1.25\,\text{kg})(0.25\ \text{m})^2(235\,\text{rad/s})^2 = 1.08\times10^3\ \text{J} \approx 1.1\times10^3\ \text{J}.$$

(b) For the larger cylinder, we obtain

$$K_2 = \frac{1}{4}(1.25\,\text{kg})(0.75\ \text{m})^2(235\,\text{rad/s})^2 = 9.71\times10^3\ \text{J} \approx 9.7\times10^3\ \text{J}.$$

36. The parallel axis theorem (Eq. 10-36) shows that I increases with h. The phrase "out to the edge of the disk" (in the problem statement) implies that the maximum h in the graph is, in fact, the radius R of the disk. Thus, $R = 0.20$ m. Now we can examine, say, the $h = 0$ datum and use the formula for I_{com} (see Table 10-2(c)) for a solid disk, or (which might be a little better, since this is independent of whether it is really a solid disk) we can the difference between the $h = 0$ datum and the $h = h_{max} = R$ datum and relate that difference to the parallel axis theorem (thus the difference is $M(h_{max})^2 = 0.10\ \text{kg}\cdot\text{m}^2$). In either case, we arrive at $M = 2.5$ kg.

37. We use the parallel axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h = 0.50$ m – 0.20 m = 0.30 m. We find

$$I_{com} = \frac{1}{12}ML^2 = \frac{1}{12}(0.56\ \text{kg})(1.0\ \text{m})^2 = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

Consequently, the parallel axis theorem yields

$$I = 4.67\times10^{-2}\ \text{kg}\cdot\text{m}^2 + (0.56\ \text{kg})(0.30\ \text{m})^2 = 9.7\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

38. (a) Equation 10-33 gives

$$I_{total} = md^2 + m(2d)^2 + m(3d)^2 = 14\,md^2.$$

If the innermost one is removed then we would only obtain $m(2d)^2 + m(3d)^2 = 13\,md^2$. The percentage difference between these is (13 – 14)/14 = 0.0714 ≈ 7.1%.

(b) If, instead, the outermost particle is removed, we would have $md^2 + m(2d)^2 = 5\,md^2$. The percentage difference in this case is 0.643 ≈ 64%.

39. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(500\,\text{kg})(200\pi\ \text{rad/s})^2(1.0\,\text{m})^2 = 4.9\times10^7\ \text{J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9\times10^7\,\text{J}}{8.0\times10^3\ \text{W}} = 6.2\times10^3\ \text{s}$$

which we rewrite as $t \approx 1.0 \times 10^2$ min.

40. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O, shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = \frac{1}{2}mR^2$. The next one (using the parallel-axis theorem) has

$$I = \tfrac{1}{2}mR^2 + mh^2$$

where $h = 2R$. The third one has $I = \frac{1}{2}mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = \tfrac{2255}{2}mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = \tfrac{1}{6}(2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000\ \text{kg})(1.0000\ \text{m})^2 = 8.352\times10^{-3}\ \text{kg}\cdot\text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\,ML^2$), we find our answer to part (a) is 0.22% lower.

50. The rotational inertia is found from Eq. 10-45.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28\ \text{kg}\cdot\text{m}^2$$

51. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block m_2, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\ \text{m})}{(5.00\ \text{s})^2} = 6.00\times10^{-2}\ \text{m/s}^2.$$

Block 1 has an acceleration of $6.00\times10^{-2}\ \text{m/s}^2$ upward.

(b) Newton's second law for block 2 is $m_2 g - T_2 = m_2 a$, where m_2 is its mass and T_2 is the tension force on the block. Thus,

$$T_2 = m_2(g-a) = (0.500\ \text{kg})\left(9.8\ \text{m/s}^2 - 6.00\times10^{-2}\ \text{m/s}^2\right) = 4.87\ \text{N}.$$

(c) Newton's second law for block 1 is $m_1 g - T_1 = -m_1 a$, where T_1 is the tension force on the block. Thus,

$$T_1 = m_1(g+a) = (0.460\ \text{kg})\left(9.8\ \text{m/s}^2 + 6.00\times10^{-2}\ \text{m/s}^2\right) = 4.54\ \text{N}.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00\times10^{-2}\ \text{m/s}^2}{5.00\times10^{-2}\ \text{m}} = 1.20\ \text{rad/s}^2.$$

(e) The net torque acting on the pulley is $\tau = (T_2 - T_1)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = \frac{(T_2 - T_1)R}{\alpha} = \frac{(4.87\ \text{N} - 4.54\ \text{N})\left(5.00\times10^{-2}\ \text{m}\right)}{1.20\ \text{rad/s}^2} = 1.38\times10^{-2}\ \text{kg}\cdot\text{m}^2.$$

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\ \text{m/s}^2)}{55.0\ \text{m}}(1-\cos 35.0°)} = 0.311\ \text{rad/s}.$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g\,(1 - \cos\,\theta) = 3\,(9.80\ \text{m/s}^2)(1 - \cos 35.0°) = 5.32\ \text{m/s}^2\ .$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1 - \cos\,\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\ \text{m/s}^2)}{2}\sin 35.0° = 8.43\ \text{m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

66. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4\ \text{m/s}.$$