Name:____________________ Major:____________________ Assignment No: ____________Grade:__________

1. Find the $\lim_{x\to+\infty}\left(\frac{x+2}{x-2}\right)^{\sin x} =$ ___1___.

2. Find the inflection point of the curve $y=|\ln x|$ ___（1，0）___.

3. Let $f(x)$ is derivative at x_0, then $\lim_{h\to0}\frac{f(x_0+2h)-f(x_0-3h)}{h} =$ ___$5f'(x_0)$___.

4. If $\int_0^{x^3-1} f(t)\,dt = x$, then $f(7)=$ ___$\frac{1}{12}$___.

5. If $y=\ln\left(\sqrt{(\sin x+1)^3}\right)$, then $dy=$ ___$\frac{3}{2}\frac{\cos x}{\sin x+1}dx$___.

6. If $y=\frac{1}{1+2x}$, then $y^{(6)}(x)=$ ___$(-2)^6\frac{6!}{(1+2x)^7}$___.

7. If $f(x)=x(x-1)(x-2)(x-3)\cdots(x-2008)$, $f'(0)=$ ___2008!___.

8. $f(x)$ has one-order continuous derivative, and $f(0)=f'(0)=1$, find $\lim_{x\to0}\frac{f(\sin x)-1}{\ln f(x)} =$ ___1___.

Solu:Limit= $\lim_{x\to0}\frac{f(\sin x)-f(0)}{\sin x-0}\frac{\sin x}{x}\frac{1}{\frac{\ln f(x)-\ln f(0)}{x-0}} = f'(0)\cdot1\cdot\frac{1}{\left[\ln f(x)\right]'\big|_{x=0}} = 1$

9. Find $\lim_{x\to0}\left(1+\frac{a_1^x+a_2^x+\cdots+a_n^x-n}{n}\right)^{\frac{1}{x}}$.

Solution 1：

$$Limit = e^{\lim_{x\to0}\frac{\ln\frac{a_1^x+a_2^x+\cdots+a_n^x}{n}}{x}} = e^{\lim_{x\to0}\frac{\frac{a_1^x\ln a_1+a_2^x\ln a_2+\cdots+a_n^x\ln a_n}{a_1^x+a_2^x+\cdots+a_n^x}}{1}}$$

$$= e^{\frac{\frac{\ln a_1+\ln a_2+\cdots+\ln a_n}{n}}{1}} = \sqrt[n]{a_1\bullet a_2\bullet\cdots\bullet a_n}$$

Solution 2：

$$\because \lim_{x\to0}\frac{a_1^x+a_2^x+\cdots+a_n^x-n}{n}\bullet\frac{1}{x}$$

$$=\lim_{x\to0}\frac{a_1^x\ln a_1+\cdots+a_n^x\ln a_n}{n} = \frac{\ln a_1\bullet a_2\bullet\cdots\bullet a_n}{n}$$

$$=\ln\sqrt[n]{a_1\bullet a_2\bullet\cdots\bullet a_n}$$

$$Limit = \sqrt[n]{a_1\bullet a_2\bullet\cdots\bullet a_n}$$

10. Find $\lim\limits_{n\to\infty}\left(\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\cdots+\frac{n}{(n+n)^2}\right)$

$$\lim_{n\to\infty}\left(\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\cdots+\frac{n}{(n+n)^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{\left(1+\frac{1}{n}\right)^2}+\frac{1}{\left(1+\frac{2}{n}\right)^2}+\cdots+\frac{1}{\left(1+\frac{n}{n}\right)^2}\right)\cdot\frac{1}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{1}{\left(1+\frac{i}{n}\right)^2}\right)\cdot\frac{1}{n}$$

$$=\int_0^1\frac{1}{(1+x)^2}dx$$.

$$=\int_0^1\frac{1}{(1+x)^2}d(x+1)$$

$$=-\frac{1}{1+x}\Big|_0^1$$

$$=\frac{1}{2}.$$

11. Find $\lim\limits_{x\to+\infty}\frac{\int_0^x(\arctan t)^2dt}{\sqrt{x^2+1}}$.

An= $\frac{\pi^2}{4}$

12. Find $\lim\limits_{x\to 0}\frac{e^{\sin x}-e^{x\cos x}}{x^3}$.

$$\lim_{x\to 0}\frac{e^{\sin x}-e^{x\cos x}}{x^3}$$

$$=\lim_{x\to 0}\frac{e^{x\cos x}(e^{\sin x-x\cos x}-1)}{x^3}$$

$$=\lim_{x\to 0}e^{x\cos x}\cdot\lim_{x\to 0}\frac{(e^{\sin x-x\cos x}-1)}{x^3}$$

$$=\lim_{x\to 0}\frac{\sin x-x\cos x}{x^3}$$

$$=\lim_{x\to 0}\frac{\cos x-\cos x+x\sin x}{3x^2}$$

$$=\frac{1}{3}.$$

13. Find the maximum value of the sequence of $\{\sqrt[n]{n}\}$, and say the reason.

Solu:Let $f(x)=x^{\frac{1}{x}}$, $f'(x)=x^{\frac{1}{x}}(1-\ln x)/x_2$,

so $f'(e)=0$

when $0<x<e, f'(x)>0, f(x)$ Monotonically increasing;

when $x>e, f'(x)<0, f(x)$ Monotonically decreasing

and $2<e<3$, $\sqrt{2}<\sqrt[3]{3}$,

so the maximum value of the sequence of $\{\sqrt[n]{n}\}$ is $\sqrt[3]{3}$

14. Find the point M of the curve $y=x^2+1$, such that the distance of M and $P(5,0)$ is the shortest.

Solu: Let $|MM_0|=\rho, u=\rho^2=y^2+(x-5)^2=(x^2+1)^2+(x-5)^2$

$$\frac{du}{dx}=2(x^2+1)\cdot 2x+2(x-5)=4x^3+6x-10=0$$

We can get the unique solution $x_1=1$

since $\dfrac{d^2u}{dx^2}=12x^2+6>0$

then u Get the minimum at $x_1=1$ and $y_1=1^2+1=2$

the point $M\ (1,2)$

15. If $0<x<y<\frac{\pi}{2}$, prove $(y-x)\cos^2 y<(\tan y-\tan x)\cos^2 y\cos^2 x<(y-x)\cos^2 x$

Proof: On interval $[x,y]$, function $f(t)=\tan t$ satisfies the condition of Lagrange's theorem, so there exists

$\xi\in(x,y)$,s.t. $\tan y-\tan x=(y-x)\sec^2\xi=\dfrac{y-x}{\cos^2\xi}\in\left(\dfrac{y-x}{\cos^2 x},\dfrac{y-x}{\cos^2 y}\right)$

So $(y-x)\cos^2 y<(\tan y-\tan x)\cos^2 y\cos^2 x<(y-x)\cos^2 x$

when $0<x<y<\frac{\pi}{2}$, by the integral mean value theorem and monotonicity

$$(y-x)\frac{1}{\cos^2 x}<\int_x^y\frac{1}{\cos^2 t}dt=\frac{y-x}{\cos^2\xi}=\tan y-\tan x<\frac{1}{\cos^2 y}(y-x),\xi\in[x,y]$$

16. $\varphi(x)$ has two-order continuous derivative and $\varphi(0)=0$, and $f(x)=\begin{cases}\dfrac{\varphi(x)}{x}, x\neq 0\\ a, x=0\end{cases}$

(1) Find a, such that $f(x)$ is continuous in $(-\infty,+\infty)$; (2) Find $f'(x)$

Solu: (1)since $f(x)$ is continuous in $(-\infty,+\infty)$

$$a=f(0)=\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\varphi(x)-\varphi(0)}{x-0}=\varphi'(0)$$

(2)when $x\neq 0$, $f'(x)=\dfrac{x\varphi'(x)-\varphi(x)}{x^2}$

and $f'(0)=\lim\limits_{x\to 0}\dfrac{f(x)-f(0)}{x-0}=\lim\limits_{x\to 0}\dfrac{\dfrac{\varphi(x)}{x}-\varphi'(0)}{x-0}=\lim\limits_{x\to 0}\dfrac{\varphi(x)-x\varphi'(0)}{x^2}$

$$=\lim_{x\to 0}\frac{\varphi'(x)-\varphi'(0)}{2x}=\frac{1}{2}\varphi''(0)$$

So $f'(x)\begin{cases}\dfrac{x\varphi'(x)-\varphi(x)}{x^2}, x\neq 0\\ \dfrac{1}{2}\varphi''(0), x=0\end{cases}$

17. If $f(x)=\begin{cases}\dfrac{e^{2x}-1}{x}, x\neq 0\\ 2, x=0\end{cases}$, determine $f(x)$ and $f'(x)$ is continuous at $x=0$ or not.

Solu : since $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}=2=f(0)$, $f(x)$ continuous at $x=0$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\dfrac{e^{2x}-1}{x}-2}{x}=\lim_{x\to 0}\frac{e^{2x}-1-2x}{x^2}=\lim_{x\to 0}\frac{2e^{2x}-2}{2x}=2$$

when $x\neq 0$,

$$f'(x)=\frac{2xe^{2x}-(e^{2x}-1)}{x^2}, \lim_{x\to 0}f'(x)=\lim_{x\to 0}\frac{2xe^{2x}-(e^{2x}-1)}{x^2}=\lim_{x\to 0}\frac{4xe^{2x}}{2x}=2=f'(0)$$

so $f'(x)$ continuous at $x=0$