Sit NO. ____ Major ____ School ____ Student ID ____ Chinese Name ____

**Be honest, cheating on exams will lead to serious consequences!**

# Middle Test for 《Calculus II》

2022-2023 2nd Semester

**Note:1. Please clearly write down the private information needed before exam.**

**2.Please clearly write down the answers on the paper or answer sheet.**

**3.Full mark: 100**

**4.Duration of Exam: 95 minutes**

**Class: ____ : Name: ____ Homework No.: ____**

| Question Number | 1-5 | 6-9 | 10-113 | 14-15 | Total score |
|---|---|---|---|---|---|
| Score | $6'\times 5=30'$ | $6'\times 4=24'$ | $6'\times 4=24'$ | $11'\times 2=22'$ | |
| Score | | | | | |

1. In the parametric equation $\begin{cases} x=2e^t+t+1 \\ y=4(t-1)e^t+t^2 \end{cases}$, please find $\left.\dfrac{d^2y}{dx^2}\right|_{t=0}$ .

2. Find the tangent plane of the surface $z=x+2y+\ln(1+x^2+y^2)$ at point $(0,0,0)$ .

3. The maximum directional derivative of $f = x^2 + 2y^2$ at $(0,1)$

4. Find the convergence field and summation function of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$.

5. Find the power series of $x$ for the function $f(x) = \ln \frac{1+x}{1-x}$.

6. Find the maximum and minimum values of $u = x - 2y + 2z$ subject to the condition $x^2 + y^2 + z^2 = 1$.

7. Let $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & x^2 + y^2 \neq 0 \\ 0 & x^2 + y^2 = 0 \end{cases}$

Prove：1）the partial derivatives of $f(x, y)$ exist at (0,0);

2） $f(x, y)$ is not differentiable at $(0, 0)$.

8. Let $z = xf\left(xy\ ,\ \dfrac{y}{x}\right)$, $f$ has second-order continuous partial derivatives, find $\dfrac{\partial z}{\partial y}$ , $\dfrac{\partial^2 z}{\partial y \partial x}$

9. Let $z = f(u, x, y), \quad u = xe^y$, and $f$ has second-order continuous partial derivatives, find $\dfrac{\partial^2 z}{\partial x \partial y}$.

10. Let $z = f(2x - y, y\sin x)$, and $f$ has second-order continuous partial derivatives，find $\dfrac{\partial^2 z}{\partial x \partial y}$.

11. Let $G(u\ ,\ v)$ is differentiable, and the equation $G\left(\dfrac{x}{z}\ ,\ \dfrac{y}{z}\right) = 0$ implying $z = z(x, y)$, compute $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$.

12. Let $z = xyf\left(\frac{y}{x}\right)$, and $f$ is derivative, if $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = y^2(\ln y - \ln x)$ find $f(1), f'(1)$.

13. Let $f$ is differential, and $f(x+1, e^x) = x(x+1)^2$, $f(x, x^2) = 2x^2 \ln x$, find $df(1,1)$.

14.The convergence field is $(a,+\infty)$ of series $\sum_{n=1}^{\infty}\frac{n!}{n^n}e^{-nx}$, find $a$.

15. Let $u_n(\mathrm{x}) = e^{-nx} + \frac{1}{n(n+1)}x^{n+1}(n=1,2,\cdots)$, find convergence field and sum function of $\sum_{n=1}^{\infty}u_n(x)$.