Name______ Student ID ______ School______ Major/Class______ Seat No.______

(DONNOT WRITE YOUR ANSWER IN THIS AREA)

Seal Line

WARNING: MISBEHAVIOR AT EXAM TIME WILL LEAD TO SERIOUS CONSEQUENCE.

[redacted]

Linear Algebra and Geometry [redacted]

Notice:
1. Make sure that you have filled the form on the left side of seal line.
2. Write your answers on the exam paper.
3. This is a close-book exam.
4. The exam with full score of 100 points lasts 120 minutes.

Question No.	I	II	III	IV	V	VI	VII	VIII	Sum
Score									

1. (12 points) Let

$$A = \begin{bmatrix} 1 & 3 & 2 & -7 \\ -2 & -2 & -8 & 6 \\ 2 & 3 & 7 & 1 \\ 3 & 4 & 11 & -7 \end{bmatrix}.$$

(1) (6 points) Find a set of basis for col A.

(2) (3 points) What's rank A?

(3) (3 points) Is the matrix equation $A\mathbf{X} = \mathbf{b}$ consistent for all possible $\mathbf{b}$?

2. (10 points) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2, x_3) = (x_1 - 3x_2 + 2x_3, -2x_1 + 7x_2 + x_3, -4x_1 + 6x_2 + hx_3)$ with a scalar h. Note that x_1, x_2 and x_3 are entries in a $\mathbb{R}^3$ vector.

(1) (4 points) Find the standard matrix of T.

(2) (6 points) For what values of h the linear transformation T maps $\mathbb{R}^3$ onto $\mathbb{R}^3$.

3. (10 points) Let $A = \begin{bmatrix} 0 & 4 & 5 & -6 \\ -3 & -6 & 2 & 3 \\ 3 & 10 & 0 & 1 \\ 3 & 14 & 6 & -8 \end{bmatrix}$

(1) (7 points) Find the determinant of A.

(2) (3points) Is adj A invertible?

4. (20 points) Let $\boldsymbol{u}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $\boldsymbol{u}_2 = \begin{bmatrix}-4\\-1\\2\end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix}1\\3\\-7\end{bmatrix}$ and $W = \text{span}\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$.

(1) (4 points) what's the distance between $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$?

(2) (4 points) Are $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ orthogonal?

(3) (6 points) Find the distance from $\boldsymbol{y}$ to the subspace W.

(4) (6 points) Find a set a basis for the orthogonal compliment $\mathrm{W}^{\perp}$ of W.

5. (15 points) Let $\boldsymbol{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{b}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\boldsymbol{b}_3 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, $\boldsymbol{c}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\boldsymbol{c}_2 = \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$, $\boldsymbol{c}_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$ and consider the basis for $\mathbb{R}^3$ given by $\mathrm{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$ and $\mathrm{C} = \{\boldsymbol{c}_1, \boldsymbol{c}_2, \boldsymbol{c}_3\}$.

(1) (6 points) Find $[\boldsymbol{X}]_B$, the B-coordinate vector of $\mathbf{X} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$.

(2) (6 points) Find the change-of-coordinate matrix from B to C: $P_{C \leftarrow B}$

(3) (3 points) Find $[\boldsymbol{X}]_C$, the C-coordinate vector of $\mathbf{X} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

6. (16 points) For the following quadratic form

$$\boldsymbol{x}^T A\boldsymbol{x} = 2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3$$

(1) (4 points)Give the matrix A of the quadratic form and indicate which type this quadratic form is? (For example, positive definite, negative definite or indefinite).

(2) (12 points) Find an orthogonal matrix P such that the change of variable $\boldsymbol{x} = P\boldsymbol{y}$ transforms $\boldsymbol{x}^T A\boldsymbol{x}$ into a new quadratic form with no cross-product term.

7. (9 points) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -5 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 4 & 3 \end{bmatrix}$

(1) (6 points) Compute A^{-1} and B^{-1}.

(2) (3 points) Find a matrix X such that $AXB = C$.

8. (8 points) Let A be an $n \times n$ matrix. Show that if A has an eigenvalue 0, so is A^2.