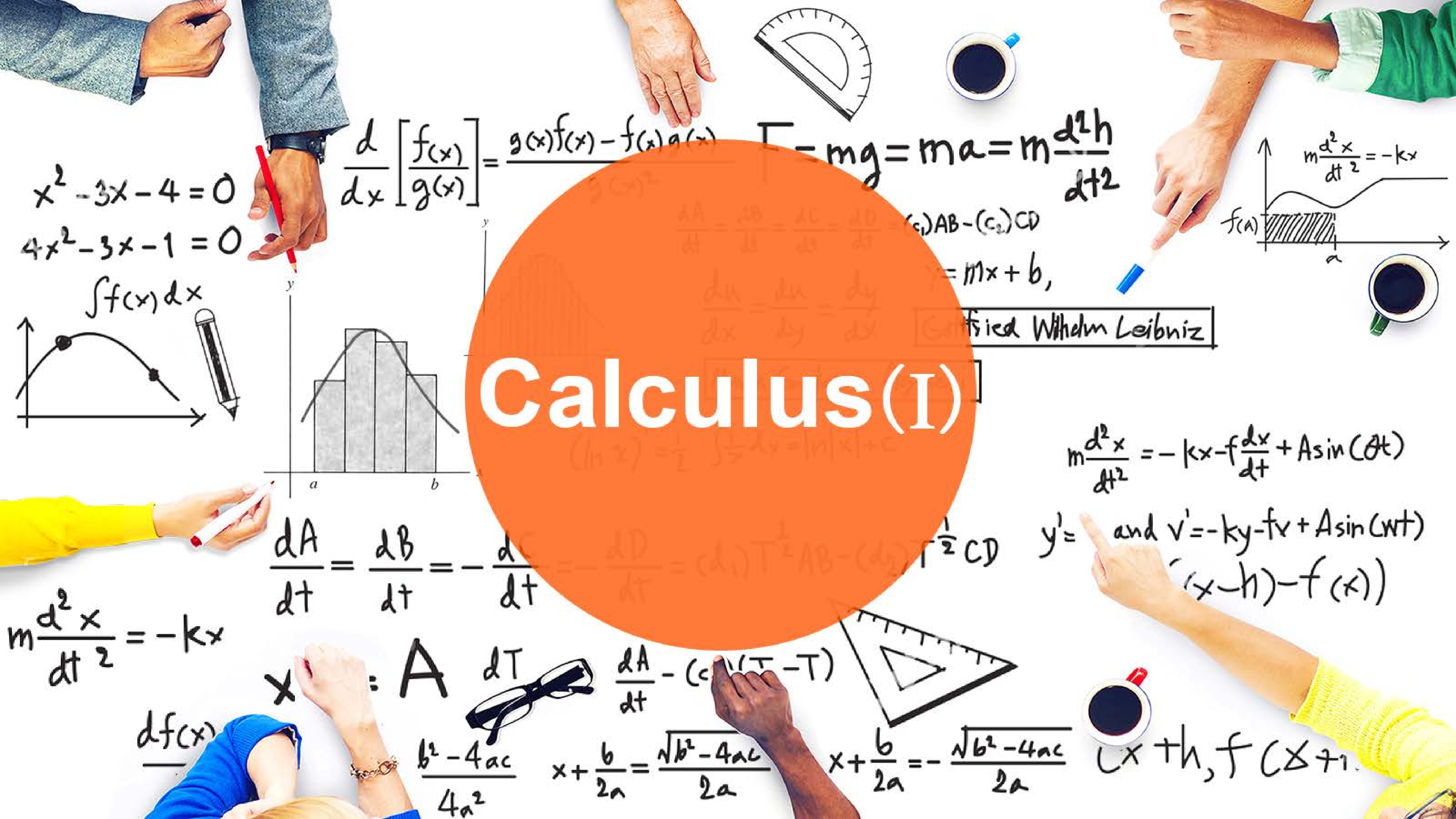
$x^2-3x-4=0$
$4x^2-3x-1=0$
$\int f(x)\,dx$
$m\frac{d^2x}{dt^2}=-kx$
$m\frac{d^2x}{dt^2}=-kx-f\frac{dx}{dt}+A\sin(\theta t)$
$\frac{dA}{dt}=\frac{dB}{dt}=-$

Calculus(I)

Summary

Lecturer: Xue Deng

Summary

1. 极限

- 等价无穷小，
- 特殊重要极限，
- 极限定义，
- 洛必达法则，
- 数学语言证明。

2. 导数：

- 导数定义，
- 参数方程求导，
- 隐函数求导法，
- 微分，
- 高阶求导。

导数应用：

- 极大极小值，
- 单调和凹凸性，
- 微分中值定理，
- 函数性质（凹凸性）
- +描图。

3. 积分

不定积分:

- 换元积分,
- 三角函数积分,
- 分部积分,
- 根号函数积分。

定积分:

- 积分中值定理,
- 定积分计算，Sinx,
- 积分上限函数。

定积分应用:

- 面积、体积、弧长,
- 广义积分

（无穷区间，无界函数）。

4. 综合

- 积分和导数（证明）；
- 连续和极限定义;
- 连续和导数的定义。

18=Questions+Proof (2)
60+25+15

Proof

$$\lim_{x\to a} f(x) = L(>0) \Rightarrow \lim_{x\to a} \frac{1}{f(x)} = \frac{1}{L}.$$

Proof :

$$\left(\begin{array}{l} \lim_{x\to a} f(x) \\ \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, s.t. 0 < |x-a| < \delta, \text{we have } |f(x) - L| < \varepsilon. \\ \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, s.t. 0 < |x-a| < \delta, \text{we have } |f(x) - L| < 2\varepsilon. \\ \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, s.t. 0 < |x-a| < \delta, \text{we have } |f(x) - L| < 3\varepsilon. \\ \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0, s.t. 0 < |x-a| < \delta, \text{we have } |f(x) - L| < M\varepsilon. \end{array} \right)$$

Proof

$\because \lim\limits_{x \to a} f(x),$

$\therefore \forall \varepsilon > 0, \exists \delta_1 > 0, s.t. 0 < |x-a| < \delta_1,$ we have $|f(x)-L| < \varepsilon.$

$\because$ take $\varepsilon = \dfrac{L}{2} > 0, \exists \delta_2 > 0, s.t. 0 < |x-a| < \delta_2,$ we have $|f(x)-L| < \varepsilon = \dfrac{L}{2}.$

$$|f(x)-L| < \varepsilon = \frac{L}{2} \Rightarrow |f(x)| > \frac{L}{2} \Rightarrow \frac{1}{|f(x)|} < \frac{2}{L}, \delta = \min\{\delta_1, \delta_2\}$$

$$\therefore \left|\frac{1}{f(x)} - \frac{1}{L}\right| = \left|\frac{f(x)-L}{f(x)L}\right| = \frac{|f(x)-L|}{|f(x)|L} < |f(x)-L| \cdot \frac{1}{L} \cdot \frac{2}{L} = \frac{2}{L^2}|f(x)-L| < \frac{2}{L^2}\varepsilon,$$

namely, $\lim\limits_{x \to a} \dfrac{1}{f(x)} = \dfrac{1}{L}.$

(1) $\lim\limits_{x\to 4}(3x-7)=5$

$$\lim_{x\to 4}(3x-7)=5$$

$\left(Analysis:\right)\forall\varepsilon>0$, we need $|3x-7-5|<\varepsilon\Rightarrow 3|x-4|<\varepsilon\Rightarrow|x-4|<\dfrac{\varepsilon}{3}$, Take $\delta=\dfrac{\varepsilon}{3}$.

$\forall\varepsilon>0$, we take $\delta=\dfrac{\varepsilon}{3}$, when $|x-4|<\delta=\dfrac{\varepsilon}{3}$, we always have

$|3x-7-5|=3|x-4|<3\cdot\dfrac{\varepsilon}{3}=\varepsilon$, namely

$$\lim_{x\to 4}(3x-7)=5.$$

(2)

$$\cos t \le \frac{t}{\sin t} \le 2-\cos t \qquad \lim_{t\to 0}\frac{e^{\sin^2 t}-e^{t^2}}{\sin^2 t - t^2}$$

$$\lim_{t\to 0}\cos t \le \lim_{t\to 0}\frac{t}{\sin t} \le \lim_{t\to 0}(2-\cos t) \Rightarrow \lim_{t\to 0}\frac{t}{\sin t}=1.$$

$$\lim_{t\to 0}\frac{e^{\sin^2 t}-e^{t^2}}{\sin^2 t - t^2}=\lim_{t\to 0}\frac{e^{t^2}(e^{\sin^2 t - t^2}-1)}{\sin^2 t - t^2}$$

$$=\lim_{t\to 0}e^{t^2}\cdot\lim_{t\to 0}\frac{e^{\sin^2 t - t^2}-1}{\sin^2 t - t^2}=1\cdot 1=1.$$

(3)

$$\lim_{x\to\infty}\left(\frac{x-5}{x+5}\right)^{x}$$

$$=\lim_{x\to\infty}\left(1+\frac{x-5}{x+5}-1\right)^{x}$$

$$=\lim_{x\to\infty}\left(1+\frac{-10}{x+5}\right)^{\frac{x+5}{-10}\cdot\frac{-10}{x+5}\cdot x}$$

$$=e^{\lim\limits_{x\to\infty}\frac{-10}{x+5}\cdot x}=e^{-10}.$$

(4) $f(x)$, $f(a)>0$ continuous and differentiable

$$\lim_{n\to\infty}\left[\frac{f(a+\frac{1}{n})}{f(a)}\right]^{n}=\lim_{x\to+\infty}\left[\frac{f(a+\frac{1}{x})}{f(a)}\right]^{x}=\lim_{x\to+\infty}\left[1+\frac{f(a+\frac{1}{x})-f(a)}{f(a)}\right]^{x}$$

$$=\lim_{x\to+\infty}\left[1+\frac{f(a+\frac{1}{x})-f(a)}{f(a)}\right]^{\frac{f(a)}{f(a+\frac{1}{x})-f(a)}\cdot\frac{f(a+\frac{1}{x})-f(a)}{f(a)}\cdot x}$$

$$=e^{\lim\limits_{x\to+\infty}\frac{f(a+\frac{1}{x})-f(a)}{f(a)\frac{1}{x}}}=e^{\lim\limits_{h\to0}\frac{f(a+h)-f(a)}{h}\cdot\frac{1}{f(a)}}=e^{\frac{f'(a)}{f(a)}}.$$

(5)

$$f(x)=\begin{cases}\dfrac{x}{1+e^{\frac{1}{x}}}, & x\neq 0\\ 0, & x=0\end{cases}$$

(1) $f(0)=0$;

(2) $\lim\limits_{x\to 0+} f(x)=\lim\limits_{x\to 0+}\dfrac{x}{1+e^{\frac{1}{x}}}=0$

$\lim\limits_{x\to 0-} f(x)=\lim\limits_{x\to 0-}\dfrac{x}{1+e^{\frac{1}{x}}}=\dfrac{0}{1}=0$

$\lim\limits_{x\to 0} f(x)=0$;

(3) $\lim\limits_{x\to 0} f(x)=0=f(0)$.

It is continuous at point 0.

(6)

$$f(x)=\begin{cases} e^{-1/x^2} & x\neq 0 \\ 0 & x=0 \end{cases}$$

(1) $x=0$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}=\lim_{x\to 0}\frac{e^{-1/x^2}}{x}=\lim_{t\to\infty} t\cdot e^{-t^2}$$

$$=\lim_{t\to\infty}\frac{t}{e^{t^2}}=\lim_{t\to\infty}\frac{1}{2te^{t^2}}=0.$$

(2) $x\neq 0$

$$f'(x)=\left(e^{-1/x^2}\right)'=2x^{-3}e^{-1/x^2}.$$

(3) $\lim\limits_{x\to 0} f'(x)=\lim\limits_{x\to 0} 2x^{-3}e^{-1/x^2}=\lim\limits_{t\to\infty} 2t^3e^{-t^2}=\lim\limits_{t\to\infty}\frac{2t^3}{e^{t^2}}=0=f'(0).$

It is continuous at 0.

(7)

$$\lim_{n\to\infty}\left(\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\ldots+\frac{n}{n^2+n+n}\right)^{\frac{1}{n}}$$

$$f(n)=\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\ldots+\frac{n}{n^2+n+n}$$

$$\geq\frac{1}{n^2+n+n}+\frac{2}{n^2+n+n}+\ldots+\frac{n}{n^2+n+n}$$

$$=\frac{1+2+\ldots+n}{n^2+n+n}=\frac{\frac{1}{2}n(n+1)}{n^2+n+n}\to\frac{1}{2}.$$

$$f(n)=\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\ldots+\frac{n}{n^2+n+n}$$

$$\leq\frac{1}{n^2+n+1}+\frac{2}{n^2+n+1}+\ldots+\frac{n}{n^2+n+1}$$

$$=\frac{1+2+\ldots+n}{n^2+n+1}=\frac{\frac{1}{2}n(n+1)}{n^2+n+1}\to\frac{1}{2}.$$

By squeeze Theorem,

$$\lim_{n\to\infty}\left(\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\ldots+\frac{n}{n^2+n+n}\right)=\frac{1}{2}$$

$$\lim_{n\to\infty}\left(\frac{1}{n^2+n+1}+\frac{2}{n^2+n+2}+\ldots+\frac{n}{n^2+n+n}\right)^{\frac{1}{n}}$$

$$=\lim_{n\to\infty}\left(\frac{1}{2}\right)^{\frac{1}{n}}=1.$$

(8)

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2+1^2}}+\frac{1}{\sqrt{n^2+2^2}}+\dots+\frac{1}{\sqrt{n^2+n^2}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{1}{\sqrt{1+\left(\frac{1}{n}\right)^2}}+\frac{1}{\sqrt{1+\left(\frac{2}{n}\right)^2}}+\dots+\frac{1}{\sqrt{1+\left(\frac{n}{n}\right)^2}}\right)\frac{1}{n}$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{\sqrt{1+\left(\frac{i}{n}\right)^2}}\cdot\frac{1}{n}$$

$$=\int_0^1\frac{1}{\sqrt{1+x^2}}dx\left(x=\tan t, x:0\to 1, t:0\to\frac{\pi}{4}\right)$$

$$=\int_0^{\frac{\pi}{4}}\frac{1}{\sqrt{1+\tan^2 t}}d\tan t=\int_0^{\frac{\pi}{4}}\frac{\sec^2 t}{\sec t}dt=\int_0^{\frac{\pi}{4}}\frac{1}{\cos t}dt=\int_0^{\frac{\pi}{4}}\frac{\cos t}{\cos^2 t}dt$$

$$=\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin^2 t}d\sin t=\int_0^{\frac{\pi}{4}}\frac{1}{(1+\sin t)(1-\sin t)}d\sin t$$

$$=\frac{1}{2}\int_0^{\frac{\pi}{4}}\left(\frac{1}{1+\sin t}+\frac{1}{1-\sin t}\right)d\sin t=\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{1}{1+\sin t}d(1+\sin t)-\frac{1}{2}\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin t}d(1-\sin t)$$

$$=\frac{1}{2}\ln\frac{1+\sin t}{1-\sin t}\Bigg|_0^{\frac{\pi}{4}}=\ln(1+\sqrt{2}).$$

OR $x=\tan t\Rightarrow t=\arctan x$

$$\frac{1}{2}\ln\frac{1+\sin t}{1-\sin t}=\frac{1}{2}\ln\frac{(1+\sin t)^2}{\cos^2 t}=\ln\frac{1+\sin t}{\cos t}=\ln(\mathrm{sect}+\mathrm{tant})$$

$$=\ln(\sqrt{1+\tan^2 t}+\mathrm{tant})=\ln(\sqrt{1+x^2}+x)$$

$$\Rightarrow\ln(\sqrt{1+x^2}+x)\Big|_0^1=\ln(1+\sqrt{2}).$$

(9)

$$\lim_{x\to 0}\frac{\int_0^{x^2} te^t\,dt}{\int_0^x x^2\sin t\,dt}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\left(\int_0^{x^2} te^t\,dt\right)'}{\left(x^2\cdot\int_0^x \sin t\,dt\right)'}=\lim_{x\to 0}\frac{x^2e^{x^2}\cdot 2x}{2x\cdot\int_0^x \sin t\,dt+x^2\cdot\sin x}$$

$$=\lim_{x\to 0}\frac{2x^2e^{x^2}}{2\int_0^x \sin t\,dt+x\cdot\sin x}=\lim_{x\to 0}\frac{4xe^{x^2}+4x^3e^{x^2}}{2\sin x+\sin x+x\cos x}$$

$$=\lim_{x\to 0}\frac{4e^{x^2}(x+x^3)}{3\sin x+x\cos x}=4\lim_{x\to 0}\frac{x+x^3}{3\sin x+x\cos x}$$

$$=4\lim_{x\to 0}\frac{1+x^2}{3\frac{\sin x}{x}+\cos x}=4\cdot\frac{1}{4}=1.$$

(10) $\lim\limits_{x\to 0}\left(\frac{\ln(x+1)}{x}\right)^{\frac{1}{\sin x}}$

$$=\lim_{x\to 0}\left(1+\frac{\ln(x+1)-x}{x}\right)^{\frac{1}{\sin x}}$$

$$=\lim_{x\to 0}\left(1+\frac{\ln(x+1)-x}{x}\right)^{\frac{x}{\ln(x+1)-x}\cdot\frac{\ln(x+1)-x}{x}\cdot\frac{1}{\sin x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{\ln(x+1)-x}{x}\cdot\frac{1}{\sin x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{\ln(x+1)-x}{x^2}}=e^{\lim\limits_{x\to 0}\frac{\frac{1}{x+1}-1}{2x}}=e^{\lim\limits_{x\to 0}\frac{\frac{-x}{x+1}}{2x}}$$

$$=e^{\lim\limits_{x\to 0}\frac{-1}{2(x+1)}}=e^{-\frac{1}{2}}.$$

(12) f is continuous $\lim_{x\to1}\frac{f(x)}{x-1}=5$ $f'(1)$ *and* $\lim_{x\to0}\frac{f(\frac{\sin x}{x})}{\ln(x^2+1)}$

$$\lim_{x\to1}\frac{f(x)}{x-1}=5\Rightarrow f(1)=\lim_{x\to1}f(x)=0$$

$$(1)\ f'(1)=\lim_{x\to1}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1}\frac{f(x)}{x-1}=5.$$

$$(2)\lim_{x\to0}\frac{f(\frac{\sin x}{x})}{\ln(x^2+1)}=\lim_{x\to0}\frac{f(\frac{\sin x}{x})}{\frac{\sin x}{x}-1}\cdot\frac{\frac{\sin x}{x}-1}{\ln(x^2+1)}=\lim_{x\to0}\frac{f(\frac{\sin x}{x})-f(1)}{\frac{\sin x}{x}-1}\cdot\frac{\frac{\sin x}{x}-1}{x^2}=f'(1)\lim_{x\to0}\frac{\sin x-x}{x^3}=-\frac{5}{6}.$$

(13) $y = x + e^{x(2-y)}$ $\lim\limits_{n\to\infty} n[f(\frac{1}{n}) - 1]$

$y = x + e^{x(2-y)}$

$(1)\, y(0) = 0 + e^{0(2-y)} = 1 \Rightarrow f(0) = 1.$

$(2)\, y' = 1 + [2 - y + x(-y')]e^{x(2-y)}$

$y'(0) = 1 + [2 - y(0) + 0(-y')]e^{0(2-y)}$

$\Rightarrow y'(0) = 1 + [2-1] = 2 \Rightarrow f'(0) = 2.$

$\lim\limits_{x\to+\infty} x[f(\frac{1}{x}) - 1] (t = \frac{1}{x} \to 0)$

$= \lim\limits_{t\to 0} \frac{[f(t) - 1]}{t} = \lim\limits_{t\to 0} \frac{[f(t) - f(0)]}{t - 0}$

$= f'(0) = 2.$

$\lim\limits_{n\to\infty} n[f(\frac{1}{n}) - 1] = 2.$

(14)

$$\lim_{x\to 0}\frac{\int_0^{\sin x}\frac{t^2}{\sqrt{a+t^2}}dt}{bx-\sin x}=2$$

We have to check $\lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}\cdot\cos x}{b-\cos x}$ *exist*

$\left(note: If \lim_{x\to 0}\frac{\int_0^{\sin x}\frac{t^2}{\sqrt{a+t^2}}dt}{bx-\sin x}=-1, \lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}\cdot\cos x}{b-\cos x}\right.$ $\left. not\ exist,\ but\ we\ have\ solution\ a=0, b=0\right)$

If b $\neq 1, \lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}\cdot\cos x}{b-\cos x}=0$

If b=1,a $\neq 0, \lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}\cdot\cos x}{b-\cos x}=\frac{2}{\sqrt{a}}$

If b=1,a=0, $\lim_{x\to 0}\frac{\int_0^{\sin x}\frac{t^2}{\sqrt{a+t^2}}dt}{bx-\sin x}\neq 2(contradicts)$

(14)

$$\therefore \lim_{x\to 0}\frac{\int_0^{\sin x}\frac{t^2}{\sqrt{a+t^2}}dt}{bx-\sin x}=\lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}\cdot\cos x}{b-\cos x}=2$$

$$\lim_{x\to 0}\cos x\cdot\lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}}{b-\cos x}=2$$

$$1\cdot\lim_{x\to 0}\frac{\frac{\sin^2 x}{\sqrt{a+\sin^2 x}}}{b-\cos x}=2$$

$$\therefore \lim_{x\to 0}\frac{x^2}{\sqrt{a+\sin^2 x}\cdot(b-\cos x)}=2$$

$$\therefore \lim_{x\to 0}\frac{x^2}{\sqrt{a}\cdot(b-\cos x)}=2$$

$$\Rightarrow b-\cos x\to 0\Rightarrow b=1;$$

$$\Rightarrow \lim_{x\to 0}\frac{x^2}{\sqrt{a}\cdot(1-\cos x)}=\lim_{x\to 0}\frac{x^2}{\sqrt{a}\cdot\frac{x^2}{2}}=2$$

$$\Rightarrow \lim_{x\to 0}\sqrt{a}=1\Rightarrow a=1.$$

(15) $\lim\limits_{x\to\infty}\left(\dfrac{x+a}{x-a}\right)^x=\displaystyle\int_{-\infty}^{a} te^{2t}dt$

$$\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x=\int_{-\infty}^{a} te^{2t}dt$$

$$\because \lim_{x\to\infty}\left(1+\frac{2a}{x-a}\right)^{\frac{x-a}{2a}\cdot\frac{2a}{x-a}\cdot x}=\lim_{s\to-\infty}\int_{s}^{a} te^{2t}dt$$

$$\text{Left}=e^{\lim\limits_{x\to\infty}\frac{2a}{x-a}\cdot x}=e^{2a};$$

$$\because \frac{1}{4}\int_{s}^{a} 2te^{2t}d2t=\frac{1}{4}\int_{2s}^{2a} ue^{u}du$$

$$=\frac{1}{4}\int_{2s}^{2a} ude^{u}=\frac{1}{4}\left(ue^{u}\Big|_{2s}^{2a}-\int_{2s}^{2a} e^{u}du\right)$$

$$=\frac{1}{4}e^{u}(u-1)\Big|_{2s}^{2a}$$

$$=\frac{1}{4}e^{2a}(2a-1)-\frac{1}{4}e^{2s}(2s-1)$$

$$\text{Right}=\lim_{s\to-\infty}\left(\frac{1}{4}e^{2a}(2a-1)-\frac{1}{4}e^{2s}(2s-1)\right)$$

$$=\frac{1}{4}e^{2a}(2a-1)-\frac{1}{4}\lim_{s\to-\infty}\frac{2s-1}{e^{-2s}}$$

$$=\frac{1}{4}e^{2a}(2a-1)-\frac{1}{4}\lim_{s\to-\infty}\frac{2}{-2e^{-2s}}$$

$$=\frac{1}{4}e^{2a}(2a-1)-0=\frac{1}{4}e^{2a}(2a-1).$$

$$\text{Left}=\text{Right}$$

$$\Rightarrow e^{2a}=\frac{1}{4}e^{2a}(2a-1)\Rightarrow a=\frac{5}{2}.$$

(16)

$$\int_0^{\frac{\pi}{2}} \cos^n t dt$$

$$I_n = \int_0^{\frac{\pi}{2}} \cos^n x dx = \int_0^{\frac{\pi}{2}} \cos^{n-1} x d\sin x$$

$$= \left[\sin x \cos^{n-1} x\right]_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \sin^2 x \cos^{n-2} x dx$$

$$= (n-1)\int_0^{\frac{\pi}{2}} (1-\cos^2 x)\cos^{n-2} x dx$$

$$= (n-1)\int_0^{\frac{\pi}{2}} \cos^{n-2} x dx - (n-1)\int_0^{\frac{\pi}{2}} \cos^n x dx$$

$$= (n-1)I_{n-2} - (n-1)I_n$$

$$I_n = \frac{n-1}{n} I_{n-2}, I_0 = \int_0^{\frac{\pi}{2}} 1 dx = \frac{\pi}{2}, I_1 = \int_0^{\frac{\pi}{2}} \cos x dx = 1.$$

$$I_n = \begin{cases} \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot I_0 = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} (even) \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{2}{3}\cdot I_1 = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdot\cdots\cdot\frac{2}{3}\cdot 1. \quad (odd) \end{cases}$$

(17)

$$\lim_{x\to\infty}\left[x+x^2\ln(1-\frac{1}{x})\right](t=\frac{1}{x}\to 0)$$

$$=\lim_{t\to 0}\left[\frac{1}{t}+\frac{\ln(1-t)}{t^2}\right]$$

$$=\lim_{t\to 0}\frac{t+\ln(1-t)}{t^2}$$

$$=\lim_{t\to 0}\frac{1+\frac{-1}{1-t}}{2t}$$

$$=\lim_{t\to 0}\frac{\frac{-t}{1-t}}{2t}=\lim_{t\to 0}\frac{1}{2(t-1)}=-\frac{1}{2}.$$

(18)

$$\lim_{x\to\infty}\left[3x-\sqrt{ax^2+bx+1}\right]=2$$

$$\lim_{x\to\infty}\frac{\left[3x-\sqrt{ax^2+bx+1}\right]\left[3x+\sqrt{ax^2+bx+1}\right]}{\left[3x+\sqrt{ax^2+bx+1}\right]}=2$$

$$\lim_{x\to\infty}\frac{9x^2-ax^2-bx-1}{\left[3x+\sqrt{ax^2+bx+1}\right]}=2$$

$$\Rightarrow 9x^2-ax^2=0\Rightarrow a=9;$$

$$\lim_{x\to\infty}\frac{9x^2-9x^2-bx-1}{\left[3x+\sqrt{9x^2+bx+1}\right]}=2$$

$$\Rightarrow\lim_{x\to\infty}\frac{-bx-1}{\left[3x+\sqrt{9x^2+bx+1}\right]}=2$$

$$\Rightarrow\lim_{x\to\infty}\frac{-bx}{6x}=2\Rightarrow b=-12.$$

$$\Rightarrow\begin{cases}a=9,\\ b=-12.\end{cases}$$

(19)

$$\because \lim_{x\to\infty}[\sqrt[3]{1-x^3}-(\alpha x+\beta)]=0$$

$$\therefore \lim_{x\to\infty}[\sqrt[3]{1-x^3}-(\alpha x+\beta)]\cdot\frac{1}{x}=0$$

$$\lim_{x\to\infty}[\frac{\sqrt[3]{1-x^3}}{x}-\alpha-\frac{\beta}{x}]=0$$

$$\Rightarrow \alpha=\lim_{x\to\infty}\frac{\sqrt[3]{1-x^3}}{x}=\lim_{x\to\infty}\sqrt[3]{\frac{1-x^3}{x^3}}=-1.$$

$$\Rightarrow \beta=\lim_{x\to\infty}[\sqrt[3]{1-x^3}+x]=\lim_{t\to0}[\sqrt[3]{\frac{t^3-1}{t^3}}+\frac{1}{t}]$$

$$=\lim_{t\to0}[\frac{\sqrt[3]{t^3-1}+1}{t}]=\lim_{t\to0}[\frac{\frac{1}{3}(t^3-1)^{-\frac{2}{3}}(3t^2)}{1}]=\lim_{t\to0}[\frac{t^2}{(t^3-1)^{\frac{2}{3}}}]=0.$$

(20)

$$\lim_{x\to+\infty}\left(\frac{x+2}{x-2}\right)^{\sin x}$$

$$=\lim_{x\to+\infty}\left(1+\frac{x+2-x+2}{x-2}\right)^{\sin x}$$

$$=\lim_{x\to+\infty}\left(1+\frac{4}{x-2}\right)^{\frac{x-2}{4}\cdot\frac{4}{x-2}\cdot\sin x}$$

$$=e^{\lim\limits_{x\to+\infty}\frac{4}{x-2}\cdot\sin x}=e^{0}=1.$$

(21) $y=\left(\frac{2006^x+12^x+29^x+9^x}{4}\right)^{\frac{1}{x}}$

$$\lim_{x\to 0^+}\ln y=\lim_{x\to 0^+}\frac{\ln(2006^x+12^x+29^x+9^x)-\ln 4}{x}\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0^+}\frac{\frac{1}{2006^x+12^x+29^x+9^x}(2006^x\ln 2006+12^x\ln 12+29^x\ln 29+9^x\ln 9)}{1}$$

$$=\frac{1}{4}(\ln 2006+\ln 12+\ln 29+\ln 9)$$

$$=(\ln 2006\cdot 12\cdot 29\cdot 9)^{\frac{1}{4}}$$

$$\Rightarrow \lim_{x\to 0^+}\ln y=\ln\left(2006\cdot 12\cdot 29\cdot 9\right)^{\frac{1}{4}}$$

$$\Rightarrow \lim_{x\to 0^+}y=e^{\ln\left(2006\cdot 12\cdot 29\cdot 9\right)^{\frac{1}{4}}}=\left(2006\cdot 12\cdot 29\cdot 9\right)^{\frac{1}{4}}.$$

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