EECE5644 – Introduction to Machine Learning and Pattern Recognition

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Assignment – 3

Question 1:

For this question many multilayer perceptron(MLP) were used to approximate or estimate the class posteriors. Minimum average cross-entropy was used to train the MLP, and the trained models were then used to approximate a MAP(Maximum-a-posteriori) classification to achieve the minimum probability of error(P_e) on the validation dataset.

Here we are supposed to use 4 classes with uniform priors as given and gaussian class conditional pdfs.

P(L=*l)* = 0.2 and

l= [0,1,2,3]

$\boldsymbol{m}0$= [1 1 0], $\boldsymbol{C}0$= [1.00 0.01 0.01, 0.01 1.02 0.03, 0.01 0.03 1.06]

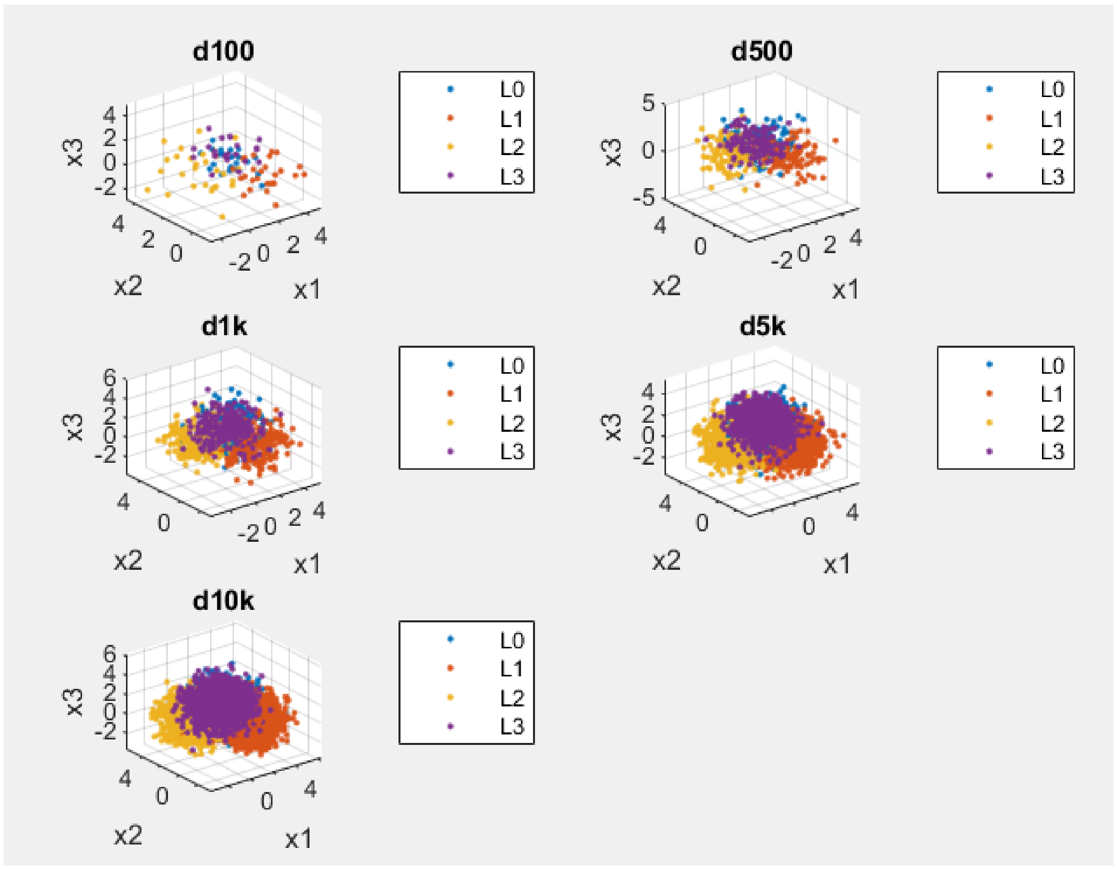
$\boldsymbol{m}1$= [1 0 0], $\boldsymbol{C}1$= [1.05 0.04 0.01, 0.04 1.04 0.01, 0.01 0.01 1]

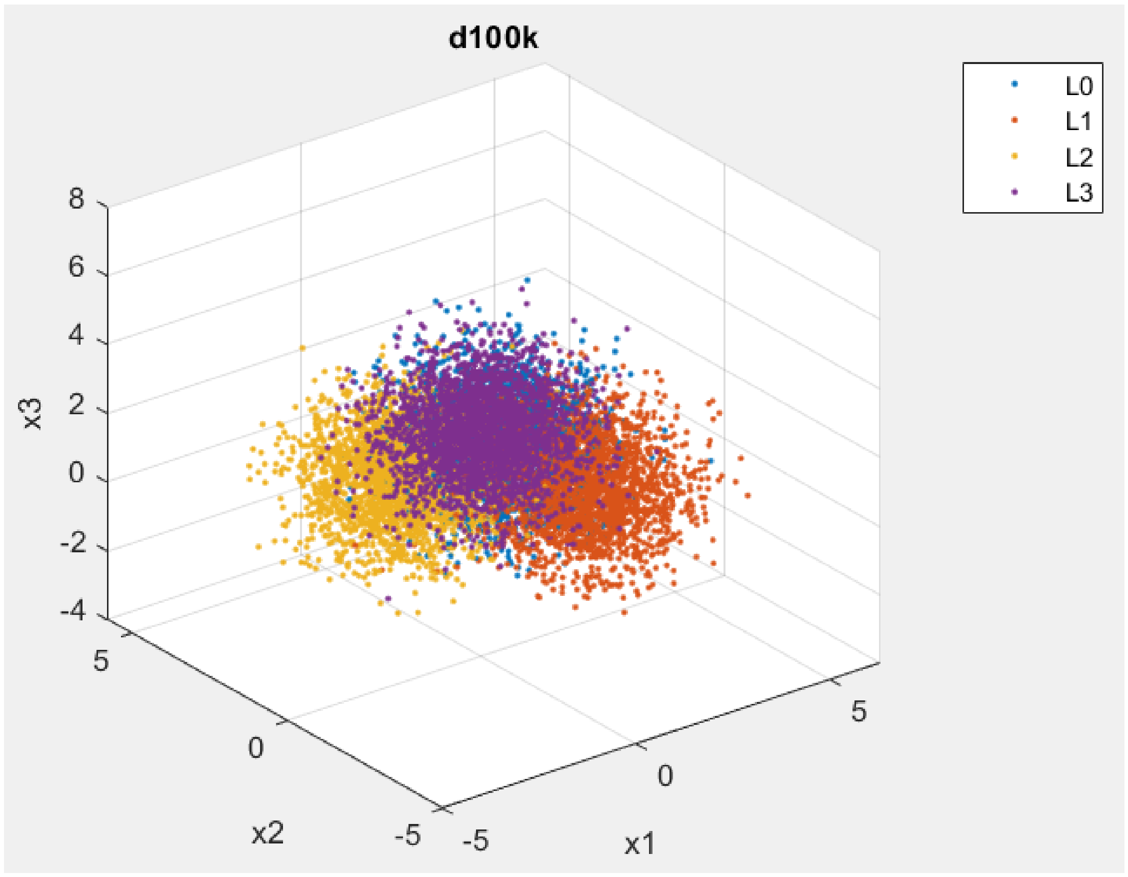
$\boldsymbol{m}2$= [0 1 0], $\boldsymbol{C}2$=[1.06 0.08 0.05 , 0.06 1.05 0.07 , 0.06 0.05 1.05]

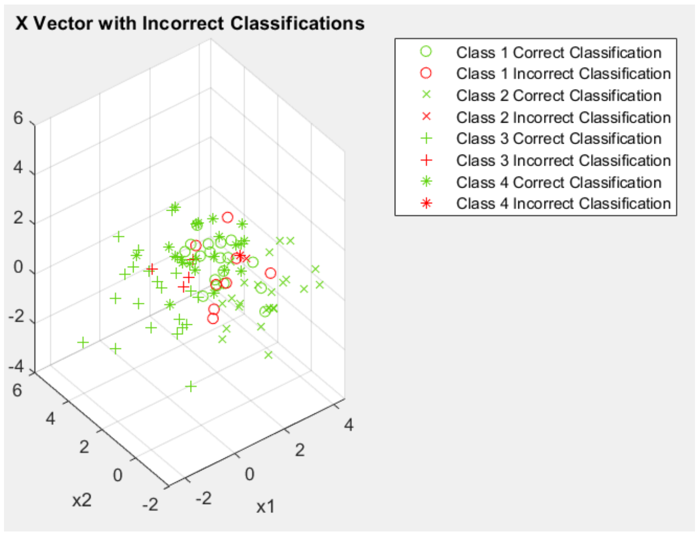
$\boldsymbol{m}3$= [0 0 1], $\boldsymbol{C}3$=[1.03 0.05 0.03 , 0.03 1.05 0.04 ,0.02 0.03 1.02]

A 2-layer Multilayer perceptron with one hidden layer and one output layer has been implemented. Here the softmax function is used to ensure all the outputs add up to 1 and that all of them are positive. To know the best number of perceptrons that will be correct for our problem we use cross-validation. Additionally, in the problem a smooth-ramp style activation function was activated.

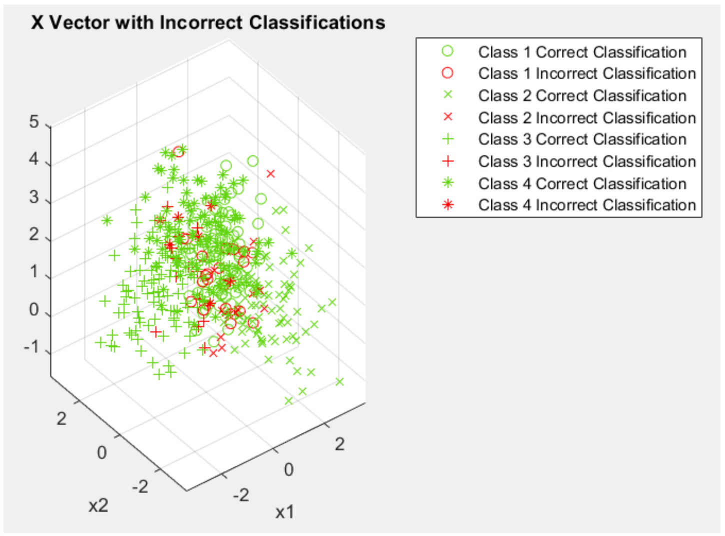
Given below are the figures of the datasets with 100, 500, 1000, 5000, 10000 respectively which were generated for training using which we could proceed with the validation of test dataset of 100,000 samples.


d100
d500
d1k
d5k
d10k
x1
x2
x3
L0
L1
L2
L3

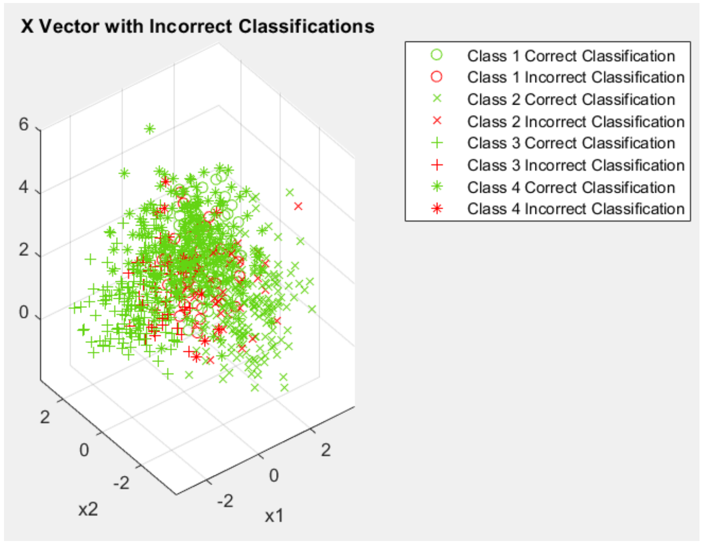

d100k
x1
x2
x3
L0
L1
L2
L3



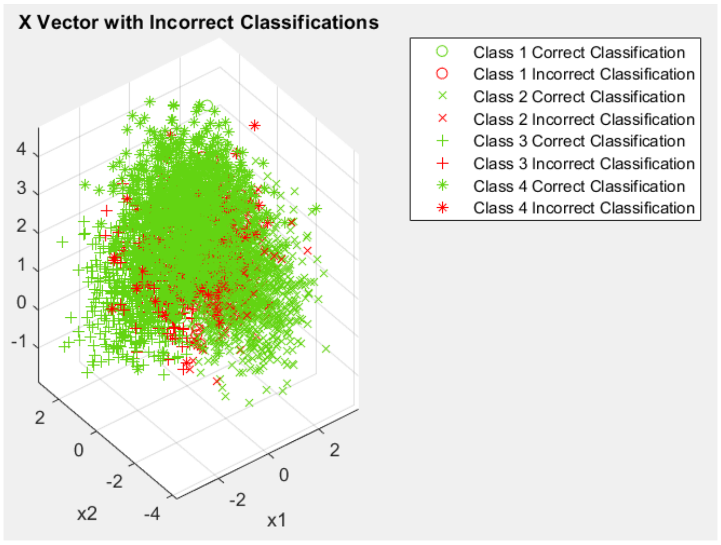
Classification of 100 Samples



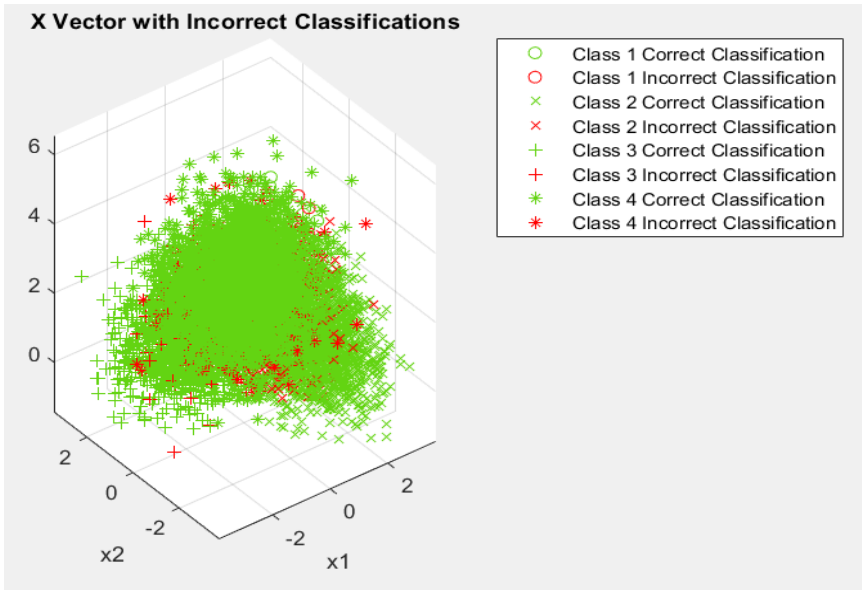
Classification of 500 Samples



Classification of 1000 Samples



Classification of 5000 Samples

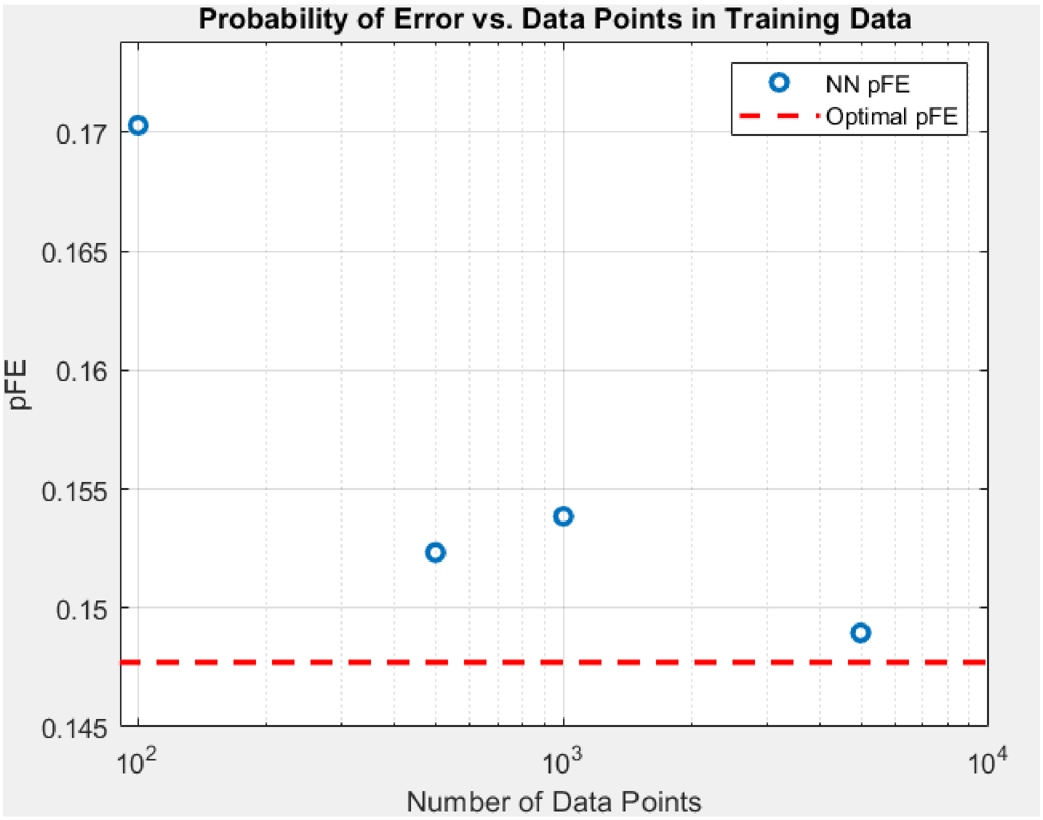


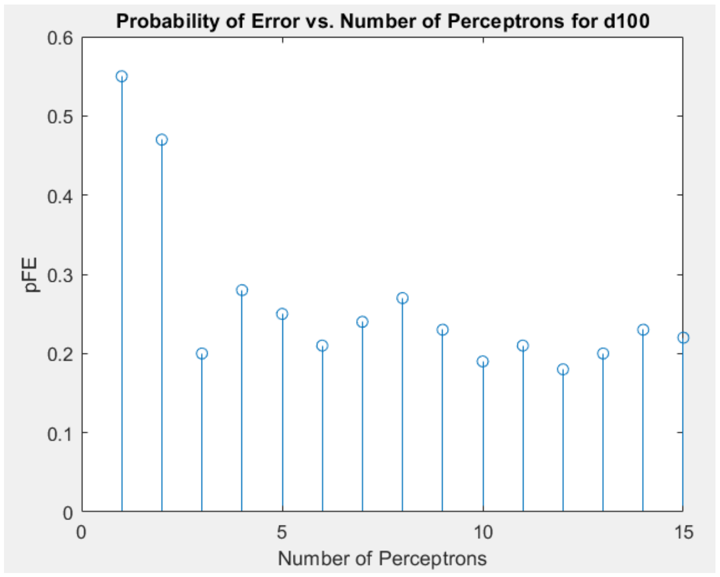
Classification of 10000 Samples

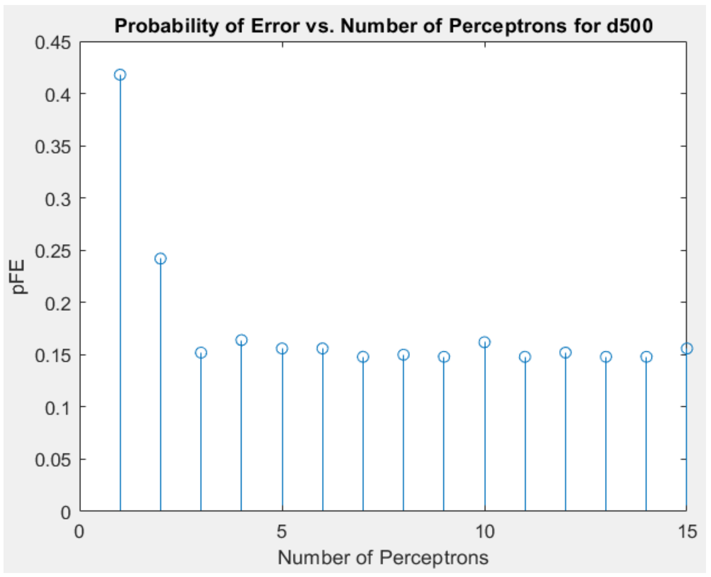
For training each of these datasets we use the 10-fold cross validation method to determine the optimal amount of perceptrons for the MLP model. In the 10 fold cross validation we would perform a fitting procedure a total of 10 times, in which the dataset is divided into 10 folds or subsets. Each iteration selects 90% of the dataset for training and the remaining 10% for the validation. With each iteration we select a different 90% of the dataset at random and take the remaining 10% for the validation and we calculate the minimum probability of error across the cross validation runs.

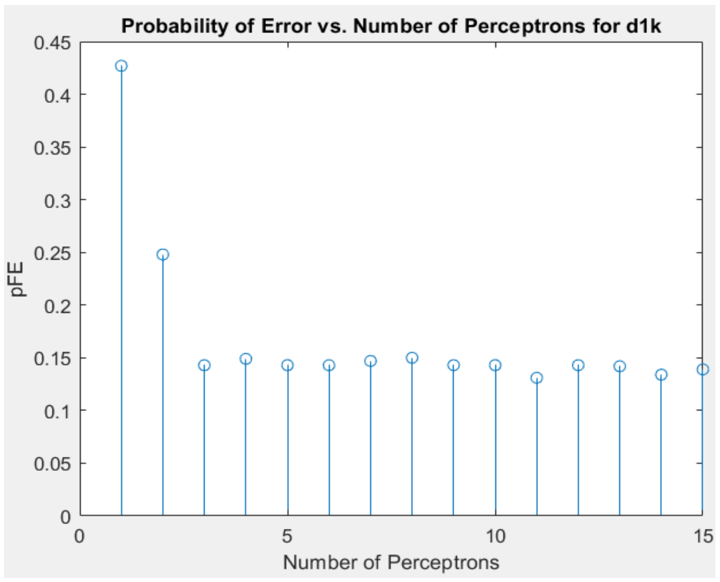
Once we conclude on the number to perceptron to use then we can run it again on the full dataset and calculate the probability of error. Finally, this trained model was evaluated using the test dataset and the probability of error was calculated as the metric of model performance.

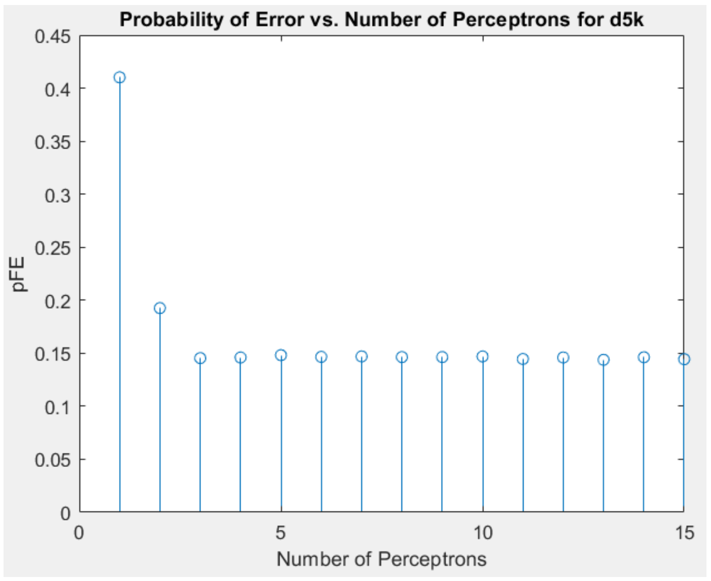
Below figure shows the results of this procedure. As can be seen in the plot, the overall probability of error is well correlated with the size of the training dataset. As the size of the dataset increases the probability of error decreases and approaches the optimal probability of error as estimated using the true pdf of the underlying data. From the graphs given below, we can say that as the number of training samples increases the model estimate can be improved which results in more accurate classifications.

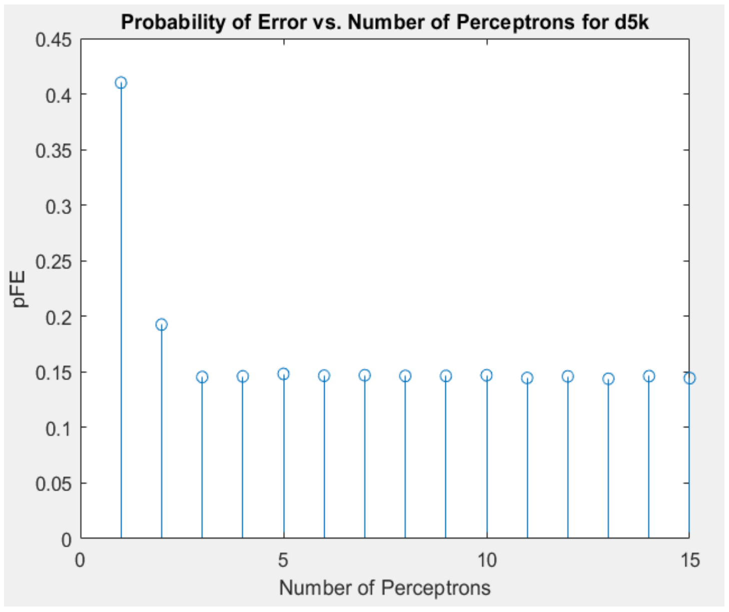


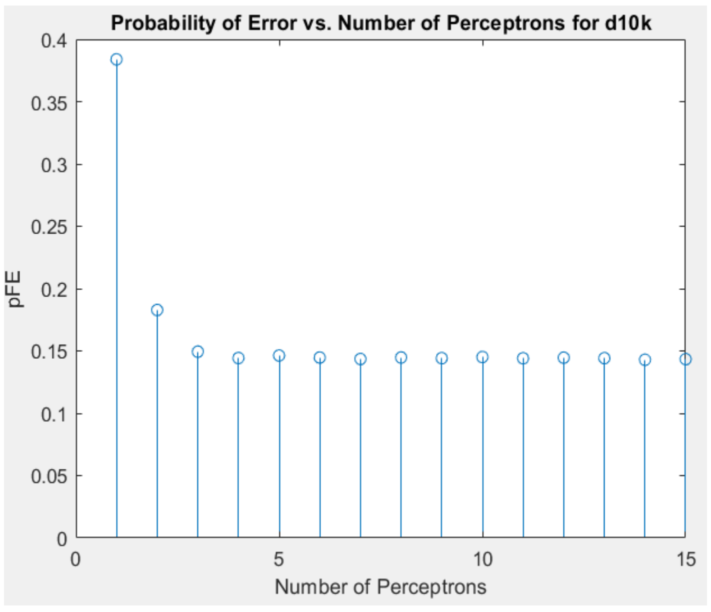

Probability of Error vs. Number of Perceptrons for d100
pFE
Number of Perceptrons
0
0.1
0.2
0.3
0.4
0.5
0.6
0
5
10
15

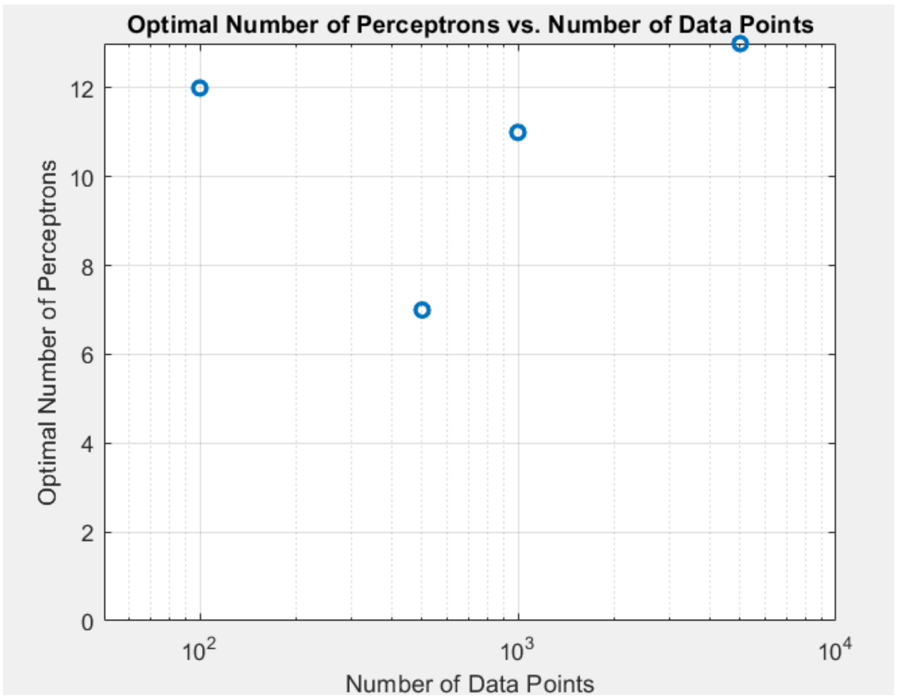

Probability of Error vs. Number of Perceptrons for d500
pFE
Number of Perceptrons
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
5
10
15


Probability of Error vs. Number of Perceptrons for d1k
pFE
Number of Perceptrons
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
5
10
15


Probability of Error vs. Number of Perceptrons for d5k
pFE
Number of Perceptrons
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
5
10
15


Probability of Error vs. Number of Perceptrons for d5k
pFE
Number of Perceptrons
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0
5
10
15


Probability of Error vs. Number of Perceptrons for d10k
pFE
Number of Perceptrons
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0
5
10
15



The probability of errors are obtained as given below:

1) They lie in the specified 10-20% range. The gaussian mixtures are also selected in a way such that the probability of error lies between 10-20%.

Optimal pFE, N=100: Error=14.00%

Optimal pFE, N=500: Error=14.00%

Optimal pFE, N=1000: Error=13.50%

Optimal pFE, N=5000: Error=14.26%

Optimal pFE, N=10000: Error=14.58%

Optimal pFE, N=100000: Error=14.42%

NN pFE, N=100: Error=17.03%

NN pFE, N=500: Error=15.38%

NN pFE, N=1000: Error=15.28%

NN pFE, N=5000: Error=14.94%

NN pFE, N=10000: Error=14.68%

Question 2:

In this question we make use of the Gaussian Mixture model(GMM) as the true probability density function for 2 – dimensional real valued data synthesis. This GMM given below has 4 different components with different mean vectors , different covariance matrices, and different probability for each Gaussian to be selected for that sample.

Here the MEAN is : [5 10 10 -10;10 10 -10 -10];

The Given covariance matrices are:

Covariance 1: [20 1;10 3]

Covariance 2:[7 1;1 2]

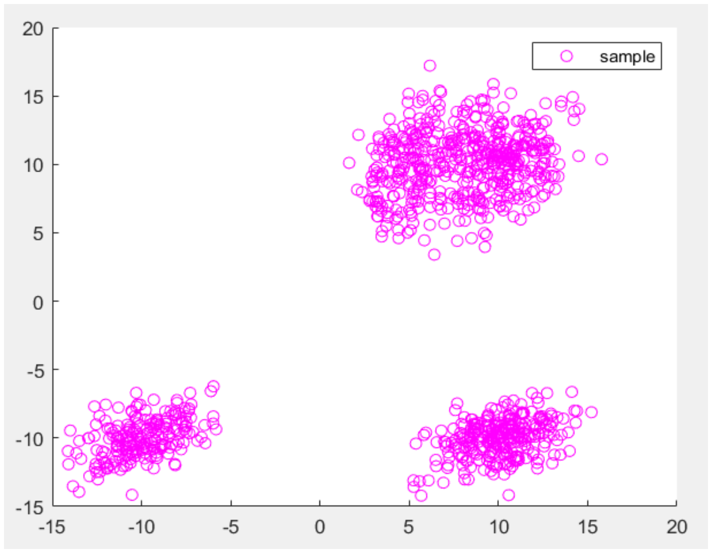
Covariance 3:[4 10;1 16]

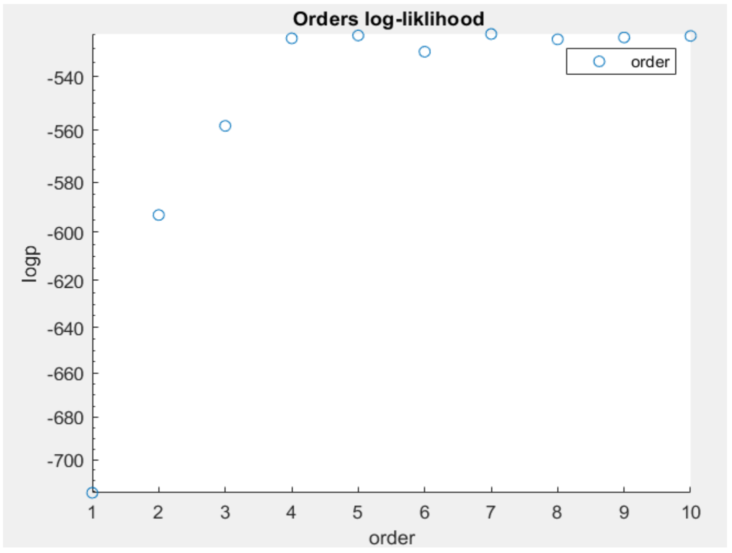
Covariance 4:[2 1;1 7]

The Gaussian model is selected of a larger value so that we can show that they are separated from each other. The K algorithm we use helps assign the new data to the correct groups once it is run.

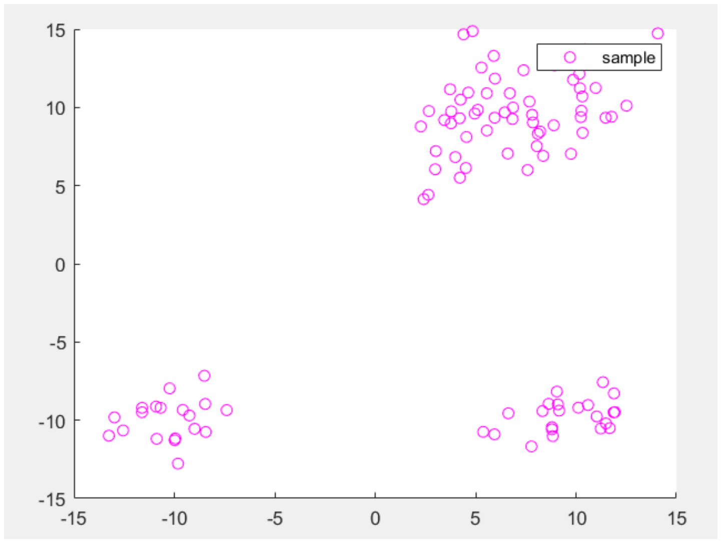
Given below are the clusters which are the GMM distribution of each sample.

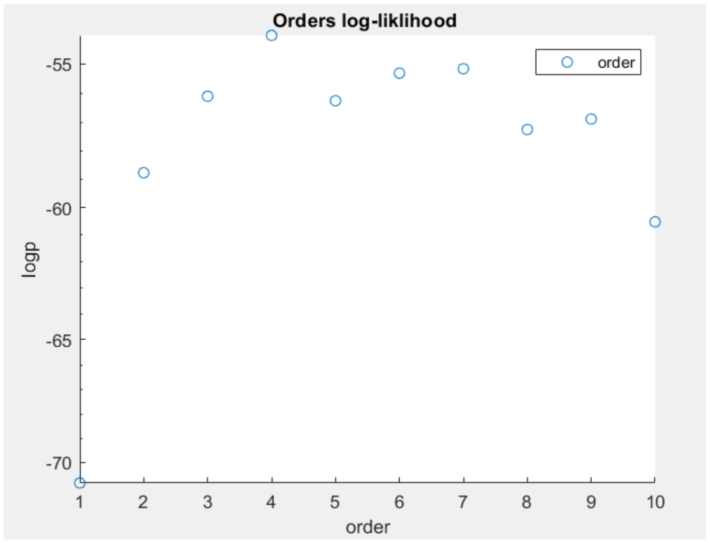
For N=1000 – the GMM distribution



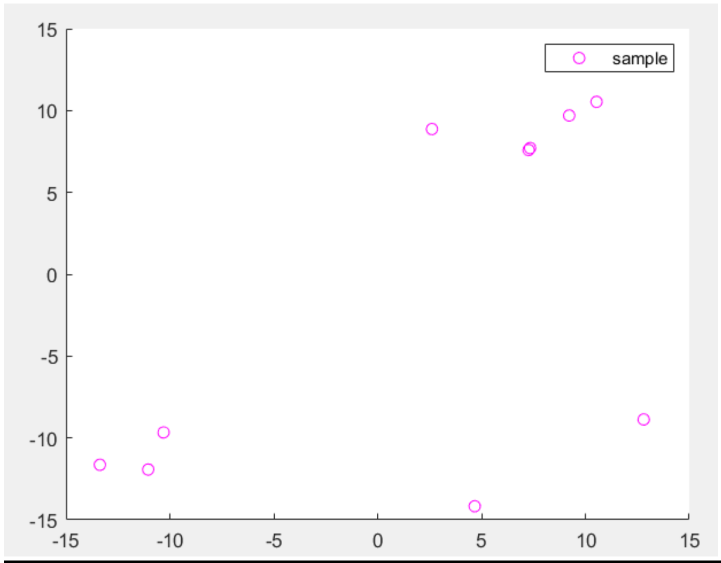


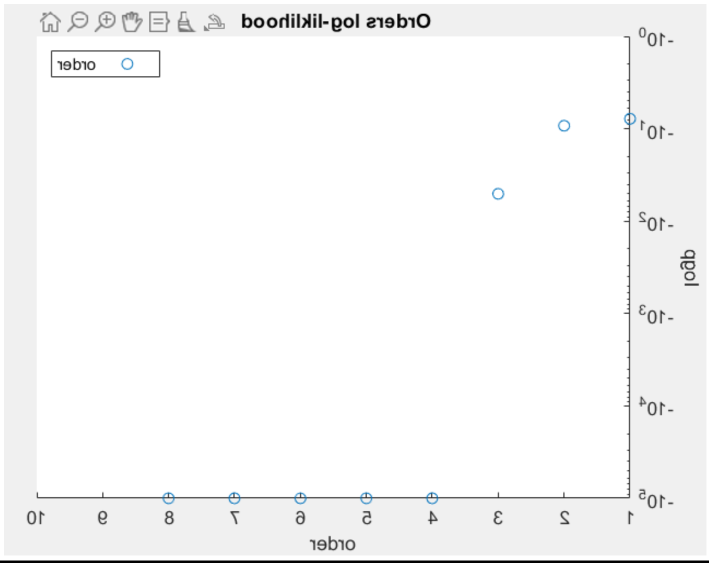
<u>For N=100 – the GMM distribution</u>





<u>For N=10 – the GMM distribution</u>





The log likelihood here indicates that 4 components would be a good fit here for this data.

(2 of the components are overlapping) as mentioned. Here in the GMM distribution the data starts to stagnate at the 4th component. In the distribution for 10 and 100 samples it is not that significant but when we take the 1000 samples distribution it becomes more significant.

Appendix:

Code:

Question 1:

```
%Expected risk minimization with 2 classes
clear;
close all;
%Switches to bypass parts 1 and 2 for debugging
dimensions=3;
numLabels=4;
Lx={'L0','L1','L2','L3'};
% For min-Perror design, use 0-1 loss
lossMatrix = ones(numLabels,numLabels)-eye(numLabels);
muScale=2.5;
SigmaScale=0.2;
%Define data
D.d100.N=100;
D.d500.N=500;
D.d1k.N=1e3;
D.d5k.N=5e3;
D.d10k.N=10e3;
D.d100k.N=100e3;
dTypes=fieldnames(D);
%Define Statistics
p=ones(1,numLabels)/numLabels; %Prior
%Label data stats
mu.L0=muScale*[1 1 0]';
RandSig=SigmaScale*rand(dimensions,dimensions);
Sigma.L0(:,:,1)=RandSig*RandSig'+eye(dimensions);
mu.L1=muScale*[1 0 0]';
RandSig=SigmaScale*rand(dimensions,dimensions);
Sigma.L1(:,:,1)=RandSig*RandSig'+eye(dimensions);
mu.L2=muScale*[0 1 0]';
RandSig=SigmaScale*rand(dimensions,dimensions);
Sigma.L2(:,:,1)=RandSig*RandSig'+eye(dimensions);
mu.L3=muScale*[0 0 1]';
RandSig=SigmaScale*rand(dimensions,dimensions);
Sigma.L3(:,:,1)=RandSig*RandSig'+eye(dimensions);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Generate Data
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for ind=1:length(dTypes)
D.(dTypes{ind}).x=zeros(dimensions,D.(dTypes{ind}).N); %Initialize Data
[D.(dTypes{ind}).x,D.(dTypes{ind}).labels,...
D.(dTypes{ind}).N_l,D.(dTypes{ind}).p_hat]=...
genData(D.(dTypes{ind}).N,p,mu,Sigma,Lx,dimensions);
end
%Plot Training Data
figure;
for ind=1:length(dTypes)-1
subplot(3,2,ind);
plotData(D.(dTypes{ind}).x,D.(dTypes{ind}).labels,Lx);
legend 'show';
title([dTypes{ind}]);
end
```

```matlab
%Plot Validation Data
figure;
plotData(D.(dTypes{ind}).x,D.(dTypes{ind}).labels,Lx);
legend 'show';
title([dTypes{end}]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Determine Theoretically Optimal Classifier
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for ind=1:length(dTypes)
[D.(dTypes{ind}).opt.PFE, D.(dTypes{ind}).opt.decisions]=...
optClass(lossMatrix,D.(dTypes{ind}).x,mu,Sigma,...
p,D.(dTypes{ind}).labels,Lx);
opPFE(ind)=D.(dTypes{ind}).opt.PFE;
fprintf('Optimal pFE, N=%1.0f: Error=%1.2f%%\n',...
D.(dTypes{ind}).N,100*D.(dTypes{ind}).opt.PFE);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Train and Validate Data
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
numPerc=15; %Max number of perceptrons to attempt to train
k=10; %number of folds for kfold validation
for ind=1:length(dTypes)-1
%kfold validation is in this function
[D.(dTypes{ind}).net,D.(dTypes{ind}).minPFE,...
D.(dTypes{ind}).optM,valData.(dTypes{ind}).stats]=...
kfoldMLP_NN(numPerc,k,D.(dTypes{ind}).x,D.(dTypes{ind}).labels,numLabels);
%Produce validation data from test dataset
valData.(dTypes{ind}).yVal=D.(dTypes{ind}).net(D.d100k.x);
[~,valData.(dTypes{ind}).decisions]=max(valData.(dTypes{ind}).yVal);
valData.(dTypes{ind}).decisions=valData.(dTypes{ind}).decisions-1;
%Probability of Error is wrong decisions/num data points
valData.(dTypes{ind}).pFE=...
sum(valData.(dTypes{ind}).decisions~=D.d100k.labels)/D.d100k.N;
outpFE(ind,1)=D.(dTypes{ind}).N;
outpFE(ind,2)=valData.(dTypes{ind}).pFE;
outpFE(ind,3)=D.(dTypes{ind}).optM;
fprintf('NN pFE, N=%1.0f: Error=%1.2f%%\n',...
D.(dTypes{ind}).N,100*valData.(dTypes{ind}).pFE);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This code was used to plot the results from the data generated in the main
%function
%Extract cross validation results from structure
for ind=1:length(dTypes)-1
[~,select]=min(valData.(dTypes{ind}).stats.mPFE);
M(ind)=(valData.(dTypes{ind}).stats.M(select));
N(ind)=D.(dTypes{ind}).N;
end
%Plot number of perceptrons vs. pFE for the cross validation runs
for ind=1:length(dTypes)-1
figure;
stem(valData.(dTypes{ind}).stats.M,valData.(dTypes{ind}).stats.mPFE);
xlabel('Number of Perceptrons');
ylabel('pFE');
title(['Probability of Error vs. Number of Perceptrons for ' ,dTypes{ind}]);
end
%Number of perceptrons vs. size of training dataset
figure,semilogx(N(1:end-1),M(1:end-1),'o','LineWidth',2)
grid on;
xlabel('Number of Data Points')
ylabel('Optimal Number of Perceptrons')
```

```matlab
ylim([0 10]);
xlim([50 10^4]);
title('Optimal Number of Perceptrons vs. Number of Data Points');
%Prob. of Error vs. size of training data set
figure,semilogx(outpFE(1:end-1,1),outpFE(1:end-1,2),'o','LineWidth',2)
xlim([90 10^4]);
hold all;semilogx(xlim,[opPFE(end) opPFE(end)],'r--','LineWidth',2)
legend('NN pFE','Optimal pFE')
grid on
xlabel('Number of Data Points')
ylabel('pFE')
title('Probability of Error vs. Data Points in Training Data');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [x,labels,N_l,p_hat]= genData(N,p,mu,Sigma,Lx,d)
%Generates data and labels for random variable x from multiple gaussian
%distributions
numD = length(Lx);
cum_p = [0,cumsum(p)];
u = rand(1,N);
x = zeros(d,N);
labels = zeros(1,N);
for ind=1:numD
pts = find(cum_p(ind)<u & u<=cum_p(ind+1));
N_l(ind)=length(pts);
x(:,pts) = mvnrnd(mu.(Lx{ind}),Sigma.(Lx{ind}),N_l(ind))';
labels(pts)=ind-1;
p_hat(ind)=N_l(ind)/N;
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function plotData(x,labels,Lx)
%Plots data
for ind=1:length(Lx)
pindex=labels==ind-1;
plot3(x(1,pindex),x(2,pindex),x(3,pindex),'.','DisplayName',Lx{ind});
hold all;
end
grid on;
xlabel('x1');
ylabel('x2');
zlabel('x3');
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function g = evalGaussian(x,mu,Sigma)
% Evaluates the Gaussian pdf N(mu,Sigma) at each coumn of X
[n,N] = size(x);
invSigma = inv(Sigma);
C = (2*pi)^(-n/2) * det(invSigma)^(1/2);
E = -0.5*sum((x-repmat(mu,1,N)).*(invSigma*(x-repmat(mu,1,N))),1);
g = C*exp(E);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [minPFE,decisions]=optClass(lossMatrix,x,mu,Sigma,p,labels,Lx)
% Determine optimal probability of error
symbols='ox+*v';
numLabels=length(Lx);
N=length(x);
for ind = 1:numLabels
pxgivenl(ind,:) =...
evalGaussian(x,mu.(Lx{ind}),Sigma.(Lx{ind})); % Evaluate p(x|L=l)
```

```
end
px = p*pxgivenl; % Total probability theorem
classPosteriors = pxgivenl.*repmat(p',1,N)./repmat(px,numLabels,1); %P(L=l|x)
% Expected Risk for each label (rows) for each sample (columns)
expectedRisks =lossMatrix*classPosteriors;
% Minimum expected risk decision with 0-1 loss is the same as MAP
[~,decisions] = min(expectedRisks,[],1);
decisions=decisions-1; %Adjust to account for L0 label
fDecision_ind=(decisions~=labels);%Incorrect classificiation vector
minPFE=sum(fDecision_ind)/N;
%Plot Decisions with Incorrect Results
figure;
for ind=1:numLabels
class_ind=decisions==ind-1;
plot3(x(1,class_ind & ~fDecision_ind),...
x(2,class_ind & ~fDecision_ind),...
x(3,class_ind & ~fDecision_ind),...
symbols(ind),'Color',[0.39 0.83 0.07],'DisplayName',...
['Class ' num2str(ind) ' Correct Classification']);
hold on;
plot3(x(1,class_ind & fDecision_ind),...
x(2,class_ind & fDecision_ind),...
x(3,class_ind & fDecision_ind),...
['r' symbols(ind)],'DisplayName',...
['Class ' num2str(ind) ' Incorrect Classification']);
hold on;
end
xlabel('x1');
ylabel('x2');
grid on;
title('X Vector with Incorrect Classifications');
legend 'show';
if 0
%Plot Decisions with Incorrect Decisions
figure;
for ind2=1:numLabels
subplot(3,2,ind2);
for ind=1:numLabels
class_ind=decisions==ind-1;
plot3(x(1,class_ind),x(2,class_ind),x(3,class_ind),...
'.','DisplayName',['Class ' num2str(ind)]);
hold on;
end
plot3(x(1,fDecision_ind & labels==ind2),...
x(2,fDecision_ind & labels==ind2),...
x(3,fDecision_ind & labels==ind2),...
'kx','DisplayName','Incorrectly Classified','LineWidth',2);
ylabel('x2');
grid on;
title(['X Vector with Incorrect Decisions for Class '
num2str(ind2)]);
if ind2==1
legend 'show';
elseif ind2==4
xlabel('x1');
end
end
end
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%This function performs the cross validation and model selection
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [outputNet,outputPFE, optM,stats]=kfoldMLP_NN(numPerc,k,x,labels,numLabels)
%Assumes data is evenly divisible by partition choice which it should be
N=length(x);
numValIters=10;
%Create output matrices from labels
y=zeros(numLabels,length(x));
for ind=1:numLabels
y(ind,:)=(labels==ind-1);
end
%Setup cross validation on training data
partSize=N/k;
partInd=[1:partSize:N length(x)];
%Perform cross validation to select number of perceptrons
for M=1:numPerc
for ind=1:k
index.val=partInd(ind):partInd(ind+1);
index.train=setdiff(1:N,index.val);
%Create object with M perceptrons in hidden layer
net=patternnet(M);
% net.layers{1}.transferFcn = 'softplus';%didn't work
%Train using training data
net=train(net,x(:,index.train),y(:,index.train));
%Validate with remaining data
yVal=net(x(:,index.val));
[~,labelVal]=max(yVal);
labelVal=labelVal-1;
pFE(ind)=sum(labelVal~=labels(index.val))/partSize;
end
%Determine average probability of error for a number of perceptrons
avgPFE(M)=mean(pFE);
stats.M=1:M;
stats.mPFE=avgPFE;
end
%Determine optimal number of perceptrons
[~,optM]=min(avgPFE);
%Train one final time on all the data
for ind=1:numValIters
netName(ind)={['net' num2str(ind)]};
finalnet.(netName{ind})=patternnet(optM);
% finalnet.layers{1}.transferFcn = 'softplus';%Set to RELU
finalnet.(netName{ind})=train(net,x,y);
yVal=finalnet.(netName{ind})(x);
[~,labelVal]=max(yVal);
labelVal=labelVal-1;
pFEFinal(ind)=sum(labelVal~=labels)/length(x);
end
[minPFE,outInd]=min(pFEFinal);
stats.finalPFE=pFEFinal;
outputPFE=minPFE;
outputNet=finalnet.(netName{outInd});
end
```

Question 2:

```
clear;
close all;
%N=10; % taking the values and trying each time with the N as
N=100;
%N=1000;
%10,100,1000
delta = 1e0; % tolerance for EM stopping criterion
regWeight = 1e-10;
%Replicating it 100 times
% Generate samples from a 4-component GMM
alpha_true = [0.2,0.3,0.3,0.2];
mu_true = [5 10 10 -10;10 10 -10 -10];
Sigma_true(:,:,1) = [2 1;1 6];
Sigma_true(:,:,2) = [3 1;1 5];
Sigma_true(:,:,3) = [4 1;1 2];
Sigma_true(:,:,4) = [3 1;1 2];
x = randGMM(N,alpha_true,mu_true,Sigma_true);
figure(1);
figure(1),scatter(x(1,:),x(2,:),'om'), hold on,
figure(1),legend('sample')
d = 2;
K = 10;
dummy = ceil(linspace(0,N,K+1));
for k = 1:K
indPartLimits(k,:) = [dummy(k)+1,dummy(k+1)];
end
avgp = zeros(1,10);
for M = 1:10
psum = zeros(1,10);
for k = 1:K
    indValidate = [indPartLimits(k,1):indPartLimits(k,2)];
    xValidate = x(:,indValidate); % Using folk k as validation set
    if k == 1
        indTrain = [indPartLimits(k,2)+1:N];
    elseif k == K
        indTrain = [1:indPartLimits(k,1)-1];
else
        indTrain = [[1:indPartLimits(k-1,2)],[indPartLimits(k+1,1):N]];
end
    xTrain = x(:,indTrain); % using all other folds as training set
    Ntrain = length(indTrain);
    Nvalidate = length(indValidate);
    [alpha,mu,Sigma] = EMforGMM(Ntrain,xTrain,M,d,delta,regWeight);% determine dimensionality of
samples and number of GMM components
    p = zeros(1,Nvalidate);
    for j = 1:Nvalidate
        for i = 1:M
            p(j) = p(j) +alpha(i)*evalGaussian(xValidate(:,j),mu(:,i),Sigma(:,:,i));
end
        p(j) = log(p(j));
end
    psum(k) = sum(p);
    dummy(k,M)=sum(p);
end
avgp(M) = sum(psum)/10;
if (avgp(M)== -inf)
    avgp(M) = -1e5;
end
end
```

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figure(2),scatter([1,2,3,4,5,6,7,8,9,10],avgp),set(gca,'yscale','log'),
figure(2),legend('order'),title('Orders log-liklihood '),
xlabel('order'), ylabel('logp')
function [alpha,mu,Sigma] = EMforGMM(N,x,M,d,delta,regWeight)
% Initialize the GMM to randomly selected samples
alpha = ones(1,M)/M;
shuffledIndices = randperm(N);
mu = x(:,shuffledIndices(1:M)); % pick M random samples as initial mean estimates
[~,assignedCentroidLabels] = min(pdist2(mu',x'),[],1); % assign each sample to the
nearest mean
for m = 1:M
Sigma(:,:,m) = cov(x(:,find(assignedCentroidLabels==m))') + regWeight*eye(d,d);
end
t = 0; %displayProgress(t,x,alpha,mu,Sigma);

Converged = 0; % Not converged at the beginning
while ~Converged
for l = 1:M
    temp(l,:) = repmat(alpha(l),1,N).*evalGaussian(x,mu(:,l),Sigma(:,:,l));
end
plgivenx = temp./sum(temp,1);
alphaNew = mean(plgivenx,2);
w = plgivenx./repmat(sum(plgivenx,2),1,N);
muNew = x*w';
for l = 1:M
    v = x-repmat(muNew(:,l),1,N);
    u = repmat(w(l,:),d,1).*v;
    SigmaNew(:,:,l) = u*v' + regWeight*eye(d,d); % adding a small regularization term
end
Dalpha = sum(abs(alphaNew-alpha'));
Dmu = sum(sum(abs(muNew-mu)));
DSigma = sum(sum(abs(abs(SigmaNew-Sigma))));
Converged = ((Dalpha+Dmu+DSigma)<delta); % Check if converged
alpha = alphaNew; mu = muNew; Sigma = SigmaNew;
t = t+1;
end
end
function x = randGMM(N,alpha,mu,Sigma)
d = size(mu,1); % dimensionality of samples
cum_alpha = [0,cumsum(alpha)];
u = rand(1,N); x = zeros(d,N); labels = zeros(1,N);
for m = 1:length(alpha)
ind = find(cum_alpha(m)<u & u<=cum_alpha(m+1));
x(:,ind) = randGaussian(length(ind),mu(:,m),Sigma(:,:,m));
end
end
%%%
function x = randGaussian(N,mu,Sigma)
% Generates N samples from a Gaussian pdf with mean mu covariance Sigma
n = length(mu);
z =  randn(n,N);
A = Sigma^(1/2);
x = A*z + repmat(mu,1,N);
end
function g = evalGaussian(x,mu,Sigma)
[n,N] = size(x);
invSigma = inv(Sigma);
C = (2*pi)^(-n/2) * det(invSigma)^(1/2);
E = -0.5*sum((x-repmat(mu,1,N)).*(invSigma*(x-repmat(mu,1,N))),1);
g=C*exp(E);
end
```

Credits: Professor Deniz Notes and GitHub