Fast Capture—Recapture Approach for Mitigating the Problem of Missing RFID Tags

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Abstract—The technology of Radio Frequency IDentification (RFID) enables many applications that rely on passive, battery-less wireless devices. If a RFID reader needs to gather the ID from multiple tags in its range, then it needs to run an anticollision protocol. Due to errors on the wireless link, a single reader session, which contains one full execution of the anticollision protocol, may not be sufficient to retrieve the ID of all tags. This problem can be mitigated by running multiple, redundant reader sessions and use the statistical relationship between these sessions. On the other hand, each session is time consuming and therefore the number of sessions should be kept minimal. We optimize the process of running multiple reader sessions, by allowing only some of the tags already discovered to reply in subsequent reader sessions. The estimation procedure is integrated with an actual tree-based anticollision protocol, and numerical results show that the reliable tag resolution algorithm attain high speed of protocol execution, while not sacrificing the reliability of the estimators used to assess the probability of missing tags.

Index Terms—RFID, reliable arbitration process, anticollision protocols.

1 Introduction

With concepts such as Internet of Things [1] and Smart Dust [2], the interest in Radio Frequency IDentification (RFID) has markedly increased. Originally perceived as a technology for inventorying [3], RFID has gradually evolved into an enabler of ubiquitous computing, by bridging the physical and digital world. The most interesting category is the one utilizing passive RFID tags, which do not have their own power supply. Passive tags are powered by the signal sent from a reader, which energizes their circuitry and enables them to respond by backscattering the signal [4].

A passive RFID system is based on request/response: first, a reader sends an interrogation signal to all tags within range. Then, each tag responds to the reader by backscattering the interrogation signal, modulated by the tag in a way so it conveys information from the tag to the reader. Should multiple tags respond at the same time, the reader experiences tag collision and must run a certain anticollision protocol, also called collision resolution or arbitration protocol. The goal of such protocol is to resolve each tag in a reader's range, i.e., to enable each tag to send its ID to the reader in a successful, collision-free manner [4]. Anticollision protocols are normally divided into two groups: ALOHA based [5], [6] and tree based [7], [8]. These protocols have been designed to successfully resolve a set of tags in an otherwise error-free environment. We define a *reader session* as a single protocol execution that, in absence of errors, gathers the ID of all tags in the reader's range.

However, as the wireless medium is far from error free, this assumption does not always hold. Errors occur on both the reader-to-tag link (a tag does not receive a query and therefore does not reply) and on the tag-to-reader link (a tag replies to a query, but the reader does not receive the reply). Most importantly, if a tag is not resolved during an arbitration protocol run, the tag may be missed entirely, which is defined as the *missing tag problem* [9]. Note that a reader is not aware of the existence of the missing tag before arbitration, as it has not yet gathered the tag IDs in its interrogation zone.

In [9], a *sequential decision process* is proposed to deal with this problem. This process is used to obtain *reliable arbitration* by performing sequential runs of an arbitration protocol (reader sessions), until the estimated probability of missing tags is below some user defined threshold. The sequential decision process harnesses capture-recapture techniques, and, conceptually, this decision process runs at a *reliability layer*, on top of an *arbitration layer* that performs arbitration/collision resolution [10].

The algorithms described in [9] and [10] are not designed for *fast* reliable arbitration. They both run multiple reader sessions in which they arbitrate the full tag set, not taking into account that previously discovered tags are participating in subsequent reader sessions. In order to improve time efficiency (speed), this paper investigates the effect of *silencing* tags in some reader sessions during the reliable arbitration. In other words, not all of the tags that have been discovered hitherto are allowed to participate in the next reader session. Tag silencing is a practical mechanism and provisions have been made for it in the existing standards, see the use of the select command for Gen 2 tags [5].

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The contribution of this paper can be summarized as follows: we improve the procedure for estimating the probability of missing tags by disabling the response from some of the tags to be arbitrated that have already been discovered. Usage of less tags in the estimation process decreases the accuracy of the estimators. We, therefore, derive a criteria for the minimum number of enabled tags without significantly affecting the estimation of the probability of missing tags. Our analytical and numerical results show how our proposed method obtains the same reliability as the current state of the art techniques, but does so more efficiently with respect to time.

The paper is organized as follows: the next section provides background information on arbitration protocols, the system model, and an overview of the basic ideas in statistical tag set estimation. In Section 3, we describe the new *Reduced Sets Estimator* and we determine its optimized parameters, so the maximum arbitration speed is achieved while maintaining estimation accuracy. Usage of the proposed estimator when there are dynamic errors is described in Section 4. Numerical results are in Section 5, and the paper is concluded in Section 6.

2 Background

2.1 Arbitration Protocols

As mentioned in the introduction, anticollision protocols are normally divided into two groups; ALOHA based and tree based. In protocols based on framed ALOHA, the query sent by a reader informs about the length of the frame, and each tag independently and randomly picks a slot in the frame to transmit. The key design ingredient is the choice of the frame size, which should dynamically adapt to the population of contending tags [11], such that the probability to obtain a response from a single tag in a given slot is maximized. Recall from the introduction that a single reader session is defined as a procedure that guarantees to gather all tags in the absence of errors. Hence, a single protocol run, or reader session, with an ALOHA-based protocol may contain several frames: if there is one or more collisions in the frame, then another frame is initiated.

In the tree-based arbitration protocols [12], the reader identifies a group of tags that should transmit in a given slot based on the outcomes of previous slots. In determining the group of transmitting tags, the reader probes the population of tags by traversing a binary tree. It is assumed that tags progressively generate a random *bit-array* in a reader session, which is used by the reader to select and deselect tags. The bit-array should be random and reset between each reader session to mitigate the correlation across tags introduced by the arbitration protocol during a protocol run [10].

Fig. 1 depicts an example of the basic variant of the tree protocol in the absence of errors. Initially, in slot s_1, all eight tags are probed by the reader and they transmit, resulting in collision (C). In s_2, only the tags with bit-array "0*" (i.e., the bit-array prefix is "0") are probed and enabled to transmit, in s_3 only the tags with bit-array "00*," in s_4 only the tags with bit-array "01*," etc. For proper operation, it is important to assume that the tag bit-arrays are random

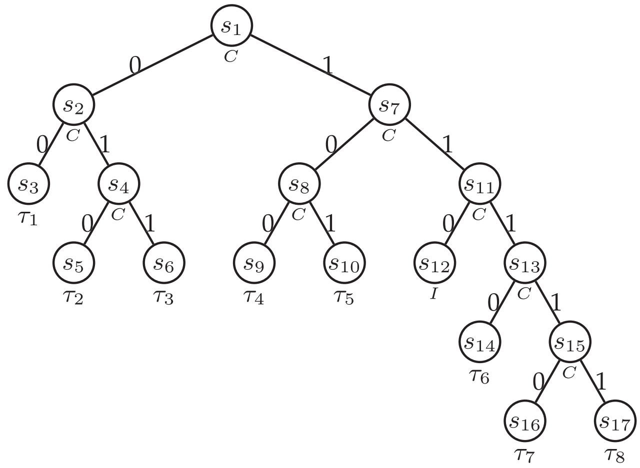


Fig. 1. An instance of the binary tree algorithm for $N = 8$ tags. The vertices represent a slot, which state can be Idle (I), Single (S) or Collision (C). For channel state "S," τ_i denotes the resolved tag.

and i.i.d., i.e., the probability that a tag has, at any position in the bit-array, a 0 or 1 is $\frac{1}{2}$.

Important for both protocol categories is a fast resolution process. In ALOHA, this means resolving the set with as few slots as possible, and for the tree based it means traversing the binary tree with as few probes as possible. As probes and slots are basically the same, a time slot in which one or more tags may transmit, we hereafter use slots to denote both. We therefore measure the performance of the sequential decision process in how many slots it requires.

2.2 System Model

A reliable arbitration is performed on a set with N tags and consists of a sequence of reader sessions $(r_1, r_2, \ldots, r_R)$, each defined as one run of a certain arbitration protocol. If sessions are executed by different readers, we assume that the readers are cooperative, in a sense where they exchange information about the IDs of the tags they have found and that they cooperate toward inferring information about the entire tag set. This also eliminates the reader collision problem [14].

The sessions are assumed independent in the sense that the event of a tag being read in a given reader session is independent of whether it or any other tag has been read in the previous reader sessions.

We put a MAC-layer view on the errors introduced in the system: *static errors* and *dynamic errors*. A *static error* occurs whenever the tag is at a blind spot [13] during an entire reader session. Each tag experiences static error with probability p, independent in each session and independent of the other tags. Note that a tag that experiences static errors is missed in that reader session, i.e., p is also the probability of missing a tag in one reader session.

Dynamic errors account for noise-induced random errors which occur independently for each query/response in a given reader session. We let this error occur with probability q, and, if in a given session there are only dynamic errors, then the probability that a tag is not read in that reader session is $p = f(q, N)$, an increasing function of q and N, but the precise form depends on the used arbitration protocol. Static errors are therefore errors in which a tag is unreachable in an entire reader session, due to, e.g., physical circumstances or so severe a channel, so that no reply can get through. Dynamic errors, on the other hand, occur independently for each query and reply between a tag and a reader. We further develop the model for dynamic errors and investigate the relation $p = f(q, N)$ in Section 4.

We will present estimators by assuming that only static errors occur, and by estimating $f(q, N)$ we show that the estimators are also valid in the case where dynamic errors occur. The estimators can be applied to the case when there is a combination of static and dynamic errors, and in that case the probability to miss a tag is $p + (1 - p)f(q, N)$.

Throughout the paper, we assume that the probabilities p and q are identical for all tags. This assumption and the assumption of independent reader sessions are limiting ones, but can be justified for scenarios in which the physical setup for each reader session is randomly changed. An example of such scenario is the case in which the tags are put in a box and a person with handheld reader randomly changes the position for each reader session. In [10], the estimators proposed there are numerically evaluated in scenarios with dependent reader sessions and are shown to exhibit high robustness toward this dependence. The focus of this work is in the reduced sets and therefore investigation of the case with correlated sessions is outside the scope. However, as the estimators proposed here are derived from the same basic idea as in [10], there is a strong argument to expect these estimators will exhibit the same robustness with respect to correlated sessions. A more elaborate discussion on the issue of varying p or q across tags in a given reader session is outside the scope of this paper and is a subject for future work.

The speed of the reliable arbitration is measured as the number of slots used by the series of arbitration protocol runs in a reliable arbitration. We do not explicitly account for the cost of silencing tags, but we assume that such mechanism is incorporated in the arbitration protocol used.

2.3 Basic Ideas in Estimation of Missing Tags

From [9], we have the following instructive scenario, where we consider one way to determine estimates of p and N (this without the optimization presented in this paper). A set of tags is read by a reader in two separate reader sessions r_1 and r_2. The set has N tags and the probability that a tag cannot be read during a reader session is p, both N and p are not known a priori by the reader. The tag set is first read in r_1 and then in r_2. Let k_1 denote the subset of tags that have been read in both reader session r_1 and r_2. Let $k_{2a}(k_{2b})$ be the subset of tags that are read only in $r_1(r_2)$. There is also a set of k_3 tags that are not read in either of the reader sessions. Let $\hat{p}$ and $\hat{N}$ denote the estimates of p and N, respectively. Based on the expected values for k_1, k_{2a}, and k_{2b}, one can write

$$\begin{aligned} k_1 &= \hat{N}(1-\hat{p})^2 \\ k_{2a} + k_{2b} &= 2\hat{N}(1-\hat{p})\hat{p}. \end{aligned} \tag{1}$$

Using these two equations, one can obtain values for $\hat{p}$ and $\hat{N}$. In [10], three estimation approaches, all estimating p and N, are presented utilizing the statistical relationship between the sets which is valid when the sets are assumed

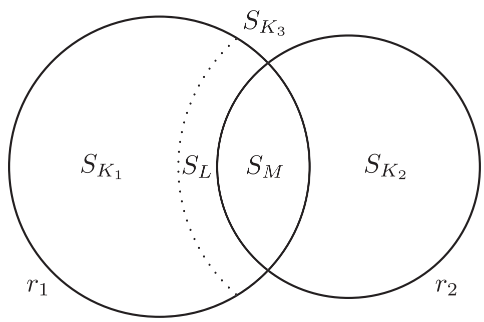


Fig. 2.Venn diagram of the observable sets S_{K_1}, S_L, S_M, and S_{K_2} and of the unobservable set S_{K_3}.

independent. One is the Venn estimator which uses the relations from (1) to estimate p and then N by using the relation $\frac{k_1}{k_{2a}+k_{2b}}$, the second is the Schnabel estimator which uses the relation $\frac{(k_{2a}+k_1)(k_{2b}+k_1)}{k_1}$ to estimate N, and an estimate of p can be found based on this estimate. The last estimator, the Combined Estimator, is a combination of the two, which utilizes that the two estimation methods use different information in their estimates, and a fusion of this information can provide better estimates.

This work is continued with an estimator for the probability of missing tags after R reader sessions that relies on the estimates of p and N. For one reader session, the probability of not missing $\hat{N}$ tags in R reader sessions is $(1 - \hat{p}^R)^{\hat{N}}$. This gives the estimate of the probability of missing at least one tag as

$$\hat{p}_M = 1 - (1 - \hat{p}^R)^{\hat{N}}. \tag{2}$$

If this estimate is large, it is likely that tags are left unread. This gives the main role of the reliability layer, that is, govern the following sequential decision process: after the Rth protocol run is finished, use (2) to estimate the probability that there are missing tags and, if this probability is higher than a predefined value, then another protocol run is initiated.

In this paper, we propose a novel alternative to the Combined Estimator to estimate p and N, where tags participate in the next reader session with probability v. By having tags silenced, the tag set to be resolved is reduced and the resolution can be done with fewer slots. The chosen v must be balanced so as many tags as possible are silenced, while still maintaining enough tags for the estimates to be accurate. The estimator is found in the next section.

3 Reduced Sets Estimator

We first show how the estimator works for two reader sessions, after which we extend the approach to more reader sessions. Five random variables, K_1, L, M, K_2, and K_3, follow the multinomial distribution, and describe the number of tags in the sets S_{K_1}, S_L, S_M, S_{K_2}, and the *unobservable* S_{K_3}, respectively, as can be seen in Fig. 2. Let k_1, l, m, k_2, and k_3 denote the realizations of K_1, L, M, K_2, and K_3, respectively. The variables are explained as follows:

- k_1—The number of tags found in r_1, which are silenced in r_2.

- l—The number of tags found in r_1, reused for r_2, but not found.
- m—The number of tags found in r_1 and reused and found in r_2.
- k_2—The number of tags not found in r_1, but in r_2.
- k_3—The *unobservable* number of tags not found in either r_1 or r_2.

These five numbers sum to N. The expected values, denoted as $E[\cdot]$, of the five random variables are, when introducing the "reuse" probability v:

$$\begin{aligned} E[K_1] &= N(1-p)(1-v), \\ E[L] &= N(1-p)vp, \\ E[M] &= N(1-p)^2 v, \\ E[K_2] &= Np(1-p), \\ E[K_3] &= Np^2. \end{aligned}$$

An estimator for p, the static error probability, can be found by using the relationship between l and m, if these sets are assumed to be approximations of the expected values of L and M, respectively. The relation $\frac{m}{m+l}$ then denotes the fraction of reused tags that were found in r_2 and is approximately

$$\frac{m}{m+l} \approx \frac{E[M]}{E[M]+E[L]} = \frac{(1-p)^2 v}{(1-p)^2 v + (1-p)vp} = 1-p.$$

By rearranging the formula, we obtain an estimate of p, denoted $\hat{p}$

$$\hat{p} = \frac{l}{l+m}.$$

This estimator has one special case, when $l+m=0$. In that case, the estimate of the static error probability is set to 1

$$g(l,m) = \hat{p} = \begin{cases} 1 & l+m=0, \\ \dfrac{l}{l+m} & \text{otherwise.} \end{cases} \tag{3}$$

The expected value of the estimator is

Lemma 1. *Let the estimate of* p *be defined as in (3), then the expected value of* $\hat{p}$ *for known* N, p, *and* v *is*

$$E[g(l,m)|N,p,v] = p + (1-p)(p + (1-p)(1-v))^N.$$

Proof. Let the estimate of p be defined as in (3). Also, let the tags be distributed as shown in Fig. 3. Here, there are N tags which are either missed with probability p, thereby ending in the set K or found with probability $1-p$, thereby ending in the set A. From the set A, tags are reused with probability v or silenced with probability $1-v$, thereby ending in either the set B or K_1, respectively. The tags in B are then refound in the second reader session with probability $1-p$ or missed with probability p, thereby ending in either the set M or L, respectively. This decision tree imposes some restrictions on the random values in the Venn diagram. As all the possible sets must sum to N, the following must hold:

$$N = k + A = k + k_1 + B = k + k_1 + l + m. \tag{4}$$

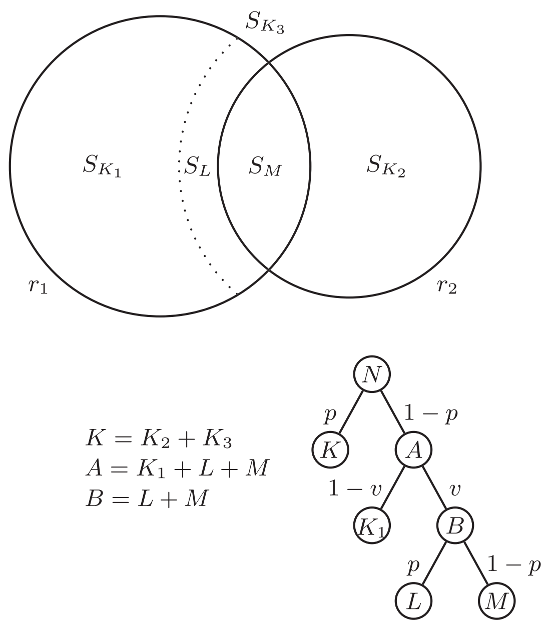


Fig. 3. The Venn diagram and the decision process on how the tags are distributed. The abstraction of the decision process is useful in calculating the bias of the estimator. Also listed are three helpful abstractions that aid in understanding how the decision process is generated from the Venn diagram. Since K_2 does not matter for the estimator it has been merged with K_3 in the set denoted K.

This must be taken into account in the expected value at some point, but for ease of notation, we begin by defining the expected value of the estimator $E[g(L,M)|N,p,v]$ without it. In the following, we leave out the conditioning on N, p, and v in the notation of the expected value also for ease of notation:

$$E[g(L,M)] = \sum_{k,k_1,l} g(l,m)\Pr[K=k, K_1=k_1, L=l, M=m].$$

As the function in (3) has two cases, this sum can be split in two:

$$\begin{aligned} &E[g(L,M)] \\ &\quad= \sum_{\substack{k,k_1,l \\ l+m\neq 0}} \frac{l}{l+m}\Pr[K=k, K_1=k_1, L=l, M=m] \\ &\quad+ \sum_{\substack{k,k_1,l \\ l+m=0}} 1\Pr[K=k, K_1=k_1, L=l, M=m]. \end{aligned} \tag{5}$$

Using the restrictions imposed by the decision tree in (4), we now expand the sums. Because all set cardinalities must sum to N, we loose one degree of freedom and must replace m with $N-k-k_1-l$ in the expectation for the general case. The restriction of the general case is that $l+m \neq 0$, which also has implications, because of (4)

$$\begin{aligned} &l+m = N-k-k_1 \neq 0 \quad \wedge \quad m = N-k-k_1-l \quad \Rightarrow \\ &k \neq N \quad \wedge \quad k_1 \neq N-k \quad \Rightarrow \\ &k = 0,1,\ldots,N-1 \quad \wedge \quad k_1 = 0,1,\ldots,N-k-1 \quad \wedge \\ &l = 0,1,\ldots,N-k-k_1 \quad \wedge \quad m = N-k-k_1-l. \end{aligned}$$

In the special case, we know that $l+m=0$ and that (4) must hold, which results in the following:

$$
\begin{aligned}
&l + m = N - k - k_1 = 0\\
&l = m = 0 \ \wedge \ k_1 = N - k \ \Rightarrow\\
&k = 0, 1, \ldots, N \quad \wedge \quad k_1 = N - k \ \wedge \ l = 0 \ \wedge \ m = 0.
\end{aligned}
$$

We insert these results in (5)

$$
\begin{aligned}
&E[g(L, M)]\\
&\quad = \sum_{k=0}^{N-1} \sum_{k_1=0}^{N-k-1} \sum_{l=0}^{N-k-k_1} \frac{l}{l + N - k - k_1 - l}\\
&\qquad \cdot \underbrace{\Pr[K = k, K_1 = k_1, L = l, M = N - k - k_1 - l]}_{X}\\
&\qquad + \underbrace{\sum_{k=0}^{N} 1 \Pr[K = k, K_1 = N - k, L = 0, M = 0]}_{Y}.
\end{aligned} \tag{6}
$$

In the following, we first solve the general case (X), after which we solve for the special case (Y). For the general case, we insert the multinomial distribution with the probabilities for each set

$$
\begin{aligned}
X &= \sum_{k=0}^{N-1} \sum_{k_1=0}^{N-k-1} \sum_{l=0}^{N-k-k_1} \frac{l}{l + N - k - k_1 - l}\\
&\quad \cdot \Pr[K = k, K_1 = k_1, L = l, M = N - k - k_1 - l]\\
&= \sum_{k=0}^{N-1} \sum_{k_1=0}^{N-k-1} \sum_{l=0}^{N-k-k_1} \frac{l}{l + N - k - k_1 - l} \binom{N}{k, k_1, l}\\
&\quad \cdot p^k((1-p)(1-v))^{k_1}((1-p)vp)^l((1-p)^2 v)^{N-k-k_1-l}\\
&= \sum_{k=0}^{N-1} \sum_{k_1=0}^{N-k-1} \sum_{l=0}^{N-k-k_1} \frac{l}{l + N - k - k_1 - l}\\
&\quad \cdot \binom{N}{k} \binom{N-k}{k_1} \binom{N-k-k_1}{l}.\\
&\quad p^k((1-p)(1-v))^{k_1}((1-p)vp)^l((1-p)^2 v)^{N-k-k_1-l}.
\end{aligned}
$$

Now, it becomes beneficial to replace many of the terms in the sums, with the helpful abstractions introduced in Fig. 3. Recall from (4) that $A = N - k$ and $B = N - k - k_1 = l + m$

$$
\begin{aligned}
X &= \sum_{k=0}^{N-1} \sum_{k_1=0}^{A-1} \sum_{l=0}^{B} \frac{l}{B} \binom{N}{k} \binom{A}{k_1} \binom{B}{l}\\
&\quad \cdot p^k((1-p)(1-v))^{k_1}((1-p)vp)^l((1-p)^2 v)^{B-l}\\
&= \sum_{k=0}^{N-1} \binom{N}{k} p^k \sum_{k_1=0}^{A-1} \binom{A}{k_1} (1-p)^{k_1}(1-v)^{k_1}(1-p)^B v^B\\
&\quad \cdot \frac{1}{B} \sum_{l=0}^{B} l \binom{B}{l} p^l (1-p)^{B-l}.
\end{aligned}
$$

Notice that the last sum can now be replaced with the expected value of a binomially distributed random variable, $E[L] = Bp$

$$
X = \sum_{k=0}^{N-1} \binom{N}{k} p^k \sum_{k_1=0}^{A-1} \binom{A}{k_1} (1-p)^{B+k_1}(1-v)^{k_1} v^B \frac{1}{B} Bp.
$$

Now isolate the terms with k_1 and A in a binomial distribution (recall from (4) that $B + k_1 = N - k$ and $B = A - k_1$)

$$
\begin{aligned}
X &= p \sum_{k=0}^{N-1} \binom{N}{k} p^k (1-p)^{N-k} \sum_{k_1=0}^{A-1} \binom{A}{k_1} (1-v)^{k_1} v^{A-k_1}\\
&= p \sum_{k=0}^{N-1} \binom{N}{k} p^k (1-p)^{N-k}\\
&\quad \cdot \left(\sum_{k_1=0}^{A} \binom{A}{k_1} (1-v)^{k_1} v^{A-k_1} - (1-v)^A \right)\\
&= p \sum_{k=0}^{N-1} \binom{N}{k} p^k (1-p)^{N-k} (1 - (1-v)^A)\\
&= p \sum_{k=0}^{N-1} \binom{N}{k} p^k (1-p)^{N-k}\\
&\quad - p \left(\sum_{k=0}^{N-1} \binom{N}{k} p^k (1-p)^{N-k} (1-v)^A \right).
\end{aligned}
$$

As $A = N - k$, the last term $(1-v)^A$ can be merged into the binomial distribution. Also, in the first sum we add the term for $k = N$ to fix the limits on the sum

$$
\begin{aligned}
X &= p \left(\sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} - p^N \right)\\
&\quad - p \left(\sum_{k=0}^{N-1} \binom{N}{k} p^k ((1-p)(1-v))^{N-k} \right)\\
&= p(1 - p^N) - p \left(\sum_{k=0}^{N-1} \binom{N}{k} p^k ((1-p)(1-v))^{N-k} \right)\\
&= p - p \left(\sum_{k=0}^{N-1} \binom{N}{k} p^k ((1-p)(1-v))^{N-k} + p^N \right).
\end{aligned}
$$

Now, we use the binomial theorem to simplify the equation

$$
\begin{aligned}
X &= p - p \sum_{k=0}^{N} \binom{N}{k} p^k ((1-p)(1-v))^{N-k}\\
&= p - p(p + (1-p)(1-v))^N.
\end{aligned} \tag{7}
$$

This is part X of the expected value $E[g(L, M)]$ in (6). Now, we continue by solving part Y. If we insert $l = m = 0$ and the multinomial distribution for the probability for each set, we get

$$
\begin{aligned}
Y &= \sum_{k=0}^{N} 1 \Pr[K = k, K_1 = N - k, L = 0, M = 0]\\
&= \sum_{k=0}^{N} \binom{N}{k} p^k ((1-p)(1-v))^{N-k} ((1-p)vp)^0 ((1-p)^2 v)^0\\
&= \sum_{k=0}^{N} \binom{N}{k} p^k ((1-p)(1-v))^{N-k}.
\end{aligned}
$$

Using the binomial theorem, this corresponds to

$$
Y = (p + (1-p)(1-v))^N. \tag{8}
$$

The expected value of the estimator therefore is the addition of the (7) and (8)

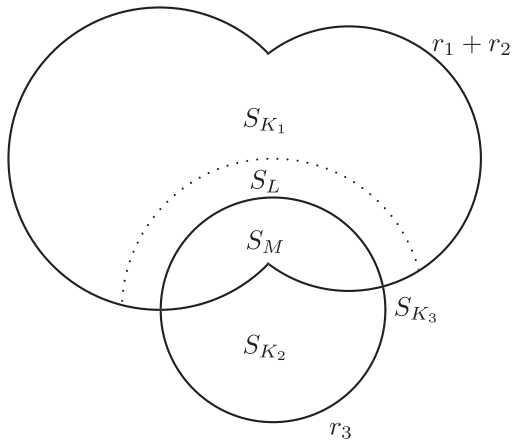


Fig. 4. Venn diagram of the observable sets S_{K_1}, S_L, S_M, and S_{K_2} and of the unobservable set S_{K_3}.

$$\begin{aligned}E[g(L,M)|N,p,v] &= p - p(p+(1-p)(1-v))^N \\ &\quad + (p+(1-p)(1-v))^N \\ &= p + (1-p)(p+(1-p)(1-v))^N \\ &= p + (1-p)(1-v+vp)^N.\end{aligned}$$

□

Note that if $v = 1$ (i.e., we reuse all tags) the bias is only $(1-p)p^N$, which is a smaller bias then the one found for a similar estimator in [9].

The found Reduced Sets Estimator of p in (3) can then be used in the following estimator of N, which is inspired by a similar, unbiased estimator for N from [9]:

$$\hat{N} = \frac{k_1 + l + m + k_2}{1 - \hat{p}^2}.$$

Together, these two estimates, $\hat{p}$ and $\hat{N}$, can be used in the estimator for p_M in (2).

One could ask, in (2), why $(1 - \hat{p}^R)$ is raised to the power of $\hat{N}$, when it is not N tags that participate in all reader sessions? But (2) is based on the *probability* of N tags *not* being missed in R reader sessions ($(1-\hat{p}^R)^{\hat{N}}$), which is not influenced by the reduction of the sets to be resolved. This is because the probability of any given tag being missed in R reader sessions is the same for all tags (independently on whether they were used in the reader session or not).

3.1 Extending the Approach to More than Two Reader Sessions

If the estimate of the probability of missing tags $\hat{p}_M$ (found using (2)) after two reader sessions is above some user defined threshold, another reader session may be performed. This is done by 1) silencing all found tags with probability $1 - v$, 2) performing another reader session, and 3) estimating the probability of missing tags as before. The new sets are then distributed as shown in Fig. 4, and the estimates $\hat{p}$, $\hat{N}$, and $\hat{p}_M$ are calculated as before. This continues until $\hat{p}_M$ is below the chosen threshold.

The algorithm can be summarized as

1. Perform first reader session.
2. Silence all found tags with probability $1 - v$.
3. Perform another reader session.
4. Estimate the probability of missing tags.
5. If $\hat{p}_M$ is above the chosen threshold, repeat steps 2-5, otherwise stop.

In this approach, we make a new estimate of p and N for each reader session, and discard the old estimates. As $\hat{N}$ depends on $\hat{p}$, it is most important that $\hat{p}$ is accurate. Some reader sessions may produce inaccurate estimates of p due to variance, which affects the estimates of N and p_M. To avoid this, we can take the average of the estimates of the static error probability from all the performed reader sessions, that is, instead of letting the used estimate of p after reader session R be $\hat{p} = \hat{p}_R$, we let the estimate be $\hat{p} = (\hat{p}_1 + \cdots + \hat{p}_R)/R$. We investigate the effect of averaging in the numerical results and show that averaging over all reader sessions decreases the variance of the estimate.

3.2 Optimized Relation between Performance and Accuracy

Until now, we have considered v a value that is fixed before the sequential decision process is initiated. Now, we analyze the relation between the desired accuracy of $\hat{p}$ and the number of tags that shall participate. The accuracy of $\hat{p}$ is important for the reliability of the other estimators and the bias must therefore be given an upper bound. This leads to finding the minimum number of tags, denoted N_{min}, that must participate in a reader session for estimation with desired accuracy. This number is therefore the optimum choice for the number of tags to reuse for the estimator in (3). To find N_{min}, we need to establish what acceptable accuracy means. Let us define an acceptable estimate of p as being an estimate which expected value differs from the true value with an error less or equal to 0.01. Then, we can use the expected value of the estimator in (3) and the following must hold:

$$\begin{aligned}|\hat{p} - p| &\le 0.01 \\ |p + (1-p)(p+(1-p)(1-v))^{N_{min}} - p| &\le 0.01 \\ |(1-p)(p+(1-p)(1-v))^{N_{min}}| &\le 0.01.\end{aligned} \quad (9)$$

By finding the minimum value of N_{min} that satisfies this equation for all p and v, we find the optimum choice of N_{min}. Let us ensure that we always reuse at least N_{min} tags, therefore, we set $v = 1$ in the above equation

$$\begin{aligned}(1-p)p^{N_{min}} &\le 0.01 \\ N_{min} &\ge \frac{\ln(0.01) - \ln(1-p)}{\ln(p)},\end{aligned}$$

where $\ln$ denotes the natural logarithm. As we want to find the minimum number of tags to reuse for the estimation process, we bound N_{min} by solving for all $p \in [0, 1)$ in the following way:

$$\begin{aligned}N_{min} &\ge \left\lceil \max_{p\in[0,1)} \frac{\ln(0.01) - \ln(1-p)}{\ln(p)} \right\rceil \\ N_{min} &\ge 37.\end{aligned}$$

This gives, for optimized speed versus the desired accuracy, we must reuse 37 tags.

In summary, in the previous section we introduced the parameter v as being the probability that tags participate in the next reader session. Now, instead of reusing found tags

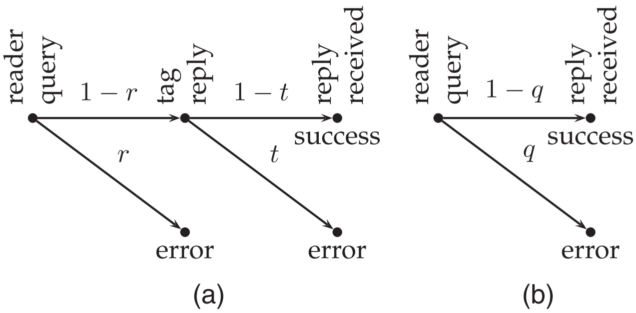


Fig. 5. Model for dynamic errors. (a) Model with errors on each link. (b) Interpretation of link errors where $q = r + (1 - r)t$.

with probability v, we say that we should always reuse exactly 37 tags from the population of found tags as it will provide the sufficient estimation accuracy. These 37 tags should be drawn independently from the found tags between each reader session to avoid correlation between the sets used for estimation. If the set of resolved tags does not contain 37 tags, we reuse them all, to get the best possible base for estimation after the next reader session.

4 Dynamic Errors

As we have stated, $p = f(q, N)$, which prompts the question: When we silence tags in between reader sessions, will the static error probability also change? In the following, we show for the binary tree protocol that the static error probability p remains the same in all reader sessions (within a small margin), independently of the tag set cardinality N, when the number of tags is $N \geq 13$. Technically, we show that the static error probability $p = f(q, N)$ can be well approximated as $f(q)$ for $N \geq 13$; this yields that the estimation becomes independent of the number of participating tags. We do this to show that in the case with dynamic errors with 13 or more tags, then (2) is justified and still holds when the Reduced Sets Estimator is used to estimate p and N.

We have previously shown that we always reuse 37 tags (if possible); if the estimation is performed in the case of dynamic errors, we need at least 13 tags. Therefore, when the Reduced Sets Estimator is used in its optimized setting, it will work in the case of dynamic errors.

4.1 Extended System Model

A reader sends multiple queries to a set of tags, and receives replies accordingly. This communication is subject to *dynamic errors*; noise-induced random errors, which can happen on 1) the *reader-to-tag* link and 2) the *tag-to-reader* link. Let the first link be in error with probability r and the second with probability t, then the probability that a reader does not receive a reply on a query to a given tag is (see Fig. 5)

$$q = r + (1 - r)t.$$

Note that we only consider dynamic errors which may contribute to the problem of missing tags. For example, errors in which a single slot is interpreted at the reader as a collision slot does not contribute to the probability of missing tags (although it may increase the arbitration time). Additionally, we assume that the dynamic error probability q is the same for all queries during an arbitration protocol run; that is, all tags have equal probability of being in error, with no correlation on whether the link was previously in error/not in error. Also, let p continue to denote the *static error*, i.e., the probability of missing a tag after a given reader session.

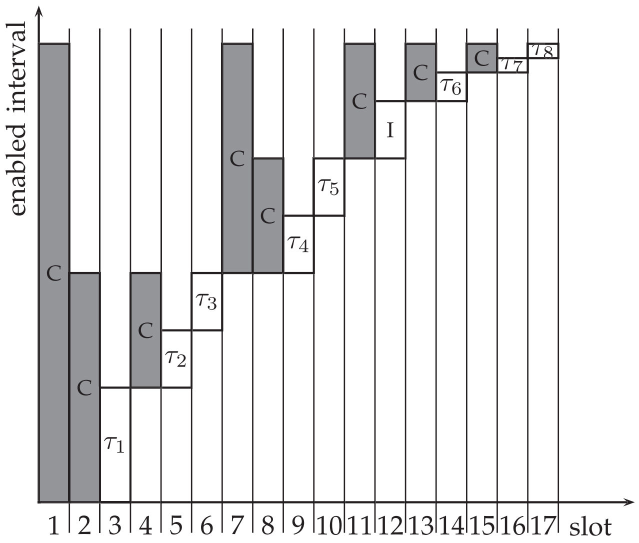


Fig. 6. Alternative representation of the tree in Fig. 1 where each slot is specified as intervals. C refers to collision.

In Section 2.1, we introduced the tree-based algorithm for collision resolution of tag sets, where each tag generated a *bit-array*. For the following derivation, it is instructive to utilize the alternative representation of the tree algorithms, as suggested in [15]. Each bit-array $x_1 x_2 \ldots$ is then uniquely represented by a *token* in the interval $[0, 1)$, when using the interval notation introduced in [15]. The token is the real number that has a binary representation $0.x_1 x_2 x_3 \ldots$ The mapped token provides a different representation of the arbitration process by the binary tree. Thus, when the tags with bit-array "0^*" are allowed to transmit, it is equivalent to state that the tags that have tokens in $[0, 0.5)$ are allowed to transmit. In short, we say that "$[0, 0.5)$ is enabled." Therefore, instead of traversing a binary tree, now the arbitration process can be represented by using a sequence of enabled intervals. Continuing the example in Fig. 1, a graphical illustration of the enabled intervals is in Fig. 6.

4.2 Relation between Static and Dynamic Errors

The relation between the static error probability p (the probability to miss a tag in a reader session), and the dynamic error probability q (the probability of error on the *reader-tag-reader* link) for the binary tree algorithm is analyzed and found in this section. The analysis is separated in two steps, where we *first* analyze the binary tree and find an expression for the expected number of missed tags in the interval $[a, b)$ given the number of tags in that interval. The analysis is performed for intervals containing $L = 0, 1, 2, 3$ tags, and we state the general recursive algorithm for $L = 0, 1, \ldots$ tags. *Next*, we use the expression for the expected number of missed tags together with the actual number of tags in the interval to find an expression for the static error probability. This expression is evaluated numerically, and the conclusion of invariance on N is drawn.

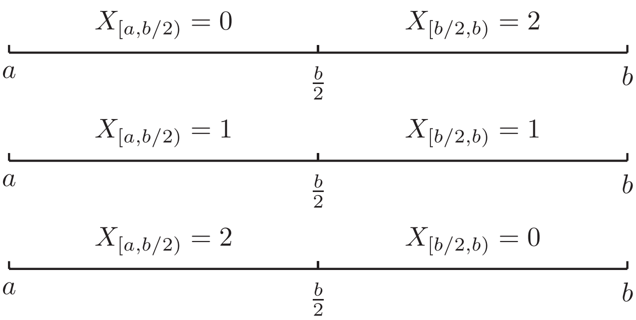


Fig. 7. The possible distributions in $[a, b)$ when $L = 2$.

4.2.1 Expected Number of Missed Tags in Interval $[a, b)$

Let the expected number of missed tags in an interval containing L tags be denoted M_L, then we have the following expected number of missed tags for $L = 0, 1$:

$$M_0 = 0, \qquad M_1 = q,$$

which is fairly simple as no collisions can occur. For $L = 2$, we have the possible distributions in the interval $[a, b)$ shown in Fig. 7, where $X_{[u,v)}$ is a random variable signifying the number of tags having tag tokens in the interval $[u, v)$. The probabilities for each distribution are (tag tokens are considered i.i.d.)

$$\begin{aligned}\Pr\left[X_{[a,b/2)} = 0\right] &= \Pr\left[X_{[a,b/2)} = 2\right] = \frac{1}{4},\\ \Pr\left[X_{[a,b/2)} = 1\right] &= \binom{2}{1}\frac{1}{4}.\end{aligned}$$

The expected number of missed tags is now found for each of the possible distributions. If the distribution is $X_{[a,b/2)} = 0$, then the expected number of missed tags is

$$M_{2|X_{[a,b/2)}=0} = 2q^2 + 1 \cdot 2(1-q)q + (1-q)^2 M_2;$$

we miss two tags if the outcome is idle, one tag if the outcome is single, and the expected value M_2 if the outcome is collision. The reason for this is: if a collision is detected, then the reader first enables the interval $[a, b/2)$ and then $[b/2, b)$. There are no tags in the interval $[a, b/2)$ according to the condition $X_{[a,b/2)} = 0$, so the outcome in this interval is idle and the expected number of missed tags is zero. In the next enabled interval $[b/2, b)$, there are two unread tags, and the expected number of missed tags in this interval is the unconditioned M_2, as we have no knowledge about where in the interval $[b/2, b)$ the two tags are distributed.

In the same way as for the first interval, we find the expected number of missed tags for the other two distributions to be

$$\begin{aligned}M_{2|X_{[a,b/2)}=1} &= 2q^2 + 2(1-q)q + (1-q)^2(M_1 + M_1),\\ M_{2|X_{[a,b/2)}=2} &= 2q^2 + 2(1-q)q + (1-q)^2 M_2.\end{aligned}$$

Here, for $X_{[a,b/2)} = 1$, if we observe collision and as we have conditioned that we have one tag in $[a, b/2)$ and one tag in $[b/2, b)$, then the binary tree algorithm first enables the interval $[a, b/2)$ where the expected number of missed tags is M_1, it then continues to $[b/2, b)$ and the expected number of missed tags is again M_1.

We now multiply the expected values with the respective probabilities, and we find, because of symmetry in the distributions for $X_{[a,b/2)} = 0$ and $X_{[a,b/2)} = 2$, the unconditioned M_2 as

$$\begin{aligned}M_2 &= 2\Pr\left[X_{[a,b/2)} = 0\right] M_{2|X_{[a,b/2)}=0}\\ &\quad + \Pr\left[X_{[a,b/2)} = 1\right] M_{2|X_{[a,b/2)}=1}\\ &= 2q^2 + \binom{2}{1}(1-q)q\\ &\quad + (1-q)^2\left\{2\frac{1}{4}M_2 + \binom{2}{1}\frac{1}{4}(M_1 + M_1)\right\}.\end{aligned}$$

In a similar way, we find for $L = 3$ the probabilities for the distributions to be

$$\begin{aligned}\Pr\left[X_{[a,b/2)} = 0\right] &= \Pr\left[X_{[a,b/2)} = 3\right] = \frac{1}{8},\\ \Pr\left[X_{[a,b/2)} = 1\right] &= \Pr\left[X_{[a,b/2)} = 2\right] = \binom{3}{1}\frac{1}{8}.\end{aligned}$$

The conditioned expected number of missing tags for the respective probabilities are

$$\begin{aligned}M_{3|X_{[a,b/2)}=0} = M_{3|X_{[a,b/2)}=3} &= 3q^3 + 2 \cdot 3(1-q)q^2\\ &\quad + \left[\binom{3}{2}(1-q)^2 q + (1-q)^3\right] M_3,\\ M_{3|X_{[a,b/2)}=1} = M_{3|X_{[a,b/2)}=2} &= 3q^3 + 2 \cdot 3(1-q)q^2\\ &\quad + \left[\binom{3}{2}(1-q)^2 q + (1-q)^3\right](M_1 + M_2).\end{aligned}$$

We again multiply the conditioned expected number of missed tags with their respective probabilities, and the expected number of missed tags for $L = 3$ is

$$\begin{aligned}M_3 &= 3q^3 + 6(1-q)q^2\\ &\quad + \left[\binom{3}{2}(1-q)^2 q + (1-q)^3\right]\\ &\quad \cdot \left\{2\frac{1}{8}M_3 + 2\binom{3}{1}\frac{1}{8}(M_1 + M_2)\right\}.\end{aligned}$$

It can be shown that for $L = 0, 1, \ldots$ the general expression is

$$\begin{aligned}M_L &= Lq^L + (L-1)L(1-q)q^{L-1}\\ &\quad + \sum_{i=2}^{L}\binom{L}{i}(1-q)^i q^{L-i}\sum_{j=1}^{L}\binom{L}{j}\frac{2}{2^L}M_j,\end{aligned}$$

which can be rewritten to the following recursive expression:

$$M_L = \frac{Lq^L + (L-1)L(1-q)q^{L-1} + p_C(L)\sum_{j=1}^{L-1}\binom{L}{j}\frac{2}{2^L}M_j}{\left(1 - \frac{2}{2^L}p_C(L)\right)}, \tag{10}$$

where

$$p_C(L) = \sum_{i=2}^{L}\binom{L}{i}(1-q)^i q^{L-i}$$

is the probability of collision for L participating tags.

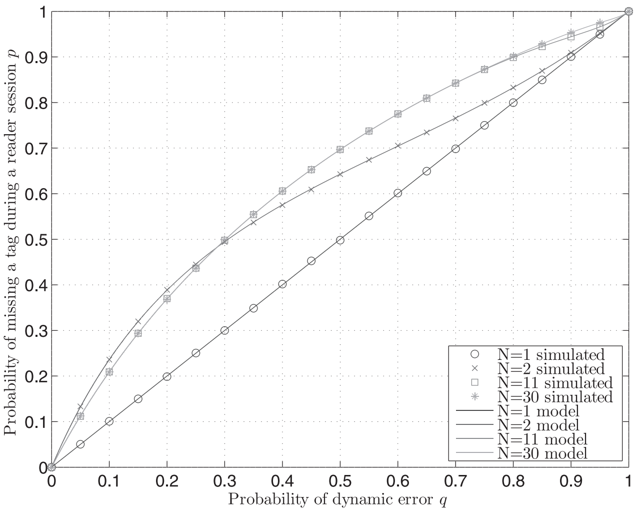


Fig. 8. Calculated and simulated probability of dynamic error q versus the probability of error in a reader session p for different choices of N.

4.2.2 Analysis of Estimation Error

Given the expected number of missed tags in the interval $[0, 1)$ is M_N (found using (10)), then, as the probability of missing a tag in a reader session is p, we have

$$NE[p] = M_N,$$

and an estimate of the static error probability when N tags participate is

$$\hat{p}_N = \frac{M_N}{N}. \tag{11}$$

It is interesting how the tag set cardinality affects the relation between p and q (recall that M_N is a function of q and N). The relation is illustrated in Fig. 8 for different values of N, where the expected value of p can be compared with simulated results. The figure shows: 1) that p is a function of q and that the dependence on N decreases for large N, and 2) that the expected static error probability is similar to the averaged simulations. The simulated results are found by averaging over 100,000 runs of the binary tree algorithm, where the tag tokens are randomized between each run. One can show that such a randomization removes the correlation of errors across the tags. Instead of showing it formally, we only illustrate the concept. For the example on Fig. 6, one can see that if both τ_2 and τ_3 do not receive the query, then there is idle response in slot 4, such that both will be missed. If the tokens of τ_2 and τ_3 are not randomly chosen for the next reader session, then the probability that τ_2 and τ_3 are again jointly missed is higher than the case in which their tokens are randomized.

It is relevant to ask how large N should be before the relation becomes independent of N. We have investigated this issue numerically where we want to find the minimum N where the expected error in the relation between p and q compared to any larger tag set cardinality is less than some threshold.

Let Q be the smallest value of N where the estimation error made for this and any larger N is less than 10^{-2}. This states, from (11)

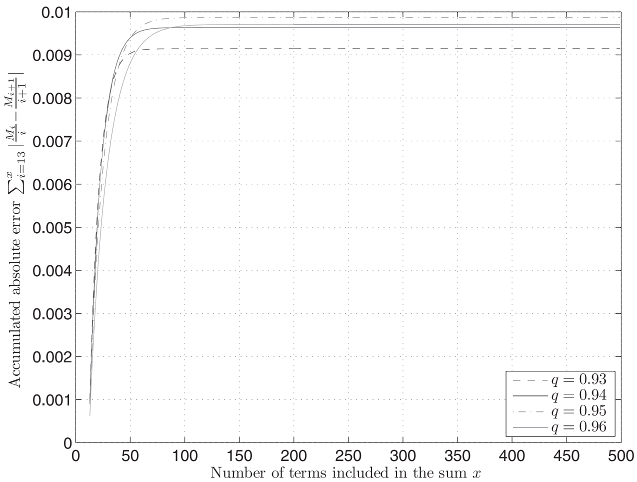


Fig. 9. Calculated accumulative absolute error between p_{13} and p_L versus the number of terms included in the sum, L, where $L = 14, 15, \ldots, 500$. We conjecture that for any larger values of L the error will never exceed 10^{-2}.

$$\hat{p}_Q - \hat{p}_L = \frac{M_Q}{Q} - \frac{M_L}{L} \le 10^{-2}, \quad L = Q, Q+1, \ldots$$

The errors are cumulative, so

$$\hat{p}_Q - \hat{p}_L = \sum_{i=Q}^{L} \left(\frac{M_i}{i} - \frac{M_{i+1}}{i+1} \right).$$

Being more pessimistic, we say that

$$\hat{p}_Q - \hat{p}_L \le \sum_{i=Q}^{L} \left| \frac{M_i}{i} - \frac{M_{i+1}}{i+1} \right|, \tag{12}$$

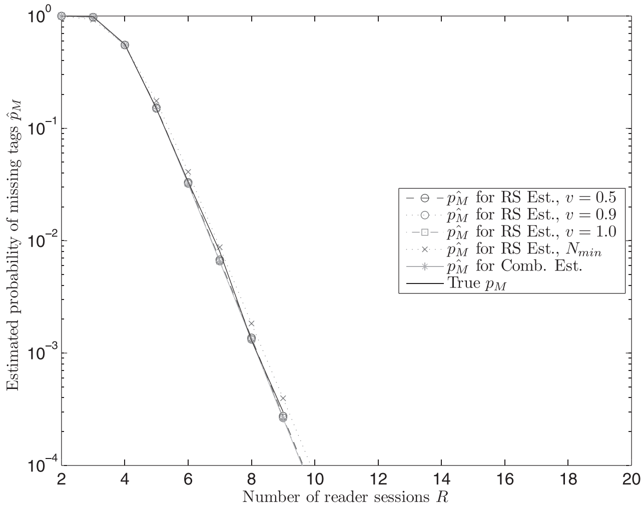
as then we know that the error is always equal or decreases as Q increases. From the relation in Fig. 8, it can be seen that the dependence on N is largest for high values of q, more precisely in the region $q = [0.9, 1)$, as the difference between the lines for high values of N is largest in this region. The expression for incremental errors in (12) can be plotted for different values of Q. In Fig. 9, the error made is plotted for values of q in the critical region with $Q = 13$ versus the number of terms included in the sum L. The figure indicates that the error peaks for $q \approx 0.95$, and that the error does not exceed the specified margin at 10^{-2} no matter how many terms are included in the sum. Based on this, we conjecture that for $N \ge 13$ the relation between p, q, and N becomes independent of N.

5 Numerical Results

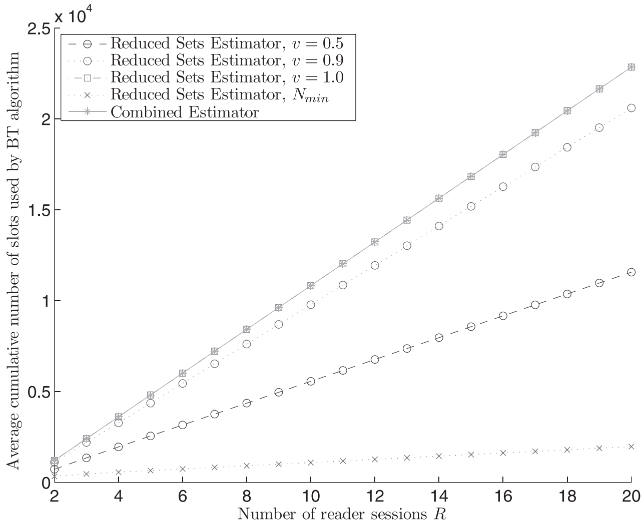
In this section, we show that the new estimator working on reduced sets obtain the same reliability as the Combined Estimator, and that it does so using fewer slots. We first compare the reliability of the estimators alongside with the number of slots used for arbitration. Then, we show the advantage of averaging the estimates of the static error probability over all performed reader sessions.

5.1 Method for Numerical Simulations

In one simulation run, we generate a set of random tag IDs and use the proposed estimation methods together with the



(a)



(b)

Fig. 10. Numerical simulation results: (a) The estimate of p_M for the Reduced Sets Estimator with $v = 0.5$, $v = 0.9$, $v = 1$, optimum N_{min} and the Combined Estimator for comparison ($N = 500$). (b) The cumulative average number of slots used per reader sessions.

basic tree protocol to arbitrate the set. The estimators in the reliability layer operate on the resolved tag sets from the arbitration layer, where the basic tree protocol is used for collision resolution. The basic tree protocol is used as in [10] with dynamic errors, i.e., noise-induced random errors, where the static error probability p is set to $p = 0.2$.[1] Each performed simulation is run 1,000 times and the results are averaged over all the runs. We also use the optimization of averaging over all found estimates of p in previous reader sessions, as mentioned in Section 3.1, which decreases the variance of the estimates.

To compare and validate the results, we estimate the value to which p_M converges by performing 100,000 simulations, each consisting of 20 reader sessions with the binary tree algorithm. We denote this the *true* p_M. For each of the 20 reader sessions, it is calculated in how many of the simulations one or more tags were missing, and this is then used to calculate the probability of missing tags.

1. Conversion to the dynamic error probability for use with the binary tree algorithm is found using Fig. 8.

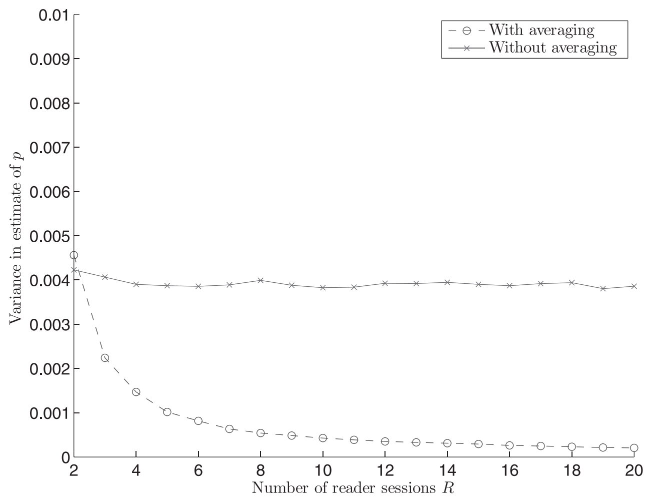


Fig. 11. The variance in the estimate over 1,000 runs per reader session.

5.2 Results

In Fig. 10, the estimated probability of missing tags with different estimators is shown in a scenario with $N = 500$ and $p = 0.2$, and it can be seen that the Reduced Sets Estimator performs almost identically to the Combined Estimator from previous work. Fig. 10 shows how the number of slots used per reader session by the arbitration protocol quickly decreases, when using the proposed technique with reduced sets. Also, it can be seen from $v = 0.5$, $v = 0.9$, and $v = 1$ that the decrease in used slots is proportional to the choice of v and that for $v = 1$, the Reduced Sets Estimator performs identically to the Combined Estimator.

To demonstrate the effect of averaging over the estimates of p accumulated over the reader sessions, we show in Fig. 11 how the variance of the estimate of the static error probability behaves. As can be seen the estimate of p with averaging in between reader sessions estimates p with less variance. This is important, because a single bad estimate of p may cause the sequential decision process to terminate before the true probability of missing tags is below the chosen threshold. This is less likely to happen when the estimate of p does not only depend on the current reader session, but on all performed reader sessions.

6 Conclusion

We have presented a novel type of estimator for the probability of missing tags, which can be used in a sequential decision process for reliable reading of RFID tag sets. The current state of the art estimator, the Combined Estimator, finds the same tags several times, resulting in a decrease in performance in terms of used slots by the arbitration protocol. The new Reduced Sets Estimator reduces this drop in performance by silencing many of the already found tags, while maintaining the same accuracy as the old estimator. Using the analytically found expression for the expected value, a relation is established between the desired accuracy of the estimate of static error and the minimum number of tags to reuse. This relation is used to find the optimum number of tags to reuse when applying the Reduced Sets Estimator proposed in this paper. We then show that for dynamic errors, our estimator is still valid, when using the binary tree protocol, as p becomes independent of N, for $N \geq 13$.

Numerical simulations show that the Reduced Sets Estimator for three different choices of probability of reusing tags and when reusing exactly N_{min} tags is as reliable as the estimators proposed in other work. The simulations also show that for a low probability of reusing a tag, less slots are used for arbitration, which speeds up the arbitration process.

The most important step in future work is to apply the presented ideas for estimation, with a suitable modification, to the case in which the probability of missing a tag is not equal for all the tags. Such a model needs to consider the physical scenario of tag deployment and propagation effects. In addition, the performance evaluation should be conducted by using real-life protocol parameters and durations and explicitly account for the cost of the procedure for tag silencing.

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