Learning without feedback: Fixed random learning signals allow for feedforward training of deep neural networks

Charlotte Frenkel*‡, Martin Lefebvre‡ and David Bol

ICTEAM, Université catholique de Louvain, Belgium

‡ Equal contributions
*Now with Institute of Neuroinformatics, UZH and ETH Zürich, Switzerland
charlotte@ini.uzh.ch, martin.lefebvre@uclouvain.be

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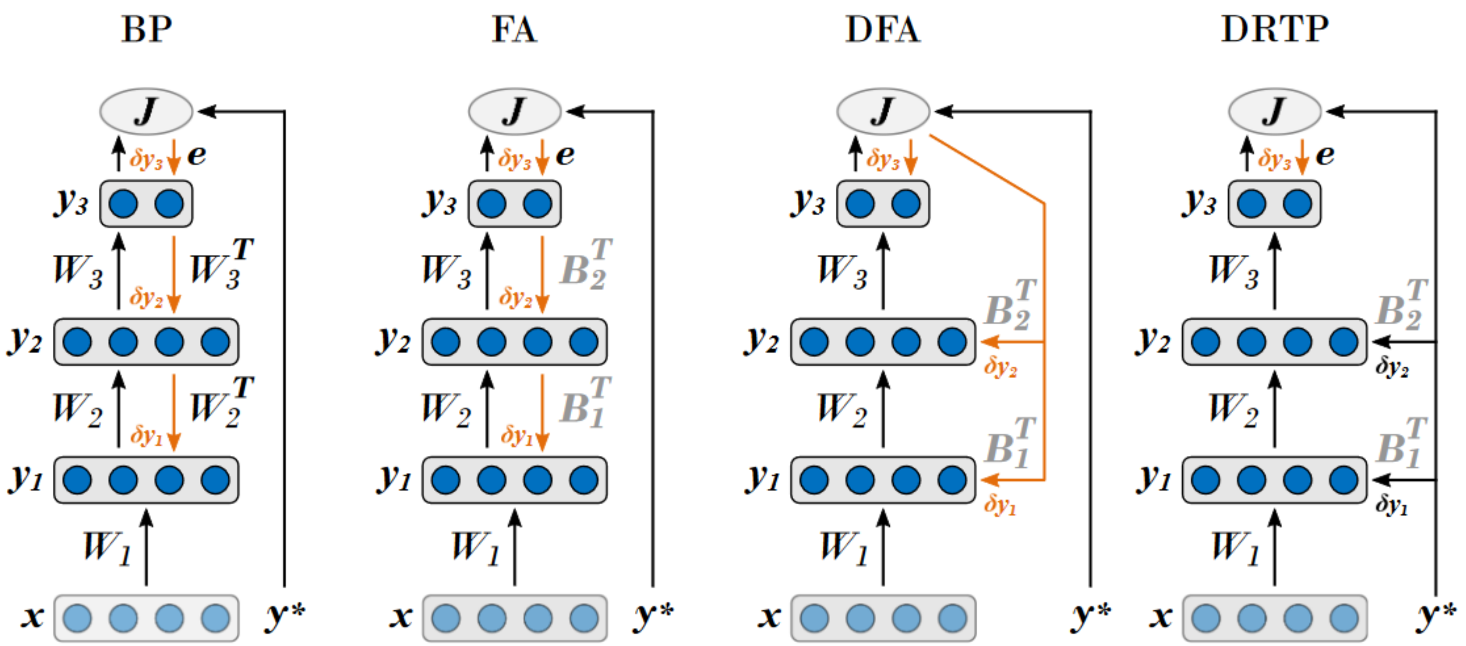
Overview – From feedback alignment to direct random target projection

Releasing the weight transport and update locking of backprop



δy_k	$\frac{\partial J}{\partial y_k} = W_{k+1}^T \delta z_{k+1}$	$B_k^T \delta z_{k+1}$	$B_k^T e$	$B_k^T y^*$
Weight-transport-free	×	✓	✓	✓
Update-unlocked	×	×	×	✓

Feedforward local training

↘ Computational and memory cost ↘

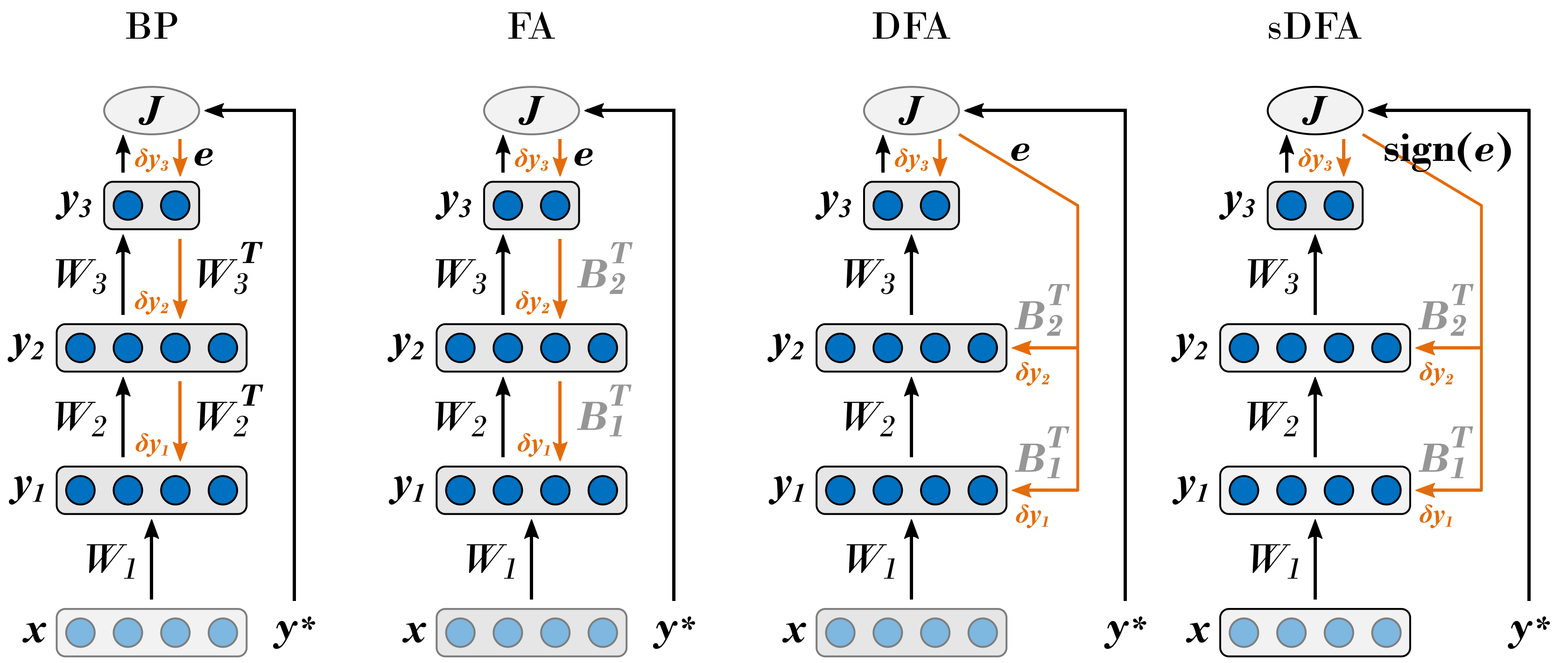
Outline

- Sign-based DFA (sDFA) solves synthetic regression and classification tasks
- From sDFA to DRTP: releasing update locking for classification tasks
- DRTP solves classification tasks: MNIST and CIFAR-10 benchmarking
- Conclusion and perspectives

Empirical results on synthetic datasets

Training algorithms

Claim: Weight updates based only on the error sign provide learning to multi-layer networks

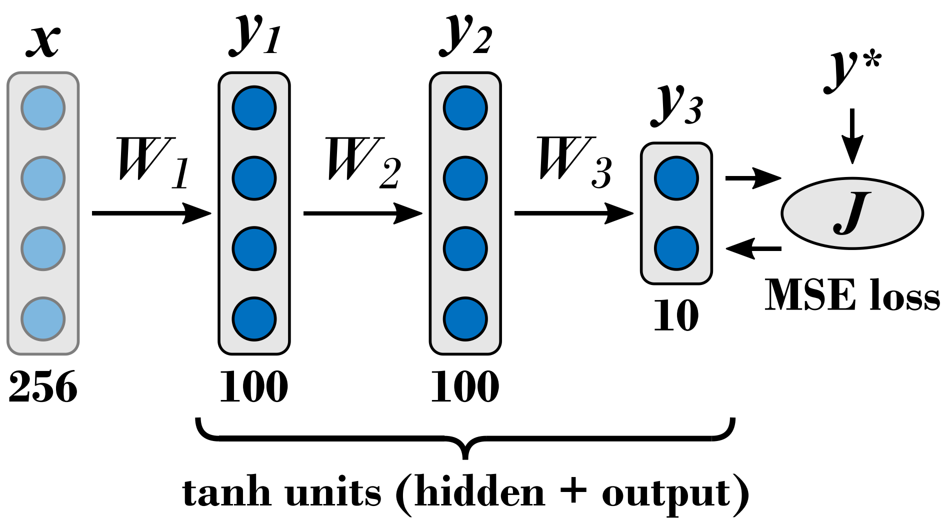


δy_k $\quad \frac{\partial J}{\partial y_k} = W_{k+1}^T \delta z_{k+1}$ $\quad B_k^T \delta z_{k+1}$ $\quad B_k^T e$ $\quad B_k^T \mathrm{sign}(e)$

Sign-based DFA solves regression tasks

Setup

Goal: Approximate 10 non-linear cosine functions



$y_i^* = \cos(\bar{x} + \phi_i)$ with $i \in \mathbb{Z}, i \in [0; 9]$

where $\phi_i = \frac{-\pi}{2} + \frac{i\pi}{9}$

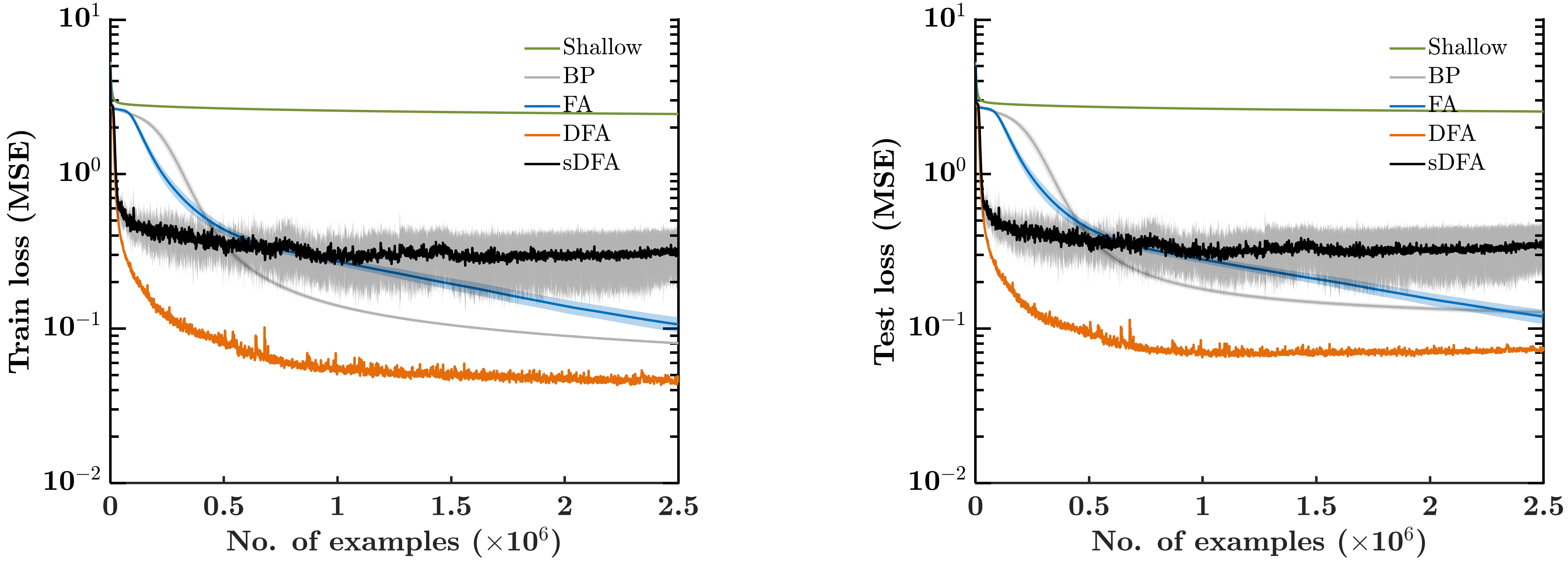
$\bar{x} = \text{mean}(x)$

$x_j \sim N(\mu_x, 1)$ with $j \in \mathbb{Z}, j \in [0; 255]$

$\mu_x \sim U(-\pi, \pi)$

Sign-based DFA solves regression tasks

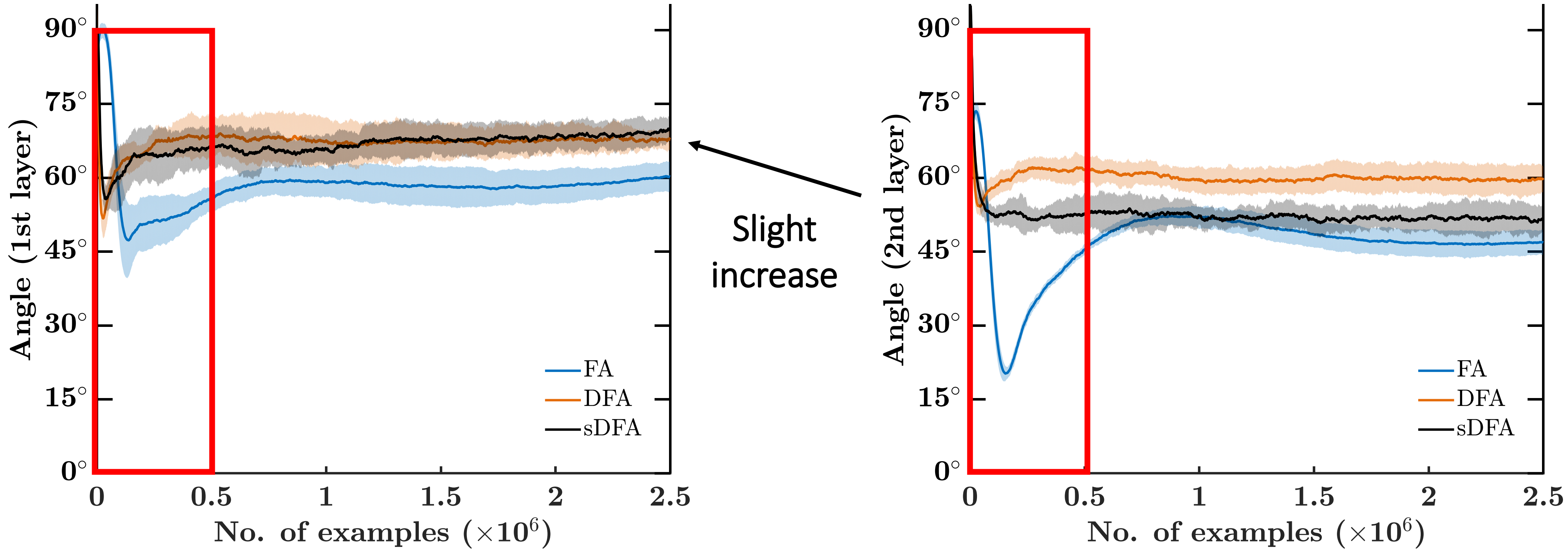
Loss



- DFA > BP/FA > sDFA > shallow
- sDFA performance drop due to lack of error magnitude information
  - No reduction of effective learning rate as training progresses
  - Class-dependent error magnitude is lost

Sign-based DFA solves regression tasks

Angle

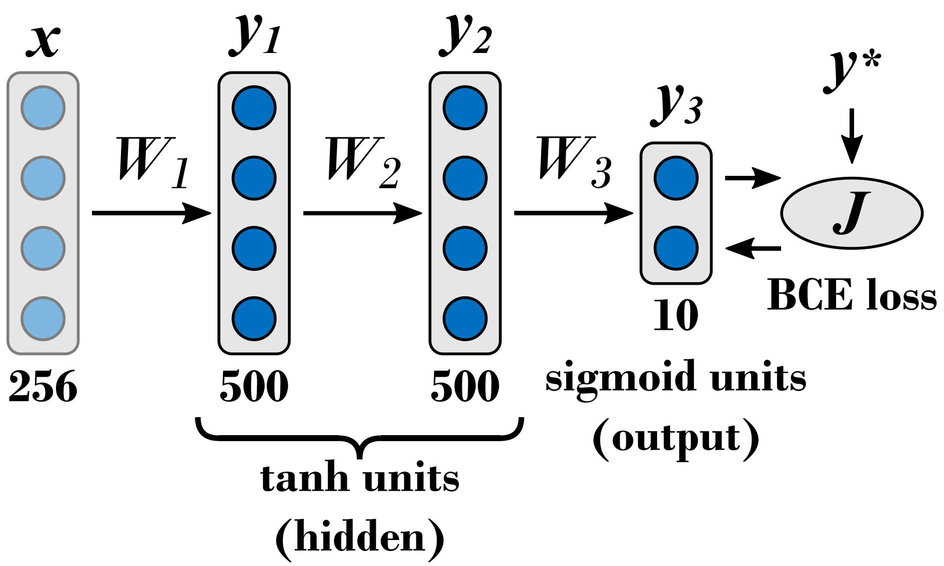


- FA alignment better during 100 first epochs
- sDFA alignment similar to DFA
- Alignment slightly **degrades** away from the network output

Sign-based DFA solves classification tasks

Setup

Goal: Classify 256-dimensional vectors into 10 classes

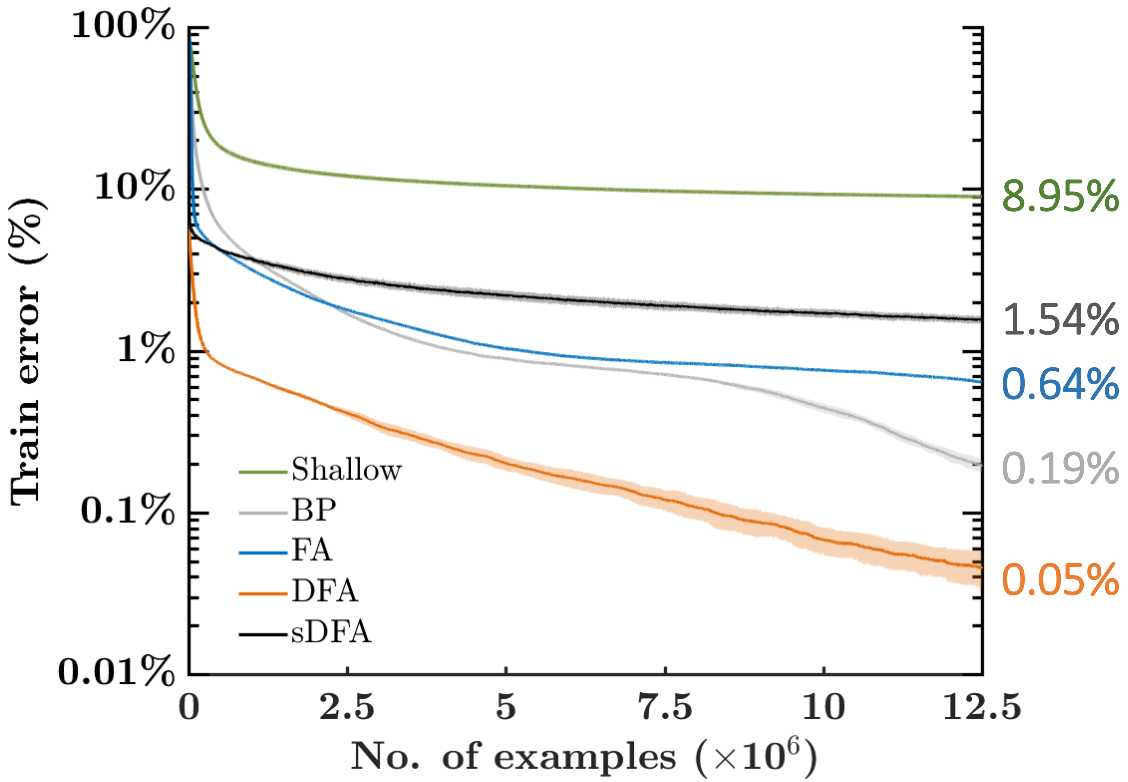


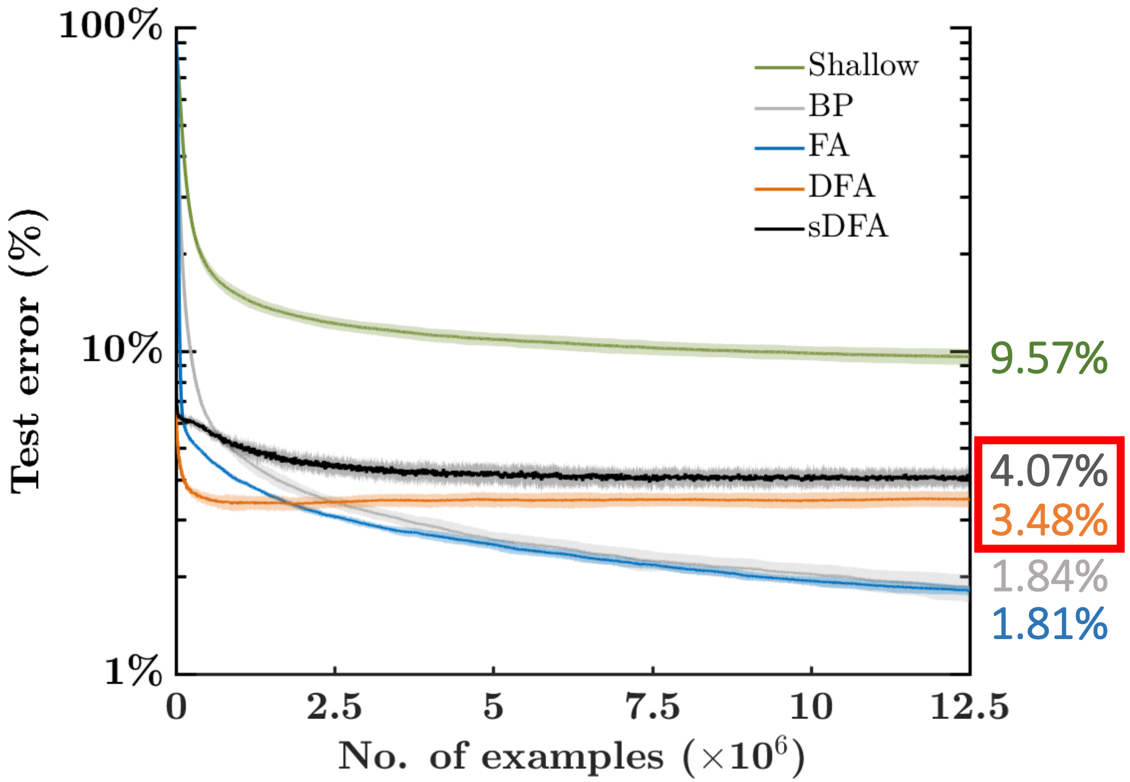
(x, y^*) pairs generated by sklearn library

(make_classification function)

Sign-based DFA solves classification tasks

Accuracy

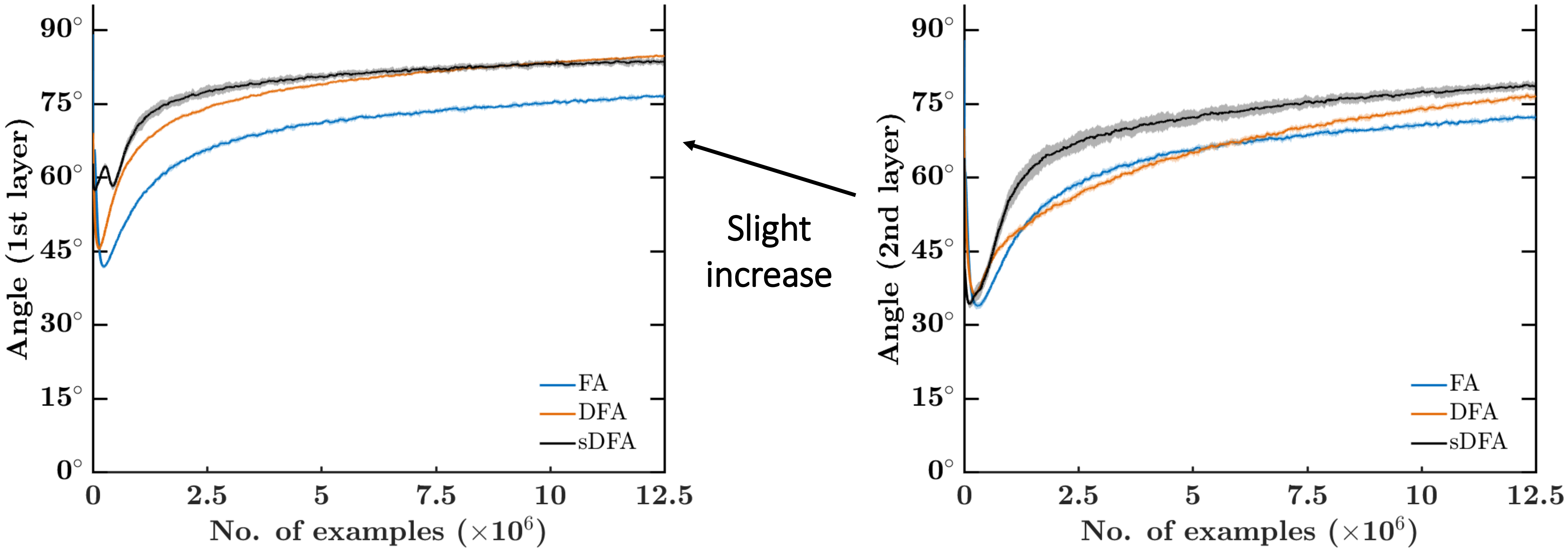




- **Training set**: DFA > BP > FA > sDFA > shallow
- **Test set**: BP/FA > DFA/sDFA > shallow

Sign-based DFA solves classification tasks

Angle



- FA > DFA/sDFA

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- Sign-based DFA (sDFA) solves synthetic regression and classification tasks
- From sDFA to DRTP: releasing update locking for classification tasks
- DRTP solves classification tasks: MNIST and CIFAR-10 benchmarking
- Conclusion and perspectives

The error sign is known in advance

Claim: For classification, a feedback pathway is no longer needed as the error sign is known in advance

$$e = y^* - y_K \longrightarrow e_c = \begin{cases} 1 - y_{Kc} & \text{if} \quad c = c^* \\ -y_{Kc} & \text{otherwise} \end{cases}$$

Correct class
Incorrect classes

$y_{Kc} \in [0; 1]$ softmax/sigmoid

$$\downarrow$$

$$\text{sign}(e_c) = \begin{cases} +1 & \text{if} \quad c = c^* \\ -1 & \text{otherwise} \end{cases}$$

Correct class
Incorrect classes

For a given example, the error sign does not change during training

DRTP solves classification tasks without feedback

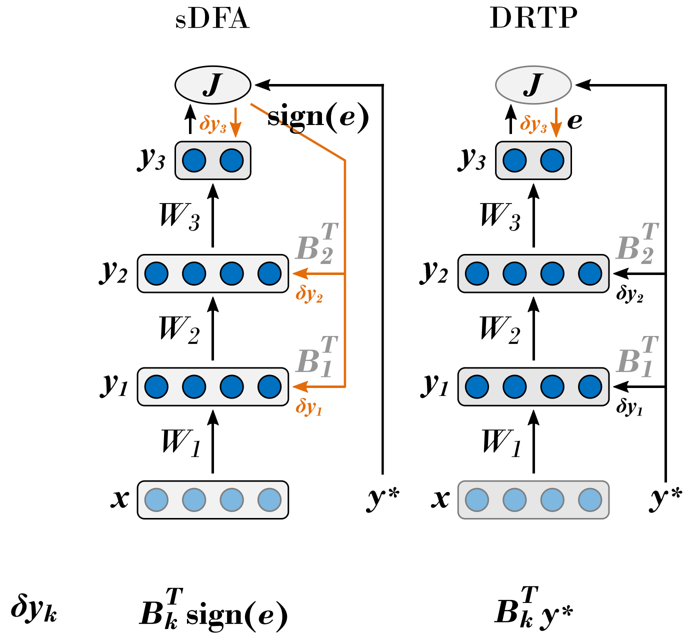
Link between sDFA and DRTP

Claim: Direct random target projection (DRTP) delivers useful modulatory signals

DRTP is a simplified version of sDFA

1. DRTP is computationally cheaper than sDFA
   DRTP: Label-dependent selection of layerwise random vector
   sDFA: Matrix product between error vector and fixed random matrix

2. DRTP systemically outperforms sDFA on MNIST and CIFAR-10 datasets
   DRTP: Only the correct class has an impact
   sDFA: The C-1 incorrect classes outweigh the correct class



The directions of DRTP and BP modulatory signals are within 90° of each other

➡ See full proof in the paper

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DRTP benchmarking on classification tasks

MNIST assessment

MNIST

Network	BP	FA	DFA	DRTP
784-500-10	1.72±0.08%	1.92±0.08%	2.59±0.11%	4.58±0.12%
784-1000-10	1.76±0.06%	1.90±0.06%	2.12±0.05%	4.03±0.13%
784-500-500-10	1.62±0.12%	1.95±0.07%	4.35±0.30%	4.57±0.13%
784-1000-1000-10	1.67±0.07%	1.90±0.07%	3.46±0.25%	4.04±0.12%
CONV* (random)	1.31±0.08%	1.55±0.04%	1.66±0.11%	1.87±0.12%
CONV* (trained)	0.99±0.05%	1.38±0.06%	2.38±0.39%	1.81±0.14%

* 28x28-32c5-2p-1000-10

Fully-connected networks

Performance degrades as the BP constraints are relaxed

DRTP is still competitive

DRTP benchmarking on classification tasks

MNIST assessment

MNIST

Network	BP	FA	DFA	DRTP
784-500-10	1.72±0.08%	1.92±0.08%	2.59±0.11%	4.58±0.12%
784-1000-10	1.76±0.06%	1.90±0.06%	2.12±0.05%	4.03±0.13%
784-500-500-10	1.62±0.12%	1.95±0.07%	4.35±0.30%	4.57±0.13%
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* 28x28-32c5-2p-1000-10

Convolutional neural networks (fixed random convolutional layers)

All training algorithms lie close to each other on MNIST

DRTP benchmarking on classification tasks

MNIST assessment

MNIST

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784-500-10	1.72±0.08%	1.92±0.08%	2.59±0.11%	4.58±0.12%
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* 28x28-32c5-2p-1000-10

Convolutional neural networks (trained convolutional layers)

Only BP allows leveraging training for convolutional layers.
Feedback-alignment-based algorithms require parameter redundancy, which is not offered in convolutional layers.

DRTP benchmarking on classification tasks

MNIST and CIFAR-10 assessment

MNIST

Network	BP	FA	DFA	DRTP
784-500-10	1.72±0.08%	1.92±0.08%	2.59±0.11%	4.58±0.12%
784-1000-10	1.76±0.06%	1.90±0.06%	2.12±0.05%	4.03±0.13%
784-500-500-10	1.62±0.12%	1.95±0.07%	4.35±0.30%	4.57±0.13%
784-1000-1000-10	1.67±0.07%	1.90±0.07%	3.46±0.25%	4.04±0.12%
CONV* (random)	1.31±0.08%	1.55±0.04%	1.66±0.11%	1.87±0.12%
CONV* (trained)	0.99±0.05%	1.38±0.06%	2.38±0.39%	1.81±0.14%

* 28x28-32c5-2p-1000-10

CIFAR-10

Network	BP	FA	DFA	DRTP
784-500-10	48.43±0.30%	49.59±0.25%	49.73±0.24%	53.72±0.30%
784-1000-10	47.58±0.21%	48.56±0.28%	48.45±0.17%	52.99±0.22%
784-500-500-10	49.23±0.24%	50.83±0.20%	50.76±0.24%	53.46±0.16%
784-1000-1000-10	49.00±0.22%	50.35±0.18%	50.51±0.24%	52.83±0.44%
CONV* (random)	30.13±0.31%	30.28±0.37%	30.40±0.46%	32.69±0.38%
CONV* (trained)	27.45±0.28%	29.84±0.31%	32.06±0.29%	35.45±0.76%

* 32x32x3-64c3-2p-256c3-2p-1000-1000-10

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Take-home messages

1. The error sign is sufficient to provide learning to multi-layer networks

2. For classification problems, the error sign is known in advance
   Solves the update locking problem!

3. ‘Soft’ alignment between forward and backward weights
   $\Leftrightarrow$ Modulatory signals within 90° of those prescribed by BP

4. DRTP is demonstrated on the MNIST and CIFAR-10 datasets

Outlook

Neuroscience

DRTP could come in line with recent findings in cortical areas that reveal the existence of output-independent target signals in the dendritic instructive pathways of intermediate-layer neurons.

[Magee & Grienberger, *Annual Review of Neuroscience*, 2020]

Circuit implementation

Can lead to record low silicon area and energy overheads to embed on-chip online learning for edge computing devices.

[Frenkel, ISCAS, 2020]

Thank you!

Further resources:

The DRTP preprint: https://arxiv.org/pdf/1909.01311.pdf

Open-source DRTP PyTorch code: https://github.com/chfrenkel

ISCAS paper for the silicon implementation: https://arxiv.org/pdf/2005.06318.pdf

Supplementary – Synthetic datasets

Learning parameters

Regression task: 5k training set, 1k test set
Classification task: 25k training set, 5k test set

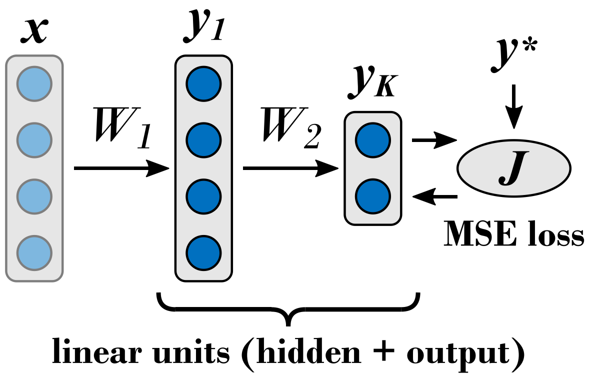
- 500 epochs
- Mini-batches of size 50
- Learning rate of 5×10^{-4}, similar for all training algorithms
- Forward weights (W_k) are drawn from He distributions (BP) or zero-initialized for FA-based training algorithms
- Feedback weights (B_k) are drawn from He distributions

Supplementary - Classification problems

DRTP proof of alignment

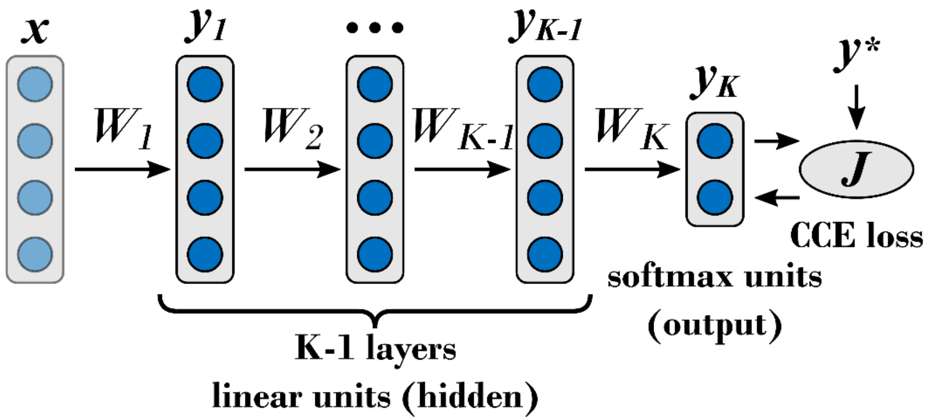
Claim: Direct random target projection delivers useful modulatory signals
The directions of DRTP and BP modulatory signals are within 90° of each other

FA proof [Lillicrap, *Nat. Comms.*, 2016]



- Single example
- Forward weights zero-initialized
- 1 linear hidden layer
- Linear output layer
- MSE loss

DRTP proof



- Single example
- Forward weights zero-initialized
- K-1 (arbitrary no.) linear hidden layers
- Softmax/sigmoid output layer
- CCE/BCE loss

Supplementary - Classification problems

DRTP proof of alignment

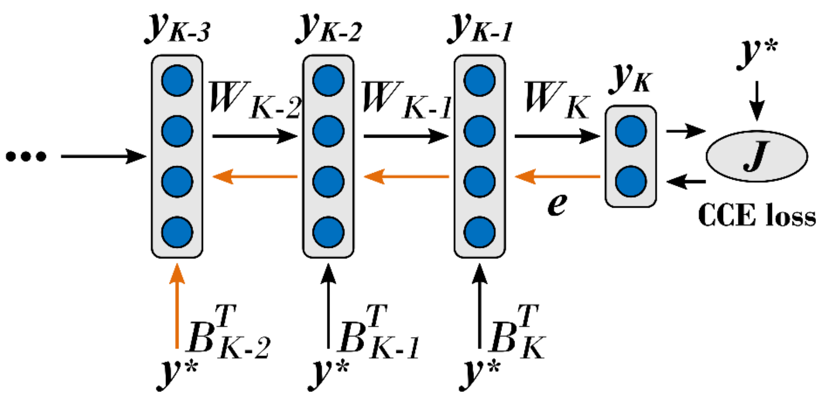
Claim: Direct random target projection delivers useful modulatory signals
The directions of DRTP and BP modulatory signals are within 90° of each other

BP modulatory signals

$$\delta y_k = \delta z_k = -\frac{1}{C}\left(\prod_{i=k+1}^{K} W_i^T\right) e$$

DRTP modulatory signals

$$\delta y_k = \delta z_k = B_k^T y^*$$



Alignment

$$-\frac{1}{C} e^T \left(\prod_{i=k+1}^{K} W_i^T\right)^T B_k^T y^* > 0$$

Theorem

$$B_k^T y^* = -\alpha_k^t \left(\prod_{i=K}^{k+1} W_i^T\right)^+ e$$

where $\alpha_k^t > 0$