

AE 625 - Particles Methods for Fluid Flow Simulation

SPH function and derivative approximation

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- 1 Plot the L2 error in the approximation with respect to the exact value as a function of the number of points used and the h value chosen. Use both a cubic spline kernel and the Gaussian Kernels. The L2 norm of a vector y is $|y| = (\sum y_i^2)^{0.5}$. Perform the same for the derivative of the function and compare with the exact solution using the formula discussed..
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- 2 Finally, pick one of the kernels and add a small amount of noise to the positions of the particles (by adding a small random displacement, uniformly distributed), and see how this affects the accuracy. 13

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Consider the function $\sin(\pi x)$ in the region $[-1, 1]$. Sample this function at equal spaced intervals and perform an SPH approximation of the exact function.

The report is generated through the command `` sh a7-140010042.sh ``

- 1 Plot the L2 error in the approximation with respect to the exact value as a function of the number of points used and the h value chosen. Use both a cubic spline kernel and the Gaussian Kernels. The L2 norm of a vector y is $|y|=(\sum iy_i)^{0.5}$. Perform the same for the derivative of the function and compare with the exact solution using the formula discussed..

Results:

Cubic spline kernel

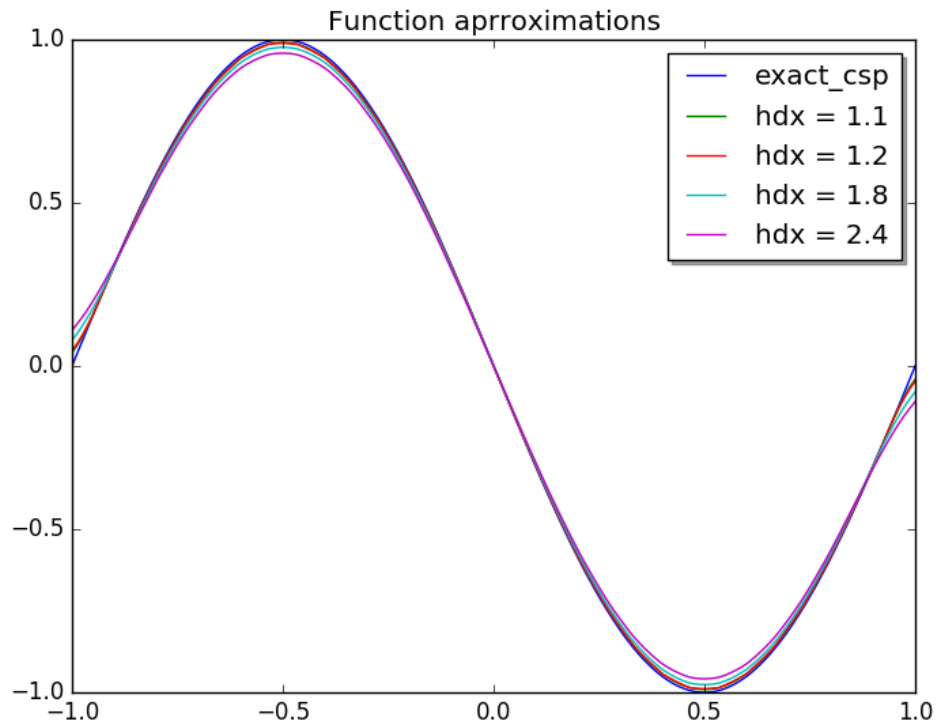


Figure 1: Cubic spline - function approx.

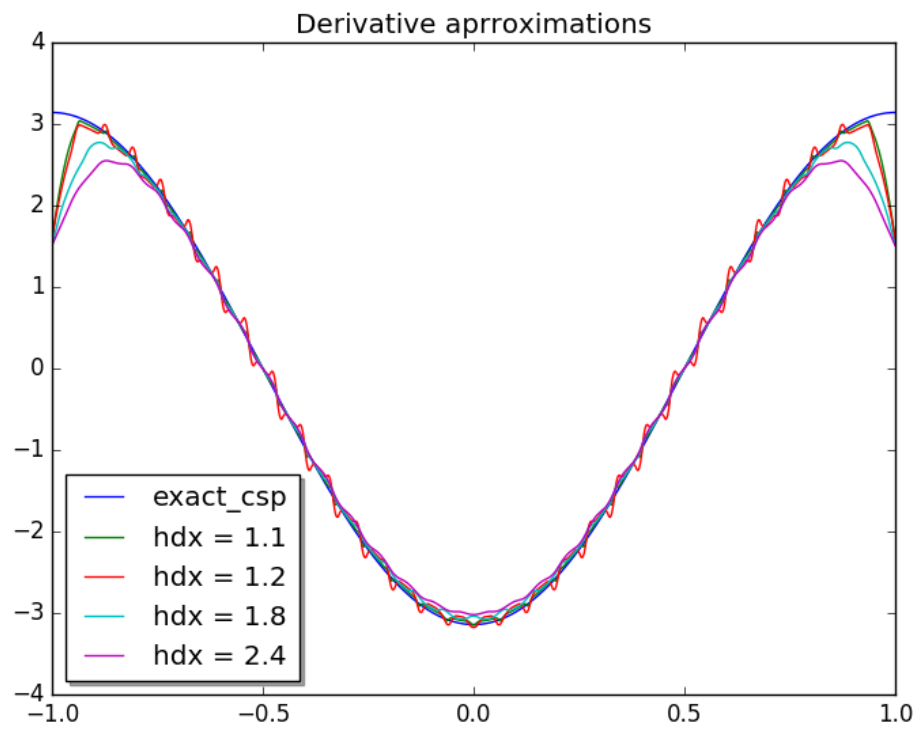


Figure 2: Cubic spline - derivative approx.

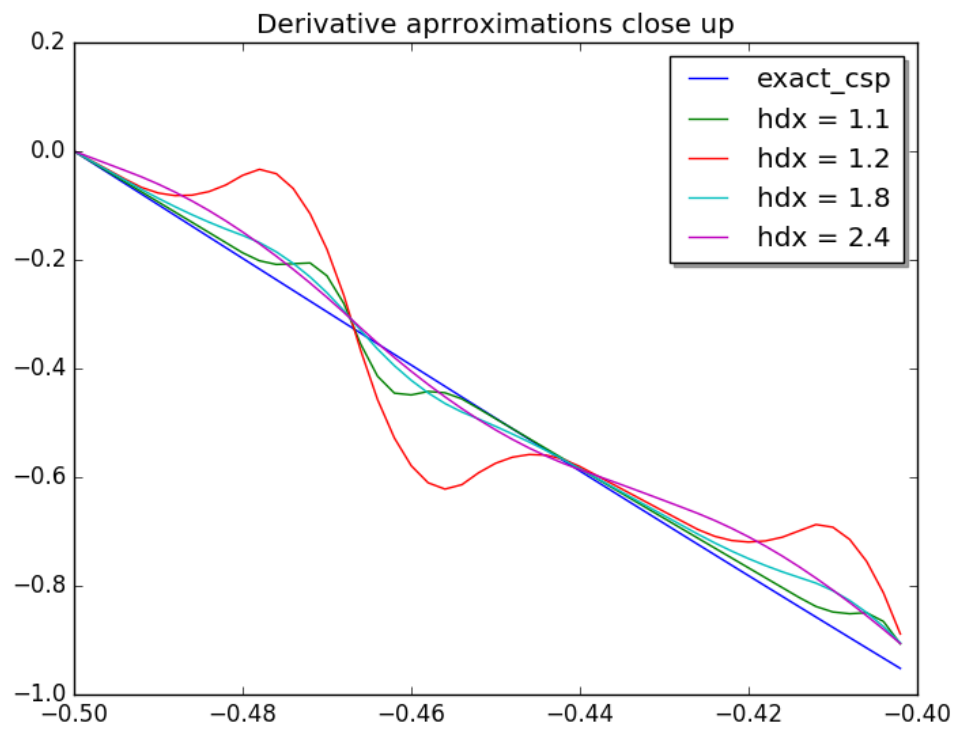


Figure 3: Cubic spline - derivative approx close up.

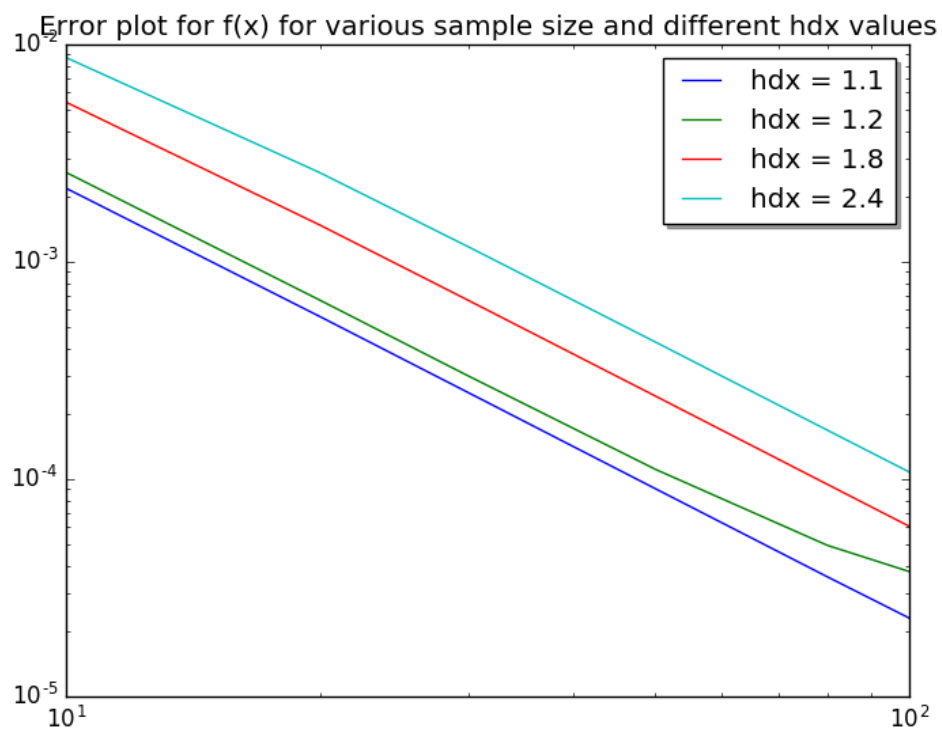


Figure 4: Cubic spline - function approx. error.

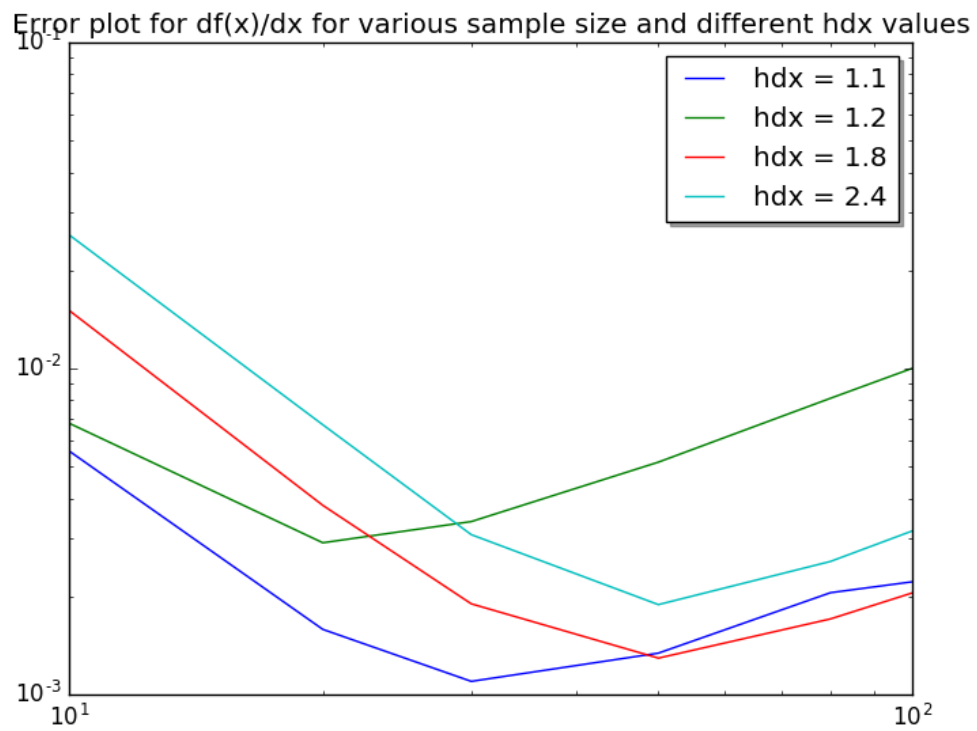


Figure 5: Cubic spline - derivative approx. error.

Gaussian kernel

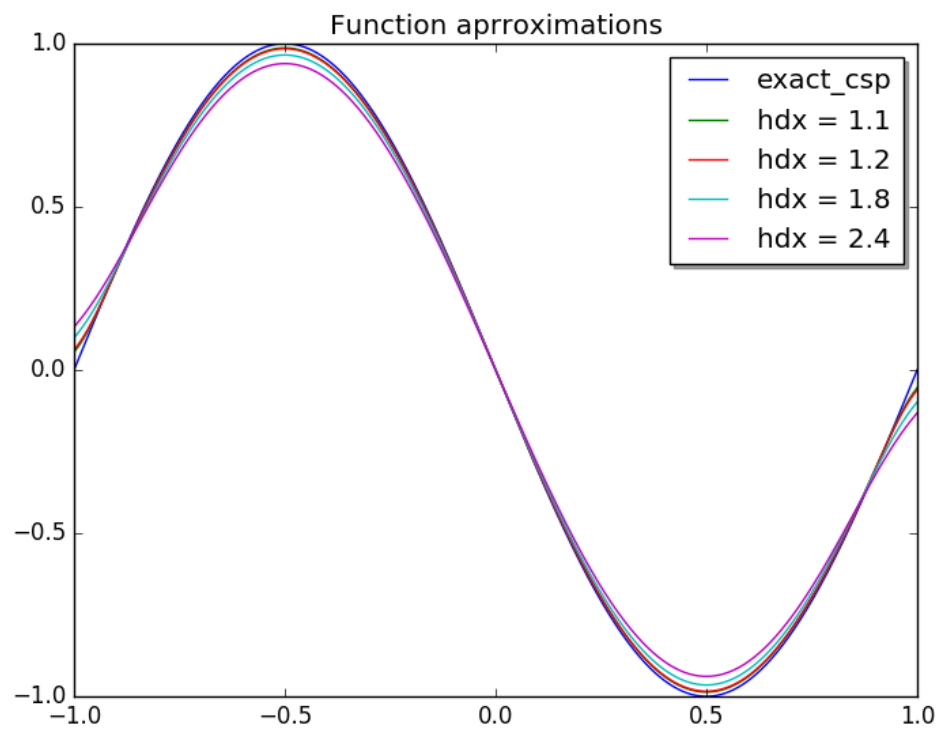


Figure 6: Gaussian - function approx.

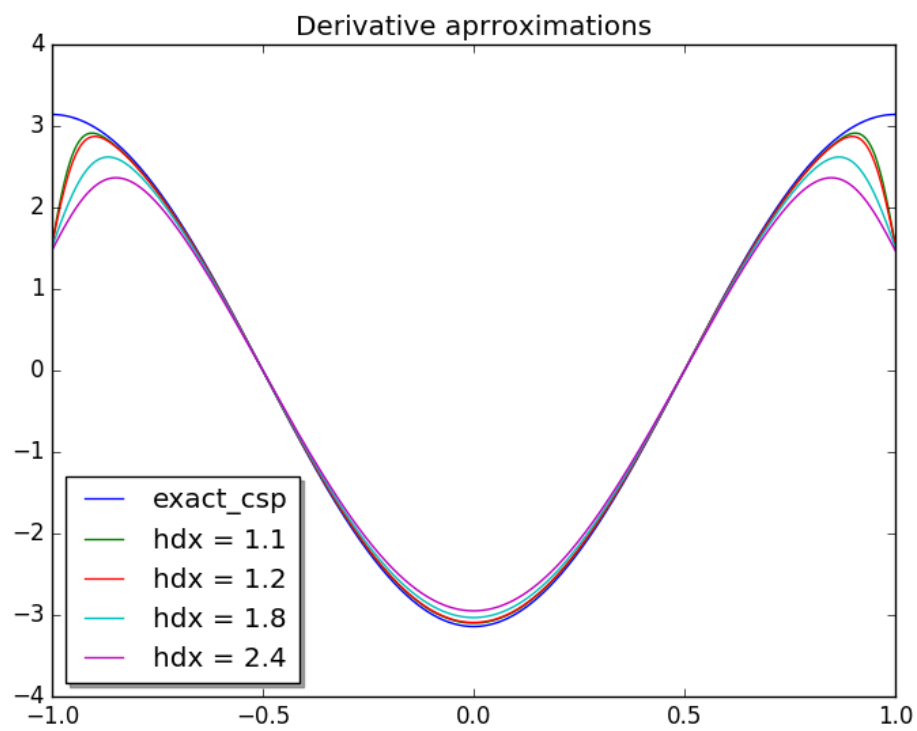


Figure 7: Gaussian - derivative approx.

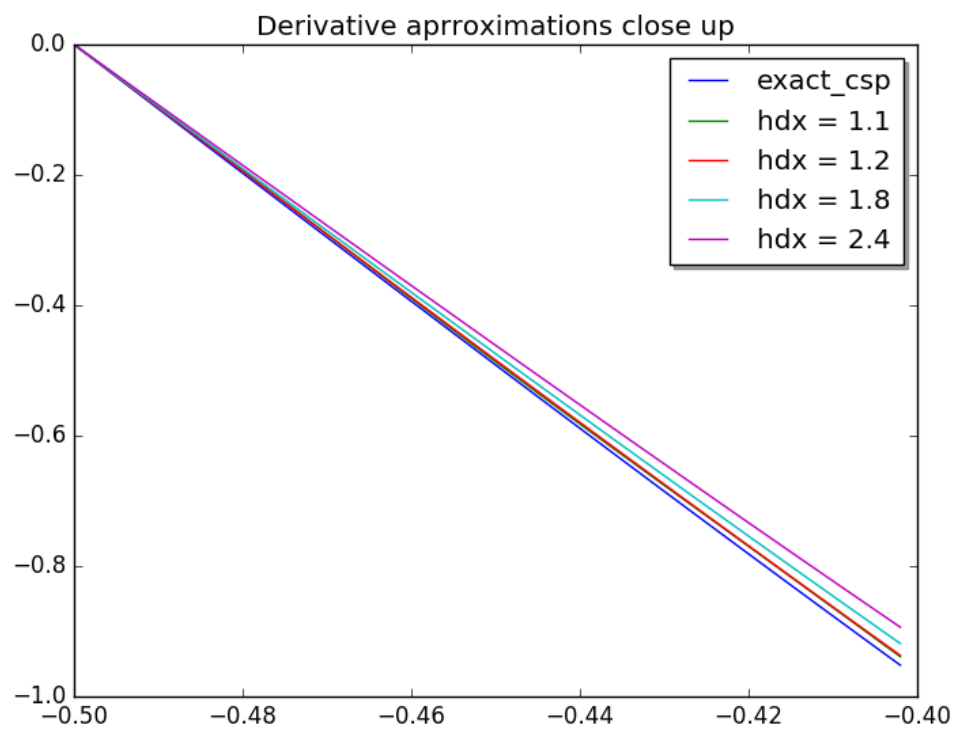


Figure 8: Gaussian - derivative approx closeup.

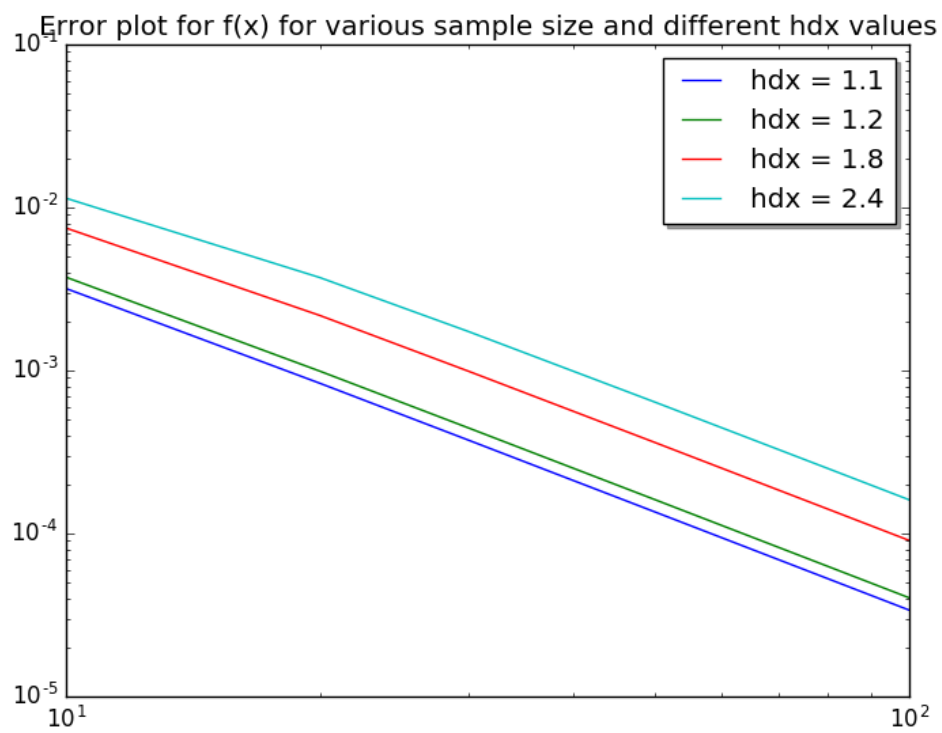


Figure 9: Gaussian - function approx. error.

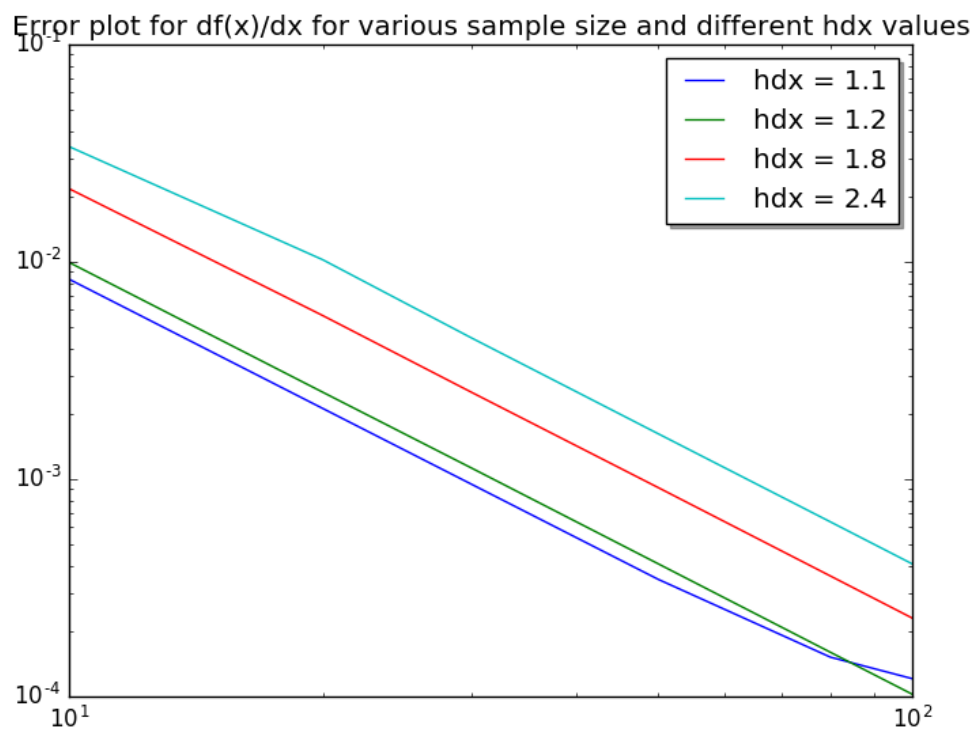


Figure 10: Gaussian - derivative approx. error.

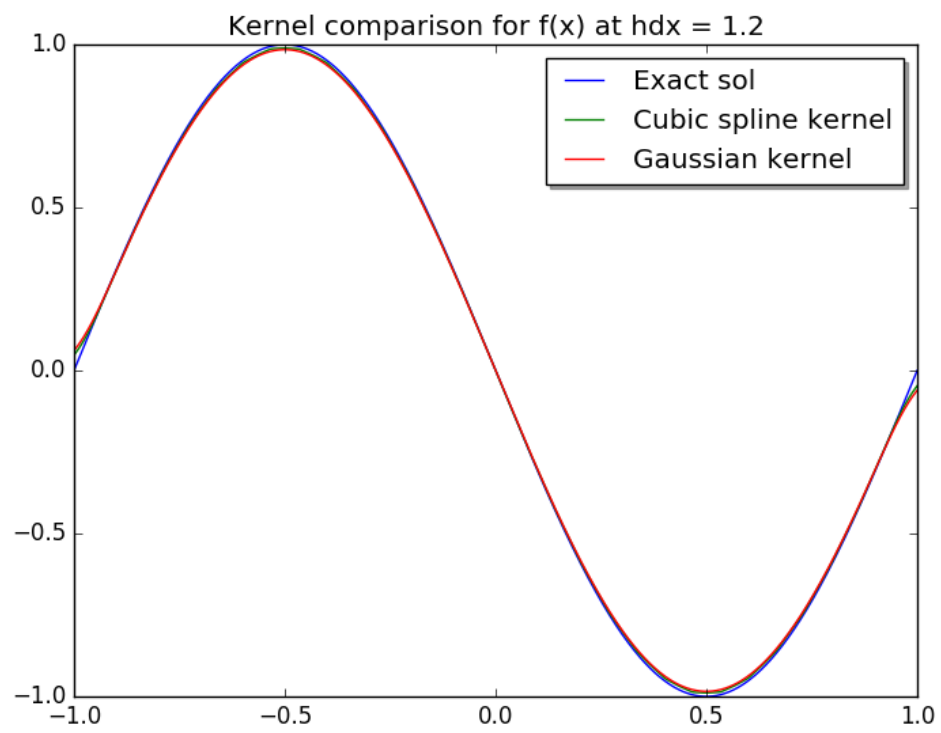


Figure 11: Kernel comparison - function approx.

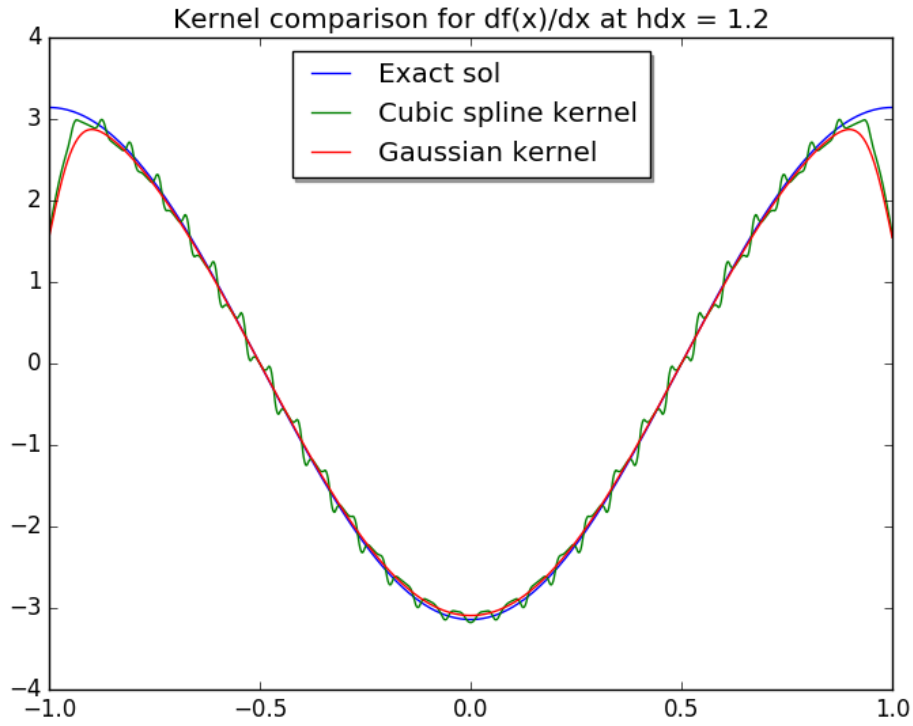


Figure 12: Kernel comparison - derivative approx.

- 2 Finally, pick one of the kernels and add a small amount of noise to the positions of the particles (by adding a small random displacement, uniformly distributed), and see how this affects the accuracy.

Results:

Approximation without noise for $hdx = 1.1$.

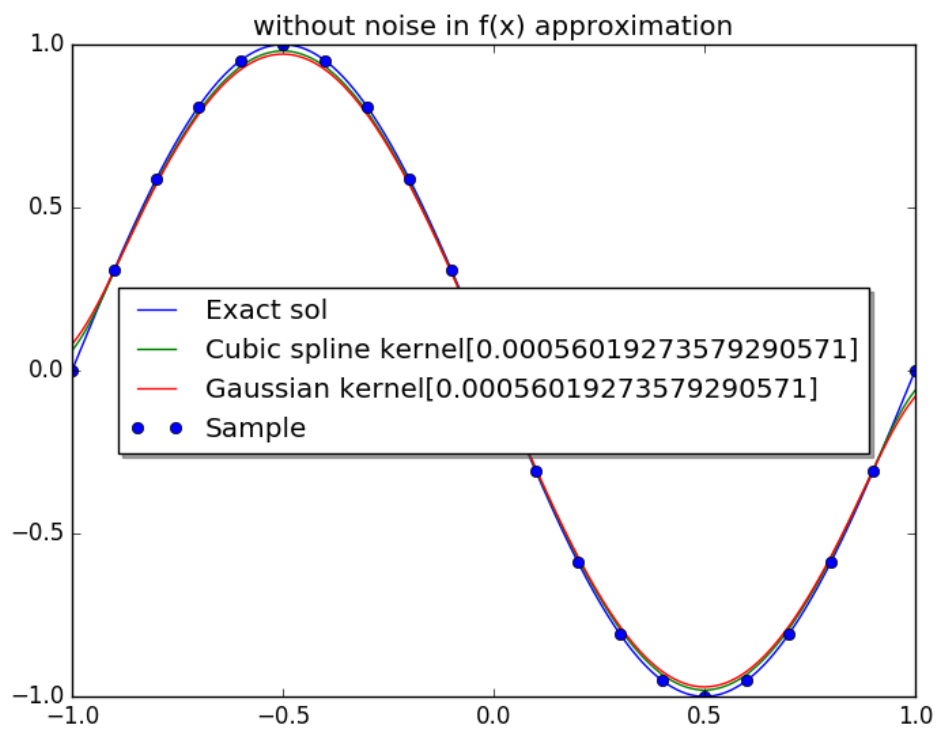


Figure 13: function approx without noise

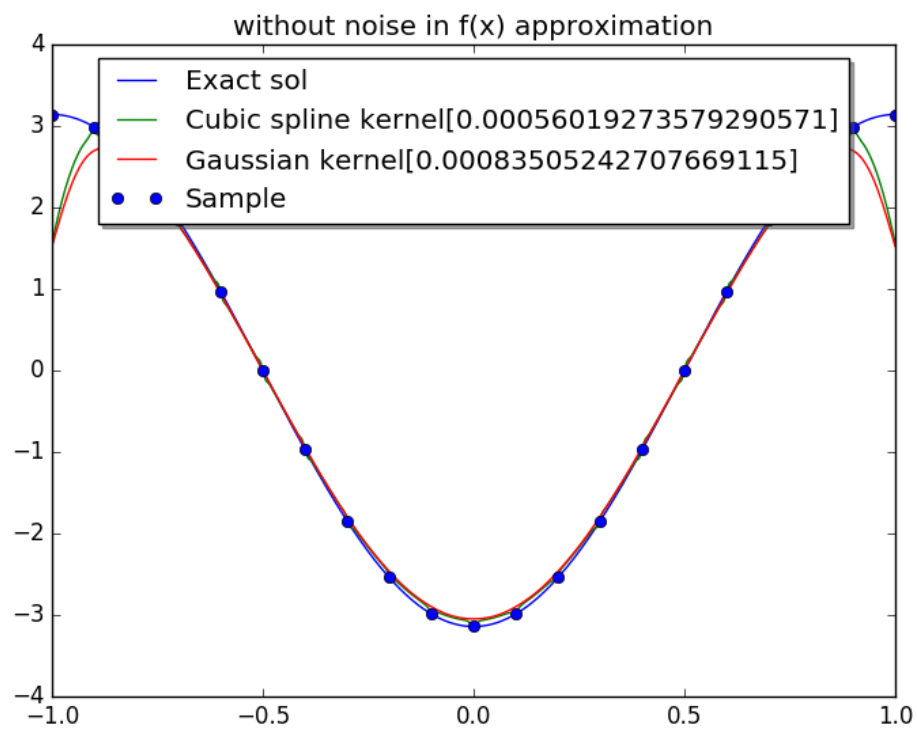


Figure 14: derivative approx without noise

Approximation with noise for $h_{dx} = 1.1$.

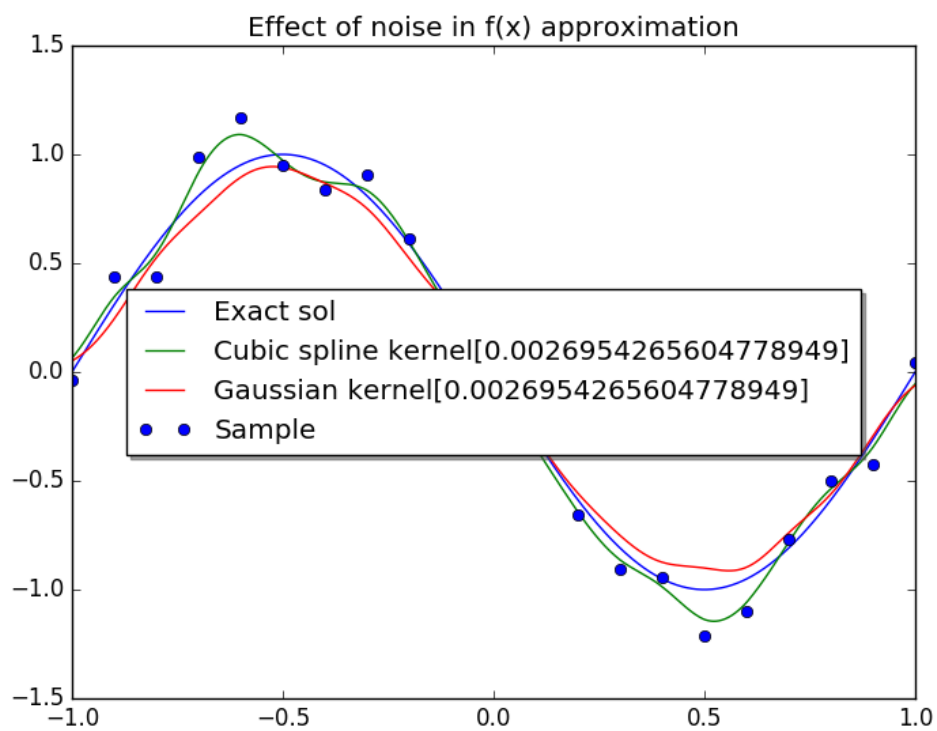


Figure 15: function approx with noise

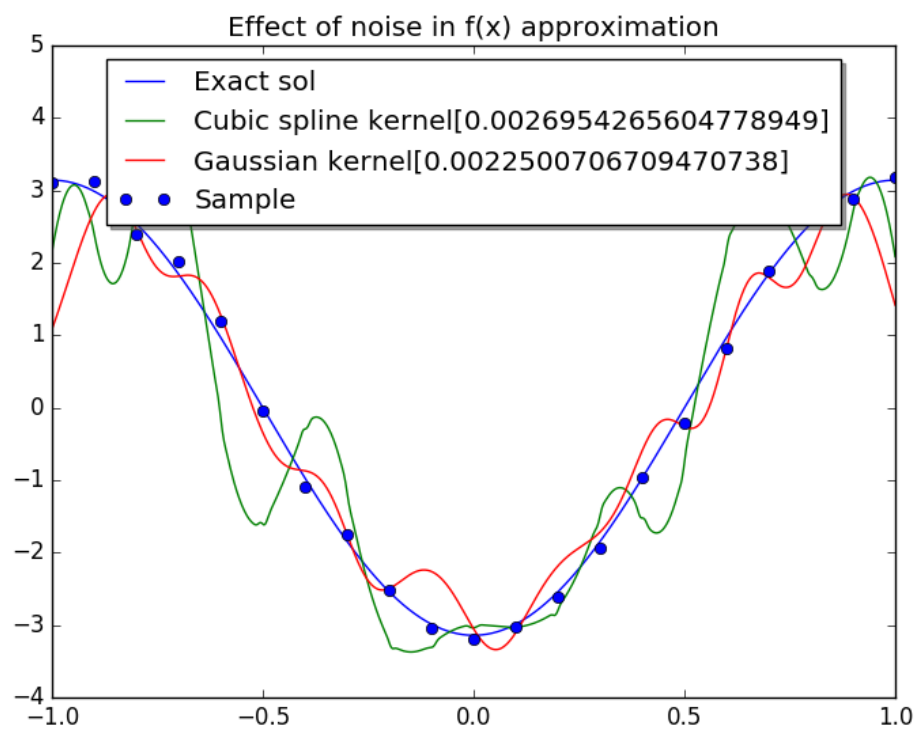


Figure 16: derivative approx with noise