

8 Tuesday, February 22, 2011

Outline:

I. playing with curves in \mathbb{P}^2 II. Affine algebraic geometry (structure of $\text{Spec } A$)

III. Projective geometry

IV cohomology

thm: Nullstellensatz:

Maximal ideals of $\mathbb{C}[x_1, \dots, x_n] \leftrightarrow$ points in $A^n (= \mathbb{C}^n)$ $(a_1, \dots, a_n) \in A^n$ max ideal $\mathcal{M}_a = \text{kernel of hom. } \mathbb{C}[x] \rightarrow \mathbb{C}$
 $f(x) \mapsto f(a)$ $\mathcal{M}_a = (x_1 - a_1, \dots, x_n - a_n)$ $A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I = (f_1, \dots, f_n)$, $f_i \in \mathbb{C}[x]$ Cor: Then $\text{Spec } A = \{\text{max ideals}\} \xleftrightarrow{\text{bij}} V(I) = \text{locus of zeros}$

Why? b/c

thm Correspondence Thm: ideals of $A \xleftrightarrow{\text{bij}}$ ideals of $\mathbb{C}[x]$ that contain I
max ideals $\xleftrightarrow{\text{bij}}$ max ideals containing I If A is only finitely generated \mathbb{C} -algebra (ring that contains \mathbb{C}),then $A \overset{\text{isomorphic}}{\sim} \mathbb{C}[x]/I$ $\text{Spec } A \xleftrightarrow{\text{max ideals}} V(I)$ variety in A^n

$$\text{If } A \xrightarrow{\varphi} B \xrightarrow{\pi} \mathbb{A}^n$$

get $\pi \circ \varphi$

Topics for Affine Algebraic Geometry

- localization (adjoining inverses)
- integral extensions (B a finite A -module)
- prime ideals
- dimension

Ex: $A = \mathbb{C}[x]$, $\text{spec } A = \mathbb{A}^1$

$B = A[g^{-1}]$ g some non-zero polynomial

Spec B ?

$$B = A[y]/(yg-1) = \mathbb{C}[x, y]/(yg(x)-1)$$

Spec $B = \text{locus } yg=1 \text{ in } \mathbb{A}_{x,y}^{n+1}$

Say $(x^0, y^0) \in \text{Spec } B$. So $y^0 g(x^0) = 1$.

Given x^0 , can solve uniquely for y^0 , provided $g(x^0) \neq 0$

If $g(x^0) = 0$, no solution.

Cor: $\text{spec } \mathbb{C}[x][g^{-1}] \xleftrightarrow{\text{bij}} \text{points of } \mathbb{A}_x^n \text{ where } g(x) \neq 0$