

## 8 Tuesday, February 22, 2011

Outline:

I. playing with curves in  $\mathbb{P}^2$ II. Affine algebraic geometry (structure of  $\text{Spec } A$ )

III. Projective geometry

IV cohomology

thm: Nullstellensatz:

Maximal ideals of  $\mathbb{C}[x_1, \dots, x_n] \leftrightarrow$  points in  $A^n (= \mathbb{C}^n)$  $(a_1, \dots, a_n) \in A^n$  max ideal  $\mathcal{M}_a = \text{kernel of hom. } \mathbb{C}[x] \rightarrow \mathbb{C}$   
 $f(x) \mapsto f(a)$  $\mathcal{M}_a = (x_1 - a_1, \dots, x_n - a_n)$  $A = \mathbb{C}[x]/I$  quotient of  $\mathbb{C}[x]$ , say  $I = (f_1, \dots, f_n)$ ,  $f_i \in \mathbb{C}[x]$ Cor: Then  $\text{Spec } A = \{\text{max ideals}\} \xleftrightarrow{\text{bij}} V(I) = \text{locus of zeros}$ 

Why? b/c

thm Correspondence Thm: ideals of  $A \xleftrightarrow{\text{bij}}$  ideals of  $\mathbb{C}[x]$  that contain  $I$ max ideals  $\xleftrightarrow{\text{bij}}$  max ideals containing  $I$ If  $A$  is finitely generated  $\mathbb{C}$ -algebra (ring that contains  $\mathbb{C}$ ),then  $A \overset{\text{isomorphic}}{\sim} \mathbb{C}[x]/I$  $\text{Spec } A \xleftrightarrow{\text{max ideals}} V(I)$  variety in  $A^n$

$$\text{If } A \xrightarrow{\varphi} B \xrightarrow{\pi} \mathbb{A}^n$$

get  $\pi \circ \varphi$

### Topics for Affine Algebraic Geometry

- localization (adjoining inverses)
- integral extensions ( $B$  a finite  $A$ -module)
- prime ideals
- dimension

Ex:  $A = \mathbb{C}[x]$ ,  $\text{spec } A = \mathbb{A}^1$

$B = A[g^{-1}]$   $g$  some non-zero polynomial

Spec  $B$ ?

$$B = A[y]/(yg-1) = \mathbb{C}[x, y]/(yg(x)-1)$$

$x_1, \dots, x_n$   
↓

$\text{Spec } B = \text{locus } yg=1 \text{ in } \mathbb{A}_{x,y}^{n+1}$

Say  $(x^0, y^0) \in \text{Spec } B$ . So  $y^0 g(x^0) = 1$ .

Given  $x^0$ , can solve uniquely for  $y^0$ , provided  $g(x^0) \neq 0$

If  $g(x^0) = 0$ , no solution.

Cor:  $\text{spec } \mathbb{C}[x][g^{-1}] \xleftrightarrow{\text{bij}} \text{points of } \mathbb{A}_x^n \text{ where } g(x) \neq 0$