Outline:

I. playing with curves in $\mathbb{P}^2$

II. Affine algebraic geometry (structure of spec A)

III. Projective geometry

IV cohomology

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thm: Nullstellensatz:

Maximal ideals of $\mathbb{C}[x_1, \dots, x_n] \longleftrightarrow$ points in $\mathbb{A}^n (= \mathbb{C}^n)$

$(a_1, \dots, a_n) \in \mathbb{A}^n$ max idea $\mathfrak{m}_a$ = kernel of hom. $\mathbb{C}[x] \to \mathbb{C}$
$f(x) \mapsto f(a)$

$\mathfrak{m}_a = (x_1 - a_1, \dots, x_n - a_n)$

$A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I = (f_1, \dots, f_n)$, $f_i \in \mathbb{C}[x]$

Cor: Then Spec $A = \{$max ideals$\} \underset{bij}{\longleftrightarrow} V(I)$ = locus of zeros

Why? B/C

thm <u>Correspondence Thm</u>: ideals of $A \underset{bij}{\longleftrightarrow}$ ideals of $\mathbb{C}[x]$ that contain I

max ideals $\underset{bij}{\longleftrightarrow}$ max ideals containing I

If A is a finitely generated $\mathbb{C}$-algebra (ring that contains $\mathbb{C}$),

then $A \overset{\text{isomorphic}}{\approx} \mathbb{C}[x]/I$

$\underset{\text{Max ideals}}{\text{Spec } A} \longleftrightarrow V(I)$ variety in $\mathbb{A}^n$

If
$$\begin{tikzcd} A \arrow[r, "\varphi"] \arrow[rr, bend right, "\pi\varphi"'] & B \arrow[r, "\pi"] & \mathbb{C} \end{tikzcd}$$
get $\pi\varphi$

Topics for Affine Algebraic Geometry

- localization (adjoining inverses)
- integral extensions (B a finite A-module)
- prime ideals
- dimension

Ex: $A = \mathbb{C}[x]$. $\operatorname{spec} A = \mathbb{A}^n$

$B = A[g^{-1}]$ g some non-zero polynomial

Spec B?

$$B = A[y]/(yg-1) = \mathbb{C}[\overset{x_1,\dots,x_n}{\overset{\downarrow}{x}}, y]/(yg(x)-1)$$

$\operatorname{spec} B =$ locus $yg = 1$ in $\mathbb{A}^{n+1}_{x,y}$

Say $(x^\circ, y^\circ) \in \operatorname{Spec} B$. So $y g(x^\circ) = 1$.

Given x°, can solve uniquely for y°, provided $g(x^\circ) \neq 0$

If $g(x^\circ) = 0$, no solution.

Cor: $\operatorname{spec} \mathbb{C}[x][g^{-1}] \xleftrightarrow[\text{bij}]{}$ points of $\mathbb{A}^n_x$ where $g(x) \neq 0$