18.721 Problem Set 1 (due Friday Feb 11)

Plane curves should be taken to be irreducible.

1. Show that if Z is a proper closed subset of affine space $\mathbb{A}^n$, then $\mathbb{A}^n \backslash Z$ is path-connected.

2. Show that a homogeneous quadratic form in three variables is a product of two linear forms iff the determinant of its Hessian matrix is zero.

3. Let C be a plane curve defined as the zero locus of a homogeneous form f of degree $d \geq 2$, and let P be a nonsingular point on C. Show that the Hessian form $H(f)$ vanishes at P iff the curve C has a flex at P.

4. Find all the prime ideals in the polynomial ring $\mathbb{C}[x, y]$.

5. Prove that two distinct plane curves intersect in finitely many points. (See Prop 8.8 of Chapter 10 in "Algebra".)

6. Prove that a cubic plane curve has at most one singular point.