Outline:

I. playing with curves in $\mathbb{P}^2$

II. Affine algebraic geometry (structure of spec A)

III. Projective geometry

IV cohomology

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thm: Nullstellensatz:

Maximal ideals of $\mathbb{C}[x_1,\dots,x_n] \longleftrightarrow$ points in $\mathbb{A}^n (=\mathbb{C}^n)$

$(a_1,\dots,a_n) \in \mathbb{A}^n$ max ideal $\mathfrak{M}_a$ = kernel of hom. $\mathbb{C}[x] \to \mathbb{C}$, $f(x) \rightsquigarrow f(a)$

$\mathfrak{M}_a = (x_1-a_1,\dots,x_n-a_n)$

$A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I=(f_1,\dots,f_n)$, $f_i \in \mathbb{C}[x]$

cor: Then $\operatorname{Spec} A = \{\text{max ideals}\} \overset{\text{bij}}{\longleftrightarrow} V(I)$ = locus of zeros

Why? B/c

thm Correspondence Thm: ideals of $A \overset{\text{bij}}{\longleftrightarrow}$ ideals of $\mathbb{C}[x]$ that contain I

max ideals $\overset{\text{bij}}{\longleftrightarrow}$ max ideals containing I

If A is a finitely generated $\mathbb{C}$-algebra (ring that contains $\mathbb{C}$),

then $A \overset{\text{isomorphic}}{\cong} \mathbb{C}[x]/I$

$\operatorname{Spec} A \longleftrightarrow V(I)$ variety in $\mathbb{A}^n$

max ideals

Zariski topology: closed sets are $V(I)$, I an ideal

Affine Algebraic Geometry

$\updownarrow$ finitely generated $\mathbb{C}$-algebra

Say $A \xrightarrow{\varphi} B$ homomorphism of finitely generated algebras.

Then the map goes $\operatorname{spec} A \longleftarrow \operatorname{spec} B$

Equivalent Sets: A a fin.-gen. $\mathbb{C}$-alg.

- (max ideals of A)
- (homomorphisms $A \to \mathbb{C}$)
- ($V(I)$ = locus of zeros of I if $A = \mathbb{C}[x]/I$)