Outline:
I. playing with curves in $\mathbb{P}^2$

II Affine algebraic geometry (structure of spec A)
III. Projective geometry
IV cohomology

Thm: Nullstellensatz:
Maximal ideals of $\mathbb{C}[x_1,\dots,x_n] \longleftrightarrow$ points in $\mathbb{A}^n/\mathbb{C}^n$
$(a_1,\dots,a_n) \in \mathbb{A}^n$ max ideal $\mathfrak{m}_a$ = kernel of hom. $\mathbb{C}[x] \to \mathbb{C}$, $f(x) \mapsto f(a)$
$\mathfrak{m}_a = (x_1-a_1,\dots,x_n-a_n)$

$A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I = (f_1,\dots,f_n)$, $f_i \in \mathbb{C}[x]$
Cor: Then $\operatorname{Spec} A = \{\text{maximal ideals}\} \overset{\text{bij}}{\longleftrightarrow} V(I)$ = locus of zeros

Why (cor)
Correspondence Thm: ideals of $A \overset{\text{bij}}{\longleftrightarrow}$ ideals of $\mathbb{C}[x]$ that contain I
max ideals $\overset{\text{bij}}{\longleftrightarrow}$ max ideals containing I

If A is finitely generated $\mathbb{C}$-algebra (ring that contains $\mathbb{C}$), then $A \cong \mathbb{C}[x]/I$ (isomorphic)
$\operatorname{Spec} A \longleftrightarrow V(I)$ variety in $\mathbb{A}^n$
(max ideals)

Zariski topology: closed sets are $V(S)$, S any

Affine Algebraic Geometry
$\Updownarrow$ finitely generated $\mathbb{C}$ algebra
Say $A \xrightarrow{\varphi} B$ homomorphism of finitely generated algebras.
Then the map goes $\operatorname{Spec} A \longleftarrow \operatorname{Spec} B$

Equivalent Sets: A a fin. gen. $\mathbb{C}$-alg.
- (max ideals of A)
- (homomorphisms $A \to \mathbb{C}$)
- ($V(I)$ = locus of zeros of I if $A = \mathbb{C}[x]/I$)