

$$\text{If } A \xrightarrow{\varphi} B \xrightarrow{\pi} C$$

get $\pi \varphi$

Topics for Affine Algebraic Geometry

- localization (adjoining inverses)
- integral extensions (B a finite A -module)
- prime ideals
- dimension

$$\text{Ex: } A = \mathbb{C}[x], \text{ spec } A = \mathbb{A}^1$$

$$B = A[g^{-1}] \text{ } g \text{ some non-zero polynomial}$$

Spec B ?

$$B = A[y]/(yg-1) = \mathbb{C}[x, y]/(yg(x)-1)$$

x_1, \dots, x_n
 \downarrow

$$\text{Spec } B = \text{locus } yg=1 \text{ in } \mathbb{A}_{x,y}^{n+1}$$

Say $(x^0, y^0) \in \text{Spec } B$. So $y^0 g(x^0) = 1$.

Given x^0 , can solve uniquely for y^0 , provided $g(x^0) \neq 0$

If $g(x^0) = 0$, no solution.

Cor: $\text{spec } \mathbb{C}[x][g^{-1}] \xleftrightarrow{\text{bij}} \text{points of } \mathbb{A}_x^n \text{ where } g(x) \neq 0$