Outline.

I. playing with curves in $\mathbb{P}^2$

II Affine algebraic geometry (structure of Spec A)

III. Projective geometry

IV cohomology

thm: Nullstellensatz:

Maximal ideals of $\mathbb{C}[x_1, \ldots, x_n] \longleftrightarrow$ points in $\mathbb{A}^n$ ($= \mathbb{C}^n$)

$(a_1, \ldots, a_n) \in \mathbb{A}^n$ max idea $\mathfrak{m}_a$ = kernel of hom. $\mathbb{C}[x] \to \mathbb{C}$, $f(x) \mapsto f(a)$

$\mathfrak{m}_a = (x_1 - a_1, \ldots, x_n - a_n)$

$A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I = (f_1, \ldots, f_n)$, $f_i \in \mathbb{C}[x]$

cor: Then Spec A = {max ideals} $\xleftrightarrow[\text{bij}]{}$ $V(I)$ = locus of zeros

Why? bec

... Correspondence Thm: ideals of A $\xleftrightarrow[\text{bij}]{}$ ideals of $\mathbb{C}[x]$ that contain I

max ideals $\xleftrightarrow[\text{bij}]{}$ max ideals containing I

If A is any finitely generated $\mathbb{C}$-algebra (ring that contains $\mathbb{C}$),

then $A \overset{\text{isomorphic}}{\approx} \mathbb{C}[x]/I$

$\underset{\text{max ideals}}{\text{Spec } A} \longleftrightarrow V(I)$ variety in $\mathbb{A}^n$

Zariski topology: Closed sets are $V(I)$, I an ideal

<u>Affine Algebraic Geometry</u>

$\Leftrightarrow$ finitely generated $\mathbb{C}$ algebra

Say $A \xrightarrow{\varphi} B$ homomorphism of finitely generated algebras.

Then the map goes Spec $A \longleftarrow$ Spec B

<u>Equivalent Sets:</u> A a fin. gen. $\mathbb{C}$-alg.

- (max ideals of A)
- (homomorphisms $A \to \mathbb{C}$)
- ($V(I)$ = locus of zeros of I if $A = \mathbb{C}[x]/I$)

If $A \xrightarrow{\varphi} B \xrightarrow{\pi} \mathbb{C}$

get $\pi\varphi$

Topics for Affine Algebraic Geometry

- localization (adjoining inverses)
- integral extensions (B a finite A-module)
- prime ideals
- dimension

Ex: $A = \mathbb{C}[x]$, $\operatorname{Spec} A = \mathbb{A}^n$

$B = A[g^{-1}]$ g some non-zero polynomial

Spec B?

$$B = A[y]/(yg-1) = \mathbb{C}[\overset{x_1,\dots,x_n}{x}, y]/(yg(x)-1)$$

$\operatorname{Spec} B =$ locus $yg = 1$ in $\mathbb{A}^{n+1}_{x,y}$

Say $(x^0, y^0) \in \operatorname{Spec} B$. So $y g(x^0) = 1$.

Given x^0 can solve uniquely for y^0, provided $g(x^0) \neq 0$

If $g(x^0) = 0$, no solution.

Cor: $\operatorname{Spec} \mathbb{C}[x][g^{-1}] \xleftrightarrow{\text{bij}}$ points of $\mathbb{A}^n_x$ where $g(x) \neq 0$