Outline:

I. playing with curves in $\mathbb{P}^2$

II. Affine algebraic geometry (structure of spec A)

III. Projective geometry

IV cohomology

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thm: Nullstellensatz:

Maximal ideals of $\mathbb{C}[x_1, \dots, x_n] \longleftrightarrow$ points in $\mathbb{A}^n (= \mathbb{C}^n)$

$(a_1, \dots, a_n) \in \mathbb{A}^n$ max ideal $\mathfrak{m}_a$ = kernel of hom. $\mathbb{C}[x] \to \mathbb{C}$, $f(x) \rightsquigarrow f(a)$

$\mathfrak{m}_a = (x_1 - a_1, \dots, x_n - a_n)$

$A = \mathbb{C}[x]/I$ quotient of $\mathbb{C}[x]$, say $I = (f_1, \dots, f_n)$, $f_i \in \mathbb{C}[x]$

cor: Then $\operatorname{Spec} A = \{\text{max ideals}\} \overset{\text{bij}}{\longleftrightarrow} V(I) = \text{locus of zeros}$

Why? B/C

thm Correspondence Thm: ideals of $A \overset{\text{bij}}{\longleftrightarrow}$ ideals of $\mathbb{C}[x]$ that contain I

max ideals $\overset{\text{bij}}{\longleftrightarrow}$ max ideals containing I

If A is any finitely generated $\mathbb{C}$-algebra (ring that contains $\mathbb{C}$),

then $A \overset{\text{isomorphic}}{\approx} \mathbb{C}[x]/I$

$\operatorname{Spec} A \overset{\text{max ideals}}{\longleftrightarrow} V(I)$ variety in $\mathbb{A}^n$