

| Q.No. | Sub. Sec. | Question Description | Marks |
|-------|-----------|--|-------|
| 1. | | Expand $f(x, y) = e^x \sin y$ in Taylor Series about $\left(1, \frac{\pi}{2}\right)$ up to third degree terms and hence find the approximate value of $e^{0.5} \sin\left(\frac{\pi+2}{2}\right)$. | [10] |
| 2. | | Find the absolute extrema of $z = 2x^2 + y - 3xy$ in the plane region D bounded by the lines $y = 1 - x$, $y = 1 + x$, $y = -1 - x$ and $y = -1 + x$. | [10] |
| 3 | | Sketch the region of integration and evaluate $\int_0^3 \int_{4y/3}^{\sqrt{25-y^2}} x \, dx \, dy$ after changing the order of integration. | [10] |
| 4. | [a] | Using spherical coordinates, evaluate $\iiint_E (x^2 + y^2) \, dv$ where E lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$. | [7] |
| | [b] | Evaluate $\iiint_R (x - y - z) \, dx \, dy \, dz$, where $R : 1 \leq x \leq 2; 2 \leq y \leq 3; 1 \leq z \leq 3$. | [3] |
| 5. | [a] | If n is positive integer and $m > -1$, then prove that $\int_0^1 x^m (\log x)^n \, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ | [5] |
| | [b] | Using Beta and Gamma function, evaluate $\int_0^a y^4 \sqrt{a^2 - y^2} \, dy$. | [5] |

Answer all the Questions

| | | |
|----|--|----|
| 1. | <p>(i) Find all critical points of the function $f(x, y) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy$ and check whether the function attains maximum or minimum at each of these points.</p> <p>(ii) Show that point $(0, 0)$ is neither a point of local minimum nor a point of local maximum for the function given by $f(x, y) = 3x^4 - 4x^2y + y^2$ for $(x, y) \in \mathbb{R}^2$.</p> | 10 |
| 2. | <p>(i) If x, y and z are positive real numbers, then find the minimum value of function $x^2 + 8y^2 + 27z^2$, where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.</p> <p>(ii) Find the Taylor series expansion of $f(x, y) = \sin xy + x^2y + e^x$ in the power of $(x - 1)$ and $(y - \pi)$ up to second degree terms.</p> | 10 |

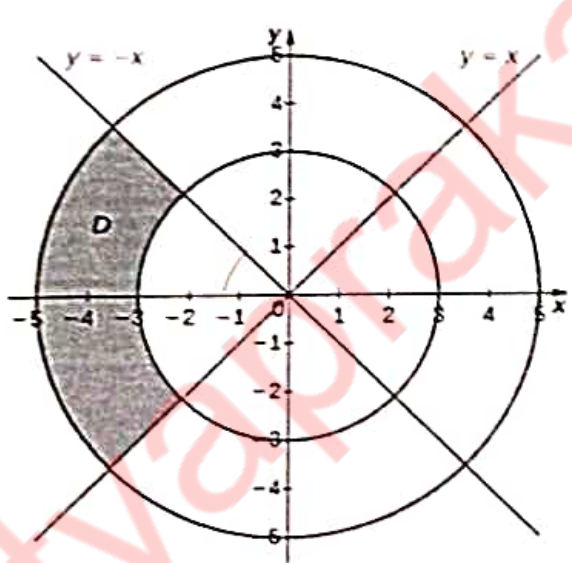
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|----|---|----|
| 3. | <p>(i) Find the value of integral by using the polar coordinates.</p> $I = \iint_D \sqrt{x^2 + y^2} \, dydx \quad \text{where } D = \{(x, y) \in \mathbb{R}^2 : x \leq x^2 + y^2 \leq 2x\}$ <p>(ii) Find the value of integral by changing the order of integration</p> $I = \int_0^4 \int_{(4-x)^{\frac{1}{2}}}^2 e^{y^3} \, dydx$ | 10 |
| 4. | Using multiple integrals, find the volume of the solid region bounded above by hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$. | 10 |
| 5. | <p>Solve the following integrals by using Beta and Gamma Function:</p> <p>(i) $I = \int_0^{\infty} \frac{e^{-\frac{k}{x^2}}}{x^6} dx \quad \text{where } k \neq 0$</p> <p>(ii) $I = \int_0^1 x^4 \sqrt{1-x^2} \, dx$</p> | 10 |



Continuous Assessment Test (CAT)- II- December 2022

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|--------------|--|------------|--|
| Programme | : B.Tech. | Semester | : Fall Semester Year I 2022-2023 |
| Course Title | : Calculus | Code | : BMAT101L |
| | | Slot | : E2+TE2 |
| Faculty | : Dr. Berin Greeni A, Dr. Prosenjit Paul, Dr. Srutha Keerthi B, Dr. Dhivya P, Dr. Saurabh Chandra Maury, Dr. Karan Kumar Pradhan | Class No. | : CH2022231700201, 197, 199, 202, 198, 200 |
| Duration | : 1 ½ Hours | Max. Marks | : 50 |

Answer all the Questions (50 marks)

| Q. No. | Question Description | Marks |
|--------|--|-------|
| 1. ✓ | (a) Find the Taylor series expansion of x^y about (1, 1) up to second degree terms. (b) Obtain the critical points of $(x^2 + 3y^2)e^{-(x^2+y^2)}$. | [5+5] |
| 2. ✓ | A wire of length L is cut into two parts (not necessarily equal) which are bent to in the form of a square and a circle respectively. Find the least value of the sum of areas so found. | [10] |
| 3. ✓ | If $f(x, y) = (x^2 + y^2)$ represents the population density of a planar region on the Earth, where x and y are measured in miles, find the number of people in the region shown below. | [10] |
| |  | [10] |
| 4. ✓ | Evaluate $\iiint_R (x^2 + y^2 + z^2) dV$, where R is the region above the xy-plane bounded by the cone $z^2 = 3(x^2 + y^2)$ and by the sphere $x^2 + y^2 + z^2 = 1$. | [10] |
| 5. | Using special functions compute the integral $\iint_R x^2 y^2 dx dy$, where R is the region bounded by the curve $x^{2/3} + y^{2/3} - 4 = 0$. | [10] |



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Continuous Assessment Test (CAT)- II- December, 2022

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|--------------|--|------------|--|
| Programme | : B.Tech. | Semester | : Fall Semester I 2022-2023 |
| Course Title | : Calculus | Code | : BMAT101L |
| | | Slot | : E1+TE1 |
| Faculty | : Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, Dr. P. S. S. S. S. Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya | Class Nbr | : CH202223170189, 191, 192, 194, 257, 323, 324 |
| Duration | : 1 ½ Hours | Max. Marks | : 50 |

Answer all the Questions (50 marks)

| Q.No. | Question Description | Marks |
|-------|---|-------|
| 1. | A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x, y, z) on the probe's surface is $T = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's. | [10] |
| 2. | Find the absolute maximum and minimum of $f(x, y) = x + y - xy$ on the triangle ABC with vertices A (0,50), B(50,0) and C(-50,-50). | [10] |
| 3. | a) Change the order of integration, evaluate $\int_0^2 \int_0^{9-x^2} x \, dy \, dx$. | [7] |
| | b) Find the area of $r = \sin \theta$ in polar coordinates. | [3] |
| 4. | A spherical tank of radius 3 meters is filled with water to a height of 2 meters. Find the volume of the water using the cylindrical coordinates. | [10] |
| 5. | a) Evaluate $\int_0^\infty \sqrt{x} e^{-x^5} \, dx$. | [5] |
| | b) Evaluate $\int_0^1 x^7 (1-x^2)^6 \, dx$. | [5] |



**VIT[®]**Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1976)**Continuous Assessment Test (CAT)- II- December 2022**

| | | | |
|--------------|---|------------|--|
| Programme | : B.Tech. | Semester | : Fall 2022-2023 |
| Course Title | : Calculus | Code | : BMAT101L |
| | | Slot | : A2+TA2 |
| Faculty | : Dr. Balamurugan, Dr. Saroj Kumar Dash, Dr. Mini Ghosh, Dr. Manimaran, Dr. Sowndarrajan, Dr. Prabhakar, Dr. Rajesh Kumar, Dr. Soumendu Roy | Class ID | : CH2022231700410, 416,429,440,443,573,604,610 |
| Duration | : 1 ½ Hours | Max. Marks | : 50 |

Answer all the Questions (50 marks)

| Q.No. | Question Description | Marks |
|-------|---|-------|
| 1. | (a). Find the absolute maximum and minimum values of $f(x, y)$ on the region R where $f(x, y) = x^2 + xy + y^2 - 3x + 3y$ and R is the triangular region cut from the first quadrant by the line $x + y = 4$. | [5] |
| | (b). For what values of the constant k does the second derivative test guarantee that $f(x, y) = x^2 + kxy + y^2$ will have a saddle point at $(0, 0)$? A local minimum at $(0, 0)$? For what values of k is the second derivative test inconclusive? Give reasons of your results. | [5] |
| 2. | (a). Find three positive numbers whose sum is 50 and whose product is maximum. | [5] |
| | (b). A flat circular plate has the shape of the region $x^2 + y^2 \leq 4$. The plate, including the boundary where $x^2 + y^2 = 4$, is heated so that the temperature at the point (x, y) is $T(x, y) = x^2 + 2y^2 - x$. Find the temperatures at the hottest and coldest points on the plate. | [5] |
| 3. | Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(1+e^y)\sqrt{1-x^2-y^2}} dy dx$. | [10] |
| 4. | Find the volume of the solid bounded by the xy -plane, the paraboloid $2z = x^2 + y^2$ and cylinder $x^2 + y^2 = 4$. | [10] |
| 5. | (a). Find the value of $\int_0^\pi \sin^2 x (1 + \cos x)^4 dx$ in terms of gamma function. | [5] |
| | (b). Evaluate the integral $\int_0^\infty x^4 e^{(-x^8)} dx$. | [5] |

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Continuous Assessment Test II – December 2022

| | | | |
|-----------|--|--------------|--|
| Programme | : B.Tech | Semester | : FALLSEM 2022-23 |
| Course | : Calculus | Code | : BMAT1011 |
| Faculty | : Dr. S. Radha Dr. N. Nathiya Dr. Sowndarrajan P T Dr. Manoj Kumar Singh Dr. Harshavarthini Shanmugam Dr. Manimaran J | Slot | : AI+TA1 |
| | | Class Number | : CH2022231700297 CH2022231700423 CH2022231700424 CH2022231700298 CH2022231700617 CH2022231700608 |
| Time | : 1½ hours | Max. Marks | : 50 |

Answer ALL the Questions (5 x 10 = 50 marks)

| Q.No. Sec | Question Description | Marks |
|-----------|---|-------|
| 1. a. | Find the critical points of the function $f(x, y) = x^3 + y^3 - 12x - 6y + 40$. Test each of these for maximum and minimum. | 5 |
| b. | Use Taylor's formula to find a quadratic approximation of $f(x, y) = xe^y + 1$ at $(1, 4)$. | 5 |
| 2. | The temperature T at any point (x, y, z) in space is $T = 625xzy^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. | 10 |
| 3. | Find the volume of the region using double integration which lies under the paraboloid $z = 4 - x^2 - y^2$ and above the disk $(x - 1)^2 + y^2 = 1$ on the xy -plane. | 10 |
| 4. | Evaluate $\iiint_R e^{-x^2-z^2} dV$ where R is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \leq y \leq 5$ and $z \leq 0$. | 10 |
| 5. a. | Evaluate $\int_0^1 x^{\frac{7}{2}} \left(1 - x^{\frac{3}{2}}\right)^{11} dx$. | 5 |
| b. | Evaluate $I = \int_0^\infty x^4 e^{-x^4} dx$. | 5 |

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