VIT

Vellore Institute of Technology

Reg. No. : 20BCE1707

Final Assessment Test (FAT) – November/December 2022

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**APPLIED LINEAR ALGEBRA**	Course Code	**MAT3004**
Faculty Name	**Prof. Hannah Grace G**	Slot	C2+TC2+TCC2
		Class Nbr	**CH2022231000391**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)

Answer <u>any 10</u> questions

1. Find the inverse of the coefficient matrix of the following system using Gauss-Jordan elimination and hence solve it. [10]

$5x + 3y + 9z = -1$

$-2x + 3y - z = -2$

$-x - 4y + 5z = 1$

2. (a) Find the values of a and b for which the system of equations [10]

$x + y + z = 3$

$x + 2y + 2z = 6$

$x + 9y + az = b$

will have (i) no solution (ii) unique solution.

(b) Use appropriate permutation matrix P to factorize PA such that PA = LU, where

$$A = \begin{pmatrix} 0 & 2 & 4 \\ 0 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

3. (a)Verify whether first vector is in the span of the second and third vectors in the following: [10]

(i) (1,2,-1), (1,0,2), (2,1,1), (ii) $x^3 - x$,$2x^2 + 4$, $-2x^3 + 3x^2 + 2x + 6$

(b)Verify whether the set $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad \geq bc \right\}$ is a subspace of V = M_{22}.

4. Find the bases for column space, row space and null space for [10]

$$A = \begin{pmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{pmatrix}.$$

Also discuss the relation between rank and nullity of A.

5. (a) Construct a second degree polynomial for the data points (0,1),(1,14),(2,15) . [10]

(b) Find the coordinate vector of $p(x) = 2 - x + 3x^2$ with respect to the basis $B = \{1, 1+x, x^2-1\}$ of P_2.

6. Check whether the following linear transformation $T : P_3(R) \to P_3(R)$ defined by $T(f(x)) = f(x) + f'(2) + f''(0)$ is invertible. If so find T^{-1} and its standard matrix. [10]

7. Compute the matrix of the linear transformation $T : R^3 \to R^3$ with respect to the basis $B' = \{v_1, v_2, v_3\}$ where [10]

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

$B = \{e_1, e_2, e_3\}$ is the standard basis for R^3 and $T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 2y - z \\ -y \\ x + 7z \end{pmatrix}$ using similarity transformation.

8. Suppose that **u**, **v** and **w** are vectors in an inner product space such that $\langle \mathbf{u},\mathbf{v}\rangle=1$, $\langle \mathbf{u},\mathbf{w}\rangle=5$, $\langle \mathbf{v},\mathbf{w}\rangle=0$. $\|\mathbf{u}\|=1$, $\|\mathbf{v}\|=\sqrt{3}$, $\|\mathbf{w}\|=2$. [10]
Evaluate the expressions (i)$\langle \mathbf{u}+\mathbf{w}, \mathbf{v}-\mathbf{w}\rangle$ (ii)$\langle 2\mathbf{v}-\mathbf{w}, 3\mathbf{u}+2\mathbf{w}\rangle$,(iii) $\|\mathbf{u}-\mathbf{v}\|$,(iv) $\|\mathbf{u}+\mathbf{v}\|$ and (v) $\|2\mathbf{u}-3\mathbf{v}+\mathbf{w}\|$.

9. Consider the following system AX=B of linear equations [10]

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & -1 & 4 \\ 3 & 1 & 2 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

Find the least square solution..

10. Find the QR decomposition of the following matrix [10]

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

11. The alphabets A to Z are encoded using A↔0, B↔1,...,Z↔25. The encrypted ciphertext is the sequence of numbers " 19, 95, 209, 10, 26, 54 ". Matrix [10]

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 3 & 2 \\ 6 & 7 & 4 \end{pmatrix}$$

is used to encrypt this message. (i) Find the original message (ii) Encrypt the message "LINEAR" in ciphertext.

12. (a) Let $V=P_2[0,1]$ and $B=\{1, 1+x, 1+x+x^2\}$ be a basis of V. Apply Gram-Schmidt Process to the basis B to obtain orthogonal basis using the inner product [10]
$\langle f, g\rangle = \int_0^1 f(x)g(x)dx$
(b) For the 1-D signal f = [1,2,2,8,8,10,10,8], find its1-level , 2-level and 3-level Harr transforms. Also discuss the conservation of energy for1-level Harr transform.