VIT
Vellore Institute of Technology

Reg. No. : 21BLC1488

Final Assessment Test (FAT) - July/August 2023

Programme	**B.Tech.**	Semester	**Fall Inter Semester 22-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Jaganathan B**	Slot	**D2+TD2+TDD2**
		Class Nbr	**CH2022232500609**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)

Answer any 10 questions

01. Prove that $u = e^x(x\cos y - y\sin y)$ is harmonic. Also find a function v such that $f(z) = u + iv$ is analytic and express $f(z)$ in terms of z. **[10]**

02. Show that the function **[10]**
$f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0, \\ 0, & z = 0, \end{cases}$ satisfies the Cauchy-Riemann equations at origin but not analytic at $z = 0$.

03. Under the transformation $w = \frac{1}{z}$, **[10]**
i) Find the image of $1 < x < 2$,
ii) Find the image of $|z - 2i| = 2$.

04. Find a bilinear transformation that maps the points $1, i, -i$ into the points $0, 1, \infty$. respectively. And also find the image of interior and exterior of the unit circle. **[10]**

05. Expand the function $f(z) = \frac{1}{(z+3)(1+z)}$ as a Laurent's series in suitable domains: **[10]**
i) $0 < |z+1| < 2$,
ii) $1 < |z| < 3$,
iii) $|z| > 3$.

06. Using complex variable techniques, evaluate $\int_0^{2\pi} \frac{\sin^2\theta}{5-4\cos\theta}\,d\theta$. [10]

07. [10]

Let $A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{bmatrix}$,

i) Find a basis for the row space of the matrix A,

ii) Find a basis for the column space of the matrix A,

iii) Find a basis for the null space of the homogenous linear system $AX = 0$.

08. i) Check whether $T : P_2(\mathbb{R}) \to \mathbb{R}$ defined as $T(a + bx + cx^2) = a + b + c$ is a linear transformation or not. Also verify the given transformation is invertible or not? [10]

ii) Determine, whether the vectors $u_1 = (1, 1, 1), u_2 = (1, 2, 3), u_3 = (2, -1, 1)$ form a basis of $\mathbb{R}^3$ or not.

09. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation given by $T(1, 1, 0) = (0, 1), T(1, 0, 1) = (2, 1)$ and $T(0, 1, 1) = (0, 2)$. [10]

i) Find the general transformation $T(x, y, z)$,

ii) Find $[T]_\alpha^\beta$, where $\alpha = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ and $\beta = \{(1, 1), (0, 1)\}$.

10. Find an orthonormal basis for $\mathbb{R}^3$ with the Euclidean inner product by applying the Gram-Schmidt orthonormalisation to the vectors $x_1 = (1, 0, 1), x_2 = (1, 0, -1), x_3 = (0, 3, 4)$. [10]

11. i) Consider the inner product space $C[0,1]$ with the inner product $< f, g >= \int_0^1 f(x)g(x)dx$, where $f(x) = 1, g(x) = x$, then find angle between f and g. [10]

ii) Verify the Cayley-Hamilton theorem and find the inverse for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

12. Find the eigen values and eigen vectors of the following matrix [10]

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

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Reg. No. : 21BLC1671

Final Assessment Test (FAT) - July/August 2023

Programme	**B.Tech.**	Semester	**Fall Inter Semester 22-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Rajivganthi Chinnathambi**	Slot	**D1+TD1+TDD1**
		Class Nbr	**CH2022232500602**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)

Answer any 10 questions

01. Find the analytic function $f(z) = u + iv$, where $u - v = (x - y)(x^2 + 4xy + y^2)$. [10]

02. If $u = x^2 - y^2, v = \frac{-y}{x^2+y^2}$, then show that u and v satisfy the Laplace's equation but $u + iv$ is not an analytic function of z. [10]

03. Find the image of the triangular region bounded by the lines $x = 1, y = 1$ and $x + y = 1$ in the z-plane under the transformation $w = z^2$. [10]

04. Find the bilinear transformation w=f(z) which maps the points -1, 0, 1 of the z-plane onto the points -1, i, 1 of the w-plane. Hence find the fixed points and critical points of this transformation. [10]

05. Expand f(z)=(z-1)/((z+2)(z+3)) as an infinite power series in the following regions: (i) |z|<2 (ii) 2<|z|<3 and (iii) |z|>3. [10]

06. Evaluate $\oint_c z^2/((z-1)^2(z+2))dz$, where C is the circle |z|=3 using (i) Cauchy integral formula (ii) Cauchy Residue theorem. [10]

07. Find the rank and nullity of the given matrix [10]

$$\begin{bmatrix} 1 & 3 & 1 & 7 \\ 2 & 3 & -1 & 9 \\ -1 & -2 & 0 & -5 \end{bmatrix}$$

Show that $dimR(A) = dimC(A)$ by finding the basis.

08. (a)Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $\{(1,-2,5,-3),(0,1,1,4),(1,0,1,0)\}$. Find a basis for W and extend it to a basis for $\mathbb{R}^4$. [10]

(b)Construct a linear transformation $T : P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$.

09. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,y,z) = (x+2y+z, -y, x+4z)$. Let α be the standard basis and let $\beta = \{(1,0,0),(1,-1,0),(1,1,2)\}$ be an another ordered basis for $\mathbb{R}^3$. Find the matrices $[T]_\alpha$ and $[T]_\beta$. [10]

10. Find an orthonormal basis for $\mathbb{R}^3$ with the Euclidean inner product by applying the Gram Schmidt normalization to the vectors $x_1 = (1,1,0), x_2 = (1,0,1), x_3 = (0,1,1)$. [10]

11. (a) Verify the following $< x, y >= x_1y_1 + 4x_2y_2$. where $x = (x_1, x_2), y = (y_1, y_2)$ on $\mathbb{R}^2$ is inner product or not? [10]

(b) The eigen values of a matrix A are 1,2,-3. find the determinant and trace of that matrix A

12. Find the eigen values and eigen vectors of the matrix [10]

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

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Reg. No. : 21BCE6185

Final Assessment Test (FAT) – November/December 2022

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. ASHISH KUMAR**	Slot	A1+TA1+TAA1
		Class Nbr	CH2022231001167
Time	**3 Hours**	Max. Marks	**100**

Part-A (10 X 10 Marks)

Answer any 10 questions

1. Determine the analytic function $f(z) = u + iv$ given that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ and $f(\frac{\pi}{2}) = 0$. [10]

2. Find the image of the rectangle with vertices $-1+i, 1+i, 1+2i$ and $-1+2i$ under the linear mapping $f(z) = 4iz + 2 + 3i$. [10]
   Sketch the rectangle and its image.

3. a) Find the bilinear transformation which maps the points $z = i, z = -1$ and $z = 1$ into the points $w = 0, w = 1$ and $w = \infty$. [10]
   b) Find the fixed points and image of the interior of the circle $|z| = 1$ under the transformation $w = \frac{z-i}{1-iz}$.

4. Using Contour integration, evaluate the real integral $\int_0^{2\pi} \frac{1+2\cos\theta}{10+8\cos\theta} d\theta$. [10]

5. a) Find Taylor's series expansion to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| = 2$. [10]
   b) Find the nature of singularity and find the residue for i) $f(z) = \frac{z-\sin z}{z^3}$ ii) $f(z) = \frac{1-\cos z}{z}$.

2.

6. Find a basis for the row space and null space of $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 1 & 5 & -2 \end{pmatrix}$. [10]

7. Let V be the vector space of functions $f : \mathbb{R} \to \mathbb{R}$. Verify whether the following are subspaces of V or not. Justify your answer. [10]
i) $W_1 = \{f \in V : f(1) = 3\}$,
ii) $W_2 = \{f \in V : f(3) = f(1)\}$,
iii) $W_3 = \{f \in V : f(-x) = -f(x)\}$.

8. A mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by [10]

$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3), \quad x_1, x_2, x_3 \in \mathbb{R}.$$

Show that T is a linear mapping. Find Ker T and the dimension of Ker T.

9. Let $S = \{v_1 = (1, 2, 0), v_2 = (1, 3, 2), v_3 = (0, 1, 3)\}$ and $S' = \{u_1 = (1, 2, 1), u_2 = (0, 1, 2), u_3 = (1, 4, 6)\}$, [10]
i) Find the change of basis matrix P from S to S',
ii) Find the change of basis matrix Q from S' to S,
iii) verify $Q = P^{-1}$.

10. Let $\mathbb{R}^3$ have the Euclidean inner product. Use the Gram Schmidt process to transfer the basis $\{u, v, w\}$ into an orthonormal basis, where [10]
$u = (1, 0, -1), v = (-7, 4, -2), w = (-3, 0, -1)$.

11. Solve the following system, by using Gauss elimination method [10]

$2y - z = 1,$

$4x - 10y + 3z = 5,$

$3x - 3y = 6.$

12. Let A and B be two matrices such that $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, $B = I - \frac{1}{2}A$. If λ_i and λ_i' are the eigenvalues of A and B respectively. Find $\lambda_i, \lambda_i', i = 1, 2, 3$, hence verify that $\lambda_i' + \frac{1}{2}\lambda_i = 1$ [10]

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Vellore Institute of Technology

Reg. No. :

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	**B.Tech**	Semester	**Winter Semester 2022-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. Om Namha Shivay**	Slot	**D2+TD2+TDD2**
		Class Nbr	**CH2022235001058**
Time	**3 Hours**	Max. Marks	**100**

Section I (10 X 10 Marks)

Answer any 10 questions

01. Define harmonic function and state Cauchy-Riemann equations. Show that the function $e^{x}(\cos y + i \sin y)$ is analytic, and find its derivative. [10]

02. If $f(z) = U + iV$ is an analytic function of z and $U - V = \frac{\cos x + \sin x - e^{-y}}{2\cos x - 2\cosh y}$. Then by using the Milnes method prove that $f(z) = \frac{1}{2}\left[1 - \cot\frac{z}{2}\right]$, when $f\left(\frac{\pi}{2}\right)=0$. [10]

03. Find the bilinear transformation $w = f(z)$ which maps the points $z = 0, 1, \infty$ into the points $w = i, 1, -i$ respectively. Find the invariant points and critical points of this transformation. [10]

04. Expand $f(z) = \frac{z}{(z+1)(z-2)}$ in a Laurent series valid for the following annular domain. [10]

(a) $1 < |z| < 2$

(b) $0 < |z-2| < 1$

05. Evaluate $\int_{-\infty}^{\infty} \frac{x}{(x^2+1)(x^4+1)} dx$ using contour integration. [10]

06. Show that vector $u = (-2, 1, 0, 5)^t$ is in the column space of the matrix below while $v = (1, -2, 0, 5)^t$ is not: [10]

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & -3 \\ -6 & -2 & -4 \end{pmatrix}$$

07. (a) Find 3 linearly independent vectors u, v, w in R^3 such that u, v lie in the plane $2x - 3y - z = 0$ [10]

(b) Find 3 linearly independent vectors u', v', w' in R^3 such that none of them lie in the plane $x - 2y + z = 0$

08. (a) Find the image of the strip $0 < y < 1/2$ under the map $w = 1/z$. Sketch the image roughly. [10]

(b) Let $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Consider the linear map $T_A : M_{2x2}(R) \to M_{2X2}(R)$ (set of all square matrices of order 2), defined by $T_A(X) = AX - XA$; for all X $\in M_{2X2}(R)$. Find the kernel of T_A, and also verify whether the given transformation is invertible or not.

09. Let $T : R^3 \to R^3$ be a linear transformation, defined as $T(x, y, z) = (x + 2y + z, -y - z, x + 4z)$. Let α be the standard basis of R^3 and $\beta = \{(1, 1 - 1), (1, -1, 1), (-1, 1, 1)\}$ be another basis of R^3. Then show that the matrix of transformation $[T]_\beta$ is similar to $[T]_\alpha$ using similarity transform. [10]

10. Apply the Gram-Schmidt orthogonalization process to the vectors $v_1 = (1, -1, 1)$, $v_2 = (1, 0, 1)$ and $v_3 = (1, 1, 2)$ to get an orthogonal basis. [10]

11. Solve the following system of equations by Gauss Jordan method: [10]

$$\begin{aligned} p - 2q + 3r &= 9 \\ -p + 3q &= -4 \\ 2p - 5q + 5r &= 17 \end{aligned}$$

12. Find the eigen values and eigen vectors of $A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & 1 \\ 2 & 0 & -2 \end{pmatrix}$ [10]

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PART A (10 X 10 Marks)

Answer <u>any 10</u> questions

01. Find the analytic function $w = u + iv$. Given that $v = e^{-2xy}\sin(x^2 - y^2)$. [10]

02. (a) Show that $\log|f(z)|$ is harmonic where $f(z) = u(x,y) + iv(x,y)$ [5 marks] [10]

(b) Find the image of the circle $|z| = 2$ under the transformation $w = \left(\sqrt{2}e^{\frac{i\pi}{4}}\right)z$. [5 marks]

03. Determine the bilinear transformation which maps the points $z = 0, -i, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively. Find all the invariant points of this transformation. Sketch the image of $|z - i| < 1$ under the obtained transformation. [10]

04. Evaluate $\int_0^{\infty} \frac{dx}{x^4+1}$ using contour integration. [10]

05. (a) Find the Laurent's series for the function $\frac{z^2-1}{z^2+5z+6}$ in the region $2 < |z| < 3$. [5 Marks] [10]

(b) Evaluate $\int_C \frac{\tan\frac{z}{2}}{(z-1-i)^2}dz$, where C is the boundary of the square whose sides are the lines $x = \pm 2$ and the y-lines as $y = \pm 2$. [5 Marks]

06. (a) Do the matrices $\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, and $\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$ span $M_{2\times 2}$? Justify your answer. [5 Marks] [10]

(b) Does $W = \{(x, y, z) \mid \text{either } y = 0 \text{ or } z = 0\}$ subspace of $\mathbb{R}^3$? Justify your answer [5 Marks]

07. Find bases for the row space, column space, null space, rank and nullity for the given matrix [10]

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & -1 & 1 \end{bmatrix}$$

08. Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping given by $F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find the basis, dimension of kernel of F, Image of F and the dimension of Image of F. [10]

09. Find the matrix of change of basis P from the basis $\alpha = \{(1, 2, 1), (-1, 2, 1), (1, 1, 3)\}$ to $\beta = \{(-3, 2, -3), (1, -1, -1), (5, 4, 9)\}$ for $\mathbb{R}^3$. Also find the matrix of change of basis from β to α. [10]

10. Consider $P_2(R)$ with basis $\{1, t, t^2\}$ and inner product $< f(t), g(t) >= \int_0^1 f(t)g(t)dt$. Find an orthogonal basis of $P_2(R)$ using Gram-Schmidt process. [10]

11. (a) For $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in $\mathbb{R}^2$ defined as [10]

$\langle \alpha, \beta \rangle = (x_1 + x_2)(y_1 + y_2) + (2x_1 + x_2)(2y_1 + y_2)$.

Show that $\langle , \rangle$ is an inner product in $\mathbb{R}^2$. [5 Marks]

(b) Verify the Cayley-Hamilton theorem for $A = \begin{bmatrix} 1/5 & 4 \\ 3/5 & -2 \end{bmatrix}$ and hence, find the inverse of A.

[5 Marks]

12. Find for what real value(s) of c, the following system of equations has non-trivial solution and hence solve: [10]

$x + 2y + 3z = cx$; $3x + y + 2z = cy$; $2x + 3y + z = cz$ using Gauss elimination

VIT

Vellore Institute of Technology

Reg. No. : 22BCE1351

Final Assessment Test (FAT) - November/December 2023

Programme	B.Tech.	Semester	FALL SEMESTER 2023 - 24
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
Faculty Name	Prof. Abhishek Kumar Singh	Slot	A2+TA2+TAA2
		Class Nbr	CH2023240101013
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

1/2 01. Find the analytic function $f(z) = u + iv$, where $2u + v = e^x[\cos y - \sin y]$. [10]

02. (a) Find the constants a, b if $f(z) = (x^2 - y^2 - axy) + i(x^2 - y^2 + bxy)$ is analytic. (5 Marks) [10]

(b) Find the image of the circle with radius 2 and the centre at $(3, -5)$ under the transformation $f(z) = 2iz + 3 - i$. (5 Marks)

1/2 03. Find the bilinear transformation that interchanges the points 0 and i and sends ∞ to $i/2$. Determine its invariant points. Also find the image of the horizontal line $y = \frac{1}{2}$. [10]

X Y 04. Evaluate $\int_0^{2\pi} \frac{1}{5+4\cos\theta} d\theta$, using contour integration. [10]

Y 05. (a) Classify the singularities of the function $f(z) = \frac{\cot \pi z}{(z-a)^3}$. Also find the residue at $z = a$. [10]

(5 Marks)

(b) Evaluate the integral $\int_C \frac{e^z}{(z+2)(z+1)^2} dz$, where C is $|z| = 3$ using Cauchy's integral formula.

(5 Marks)

06. (a) Find the basis and dimension of the solution space W of this homogeneous system: [10]

(5 Marks)

$$x + 2y + 2z - s + 3t = 0$$
$$x + 2y + 3z + s + t = 0$$
$$3x + 6y + 8z + s + 5t = 0$$

1/2 (b) Does $W = \{(a, b, c, d) \mid a + b - c + d = 0)\}$ is subspace of $\mathbb{R}^4$. If yes, find the basis.

(5 Marks)

07. Verify Rank Nullity theorem for $A = \begin{pmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{pmatrix}$. [10]

08. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by [10]

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$$T(x, y) = (2x + 3y, 4x - 5y).$$

4 + 5 = 8 − 25

Find the matrix representation of T

(a) with respect to the standard basis $E = \{(1, 0), (0, 1)\}$. (5 Marks)

(b) with respect to the basis $S = \{(1, 2), (2, 5)\}$. (5 Marks)

09. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$ and let $\alpha = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $\beta = \{(1,3), (2,5)\}$ be the bases for $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively. Then find the associated matrix T with respect to α and β. **[10]**

10. Consider $\mathbb{R}^4$ with the usual dot product. Find an orthonormal basis for the subspace spanned by $\{(1,1,1,0), (1,1,0,0), (1,0,0,0), (2,1,3,0)\}$. **[10]**

11. (a) Use the standard inner product on P_2 to find the angle between vectors $f(x) = 1 - x$ and $g(x) = x^2$. **(5 Marks)** **[10]**

Soln 1/2 ? (b) Two eigenvalues of a matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double of third eigenvalue, hence find the eigenvalues of A^{-1} and A^4. **(5 Marks)**

12. Solve the following system by Gaussian Elimination method **[10]**

$$x + y + z - w = -2$$

$$2x - y + z + w = 0$$

$$3x + 2y - z - w = 1$$

$$x + y + 3z - 3w = -8$$

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