VIT

Reg. No.: 21BEC1550

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	B.Tech	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Dr.Durga Nagarajan	Slot	C1+TC1+TCC1
		Class Nbr	CH2022235002169
Time	3 Hours	Max. Marks	100

Part-I (10 X 10 Marks)

Answer any 10 questions

01. (a) Solve the following differential equation using method of variation of parameters $y'' - y' - 6y = 20e^{-2x}$. [10]

(b) Solve the nonlinear partial differential equation $q^2 = z^2p^2(1-p^2)$.

02. Solve $x^2y'' - 5xy' + 8y = 8x^6$, $y(\frac{1}{2}) = 0$, $y'(\frac{1}{2}) = 0$. [10]

03. Solve the linear partial differential equation $x(y^2 - z^2)p - y(z^2 + x^2)q = z(x^2 + y^2)$. [10]

04. Find $L^{-1}\left(\dfrac{s^2+s-2}{s(s+3)(s-2)}\right)$. [10]

05. (a) Find the Laplace transform of $f(t) = \begin{cases} t & \text{for} \quad 0 < t < 4 \\ 5 & \text{for} \quad t \geq 4 \end{cases}$. [10]

(b) Solve the PDE $u_x + 2u_t = xt$, for $x > 0$, $t > 0$, given that $u(x,0) = 0$, $u(0,t) = 0$ using Laplace transform.

06. Solve $y'' + 3y' + 2y = \delta(t-2)$, given $y(0) = 0$, $y'(0) = 0$ using Laplace transform. Here, δ corresponds to Dirac delta function. [10]

07. Find the Fourier series of $f(x) = \dfrac{\pi - x}{2}$ in the interval $(0, 2\pi)$. Hence, deduce that $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$. [10]

08. Find the half range Fourier sine series and half range Fourier cosine series for $f(x) = x$ in the interval $0 < x < 2$. [10]

09. Find the Fourier transform of the function $f(x) = \begin{cases} 1 - x^2 & \text{for} \quad |x| < 1 \\ 0 & \text{for} \quad |x| \geq 1 \end{cases}$. Hence, show that [10]

$$\int_0^\infty \frac{\sin s - s\cos s}{s^2} \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}.$$

10. Evaluate $\displaystyle\int_0^\infty \frac{1}{(x^2+1)(x^2+4)}dx$ using Fourier cosine transform. [10]

11. Use Z-transform to solve $y_{n+2} + 4y_{n+1} - 5y_n = 24n - 8$ given that $y_0 = 3$, $y_1 = -5$. [10]

12. (a) Evaluate $Z^{-1}\left(\dfrac{20z}{(z-7)(z-11)}\right)$. [10]

(b) Find (i) $Z[{}^nC_2]$, (ii) $Z[(n+1)(n+2)]$.

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VIT

Reg. No. : 22BCE1351

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Sowndar Rajan P.T.	Slot	A1+TA1+TAA1
		Class Nbr	CH2022232300429
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

01. Solve $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin\{2\{\log(1+x)\}$ [10]

02. a)Determine the charge on the capacitor at any time $t>0$ in circuit series having an emf $E(t)=100\sin 60t$, a resistor of 2Ω, an inductor of 0.1H and capacitor of $\dfrac{1}{260}$ F, if the initial current and charge on the capacitor are both zero. [5 marks] [10]

b)Solve $p^2+q^2=x+y$ where p and q are $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ respectively. [5 marks]

03. A heat transfer problem is described by the first-order linear partial differential equation: [10]

$$4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u.$$

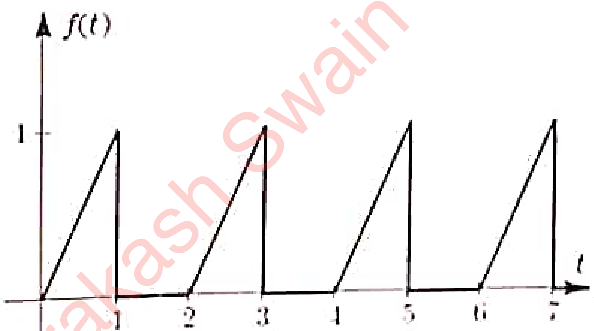
Initially, the temperature distribution is given as $u=3e^{-y}-e^{-5y}$ when $x=0$.Determine the temperature distribution $u(x,y)$ for the entire domain using the method of separation of variables.

04. Consider the function [10]

$$g(t)=\begin{cases}\sin t & 0\le t<1\\ t & 1<t<5\\ 1 & 5<t\end{cases}$$

Write $g(t)$ in terms of the unit step function. Find the Laplace transform of $g(t)$..

05. a) Find the Laplace transform of the periodic function: [5 marks] [10]



Graph of periodic unit ramp function

b)Find the Fourier cosine series of the function

$f(x) = \begin{cases} x^2 & \text{if } 0 \le x \le 2 \\ 4 & \text{if } 2 \le x \le 4. \end{cases}$ [5 marks]

06. Solve $\dfrac{\partial u(x,t)}{\partial t} + \dfrac{\partial u(x,t)}{\partial x} + 2u(x,t) = 0$ with the initial and boundary conditions $u(0,t) = -\cos 2t$ and $u(x,0) = -e^{-2x}cos(2x) + 1$. [10]

07. Solve the following sytem of differential equations $\dfrac{dx}{dt} - y = e^t$ and $\dfrac{dy}{dt} + 4x = 2$ with the initial conditions $x = y = 0$ when $t = 0$. [10]

08. Find the Fourier series expansion of $f(x) = x^2$ in $(-1, 1)$ and hence deduce the value of $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$. [10]

09. Find the Fourier sine and cosine transform of the function [10]

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 3 - x & 2 < x < 3 \\ 0 & x \ge 3 \end{cases}$$

10. Find the Fourier transform of $F(e^{-x^2})$ and hence find the Fourier transform of $e^{-7(x-3)^2}$ [10]

11. Evaluate [10]

$$Z\left(\frac{1}{n(n+1)}\right) \text{ and } Z^{-1}\left(\frac{4 - 8z^{-1} + 6z^{-2}}{(1+z^{-1})(1-2z^{-1})^2}\right)$$

12. Solve the difference equation $y(n+2) - 5y(n+1) + 6y(n) = n$ using Z-transform with the initial conditions $y_0 = 1$ and $y_1 = 2$ [10]

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Abhishek Kumar Singh	Slot	A2+TA2+TAA2
		Class Nbr	CH2022232300446
Time	3 Hours	Max. Marks	100

PART-A (10 X 10 Marks)
Answer any 10 questions

01. Solve the ODE $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=(\ln x)\sin(\ln x)$. [10]

02. (a) Solve the ODE $\dfrac{d^2y}{dx^2}-y=e^x$ by using undetermined coefficient method. [5-marks] [10]
(b) Solve $y^2p-xyq=x(z-2y)$. [5-marks]

03. (a) Solve $p^2z^2+q^2=p^2q$. [5-marks] [10]
(b) Solve $p-q=\ln(x+y)$. [5-marks]

04. Evaluate $L^{-1}\left[\dfrac{s^2-5s+7}{(s+2)^2}\right]$. [10]

05. (a) Find $L[tH(t-1)+e^{2t}\delta(t-2)]$. [5-marks] [10]
(Note: $H(t-a)$ is the unit step function at the point a and $\delta(t-b)$ is the impulse function at the point b.)

(b) Find the Fourier sine series of $f(x)=\begin{cases}\dfrac{\pi x}{4}, & 0\le x<\pi/2\\ \dfrac{\pi(\pi-x)}{4}, & \pi/2\le x<\pi\end{cases}$. [5-marks]

06. Use Laplace transform to solve $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial t}=x$, $u(0,t)=0$ and $u(x,0)=0$ for $x>0$, $t>0$. [10]

07. Use Laplace transform to solve $\dfrac{d^2x}{dt^2}-2\dfrac{dx}{dt}+x=e^t$ with $x=2$, $\dfrac{dx}{dt}=-1$ at $t=0$. [10]

08. Obtain the Fourier cosine series of $x\sin(x)$ in $(0,\pi)$. Hence show that [10]
$\dfrac{1}{(1)(3)}-\dfrac{1}{(3)(5)}+\dfrac{1}{(5)(7)}-\cdots=\dfrac{\pi-2}{4}$.

09. Find the Fourier cosine transform of $f(x)=e^{-x}$. Hence evaluate $\displaystyle\int_0^\infty \frac{\cos mx}{1+x^2}dx$ for $m>0$. [10]

10. Solve $\dfrac{\partial u}{\partial t}=3\dfrac{\partial^2 u}{\partial x^2}$, by using Fourier sine transform, where the initial and boundary conditions are $u(x,0)=e^{-2x}$, $u(0,t)=0$ for $x>0$ and $t>0$. [10]

11. Use Z-transform to solve $u_{n+2}-6u_{n+1}+8u_n=4^n$ such that $u_0=0$ and $u_1=1$. [10]

12. (a) Find $Z[\sinh 3n]$. [5-marks] [10]
(b) Find $Z^{-1}\left[\dfrac{z^3}{(z-1)^2(z-2)}\right]$. [5-marks]

Final Assessment Test – June 2023

VIT® Vellore Institute of Technology

Course: BMAT102L - Differential Equations and Transfor[illegible]

Class NBR(s): 0378 / 0407 / 4432 / 4460 / 4503 / 4505 / 4593 / 4595 / 4666 / 4668 / 4670 / 4869 / 4873 / 4875 / 4877 / 4879 / 4905 / 4907

Slot:

Time: Three Hours

Max.

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MA[illegible]

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Solve the differential equation $y''(x) - 2y'(x) + 2y(x) = e^x \tan x$, using the method of variation of parameters.

2. Find the solution of $x^2y''(x) - xy'(x) - 3y(x) = x^2 \log x$.

3. Solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$; $u(0,y) = 8e^{-3y}$ using method of separation of variables.

4. (i) Form the partial differential equation of all the spheres having centre lying in xy - plane and having constant radius. [5]

   (ii) Solve $p^2 + q^2 = x + y$. [5]

5. Find the Laplace Transform of the periodic function $f(t) = \begin{cases} t & ; 0 < t < a \\ 2a - t & ; a < t < 2a \end{cases}$ with period $2a$.

6. (i) Find the Inverse Laplace Transform of $\frac{1}{s^3(s^2+1)}$ by using convolution theorem. [6]

   (ii) Evaluate $L\{f(t)\}$ where $f(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right) & ; \ t > \frac{2\pi}{3} \\ 0 & ; \ t < \frac{2\pi}{3} \end{cases}$ [4]

   using second shifting property.

7. Solve the initial value problem $y'' + 3y' + 2y = g(t)$ with $y(0) = 0$, $y'(0) = 1$ where $g(t) = \begin{cases} 1 & ; \ if \quad 0 \le t < 1 \\ 0 & ; \ if \quad t \ge 1 \end{cases}$ using Laplace Transform.

8. Expand $f(x)$ as Fourier series if $f(x) = \begin{cases} \pi x & ; 0 < x < 1 \\ \pi(2-x) & ; 1 < x < 2 \end{cases}$.

   Hence deduce that $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - .... = \frac{\pi^2}{8}$.

9. Find a series of cosines of multiples of x which will represent $x \sin x$ in the interval $(0, \pi)$ and hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} + ... = \frac{\pi - 2}{4}$.

10. Find the Fourier Transform of $f(x)=\begin{cases}1-|x| & ; |x|<1 \\ 0 & ; |x|>1\end{cases}$ and hence find the value of $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt.$

11. Find the inverse Z-transform of $\frac{z^2}{(z-1)(z-3)}$ using convolution theorem.

12. Solve the difference equation $y_{n+2}+6y_{n+1}+9y_n=2^n$ with $y_0=y_1=0$ using Z-transform.

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Reg. No. : 22BEE1136

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	**B.Tech.**	Semester	**Winter Semester 2022-23**
Course Title	**DIFFERENTIAL EQUATIONS AND TRANSFORMS**	Course Code	**BMAT102L**
Faculty Name	**Prof. KRITI ARYA**	Slot	C1+TC1+TCC1
		Class Nbr	**CH2022232300682**
Time	**3 Hours**	Max. Marks	**100**

Section A (10 X 10 Marks)
Answer <u>any 10</u> questions

01. Solve the ODE $2x^2y'' + 5xy' + y = \frac{x-1}{2x}$. [10]

02. (a) Find the solution of the differential equation $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$. Hence find the Wronskian of the solutions and conclude that the solutions are linearly independent or not. [5] [10]
(b) Form a partial differential equation by eliminating the arbitrary function from the family of curves $z = f(x + ct) + g(x - ct)$. [5]

03. Solve the PDE: $(x^2 - yz)p + (y^2 - zx)q = (z^2 - xy)$. [10]

04. Find the Laplace transforms of the following functions. [10]
(a) $f(t) = \begin{cases} \text{sint}, & \text{if } 0 < t < \pi \\ \text{sin2t}, & \text{if } \pi < t < 2\pi \\ \text{sin3t}, & \text{if } t > 2\pi \end{cases}$ [5]
(b) $f(t) = e^{-5t}\int_0^t \frac{\text{sint}}{t}\,dt$. [5]

05. Solve the following partial differential equation by the method of Laplace transform [10]
$\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial t} = -8t; x, t > 0$
with the initial and boundary conditions $u(x, 0) = 0$ and $u(0, t) = 2t^2$.

06. Solve the following ODE using Laplace transform. [10]
$y'' - 4y' - 5y = 30u(t - 1)$, given that $y = 0, y' = 6$, at $t = 0$ and $u(t)$ denotes the unit step function.

07. (a) Find the Laplace inverse of the function $\frac{1}{(s^2+a^2)^2}$, where a is a constant. [5] [10]
(b) Let $f(x) = (\pi - x)^2, x \in (0, \pi)$. Find the half range cosine series of $f(x)$ and hence find the value of $\sum_{n=1}^{\infty} \frac{1}{n^4}$. [5]

08. Let $f(x) = \begin{cases} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{cases}$ and $f(x + 2\pi) = f(x)$, for all real number x. Find the Fourier series of $f(x)$ and hence find the value $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$. [10]

09. Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence evaluate the Fourier sine transform of $xe^{-a^2x^2}$. [10]

10. Find the Fourier sine transform of $e^{-2x}, x > 0$ Hence evaluate $\int_0^{\infty} \frac{x^2}{(x^2+4)^2}\,dx$. [10]

11. (a) Find the Z-transform of the function $f(n) = ke^{-an} - 2\sin(bn) + 2^n(n^2 - n)$. [5] [10]

(b) Find the inverse Z-transform of the function $\frac{4z^2}{4z^2-2z\sqrt{3}+1}$. [5]

12. Solve the following difference equation. [10]

$8y(n+2)+6y(n+1)+y(n)=5$, where $y(0)=0,\ y(1)=-1$.

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Berin Greeni A	Slot	C2+TC2+TCC2
		Class Nbr	CH2022232300619
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)

Answer <u>any 10</u> questions

a^n

01. Solve by using the method of variation of parameter $4\frac{d^2y}{dx^2} - y = \frac{1}{4}xe^{(\frac{x}{2})}; y(0) = 1, y'(0) = 0.$ [10]

02. A) Solve the differential equation $\frac{d^2y}{dx^2} + y = 12e^{2x}$ by using the method of undetermined coefficients. [10]

    B) Form a partial differential equation by eliminating f and g from the following equation $z = f(x^2 - y) + g(x^2 + y)$.

    (5+5 Marks)

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03. A. Solve $z^2(p^2z^2 + q^2) = 1$. Does singular solution exist? $1-\frac{\pi}{2n}$ [10]

    B. Obtain the general solution of the following partial differential equation $\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$.

    (5+5 Marks)

2^n $\cos(\pi+\pi)$ $\frac{3!}{s^3}$

2 $\frac{1}{2}\times\frac{\sqrt{3}}{2}$

04. Find $L^{-1}\left[\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right]$ [10]

05. A. Find the Laplace transform of $f(t) = \begin{cases} t & if\ t < 6 \\ -8 + (t-6)^2 & if\ t \geq 6 \end{cases}$ [10]

B. Using the Fourier series of $f(x) = x$ in the interval $(0, 2\pi)$, show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(5 +5 Marks)

06. Solve the following differential equation using Laplace transform $y'' + 3y' + 2y = \delta(t-1)$ with the initial condition $y(0) = y'(0) = 0$, where $\delta(t-1)$ is the unit impulse at time $t = 1$. [10]

07. Find the bounded solution $y(x,t), x, t > 0$ of following partial differential equation by the method of Laplace transform $\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-t}; x, t > 0$ with $y(x, 0) = 0$. [10]

08. Find the half range sine series for $f(x) = \begin{cases} x & 0 \leq x < \pi/2 \\ \pi - x & \pi/2 \leq x \leq \pi \end{cases}$. [10]

Deduce (i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

(ii) $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{96}$.

09. Find $f(x)$, if its Fourier sine transform is $\frac{e^{-a\omega}}{\omega}$. Hence, deduce $F_s^{-1}\left(\frac{1}{\omega}\right)$. [10]

10. Find the Fourier transform of $f(x) = \begin{cases} 4 - |x|, & if\ |x| < 4 \\ 0, & if\ |x| > 4 \end{cases}$. Hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$. [10]

11. A. Find the $Z-$ transform and the radius of convergence of $f(n) = 2^n, n < 0$ [10]

B. If $U(z) = \frac{2z^2 - 5z + 14}{(z-1)^4}$, then evaluate u_2 and u_3.

(5–5 Marks)

12. Solve the difference equation $u_{n+2} - 2u_{n+1} + u_n = 3n + 5$ using $Z-$ transform.

Reg. No. : 21BAS1085

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. PROSENJIT	Slot	D1+TD1+TDD1
		Class Nbr	CH2022232300628
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

01. a) Solve the differential equation $\frac{d^2y}{dx^2}+16y=8e^{x}$ by method of undetermined coefficients. [5 marks] [10]

b) Find the singular integral of $z=px+qy-2\sqrt{(pq)}$. [5 marks]

02. An LCR circuit connected in series has $R=10$ ohms, $C=10^{-2}$ farad, $L=\frac{1}{2}$ henry and an applied voltage $E=12$ volts. Assuming no initial current and no initial charge at $t=0$ when the voltage is first applied. Find the subsequent current in the system at any time t without finding the charge. [10]

03. a) Solve by using Lagrange multiplier method $y^2p+x^2q=x^2y^2z^2$. [7 marks] [10]

b) Form a partial differential equation by eliminating the arbitrary constants a and b from $z=a(x+y)+b$. [3 marks]

04. Find the Laplace transform of $f(t)=tu(t-2)+\delta(t-\frac{\pi}{2})cost+\int_0^t \frac{sint}{t}dt$, where $u(t-2)$ and $\delta(t-\frac{\pi}{2})$ represent Unit step function and Impulse function respectively. [10]

05. Solve the given partial differential equation by using Laplace transform $\frac{\partial u}{\partial x}+x\frac{\partial u}{\partial t}=0$ with boundary conditions $u(0,t)=t$ and $u(x,0)=0$. [10]

06. Solve the given differential equation using Laplace transform $\frac{d^2y}{dt^2}-\frac{dy}{dt}=cos(2t)+cos(2t-12)u(t-6)$ with the boundary conditions $y(0)=0$ and $y'(0)=0$. [10]

07. Find the half range Fourier cosine series of $f(x)=(\pi-x^2)$ in the interval $(0,\pi)$. Hence find the sum of the infinite series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots$ by using Parseval's identity. [10]

08. a) Determine the Fourier series of the periodic function $f(x)=x-1$ defined on $(-\pi,\pi)$. [5 marks] [10]

b) Using convolution theorem find $f(t)$ if $L(f(t))=\frac{1}{s(s^2+1)}$. [5 marks]

09. Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$ using Fourier transform. [10]

10. Find the Fourier transform of $e^{-2|x|}$ and hence evaluate [10]

$$\int_0^\infty \frac{dx}{(x^2+4)^2}.$$

11. a) Find $Z[(n+1)^2e^{-5n+7}]$. [5 marks] [10]

b) Find $Z^{-1}[\frac{z^2}{(z-0.5)(z+3)}]$ using convolution theorem. [5 marks]

12. Solve $y_{n+2}-3y_{n+1}+2y_n=3^n+3$ with $y_0=1$ and $y_1=1$. [10]

VIT Vellore Institute of Technology

Reg. No. : 22BME1062

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	B.Tech.	Semester	Winter Semester 2022-23
Course Title	DIFFERENTIAL EQUATIONS AND TRANSFORMS	Course Code	BMAT102L
Faculty Name	Prof. Nathiya N	Slot	D1+TD1+TDD1
		Class Nbr	CH2022232300594
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

01. a) Solve the differential equation $\frac{d^2y}{dx^2} + 16y = 8e^x$ by method of undetermined coefficients. **[5 marks]** [10]

b) Find the singular integral of $z = px + qy - 2\sqrt{(pq)}$. **[5 marks]**

02. An LCR circuit connected in series has $R = 10$ ohms, $C = 10^{-2}$ farad, $L = \frac{1}{2}$ henry and an applied voltage $E = 12$ volts. Assuming no initial current and no initial charge at $t = 0$ when the voltage is first applied. Find the subsequent current in the system at any time t without finding the charge. [10]

03. a) Solve by using Lagrange multiplier method $y^2p + x^2q = x^2y^2z^2$. **[7 marks]** [10]

b) Form a partial differential equation by eliminating the arbitrary constants a and b from $z = a(x+y) + b$. **[3 marks]**

04. Find the Laplace transform of $f(t) = tu(t-2) + \delta(t - \frac{\pi}{2})cost + \int_0^t \frac{sint}{t} dt$, where $u(t-2)$ and $\delta(t - \frac{\pi}{2})$ represent Unit step function and Impulse function respectively. [10]

05. Solve the given partial differential equation by using Laplace transform $\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial t} = 0$ with boundary conditions $u(0,t) = t$ and $u(x,0) = 0$. [10]

06. Solve the given differential equation using Laplace transform $\frac{d^2y}{dt^2} - \frac{dy}{dt} = cos(2t) + cos(2t - 12)u(t-6)$ with the boundary conditions $y(0) = 0$ and $y'(0) = 0$. [10]

07. Find the half range Fourier cosine series of $f(x) = (\pi - x^2)$ in the interval $(0, \pi)$. Hence find the sum of the infinite series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$ by using Parseval's identity. [10]

08. a) Determine the Fourier series of the periodic function $f(x) = x - 1$ defined on $(-\pi, \pi)$. **[5 marks]** [10]

b) Using convolution theorem find $f(t)$ if $L(f(t)) = \frac{1}{s(s^2+1)}$. **[5 marks]**

09. Evaluate $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$ using Fourier transform. [10]

10. Find the Fourier transform of $e^{-2|x|}$ and hence evaluate [10]

$$\int_0^\infty \frac{dx}{(x^2+4)^2}.$$

11. a) Find $Z[(n+1)^2 e^{-5n+7}]$. **[5 marks]**

b) Find $Z^{-1}[\frac{z^2}{(z-0.5)(z+3)}]$ using convolution theorem. [5 marks]

12. Solve $y_{n-2} - 3y_{n+1} + 2y_n = 3^n + 3$ with $y_0 = 1$ and $y_1 = 1$.

VIT

Vellore Institute of Technology

Reg. No. : 22BPS1143

Final Assessment Test (FAT) - JUNE/JULY 2023

Programme	**B.Tech.**	Semester	**Winter Semester 2022-23**
Course Title	**DIFFERENTIAL EQUATIONS AND TRANSFORMS**	Course Code	**BMAT102L**
Faculty Name	**Prof. Manimaran J**	Slot	D2+TD2+TDD2
		Class Nbr	CH2022232300626
Time	**3 Hours**	Max. Marks	100

Section-A (10 X 10 Marks)

Answer any 10 questions

01. Consider an LRC circuit where $L = 1$, $R = 2$ and $C = 1$. The current $I(t)$ is driven by an electromagnetic force $2\sin 2t$. The circuit equation for the voltage $V(t)$ across the capacitor is $\frac{d^2V}{dt^2} + 2\frac{dV}{dt} + V = 2\sin 2t$ with $V(0) = 0$ and $V'(0) = 0$. Describe the subsequent voltage oscillations by using the method of undetermined coefficients. **[10]**

02. (a) Consider the initial value problem $\frac{d^2y}{dx^2} - 6y = 0$ with $y(1) = \alpha$ and $y'(1) = 6$. If $y(x) \to 0$ as $x \to 0$, then find the value of α. **[10]**

    (b) Solve the PDE: $py + qx = xyz^2(x^2 - y^2)$. - Lagrange

03. Find the complete and singular integrals of the PDE: $z = px + qy + \sqrt{1 + p^2 + q^2}$. **[10]**

04. Find the Laplace transform of $f(t) = u(t - 2\pi)\sin t + t^4\delta(t - 3) + \frac{\sin t}{t} + \sqrt{t}$. **[10]**

    cos

05. (a) Using convolution theorem find $f(t)$ if $\mathcal{L}\{f(t)\} = \frac{9}{s(s+3)^2}$. **[10]**

    (b) Evaluate the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ from the Fourier series expansion of $f(x) = x$ in the interval $(-\pi, \pi)$.

06. Solve the simultaneous equations $\frac{dx}{dt} - y = e^t$ and $\frac{dy}{dt} + x = \sin t$ with $x(0) = 1$ and $y(0) = 0$. **[10]**

07. Using the Laplace transform method solve the following PDE: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ with $u(x, 0) = 6e^{-3x}$ and solution being bounded for $x > 0$, $t > 0$. **[10]**

08. Find the Fourier series expansion of the following periodic function in the interval $(-\pi, \pi)$ with period 2π: **[10]**

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}.$$

Hence, obtain the value of $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$.

09. Find the Fourier transform of the function $f(x) = e^{-|x|}$, $-\infty < x < \infty$. Hence, (a) derive that $\int_0^{\infty} \frac{\cos \omega x}{1-\omega^2} d\omega = \frac{\pi e^{-|x|}}{2}$; and (b) obtain the Fourier transform of $xe^{-|x|}$. **[10]**

10. Using Fourier transform, solve the PDE $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$, subject to the conditions: (i) $u(0, t) = u_0$, $t > 0$; (ii) $u(x, 0) = 0$, $x > 0$; (iii) u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \to \infty$. **[10]**

11. (a) Find the Z-transform of $2^n \cos(n + 3)$. **[10]**

    (b) Find the inverse Z-transform of $\frac{2z^2 - 3z}{z^2 - 6z + 8}$.

12. Solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$ and $u_1 = 1$ by using Z-transform. **[10]**