Final Assessment Test (FAT) - November/December 2022

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
Faculty Name	Prof. AMIT KUMAR RAHUL	Slot	A1+TA1+TAA1
		Class Nbr	CH2022231001182
Time	3 Hours	Max. Marks	100

Part-A (10 X 10 Marks) Answer any 10 questions

-]. Determine the analytic function f(z)=u+iv given that $u-v=rac{\cos x+\sin x-e^{-u}}{2\cos x-e^{u}-e^{-u}}$ and [10] $f(\frac{\pi}{2}) = 0.$
- Find the image of the rectangle with vertices -1+i, 1+i, 1+2i and -1+2i under the linear [10]mapping f(z) = 4iz + 2 + 3i.

Sketch the rectangle and its image. a) Find the bilinear transformation which maps the points z = i, z = -1 and z = 1 into the [10]points w = 0, w = 1 and $w = \infty$.

b) Find the fixed points and image of the interior of the circle |z|=1 under the transformation $w = \frac{z-i}{1-iz}$.

4. Using Contour integration, evaluate the real integral $\int_0^{2\pi} \frac{1+2\cos\theta}{10+8\cos\theta} d\theta$. [10]

5. a) Find Taylor's series expansion to represent $\frac{z^2-1}{(z+2)(z+3)}$ in |z|=2. [10]

b) Find the nature of singularity and find the residue for i) $f(z) = \frac{z - \sin z}{z^3}$ ii) $f(z) = \frac{1 - \cos z}{z}$.

Find a basis for the row space and null space of $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 1 & 5 & -2 \end{pmatrix}$. [10]6.

7. Let V be the vector space of functions $f: \mathbb{R} \to \mathbb{R}$. Verify whether the following are subspaces of [10] V or not. Justify your answer.

i) $W_1 = \{ f \in V : f(1) = 3 \},$

ii) $W_2 = \{ f \in V : f(3) = f(1) \}.$

iii) $W_3 = \{ f \in V : f(-x) = -f(x) \}.$

[10]8. A mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by

 $T(x_1,x_2,x_3)=(x_1+x_2+x_3,2x_1+x_2+2x_3,x_1+2x_2+x_3),\quad x_1,x_2,x_3\in\mathbb{R}.$

Show that T is a linear mapping. Find Ker T and the dimension of Ker T.

[10]9. Let $S = \{v_1 = (1,2,0), v_2 = (1,3,2), v_3 = (0,1,3)\}$ and $S' = \{u_1 = (1, 2, 1), u_2 = (0, 1, 2), u_3 = (1, 4, 6)\},$

i) Find the change of basis matrix P from S to S',

ii) Find the change of basis matrix Q from S' to S,

iii) verify $Q = P^{-1}$. [10]

Let \mathbb{R}^3 have the Euclidean inner product. Use the Gram Schmidt process to transfer the basis

 $\{u,v,w\}$ into an orthonormal basis, where u = (1, 0, -1), v = (-7, 4, -2), w = (-3, 0, -1). $V_1 = V_1 = V_1 - \langle V_2, U_1 \rangle \cdot \langle \overline{U_1} \rangle_2$ $||U_1||^2$

- 1. Solve the following system, by using Gauss elimination method
 - 2y z = 1.
 - 4x 10y + 3z = 5.



- 3x 3y = 6.Let A and B be two matrices such that $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, $B = I \frac{1}{2}A$. If λ_i and λ_i' are
 - the eigenvalues of A and B respectively. Find $\lambda_i,\lambda_i',i=1,2,3$, hence verify that $\lambda_i'+\frac{1}{2}\lambda_i=1$



10

[10]

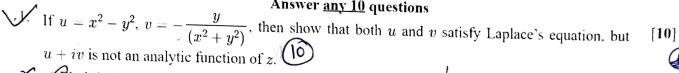


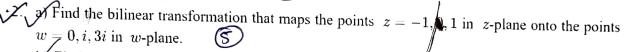
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	COMPLEX VARIABLES AND LINEAR ALGEBRA	Course Code	BMAT201L
Faculty Name	Prof. A Felix	Slot	A2+TA2+TAA2
Time		Class Nbr	CH2022231001187
	3 Hours	Max. Marks	100

Part A (10 X 10 Marks) Answer any 10 questions





Find the bilinear transformation that maps the points
$$z = -1$$
, 1 in z-plane onto the points $w = 0, i, 3i$ in w-plane. S

Find the fixed points and the image of upper half of the z-plane under the transformation $w = \frac{1}{1-z}$. [5+5 Marks]

$$\tilde{z}$$
 in Find the image of the line segment from 1 to i under the complex mapping $w=-i\bar{z}$, where \bar{z} represents the conjugate of z .

b) Find all the points where the mapping
$$f(z) = \sin(z)$$
 is conformal. [7+3 Marks]

Expand
$$f(z) = \frac{1}{(z-1)^2(z-3)}$$
 in a Laurent's series valid for $0 < |z-1| < 2$.

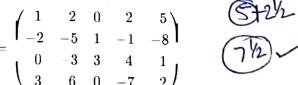
Define singularity of a complex function and determine the type of singularity of

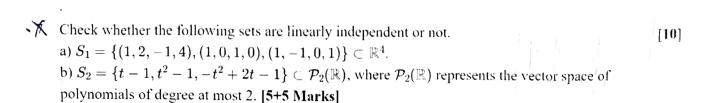
$$f(z) = ze^{\frac{z-1}{z^3 - 1 - 3z^2 + 3z}} - \frac{e^{\frac{1}{z-1}}}{\frac{1}{e^{\frac{1}{1-z}}}}.$$
 [5+5 Marks]

Evaluate
$$\int \frac{z \cosh(z)}{z^5 + 2iz^4} dz$$
 over a closed curve C , where C is a unit circle, using Cauchy's integral formula.

Find a basis of row space, column space and null space for the given matrix
$$A$$
 and hence, verify the rank-nullity theorem

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{pmatrix}$$





8. Consider the following matrix

$$A = \begin{bmatrix} -8 & 5\\ -41 & 24\\ -3 & 2 \end{bmatrix}$$

- a) Find the unique linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^3$ so that A is the associated matrix of Twith respect to bases, $\alpha = \{(1,3),(2,5)\}$ and $\beta = \{(1,0,0),(1,1,0),(1,1,1)\}$. b) Find T(2, 3).
- 9. Let $F: \mathbb{R}^4 \to \mathbb{R}^3$ be linear mapping defined by F(x,y,s,t)=(x-y+s+t,x+2s-t,x+y+3s-3t). Find basis and dimension of image of F.
- Find an orthonormal basis for the solution space of the homogeneous system of linear equations $x + y - 2z + w = 0; \ y + 2z + w = 0.$
- Solve the system of linear equations x + 2y z = -1, 3x + 8y + 2z = 28, 4x + 9y z = 14[10]using the Gauss-Jordan method.
- Find the eigenvalues and eigenvectors of the given matrix



$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \qquad \bigcirc$$



[10]

[10]

[10]