## Final Assessment Test(FAT) - Nov/Dec 2024

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|---|---|---|---|
| Programme | B.Tech. | Semester | Fall Semester 2024-25 |
| Course Code | BMAT205L | Faculty Name | Prof. Aarthy B |
| Course Title | Discrete Mathematics and Graph Theory | Slot | C2+TC2+TCC2 |
| | | Class Nbr | CH2024250102066 |
| Time | 3 hours | Max. Marks | 100 |

**General Instructions**

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

## Course Outcomes

CO 1. Learn proof techniques and concepts of inference theory

CO 2. Use algebraic structures in applications

CO 3. Counting techniques in engineering problems.

CO 4. Use lattice and Boolean algebra properties in Digital circuits.

CO 5. Solve Science and Engineering problems using Graph theory.

### Section - I
### Answer any 10 Questions (10 × 10 Marks)

*M - Marks

| Q.No | Question | *M | CO | BL |
|---|---|---|---|---|
| 01. | [(a)] Check the validity of the following argument using the inference rules ``It is not sunny this afternoon and it is colder than yesterday. We will go swimming only if it is sunny. If we do not go swimming, then we will take a trip. If we take a trip, then we will be home by sunset. We will be home by sunset." **[7 marks]** <br> [(b)] Over the universe of books, define the propositions: M(x): x is a mathematics book, U(x): x is published in the United States, and R(x,y) : The bibliography of x includes y. **[3 Marks]** <br> (i) Translate $(\exists y)((\forall x)(M(x) \rightarrow R(x,y)))$ in word. <br> (ii) Express using quantifiers: There are mathematics books that are published outside the United States. | 10 | 1 | 1 |
| 02. | (a) Find the PDNF and PCNF of the following propositioin: $X \vee (\neg X \rightarrow (Y \vee (\neg Y \rightarrow Z)))$. **[6 marks]** <br> (b) Check whether the following compound proposition is a tautology or contradiction: $\neg q \wedge (p \rightarrow q) \rightarrow \neg p$. **[4 Marks]** | 10 | 1 | 2 |
| Q3. | (a) Prove that $G = (Z_7 - \{0\}, X_7)$ is a group where $X_7$ denotes multiplication modulo 7. **[5 Marks]** <br> (b) Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$, corresponding to the encoding function $e : B^4 \rightarrow B^7$, find the corresponding parity check matrix and use it to decode the following received words and hence, to find the original message: <br> (i) 1100001 (ii) 1110111 (iii) 0010001 (iv) 0011100 **[5 Marks]** | 10 | 2 | 5 |

**04.** (a) If $*$ is defined on $S = \mathbb{N} \times \mathbb{N}$ by $(a, b) * (c, d) = (a + c, b + d)$ and if the mapping $f : (S, *) \to \mathbb{Z}, +)$ is defined by $f(a, b) = a - b$. Check if $f$ is a homomorphism. If yes, find its kernel. **[5 marks]**

(b) (i) In a course, a professor gives five grades $A, B, C, D, E$. What is the minimum number of students required so that four of them are guaranteed to get the same grade? **[2 marks]**

(ii) How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once? **[3 marks]**

`10  2,3  4`

**05.** Use the method of generating function to solve the recurrence relation
$a_n = 3a_{n-1} + 1; \quad n \geq 1$, given that $a_0 = 1$.

`10  3  4`

**06.** With respect to the poset $(D_{30}, /)$ where $D_{30}$ is the set of all divisors of 30 and $/$ represents the divisibility relation, answer the following questions:

(i) Draw the Hasse diagram. **[3 marks]**

(ii) Do there exist greatest and smallest element in this poset? Justify. **[2 marks]**

(iii) If $a \vee b$ represent l.u.b of $a$ and $b$, and $a \wedge b$ represent g.l.b. of $a$ and $b$, does there exists an element $x \in D_{30}$ such that $3 \wedge x = 1$ and $3 \vee x = 30$? Justify. **[3 marks]**

(iv) Is the poset a totally ordered set? Justify. **[2 marks]**

`10  4  1`

**07.** (a) Draw the Boolean circuit for the Boolean function $f(x, y, z) = (x' y)'(x + y)$ and also convert it to the complete product of sums (POS) and sum of product (SOP) form. **[5 Marks]**
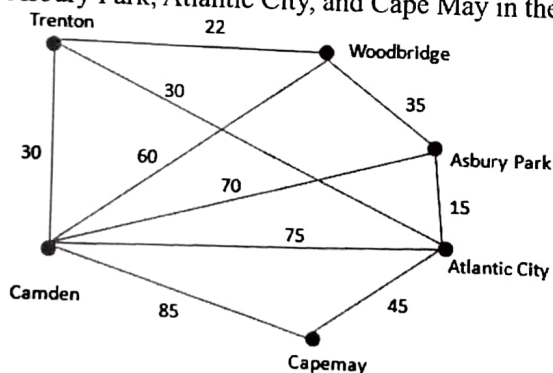
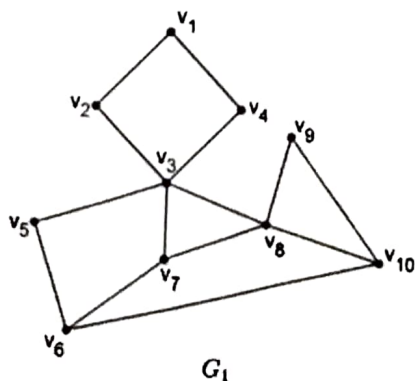(b) Check whether the following graphs are isomorphic: **[5 Marks]**

`10  4,5  1`
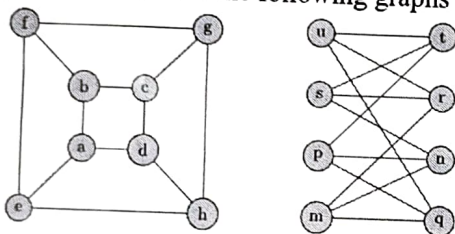


**08.** (a) Find the shortest path from Camden to each of the cities: Trenton, Woodbridge, Asbury Park, Atlantic City, and Cape May in the given figure. **[7 marks]**
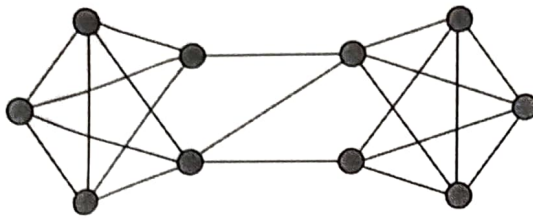
`10  5  3`



(b) Determine which of the following graphs $G_1$ an have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one. **[3 marks]**



$G_1$
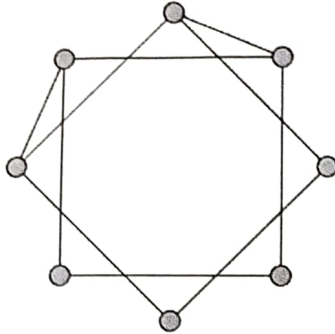
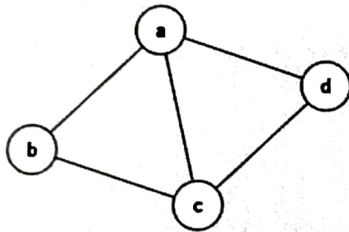09.   (a) (i) Check the Whitney's inequality for the given graph and draw the components: [3 Marks]

(ii) Check whether the following graph is Hamiltonian. If yes, mention the Hamiltonian cycle. [2 Marks]



(b) Find the chromatic polynomial of the following graph. [5 marks]



10.   A national power grid company needs to set up a high-voltage electrical network connecting seven different substations, labelled A to G. The costs (in millions of dollars) for constructing power lines between any two substations are listed in the table below:

| Edge | Cost (in million dollars) |
|------|---------------------------|
| A-B  | 2 |
| A-C  | 1 |
| B-C  | 3 |
| B-E  | 7 |
| C-D  | 1 |
| C-E  | 9 |
| C-F  | 2 |
| D-A  | 4 |
| D-F  | 6 |
| E-G  | 3 |
| F-G  | 1 |

The company aims to minimize the total cost of constructing the power lines while ensuring that all substations are connected, either directly or indirectly. The goal is to use the minimum number of power lines required to connect all substations. Using the information provided in the table, how can the company construct the most cost-effective electrical network? Illustrate your solution by using Kruskal's Algorithm to find the Minimum Spanning Tree (MST) and calculate the total cost of constructing the network. [10 Marks]

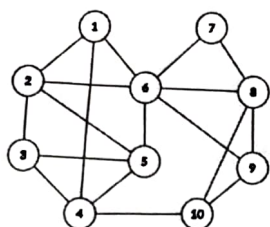11. (a) For the following tree find the preorder, inorder and postorder traversal. [5 marks]  10  5  5
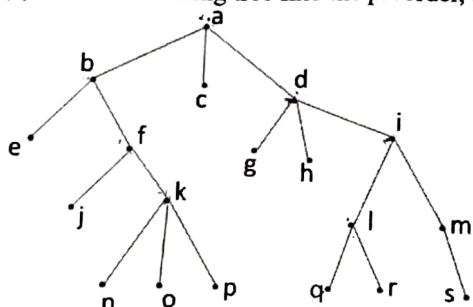


(b) Find the eccentricity of the following vertices: k, d, f, p and a. Also, find the diameter and center of the tree. [5 marks]
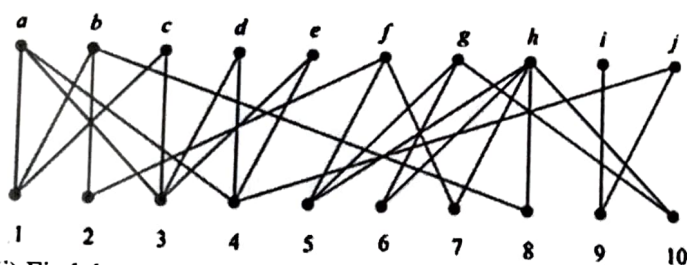
12. (a) Answer the following questions in context of the given graph: [5 marks]  10  5  1



(i) Is the graph bipartite? [1 marks]
(ii) Color all the vertices using minimum colors. What is the chromatic number? Hence form a chromatic partition of the graph.[2 marks]
(iii) Validate the Euler's formula for the given polyhedral. [2 marks]

(b) For the following graph G,



(i) Find the maximum matching of G. [2 marks]
(ii) Does the graph have a perfect matching? Justify. [3 marks]

BL-Bloom's Taxonomy Levels - (1.Remembering, 2.Understanding, 3.Applying, 4.Analysing, 5.Evaluating, 6.Creating)

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