VIT

Vellore Institute of Technology

Reg. No. : 21BCE1964

Final Assessment Test (FAT) – November/December 2022

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. AMIT KUMAR RAHUL**	Slot	**A1+TA1+TAA1**
		Class Nbr	**CH2022231001182**
Time	**3 Hours**	Max. Marks	**100**

Part-A (10 X 10 Marks)
Answer <u>any 10</u> questions

1. Determine the analytic function $f(z) = u + iv$ given that $u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ and $f(\frac{\pi}{2}) = 0$. **[10]**

2. Find the image of the rectangle with vertices $-1+i, 1+i, 1+2i$ and $-1+2i$ under the linear mapping $f(z) = 4iz + 2 + 3i$. **[10]**
   Sketch the rectangle and its image.

3. a) Find the bilinear transformation which maps the points $z = i, z = -1$ and $z = 1$ into the points $w = 0, w = 1$ and $w = \infty$. **[10]**
   b) Find the fixed points and image of the interior of the circle $|z| = 1$ under the transformation $w = \frac{z-i}{1-iz}$.

4. Using Contour integration, evaluate the real integral $\int_0^{2\pi} \frac{1+2\cos\theta}{10+8\cos\theta}\, d\theta$. **[10]**

5. a) Find Taylor's series expansion to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| = 2$. **[10]**
   b) Find the nature of singularity and find the residue for i) $f(z) = \frac{z-\sin z}{z^3}$ ii) $f(z) = \frac{1-\cos z}{z}$.

6. Find a basis for the row space and null space of $A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 2 & 1 & 5 & -2 \end{pmatrix}$. **[10]**

7. Let V be the vector space of functions $f : \mathbb{R} \to \mathbb{R}$. Verify whether the following are subspaces of V or not. Justify your answer. **[10]**
   i) $W_1 = \{f \in V : f(1) = 3\}$,
   ii) $W_2 = \{f \in V : f(3) = f(1)\}$,
   iii) $W_3 = \{f \in V : f(-x) = -f(x)\}$.

8. A mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by **[10]**
$$T(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 + x_2 + 2x_3, x_1 + 2x_2 + x_3), \quad x_1, x_2, x_3 \in \mathbb{R}.$$
   Show that T is a linear mapping. Find Ker T and the dimension of Ker T.

9. Let $S = \{v_1 = (1,2,0), v_2 = (1,3,2), v_3 = (0,1,3)\}$ and $S' = \{u_1 = (1,2,1), u_2 = (0,1,2), u_3 = (1,4,6)\}$, **[10]**
   i) Find the change of basis matrix P from S to S',
   ii) Find the change of basis matrix Q from S' to S,
   iii) verify $Q = P^{-1}$.

10. Let $\mathbb{R}^3$ have the Euclidean inner product. Use the Gram Schmidt process to transfer the basis $\{u, v, w\}$ into an orthonormal basis, where $u = (1, 0, -1), v = (-7, 4, -2), w = (-3, 0, -1)$. **[10]**

$U_1 = V_1$

V_1

$V_1 = U \quad U_2 = V_2 - \frac{\langle V_2, U_1\rangle}{\|U_1\|^2} \cdot \langle U_1 \rangle$

11. Solve the following system, by using Gauss elimination method **[10]**

$2y - z = 1,$

$4x - 10y + 3z = 5,$

$3x - 3y = 6.$

12. **[10]**

Let A and B be two matrices such that $A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, $B = I - \frac{1}{2}A$. If λ_i and λ_i' are the eigenvalues of A and B respectively. Find $\lambda_i, \lambda_i', i = 1, 2, 3$, hence verify that $\lambda_i' + \frac{1}{2}\lambda_i = 1$

.

50–55

VIT
Vellore Institute of Technology

Reg. No. : 21BCE1808

Final Assessment Test (FAT) – November/December 2022

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**COMPLEX VARIABLES AND LINEAR ALGEBRA**	Course Code	**BMAT201L**
Faculty Name	**Prof. A Felix**	Slot	**A2+TA2+TAA2**
		Class Nbr	**CH2022231001187**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)

Answer any 10 questions

1. If $u = x^2 - y^2$, $v = -\dfrac{y}{(x^2+y^2)}$, then show that both u and v satisfy Laplace's equation, but $u + iv$ is not an analytic function of z. 10 **[10]**

2. a) Find the bilinear transformation that maps the points $z = -1, 0, 1$ in z-plane onto the points $w = 0, i, 3i$ in w-plane. 5 **[10]**

   b) Find the fixed points and the image of upper half of the z-plane under the transformation $w = \dfrac{1}{1-z}$. **[5 +5 Marks]** 2½

3. a) Find the image of the line segment from 1 to i under the complex mapping $w = -i\bar{z}$, where $\bar{z}$ represents the conjugate of z. **[10]**

   b) Find all the points where the mapping $f(z) = \sin(z)$ is conformal. **[7+3 Marks]**

4. a) Expand $f(z) = \dfrac{1}{(z-1)^2(z-3)}$ in a Laurent's series valid for $0 < |z-1| < 2$. 5 **[10]**

   b) Define singularity of a complex function and determine the type of singularity of

$$f(z) = ze^{\frac{z-1}{z^3-1-3z^2+3z}} - \frac{e^{\frac{1}{z-1}}}{e^{\frac{1}{1-z}}}.$$ **[5+5 Marks]** 2½

5. Evaluate $\int \dfrac{z\cosh(z)}{z^5 + 2iz^4}dz$ over a closed curve C, where C is a unit circle, using Cauchy's integral formula. 1+2 **[10]**

6. Find a basis of row space, column space and null space for the given matrix A and hence, verify the rank-nullity theorem **[10]**

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{pmatrix}$$

5+2½

7½

7. Check whether the following sets are linearly independent or not. **[10]**

   a) $S_1 = \{(1,2,-1,4),(1,0,1,0),(1,-1,0,1)\} \subset \mathbb{R}^4$.

   b) $S_2 = \{t-1, t^2-1, -t^2+2t-1\} \subset \mathcal{P}_2(\mathbb{R})$, where $\mathcal{P}_2(\mathbb{R})$ represents the vector space of polynomials of degree at most 2. **[5+5 Marks]**

8. Consider the following matrix [10]

$$A = \begin{bmatrix} -8 & 5 \\ -41 & 24 \\ -3 & 2 \end{bmatrix}$$

a) Find the unique linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ so that A is the associated matrix of T with respect to bases, $\alpha = \{(1,3),(2,5)\}$ and $\beta = \{(1,0,0),(1,1,0),(1,1,1)\}$.

b) Find $T(2,3)$.

9. Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be linear mapping defined by [10]
$F(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t)$. Find basis and dimension of image of F.

10. Find an orthonormal basis for the solution space of the homogeneous system of linear equations [10]
$x + y - 2z + w = 0;\ y + 2z + w = 0$.

11. Solve the system of linear equations $x + 2y - z = -1, 3x + 8y + 2z = 28, 4x + 9y - z = 14$ [10]
using the Gauss-Jordan method.

12. Find the eigenvalues and eigenvectors of the given matrix [10]

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$