## VIT

Reg. No. : 22BCC1018

### Final Assessment Test (FAT) - November/December 2023

| Programme | B.Tech. | Semester | FALL SEMESTER 2023 - 24 |
|---|---|---|---|
| Course Title | PROBABILITY AND STATISTICS | Course Code | BMAT202L |
| Faculty Name | Prof. MYTHILI G Y | Slot | F2+TF2 |
| | | Class Nbr | CH2023240101042 |
| Time | 3 Hours | Max. Marks | 100 |

Statistical Tables to be provided

### Section - A (10 X 10 Marks)
### Answer any 10 questions

01. Find the missing frequency from the following distribution of daily sale of shops, given that the median sale of shops is Rs. 2500. **[10]**

| Sales (in hundred rupees) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|---|
| No. of shops | 5 | 25 | 18 | 10 | 15 | ? | 20 |

02. An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results. **[10]**

| | Firm A | Firm B |
|---|---|---|
| No. of workers | 500 | 600 |
| Average daily wages (Rs.) | 186 | 125 |
| Variance of distribution of wages | 81 | 100 |

(a) Which firm, A or B has a larger wage bill? (3 marks)

(b) In which firm, A or B is there greater variability in individual wages? (3 marks)

(c) Calculate (i) the average daily wage and (2+2 marks)

(ii) the variance of the distribution of wages of all the workers in the firms A and B taken together.

03. The joint probability distribution of two random variables X and Y is given by: **[10]**

$P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$, and $P(X = 1, Y = 1) = \frac{1}{3}$.

Find

(a) Marginal distributions of $X$ and $Y$ (4 marks)

(b) The conditional probability distribution of $X$ given $Y = 1$. (6 marks)

04. **[10]**

The probability density function of random variable $x$ is given as $\begin{cases} xe^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$

Obtain (a) the cumulative distribution function F(x), of $X$ and $P(1 < x \leq 3)$ (5 marks)

(b) the moment generating function. $M_X(t)$ of $X$ and hence obtain $E(X)$. (5 marks)

05. For given the data relating to purchases (x) and sales (y). Obtain the two regression equations by the method of least square and estimate the likely sales when the purchases equal to 100. **[10]**

| Purchases (x) | 62 | 72 | 98 | 76 | 81 | 56 | 76 | 92 | 88 | 49 |
|---|---|---|---|---|---|---|---|---|---|---|
| Sales (y) | 112 | 124 | 131 | 117 | 132 | 96 | 120 | 136 | 97 | 85 |

06. In a partially destroyed laboratory record of an analysis of a correlation data, only the following results are legible: **[10]**

Variance of $x = 9$;

Regression equations: $8x - 10y + 66 = 0$; $40x - 18y = 214$.

Determine

(a) the mean values of $x$ and $y$. (2marks)

(b) the standard deviation of $y$. (3 marks)

(c) the coefficient of correlation between $x$ and $y$. (5 marks)

07. The random variable $X$ has a gamma distribution with cumulative distribution function $(C.D.F)$ **[10]**

$$F(x) = \begin{cases} 1 - \left(1 + x + \frac{x^2}{2}\right)e^{-x}, & x \geq 0 \\ 0, & x < 0. \end{cases}$$

Find the following:

(a) the probability density function $f(x)$, of $X$ (3 marks)

(b) $P(2X + 1 > 3)$ (2 marks)

(c) $P(1 \leq 2X - 1 < 3)$ (2 marks)

(d) $E\left(3X^2\right)$ (3 marks)

08. (a) When a fair coin is tossed 100 times, determine the mean and standard deviation. (5 marks) **[10]**

(b) The daily consumption of milk in a city, in excess of 20,000 litres is approximately distributed as a Gamma variate with the parameters $\alpha = 2$ and $\beta = \frac{1}{10,000}$. The city has a daily stock of 30,000 litres. What is the probability that stock is insufficient on a particular day? (2.5 marks)

(c) If the probability that an individual suffers a bad reaction due to certain injection is 0.001. Determine the probability that out of 2000 individuals (i) exactly three (ii) more than two individuals will suffer a bad reaction. (2.5 marks)

09. (a) In a survey 246 town school children and 349 village school children were examined. Out of 246 children, 36 suffered from conductive hearing loss and among 349 children 61 suffered with hearing loss. Does this data present any evidence that conductive hearing loss is as common among town children as among village children. Test the results at 5% level of significance and interpret the results. (5 marks) **[10]**

(b) The breaking strengths of cables produced by a manufacturer have a mean of 1800 pounds (lbs) and standard deviation of 100 pounds (lbs). By a new technique in the manufacturing process, it is claimed that the breaking strength can be increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850 pounds (lbs). Can we support the claim at the 0.01 significance level? (5 marks)

10. The following table gives the number of units of production per day turned out by four different types of machines: **[10]**

| Employee | Type of machine | | | |
| --- | --- | --- | --- | --- |
| | M1 | M2 | M3 | M4 |
| E1 | 40 | 36 | 45 | 30 |
| E2 | 38 | 42 | 50 | 41 |
| E3 | 36 | 30 | 48 | 35 |
| E4 | 46 | 47 | 52 | 44 |

Using Analysis of variance (a) test the hypothesis that the mean production is the same for the four machines at 5%5% level and (5 marks) (b) test the hypothesis that the employees do not differ with respect to mean productivity at 5%5% level. (5 marks)

11. (a) The following data on vaccination were collected in a hospital to find out whether vaccination reduces the severity of any actual attack of smallpox.
Degree of Severity
Very Severe Severe Mild
Vaccinated within 10 years of attack 10 150 240
Never vaccinated 60 30 10
Use Chi-Square test for contingency to test the significance at 5% level and interpret the results. (5 marks)
(b) Two samples are drawn from two normal populations. From the following data test whether the two samples have the same variance at 5% level. (5 marks)
Sample 1 60 65 71 74 76 82
Sample 2 61 66 67 85 78 63

12. The time to failure in operating hours of a critical solid-state power unit has the hazard rate function $\lambda(t) = 0.003(\frac{t}{500})^{0.05}$, for $t \geq 0$.
(a) What is the reliability if the power unit must operate continuously for 50 hours? (2 marks)
(b) Determine the design life if a reliability of 0.90 is desired. (3 marks)
(c) Compute the mean time to failure. (2 marks)
(d) Given that the unit has operated for 50 hours, what is the probability that will survive a second 50 hours of operation? (3 marks)

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**Final Assessment Test (FAT) - January 2024**

| Programme | B.Tech. | | |
|---|---|---|---|
| Course Title | PROBABILITY AND STATISTICS | Semester | - |
| | | Course Code | BMAT202L |
| Faculty Name | - | Slot | - |
| | | Class Nbr | AR20232417000000000033 |
| Time | 3 Hours | Max. Marks | 100 |

Statistical table will be provide in the exam

## Part A (10 X 10 Marks)
### Answer any 10 questions

**01.** An incomplete distribution is given below: **[10]**

| Class Interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|---|
| Frequency | 10 | 20 | ? | 40 | ? | 25 | 15 |

(a) You are given that the median is 35. Find out missing frequencies (given the total frequency is 170).

(b) Calculate the arithmetic mean of the competed table. **(5+5=10)**

**02.** (i) The mean of five observations is 4.4 and the variance is 8.24. If three of the five observations **[10]** are 1,2 and 6 , find the values of the other two.

(ii) The arithmetic mean and standard deviation of series 20 items were calculated by a student as 20 cm and 5 cm, respectively. But while calculating them an item 13 was misread as 30 . Find the correct arithmetic mean and standard deviation. **(5+5=10)**

**03.** The joint probability density function of the random variable $(X, Y)$ is given by **[10]**

$$f(x,y) = \begin{cases} kx(x - y), & 0 < x < 2, -x < y < x \\ 0, & \text{otherwise} \end{cases}$$

(i) Find $k$ such that $f(x, y)$ is a joint probability density function.

(ii) Find that marginal densities of $X$ and $Y$.

(iii) Find the conditional distribution of $Y$ given $X = x$. **(2+5+3=10)**

**04.** A random variable $X$ has the following probability distribution: **[10]**

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $P(X)$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ | $15k$ | $17k$ |

(i) Find $k$

(ii) Find $P(X \leq 2)$

(iii) Find the distribution function of $X$

(iv) Find the mean of $X$. **(2+3+3+2=10)**

**05.** Find the mean of which two variables are involved given that the correlation coefficient between **[10]** the variables is -0.5 . The line of regression passes through the points $(4, 0)$ and $(-14, 3)$ and the other line of the regression passes through the point $(1, -1)$. **(5+5=10)**

**06.** a) Find the multiple linear regression equation of $x_3$ on $x_1$ and $x_2$, for the given correlation **[10]** coefficients $r_{12} = 0.28, r_{13} = 0.51, r_{23} = 0.49$ and standard deviations $\sigma_{x_1} = 2.7, \sigma_{x_2} = 2.4, \sigma_{x_3} = 2.7$.

b) The mean weight of many babies of age 1 year is 8.5 kg and the standard deviation is 2.4 kg. If the age distribution follows a normal distribution, then

(i) What percentage of babies had weight under 8.9 kg ?

(ii) Find the probability that the weight may lie in between 6.4 kg to 8.5 kg. (4+6=10)

07. a) The atoms of a radioactive element are randomly disintegrating. If every gram of this element, on average emits 3.9 alpha particles per second, what is the probability that during the next second the number of alpha particles emitted from 1 gram is     [10]

(i) At most 6

(ii) At least 3 and at most 6

b) Each of 6 tubes of a radio set has a length (in years) which may be considered as a random variable that follows a Weibull distribution with parameters $\alpha = 25$ and $\beta = 2$. If these tubes function independently of one another, what is the probability that no tube will have to replace during the first two months of service? (5+5=10)

08. (i) Out of a sample of 400 people in a city 200 were found to be graduated from college. In a different city, it was found that 240 out of 450 people are graduated from college. State whether there is a significant difference in the proportions of the samples at 1% and 5% level of significance.     [10]

(ii) An average number of objects created by two machines for every day are 150 and 160 with standard deviation 10 and 5 individually, based on records of 15 days creation. Would be able to respect the two machines similarly productive at 10% significance level? (5+5=10)

09. (i) A random sample size of 20 from a normal population gives a sample mean of 42 and sample standard deviation of 6 . Test the hypothesis that the population means is 44 . State clearly the alternative hypothesis you allow for, and the level of significance adopted.     [10]

(ii) The mean lifetime of a sample 13 fluorescent light bulbs produced by a company is computed to be 1400 hours with a standard deviation of 90 hours. The bulb produced by the company is 1450 hours using the level of significance of 1%. Is the claim acceptable? (5+5=10)

10. A random test is conducted to see the life (in hours) of Televisions manufactured by 3 different companies (say, A, B, and C). From each company 6 set of Televisions are used in this study. Their working history is shown in the following table:     [10]

| A | B | C |
|---|---|---|
| 8 | 8 | 10 |
| 10 | 9 | 9 |
| 9 | 9 | 10 |
| 11 | 8 | 11 |
| 10 | 10 | 9 |
| 12 | 11 | 8 |

Using the ANOVA test, discuss whether there is any significant difference between them manufactured by the aforesaid companies. (6+3+1=10)

11. The density function of the time to failure in years of the gizmos (for use on widgets) manufactured by a certain company is given by $f(t) = \frac{200}{(t+10)^3}, t \geq 0$.     [10]

(i) Derive the reliability function and determine the reliability for the first year of operation.

(ii) Compute the MTTF.

(iii) What is the design life for a reliability 0.95 ?

(iv) Will a one-year burn-in period improve the reliability in part (i)? If so, what is the new reliability? (2+2+3+3=10)

12. (i) The mean lifetime of a sample 35 fluorescent light bulbs produced by a company is computed [10] to be 1200 hours with a standard deviation of 90 hours. The bulb produced by the company is 1250 hours using the level of significance of 5%. Is the claim acceptable?

(ii) Find the system reliability for the network



(5+5=10)

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**Final Assessment Test (FAT) - July/August 2023**

| Programme | **B.Tech.** | Semester | **Fall Inter Semester 22-23** |
|---|---|---|---|
| Course Title | **PROBABILITY AND STATISTICS** | Course Code | **BMAT202L** |
| Faculty Name | **Prof. Prabhakar V** | Slot | **A1+TA1** |
| | | Class Nbr | **CH2022232500458** |
| Time | **3 Hours** | Max. Marks | **100** |

## Section 1 (10 X 10 Marks)
### Answer <u>any 10</u> questions

01. Find the mean, standard deviation, and quartile deviation of the following data  [10]

| 125-175 | 175-225 | 225-275 | 275-325 | 325-375 | 375-425 | 425-475 | 475-525 | 525-575 |
|---|---|---|---|---|---|---|---|---|
| 2 | 22 | 19 | 14 | 3 | 4 | 6 | 1 | 1 |

02. i) The Median and Mode of the following wage distribution are known to be Rs. 33.50 and Rs.  [10]
34 respectively. Find the values of $f_3$, $f_4$ and $f_5$. (5 Marks)

| Wages (in Rs.) | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | Total |
|---|---|---|---|---|---|---|---|---|
| Frequency | 4 | 16 | $f_3$ | $f_4$ | $f_5$ | 6 | 4 | 230 |

ii) Obtain the rank correlation coefficient for the following data: (5 Marks)

| X | 68 | 64 | 75 | 50 | 64 | 80 | 75 | 40 | 55 | 64 |
|---|---|---|---|---|---|---|---|---|---|---|
| Y | 62 | 58 | 68 | 45 | 81 | 60 | 68 | 48 | 50 | 70 |

03. If a random variable $X$ has a probability density function  [10]
$f(x) = \frac{1}{2}e^{-|x|}; -\infty < x < \infty$
Show that moment generating function of $X$ is $\phi(t) = \frac{1}{1-t^2}$. Hence find the variance of $X$.

04. Let $(X, Y)$ be a bivariate continuous random variable with joint PDF  [10]
$f_{XY}(x, y) = \begin{cases} cx^2y & 0 \le y \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$
a. Find the constant $c$.
b. Find marginal PDFs, $f_X(x)$ and $f_Y(y)$.
c. Find $P\left(Y \le \frac{X}{2}\right)$.
d. Find $P\left(Y \le \frac{X}{4} \mid Y \le \frac{X}{2}\right)$.

05. i) For 50 students of a class the regression equation of marks in Statistics ($X$) on marks in  [10]
Mathematics ($Y$) is $4Y - 6X + 180 = 0$. The mean mark in Mathematics is 46 and the variance
of marks in Statistics is $4/25$ th of the variance of marks in Mathematics. Find the mean marks
in Statistics and the coefficient of correlation between marks in two subjects. (7 marks)
ii) Given two regression lines $X + 3Y - 7 = 0$ and $3X + 4Y - 8 = 0$, find the regression line
of $X$ on $Y$.(3 marks)

06. The time spent waiting between the events is often modeled using the exponential distribution.  [10]
Suppose that an average of 30 customers per hour arrive at a store and the time between arrivals
is exponentially distributed.
a. On average, how many minutes elapse between two successive arrivals?

b. After a customer arrives, find the probability that it takes less than one minute for the next customer to arrive.

c. After a customer arrives, find the probability that it takes more than five minutes for the next customer to arrive.

d. Seventy percent of the customers arrive within how many minutes of the previous customer?

e. What is the probability that in any 2 minutes interval, three customers arrive at the store?

07. i) A manufacturer of balloons produces 40 percent that are oval and 60 percent that are round. **[10]** Packets of 10 balloons may be assumed to contain random samples of balloons. Determine the probability that such a packet contains:

a. An equal number of oval balloons and round balloons. (2 Marks)

b. Fewer oval balloons than round balloons. (3 Marks)

ii) A mining company needs to estimate the average amount of copper per ton mined. A random sample of 50 tons gives a sample mean of 146.75 pounds. The population standard deviation is assumed to be 35.2 pounds. Give a 95% confidence interval for the average amount of copper in the population of tons mined. (5 Marks)

08. i) A car manufacturer aims to improve the quality of the products by reducing defects and also **[10]** increasing customer satisfaction. Therefore, he monitors the efficiency of two assembly lines on the shop floor. In line A there are 18 defects reported out of 200 samples. While line B shows 25 defects out of 600 cars. At $\alpha = 5\%$, is the differences between the two assembly procedures significant? (5 Marks)

ii) The Educational Testing Service conducted a study to investigate the difference between the scores of female and male students on the Mathematics Aptitude Test. The study identified a random sample of 562 female and 852 male students who had achieved the same high score on the mathematics portion of the test. That is, female and male students are viewed as having similar high abilities in mathematics. The verbal scores for the two samples are given below:

|  | Female | Male |
|---|---|---|
| Sample mean | 547 | 525 |
| Sample standard deviation | 83 | 78 |

Do the data support the conclusion that given populations of female and male students with similar high abilities in mathematics, the female students will have a significantly high verbal ability? Test at significance level $\alpha = 5\%$. What is your conclusion? (5 Marks)

09. Three experimenters determine the water content of samples of food, each man taking a sample **[10]** from each of the consignments. The results are given below

| Experiment | Consignment | | | |
|---|---|---|---|---|
|  | I | II | III | IV |
| A | 9 | 10 | 9 | 10 |
| B | 12 | 11 | 9 | 11 |
| C | 11 | 12 | 10 | 12 |

Perform an analysis of variance on these data and discuss whether there is any significant difference between consignments or between experiments.

Given F(2, 6)=5.14, F(3, 6)=4.76 at 0.05 level of significance.

10. Samples of two types of electric light bulbs were tested for length of life (in months) are given **[10]** by: Type A: 25, 32, 30, 34, 24, 14, 32, 24, 30, 31, 35, 25.

Type B: 44, 34, 22, 10, 47, 31, 40, 30, 32, 35, 18, 21, 35, 29, 22.

Test if the mean life length of the two type of electric light bulbs differ significantly. Use the level of significance $\alpha = 0.05$.

11. The time of failure in operating hours of a critical solid-state power unit has hazard rate function **[10]**
$\lambda(t) = 0.003\left(\frac{t}{500}\right)^{0.5}$, for $t \geq 0$.

a) What is the reliability if the power unit must operate continuously for 50 hours?

b) Determine the design life if a reliability of 0.90 is desired.

c) Compute the MTTF.

d) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours operation?

12. **[10]**

Find the system reliability of the following series-parallel configurations. Component reliabilities are given
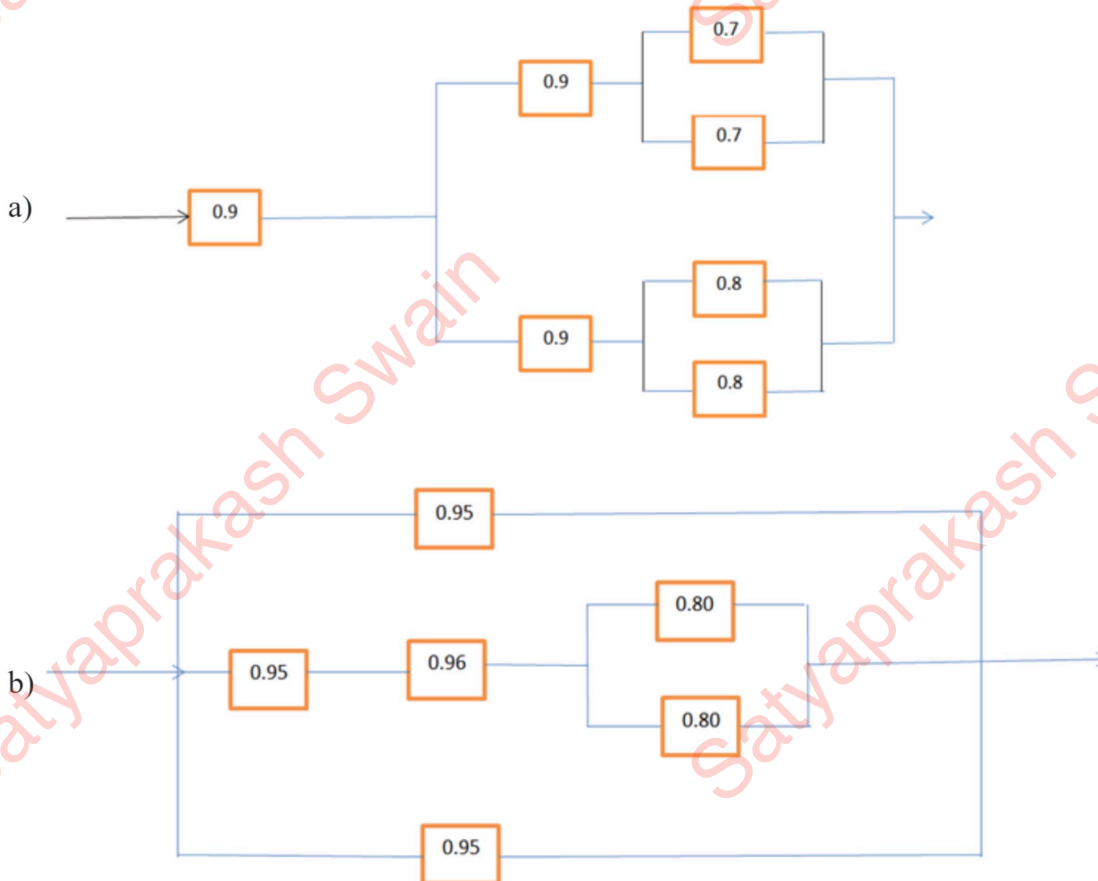


*Instruction* : Non-programmable calculators and statistical tables are permitted.

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## VIT
### Vellore Institute of Technology
(Deemed to be University under section 3 of the UGC Act 1956)

### Final Assessment Test (FAT) - May 2024

| Programme | B.Tech. | Semester | WINTER SEMESTER 2023 - 24 |
|---|---|---|---|
| Course Title | PROBABILITY AND STATISTICS | Course Code | BMAT202L |
| Faculty Name | Prof. Prabhakar V | Slot | C1+TC1 |
| | | Class Nbr | CH2023240500894 |
| Time | 3 Hours | Max. Marks | 100 |

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.
- Statistical table is required

**Answer any 10 questions (10 X 10 Marks = 100 Marks)**

01. The following table represents the scores obtained by students in a class test: [10]

| Score Range | Number of Students |
|---|---|
| 40-50 | 5 |
| 51-60 | 8 |
| 61-70 | 6 |
| 71-80 | 4 |
| 81-90 | 3 |

a) Calculate the mean and median of the scores. (5 Marks)

b) Determine the variance and standard deviation of the scores. (5 Marks)

02. (a) Find the missing frequency from the following distribution of daily sales of shops, given that the median sale of shops in Rs. 2,400. (5 Marks) [10]

| Sales (in hundred rupees) | No. of shops |
|---|---|
| 0-10 | 5 |
| 10-20 | 25 |
| 20-30 | ? |
| 30-40 | 8 |
| 40-50 | 7 |

(b) If $X$ is a random variable whose pdf is

$$f(x) = \begin{cases} \frac{x}{3}, & x = 1, 2 \\ 0, & otherwise \end{cases}$$

find the mathematical expectation of

(i) $x$ (ii) $x^2$ (5 Marks)

03. Suppose $X$ and $Y$ are two continuous random variables with the following joint probability density function: [10]

$$f_{X,Y}(x, y) = \frac{1}{\pi} e^{-x^2 - y^2}$$

a) Determine whether $f_{X,Y}(x, y)$ is a valid joint probability density function. (4 Marks)
b) Find the marginal probability density functions of $X$ and $Y$. (6 Marks)

04. Compute the correlation coefficient, using the following data: [10]

| X | 1 | 3 | 5 | 7 | 8 | 10 |
|---|---|---|---|---|---|----|
| Y | 8 | 12 | 15 | 17 | 18 | 20 |

Hence obtain the two regression lines.

05. (a) In a trivariate distribution, the correlation coefficients are [10]
$\rho_{12} = 0.87$, $\rho_{13} = 0.82$, and $\rho_{23} = 0.62$. Compute the multiple correlation coefficients $R_{1.23}$ and $R_{2.13}$. (5 Marks)

(b) A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance. (5 Marks)

06. (a) A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let $X$ denote the number of heads that come up. Calculate: [10]
(i) $P(X = 2)$, (ii) $P(X \leq 3)$, (iii) $P(1 < X \leq 5)$. (5 Marks)
(b) The number of calls coming per minute into a hotel reservation center is a Poisson random variable with mean 3. (i) Find the probability that no calls come in a given one minute period.
(ii) Find the probability that at least two calls will arrive in a given two minute period. (5 Marks)

07. Suppose that during the rainy season on a tropical island, the length of the shower has an exponential distribution, with parameter $\lambda = 2$, time being measured in minutes. [10]
(a) What is the probability that a shower will last for more than three minutes? (5 Marks)
(b) If the shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute? (5 Marks)
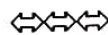
08. A sample of 100 electric bulbs produced by manufacturer A showed a mean life time of 1190 hours and a standard deviation of 90 hours. A sample of 75 bulbs produced by manufacturer B showed a mean life time of 1230 hours with a standard deviation of 120 hours. Is there a difference between the mean life time of the two brands at significance level of 5%? [10]

09. Samples of two types of electric light bulbs were tested for length of life and the following data were obtained. Type I: $n_1 = 8$, $\bar{x}_1 = 1234$ hours, $s_1 = 36$ hours Type II: $n_2 = 7$, $\bar{x}_2 = 1036$ hours, $s_2 = 40$ hours. Is the difference in mean sufficient to warrant that Type I is superior than Type II regarding the length of life? [10]

10. The price of a popular tennis racket at a national chain store is 179. Ria bought five of the same racket at an online auction site for the following prices: 155, 179, 175, 175, 161. Assuming that the auction prices of rackets are normally distributed, determine whether there is sufficient [10]

evidence in the sample, at the 5% level of significance, to conclude that the average price of the racket is less than 179 if purchased at an online auction.

11. Given that $R(t) = e^{-\sqrt{0.001t}}, t \geq 0$:  [10]
    (a) Compute the reliability for a 50-hour mission. (2.5 Marks)
    (b) Given a 10-hour wear-in period, compute the reliability for a 50-hour mission. (2.5 Marks)
    (c) What is the design life for a reliability of 0.95? (2.5 Marks)
    (d) What is the design life for a reliability of 0.95, given a 10-hour wear-in period? (2.5 Marks)

12. (a) Reliability testing has indicated that a voltage inverter has a 6 month reliability of 0.87  [10]
    without repair facility. If repair facility is made available with an MTTR of 2.2 months, compute the availability over the 6-month period by assuming constant failure and repair rate. (5 Marks)
    (b) If $\lambda(t) = (0.015 + 0.02t)$ per year, where $t$ is in years: (5 Marks)
    (i) Calculate the reliability for a 5-year design life, assuming that no maintenance is performed.
    (ii) Calculate the reliability for a 5-year design life, assuming that annual preventive maintenance restores the device to an as-good-as-new condition.

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## Final Assessment Test (FAT) - May 2024

| Programme | B.Tech. | Semester | WINTER SEMESTER 2023 - 24 |
|---|---|---|---|
| Course Title | PROBABILITY AND STATISTICS | Course Code | BMAT202L |
| Faculty Name | Prof. Prabhakar V | Slot | C1+TC1 |
| | | Class Nbr | CH2023240500894 |
| Time | 3 Hours | Max. Marks | 100 |

General Instructions:

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| 61-70 | 6 |
| 71-80 | 4 |
| 81-90 | 3 |

a) Calculate the mean and median of the scores. (5 Marks)

b) Determine the variance and standard deviation of the scores. (5 Marks)

02. (a) Find the missing frequency from the following distribution of daily sales of shops, given that the median sale of shops in Rs. 2,400. (5 Marks) **[10]**

| Sales (in hundred rupees) | No. of shops |
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(b) If $X$ is a random variable whose pdf is

$$f(x) = \begin{cases} \frac{x}{3}, & x = 1, 2 \\ 0, & otherwise \end{cases}$$

find the mathematical expectation of

(i) $x$ (ii) $x^2$ (5 Marks)

03. Suppose $X$ and $Y$ are two continuous random variables with the following joint probability density function: **[10]**

$$f_{X,Y}(x,y) = \frac{1}{\pi} e^{-x^2 - y^2}$$

a) Determine whether $f_{X,Y}(x,y)$ is a valid joint probability density function. (4 Marks)
b) Find the marginal probability density functions of $X$ and $Y$. (6 Marks)

04. Compute the correlation coefficient, using the following data: **[10]**

| X | 1 | 3 | 5 | 7 | 8 | 10 |
|---|---|---|---|---|---|----|
| Y | 8 | 12 | 15 | 17 | 18 | 20 |

Hence obtain the two regression lines.

05. (a) In a trivariate distribution, the correlation coefficients are **[10]**
$\rho_{12} = 0.87$, $\rho_{13} = 0.82$, and $\rho_{23} = 0.62$. Compute the multiple correlation coefficients $R_{1.23}$ and $R_{2.13}$. (5 Marks)
(b) A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance. (5 Marks)

06. (a) A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let $X$ denote the number of heads that come up. Calculate: **[10]**
(i) $P(X = 2)$, (ii) $P(X \leq 3)$, (iii) $P(1 < X \leq 5)$. (5 Marks)
(b) The number of calls coming per minute into a hotel reservation center is a Poisson random variable with mean 3. (i) Find the probability that no calls come in a given one minute period.
(ii) Find the probability that at least two calls will arrive in a given two minute period. (5 Marks)

07. Suppose that during the rainy season on a tropical island, the length of the shower has an **[10]** exponential distribution, with parameter $\lambda = 2$, time being measured in minutes.
(a) What is the probability that a shower will last for more than three minutes? (5 Marks)
(b) If the shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute? (5 Marks)

08. A sample of 100 electric bulbs produced by manufacturer A showed a mean life time of 1190 **[10]** hours and a standard deviation of 90 hours. A sample of 75 bulbs produced by manufacturer B showed a mean life time of 1230 hours with a standard deviation of 120 hours. Is there a difference between the mean life time of the two brands at significance level of 5%?

09. Samples of two types of electric light bulbs were tested for length of life and the following data **[10]** were obtained. Type I: $n_1 = 8$, $\bar{x}_1 = 1234$ hours, $s_1 = 36$ hours Type II: $n_2 = 7$, $\bar{x}_2 = 1036$ hours, $s_2 = 40$ hours. Is the difference in mean sufficient to warrant that Type I is superior than Type II regarding the length of life?

10. The price of a popular tennis racket at a national chain store is 179. Ria bought five of the same **[10]** racket at an online auction site for the following prices: 155, 179, 175, 175, 161. Assuming that the auction prices of rackets are normally distributed, determine whether there is sufficient

evidence in the sample, at the 5% level of significance, to conclude that the average price of the racket is less than 179 if purchased at an online auction.

11. Given that $R(t) = e^{-\sqrt{0.001t}}$, $t \geq 0$:  [10]
    (a) Compute the reliability for a 50-hour mission. (2.5 Marks)
    (b) Given a 10-hour wear-in period, compute the reliability for a 50-hour mission. (2.5 Marks)
    (c) What is the design life for a reliability of 0.95? (2.5 Marks)
    (d) What is the design life for a reliability of 0.95, given a 10-hour wear-in period? (2.5 Marks)

12. (a) Reliability testing has indicated that a voltage inverter has a 6 month reliability of 0.87  [10]
    without repair facility. If repair facility is made available with an MTTR of 2.2 months, compute the availability over the 6-month period by assuming constant failure and repair rate. (5 Marks)
    (b) If $\lambda(t) = (0.015 + 0.02t)$ per year, where $t$ is in years: (5 Marks)
    (i) Calculate the reliability for a 5-year design life, assuming that no maintenance is performed.
    (ii) Calculate the reliability for a 5-year design life, assuming that annual preventive maintenance restores the device to an as-good-as-new condition.

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