Reg. No. :

Name :

VIT
Vellore Institute of Technology

CONTINUOUS ASSESSMENT TEST II – OCTOBER 2022

Programme	:B.Tech	Semester	:Fall 2022
Course Name	:Applied Linear Algebra	Course Code	:MAT3004
Faculty	:Dr. M. Kaliyappan, Dr. Hannah Grace, Dr. David Raj Micheal, Dr. S. Dhanasekar, Dr. Om Namha Shivay	Slot/ Class No	:C2+TC2+TCC2/CH20222 31000390/391/392/ 393/394
Time	:90 mins	Max. Marks	:50

Answer all questions (5 X 10 = 50 Marks)

1. (a) Determine the dimensions of the sum and of the intersection of the vector spaces V_1 and V_2 defined by the columns of these matrices : $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & -2 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & -1 & 2 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & 3 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. 6+4

(b) Find the unique polynomial $p(x)$ of degree 2 or less, that agrees with the following data:

i	x_i	y_1
0	1	1
1	2	3
2	4	8

2. Let $T_1: P_2 \to R^3$ and $T_2: P_2 \to R^3$ be the linear transformations defined by $T_1(a+bx+cx^2) = (a+c, a+b, 0)$, $T_2(a+bx+cx^2) = (0, a+b, a-c)$ where P_2 is the vector space of all polynomials of degree at most 2. 4+6

(i) Find Ker(T_1) and Ker(T_2) and hence check the whether T_1 and T_2 are onto.

(ii) If $T_1 + T_2): P_2 \to R^3$ be another linear transformation defined as $(T_1 + T_2)(p(x)) = T_1(p(x)) + T_2(p(x))$, for every $p(x) = a + bx + cx^2 \in P_2$ and a,b,c $\in R$, find the inverse map of $(T_1 + T_2)$

3. Let $T: R^3 \to R^3$ be the linear transformation given by $T(x,y,z) = (-2x+y, -y-z, x+3z)$. 6+4

(i) Find $[T]_\alpha^\beta$ where $\alpha = \{(1,-3,1),(0,3,-1),(2,-2,1)\}$ and $\beta = \{(2,0,1),(3,-1,1),(15,-6,4)\}$

(ii) Find the transition matrix $[id]_\alpha^\beta$.

(a) Find $\|f\|$, $\|g\|$ and d(f,g) where $f = 1 + x$, $g = 1 + x + x^2$ for the inner product space V with respect to the inner product $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$. 5+5

(b) Find the value of K so that the following is an inner product space on R^2
$< u, v > = x_1y_1 - 2x_1y_2 - 2x_2y_1 + Kx_2y_2$ where $u = (x_1, x_2), v = (y_1, y_2) \in R^2$

5. Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 3 & -2 \\ 1 & 3 & 2 \\ 1 & -1 & 0 \end{bmatrix}$. 10

Also find a vector orthogonal to first two columns of A.