VIT

Reg. No.: 22BPS1179

Final Assessment Test (FAT) – January/February 2023

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	CALCULUS	Course Code	BMAT101L
Faculty Name	Prof. Radha S	Slot	A1+TA1
		Class Nbr	CH2022231700297
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

1. Determine the volume of the solid obtained by rotating the region bounded by $y = x^{\frac{1}{3}}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y axis. [10]

2. a) Determine by integration the area bounded by the three lines $y = 4 - x$, $y = 3x$ and $3y = x$. [5 marks] [10]
   b) If $u + v = e^x \cos y$ and $u - v = e^x \sin y$ then find the Jacobian of the function u and v with respect to x and y. [5 marks]

3. Consider the function $f(x, y)$ of two variables x and y defined as $f(x, y) = -\frac{xy}{x^2+y^2}$. Find the limit along the following curves as $(x, y) \to (0, 0)$. [10]
   a) the x-axis. b) the y-axis. c) the line $y = x$. d) the line $y = -x$. e) the parabola $y = x^2$.

4. A toy manufactures estimates a production function to be $f(x, y) = 100x^{\frac{3}{4}}y^{\frac{1}{4}}$, where x represents the units of labour (at Rs. 150 per unit) and y represents the units of capital (at Rs. 250 per unit). The total cost of labour and capital to Rs. 50,000. Find the maximum production level for this manufacturer. [10]

5. a) Obtain the local maximum and local minimum values of the function $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$. [5 marks] [10]
   b) Evaluate the integral $\int_0^1 x^{q-1}\left(\log \frac{1}{x}\right)^{p-1}dx$ by using Beta, Gamma functions. [5 marks]

6. Sketch the region of integration and evaluate by changing to polar coordinates $\int_0^1 \int_{x^2}^{x} (x^2 + y^2)\,dy\,dx$. [10]

7. Changing to cylindrical coordinates, find the volume of the portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$. [10]

8. Evaluate the integral $\int_0^1 \frac{x^2}{\sqrt{1-x^4}}dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}}dx$ by using Beta, Gamma functions. [10]

9. Find the points on the surface defined by $x^2 + 2y^2 + 3z^2 = 1$, where the tangent plane is parallel to the plane defined by $3x - y + 3z = 1$. [10]

10. a) Find ∇r^n, where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$. [5 marks] [10]
    b) Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $v = 3\vec{i} - 4\vec{j}$. [5 marks]

11. Verify Stoke's theorem for $F = (x - 2y)\vec{i} - 3yz^2\vec{j} - 3y^2z\vec{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on $z = 0$ plane. [10]

12. Show that $F = (2xy - z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the scalar potential and the work done by F in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$ [10]

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VIT

Reg. No.: 22BAI1303

Final Assessment Test (FAT) – January/February 2023

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	CALCULUS	Course Code	BMAT101L
Faculty Name	Prof. Sowndar Rajan P T	Slot	A2+TA2
		Class Nbr	CH2022231700443
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

1. Discuss the decreasing and increasing interval and concavity of the given function and also find the extreme values $y = \frac{9}{14}x^{\frac{1}{3}}(x^2 - 7)$. [10]

2. (a) The solid lies between planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross-section perpendicular to the x-axis are circular disk whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find the volume of the solid. [5] [10]

   (b) Find the limit of the function $f(x, y) = x \sin\frac{1}{x} + y \sin\frac{1}{y}$ at the origin (0,0). Can you redefine the same function $f(x, y)$ such that it is continuous at (0,0)? [5]

3. If $u = r^m$ where $r^2 = x^2 + y^2 + z^2$, then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ is $f(r)$, where $f(r)$ is a function of r. [10]

4. Consider the function $f(x, y) = x^2 + y^2 + 2xy - x - y + 1$ over the square $0 \leq x \leq 1$ and $0 \leq y \leq 1$. [10]

   (i) Show that f has an absolute minimum along the line segment $2x + 2y = 1$ in this square. What is the absolute minimum value?

   (ii) Find the absolute maximum value of f over the square.

5. (a) The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant? [5] [10]

   (b) Prove that $\int_0^\infty \frac{x^{(q-1)}}{(1+x)} dx = \Gamma(q)\Gamma(1-q)$. [5]

6. (a) Sketch the region of integration and evaluate $\int_0^{\ln 10} \int_{e^x}^{10} \frac{1}{\ln y} dy dx$. [5] [10]

   (b) Evaluate the following integration after converting it from rectangular coordinates to spherical coordinates. $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dy dx$. [5]

7. Determine the radius of a hole, which is drilled vertically through the center of the solid bounded by the surfaces: $z = 25e^{\frac{-(x^2+y^2)}{4}}$, xy-plane and $x^2 + y^2 = 16$, which reduces the total volume to its one-tenth. **(Note: To solve this problem it is not needed to draw this solid).** [10]

8. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, and C. Apply Dirichlet's integral to find the volume of the tetrahedron OABC. [10]

9. Find the directional derivative of $\nabla \cdot (\nabla f)$ at the point (1,-2,1) in the direction of normal to the surface $xy^2z = 3x + z^2$, where $f(x, y, z) = 2x^3y^2z^4$. [10]

10. (a) Find $curl(curl\overrightarrow{A})$, where $\overrightarrow{A}= x^2y\hat{i}$ -$2xz\hat{j}$+$2yz\hat{k}$ at the point $(1, 0, 2)$. [6] [10]

(b) The temperature of the points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(1, 1, 2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move? [4]

11. (a) Using Green's Theorem, evaluate $\int_C [\frac{1}{y}dx + \frac{1}{x}dy]$ where C is the boundary of the region bounded by the parabola $y = \sqrt{x}$ and the lines $x = 1, x = 4, y = 1$. [5] [10]

(b) Find the area of a region bounded by the parabola $y = x^2$ and the line $y = x + 2$ by expressing the area as a line integral. [5]

12. (a) Use Stoke's Theorem to compute the surface integral $\int\int_S \nabla \times \overrightarrow{F} \cdot d\overrightarrow{S}$, where $\overrightarrow{F} = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$ and S is the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z = 2$. [5] [10]

(b) Use Divergence Theorem to evaluate the surface integral $\int\int_S \overrightarrow{F} \cdot d\overrightarrow{S}$, where $\overrightarrow{F} = xy^2\hat{i} + x^2y\hat{j} + y\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$, bounded by the planes $z = 1$ and $z = -1$, and including the portions $x^2 + y^2 \leq 1$ when $z = \pm 1$. [5]

VIT

Reg. No. : 22BEE1136

Final Assessment Test (FAT) – January/February 2023

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	CALCULUS	Course Code	BMAT101L
Faculty Name	Prof. SANDIP	Slot	B1+TB1
		Class Nbr	CH2022231700484
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)

Answer any 10 questions

1. Find the area between the circle $x^2 + y^2 = 2ax$ and parabola $y^2 = ax$. **[10]**

2. (a) If $0 < a < b < 1$, then prove that, $\frac{(b-a)}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{(b-a)}{1+a^2}$, hence show that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\left(\frac{4}{3}\right) < \frac{\pi}{4} + \frac{1}{6}$. (5 Marks) **[10]**

   (b) Find $\frac{\partial z}{\partial x}$, if $yz - \ln z = x + y$, where z is a function of two independent variables x&y and prove that the partial derivative exists. (5 Marks)

3. Show that the function defined by $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is continuous at every point except the origin. **[10]**

4. Find the points on the surface $x^2 - zy = 4$ closest to the origin. **[10]**

5. (a) Using Taylor's Formula for $f(x,y)$ at the origin, find the cubic approximation of $f(x,y) = \frac{1}{1-x-y}$. (5 Marks) **[10]**

   (b) Evaluate $\int_0^\infty \frac{x^a}{a^x}\,dx$ $(a > 0)$. (5 Marks)

6. (a) Find the value of integral $I = \int_0^8 \int_{y^{\frac{1}{3}}}^2 \sqrt{x^4+1}\,dx\,dy$ by changing the order of integration. (5 Marks) **[10]**

   (b) Find the volume of the solid region bounded by the paraboloid $az = x^2 + y^2$ and cylinder $x^2 + y^2 = b^2$. (5 Marks)

7. Evaluate $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} 21xy^2\,dz\,dy\,dx.$ **[10]**

8. (a) Obtain the area of the ellipse with semi-major axis a and semi-minor axis as b respectively using gamma function. (4 Marks) **[10]**

   (b) Evaluate the Dirichlet integral $\iiint_D x^{l-1}y^{m-1}z^{n-1}\,dx\,dy\,dz$, where D is the region bounded by the first octant and $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1$. (6 Marks)

9. (a) Find the divergence of $r^n\,\vec{r}$, where $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ and $r = |\vec{r}|$. Also find the value of n for which $r^n\,\vec{r}$ is a solenoidal field. (4 Marks) **[10]**

   (b) Find the directional derivative of the scalar field $\phi = x^3 - 5x^2y - z$ at $P(1,1,1)$ in the direction of the line $\frac{x-1}{2} = \frac{y+1}{-2} = z$. In what directions does ϕ changes most rapidly at P and what are the rates of change in these directions. (6 Marks)

10. Prove the following vector identities **[10]**

(a) $div(curl(\vec{F})) = 0$ (3 Marks)

(b) $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ and $r = |\vec{r}|$. (7 Marks)

11. (a) Using Green's Theorem evaluate $\oint_C [(2xy - x^2)dx + (x^2 + y^2)dy]$ where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$. (5 Marks) [10]

(b) If $\vec{F} = (4xy - 3x^2z^2)\,\vec{i} + 2x^2\,\vec{j} - 2x^3z\,\vec{k}$, check whether the integral $\oint_C \vec{F} \cdot \vec{dr}$ is independent of the path C. (5 Marks)

12. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$ over the upper half surface $x^2 + y^2 + z^2 = 1$, bounded by its projections on the xy-plane. [10]

VIT

Reg. No. :

Final Assessment Test (FAT) – January/February 2023

Programme	B.Tech.	Semester	Fall Semester 2022-23
Course Title	CALCULUS	Course Code	BMAT101L
Faculty Name	Prof. Soumendu Roy	Slot	B2+TB2
		Class Nbr	CH2022231700547
Time	3 Hours	Max. Marks	100

Part A (10 X 10 Marks)
Answer any 10 questions

1. Find the area of the region in the first quadrant bounded by $y = 2\sqrt{x}$ and the lines $y = 2$ and $x = 4$. If the enclosed region is revolved about the line $y = 2$, then find the volume of the solid generated and also sketch the region. [10]

2. (a) Find the maximum value of $f(x) = \left(\frac{1}{x}\right)^x$. (5 Marks) [10]
   (b) If $f(x, y, z) = (xy)^{\sin z}$, then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial z}$. (5 Marks)

3. Let [10]

$$f(x) = \begin{cases} \frac{x^2 - y^2}{x + y} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Find $\left(\frac{\partial f}{\partial x}\right)_{(-2,1)}$, $\left(\frac{\partial f}{\partial y}\right)_{(0,1)}$, $\left(\frac{\partial f}{\partial x}\right)_{(0,0)}$ and $\left(\frac{\partial f}{\partial y}\right)_{(0,0)}$.

4. Find the critical points of $f(x, y) = 10xye^{-(x^2+y^2)}$ and classify them. [10]

5. (a) Find the maximum and minimum of $f(x, y) = 5x - 3y$ subject to the constraint $x^2 + y^2 = 136$. (5 Marks) [10]
   (b) Evaluate $\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$ using Beta and Gamma function. (5 Marks)

6. Evaluate the integral $\displaystyle\int_0^{4a}\int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}dxdy$ by transforming to polar coordinates. [10]

7. Evaluate the integral $\displaystyle\iiint_W \frac{dxdydz}{\sqrt{1 + x^2 + y^2 + z^2}}$ by transforming to spherical coordinates, where W is the ball $x^2 + y^2 + z^2 \leq 1$. [10]

8. Evaluate $\displaystyle\iiint_V x^{p-1}y^{q-1}z^{r-1}dxdydz$, where V denotes the closed region bounded by coordinate planes and the plane $x + y + z \leq 1$ in terms of Gamma function. [10]

9. (a) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, prove that (i) $\vec{\nabla} r = \frac{\vec{r}}{r}$ (ii) $\vec{\nabla} r^n = nr^{n-2}\vec{r}$ (6 Marks) [10]
   (b) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of the vector $2\vec{i} - \vec{j} - 2\vec{k}$. (4 Marks)

10. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is irrotational and solenoidal. Hence find the scalar potential ϕ. [10]

11. Verify Green's theorem for $\displaystyle\int_C (x + 2y^2)dx + 3xy^2dy$ where C is the boundary of the square with vertices $(-3, -2)$, $(1, -2)$, $(1, 2)$ and $(-3, 2)$. [10]

12. Evaluate $\iint_S xz^2 dydz + (x^2y - z^3)dzdx + (2xy + y^2z)dxdy$ where S is the surface of hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$. [10]

Final Assessment Test (FAT) – January/February 2023

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**CALCULUS**	Course Code	**BMAT101L**
Faculty Name	**Prof. PROSENJIT**	Slot	**E1+TE1**
		Class Nbr	**CH2022231700196**
Time	**3 Hours**	Max. Marks	**100**

Section-A (10 X 10 Marks)
Answer any 10 questions

1. For the given function $f(x) = (x^2 - 5x + 7)e^x$, find the **[10]**
   (a) critical points and classify them,
   (b) intervals where the function is increasing and decreasing,
   (c) intervals where the function is concave up and down.

2. (a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2$ and the line $x = 2$ about the axis $x = 2$. [5 marks] **[10]**
   (b) If $x + y + z = u$, $y + z = uv$, $z = uvw$, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. [5 marks]

3. Let $y = F(x, t)$, where F is a differentiable function of two independent variables x and t which are related to two variables u, v by $u = x + ct$, $v = x - ct$. Prove that **[10]**
   $\frac{\partial^2 y}{\partial u \partial v} = \frac{1}{4}\left[\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}\right]$.

4. Find the absolute minimum and maximum of $f(x, y) = x^2 - y^2 + xy - 5x$ on the region bounded by $y = 5 - x^2$ and the x-axis. **[10]**

5. (a) Find a quadratic approximation to $f(x, y) = \cos x \cos y$ near the origin. How accurate is the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$? [5 marks] **[10]**
   (b) Evaluate $\int_0^1 \frac{dx}{\sqrt[3]{x^2 - x^3}}$. [5 marks]

6. Change the order of integral and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$. **[10]**

7. Using double integration, find the area enclosed by the curves $y = 2x^2$ and $y^2 = 4x$. **[10]**

8. Prove that $\iint_R x^{l-1} y^{m-1}\, dx\, dy = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m+1)} a^{l+m}$, $a > 0$. R is the region bounded by x-axis, y-axis and the line $x + y = a$. **[10]**

9. For the function given by $f(x, y, z) = x^2 + y^2 + z^2$, find ∇f and its value at $P(2, 2, -1)$. Also, find the directional derivative of $f(x, y, z)$ at P in the direction of $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$. **[10]**

10. A fluid motion is given by $\vec{v} = (y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$. Show that the motion is irrotational and hence find the velocity potential. **[10]**

11. Verify Gauss divergence theorem for $\vec{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ taken over the region bounded by the planes $x = -1$, $x = 1$, $y = -1$, $y = 1$, $z = -1$ and $z = 1$. **[10]**

12. Verify Green's theorem for $\int_C (x^2 - y^2)\, dx + 2xy\, dy$, where C is the boundary of the area between $y = x^2$, $y = x$. **[10]**

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VIT

Vellore Institute of Technology

Final Assessment Test – Jan / Feb 2023

Course: BMAT101L - Calculus

Class NBR(s):5425/5477/5667/5701/6420 Slot: A1+TA1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACT

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. For the function $f(x)=2x^3-14x^2+22x-6$,
   i) Find the intervals on which f is increasing and on which f is decreasing.
   ii) Find the local maxima and local minima using first derivative test.
   iii) Identify the intervals where the function is concave up and concave down, hence find the points of inflection.

2. a) Find the area of the region enclosed by the curve $y=x^2-2$ and the line $y=2$. [5]

   b) Find the volume of the solid when the region enclosed by $y=\sqrt{x+1}$, $y=2\sqrt{x}$, $x=\frac{1}{3}$ and $x=1$ is revolved about x-axis. [5]

3. a) Given $u=\frac{x^2-y^2}{x^2+y^2}$ and $v=\frac{2xy}{x^2+y^2}$. Test whether u and v are functionally dependent. If so, state the relationship between them. [5]

   b) Investigate the continuity of the function [5]

$$f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}; & (x,y)\neq(0,0)\\ 0 \quad ; & (x,y)=(0,0)\end{cases}$$

4. Expand $f(x,y)=xe^y+\sin x\cos y$ as a Taylor's series in powers of x and y up to second degree terms.

5. Find the shortest and the longest distances from the point (1, 2, -1) to the sphere $x^2+y^2+z^2=24$.

6. Change the order of integration and evaluate $\int_0^a\int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{y^4-a^2x^2}}dydx$.

7. Using cylindrical coordinates find the volume intercepted between the paraboloid $x^2+y^2=2az$ and the cylinder $x^2+y^2-2ax=0$

8. a) Evaluate $\int_0^1 \frac{30x^2}{32\sqrt{1-x}}dx$ using Beta and Gamma functions. [5]

   b) Evaluate $\int_0^2 (8-x^3)^{\frac{-1}{3}}\,dx$ using Beta and Gamma functions. [5]

9. a) If $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$, find $div\vec{F}$. [5]

b) If $\vec{A} = 3xz^2\vec{i} - yz\vec{j} + (x+2z)\vec{k}$ find $curl(curl\vec{A})$. [5]

10. A fluid motion is given by
$\vec{F} = (e^x \cos y + yz)\vec{i} + (xz - e^x \sin y)\vec{j} + (xy + z)\vec{k}$. Show that the motion is irrotational and hence find its scalar potential ϕ.

11. Using Stokes' theorem evaluate $\int_C (x+y)dx + (2x-z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices (2,0,0); (0,3,0) and (0,0,6).

12. If $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ using Gauss divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}ds$ where S is the closed surface bounded by the planes $z = 0, z = 1$ and the cylinder $x^2 + y^2 = 4$.

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