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Vellore Institute of Technology

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CHENNAI

Continuous Assessment Test– January 2023

Programme	:	B.Tech.	Semester	:	Winter 2022-23
Course	:	Discrete Mathematics and Graph Theory	Code	:	BMAT205L
Faculty	:	Dr. Vidhya V, Dr. Durga Nagarajan, Dr. Sandip Dalui and Dr. Jayagopal R	Class ID's	:	CH2022235001875 CH2022235001876 CH2022235001877 CH2022235001878
			Slot		D2+TD2+TDD2
Duration	:	90 minutes	Max. Marks	:	50

Answer all the questions ($5 \times 10 = 50\ Marks$)

Q. No.	Question Description	Marks
1.	a) Let $P(x)$ the statement "x is a professional athlete" and $Q(x)$ denote the statement "x plays soccer". Write each of the following propositions in English sentences. (i) $\forall x\big(P(x) \to Q(x)\big)$ (ii) $\exists x\big(P(x) \wedge Q(x)\big)$ (iii) $\forall x\big(P(x) \wedge Q(x)\big)$ Also, write the negations of each of the above propositions both in symbols and words.	6
	b) Verify that premises $\forall x\big(P(x) \to Q(x)\big)$ and $\exists y\big(P(y)\big)$ leads to the conclusion $\exists z Q(z)$.	4
2.	a) Using algebra of logics find the PCNF and PDNF of the following proposition: $$\neg[(q \to p) \vee \neg(q \vee \neg r)] \vee (q \vee r)$$	7
	b) Write the contrapositive of the following proposition symbolically and in words: "If the network is down, then Rajiv cannot access internet"	3
3.	a) Derive the conclusion $\neg P$ from the premises $(R \to \neg Q) \wedge (R \vee S), (S \to \neg Q)\ and\ P \to Q$	4
	b) Show that these statements are inconsistent: "If Sergei takes the job offer then he will get a signing bonus." "If Sergei takes the job offer, then he will receive a higher salary." "If Sergei gets a signing bonus, then he will not receive a higher salary." "Sergei takes the job offer."	6
4.	a) Let $\mathbb{Q}$ be the set of all rational numbers. Show that $(\mathbb{Q} - \{1\},\ *)$ forms an Abelian group under the operation $*$ such that $a * b = a + b - ab,\ \forall\, a, b \in \mathbb{Q} - \{1\}$.	5
	b) Let $S = N \times N$ be a set of ordered pairs of positive integers and $*$ be an operation that is defined on the set S by $(a, b) * (c, d) = (ac, bd)$, show that $(S, *)$ is a semigroup. If $f:(S, *) \to (Q, \times)$ is defined by $f(a, b) = a/b$, show that f is a homomorphism.	5

5. a) If $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$. Solve the equation $AX = B$.　　　3

mod2　b) Given the generator matrix $G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ corresponding to the encoding

function $e: B^3 \rightarrow B^6$. Find the corresponding parity check matrix and use it to decode
the following received words and hence find the original message.　　　7
(i) 000111; (ii) 100101 and (iii) 111111

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CAT- I May 2023

Programme	: **B. Tech**	Semester	: **FALL Inter Semester 2022 – 23**
Course	: **Discrete Mathematics and Graph Theory**	Code	: **BMAT205L**
Faculty	: Dr. Kalyan Manna, Dr. Avinash Kumar Mittal, Dr. Durga Nagrajan, Dr. Vidhya V, Dr. Devi Yamini S, Dr. Om Namah Shivay, Dr. Uma Maheswari S, Dr. Rajesh Kumar Mohapatra, Dr. Manigandla Prasannalakshmi, Dr. Sandip Dalui, Dr. Pulak Konar, Dr. Surath Ghosh,Dr. Lakshmanan S	Slot(s) Class No.	: **C1+TC1+TCC1** CH2022232500280 - CH2022232500287, CH2022232500292 - CH2022232500297
Time	: **90 Mins**	Max. Marks	: **50**

Answer **ALL** the Questions

1. (a) Symbolize the statements using proposition and/or predicates. (2)
   i) All apples need not be red. Some apples are green.
   ii) Either he goes to movie or to hotel never to temple.

   (b) Show that $[(a \rightarrow b) \wedge (c \rightarrow d)] \wedge [(b \rightarrow e) \wedge (d \rightarrow f)] \wedge [\neg (e \wedge f) \wedge (a \rightarrow c)] \rightarrow (\neg a)$ is a tautology. (Without truth table) (8)

2. (a) Translate the following sentence into predicate logic: "Some student in this class has taken a course in Java" for (i) the domain is the whole class (ii) the domain is all the people. (4)

   (b) Use the indirect method to prove that the conclusion $\exists z\ Q(z)$ follows from the premises $\forall x\ (P(x) \rightarrow Q(x))$ and $\exists y\ P(y)$. (6)

3. Without using truth tables, find the PDNF and DNF of the following logical expression: (10)

$$[p \wedge (p \rightarrow q)] \rightarrow q.$$

(Note that DNF should not be the same expression as PDNF).

4. (a) If $S = \{1,2,3,6\}$ and $*$ is defined by $a * b = lcm\,(a,b)$, where $a, b \in S$. Show that $\{S,*\}$ is a monoid. (4)

(b) Prove that $Z_6 = \{0,1,2,3,4,5\}$ is a group under addition modulo 6. Find all the subgroups of Z_6. (6)

5. (a) Let $H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$ be the parity check matrix corresponding to the (3+2)

encoding function $e: B^3 \rightarrow B^6$.
   (i)   Find the code words generated by H.
   (ii)   Decode the received words (a) 111000 and (b) 001110.

(b) Let $(R, +)$ and $(R^+, \times)$ be two groups. Show that the mapping $f: R \rightarrow R^+$ (5)
defined by $f(x) = 2^x\ \forall\ x \in R$ is a homomorphism.