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Continuous Assessment Test (CAT)- I– September 2022

Programme	: B.Tech.	Semester	: Fall 2022-2023
Course Title	: Complex Variables and Linear Algebra	Code	: BMAT201L
		Slot	: A1+TA1+TAA1
Faculty	: Dr. M. Dhivya, Dr. N. Durga, Dr. M. Prasannalakshmi, Dr. C. Rajivganthi, Dr. Ashish Kumar, Dr. Prosenjit, Dr. Ankit Kumar, Dr. Surath Ghosh, Dr. Sandip Saha, Dr. Kamalesh, Dr. P. Sushmitha, Dr. Amitkumar Rahul	Class Nbr	: CH2022231001157, 60, 62, 64,67, 70, 72, 74, 76, 80, 82
Duration	: 1 ½ Hours	Max. Marks	: 50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	Prove that the function $u = e^x(x\cos y - y\sin y)$ is harmonic. Also find a function v such that $f(z) = u + iv$ is analytic and express $f(z)$ in terms of z.	[10]
2.	Show that the function $f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfies the Cauchy Riemann equations at origin but not analytic at $z = 0$.	[10]
3.	In a two-dimensional fluid flow, if $\psi(x, y) = x^2 - y^2 - 3x - 2y + 2xy$ can represent the stream function, find the velocity potential and complex potential.	[5]
4.	a) Find the points for which the following function is not conformal $f(z) = z^2 + \frac{1}{z^2}$	
	b) Find the image of the circle $\|z\| = 2$ under the transformation $w=(\sqrt{2}\, e^{\frac{i\pi}{4}})z$.	[5]
5.	Find the image of the wedge $\|z\| \leq \frac{1}{2}$, $\frac{-\pi}{8} < arg(z) < \frac{\pi}{8}$ under the mappings (i) $w = z^2$ *and* (ii) $w = iz$. Sketch the region of images.	[10]
6.	Determine the bilinear transformation which maps the points $z = 0, -i, 2i$ into the points $w = 5i, \infty, \frac{-i}{3}$ respectively. List the invariant points of this transformation. Sketch the image of $\|z - i\| < 1$ under the obtained transformation.	[10]

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$x^n = nx^{n-1}$

Continuous Assessment Test (CAT) - I – June 2023

Programme	:	B.Tech.	Semester	:	Fall Inter Semester 2022-23
			Code	:	BMAT201L
Course Title	:	Complex Variables and Linear Algebra	Slot	:	X11+X12+X21+Z21
Faculty	:	Dr. Kriti Arya	Class Nbr	:	CH2022232501343
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (5×10=50)

Q.No.	Sub. Sec.	Question Description	Marks
1.	(a)	Determine whether $\frac{1}{z}$ is analytic or not?	[5]
	(b)	Show that the function defined by $f(z) = \sqrt{\lvert xy \rvert}$ satisfies Cauchy-Riemann equation at the origin but is not analytic at that point.	[5]
2.		Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic. Also, express $f(z)$ in terms of z.	[10]
3.	(a)	If $w = \theta + i\varphi$ represents the complex potential for an electric field and $$\varphi = x^2 - y^2 + \frac{x}{x^2 + y^2},$$ determine the function θ.	[5]
	(b)	Transform the rectangular region ABCD in the z-plane bounded by $x = 1, x = 3; y = 0, and\ y = 3$, under the transformation $w = z + (2 + i)$.	[5]
4.	(a)	Show that under the transformation $w = \frac{1}{z}$, the image of the hyperbola $x^2 - y^2 = 1$ is the lemniscate $R^2 = cos2\varphi$.	[5]
	(b)	Find a bilinear transformation which maps the points $i, -i, 1$ of the z-plane into $0, 1, \infty$ of the w-plane, respectively.	[5]
5.	(a)	Find the radius of convergence of the power series: $$f(z) = \sum_{n=0}^{\infty} \frac{n!}{n^n} z^n.$$	[5]
	(b)	Find the terms in the Laurent expansion of $\frac{1}{z(e^z - 1)}$ for the region $0 < \lvert z \rvert < 2\pi$. Or Expand the function $f(z) = tan^{-1}z$ in powers of z.	[5]

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Continous Assessment Test I (CAT-1) – May 2023

Programme	: B.tech		Semester	:	Fall Inter Semster2022-23
Course Code	: BMAT 201L		Class Nbr(s)	:	CH20222325006 03,601,602,567
Course Title	: Complex Variables and Linear Algebra				
Faculty(s)	: Dr. Pankaj Shukla,Dr Prabhakar V, Dr. RajivGandhi, Dr Dhivya P		Slot	:	D1+TD1+TDD1
Time	: One and half Hours		Max. Marks	:	50

Answer all the Questions

1.		Show that the function f(z) defined by $f(z) = \frac{xy(y-ix)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$ is not analytic at the origin, though it satisfies Cauchy – Reimann equations at the origin.	10
2.		If $\varphi = (x-y)(x^2 + 4xy + y^2)$ can represent the equipotential for an electric field, find the corresponding complex potential $w = \varphi + i\psi$ and also ψ, if possible.	10
3.		Find the image of the following regions under the transformation $w = \frac{1}{z}$: (i) The half plane x>c, when c<0 (ii) The half plane y>c, when c<0 (iii) The infinite strip $\frac{1}{4} \leq y \leq \frac{1}{2}$.	10
4.		Show that the transformation $w = \frac{z-1}{z+1}$ maps the unit circle in the w – plane onto the imaginary axis in the z – plane. Find also the images of the interior and exterior of the unit circle in the z-plane.	10
5.		Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the regions: (i) $\|z+1\| < 1$, (ii) $1 < \|z+1\| < 2$ (iii) $\|z+1\| > 2$.	10

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Continuous Assessment Test (CAT)- I– May 2023

Programme	:	B.Tech.	Semester	:	Fall Inter2022-23
Course Title	:	Complex Variables and Linear Algebra	Code	:	BMAT201L
			Slot	:	D2+TD2+TDD2
Faculty	:	Dr. P. Durgaprasad, Dr. Tharasi Dilleswar Rao, Dr. C. Rajivganthi, Dr. Pankaj Shukla, Dr. B. Jaganathan	Class Nbr	:	CH2022232500604, 05, 06, 07, 09
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	a) Determine all the points where the given function $\frac{\bar{z}+1}{\lvert z+1\rvert^2}$ is differentiable b) For what values of a and b the given $v=(x-1)^3-axy^2+by^2$ is harmonic and find its harmonic conjugate.	[4+6]
2.	Find the potential and stream functions of a complex potential function, when $\phi-\psi=(x-y)(x^2+y^2+3xy)$.	[10]
3.	Show that the transformation $w=\frac{z-i}{1-iz}$ maps (i) the interior of the circle $\lvert z\rvert = 1$ onto the lower half of the w – plane and (ii) the upper half of the z – plane onto the interior of the circle $\lvert w\rvert = 1$.	[10]
4.	a) Find the bilinear transformation whose fixed points are ½ and 2. b) Find the bilinear transformation that maps the points i, $-i$, 1 into the points 0, 1, ∞, respectively.	[5+5]
5.	Find the Laurent series expansion for the function $f(z)=\frac{7z-2}{(z+1)z(z-2)}$ in the region i) $1<\lvert z\rvert<2$, ii) $1<\lvert z+1\rvert<3$.	[10]

Answer all the Questions
(5x10 = 50 marks)

Q.No.	Sub. Sec.	Question Description	Marks
1		Find the constant $'a'$ so that $u(x,y) = ax^2 - y^2 + xy$ is harmonic. Find an analytic function $f(z)$ for which u is the real part. Also find its harmonic conjugate.	10
2		(a) In a two-dimensional fluid flow, the velocity potential $\phi(x,y)$ is given as $4x(3y-4)$. Find the complex potential w= $\phi(x,y) + i\,\psi(x,y)$ where ψ is the stream function.	10
		(b) Verify whether $f(z) = \frac{1}{z-1}$ is analytic at $z = 1+i$.	[5 +5]
3		Determine the bilinear transformation which maps the points $z = 1, -1, \infty$ into the points $w = 1+i, 1-i, 1$ respectively. Find the image of the unit circle $\|z\| = 1$ under this transformation and sketch the image.	10
4		(a) Find the points where $w = e^{\cos hz}$ is not conformal.	10
		(b) Find the image of the circle $\|z-1\| = 1$ under the transformation $w = (\frac{1}{2}e^{i\frac{\pi}{2}})z$.	[5 +5]
5		(a) Find the Taylor series expansion of the given function $f(z) = \frac{z^3+1}{(z-3)(z-5)}$ about $z = 2$. Also, discuss the radius of convergence.	10
		(b) Find the poles and residues of the given function $f(z) = \frac{1}{z^4+5z^2+6}$.	[5 +5]

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Continuous Assessment Test (CAT) – 1 – September 2023

Programme	:	B.Tech	Semester	:	Fall 2023-24
Course Title	:	Complex Variables and Linear Algebra	Code	:	BMAT201L
			Slot(s)	:	A2+TA2+TAA2
Faculty	:	Dr. S. Balaji, Dr. Ashish Kumar Nandi, Dr. N. Mohana, Dr. G. Hannah Grace, Dr. Abhishek Kumar Singh, Dr. Prosenjit paul, Dr. P. Vijay Kumar, Dr. M Dhivya	Class Nbr(s)	:	CH2023240101009, 1010, 1011, 1012, 1013, 1014, 1015, 1017
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (5×10=50)

Q.No.	Sub. Sec.	Question Description	Marks
1		Check whether $u = x^2 - y^2$ and $v = \frac{-y}{x^2+y^2}$ are harmonic. Also verify if $u + iv$ is analytic. Justify?	10
2		Find the analytic function $f(z)$ given that $2u + 3v = e^x(\cos y - \sin y)$.	10
3		(a) Find all the points where the mapping $f(z) = \sin z$ is conformal. **[3 Marks]** (b) Find the image of the triangle with vertices 0, $1 + i$ and $1 - i$ under the mapping $w = z^2$. **[7 Marks]**	10
4		Find the bilinear transformation $w = f(z)$ which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Also, find the fixed point and image of $\|z\| \leq 1$ under the mapping $w = f(z)$.	10
5		(a) Using Laurent's series, expand the function $f(z) = \frac{2z+1}{z^3+z^2-2z}$ valid in the region $0 < \|z - 1\| < 1$. **[5 Marks]** (b) Find the poles and residues of the given function $f(z) = \frac{\sin z}{z \cos z}$ inside the circle $\|z\| = 5$. **[5 Marks]**	10

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