VIT

Vellore Institute of Technology

Continuous Assessment Test (CAT)- I– November 2022

Programme	: B.Tech.	Semester	: Fall 2022-2023
Course Title	: Calculus	Code	: BMAT101L
		Slot	: B2+TB2
Faculty	: Dr. Radha S Dr. Abhishek Kumar Singh Dr. Manivannan Dr. P. Vijay Kumar Dr. Ashish Kumar Dr. Kalyan Dr. Ashis Bera Dr. Surath Dr. Rajesh Kumar Mohaopatra Dr. Biswajit Mallick Dr. Soumendu Roy	Class Nbr	: CH2022231700265 CH2022231700549 CH2022231700553 CH2022231700269 CH2022231700551 CH2022231700555 CH2022231700619 CH2022231700263 CH2022231700271 CH2022231700261 CH2022231700547
Duration	: 1 ½ Hours	Max. Marks	: 50

Answer all the Questions (5×10=50)

Q.No.	Sub. Sec.	Question Description	Marks
1.	[a]	Verify Rolle's theorem for the function $f(x) = \log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a, b]$ and $0 \notin [a, b]$. Also find the value of 'c' for which $f'(c) = 0$.	[5]
	[b]	Show that the function $y = \log(1+x) - \frac{2x}{2+x}$ is an increasing function of x, when x is positive and deduce that $\log(1+x) > \frac{2x}{x+2}$	[5]
2.	[a]	Find the local maxima and local minima of the function defined by $f(x) = 3x^4 + 8x^3 - 18x^2 + 60$.	[7]
	[b]	Find the points of inflexion for the curve $y = \frac{x+1}{x^2+1}$.	[3]
3		Sketch the common region included between the curve $\frac{y+8}{x} = x - 2$ and the x-axis and find the volume of the solid generated by revolving the common area about the line $x + 5 = 0$.	[10]
4.		Show that $\lim_{(x,y)\to(0,0)} \frac{(x^3+y^3)}{\lvert x\rvert+\lvert y\rvert}$ exists by using $\varepsilon - \delta$ method.	[10]
5.	[a]	If $u = xyz, v = xy + yz + xz, w = x + y + z$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = (x-y)(y-z)(z-x)$	[5]
	[b]	If $u = x^2 + y^2 + z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as a total derivative.	[5]

⇔⇔⇔

Reg. No.:

Name :

VIT

Vellore Institute of Technology

Continuous Assessment Test (CAT)- I– October 2022

Programme	:	B.Tech.	Semester	:	Fall 2022-2023
Course Title	:	Calculus	Code	:	BMAT101L
			Slot	:	A2+TA2
Faculty	:	Dr. Balamurugan, Dr. Saroj Kumar Dash, Dr. Mini Ghosh, Dr. Manimaran, Dr. Sowndarrajan, Dr. Prabhakar, Dr. Rajesh Kumar, Dr. Soumendu Roy	Class ID	:	CH2022231700410, 416,429,440,443,57 3,604,610
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	(a). Verify Rolle's theorem for the function $f(x) = x^3 - 7x^2 + 16x - 12$ in [2,3].	[5]
	(b). Let $f(x) = \frac{x^3}{4} - 3x, x \in \mathbb{R}$. Find the intervals on which $f(x)$ is increasing (or) decreasing.	[5]
2.	Find the maximum and minimum values of $f(x) = \lvert 4 - x^2 \rvert, x \in [-4,4]$. Also find the absolute maximum and absolute minimum, if they exist.	[10]
3.	Find the area of the region in the second quadrant that is bounded above by the curve $x^2 + y^2 = -2x$ and below by x-axis and the line $y = x + 1$.	[10]
4.	Find f_x, f_y, f_{xy} and f_{yx} at each point of the domain of $f(x,y)$, where $f(x,y) = \sqrt{x^2 + y^2}$.	[10]
5.	(a). The time period 'T' of a pendulum of length 'L' is: $T = \frac{2\pi\sqrt{L}}{\sqrt{g}}$, where 'g' is the acceleration due to gravity. A pendulum is moved from the "Canal Zone", where g=32.09 feet per second square, to "Greenland", where g=32.23 feet per second square. Because of the change in temperature, the length of the pendulum changes from 2.5 feet to 2.48 feet. Find the error in the time period of the pendulum.	[5]
	(b). If $u(x,y) = xy - (\sqrt{1-x^2})(\sqrt{1-y^2})$ and $v(x,y) = cos^{-1}(x) + cos^{-1}(y)$, then find the relation between 'u' and 'v', if there is any relation between them.	[5]

⇔⇔⇔

Reg. No.:

Name :

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test I – October 2022

Programme	: B.Tech	Semester	:	FALLSEM 2022-2
Course	: Calculus	Code	:	BMAT1011
Faculty	: Dr. R. Radha Dr. N. Nathiya Dr. Sowndarrajan P T Dr. Manoj Kumar Singh Dr. Harshavarthini Shanmugam Dr. Manimaran J	Slot	:	A1+TA1
		Class Number	:	CH2022231700297 CH2022231700423 CH2022231700424 CH2022231700298 CH2022231700617 CH2022231700608
Time	: 1½ hours	Max. Marks	:	50

Answer ALL the Questions (5 x 10 = 50 marks)

Q.No.	Sec	Question Description	Marks
1.	a.	Suppose that $f(x)$ is continuous and differentiable on the interval $[-2,2]$ such that $f(-2) = 3$ and $f'(x) \leq 4$. What is the largest possible value for $f(2)$?	5
	b.	Find the intervals in which the given function $f(x) = \frac{1}{2x^2+5}$ is increasing, decreasing, concave up and concave down.	5
2.		Find the dimensions of a right circular cylinder of maximum volume that can be inspired in a sphere of radius 10 cm. What is the maximum volume?	10
3.		Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the curve $y = x^2$, below by x-axis and on the right side by $x = 1$ about the line $x = -1$.	10
		Show that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous.	10
		If $x = u - y - z$, $y = uv - z$, $z = uvw$ and $u = \frac{x_2x_3}{x_1}$, $v = \frac{x_3x_1}{x_2}$, $w = \frac{x_1x_2}{x_3}$, find $\frac{\partial(x,y,z)}{\partial(x_1,x_2,x_3)}$.	10

VIT

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test (CAT)- I– October 2022

Programme	:	B.Tech.	Semester	:	Fall Se I 2022-
Course Title	:	Calculus	Code	:	BMAT
			Slot	:	E1+TE
Faculty	:	Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, Dr. Prosenjit, Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya	Class Nbr	:	CH2022 189, 191 194, 257,
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	a) Using Mean Value Theorem (MVT) prove that $0 < \frac{1}{x}\log\left(\frac{e^x-1}{x}\right) < 1$ for $x > 0$.	[5]
	b) Find the intervals on which the function $f(x) = 3x^2 - 4x^3, x \in \mathbb{R}$ is increasing or decreasing?	[5]
2.	Examine the extreme values of the function $f(x) = x^5 - 5x^4 + 5x^3 + 12, x \in \mathbb{R}$. Also find the intervals on which the function $f(x)$ is concave up and concave down.	[10]
3.	Find the volume of the solid formed by revolving the region enclosed by the parabola $y^2 = 4ax$ and the straight line $y = x$, (i) about x-axis, (ii) about y-axis.	[10]
4.	. Let $f(x, y) = (x^2 + y^2)^{2/3}$. Find f_x , f_y, f_{xy} and f_{yx} at each point in $\mathbb{R}^2$.	[10]
5.	a) The inductance L (in microhenrys) of a straight nonmagnetic wire in free space is: $L = 0.00021\left[ln\left(\frac{2h}{r}\right) - 0.75\right]$, where '$h$' is the length of the wire in the millimetre and 'r' is the radius of the circular cross section. Find the maximum possible error of L, when $r = 2 \pm \frac{1}{16}$ millimetres and $h = 100 \pm \frac{1}{100}$ millimetres.	[5]
	b) Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u = \cos x \cosh y$, $v = \sin x \cosh y$ and $w = \sinh z$.	[5]

⇔⇔⇔