VIT

Reg. No. : 22BCE1500

Final Assessment Test (FAT) - November/December 2023

Programme	**B.Tech.**	Semester	**FALL SEMESTER 2023 - 24**
Course Title	**THEORY OF COMPUTATION**	Course Code	**BCSE304L**
Faculty Name	**Prof. Suseela S**	Slot	**F1+TF1**
		Class Nbr	**CH2023240101110**
Time	**3 Hours**	Max. Marks	**100**

PART-A (10 X 10 Marks)

Answer all questions

01. Construct an equivalent Deterministic Finite Automata for the Non-Deterministic Finite Automata with null moves represented in Figure 1. [10]

M= ({1,2,3,4,5,6,7,8,9}, {a,b,e}, starting state 1, final states 8,9).

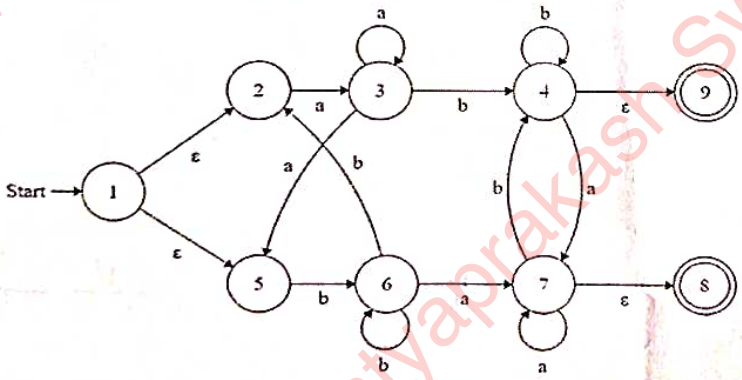

Figure 1

02. [10]

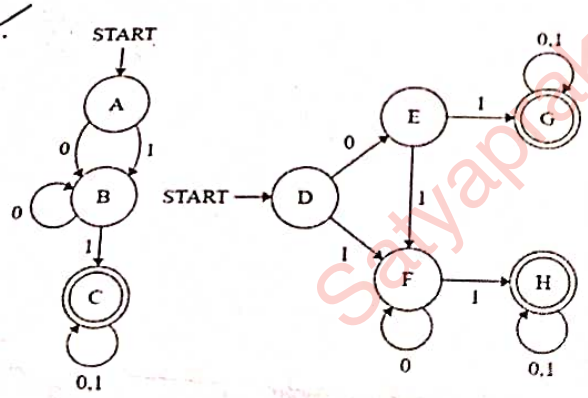

Figure 2: M1 **Figure 3: M2**

a) Using the equivalence method check whether the two given finite automata M1 & M2 are equal or not. (5 Marks)

b) Let $\sum = \{0,1\}$. Design regular expression to represent the language containing the set of all strings in $\sum^*$ having no more than three 0's or the strings with the number of 1's divisible by two. (2 Marks)

c) For the regular expression in **2(b)** design a Non-Deterministic Finite Automata. (3 Marks)

03. Construct the Context Free Grammar for the following language, [10]

a) L_1= {PQ | where P ∈{a,b}* and Q ∈{c,d}*, where P has an odd length of string and Q has an even length of string. (5 Marks)

b) L_2= {$a^i b^{2i} c^{2j} d^k e^{j+k}$, where i,j,k>0}. (5 Marks)

04. L= {ba $(ab)^{2n}$ ba $(ab)^n$ ba | n ≥1} [10]

Fit the language in Chomsky's class of hierarchy and design an automata to justify the same.

05. Check whether the following language is Context Free Language or not. [10]

a) L1 = {$a^n b^m c^n d^m$, where n, m>0}. (5 Marks)

b) L2 = {$a^n b^m$ | n > 0, m is prime}. (5 Marks)

06. a) Design a Deterministic Finite Automata, M for the language that accepts the numbers which are divisible by 3, where the numbers are taken in binary representation. (4 Marks) [10]

b) For the DFA designed in **6(a)** construct the regular expression R with proof. (6 Marks)

07. A girl collects three different shells from the seashore and arranges them in an order where blue colour comes first followed by red colour and finishes in green colour, the count of red colour shell and green colour shells may not be equal but the count of blue shells should be the total count of red and green shells. [10]

For the above given scenario construct a Context Free Grammar in Greibach Normal Form.

08. S→ABC [10]

A→aA | a

B→bB | b

C→cC | c

Construct the above grammar G into a suitable form and validate the string **aabbcc** using the CYK algorithm.

09. Is the language $L = \{a^{2m}b^{n}c^{m}d^{2n} \mid n, m>0\}$ decidable? Justify your answer. [10]

10. State whether the instances of the Post Correspondence Problem (PCP) have a solution. The following are the instances with $\sum=\{a,b\}$ [10]

Index	List A	List B
1	ab	ba
2	abb	bba
3	abb	ba
4	aaa	aa
5	ab	aba

In case the PCP has a solution, describe the post-correspondence solution with justification.

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Reg. No. : 22BCE1184

Final Assessment Test (FAT) - November/December 2023

Programme	B.Tech.	Semester	FALL SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Suseela S	Slot	F2+TF2
		Class Nbr	CH2023240101111
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)
Answer all questions

01. Design a finite automaton to recognize the language, L = {w | w ∈ {0, 1, 2} contains an odd number of 1's or contains the pattern "020"}. Check whether the string, "10202" is accepted by finite automaton or not. [10]

02. Construct an equivalent detererministic finite automaton for the non-deterministic finite automaton given below. [10]

State\Input	a	b	c	ε
→q1	-	q1	-	q2
q2	-	q3q5	-	q7
q3	q4	-	q4	-
q4	-	-	q2q7	-
q5	q6	q6	-	-
q6	-	-	q2q7	q2
q7	-	q8	-	-
*q8	-	-	-	-

03. Consider the context free grammar G: ({S, A, B, C}, {0,1,ε}, P, {S}) [10]

S->1AB|ε

A->1AC|0C

B->0S

C->1

a. Derive two strings from G. [3 marks]

b. Is the grammar ambiguous? Justify your answer. [4 marks]

c. Define the language for the given grammar G [3 marks]

04. Suppose A and B are two regular languages defined over the alphabet Σ={0,1}. Equal concatenation (EC) of A and B is defined as, EC(A,B) = {xy | xϵA, yϵB, |x| = |y|}. (Here |x| represents the number of elements in 'x' and |y| represents the number of elements in 'y'). Check whether EC(A,B) is regular or not? [10]

05. Construct an equivalent Greibach Normal Form (GNF) for the context-free grammar G = ({S, T, C, R, A}, {a, b, c, ε}, P, S) with the production sets given below. [10]

S → TC | AR

T → aTb | ε

$C \rightarrow Cc \mid \varepsilon$

$R \rightarrow bRc \mid C$

$A \rightarrow Aa \mid \varepsilon$

06. Hypertext markup language or HTML is a language used for web page designing. The programs of the language have the following structure. [10]

```
<HTML>
<HEAD>
    <TITLE>
    </TITLE >
</HEAD>
<BODY>
</BODY>
</HTML>
```

The tags within < > are called open tags and the tags within are called closing tags. Assume that the every tag has a closing tag. Generate the context free Grammar and design a pushdown automaton that can recognize programs that adhere to the HTML structure. Illustrate acceptance and rejection of a HTML structure through a sequence of stack operations

07. Construct a Push Down Automaton that accepts the language defined by the following grammar: [10]

$S \rightarrow aX \mid bY \mid a$

$X \rightarrow Xa \mid Y$

$Y \rightarrow 0 \mid 1$

Here, $V = \{S, X, Y\}$, $\sum = \{a, b, 0, 1\}$, and S is the start symbol.

08. Design a Turing machine to sort strings of 1s and 0s. The Turing machine starts scanning the leftmost symbol of an unbroken block of mixed 0s and 1s, and which halts after scanning the leftmost symbol of the block rearranged with all the 0s to the left of the 1s separated by #. [Hint: If Input String in the tape is 1010011 then, Output String should be 000#1111]. [10]

09. Design a Turing Machine to compute the function, [10]

$f(a,b) = b_1$ AND b_2

where b_1, b_2 are binary numbers of same length and 'AND' represents the logical AND operation.

b_1	b_2	b_1 AND b_2
0	0	0
0	1	0
1	0	0
1	1	1

10. a. Let L_1 be a regular language and L_2 be a context-free language. Is $L_1 \cap L_2$ regular? Justify your answer. [5 marks] [10]

b. Fit the language $L = \{uawb \mid u,w \in \{a,b\}^*, |u|=|w|\}$ in the Chomsky's hierarchy by design an appropriate grammar to generate L. [5 marks]

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. PADMA J	Slot	D2+TD2
		Class Nbr	CH2023240502748
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. (i) Let x be a string and let x^{rev} be the reverse of the string x. Prove that $(xy)^{rev} = y^{rev}x^{rev}$ for arbitrary strings x, y over an alphabet Σ using induction proof. (5 marks) [10]

(ii) If G is a Context Free Grammar in the Chomsky Normal Form (CNF), then for any string w belongs L(G) of length $n \geq 1$, it requires exactly 2n-1 steps to make any derivation of w. Prove the statement using appropriate mathematical proof method. (5 marks)

02. (i) Define a non deterministic finite automata to accept the language with set of strings over alphabet $\sum$={0, 1 , 2. . . .9} such that the final digit has appeared before. Clearly define the states, provide the transition table, and indicate the initial state, set of accepting states in the automata. (5 marks) [10]

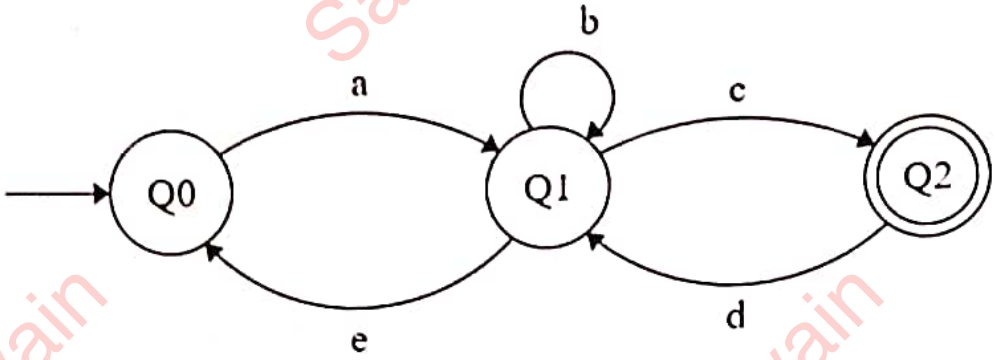
(ii) Convert the constructed non deterministic finite automata to deterministic finite automata. (5 marks)

03. Consider that you are developing a mobile application in which there is need to evaluate the strength of the password given by the user. The password must satisfy the following criteria: it should contain at least one uppercase letter, one lowercase letter, one digit, it should start with character and be at least eight characters long. [10]

(i) Design a non deterministic finite automata to validate passwords according to the given criteria. (5 marks)

(ii) Convert the NFA to DFA for the password validation and further the minimize the number of states in the resultant DFA. (5 marks)

04. (i) Transform the following deterministic finite automata to equivalent regular expression using state elimination method. (5 marks) [10]



(ii) Let $\sum = \{ 0, \# \}$ and let $L = \{ w \mid w = x_1\#x_2\#x_3\#...x_k$ for $k>=0$, $x_i \in 0^*$ and $x_i \neq x_j$ for $i \neq j \}$. Prove the language L is not regular using pumping lemma. (5 marks)

05. (i) Consider the following context free grammar with start symbol S: [10]

S -> expr
expr -> unary | binary | term
unary-> +expr | -expr
binary -> expr + expr | expr - expr
term -> a | b | c | d

Check whether the given grammar is ambiguous. (5 marks)

(ii) Determine whether the following grammar is context free using pumping lemma:
$L=\{a^n ww^R b^n : n \geq 1, w \in \{a,b\}^*\}$ (5 marks)

06. Consider the grammar G = (V, T, S, P), with V = {S, A, B, C}, T = {a, b}, S = S and the productions P defined by: [10]

S -> AB | BC
A -> BA | a
B -> CC | b
C -> AB | a

Use the CYK parsing algorithm to establish the membership or non-membership of w=abaaba in L(G).

07. (i) Develop a PDA capable of recognizing the below given context free language through acceptance by empty stack. (5 marks) [10]

$L=\{a^m b^n c^{2(m+n)} \mid m, n \geq 0 \}$

(ii) Use instantaneous description to trace the behavior of the constructed PDA for the string w=abbccccccc. (5 marks)

08. Consider the following CFG G=(V, ∑, P, S), where V={S, A, B}, ∑={a,b}, the start symbol is S and the production rules R are: [10]

S -> aSa | bSb | A | B | ε
A ->AB | aa
B ->BA | bb

Consider that the language defined by the above grammar is L(G), define a push down automata to recognize L(G).

09. (i) Construct a Turing machine that takes two non negative numbers val1, val2 as input and adds them in binary. The tape initially contains a $ followed by val1 and val2 in binary separated by #. The tape head is initially pointing the $ while in stage q_0. The Turing machine should halt once reaching state q_f with val1+val2 result stored in binary. (5 marks) [10]

(ii) Design a total Turing machine to accept the following language:

$L = \{\alpha \in \{a, b, c\}^* \mid \#a(\alpha) \leq \#b(\alpha) \leq \#c(\alpha)\}$.

Briefly describe the working of the machine (how the head moves and rewrites tape cells). You may use the string w=cbcabbcac for illustration. (5 marks)

10. Let $L_1, L_2, \ldots, L_k$ be a collection of languages over alphabet Σ such that: **[10]**

- For all $i \neq j$, $L_i \cap L_j = \emptyset$; i.e., no string is in two of the languages.
- $L_1 \cup L_2 \cup \ldots \cup L_k = \Sigma^*$; i.e., every string is in one of the languages.
- Each of the languages L_i, for $i = 1, 2, \ldots, k$ is recursively enumerable.

Prove that each of the languages is therefore recursive.

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Reg No : 22BCE3747

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Shivani Gupta	Slot	D1+TD1
		Class Nbr	CH2023240502746
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. A language is described by the regular expression: $(\Sigma^* a \Sigma^* a \cup bb)b^*$ over the $\Sigma = \{a, b\}$. [10]

a) Give a string that is in this language. [2 Marks]

b) Give a string that is NOT in this language. [2 Marks]

c) Give an Non Deterministic Finite Automata (NDFA) that recognizes this language. [6 Marks]

10

02. For the language A over alphabet $\Sigma = \{0, 1\}$ where A = {w | w starts with 0 and has odd length, or starts with 1 and has length at most 5}: [10]

a.) Draw an NFA M1 with null moves for the language L. [5 Marks]

b.) For the automata M1 construct an equivalent DFA M2. [5 Marks]

03. Consider a scenario of a bakery with three states: Bread(q0), Milk(q1) and Egg(q2) . The purchase cycles through these states using the following inputs: [10]

- a: transition from bread to milk.
- b: transition from milk to egg.
- c: transition from egg to milk.
- d: milk to bread.
- e: Milk to Milk (in Breakfast).

10

Now the scenario has to be programmed in the computer system which uses Regular Expression. Construct the Finite Automata for the above application. Convert the Finite Automata into its equivalent Regular Expression.

04. Use the CYK algorithm to determine whether or not the string $w = bbacb$ is a member of L(G), where G is the CNF grammar shown below: [10]

$S \rightarrow AB \mid b \mid c$

10

A→BB | AS | a
B→BA | b

05. A women purchased three different items from the X-Mart and arranges them in an order where paneer comes first followed by ghee and finishes in tomato. the count of ghee items and tomato may not be equal but the count of paneer items should be the total count of ghee and tomato items. [10]

For the above given scenario construct a Context Free Grammar in Greibach Normal Form (GNF).

06. According to the statement of the pumping lemma, every regular language has a pumping length p such that every string x ∈ L can be pumped if |x| ≥ p. The minimum pumping length is the smallest number p such that p is a pumping length for string A. For each of the following languages, what is the minimum pumping length p? Justify your answer, this means you must also explain why p-1 is not a pumping length for L. [10]

a. 110*11 [5 Marks]
b. (01* 0*) ∪ 1111 [5 Marks]

07 The language L over Σ = {0, 1}: {w | w is of odd length and contains at least two 0's.} [10]

a) Give a Context Free Grammar for the given language [5 Marks]
b) Design Push Down Automata for the above language. [5 Marks]

08 a) Simplify the grammar given below: [5 Marks] [10]
S → Aa | b | C
A → Ac | Sd | ε
C → CC

b) Is this grammar ambiguous? If so, prove it and construct a non-ambiguous grammar that derives the same language. [5 Marks]
S → aS | aAbB | ε
A → Ca | ε
B → b | ε

09. Design a Turing machine M that, given an input from {1}*, doubles the number of symbols and give the ID for the given M. For example, given input 111, the tape should contain 111111 when M halts. [10]

10. A queue automaton is like a PDA except that the stack is replaced by a queue. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a push) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a pull) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at anytime. Show that a language can be recognized by a deterministic queue automaton if and only if the language is Turing-recognizable. [10]

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Reg. No. : 22BCE5053

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Kavitha J C	Slot	C1+TC1
		Class Nbr	CH2023240503350
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. Consider the language, $L = L_1 L_2$ **[10]**

$L_1 = \{a^{2i} b^{2j} \mid i>0, j\geq 0\}$

$L_2 = \{c^{3k+1} d^2 \mid k>2\}$

a. Construct an automaton that accepts the language L. (5 marks)

b. Write a grammar that generates all strings in L. (5 marks)

02. Design a deterministic finite automaton to recognise the following language **[10]**

$L = \{ 0^{2m} + 1^{3n} + 2^k / m>0, n>0, k>2\}$, where $\Sigma = \{0, 1, 2, +\}$.

03. Write regular expression for each of the following languages over the alphabet $\sum = \{a,b\}$. **[10]**

i. The set of strings containing *ab* as a substring. (2.5 marks)

ii. The set of strings having at most one pair of consecutive *a's* and at most one pair of consecutive *b's*. (2.5 marks)

iii. The set of strings whose length is divisible by 6. (2.5 marks)

iv. The set of strings whose 5th symbol from the last is b. (2.5 marks)

04. In a school, every class has to be divided into 4 groups. **[10]**

Group A's strength should be greater than Group B, and the group C and D should have equal strength. Group A is followed by Group B, group B is followed by Group C and Group C is followed by Group D.

Compute a grammar for the above scenario in Chomsky Normal Form.

05. Design a Turing machine to compute the following function, **[10]**

$$f(n) = \begin{cases} n/2 & if \ \ n \bmod 2 = 0 \\ n+1 & otheriwise \end{cases}$$

where n is a positive integer, the *mod* is a function described as: **a *mod* b** is the remainder when a divides b. For example, 5 mod 2 is 1.

Validate the machine with a sample string.

06. Consider the following transition table where Q= {(1,2,3,4,5,6,7,8,9,10,11,12), ∑= (a,b,c),1 is the starting state and the final states are (4,7,10)}. Design an equivalent automaton with minimal states, [10]

δ (state, input)	a	b	c
→ 1	1	2	3
2	3	4	7
3	8	4	10
4*	1	2	Φ
5	8	10	3
6	9	7	2
7*	Φ	2	2
8	8	Φ	8
9	5	6	7
10*	5	8	Φ
11	12	7	8
12	5	8	9

07. Construct a Pushdown Automata for the following language: [10]
$L=\{a^{2p}\, b^{p+2q}\, c^{q+2} \mid p,q>0\}$. (6 marks)
Choose a valid string of minimum length 8 and justify. (4 marks)

08. a) Let $L_1= \{0^n w 00011 w^R\, 1^n \mid w \in \{0,1\}^*\}$. Check whether L_1 is regular or not? Justify your answer. (5 marks) [10]

b) Let $L_2= \{a^{2i+j}\, b^{2j} \mid i>2j, j>0\}$. Check whether L_2 is context-free or not? Justify your answer with proof. (5 marks)

09. (a) Design a Turing Machine that recognizes the following language. [10]
$L_1= \{w \in \{a, b\}^* :$ Every a is immediately followed by b $\}$ (5 marks)
(b) Design a Turing Machine that recognizes the following regular expression
$(a+b)^*aba(ab+a)^*$ (5 marks)

10. a) Let $L=\{a^i\, b^j\, c^k \mid k>i, j>2k, i>0\}$. Compute the type for the given language L through Chomsky hierarchy and suggest the possible machine which can validate the members of the language L. (5 marks) [10]

b) State whether the instances with ∑={a,b} of the Post Correspondence Problem (PCP) have a solution. The following are the instances M = (ab, bab, bbaaa) and N = (a, ba, bab). In case the PCP has a solution, describe the post-correspondence solution with justification. (5 marks)

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Reg. No. : 22BAI1226

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Renuka Devi R	Slot	F1+TF1
		Class Nbr	CH2023240503054
Time	3 Hours	Max. Marks	100

General Instructions:

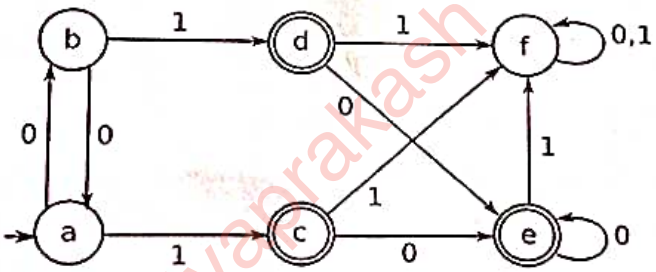
- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. (i) Prove that for every $n > 0$, $(5^n - 2^n)$ is divisible by 3. (5 Marks) [10]
(ii) Consider the following statement and prove it using structural induction " **For every tree the number of nodes is the number of edges plus one**". (5 Marks)

02. (i) Determine an FA, M accepting L, where **L={ w | w ε {0,1}*}** where every 0 in w has a 1 immediately to its right. (4 Marks) [10]
(ii) Determine an NFA accepting the Language **L = {a* + (ab)*}** . (3 Marks)
(iii) Obtain an NFA which should accept a language L, where **L = { x ε {a, b}*, |x| > =3}** and the third symbol of x from the right is "a". (3 Marks)

03. Demonstrate the algorithm for minimization of DFA. Use this algorithm to minimize the following DFA : [10]



04. a) Convert the regular expression **(01*0 + 10*1) (0+11)*** into equivalent ε-NFA and DFA. (4 Marks) [10]
b) Give a verbal description of the language. (3 Marks)
((aa)* b (aa)* + a (aa)* ba (aa)*)
c) Write the Regular Expression for the language with no consecutive 0's in the middle and no consecutive 11's at the end of the string. (3 Marks)

05. The Language L = {w = $a^N b^M$, where Σ ∈ {a, b}}. Find and prove under which inequality relationship between N and M, the language is regular and the same is not regular. [10]

06. Use the CYK method to determine if the string w = 110100 is in the language L (G) generated by the CNF grammar G. [10]

$S \rightarrow AB \mid BC$

$A \rightarrow BB \mid 0$

$B \rightarrow BA \mid 1$

$C \rightarrow AC \mid AA \mid 0$

07. Construct the CNF for the following grammar and explain the steps. [10]

$S \rightarrow pPp \mid qQq \mid \varepsilon$

$P \rightarrow R \mid p$

$Q \rightarrow R \mid q$

$R \rightarrow RTU \mid \varepsilon$

$T \rightarrow P \mid Q \mid pq$

08. Design a PDA that will recognize strings of the language $\{wcw^c \mid w \in \{0,1\}^*$ and w^c is the one's complement of w$\}$ [10]

09. Design a Turing Machine over input alphabets {0, 1} such that w is divisible by 3. Trace the output for 1001. [10]

10. Prove the following theorems: [10]

(i) If L is recursive Language, so is $\overline{L}$. (4 Marks)

(ii) State and prove that diagonalization language is not recursively enumerable. (3 Marks)

(iii) Prove that Universal language is recursively enumerable but not recursive. (3 Marks)

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Reg. No. : 22BCE1351

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Natarajan B	Slot	F2+TF2
		Class Nbr	CH2023240503349
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.
- Answers submitted without the appropriate accompanying problem-solving steps will not be awarded any credit.

Answer all questions (10 X 10 Marks = 100 Marks)

01. Imagine you are tasked with proving the following conjecture: For any sets A, B, and C, if $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, then $A \subseteq B$ or $A \subseteq C$. [10]

(a) How would you approach proving or disproving this statement? (5 Marks)

(b) Provide a step-by-step outline of your proof strategy, including the logical principles and set operations you would employ. (5 Marks)

02. Consider the following ε-Nondeterministic Finite Automata. [10]

States/Input	ε	a	b	c
→p	ϕ	{p}	{q}	{r}
q	{p}	{q}	{r}	ϕ
r*	{q}	{r}	ϕ	{p}

(a) Compute the ε–closure for each state. (3 Marks)

(b) Give all the strings of length three or less accepted by the automaton. (2 Marks)

(c) Convert the automaton to a DFA. (5 Marks)

03. Produce the Minimized Deterministic Finite Automata equivalent to the following Deterministic Finite Automata. [10]

States/Input	0	1
→A	B	E
B	C	F
*C	D	H
D	E	H
E	F	I
*F	G	B
G	H	B
H	I	C
*I	A	E

04. Prove the following languages are not regular [10]
(a) L1={0^p1^m|p≤m} is not a regular (3 Marks)
(b) L2={0^n|n is a perfect square} is not a regular (3 Marks)
(c) L3={a^{2n}|n≥1} is not a regular (4 Marks)

05. Consider two languages L1 and L2 built over alphabets {a, b}, where L1 is the set of all strings that start with either a or b and end with zero or more occurrences of a and L2 is the set of all strings that start with ba and end with zero or more occurrences of a. Construct NFAs to accept the languages [10]
(a) L1∪L2 (3 Marks)
(b) L1∩L2 (3 Marks)
(c) L1-L2. (4 Marks)

06. Let T = {0, 1, (,), ∪, *, Ø, ε}. You may think of T as the set of symbols used by regular expressions over the alphabet {0, 1}: [10]
(a) Your task is to design a CFG G with set of terminals T that generates exactly the regular expressions with alphabet {0, 1} (5 Marks).
(b) Using your CFG G, give a derivation and the corresponding parse tree for the string $(0 \cup (10)^*1)^*$ (5 Marks).

07. Convert the given Context Free Grammar into Chomsky Normal Form. [10]
S→a|aA|B
A→aBB|C
B→Aa|b

08. The school assigns tasks to students to pick two different colored balls from the basket based on the following conditions. The count of each color ball need not be the same. After picking the balls, they should place them in another bucket. [10]
(a) Construct a language and design a Push Down Automata for such a scenario and model the transition diagram (5 Marks).
(b) Verify the model using valid and invalid strings.(5 Marks)

09. Design a Turing Machine to validate strings composed of the alphabet set {a, b, c}, adhering to the language pattern where the first part consists of any combination of input symbols, followed by a second part that is the exact reverse of the first part. Analyze and create a transition diagram outlining the state transitions and actions required to fulfill these constraints. [10]

10. (a) Let A = {001, 0111,11,101} and B = {010, 111, 111, 010}. Does the instances (A,B) have a Post's Correspondence Problem solution? Does it have an [10]

Modified Post Correspondence Problem solution? (5 Marks)

(b) Find and prove the language L = {<M,w> | M is a Turing machine that accepts string w with exactly three strings} is decidable or not. (5 Marks)

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Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Sathyarajasekaran K	Slot	C2+TC2
		Class Nbr	CH2023240501827
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. Imagine you're designing a security system where users input specific sequences of binary digits to access the system. The access code system is designed to recognize valid sequences of binary digits. Construct a Deterministic Finite Automaton (DFA) to recognize valid access codes. Ensure that, the DFA rejects invalid access codes. [10]

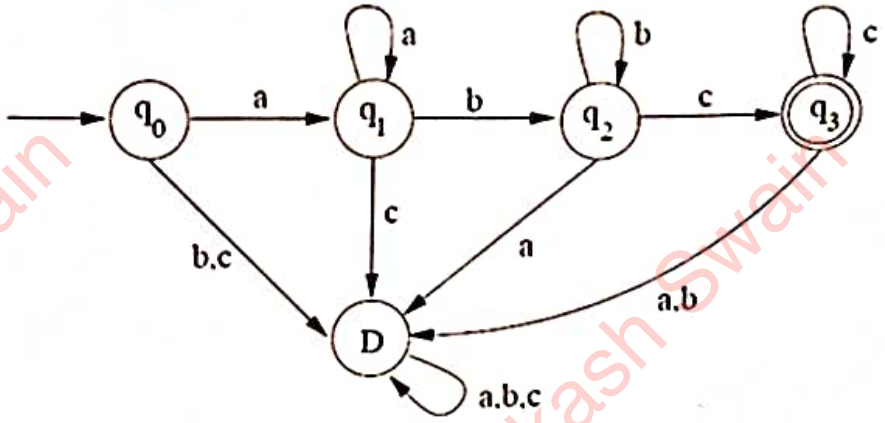
The access codes must meet the following criteria:

- They can only consist of 0s and 1s.
- They must end with either '000' or '111'.
- They must contain either '101' or '010' as a substring.

02. (a) Imagine you're a software engineer working on a project to optimize a web application's user authentication system. The current system relies on a nondeterministic finite automaton (NFA) to handle user input validation, but it's becoming increasingly inefficient as the user base grows. Your task is to improve the performance of the authentication system by converting the following NFA represented by its transition table into a deterministic finite automaton (DFA). (5 Marks) [10]

Current State	Next State(s)	
	For '0'	For '1'
→ q_0	$\{q_0, q_1\}$	$\{q_0, q_3\}$
q_1	$\{q_2\}$	ϕ
((q_2))	$\{q_2\}$	$\{q_2\}$
q_3	ϕ	$\{q_4\}$
((q_4))	$\{q_4\}$	$\{q_4\}$

(b) As a software engineer working on a project to optimize a web application's user authentication system, you are also instructed to analyse the following DFA. Describe the language accepted by the following DFA. (5 Marks)



Where 'D' is the trap state.

03. (a). Let $L, M \subseteq \sum^*$ for any $\sum$. Prove or disprove $(L^*M^*)^* = (L + M)^*$. (5 Marks) [10]
(b) Consider the language L= $\{a^{2j} b^i c^k \mid i>2, j$ is even and k is odd$\}$. Construct a grammar G to generate the language L. (5 Marks)

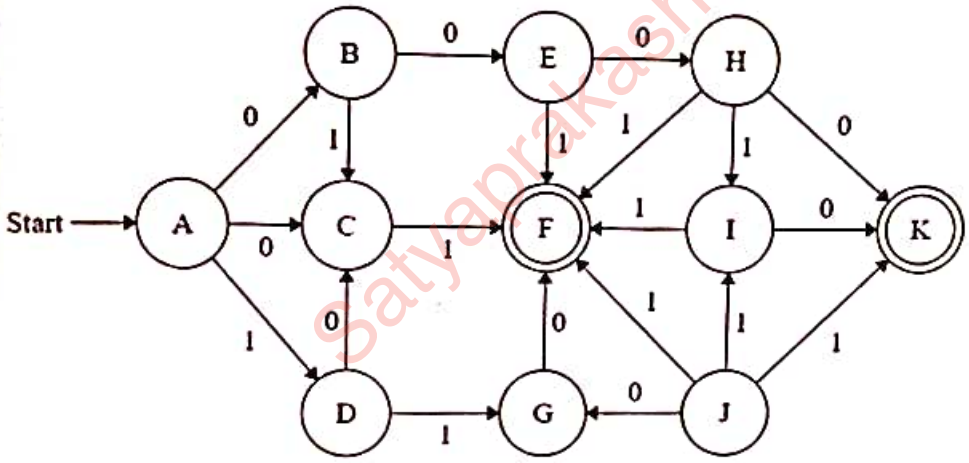
04. A juice shop owner asks his fruit vendor to pack Guava, Mango, Watermelon and Chiku fruits into baskets. After packing the baskets will be sent to different branches. [10]
The fruits are to be packed in the following count.
i. For every Watermelon, there should be three Mangoes.
ii. For every two Guavas there should be one Chiku.
The vendor should pack each basket in the above-said ratio and should ensure that the fruits are arranged in the basket in such a way that the fruits won't get smashed.
a. Express "the packing pattern where the vendor can arrange the fruits" as a language L. (2 marks)
b. Design a Turing Machine M that will recognize language L.(6 marks)
c. Verify the Turing Machine M, for valid and invalid fruit packing patterns defined in L. (2 marks)
Note: Watermelon is a heavy fruit so it has to be placed in the bottom. Over that Guava should be kept, then Mangoes should be kept, and over the Mangoes, Chiku should be arranged. The bottom to top arrangement of fruits should be viewed as a left to right arrangement for the construction of Turing machine.

05. In a kindergarten annual day celebration, the school management organizes an event for the kids. [10]
The organizers form a team of 4 kids where there are two boys and two girls.
i. Girls are asked to collect red and green color-wrapped candies.
ii. Boys are asked to collect yellow and orange color-wrapped candies.
After the collection, the kids are asked to arrange the candies in the following ratio. "The count of red candies is two times the count of green candies and the count of yellow candies is greater than orange candies."
The final arrangement should be made in a box in the following order. "The red candies are first followed by green candies. Over the green candies yellow candies are placed and in the top orange candies are to be kept.
a. Express "the arrangement of candies in a pattern" as a language L. (2 marks)

b. Design a suitable grammar G based on the type identified through the Chomsky Hierarchy. (6 marks)

c. Justify your answer by verifying a string with a minimum of 6 characters. (2 Marks)

06. Consider an Automated Teller Machine (ATM) processing, which has different states of the system before delivering the cash. The ATM designer has identified a set of states Q={A, B, C, D, E, F, G, H, I, J, K} and transitions with input {0,1}. As per the transitions, he has constructed the following Finite Automata (FA). In the implementation phase, each state has to be converted into a piece of code. He decided to reduce the number of states in the FA to reduce the number of lines. Help him in this regard to reduce the number of states. Construct Finite Automata to its equivalent DFA if needed. Further, construct its equivalent minimized DFA. *[10]*



07. Consider the formal language $L=\{0^{2a}\,1^{2b+a}\,2^{b+2c} \mid a,b,c\geq 0\}$. A Computer engineer wanted to construct a logical machine to recognize the strings in the language L over the input set {0,1}. **[10]**

a. Recommend the appropriate machine for the language. (2 Marks)

b. Help him by constructing the machine for the given language L. (8 Marks)

08. Consider the languages L1 and L2, Check whether L1 and L2 are Context-Free Languages or not. If not, prove that with appropriate pumping lemma. (Each 5 Marks) **[10]**

(a). $L1=\{a^{2p}\,b^{m}\,c^{p}\,d^{m+1} \mid p\geq 0,\ m>1\}$.

(b). $L2=\{(ab)^{2n}\,(ba)^{n}\,(ab)^{m+n} \mid n,m>2\}$.

09. Design a Turing machine to compute the following function, **[10]**

$$f(n)=\begin{cases}2n+1 & n\ mod\ 2=0\\ 2n & otheriwise\end{cases}$$

where n is the positive integer and ***mod*** is a function described as; ***a mod b*** is the remainder when *a* divides *b*. For example, *5 mod 2* is *1*.

10. Alice and Bob are participating in a programming competition where they encounter a Post Correspondence Problem (PCP). They are given two sets of tiles: **[10]**

Set A: {100, 0, 1}

Set B: {1, 100, 00}

Their task is to find a sequence of tiles from sets A and B such that when the tiles are arranged in a line, the concatenated string from Set A matches the concatenated string from Set A.

a. Describe the Post Correspondence problem and its Applications. (3 Marks)

b. Obtain the solution for the above PCP problem to help Alice and Bob win the competition. (7 Marks)

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Reg. No. : 22BAI1226

Final Assessment Test (FAT) - May 2024

Programme	B.Tech.	Semester	WINTER SEMESTER 2023 - 24
Course Title	THEORY OF COMPUTATION	Course Code	BCSE304L
Faculty Name	Prof. Renuka Devi R	Slot	F1+TF1
		Class Nbr	CH2023240503054
Time	3 Hours	Max. Marks	100

General Instructions:

- Write only Register Number in the Question Paper where space is provided (right-side at the top) & do not write any other details.

Answer all questions (10 X 10 Marks = 100 Marks)

01. (i) Prove that for every $n > 0$, $(5^n - 2^n)$ is divisible by 3. (5 Marks) [10]

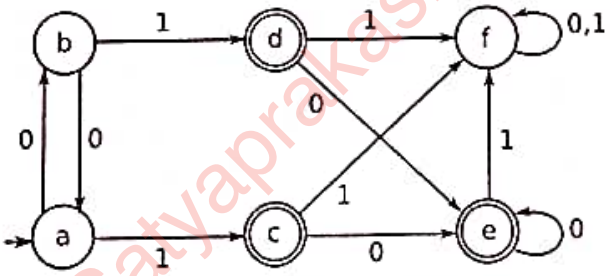
(ii) Consider the following statement and prove it using structural induction " **For every tree the number of nodes is the number of edges plus one**". (5 Marks)

02. (i) Determine an FA, M accepting L, where $L=\{ w \mid w \in \{0,1\}^* \}$ where every 0 in w has a 1 immediately to its right. (4 Marks) [10]

(ii) Determine an NFA accepting the Language $L = \{a^* + (ab)^*\}$. (3 Marks)

(iii) Obtain an NFA which should accept a language L, where $L = \{ x \in \{a, b\}^*, |x| >= 3\}$ and the third symbol of x from the right is "a". (3 Marks)

03. Demonstrate the algorithm for minimization of DFA. Use this algorithm to minimize the following DFA : [10]



04. a) Convert the regular expression (01*0 + 10*1) (0+11)* into equivalent ε-NFA and DFA. (4 Marks) [10]

b) Give a verbal description of the language. (3 Marks)

((aa)* b (aa)* + a (aa)* ba (aa)*)

c) Write the Regular Expression for the language with no consecutive 0's in the middle and no consecutive 11's at the end of the string. (3 Marks)

05. The Language $L = \{w = a^N b^M$, where $\Sigma \in \{a, b\}\}$. Find and prove under which inequality relationship between N and M, the language is regular and the same is not regular. [10]

06. Use the CYK method to determine if the string w = 110100 is in the language L (G) generated by the CNF grammar G. [10]

M4

$S \rightarrow AB \mid BC$

$A \rightarrow BB \mid 0$

$B \rightarrow BA \mid 1$

$C \rightarrow AC \mid AA \mid 0$

07. Construct the CNF for the following grammar and explain the steps. [10]

M4

$S \rightarrow pPp \mid qQq \mid \varepsilon$

$P \rightarrow R \mid p$

$Q \rightarrow R \mid q$

$R \rightarrow RTU \mid \varepsilon$

$T \rightarrow P \mid Q \mid pq$

M5 08. Design a PDA that will recognize strings of the language $\{wcw^c \mid w \in \{0,1\}^*$ and w^c is the one's complement of $w\}$ [10]

M6 09. Design a Turing Machine over input alphabets {0, 1} such that w is divisible by 3. Trace the output for 1001. [10]

10. Prove the following theorems: [10]

M7

(i) If L is recursive Language, so is $\overline{L}$. (4 Marks)

(ii) State and prove that diagonalization language is not recursively enumerable. (3 Marks)

(iii) Prove that Universal language is recursively enumerable but not recursive. (3 Marks)

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