Final Assessment Test (FAT) - July/August 2023

Programme	B.Tech.	Semester	Fall Inter Semester 22-23
Course Title	DISCRETE MATHEMATICS AND GRAPH THEORY	Course Code	BMAT205L
Faculty Name	Prof. Avinash Kumar Mittal	Slot	C1+TC1+TCC1
		Class Nbr	CH2022232500281
Time	3 Hours	Max. Marks	100

Section A (10 X 10 Marks)
Answer any 10 questions

01. a) Obtain the PCNF from the PDNF of $(P \wedge Q) \vee R \to \neg P$ using laws and validate the result with truth table. (7 Marks)   [10]

b) Symbolize the following: (3 Marks)
(i)   Whenever there is an active alert, all the queued messages are transmitted or rejected.
(ii) When everyone is cautious, no one can cheat.

02. Show that the following set of premises are inconsistent.   [10]
If the contract is valid, then Harry is liable for a penalty. If Harry is liable for penalty, then he will go bankrupt. If the bank will loan him money, then he will not go bankrupt. As a matter of fact, the contract is valid, and the bank will loan him money.

03. a) Check the closure, associative, existence of identity & inverse and commutative properties for the algebraic structure $(Q^+, *)$, where Q^+ is the set of all positive rational numbers and $*$ is defined by $a * b = \frac{a}{2} + \frac{b}{4}$.   (5 Marks)   [10]

b) Determine the shortest path between the vertices **a** to **z** as shown in the figure. (5 Marks)
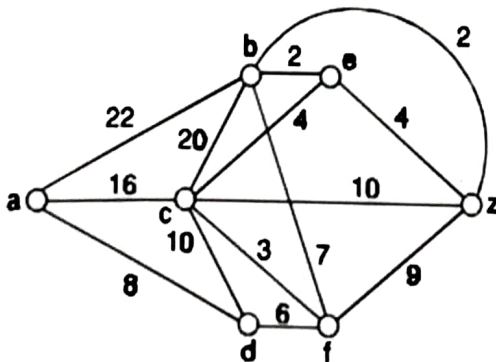
04. a) Is the set $G = \{1, 3, 7, 9\}$ an abelian group under multiplication modulo 10.   [10]
(5 Marks)

b) Find the code words generated by the parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

when the encoding function is $e : B^2 \to B^5$. (5 Marks)

05. a) A mission to Mars will consist of 4 astronauts selected from 14 available. Exactly 5 of the 14 **[10]** are trained in exobiology. If the mission requires at least 2 trained in exobiology, how many different crews can be selected? (5 Marks)
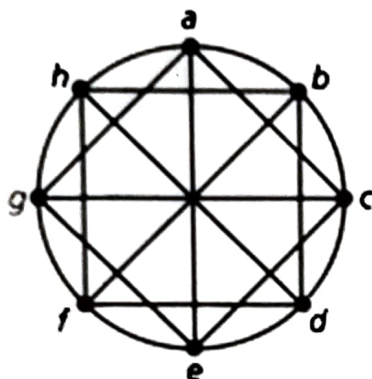
b) (i) Define bipartite and planar (ii) Check whether the following graph is (A) bipartite (B) planar. (justification required) (5 Marks)

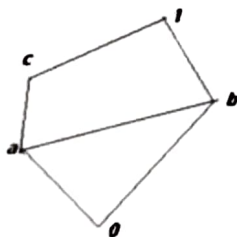06. Solve the following recurrence relation using the method of undetermined coefficients : **[10]**
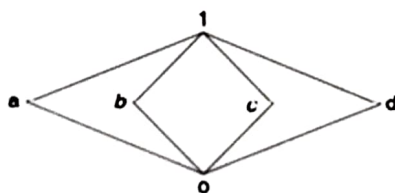$a_n - 4a_{n-1} + 4a_{n-2} = 3n + 2^n, a_0 = 1, a_1 = 1.$

07. a) Check whether the following diagrams are (i) distributive (ii) bounded (iii)complemented lattice. (5 Marks) **[10]**

(a)

(b)

b) Simplify the Boolean expression: $x[y + z(xy + xz)']$:
(5 Marks)

08. a) Draw the Hasse diagram corresponding to the divisors of 70, under divisibility and list the relations of the $POSET$ excluding the elements of the form $(1, a)$ and (a, a) where a is any divisor of 70. (5 Marks) **[10]**
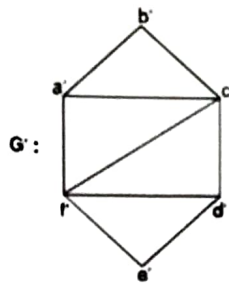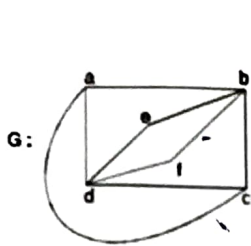
b) Are the two graphs given below isomorphic? Give a reason.
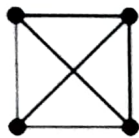(5 Marks)

G: G':

09. a) It is possible to construct a graph with 12 vertices such that 2 vertices have degree 3 and the remaining vertices have degree 4. Justify your answer. (4 Marks) **[10]**
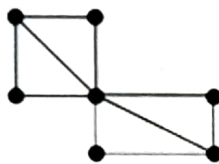
b) Which of the graphs in Figures (a), (b), (c) has (i) an Eulerian circuit (ii) an Eulerian trail but no Eulerian circuit (iii) neither an Eulerian trail nor an Eulerian circuit?

(6 Marks)

(a)   (b)   (c)

10. Eight cities A, B, C, D, E, F, G, and H are required to be connected by a new railway network. The possible tracks and the cost involved to lay them (in crores of rupees) are summarized in the following table : **[10]**
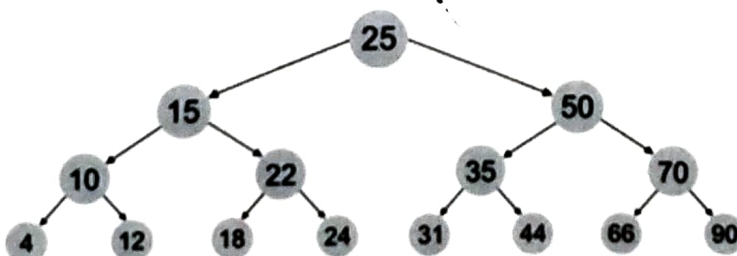
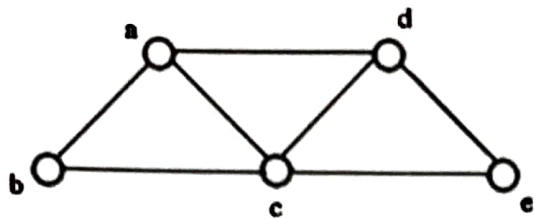| Track between	Cost	Track between	Cost
A and B	155	D and F	100
A and D	145	E and F	150
A and G	120		
B and C	145	F and G	140
C and D	150	F and H	150
C and E	95	G and H	160

Determine a railway network of minimum cost that connects all these cities.

11. a) Find preorder, postorder and inorder for the given tree (6 Marks) **[10]**
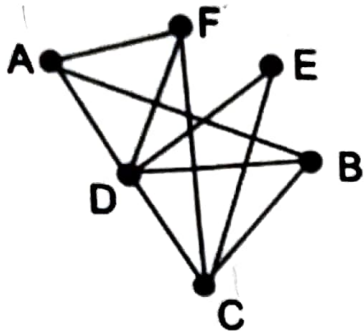

b) Draw four distinct spanning trees for the given graph G. (4 marks)

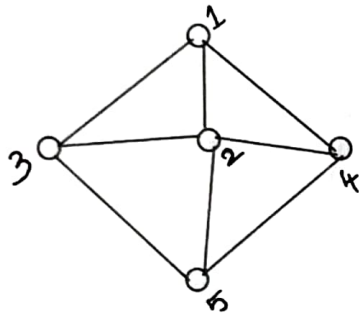12. a) Find the chromatic number of the following graph.   (3 Marks)   [10]
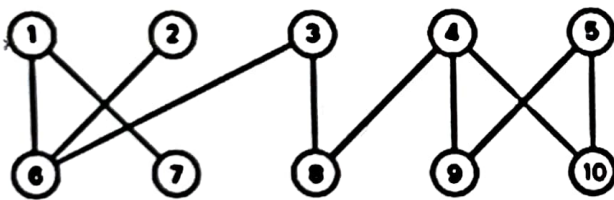

⑥

b) Find the chromatic polynomial of the following graph.   (4 Marks)

c) Find a maximum matching and minimum vertex cover for the following graph: (3 Marks)


⇔⇔⇔

Final Assessment Test (FAT) - APRIL/MAY 2023

Programme	B.Tech	Semester	Winter Semester 2022-23
Course Title	DISCRETE MATHEMATICS AND GRAPH THEORY	Course Code	BMAT205L
Faculty Name	Prof. Vidhya V	Slot	D2+TD2+TDD2
		Class Nbr	CH2022235001875
Time	3 Hours	Max. Marks	100

### Section A (10 X 10 Marks)
Answer <u>any 10</u> questions

01. Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$ and $a \wedge d$ are inconsistent without using the truth table. (mod 1)   [10]

02. An employee in my office has not completed his daily work. Everyone in my office completed his monthly files. Can we conclude, "Someone who completed his monthly files has not completed his daily work"? (mod 1)   [10]

03. (a) Let $\star$ be the binary operation on the set of all ordered pairs of rational numbers $G = \mathbb{Q} \times \mathbb{Q}$ defined by $(\alpha, \beta) \star (\mu, \delta) = (\alpha\mu, \alpha\delta + \beta\mu)$, where the pairs (α, β), $(\mu, \delta) \in G$. Prove that $(G, \star)$ is a semigroup. Check $\star$ is commutative. Find the identity and inverse element in $(G, \star)$ (mod 2) (5 Marks)   [10]

(b) Let $\Phi(n)$ be the set of all positive integers less than n and relatively prime to n. Find all the left cosets and verify Lagranges's theorem for the subgroup $\Phi(8)$ of $(Z_8, +_8)$. (5 Marks)

04. (a) Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ and the encoding function is   [10]

$e : B^3 \to B^6$. Using the parity check matrix decode and obtain the original messages for the following code words 011011, 111010, and 010100. (5 Marks) (mod 2)

(b) Twenty batteries will be put on display in an electronic store. The types of batteries are AAA, AA, C, D and 9-volt. mod 3

(i) How many ways can we choose the 20 batteries? 5^{20} ways .

(ii) How many ways can we choose the 20 batteries but be sure that at least four batteries are 9-volt batteries? $5^{20} - (4^{20} + 4^{19} + 4^{18} + 4^{17})$ ways

(iii) How many ways can we choose the 20 batteries but have no more than 2 batteries that are 9-volt batteries? (5 Marks) $\Rightarrow 4^{20} + 4^{19} + 4^{18}$ ways

05. Solve the non-homogeneous recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$ with $a_0 = 1$ and $a_1 = 4$. mod 3   [10]

06. (a) Simplify the following Boolean expression using K map (5 Marks)   [10]
$x'yz' + xyz + x'y'z + xyz' + xy'z + x'yz + xy'z'$ mod 3

(b) In relation to a spanning tree T, determine the fundamental cut-sets for graph G. (5 Marks).
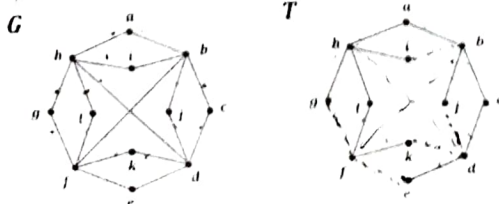
mod 5

07. (a) Consider the following POSET with the Hasse diagram: (5 Marks) [10]
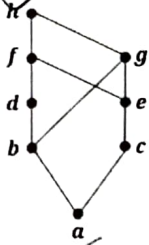
(mod4)

(i) Find the upper bound, lower bounds, least upper bound, and greatest lower bound for the set $\{b, f, e\}$.
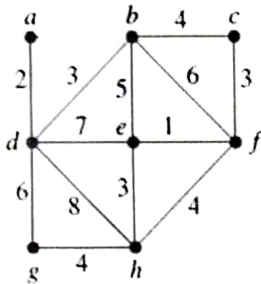
(ii) Determine whether the POSET is a lattice.

(b) Draw the Hasse diagram for the POSET

$(\{\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}, \subseteq)$ and identify its maximal and minimal elements, where $\subseteq$ denotes the subset of a set. (5 Marks)
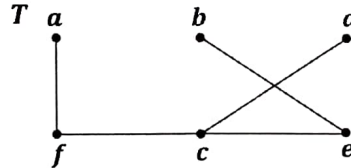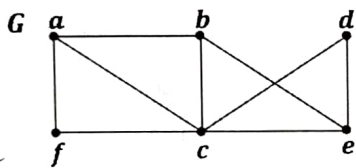
08. (a) Find the minimum spanning tree of the following weighted graph using Prim's algorithm. [10]
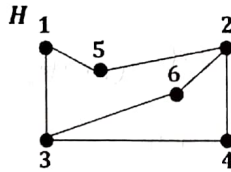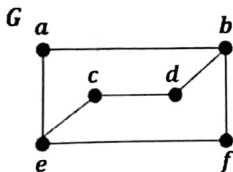(8 Marks)

(mod5)

(b) In relation to a spanning tree T, determine the fundamental cycles for graph G. (2 Marks)
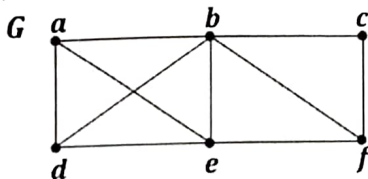

(mod5)

09. (a) Determine whether the following graphs G and H are isomorphic or not. (6 Marks) [10]
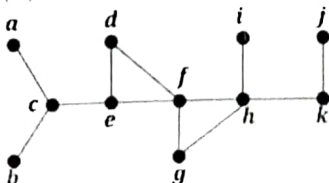

(M5)

(b) Find the number of paths of length 3 from the vertex f to the vertex g in the following graph. (4 Marks)
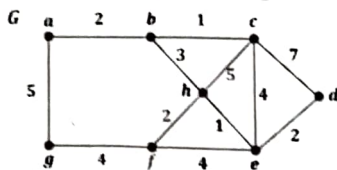

10. (a) Determine the center, radius, and diameter of the following graph. (3 Marks) [10]
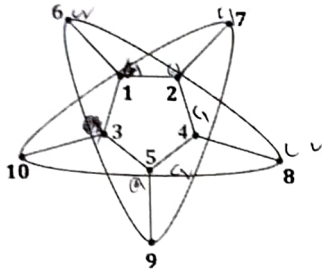
(b) Use Dijkstra's algorithm to find the shortest path between the vertex a to all other vertices in the weighted graph G given below. (7 *Marks*)
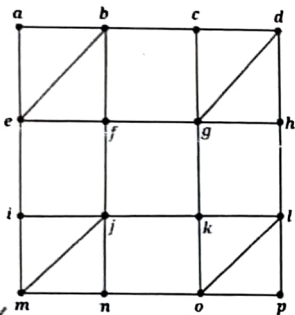

M6.

11. (a) Using the following graph, determine the chromatic number and find any three maximal matching sets. (5 *Marks*)   [10]
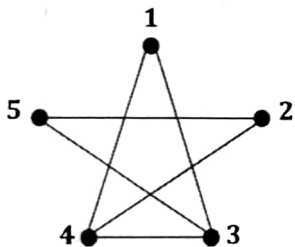

M7

(b) In the following graph, find (5 *Marks*)
   (i) two different minimal vertex covering sets
   (ii) two different minimal edge covering sets
   (iii) the vertex covering number
   (iv) the edge covering number

12. Find the chromatic polynomial of the following graph: [10]

M7.

⇔✕⇔✕⇔