Q.No.	Sub. Sec.	Question Description	Marks
1.		Expand $f(x,y)=e^x \sin y$ in Taylor Series about $\left(1,\frac{\pi}{2}\right)$ up to third degree terms and hence find the approximate value of $e^{0.5}\sin\left(\frac{\pi+2}{2}\right)$.	[10]
2.		Find the absolute extrema of $z=2x^2+y-3xy$ in the plane region D bounded by the lines $y=1-x$, $y=1+x, y=-1-x$ and $y=-1+x$.	[10]
3		Sketch the region of integration and evaluate $\int_0^3 \int_{4y/3}^{\sqrt{(25-y^2)}} x\,dxdy$ after changing the order of integration.	[10]
4.	[a]	Using spherical coordinates, evaluate $\iiint_E (x^2+y^2)dv$ where E lies between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=9$.	[7]
	[b]	Evaluate $\iiint_R (x-y-z)\,dxdydz$, where $R: 1\le x\le 2; 2\le y\le 3; 1\le z\le 3$.	[3]
5.	[a]	If n is positive integer and $m>-1$, then prove that $\int_0^1 x^m (\log x)^n\,dx=\frac{(-1)^n n!}{(m+1)^{n+1}}$	[5]
	[b]	Using Beta and Gamma function, evaluate $\int_0^a y^4\sqrt{a^2-y^2}\,dy$.	[5]

Answer all the Questions

1.		(i) Find all critical points of the function $f(x,y)=x^4+y^4-2x^2-2y^2+4xy$ and check whether the function attains maximum or minimum at each of these points. (ii) Show that point $(0,0)$ is neither a point of local minimum nor a point of local maximum for the function given by $f(x,y)=3x^4-4x^2y+y^2$ for $(x,y)\in R^2$.	10
2.		(i) If x,y and z are positive real numbers, then find the minimum value of function $x^2+8y^2+27z^2$, where $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$. (ii) Find the Taylor series expansion of $f(x,y)=\sin xy+x^2y+e^x$ in the power of $(x-1)$ and $(y-\pi)$ up to second degree terms.	10

3.		(i) Find the value of integral by using the polar coordinates. $I = \iint_D \sqrt{x^2+y^2}\,dydx \quad where\ D = \{(x,y) \in R^2 : x \le x^2+y^2 \le 2x\}$ (ii) Find the value of integral by changing the order of integration $I = \int_0^4 \int_{(4-x)^{\frac{1}{2}}}^2 e^{y^3}\,dydx$	10
4.		Using multiple integrals, find the volume of the solid region bounded above by hemisphere $z = 1+\sqrt{1-x^2-y^2}$ and bounded below by the cone $z=\sqrt{x^2+y^2}$.	10
5.		Solve the following integrals by using Beta and Gamma Function: (i) $I = \int_0^\infty \frac{e^{-\frac{k}{x^2}}}{x^6}dx \quad where\ k \ne 0$ (ii) $I = \int_0^1 x^4\sqrt{1-x^2}\,dx$	10

VIT

Vellore Institute of Technology

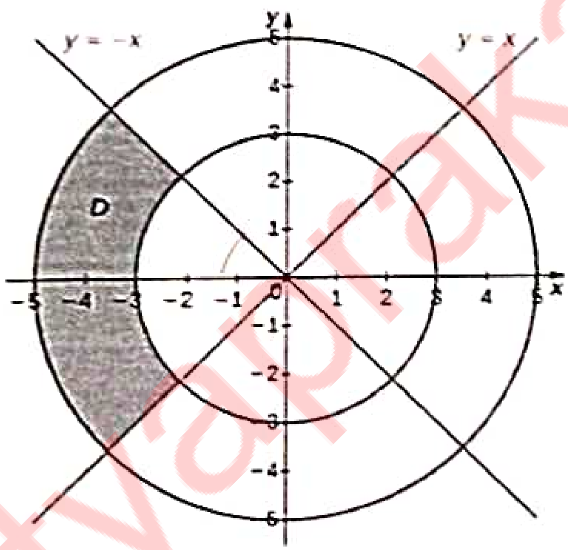
(Deemed to be University under section 3 of UGC Act, 1956)

Continuous Assessment Test (CAT)- II– December 2022

Programme	:	B.Tech.	Semester	:	Fall Semester Year I 2022-2023
Course Title	:	Calculus	Code	:	BMAT101L
			Slot	:	E2+TE2
Faculty	:	Dr. Berin Greeni A , Dr. Prosenjit Paul, Dr. Srutha Keerthi B, Dr. Dhivya P, Dr. Saurabh Chandra Maury, Dr. Karan Kumar Pradhan	Class No.	:	CH2022231700201, 197, 199, 202, 198, 200
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q. No.	Question Description	Marks
1.	(a) Find the Taylor series expansion of x^y about (1, 1) up to second degree terms. (b) Obtain the critical points of $(x^2 + 3y^2)e^{-(x^2+y^2)}$.	[5+5]
2.	A wire of length L is cut into two parts (not necessarily equal) which are bent to in the form of a square and a circle respectively. Find the least value of the sum of areas so found.	[10]
3.	If $f(x,y) = (x^2 + y^2)$ represents the population density of a planar region on the Earth, where x and y are measured in miles, find the number of people in the region shown below.	[10]
4.	Evaluate $\iiint_R (x^2+y^2+z^2)\,dV$, where R is the region above the xy-plane bounded by the cone $z^2 = 3(x^2 + y^2)$ and by the sphere $x^2 + y^2 + z^2 = 1$.	[10]
5.	Using special functions compute the integral $\iint_R x^2 y^2\,dx\,dy$, where R is the region bounded by the curve $x^{2/3} + y^{2/3} - 4 = 0$.	[10]



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Vellore Institute of Technology

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Continuous Assessment Test (CAT)- II– December, 2022

ogramme	:	B.Tech.	Semester	:	Fall Semester I 2022-2023
ourse Title	:	Calculus	Code	:	BMAT101L
			Slot	:	E1+TE1
aculty	:	Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, ~~Dr. [illegible]~~, Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya	Class Nbr	:	CH2022231700 189, 191, 192, 194, 257, 323,
uration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters Earth's atmosphere and its surface begins to heat. After 1 hour, the temperature at the point (x, y, z) on the probe's surface is $T = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe's.	[10]
2.	Find the absolute maximum and minimum of $f(x, y) = x + y - xy$ on the triangle ABC with vertices A (0,50), B(50,0) and C(-50,-50).	[10]
3.	a) Change the order of integration, evaluate $\int_0^2 \int_0^{9-x^2} x\, dy\, dx$.	[7]
	b) Find the area of $r = \sin\theta$ in polar coordinates.	[3]
4.	A spherical tank of radius 3 meters is filled with water to a height of 2 meters. Find the volume of the water using the cylindrical coordinates.	[10]
5.	a) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^5}\, dx$.	[5]
	b) Evaluate $\int_0^1 x^7(1-x^2)^6 dx$.	[5]

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Continuous Assessment Test (CAT)- II– December 2022

Programme	:	B.Tech.	Semester	:	Fall 2022-2023
Course Title	:	Calculus	Code	:	BMAT101L
			Slot	:	A2+TA2
Faculty	:	Dr. Balamurugan, Dr. Saroj Kumar Dash, Dr. Mini Ghosh, Dr. Manimaran, Dr. Sowndarrajan, Dr. Prabhakar, Dr. Rajesh Kumar, Dr. Soumendu Roy	Class ID	:	CH2022231700410, 416,429,440,443,57 3,604,610
Duration	:	1 ½ Hours	Max. Marks	:	50

Answer all the Questions (50 marks)

Q.No.	Question Description	Marks
1.	(a). Find the absolute maximum and minimum values of $f(x,y)$ on the region R where $f(x,y) = x^2 + xy + y^2 - 3x + 3y$ and R is the triangular region cut from the first quadrant by the line $x + y = 4$.	[5]
	(b). For what values of the constant k does the second derivative test guarantee that $f(x,y) = x^2 + kxy + y^2$ will have a saddle point at (0, 0)? A local minimum at (0, 0)? For what values of k is the second derivative test inconclusive? Give reasons of your results.	[5]
2.	(a). Find three positive numbers whose sum is 50 and whose product is maximum.	[5]
	(b). A flat circular plate has the shape of the region $x^2 + y^2 \leq 4$. The plate, including the boundary where $x^2 + y^2 = 4$, is heated so that the temperature at the point (x, y) is $T(x,y) = x^2 + 2y^2 - x$. Find the temperatures at the hottest and coldest points on the plate.	[5]
3.	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(1+e^y)\sqrt{1-x^2-y^2}}\, dydx$.	[10]
4.	Find the volume of the solid bounded by the xy –plane, the paraboloid $2z = x^2 + y^2$ and cylinder $x^2 + y^2 = 4$.	[10]
5.	(a). Find the value of $\int_0^\pi sin^2x\,(1 + cosx)^4 dx$ in terms of gamma function.	[5]
	(b). Evaluate the integral $\int_0^\infty x^4 e^{(-x^8)} dx$.	[5]

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Continuous Assessment Test II – December 2022

Programme	: B.Tech	Semester	:	FALLSEM 2022-23
Course	: Calculus	Code	:	BMAT101L
		Slot	:	A1+TA1
Faculty	: Dr. S. Radha Dr. N. Nathiya Dr. Sowndarrajan P T Dr. Manoj Kumar Singh Dr. Harshavarthini Shanmugam Dr. Manimaran J	Class Number	:	CH2022231700297 CH2022231700423 CH2022231700424 CH2022231700298 CH2022231700617 CH2022231700608
Time	: 1½ hours	Max. Marks	:	50

Answer ALL the Questions (5 x 10 = 50 marks)

Q.No.	Sec	Question Description	Marks
1.	a.	Find the critical points of the function $f(x, y) = x^3 + y^3 - 12x - 6y + 40$. Test each of these for maximum and minimum.	5
	b.	Use Taylor's formula to find a quadratic approximation of $f(x, y) = xe^y + 1$ at $(1, 4)$.	5
2.		The temperature T at any point (x, y, z) in space is $T = 625xzy^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.	10
3.		Find the volume of the region using double integration which lies under the paraboloid $z = 4 - x^2 - y^2$ and above the disk $(x - 1)^2 + y^2 = 1$ on the xy-plane.	10
4.		Evaluate $\iiint e^{-x^2-z^2}\, dV$ where R is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \le y \le 5$ and $z \le 0$.	10
5.	a.	Evaluate $\int_0^1 x^{\frac{7}{2}}\left(1 - x^{\frac{3}{2}}\right)^{11} dx$.	5
	b.	Evaluate $I = \int_0^{\infty} x^4\, e^{-x^4}\, dx$.	5