CS: 2210 // Handwritten Homework #3

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Section 1.4

#8. $R(x) \Rightarrow x$ is a rabbit
$H(x) \Rightarrow x$ hops.
Domain $\Rightarrow$ all animal.

a) $\forall x(R(x) \rightarrow H(x))$

Every Rabbit is hops.

b) $\forall x(R(x) \land H(x))$

All animals are rabbit and hops

c) $\exists x(R(x) \rightarrow H(x))$

There exists an animal such that, If it is a rabbit, then it hops.

$(R(x))$ $(H(x))$

d) $\exists x(R(x) \land H(x))$

Some rabbits are hops

#10. $C(x) \Rightarrow$ "x has a cat"
$D(x) \Rightarrow$ "x has a dog"
$F(x) \Rightarrow$ "x has a ferret"

a) A student in your class has a cat, a dog, and a ferret.

$\exists x(C(x) \land D(x) \land F(x))$

b) All student in your class have a cats, dogs, or a ferret.

$\forall x(C(x) \lor D(x) \lor F(x))$

c) Some student in your class has a cat and a ferret, but not a dog.

$\exists x(C(x) \land F(x) \land \neg D(x))$

d) No student in your class has a cat, a dog, and a ferret

① $\forall x \neg(C(x) \land D(x) \land F(x))$
② $\neg \exists x(C(x) \land D(x) \land F(x))$

e) * For each of the three animals cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

$(\exists x C(x)) \land (\exists x D(x)) \land (\exists x F(x))$

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#18. $P(x) \Rightarrow \{-2,-1,0,1,2\}$

write out each of these proposition.

a) $\exists x\, P(x)$ → some, or $P(x)$

$\boxed{P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2).}$

b) $\forall x\, P(x)$

And

$\boxed{P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)}$

c) $\exists x \neg P(x)$

or not $\{-2,-1,0,1,2\}$.

$\boxed{\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)}$

d) $\underbrace{\forall x}_{\text{and}}\ \underbrace{\neg P(x)}_{\neg P}$

$\boxed{\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)}$

e) $\neg \exists x\, P(x)$

$\boxed{\neg\, (P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))}$

f) $\neg \forall x\, P(x)$

$\boxed{\neg\, (P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))}$

#34. Express "negation" using "quantifiers".

a) x = drivers

$P(x) \Rightarrow$ "x obey the speed limit"

∴ Some drivers do-not obey the speed limit.

$\Rightarrow \exists x \neg P(x)$: ①

∴ Negation ⇒

$\neg \exists x \neg P(x) = \forall x \neg(\neg P(x))$

$= \boxed{\forall x\, P(x).}$ ②

All drivers obey the speed limit

b) All swedish movies are serious

① x = Swedish movies

② $P(x)$ = "x is serious"

∴ logical expression : $\forall x\, P(x)$

∴ Negation : $\neg \forall x P(x) = \boxed{\exists x \neg P(x)}$

There is a swedish movie that is not serious

c) No one can keep a secret.

$x \Rightarrow$ person

$P(x) \Rightarrow$ "x can keep secret") $\forall x \neg P(x)$

Negation : $\neg \forall x \neg P(x) = \exists x \neg(\neg P(x))$

$= \boxed{\exists x\, P(x)}$

∴ There is someone who can keep secret.

d) There is someone in this class who does not have a good attitude. ⇒ $\exists x \neg P(x)$

∴ x = people in class / $P(x)$ = x has good attitude.

Negation : $\neg \exists x \neg P(x) = \forall x \neg(\neg P(x)) = \boxed{\forall x\, P(x)}$

∴ Everyone in this class has a good attitude.

Section 1.5

#8. $Q(x,y)$: Student x has been a contestent on Quiz Show y.
x: all student at school / y: all quiz show on TV.

a) There is a Student at your school ($\exists x$) who has been a contestant on a TV quiz ($\exists y$).

$\therefore \boxed{\exists x \exists y\, Q(x,y)}$

b) No student at your school has ever been a contestant on a TV quiz show.

$\therefore \boxed{\neg \exists x \exists y\, Q(x,y)}$

c) there is a student at your school who has been a contestant on Jeopardy * and on Wheel of Fortune

$\boxed{\exists x (Q(x, \text{Jeopardy}) \land Q(x, \text{wheel of Fortune}))}$

d) Every TV quiz show ① has had a student from ② your school as a contestant.

$\boxed{\therefore \forall y \exists x\, Q(x,y)}$

e) At least two student from your school have been contestants on Jeopardy (Y)

$\therefore \boxed{\begin{array}{l}\exists x_1 \exists x_2 [Q(x_1, \text{Jeopardy}) \land Q(x_2, \text{Jeopardy})] \\ (x_1 \neq x_2)\end{array}}$

#20. domain consist "All integers"

a) The product of two negative integer (x, y) is positive.

$\boxed{\forall x \forall y ((x<0) \land (y<0) \rightarrow (x \cdot y > 0))}$

⇓ two int. ⇓ negative int. ⇓ Product ⇓ Positive

b) The average of two positive integer is positive.

$\boxed{\forall x \forall y \left((x>0) \land (y>0) \rightarrow \frac{x+y}{2} > 0\right)}$

⇓ two integer ⇓ positive integer ⇓ Average ⇓ positive

c) The difference of two negative integer is not necessarily negative.

$\boxed{\neg \forall x \forall y ((x<0) \land (y<0) \rightarrow (x-y<0))}$

⇓ not necessarily ⇓ two int. ⇓ negative ⇓ difference ⇓ negative.

d) The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.

$\boxed{\forall x \forall y (\ |x+y| \leq |x|+|y|\ )}$

⇓ Two integers. ⇓ absolute values of integers. ⇓ doesn't exceed ⇓ absolute value of the sum of two integers.

or $\boxed{\forall x \forall y (|x+y| \not> |x|+|y|)}$

Section 1.5

#32. Express negation.

all negation symbols immediately precede predicates.

a) $\exists z \forall y \forall x\, T(x, y, z)$

$\Downarrow$

$\boxed{\forall z \exists y \exists x\, \neg T(x, y, z)}$

b) $\exists x \exists y\, P(x, y) \vee \forall x \forall y\, Q(x, y)$

$\Downarrow$

$\boxed{\forall x \forall y\, \neg P(x, y) \vee \exists x \exists y\, \neg Q(x, y)}$

c) $\exists x \exists y\, (Q(x, y) \longleftrightarrow Q(y, x))$

$\Downarrow$

$\boxed{\forall x \forall y\, (Q(x, y) \longleftrightarrow \neg Q(y, x))}$

d) $\forall y \exists x \exists z\, (T(x, y, z) \vee Q(x, y))$

$\Downarrow$

$\boxed{\exists y \forall x \forall z\, (\neg T(x, y, z) \wedge \neg Q(x, y))}$

Question for Homework

Section 1.4 # 10-e.

Section 1.5 32-c

20-c, d