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11. Radiation in a Thermal Plasma

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Recombination Lines (like hydrogen in optical or radio band)

- General formulation of recombination line emissivity from k to j level in the r-ionized atom:

$$\int j_\nu \, d\nu = \frac{h\nu_{jk} n_k(X^{(r)}) A_{kj}}{4\pi}$$

- What most likely happens is that the atom is first ionized from r to r+1 and then recombined to level k in the r-ionized atom before transitioning from k to j. (e.g. HII regions where hydrogen is highly ionized).

$$4\pi \int j_\nu \, d\nu = h\nu \alpha_{mn} n_e n_p$$

$$\alpha_{mn} = \frac{b_m g_m A_{mn} e^{-E_1/m^2 kT}}{f_e}$$

Concept of Emission Measure (EM)

- To obtain the surface brightness, or specific intensity I, we must be integrated the emissivity along the line of sight.

$$\int I_\nu \, d\nu = h\nu \alpha_{mn} \frac{n_p}{n_e} \times 2.46 \times 10^{17} E_m$$

- The EM is the integrated square electron density along the ling-of-sight

$$E_m = \int_0^L n_e^2 \, ds$$

EM in HII regions

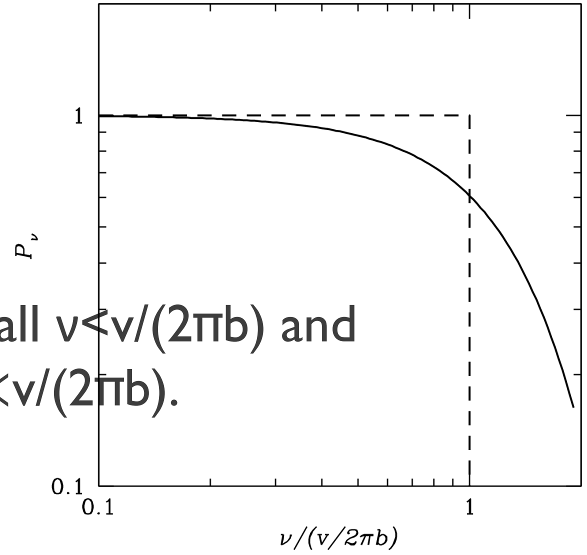
- We can obtain EM from the observed intensities of Ha and Hb emission!
- The EM changes dramatically for different lines of sight in the Galaxy!
- For the LOS through the center of the Orion nebula EM ~ 1e7 pc / cm6.
- For the LOS towards large galactic latitudes (e.g. 30^{o}), EM ~ 10 pc / cm6.

- Derive the electron density of Orion and DIG. Any caveats?

Free-Free Emission (Bremsstrahlung; “braking radiation”)

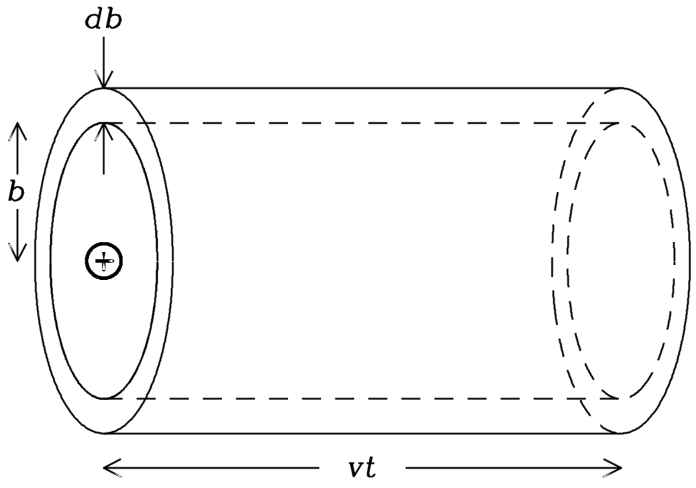
- Larmor’s formula: It implies that any charged particle radiates when accelerated and that the total radiated power is proportional to the square of the acceleration. The largest astrophysical accelerations are usually produced by electromagnetic forces, so the acceleration is proportional to the charge/mass ratio of the particle. In celestial sources, radiation from electrons is typically (mp/me)2≈1e6 times stronger than radiation from protons.

$$\boxed{P = \frac{2}{3}\frac{q^2 \dot{v}^2}{c^3}.} \qquad W_\nu \approx \frac{\pi^2}{2}\frac{Z^2 e^6}{c^3 m_e^2}\left(\frac{1}{b^2 v^2}\right)$$



- Collision -> acceleration -> power -> spectrum: The approximation Pν=1 for all ν<v/(2πb) and Pν=0 at higher frequencies (dashed line) is quite good at radio frequencies ν≪v/(2πb).

Free-Free Emissivity



- Integrate over impact parameters and velocity distribution:

$$j_{\mathrm{ff},\nu} = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} g_{\mathrm{ff},i}\,\frac{e^6}{m_e^2 c^3}\left(\frac{m_e}{kT}\right)^{1/2} \mathrm{e}^{-h\nu/kT} n_e Z_i^2 n_i$$

- If the Gaunt factor gff is independent of T (which is a good approximation), the emissivity would be independent of frequency below frequency nu<< kT/h!
- The cooling rate by free-free is then:

$$\Lambda_{\mathrm{ff}} = 4\pi\int_0^\infty j_{\mathrm{ff},\nu}\,d\nu = \frac{32\pi}{3}\left(\frac{2\pi}{3}\right)^{1/2}\frac{e^6}{m_e^2 h c^3}\left(m_e kT\right)^{1/2}\langle g_{\mathrm{ff}}\rangle_T Z_i^2 n_i n_e$$

- The cooling time is t_cool ~ T^0.5 / n_e!

Emission measure for free-free emission

- Integrate the free-free emissivity across the line-of-sight without considering free-free absorption.
- The intensity Inu is also proportional to EM: $$EM \equiv \int n_e n_p ds = \left[\frac{n_e n_p}{\kappa_{\mathrm{ff},\nu}}\right]_T \tau_\nu$$
- The physical significancy of EM is that the measurements of both line and continuum emission tie strongly to the LOS integration of the square of electron number density.
- In general, excition of both line and continuum emission is caused by collisions of free electrons with target protons/ions. As a result, the amount of emission per unit volume depends on the product of the number density of both protons/ions and electrons.

Collisionless processes in plasmas: wave-particle interaction

Electromagnetic wave propagation/oscillation in a cold plasma

- In a cold plasma with electron density n_e, electromagnetic waves propagating with E ∝ e^(ikx−iωt) must satisfy the dispersion relation:

$$k^2c^2 = \omega^2 - \omega_p^2$$

- Plasma frequency defined by the physical properties of the plasma is the frequency of the oscillation on the direction of the perturbation.

$$\omega_p \equiv \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} = 5.641 \times 10^4 \left(\frac{n_e}{\mathrm{cm}^{-3}}\right)^{1/2} \mathrm{s}^{-1}$$

Phase velocity and group velocity

- The phase velocity, the speed of propagation of a surface of constant phase, for a monochromatic wave is

$$v_{\mathrm{phase}} \equiv \frac{\omega}{k} = \frac{c}{\sqrt{1-(\omega_p/\omega)^2}} > c$$

- A "wave packet" in a dispersive medium will propagate with the group velocity:

$$v_g(\omega) = \frac{d\omega}{dk} = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}$$

- This is the speed at which information can be transmitted!
- Group velocity depends strongly on the EM wave frequency. Lower frequency wave arrive later!

Dispersion Measure (DM)

- Suppose that an astronomical object, e.g., a pulsar, emits a pulse of radiation at t = 0. If the distance to the pulsar is L, the time of arrival of energy at frequency ν = ω/2π is

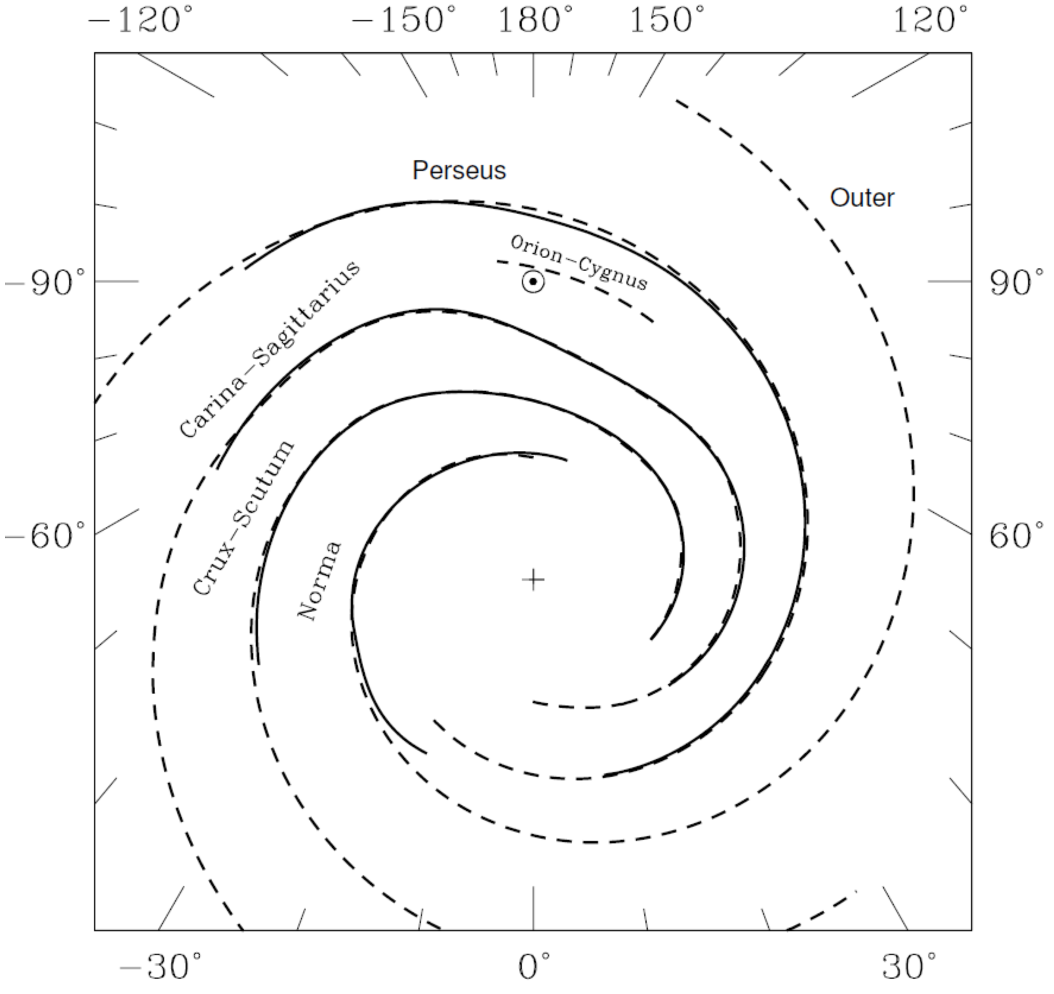
$$t_{\mathrm{arrival}} = \int_0^L \frac{dL}{v_g(\omega)}$$

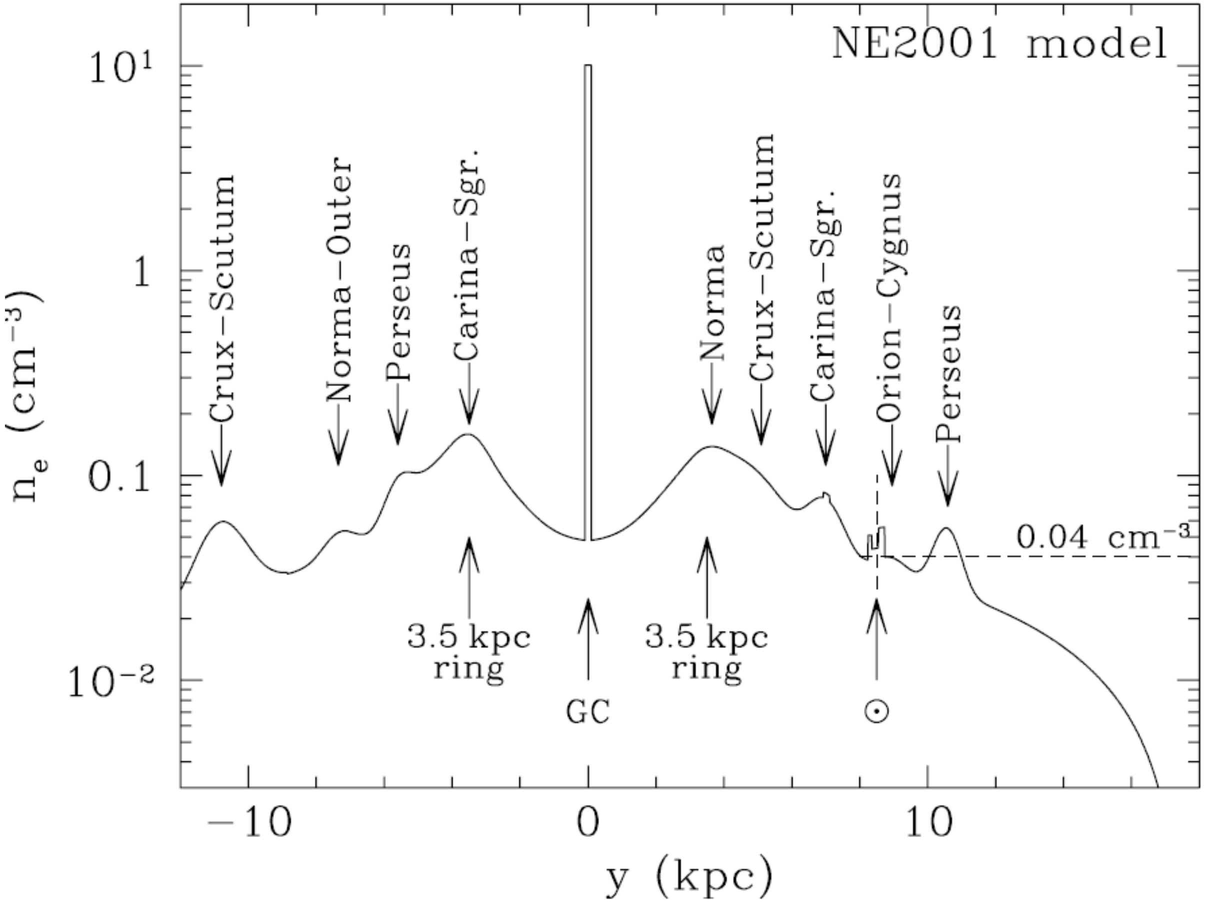
$$= \frac{L}{c} + \frac{e^2}{2\pi m_e c}\frac{1}{\nu^2} DM = \frac{L}{c} + 4.146 \times 10^{-3} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2} \frac{DM}{\mathrm{cm}^{-3}\,\mathrm{pc}}\,\mathrm{s}$$

- Dispersion measure is defined as $DM \equiv \int_0^L n_e dL$

- For a pulsar 3kpc away from the Sun, what is the radio signal travel time to the telescope? What is the delay time of the radio signal at 1 GHz?

DM as a powerful tool to obtain electron density distribution!





Electromagnetic wave propagation/oscillation in a magnetized plasma

- Now consider a static magnetic field B0 to be present in the plasma, and consider the propagation of electromagnetic waves with angular frequency ω. In cold plasma, the dispersion relation for circularly polarized waves is:

$$k^2c^2 = \omega^2 - \frac{\omega_p^2}{1 \pm \frac{\omega_B}{\omega}}$$

- In addition to plasma frequency, there exists another characteristic frequency called cyclotron frequency for the component of the magnetic field that is parallel to the direction of propagation: :

$$\omega_B \equiv \frac{eB_{\parallel}}{m_e c}$$

Rotation Measure (RM)

- The velocity of propagation of surfaces of constant phase, or phase velocity is:

$$v_{\text{phase}}(\omega) \equiv \frac{\omega}{k(\omega)} \approx c\left[1 + \frac{1}{2}\frac{\omega_p^2}{\omega^2} \mp \frac{1}{2}\frac{\omega_p^2\omega_B}{\omega^3}\right]$$

- The plus/minus sign applies to the right/left circular polarization. The two modes differ in phase velocity and wave number

$$v_{\text{phase,L}} - v_{\text{phase,R}} = c\frac{\omega_p^2\omega_B}{\omega^3} = \frac{4\pi e^3}{m_e^2}\frac{n_e B_\parallel}{\omega^3}$$

$$\Delta k \equiv k_{\text{R}} - k_{\text{L}} = \frac{\omega}{c}\frac{\omega_p^2\omega_B}{\omega^3} = \frac{4\pi e^3}{m_e^2 c^2}\frac{n_e B_\parallel}{\omega^2}$$

Rotation Measure (RM)

- The linear polarization mode obtained by adding the two circular polarization modes together will be rotated counterclockwise relative to the linear polarization of the source by a rotation angle:

$$\Psi = \frac{1}{2}\int_0^L \Delta k \, dL = \int_0^L \frac{\omega_p^2 \omega_B}{2c\omega^2} \, dL = RM \, \lambda^2$$

- The RM is defined as

$$RM \equiv \frac{1}{2\pi} \frac{e^3}{m_e^2 c^4} \int_0^L n_e B_{\parallel} dL$$

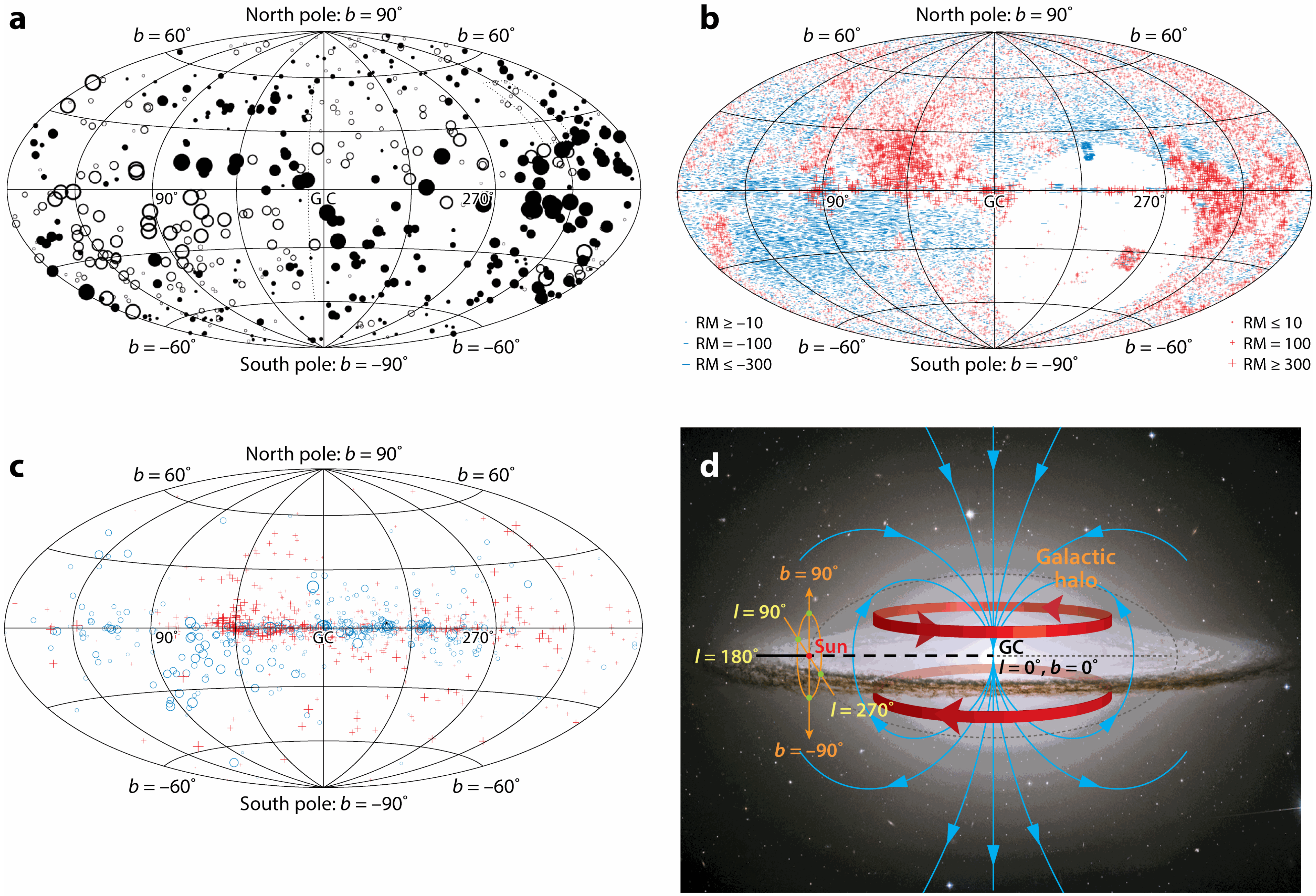
$$= 8.120 \times 10^{-5} \frac{\int_0^L n_e B_{\parallel} dL}{\mathrm{cm}^{-3} \, \mu\mathrm{G} \, \mathrm{pc}} \mathrm{rad \, cm}^{-2}$$

Measuring magnetic fields in the Milky Way

- If the dispersion measure and rotation measure can both be measured, we can determine the electron-density-weighted mean value of the line-of-sight magnetic field:

$$\langle B_{\|} \rangle = \frac{2\pi m_e^2 c^4}{e^3} \frac{RM}{DM} ,$$

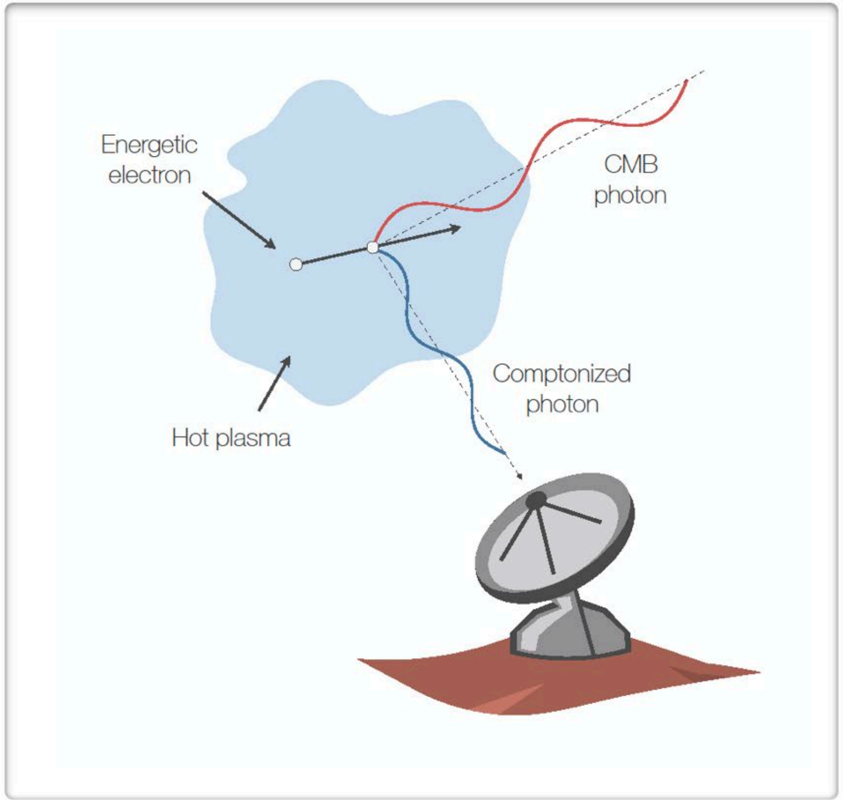
$$\frac{\langle B_{\|} \rangle}{\mu\mathrm{G}} = \frac{RM}{8.120 \times 10^{-5}\,\mathrm{rad\,cm^{-2}}} \times \frac{\mathrm{cm^{-3}\,pc}}{DM}$$

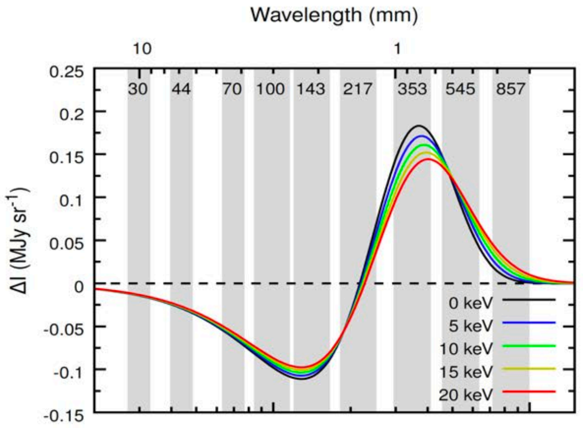


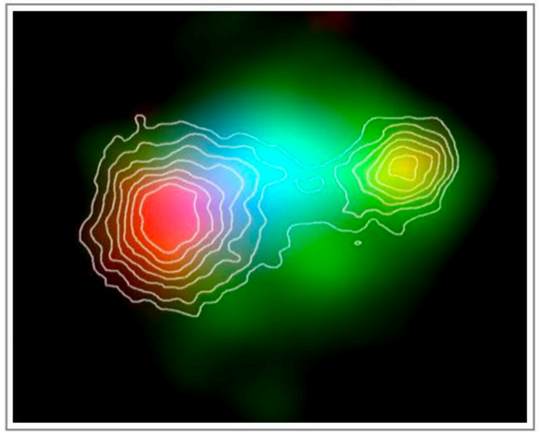
Han JL. 2017.
Annu. Rev. Astron. Astrophys. 55:111–57

Photons interacting with thermal electrons

- A fraction of CMB photons pass through massive clusters before reaching our detector, and some of these will interact with high energy electrons through inverse Compton scattering. This changes the energies of the CMB photons and produces an observable deviation from the Planck function in the CMB spectrum.
- This is call the thermal Sunyaev-Zeldovich (S-Z) effect.







Kinematic S-Z effect: Doppler and Relativistic Beaming

- When a hot cloud of electrons (such as those found in galaxy clusters) has a net velocity relative to the background light (usually the CMB), it scatters the photons isotropically in cluster rest frame. When this is transformed back to the CMB rest frame, the scattered photons are beamed, increasing the intensity in the direction of motion, and decreasing it behind the cluster.