



清华大学天文系  
Department of Astronomy, Tsinghua University

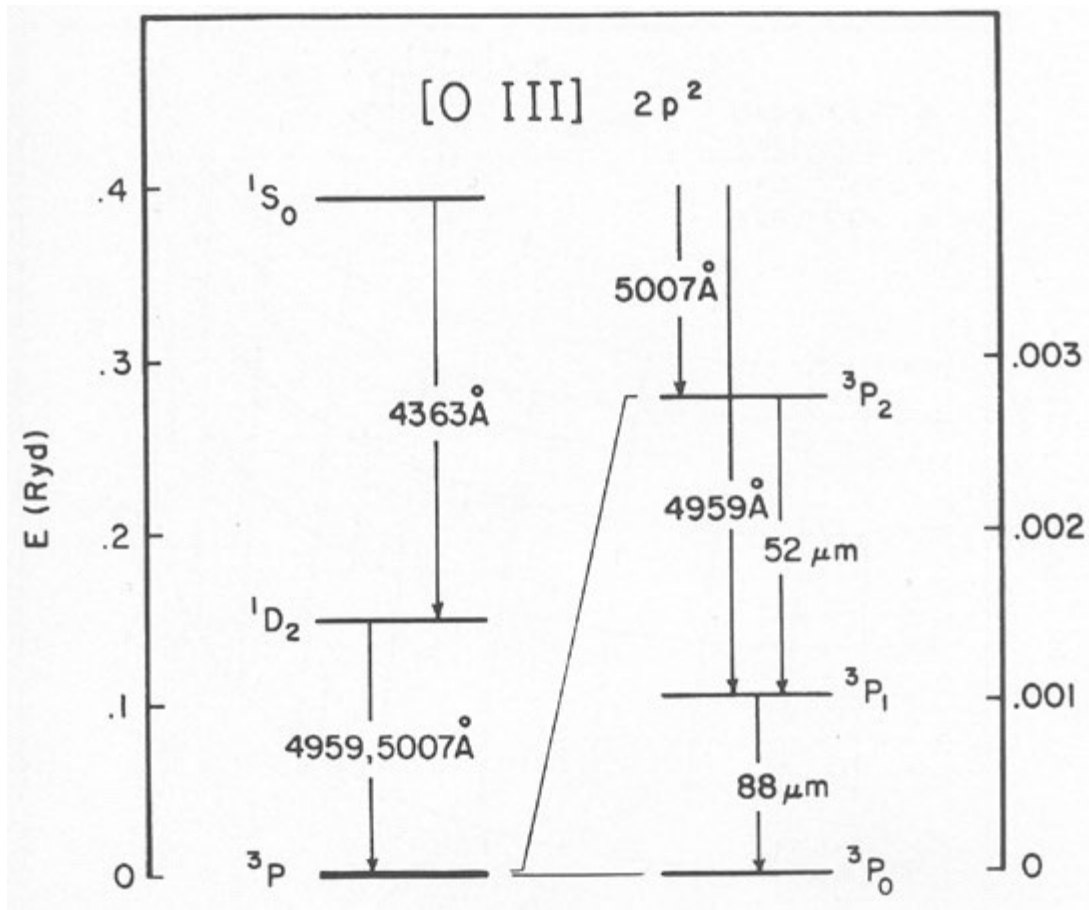
# 3. Atomic Physics

Instructor: Hui Li      TA: Chengzhe Li

Department of Astronomy

Tsinghua University

## Why do we care about atomic physics in the ISM?



[OIII]  $1D_2 \rightarrow 3P_2$  (501 nm)  
 $1D_2 \rightarrow 3P_1$  (496 nm)  
 $1S_0 \rightarrow 1D_2$  (436 nm)

# Wave Equation for Atoms

- Schrodinger equation in Spherical coordinates:

$$\left\{ -\frac{\hbar^2}{2\mu r^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right] - \frac{e^2}{4\pi\epsilon_0 r} \right\} \psi(r, \theta, \varphi) = E\psi(r, \theta, \varphi)$$

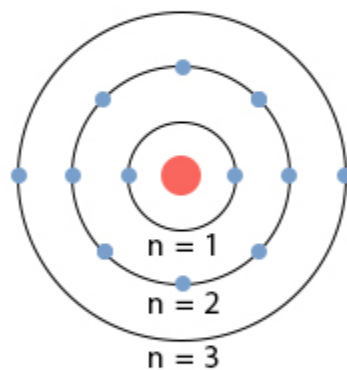
- Spherical harmonic expansion solution:

$$\psi_{(n,l,m_l)} = R_{n,l}(r) \times Y_{l,m_l}(\theta, \phi)$$

# Quantum Numbers

## Quantum Numbers

1. Principal



$n$

Distance of the  
electron from nucleus

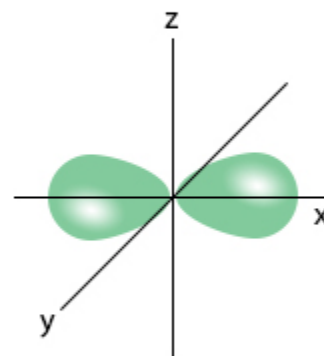
2. Azimuthal



$l$

Shape of  
the orbital

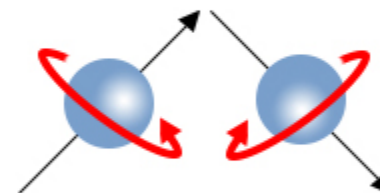
3. Magnetic



$m_l$

Orientation of  
the orbital

4. Spin



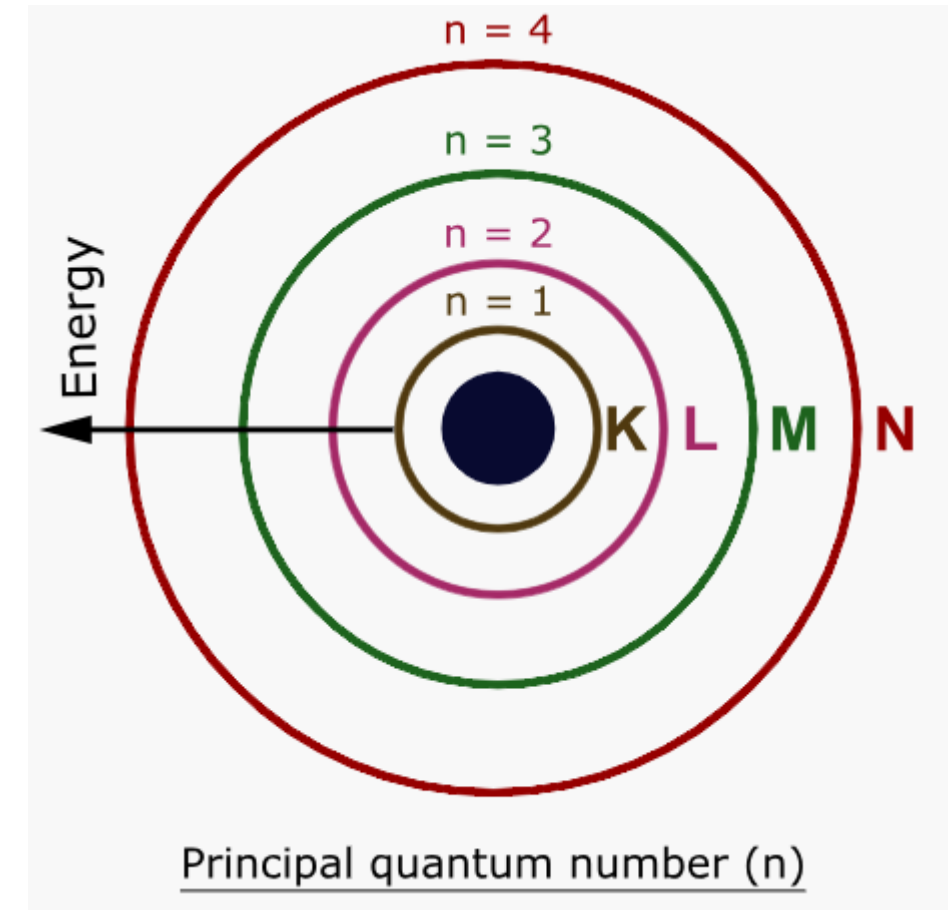
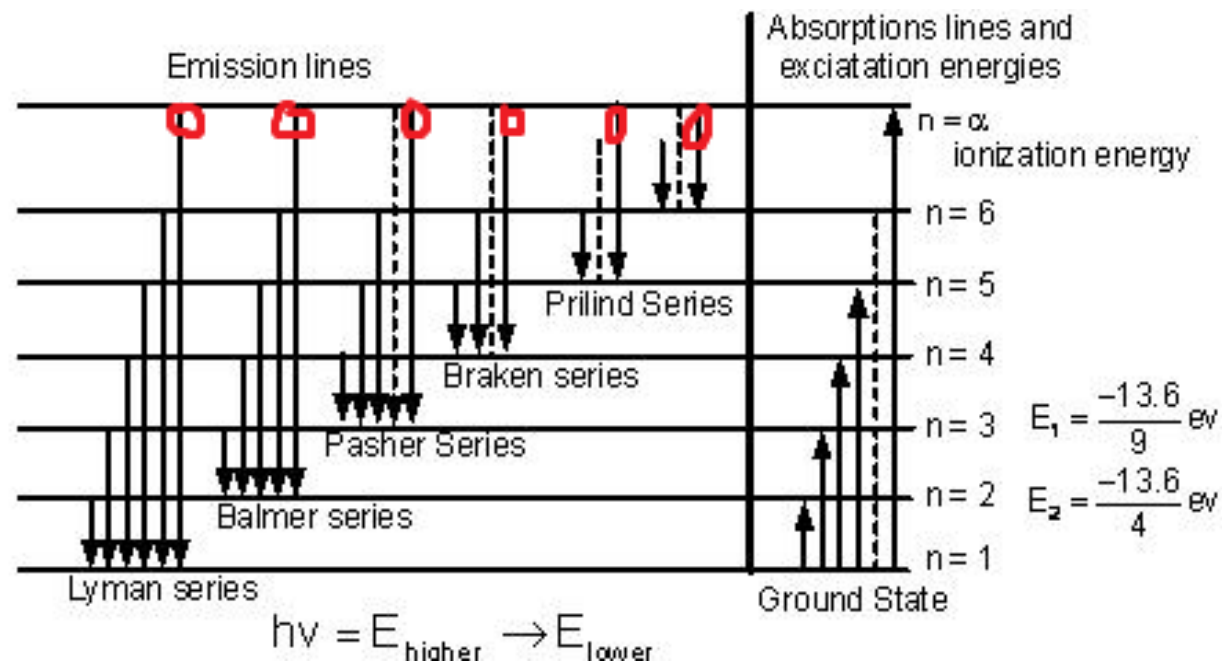
$m_s$

Orientation of  
the electron spin

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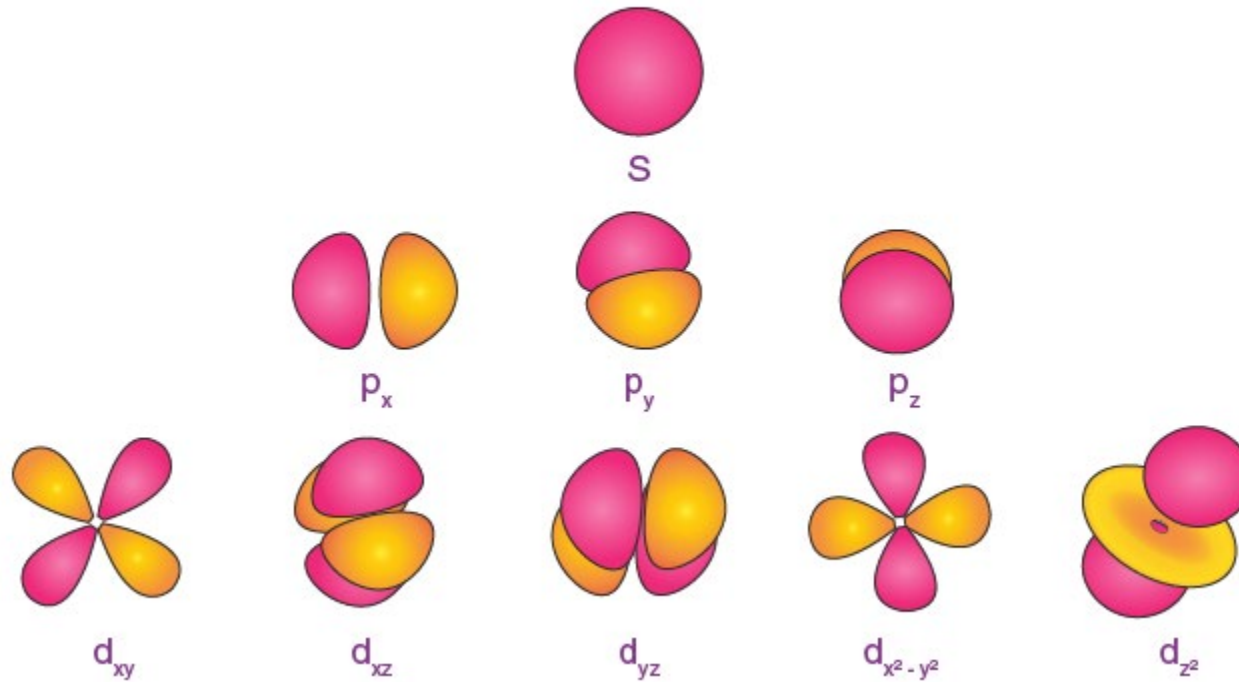
# Principal Quantum Number

- $N = 1, 2, 3, 4 \dots$
- Refer as K, L, M, N ... shells.
- Lines from these shells are called Lyman, Balmer, Paschen, Brakett ... series.



# Angular Momentum and Magnetic Quantum Numbers

## QUANTUM NUMBERS



# Put everything together!

| SYMBOL | NAME             | VALUES                       | MEANING  |
|--------|------------------|------------------------------|--|
| $n$    | principal        | 1, 2, 3...(any integer)      | energy level, shell  |
| $l$    | angular momentum | $0 \rightarrow n - 1$        | subshell, $0 = s, 1 = p, 2 = d, 3 = f \dots$<br>this is the angular dependence of the orbital, shape of the orbital<br><small>*letters have historical meaning, sharp, principle, diffuse, fundamental</small> |
| $m_l$  | magnetic         | $+l \rightarrow -l$          | orientation of angular momentum in space, orbital  |
| $m_s$  | spin             | $+\frac{1}{2}, -\frac{1}{2}$ | the imaginary property we call "spin", up or down  |

# Electronic Configuration

- Pauli exclusion principle: only one electron may occupy an (n, l, ml, ms) wavefunction at a time.
- Each l subshell can contain at most ? electrons?
- Each n shell can contain at most ? electrons?
- Can we get the electron configuration of ground state of O I and S I?

| Atom | # electrons | Configuration              |
|------|-------------|----------------------------|
| H I  | 1           | $1s^1$                     |
| He I | 2           | $1s^2$                     |
| C I  | 6           | $1s^2 2s^2 2p^2$           |
| N I  | 7           | $1s^2 2s^2 2p^3$           |
| O I  | 8           | $1s^2 2s^2 2p^4$           |
| Ne I | 10          | $1s^2 2s^2 2p^6$           |
| Mg I | 12          | $1s^2 2s^2 2p^6 3s^2$      |
| Si I | 14          | $1s^2 2s^2 2p^6 3s^2 3p^2$ |
| S I  | 16          | $1s^2 2s^2 2p^6 3s^2 3p^4$ |
| Ar I | 18          | $1s^2 2s^2 2p^6 3s^2 3p^6$ |





# Spectroscopic Terms

- The electronic configuration gives the arrangement of electrons in the atom or ion; however, the energy levels available to form spectral lines in that configuration are the result of the interactions among the electrons in the outermost (sub-)shell.
- Three total quantum numbers associated with the collective interactions of the outermost shell electrons.

$$2S+1 \mathcal{L}_J^p$$



## Some common configurations

| Configuration | Terms             | Configuration | Terms                       |
|---------------|-------------------|---------------|-----------------------------|
| $\dots ns^1$  | $^2S_{1/2}$       | $\dots np^3$  |                             |
| $\dots ns^2$  | $^1S_0$           | $\dots np^4$  | $^3P_{2,1,0}, ^1D_2, ^1S_0$ |
| $\dots np^1$  | $^2P^o_{1/2,3/2}$ | $\dots np^5$  | $^2P^o_{3/2,1/2}$           |
| $\dots np^2$  |                   | $\dots np^6$  | $^1S_0$                     |

# Energy differences between levels

- The energy differences between levels with different  $L$  quantum numbers are of order a few eV for astrophysically-interesting atomic and ionic species, and thus produce spectral lines at ultraviolet, optical, and near-infrared wavelengths.
- The energy differences between levels with different  $J$  quantum numbers within a given term (for example, between the  $J$  states in the  $3P_J$  terms of the  $\dots np^2$  configuration) are a few 0.01 eV, and thus produce spectral lines at far-infrared wavelengths.
- Spectral lines from transitions between different  $J$  quantum states within a term are called fine structure lines.

# Selection Rules

Permitted lines arise from electric dipole transitions that obey all the dipole selection rules:

1.  $\Delta L = 0, \pm 1$
2.  $\Delta J = 0, \pm 1$ , but  $J = 0 \rightarrow 0$  is forbidden
3.  $\Delta S = 0$
4. one electron  $n\ell$  changes with  $\Delta \ell = \pm 1$
5. parity changes

Permitted lines from transitions into or out of the ground state multiplet are called **resonance lines**.

## ○ III lines (permitted, semi-forbidden, forbidden)

- ○ III 3313 Å  $2p^1 3d^1 {}^3P_2^o$  to  $2p^1 3p^1 {}^3S_1$ ,  $A_{ul} = 1.18 \times 10^{10} \text{ s}^{-1}$
- ○ III] 1666 Å  $2s 2p^3 {}^5S_2^o$  to  $2s^2 2p^2 {}^3P_2$ ,  $A_{ul} = 8.06 \times 10^2 \text{ s}^{-1}$
- [○ III] 5007 Å  $2s^2 2p^2 {}^1D_2$  to  $2s^2 2p^2 {}^3P_2$ ,  $A_{ul} = 2.05 \times 10^{-2} \text{ s}^{-1}$



# Hyperfine Structure Lines

- If the nucleus has a non-zero magnetic moment, the interaction between the magnetic fields of the nucleus and electrons gives rise to a small energy difference of order  $10^{-6}$  eV, called **hyperfine structure**. A hyperfine energy level has quantum number **F**, which is the sum of the quantized nuclear spin angular momentum **I** and the electron total angular momentum **J**: **F=I+J**.
- The most astrophysically-important hyperfine structure transition is in the hydrogen  $1s\ 2S_{1/2}$  ground state, which has  $I=1/2$  and  $J=1/2$ , corresponding to hyperfine quantum numbers  $F=0$  (anti-parallel spins) and  $F=1$  (parallel spins). The energy difference between the  $F=1$  and  $F=0$  states is  $5.874 \times 10^{-6}$  eV which corresponds to a radio wavelength of 21.106 cm. The transition probability is  $A = 2.88 \times 10^{-15}/s$ . Tiny!!!