AY2024F

Junjie Mao

Radiative Processes in Astrophysics

Observation

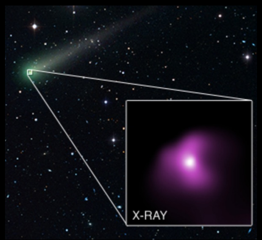
Up to cosmic size scale

Goal

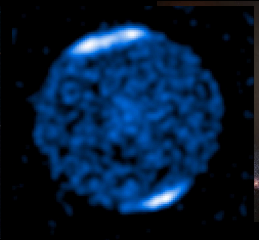
Theory

Down to atomic size scale

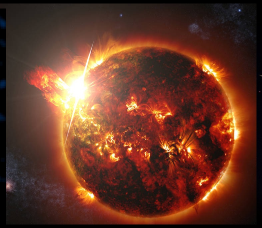
C/2012S1 (comet)



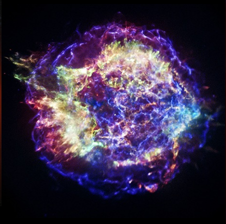
Jupiter (planet)



Sun (star)

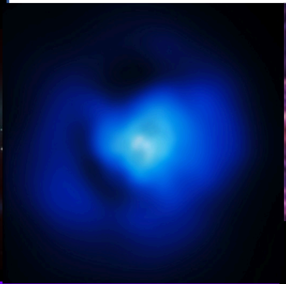


Cas A (SNR)



M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

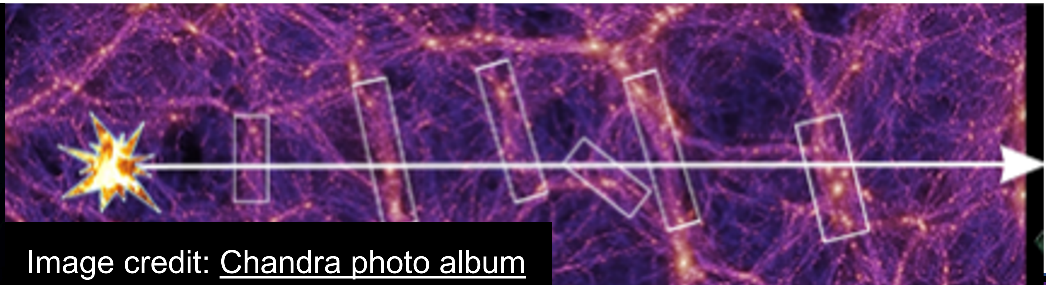
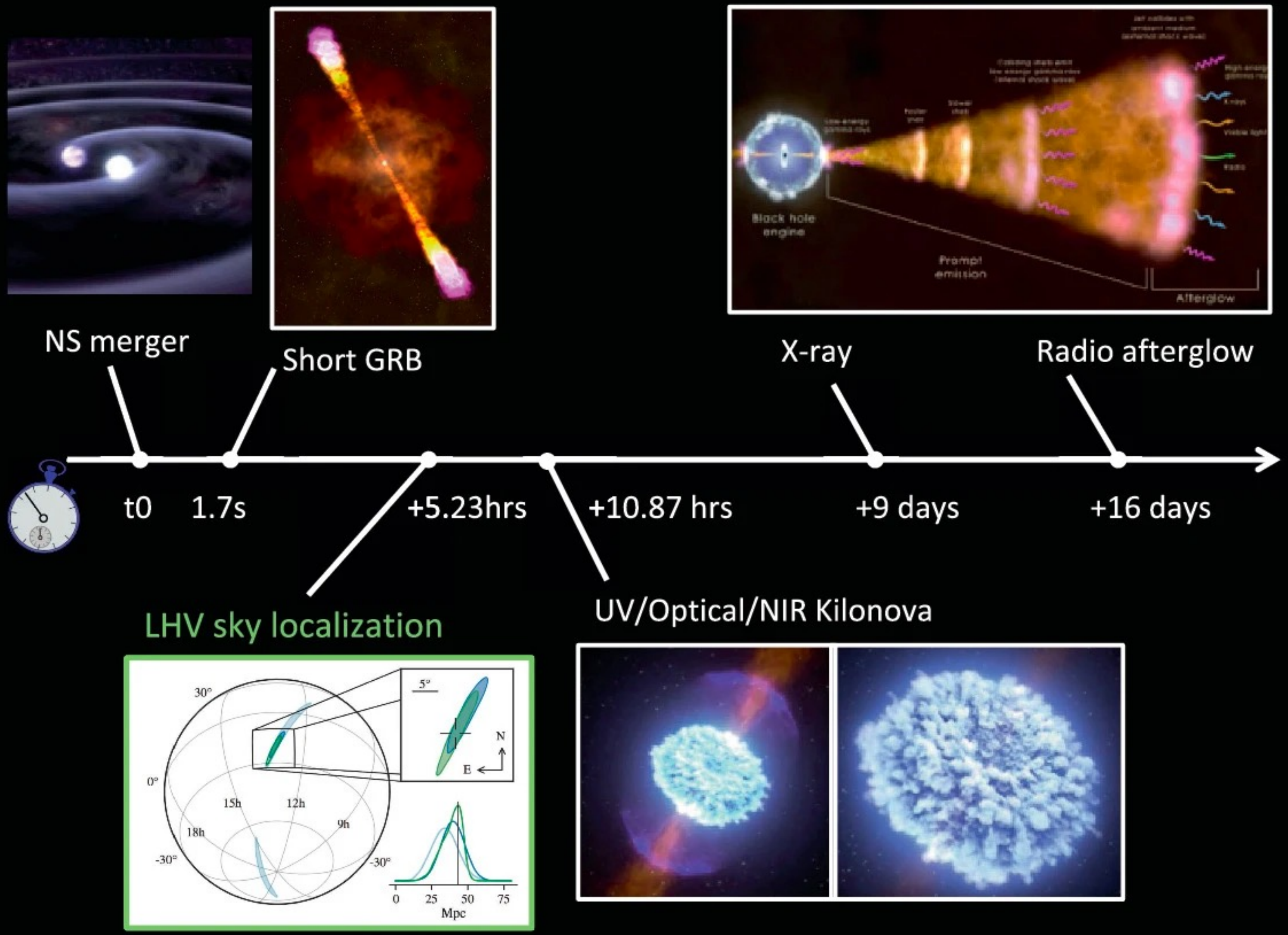


Image credit: Chandra photo album

Chpt.1 Astrophysical messengers

1.1 Electromagnetic waves
1.2 Neutrinos
1.3 Cosmic rays
1.4 Gravitational waves

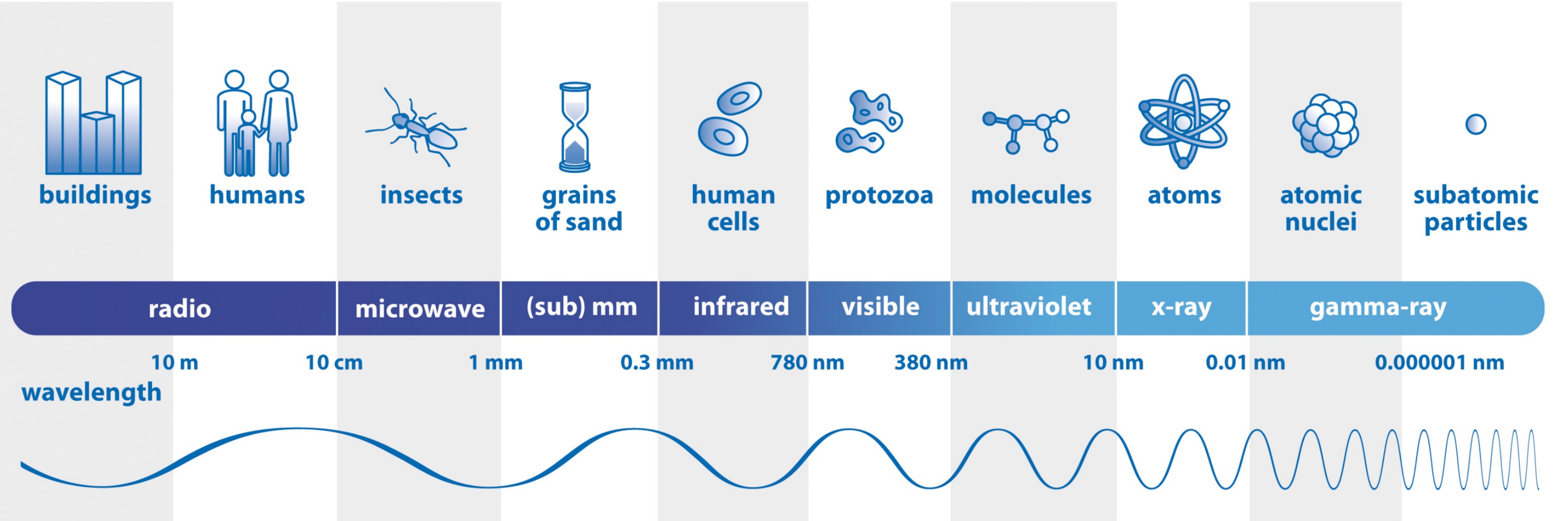


GW170817 (GW)
+GRB170817A (γ-ray)
+AT2017gfo (NIR to UV)
+ GRB afterglow (X-ray + radio)

Image credit: Branchesi (2023)

Electromagnetic waves

Image credit: ESA



speed of light

wavelength

frequency

$$c = \lambda \nu$$

Planck constant

$$E = h\nu$$

$$T = E/k$$

Boltzmann constant

Atmospheric window

Space missions (almost all UV, X-ray, and γ-ray missions)

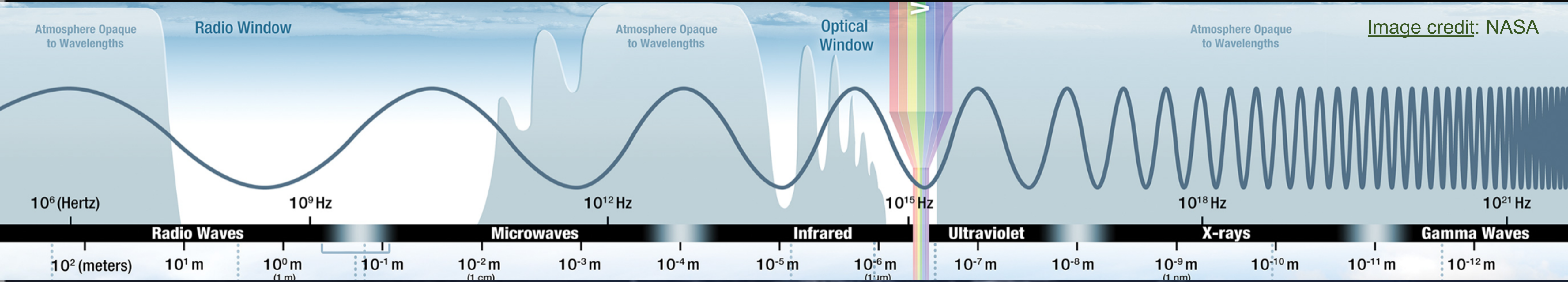
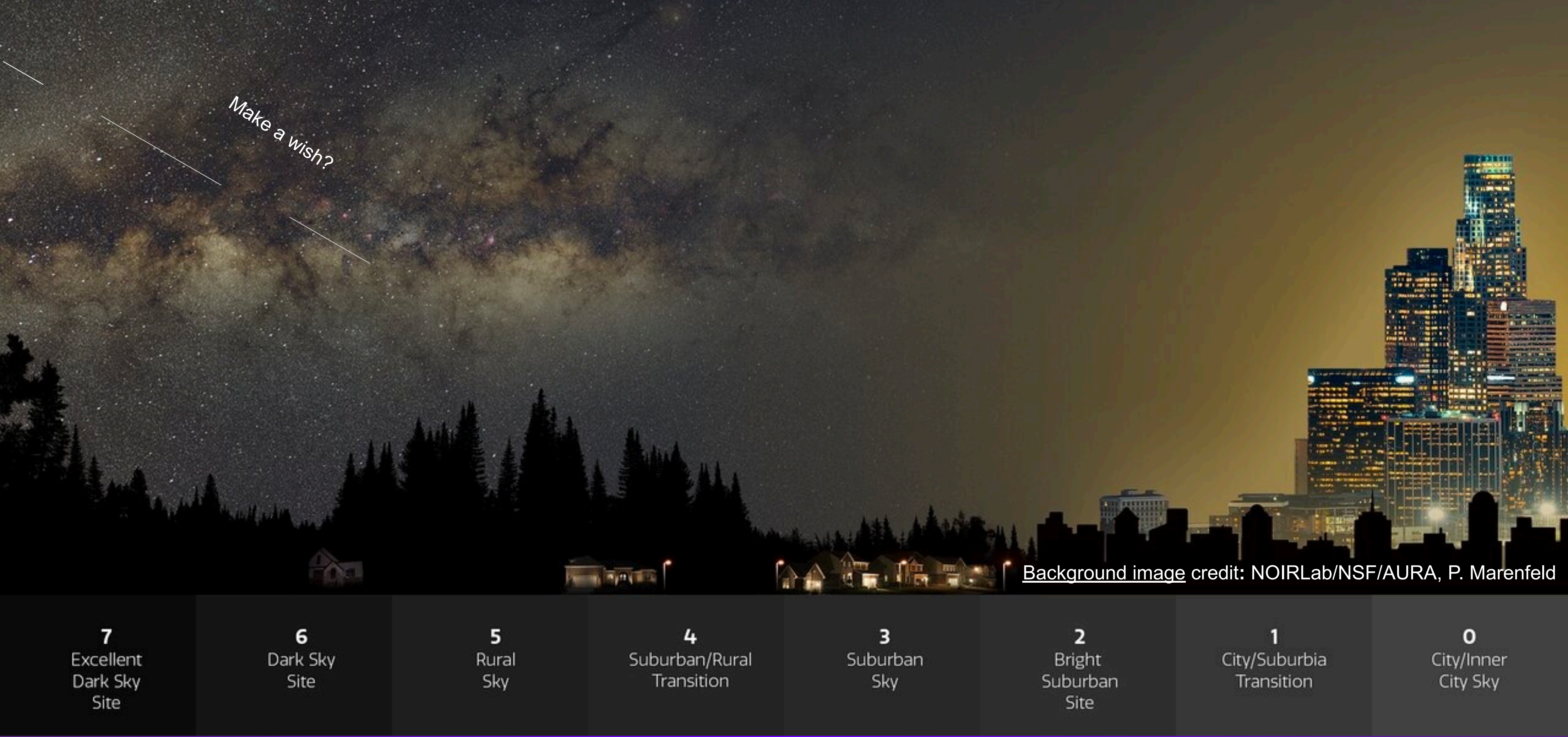


Image credit: NASA

Ground facilities (mainly radio, IR, and optical telescopes)

Background image credit: CMS

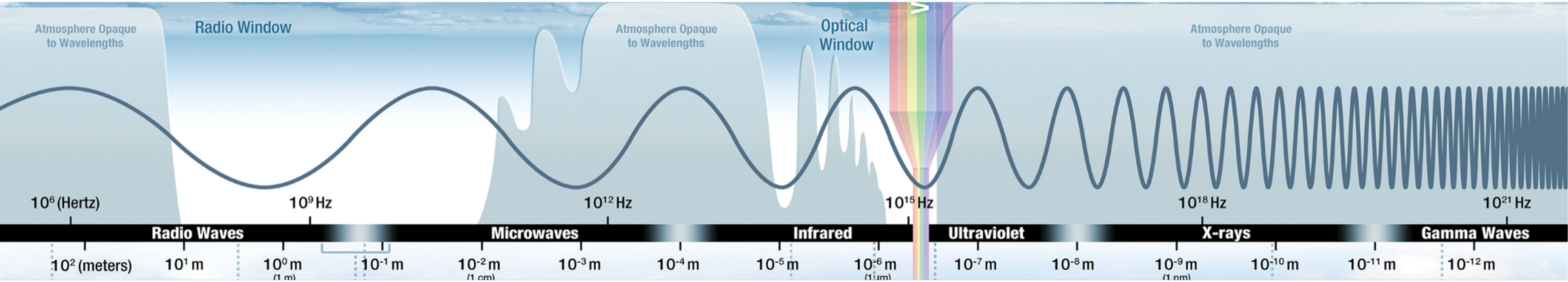
Not just atmospheric window



Background image credit: NOIRLab/NSF/AURA, P. Marenfeld

EM observing facilities

Image credit: NASA

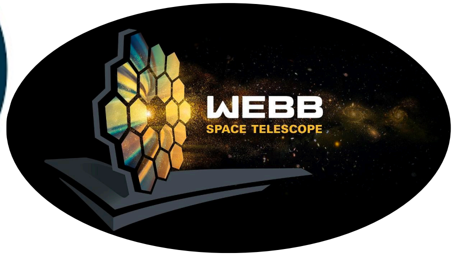


LOFAR (radio)
Image credit: ASTRON

Planck (microwave)
Image credit: ESA

HST (IR to UV)
Image credit: HST.org

Fermi (γ-ray)
Image credit: NASA



ALMA (microwave)
Image credit: ESO

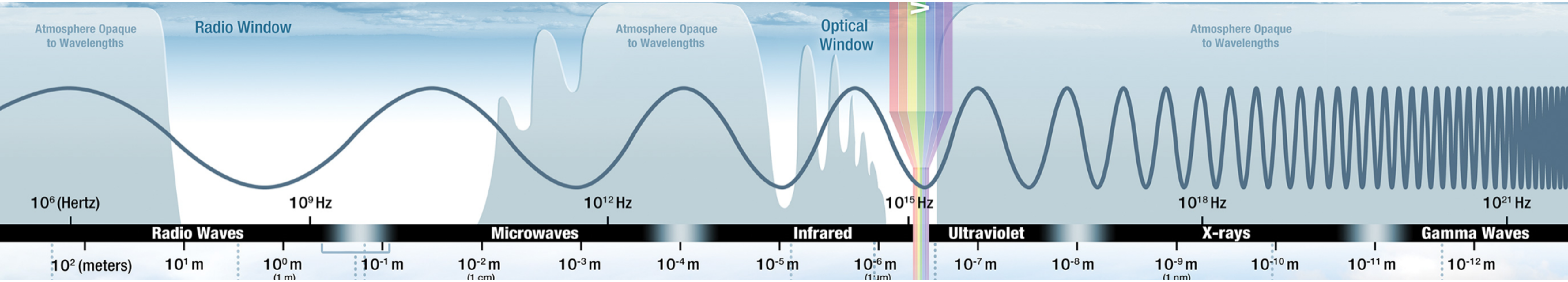
JWST (IR to optical)
Image credit: NASA

Gemini (IR to optical)
Image credit: NOIRLab

XRISM(X-ray)
Image credit: JAXA

EM observing facilities (cont.)

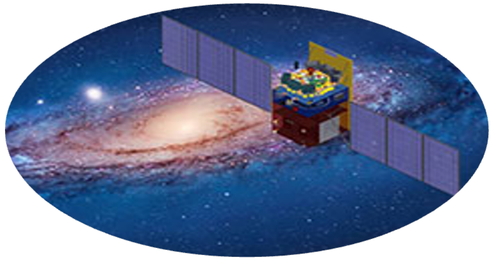
Image credit: NASA



FAST (radio)
Image credit: Nature

LAMOST (optical)
Image credit: NAOC

ASO-S (UV to X-ray)
Image credit: PMO

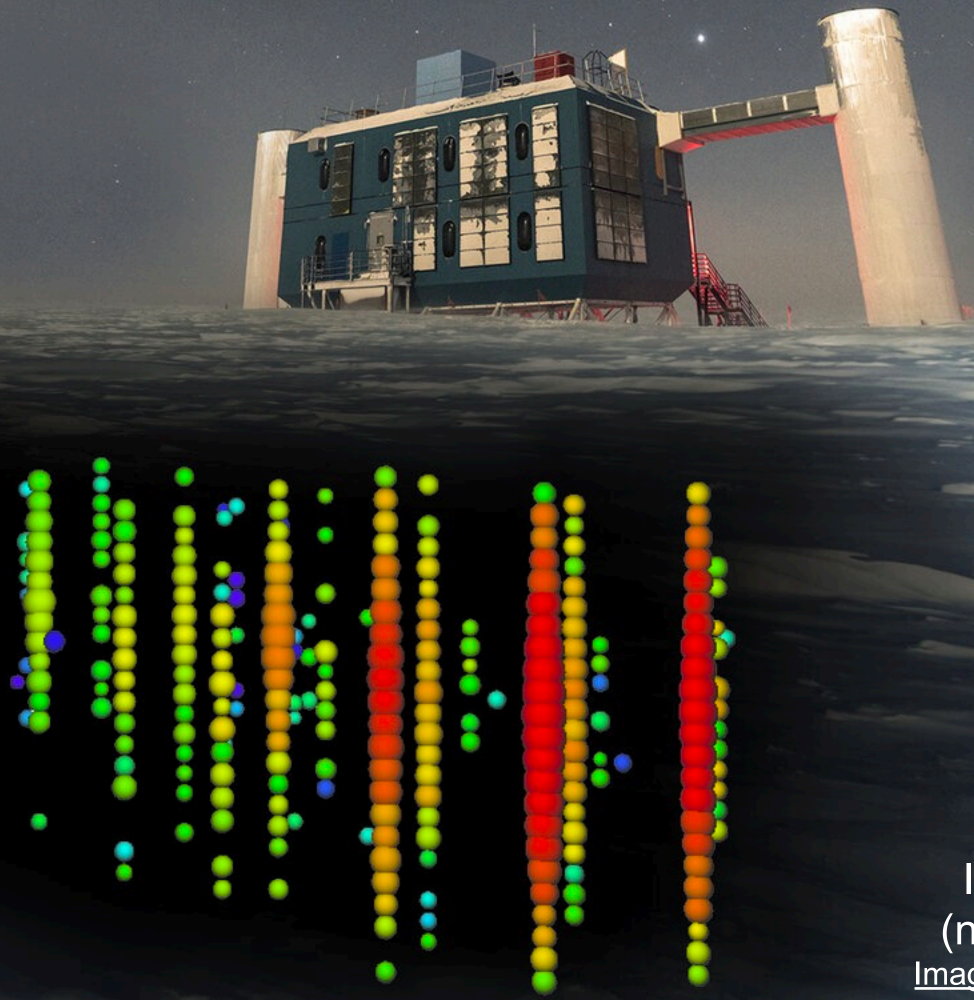


insight-HMXT
(X-ray to γ-ray)
Image credit: IHEP

Neutrinos

The neutrino was postulated by Wolfgang Pauli in the early 1930s but could only be detected for the first time in the 1950s

- ❑ Symbol is lowercase Greek letter: ν
  - Three flavors: electron ν_e, muon ν_μ, tau ν_τ
  - Oscillates between flavors
- ❑ Produced by nuclear processes
- ❑ Basic properties
  - Electrically neutral
  - Nearly massless
  - Travels close to the speed of light
- ❑ Interaction
  - Reacts to gravity and the weak nuclear force
  - Mostly passes through matter
  - Billions of neutrinos pass through our body every second



IceCube (neutrinos)
Image credit: ESA

Neutrinos (IceCube)

High energy (~290 TeV) neutrino from IceCube-170922A was detected in association with a known gamma-ray blazar TXS 0506+056 ($z \sim 0.3365$). This is the third detections of individual astrophysical sources of neutrinos, after the Sun and SN1987A.

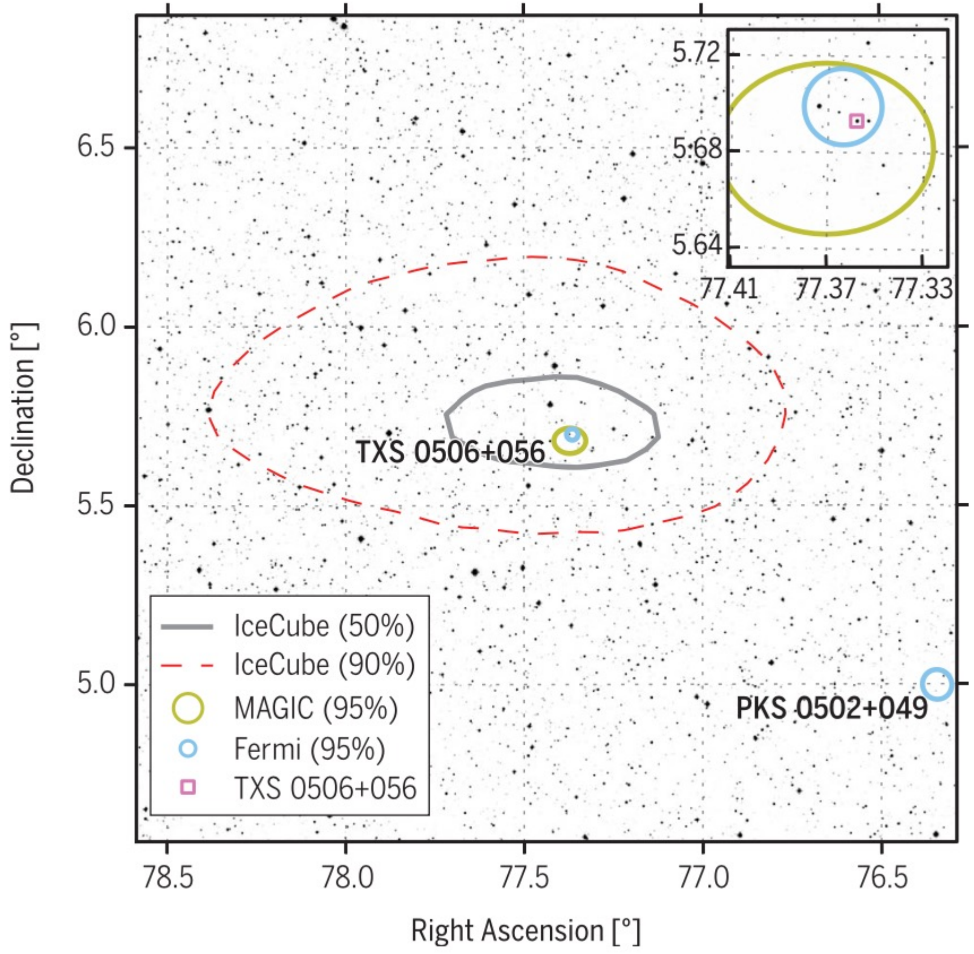
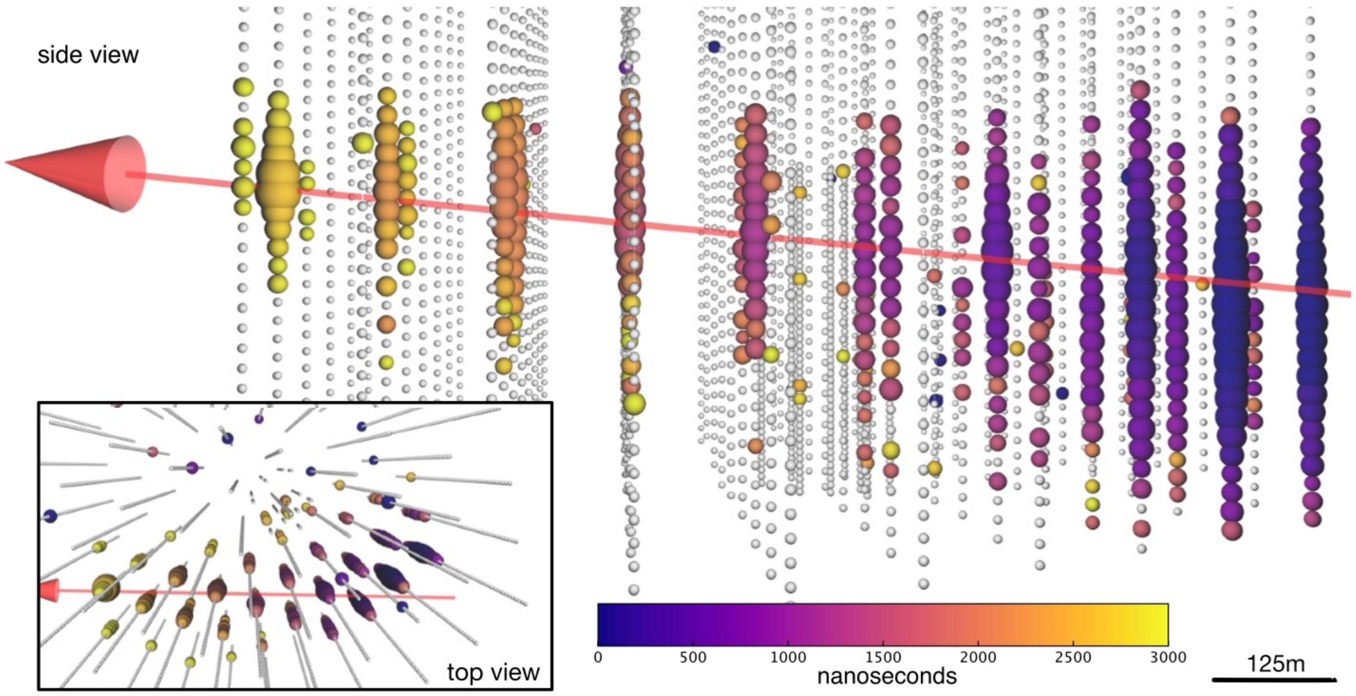


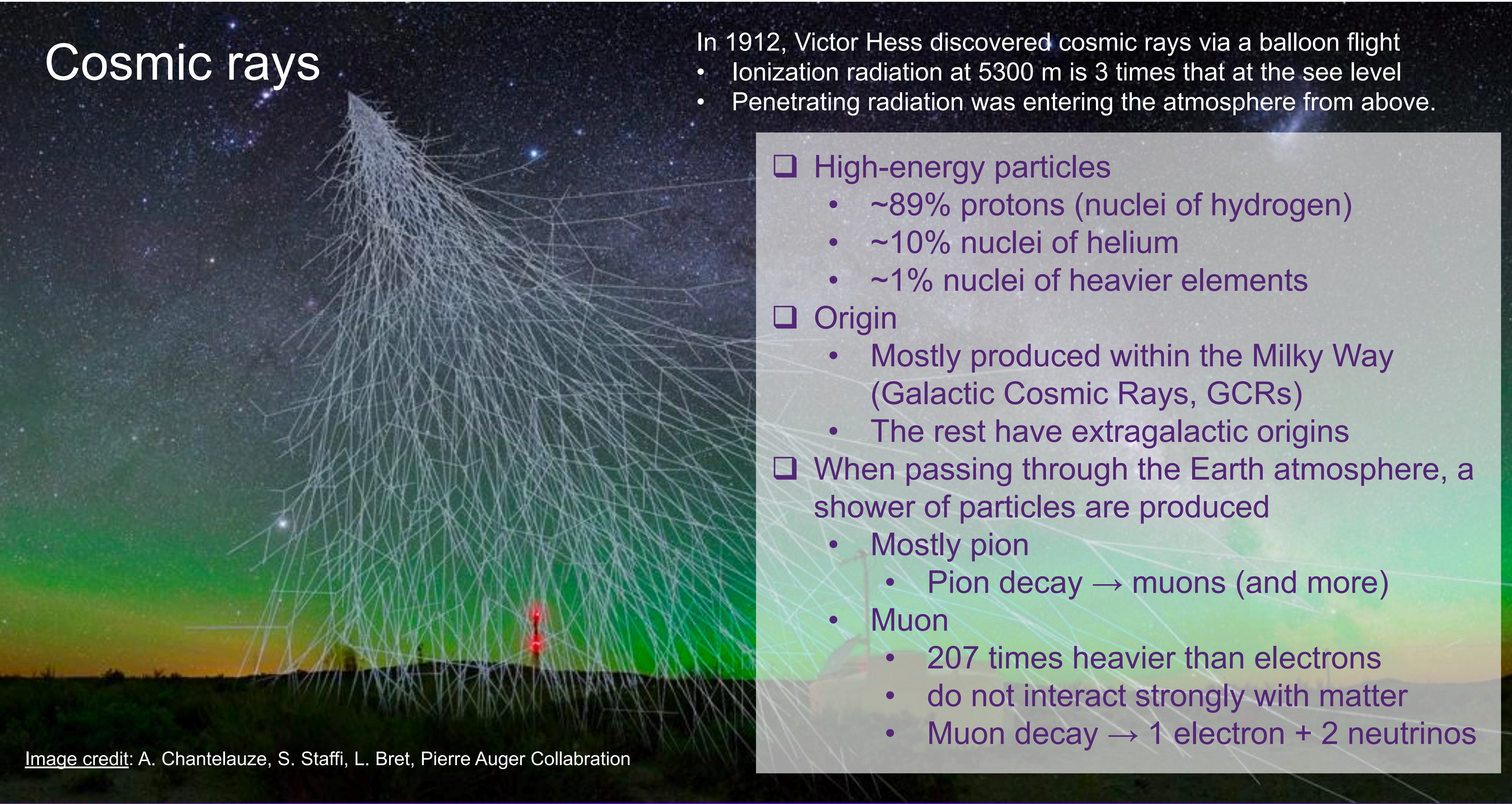
Image credit: IceCube collaboration (2023)

~5800 photoelectrons in ~3000 ns (from left to right)



Cosmic rays

In 1912, Victor Hess discovered cosmic rays via a balloon flight
- Ionization radiation at 5300 m is 3 times that at the see level
- Penetrating radiation was entering the atmosphere from above.



- ❑ High-energy particles
  - ~89% protons (nuclei of hydrogen)
  - ~10% nuclei of helium
  - ~1% nuclei of heavier elements
- ❑ Origin
  - Mostly produced within the Milky Way (Galactic Cosmic Rays, GCRs)
  - The rest have extragalactic origins
- ❑ When passing through the Earth atmosphere, a shower of particles are produced
  - Mostly pion
    - Pion decay → muons (and more)
  - Muon
    - 207 times heavier than electrons
    - do not interact strongly with matter
    - Muon decay → 1 electron + 2 neutrinos

Image credit: A. Chantelauze, S. Staffi, L. Bret, Pierre Auger Collabration

Cosmic rays (LHAASO)

Only one particle with $E > 10^{17}$ eV arrives each day in every 10^4 m^2!

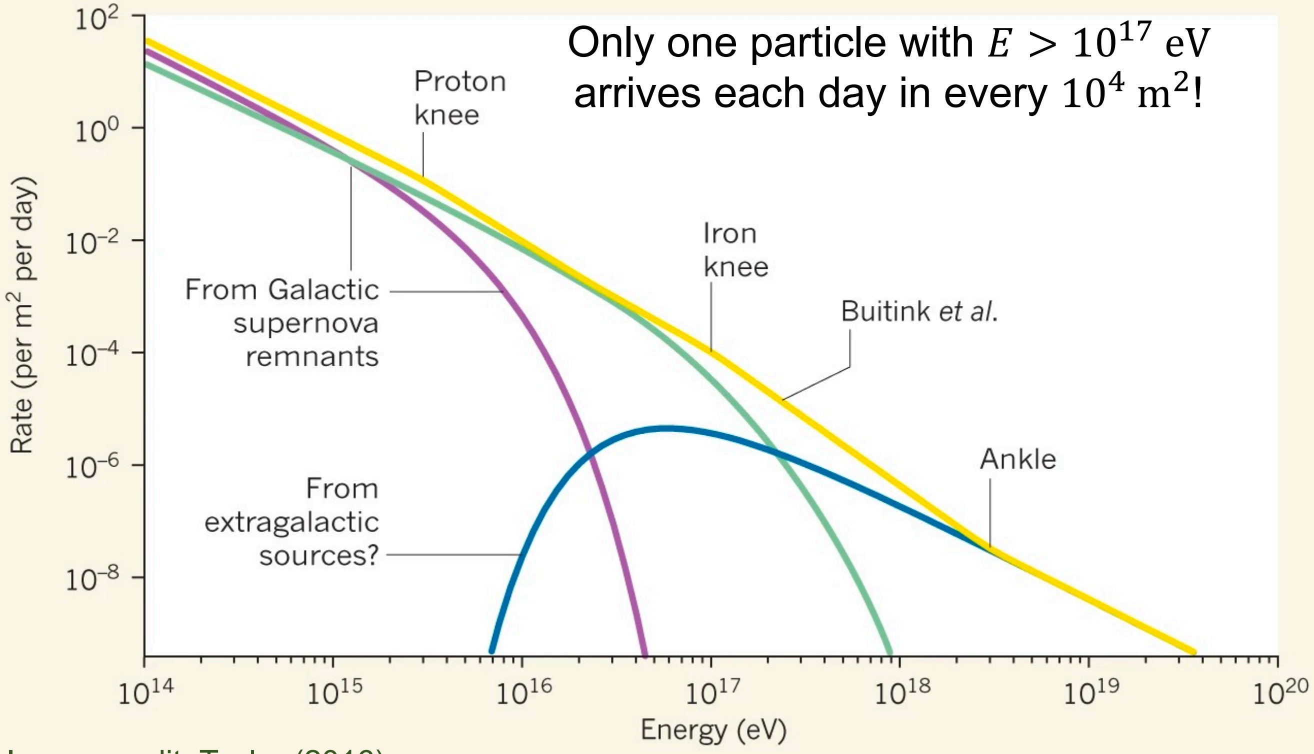
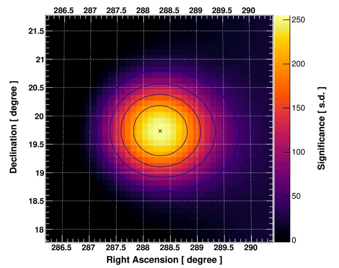


Image credit: Taylor (2016)

Large High Altitude Air Shower Observatory (LHAASO) (cosmic rays)

Image credit: Science



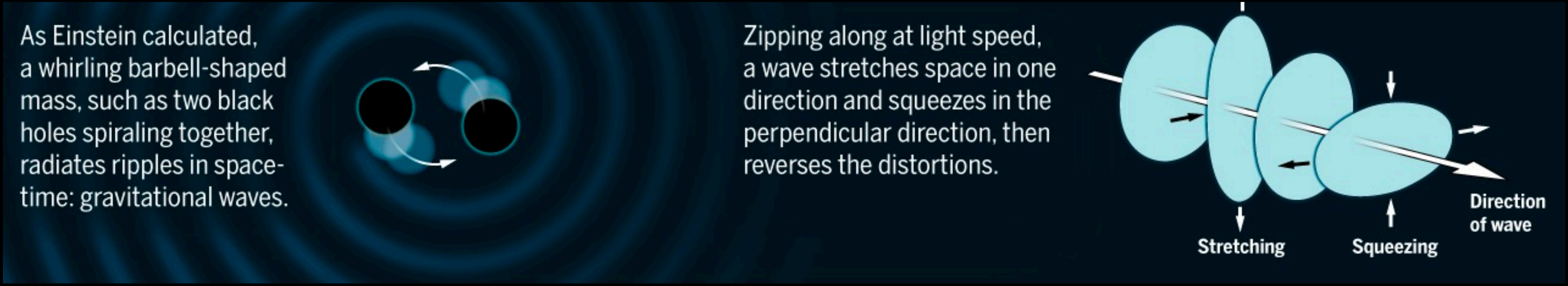
A TeV afterglow from GRB 221009A (LHAASO collaboration, 2023)

Tera (trillion): $\mathrm{TeV} = 10^{12}$ eV
Peta (quadrillion): $\mathrm{PeV} = 10^{15}$ eV
Exa (quintillion): $\mathrm{EeV} = 10^{18}$ eV
Zetta (sextillion): $\mathrm{ZeV} = 10^{21}$ eV

Clues about the origin of cosmic rays come from both their composition and their energy spectra.

Gravitational wave

Image credit: V. Altounian/Science



link: Astrobites

- Gravitational waves
  - "Ripples" in space-time caused
- Origin
  - Mostly produced by mergers of massive objects

Image credit: Caltech and R. Hurt (IPAC)

Gravitational waves (LIGO)

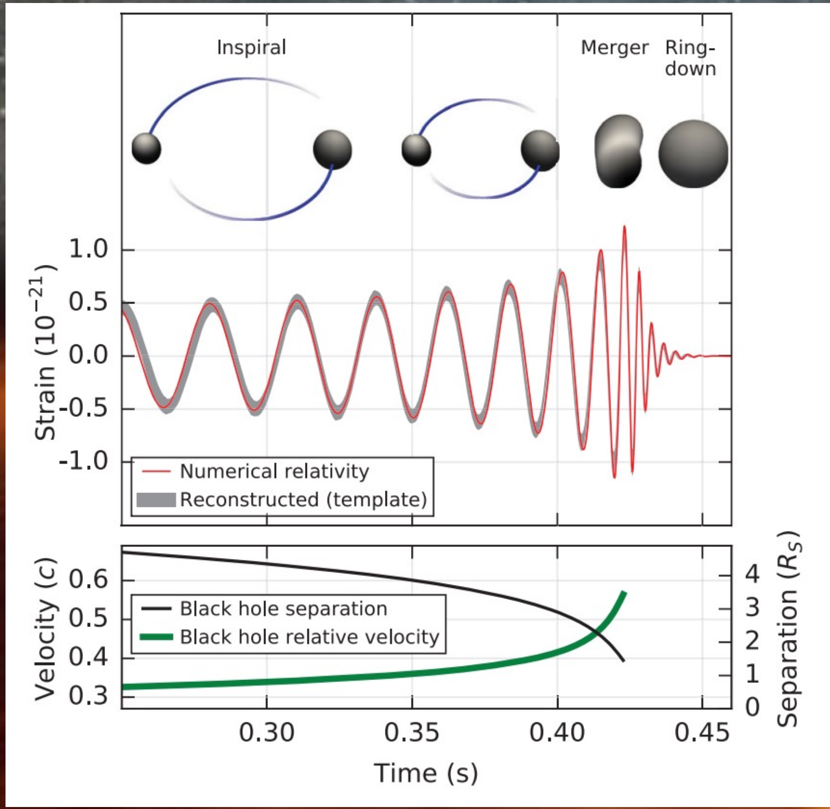
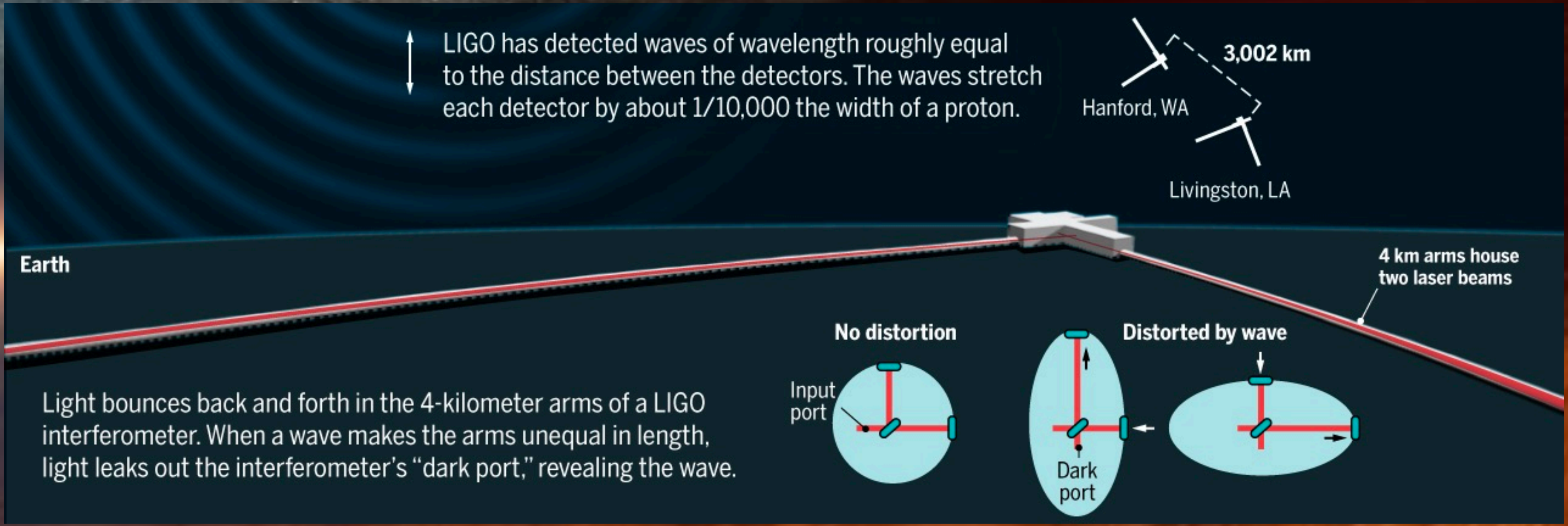


Image credit: Abbott et al. 2016



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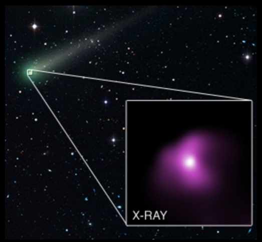
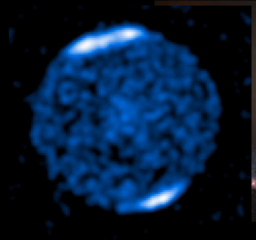
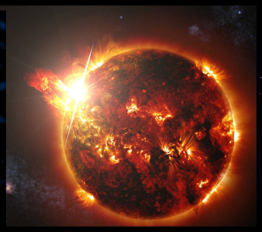


Jupiter (planet)



Sun (star)



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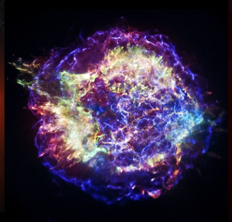
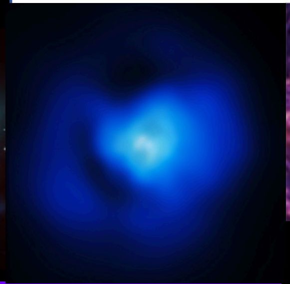


M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

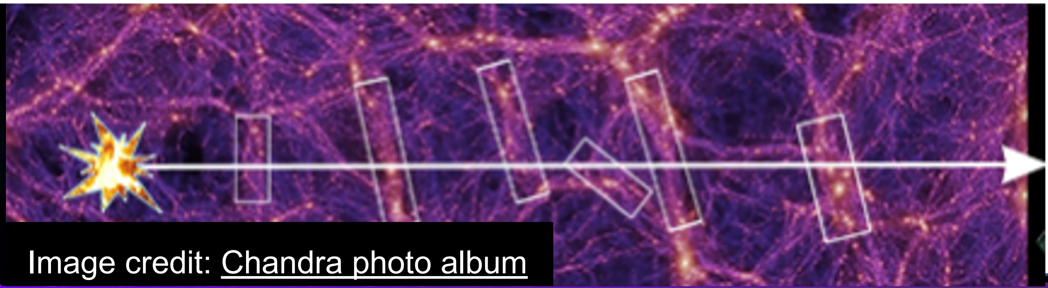


Image credit: Chandra photo album

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.1.1 Radiative flux
2.1.2 Radiative intensity
2.1.3 Specific intensity
2.1.4 Specific flux and momentum
2.1.5 Specific energy density

2.2 Radiative transfer
2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

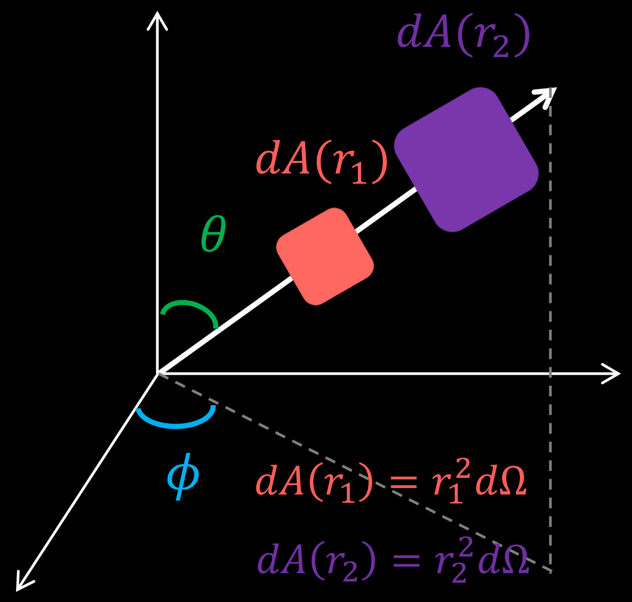


Image credit: Junjie Mao

Radiative flux

For a detector element (with a photon collecting area dA) exposed to a radiation source for a time interval dt, the number of photons or the amount of energy received is

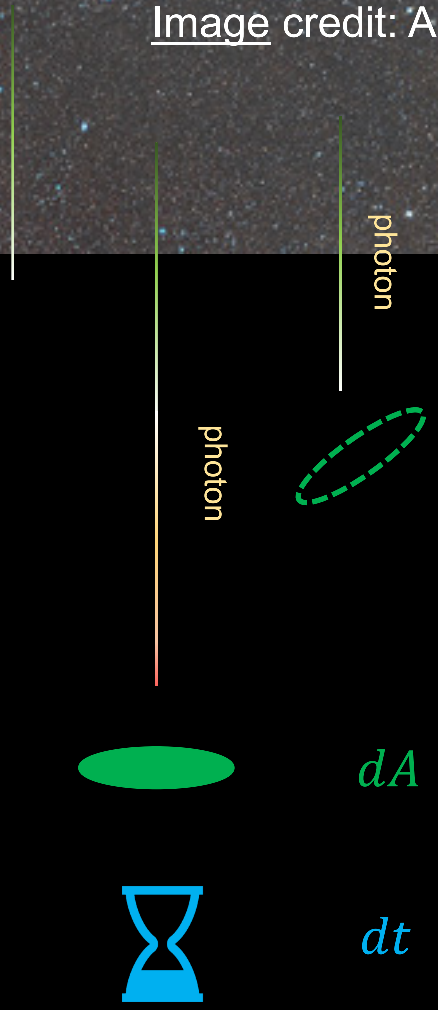
energy or photon flux

$\mathrm{erg\ cm^{-2}\ s^{-1}}$
$\mathrm{ph\ cm^{-2}\ s^{-1}}$

$$dF \equiv \frac{dE}{dA\,dt}$$

photon collecting area ($\mathrm{cm^2}$)

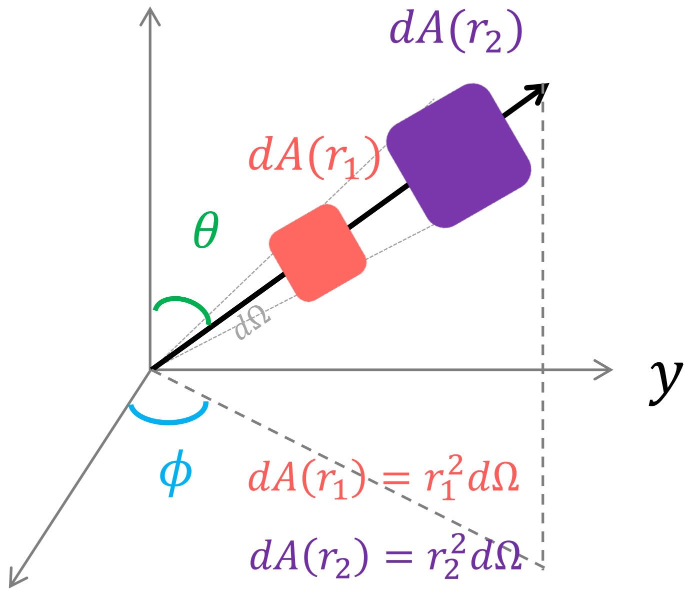
integration time (s)



F can vary for different orientations of the detector element

Inverse square law

Consider an **isotropic** source emitting photons/energy equally in all directions (e.g., a spherically symmetric and isolated star)



Conservation of energy (no loss or gain)

$$F(r) = \frac{\text{constant}}{r^2}$$

energy flux $(\mathrm{erg\ cm^{-2}\ s^{-1}})$

luminosity $(\mathrm{erg\ s^{-1}})$

$$F(r) \equiv \frac{L}{4\pi r^2}$$

distance to the source (cm)

$$dF(r) = \frac{dE}{dA(r)dt} = \frac{dE}{r^2 d\Omega dt}$$

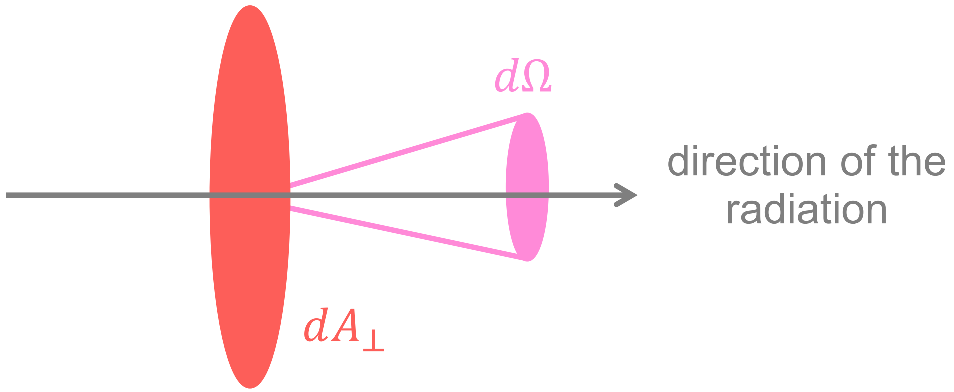
$d\Omega = \sin\theta \, d\theta \, d\phi$

Radiative intensity

Since flux changes with distance to the source, it is useful to a concept that is **intrinsic** to the properties of the source, e.g., luminosity (L), intensity (I)

Solar luminosity

$$L_\odot = 3.8 \times 10^{33}\ \mathrm{erg\ s^{-1}}$$



luminosity
($\mathrm{erg\ s^{-1}}$)

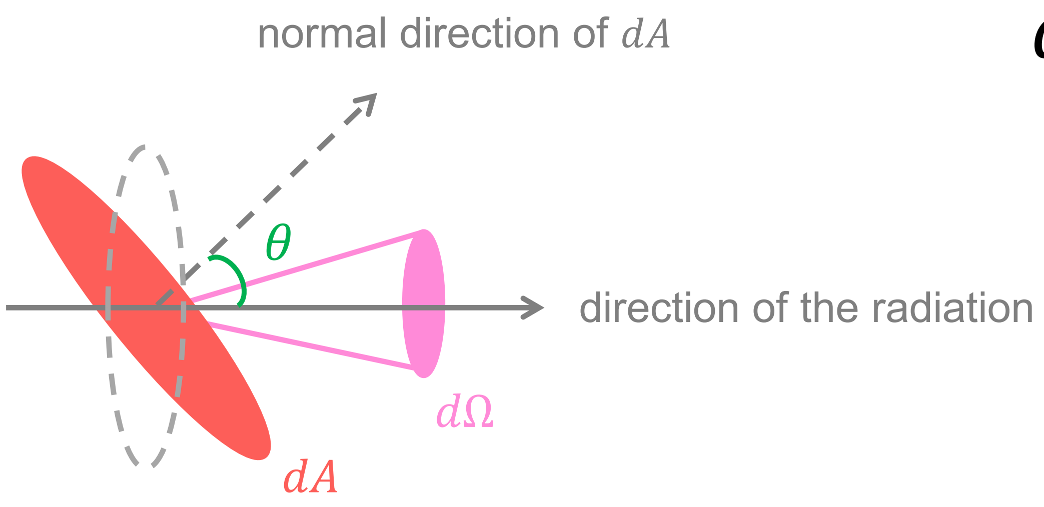
$$L \equiv \frac{dE}{dt}$$

$$I \equiv \frac{dE}{dt\, dA_\perp\, d\Omega}$$

By definition, $dA_\perp$ is normal to the direction of the radiation

Intensity and flux

Consider a radiation field (radiation in all directions), for a detector element (with photon collecting area dA) at some arbitrary orientation, we have



$$dF = I \cos\theta \, d\Omega$$

prev. sl.

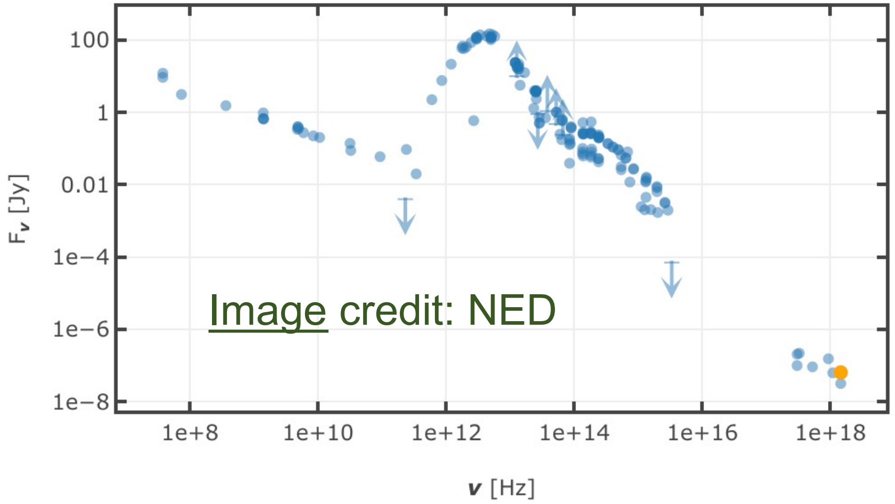
$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

prev. sl.

$$dF = \frac{dE}{dt \, dA}$$

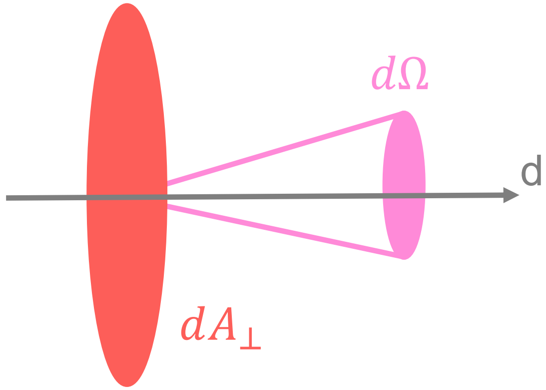
effective area: $dA_\perp = dA \cos\theta$

Specific intensity



Radiative energy in a specific frequency crossing unit area ($dA_\perp$) in unit time in unit solid angle

specific intensity



$$dE_\nu = I_\nu \, dA \, d\nu \, d\Omega \, dt$$

$$I_\nu \equiv \frac{dE_\nu}{dA_\perp \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1} sr^{-1}}$

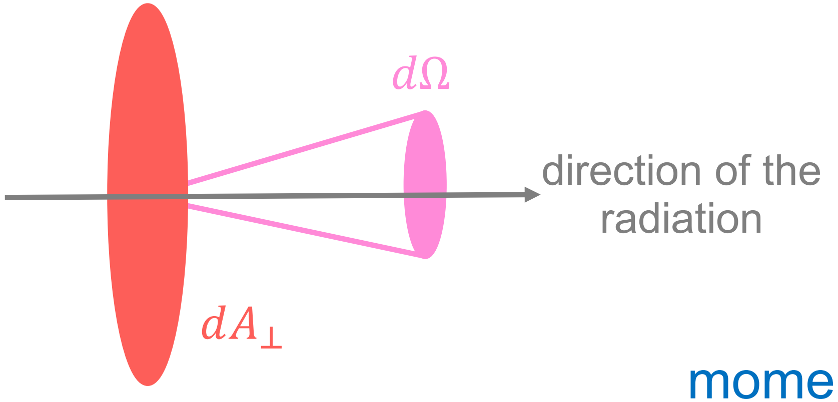
prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

$$I_\nu = \frac{\lambda}{\nu} I_\lambda$$

Specific flux and momentum

prev. sl. $F = \pi I$



$$F_\nu = \pi I_\nu$$

specific flux

specific intensity

$\mathrm{erg\ Hz^{-1}cm^{-2}s^{-1}}$

$1\ \mathrm{erg} = 1\ \mathrm{g\ cm^2\ s^{-2}}$

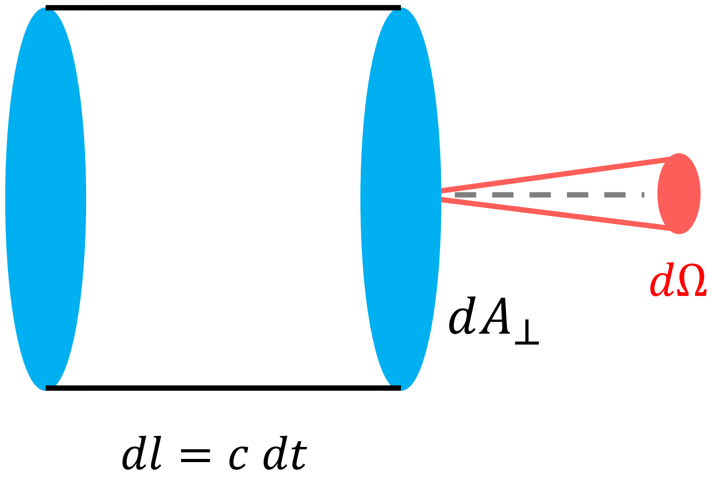
momentum $\longrightarrow$ $p_\nu = \dfrac{F_\nu}{c}$

$\mathrm{dyn\ Hz^{-1}cm^{-2}}$

$1\ \mathrm{dyn} = 1\ \mathrm{g\ cm\ s^{-2}}$

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\, dt$ and its volume $dA_\perp\, c\, dt$



$$dE_\nu = u_\nu(\Omega) d\Omega\, d\nu\, dA_\perp\, c\, dt$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$u_\nu = \frac{1}{c}\int I_\nu\, d\Omega = \begin{cases} \dfrac{4\pi}{c} I_\nu & \text{isotropic} \\ \dfrac{4\pi}{c} \bar{I}_\nu & \text{anisotropic} \end{cases}$$

$\mathrm{erg\ Hz^{-1}\ cm^{-3}}$

specific energy density

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer
2.2.1 Spontaneous emission coefficient
2.2.2 Absorption coefficient
2.2.3 Optical depth and mean free path
2.2.4 Radiative transfer equation

2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

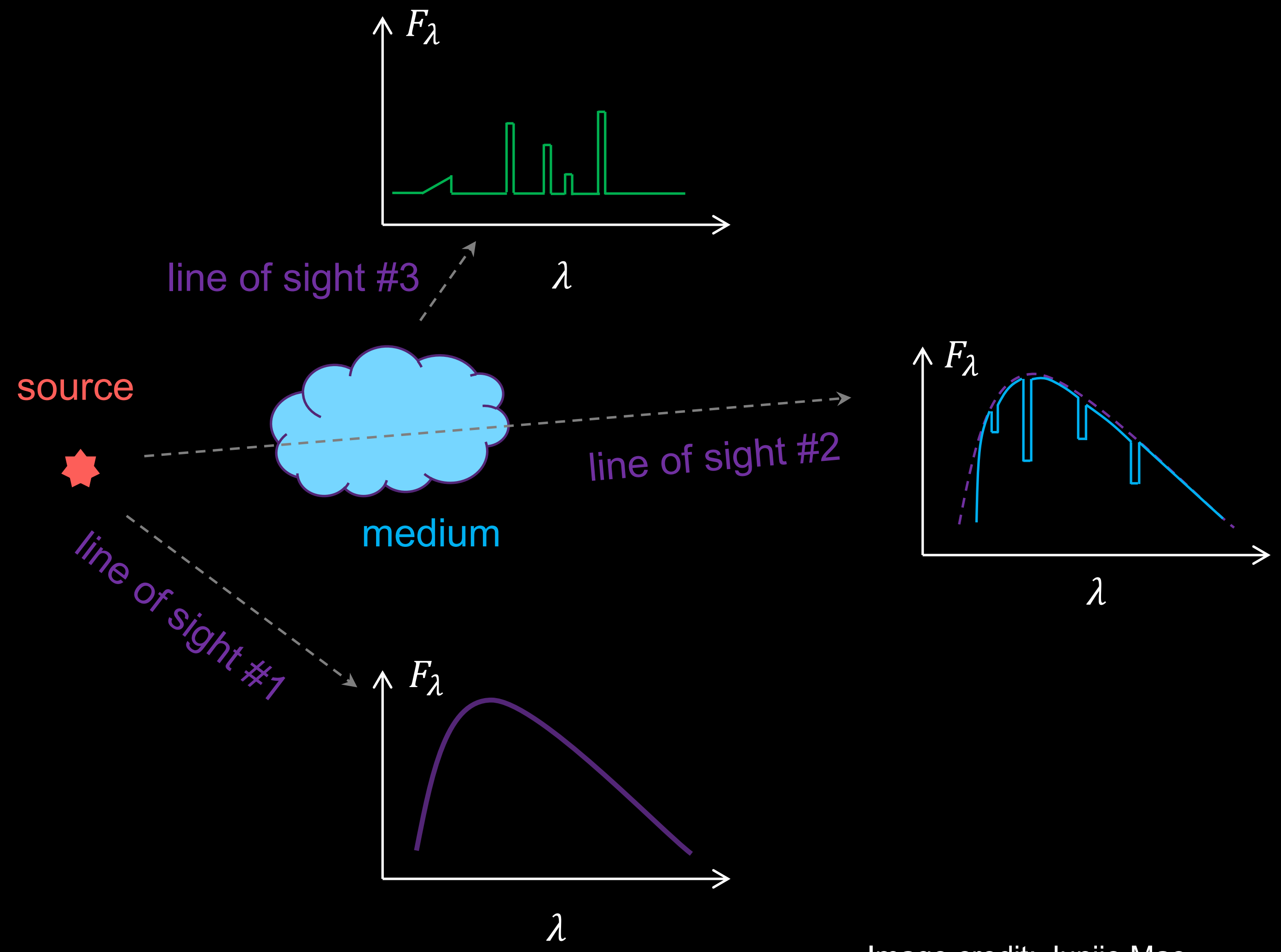


Image credit: Junjie Mao

Radiative transfer

- Radiation is one of the key processes to exchange energy for most celestial bodies
- When a ray passes through matter, the energy exchange process is referred to as radiation transfer

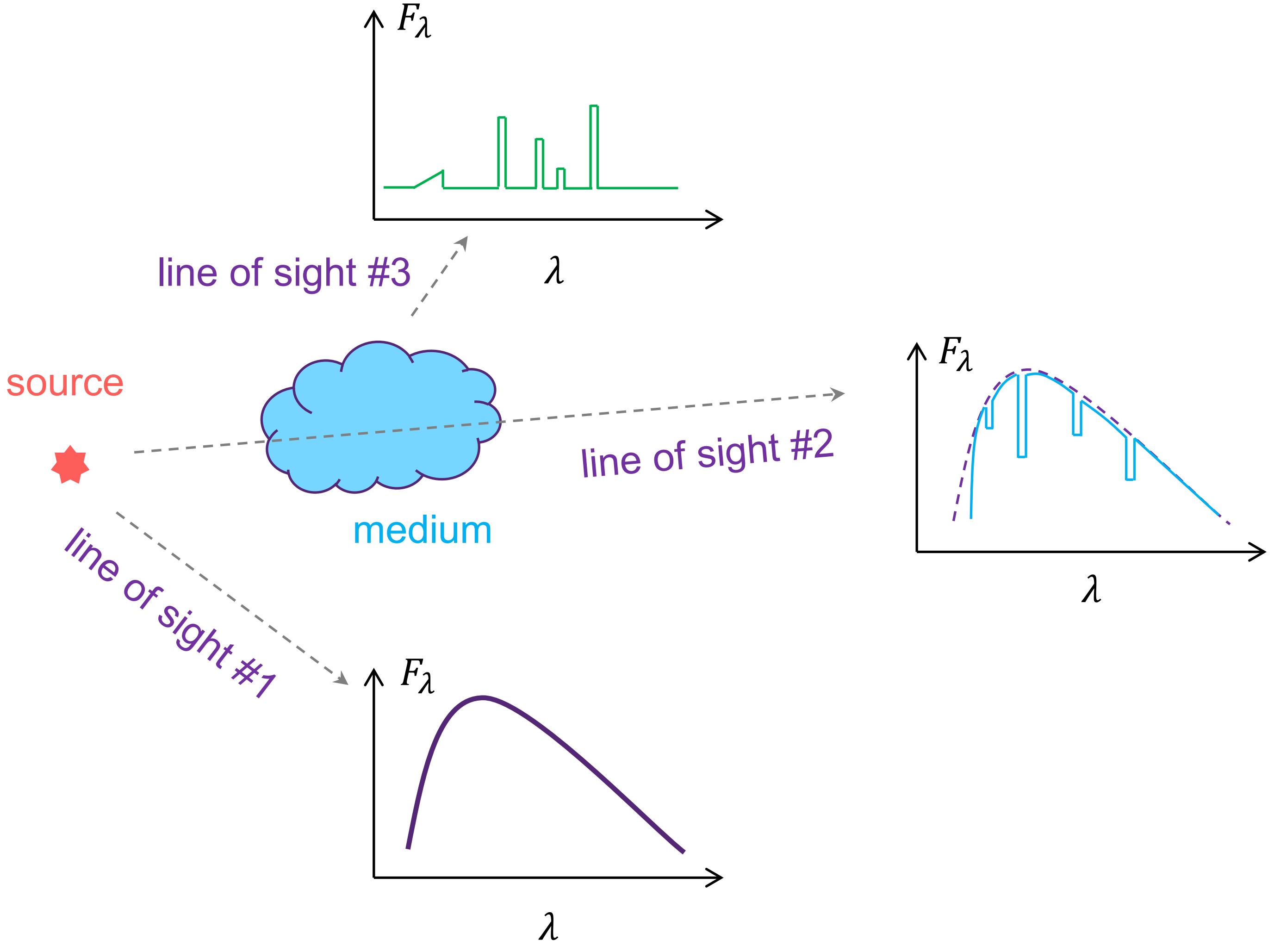
prev. sl.

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

unabsorbed flux

observed flux

$$F_{\mathrm{obs}} \leq \frac{L}{4\,\pi\, d^2}$$



Spontaneous emission coefficient

Radiative energy emitted spontaneously in a specific frequency in unit time in unit solid angle in unit volume

spontaneous emission coefficient

$$dE_\nu = j_\nu \, dV \, d\nu \, d\Omega \, dt$$

$$j_\nu \equiv \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-3} s^{-1} sr^{-1}}$

$\mathrm{Energy\ (frequency)^{-1}(volume)^{-1}(time)^{-1}(solid\ angle)^{-1}}$

Spontaneous emission coefficient (cont.)

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$dV = dA_\perp\, dl$

prev. sl.

$$I_\nu = \frac{dE_\nu}{dA_\perp\, d\nu\, d\Omega\, dt}$$

$$I_\nu = j_\nu\, dl$$

Emissivity

For an isotropic emitter or a distribution of randomly oriented emitters, the emission can also be characterized via emissivity ϵ_ν, which is the radiative energy (angle integrated) emitted spontaneously in a specific frequency per unit time per unit mass

$$dE_\nu = \epsilon_\nu \rho \, dV dt \, d\nu \, \frac{d\Omega}{4\pi}$$

normalized fraction of energy radiated into the solid angle $d\Omega$

$$\epsilon_\nu \equiv \frac{dE_\nu}{\rho \, dV dt \, d\nu \, \frac{d\Omega}{4\pi}}$$

$\mathrm{erg\,Hz^{-1} g^{-1} s^{-1}}$

mass density $\mathrm{g\,cm^{-3}}$

Spontaneous emission coefficient

For an isotropic emitter or a distribution of randomly oriented emitters,

prev. sl.

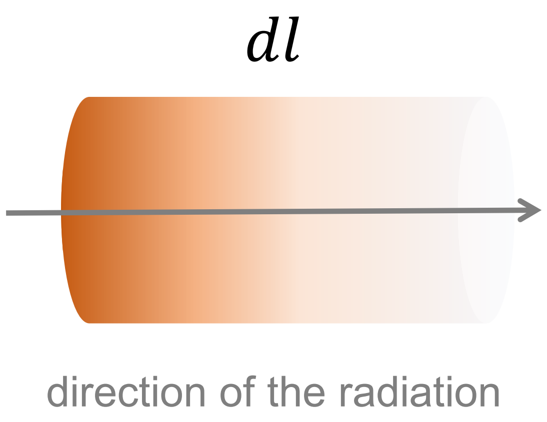
$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$$\epsilon_\nu = \frac{dE_\nu}{\rho\, dV dt\, d\nu \frac{d\Omega}{4\pi}}$$

$$j_\nu = \frac{\epsilon_\nu\, \rho}{4\pi}$$

Absorption coefficient

Loss of intensity in a beam as radiation travels a distance dl



absorption coefficient (positive values for energy taken out of the beam)

$$dI_\nu \equiv -\alpha_\nu \, I_\nu \, dl$$

$$\alpha_\nu = \rho \, \kappa_\nu$$

mass density $\mathrm{g\,cm^{-3}}$

mass absorption coefficient (a. k. a. opacity coefficient)

$$\int_{I_\nu^{\mathrm{in}}}^{I_\nu^{\mathrm{out}}} \frac{dI_\nu}{I_\nu} = -\int \alpha_\nu \, dl$$

$$\ln \frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = -\int \alpha_\nu \, dl$$

$$\frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = \exp\left(-\int \alpha_\nu \, dl\right)$$

Optical depth

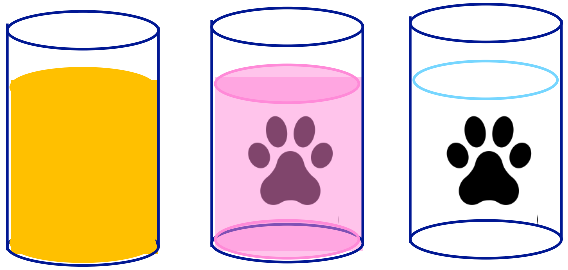
optical depth — absorption coefficient

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu) \qquad \tau_\nu \equiv \int \alpha_\nu dl$$

Optical thin $\quad \tau_\nu \ll 1, I_\nu^{\text{out}} \simeq I_\nu^{\text{in}}$

$$\tau_\nu = 1, I_\nu^{\text{out}} \sim 0.368\, I_\nu^{\text{in}}$$

Optical thick $\quad \tau_\nu \gg 1, I_\nu^{\text{out}} \sim 0$



Mean free path

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu)$$

For homogeneous medium, $\bar{\tau}_\nu = \alpha_\nu \bar{l}_\nu$

$$\bar{\tau}_\nu = \frac{\int_0^\infty \tau_\nu \exp(-\tau_\nu)\, d\tau_\nu}{\int_0^\infty \exp(-\tau_\nu)\, d\tau_\nu} = 1$$

$$\int_0^\infty x \exp(-x)\ dx = 1$$

$$\int_0^\infty \exp(-x)\ dx = 1$$

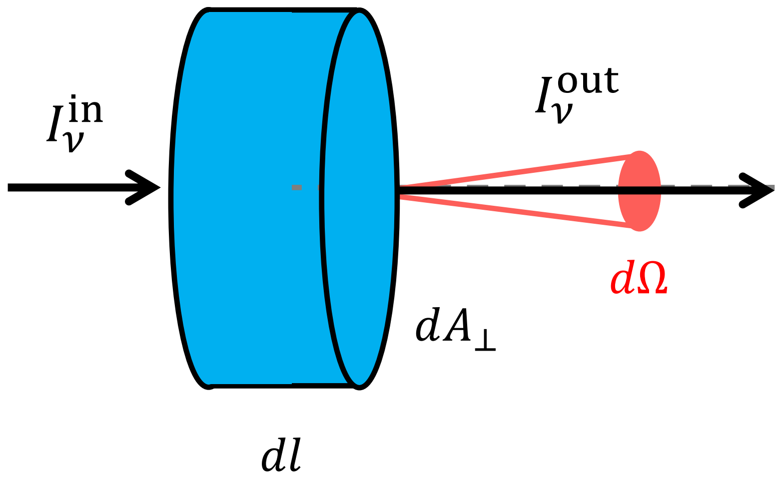
Mean free path (the average distance a photon can travel within a medium without being absorbed)

$$\bar{l}_\nu = \frac{1}{\alpha_\nu}$$

Radiative transfer equation (cont.)

Conservation of energy (no loss or gain)

$$dE^{\mathrm{out}} = dE^{\mathrm{in}} + dE^{\mathrm{abs}} + dE^{\mathrm{em}}$$



$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

prev. sl.

$$dE^{\mathrm{in}} = I_\nu \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\mathrm{out}} = (I_\nu + dI_\nu) \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\mathrm{em}} = j_\nu \, d\nu \, d\Omega \, dA_\perp \, dl \, dt$$

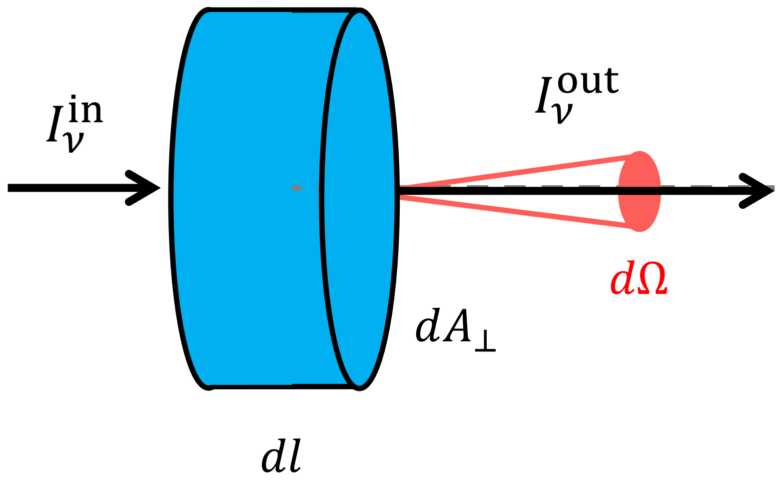
$$dE^{\mathrm{abs}} = -\alpha_\nu dE^{in} dl$$

Radiative transfer equation (cont.)

prev. sl.

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$d\tau_\nu = \alpha_\nu dl$



$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

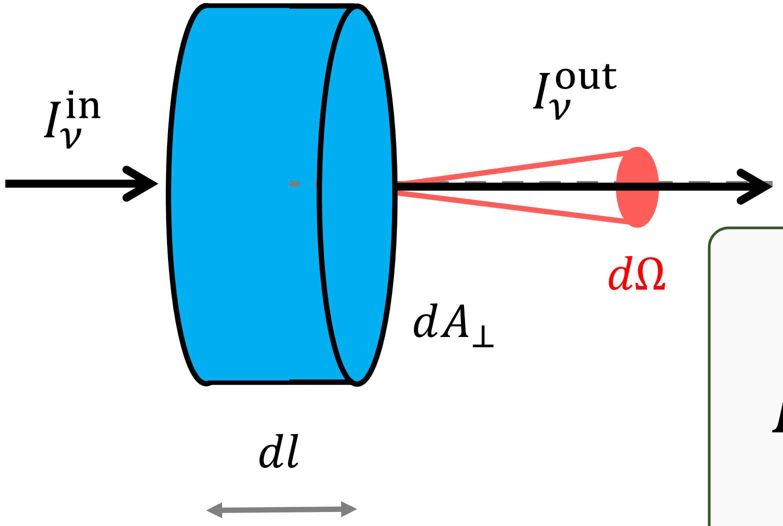
Source function

Solution to the RT equation

prev. sl. $$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} = S_\nu e^{\tau_\nu} - I_\nu e^{\tau_\nu}$$

$$e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = \frac{d}{d\tau_\nu}\left(I_\nu e^{\tau_\nu}\right) = S_\nu e^{\tau_\nu}$$



$$\tau_{\nu,0} = \int \alpha_\nu dl$$

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

see Sect. 3.2 of 《天体物理中的辐射机制》by 尤峻汉 (p81–82)

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer

2.3 Thermal radiation
2.3.1 Blackbody radiation spectrum
2.3.2 Blackbody emission intensity
2.3.3 Blackbody emission flux
2.3.4 Thermal equilibrium
2.3.5 Local thermal equilibrium

2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

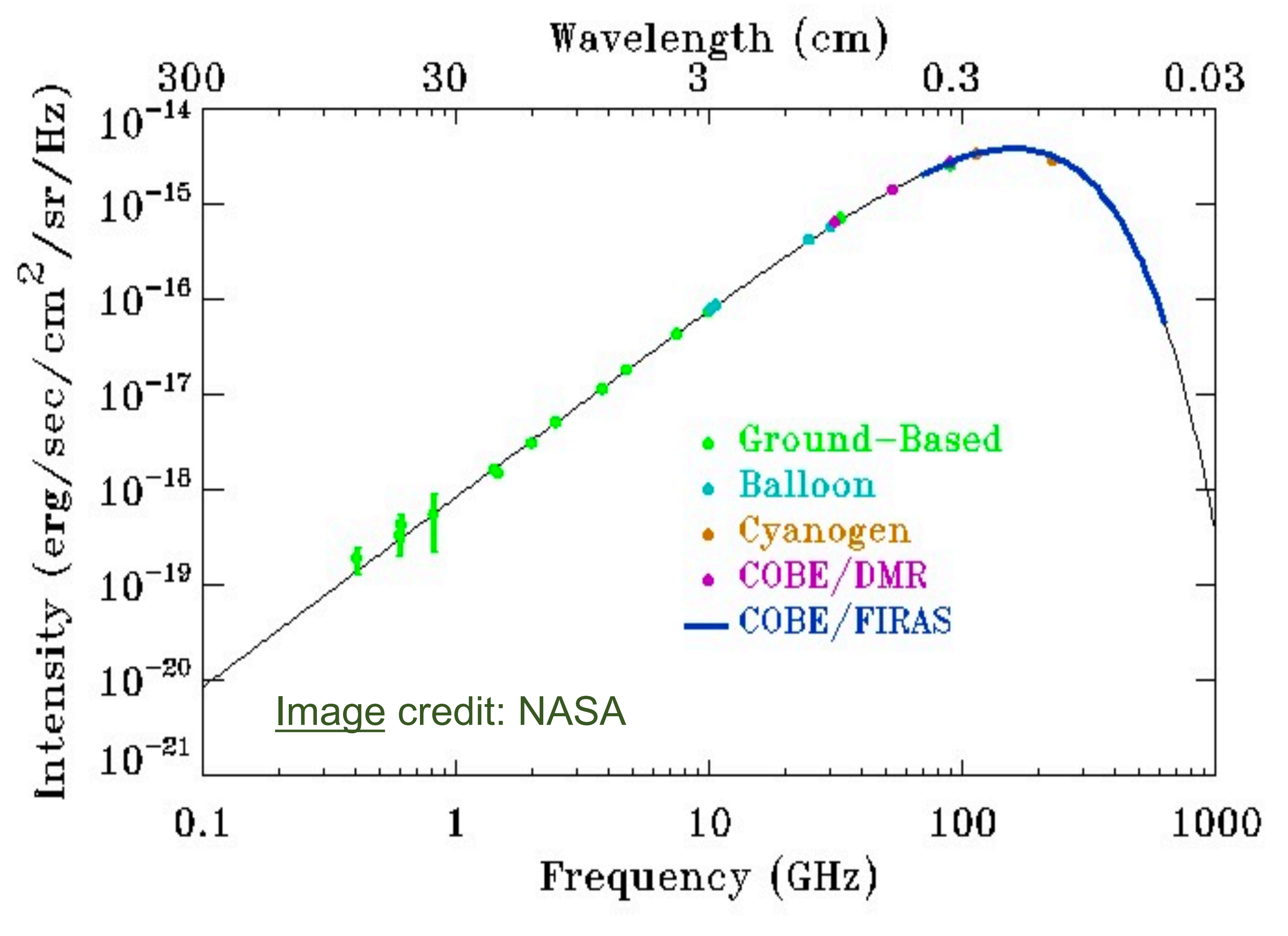


Image credit: NASA

Blackbody radiation spectrum

Thermal radiation is radiation emitted by matter in thermal equilibrium. In this case, the free electrons follows the Maxwellian distribution

$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT_e}\right)$$

Blackbody radiation spectrum is the Planck spectrum

$$h\nu_{\max} = 2.82\ kT$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)^{-1}$$

Caution that even if particles in the matter follow the Maxwellian distribution, radiation emitted by this matter does **not** necessarily have to be thermal radiation

Blackbody emission intensity

$$I_\nu^{\mathrm{BB}} \equiv B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

Blackbody emission intensity

$$I^{\mathrm{BB}} = \int_0^\infty \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} d\nu$$

$$= \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x - 1)^{-1}dx$$

$$x = \frac{h\nu}{kT}$$

Blackbody emission flux

prev. sl.

$$I^{\mathrm{BB}} = \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x - 1)^{-1}dx$$

Blackbody emission flux

$$F^{\mathrm{BB}} = \pi I^{\mathrm{BB}} = \sigma T^4$$

$$\int_0^\infty x^3(e^x - 1)^{-1}dx = \frac{\pi^4}{15}$$

$$\sigma = \frac{2\pi^5k^4}{15\, h^3c^2}$$

$$= 5.670\times10^{-5}\ \mathrm{erg\ s^{-1}cm^{-2}K^{-4}}$$

Stefan-Boltzmann constant

Blackbody radiation flux depends **only** on the temperature

Asymptotic behavior

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

$\frac{h\nu}{kT} \ll 1$ $\quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \frac{h\nu}{kT}$ $\quad I_\nu^{\mathrm{RJ}} = \frac{2\nu^2}{c^2} kT$ Rayleigh-Jeans Law

$\frac{h\nu}{kT} \gg 1$ $\quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \exp\left(\frac{h\nu}{kT}\right)$ $\quad I_\nu^{\mathrm{W}} = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$ Wien Law

Thermodynamic Equilibrium (TE)

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

For medium in thermodynamic equilibrium, Kirchoff's law links the spontaneous emission coefficient and emission source function with Planck's law

$$j_\nu \equiv \alpha_\nu B_\nu(T)$$
$$S_\nu \equiv B_\nu\,(T)$$

TE (cont.)

prev. sl.

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

$$I_\nu^{\text{out}} = B_\nu(T)\big(1 - \exp(-\tau_{\nu,0})\big)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \gg 1 \qquad I_\nu^{\text{out}} = B_\nu(T)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \ll 1 \qquad I_\nu^{\text{out}} = \tau_{\nu,0} B_\nu(T)$$

Only if the medium is in TE and it is optically thick, then its radiation follows the Planck‘s Law.
For non-thermal processes, the source function can only be derived from j_ν and α_ν.

Local Thermodynamic Equilibrium (LTE)

LTE differs from TE in that the radiation field does not need to follow Planck's law, i.e., $I_\nu \neq B_\nu(T)$.

The assumption of LTE is valid if either of the follow two applies:

- ❑ collisional processes among particles dominate photon-related processes
- ❑ collisional processes among particles are in equilibrium with photo-related processes

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer
2.3 Thermal radiation

2.4 Scattering
2.4.1 Pure scattering
2.4.2 Emission, absorption, and scattering
2.4.3 Absorption and scattering probabilities
2.4.4 Effective mean free path

2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

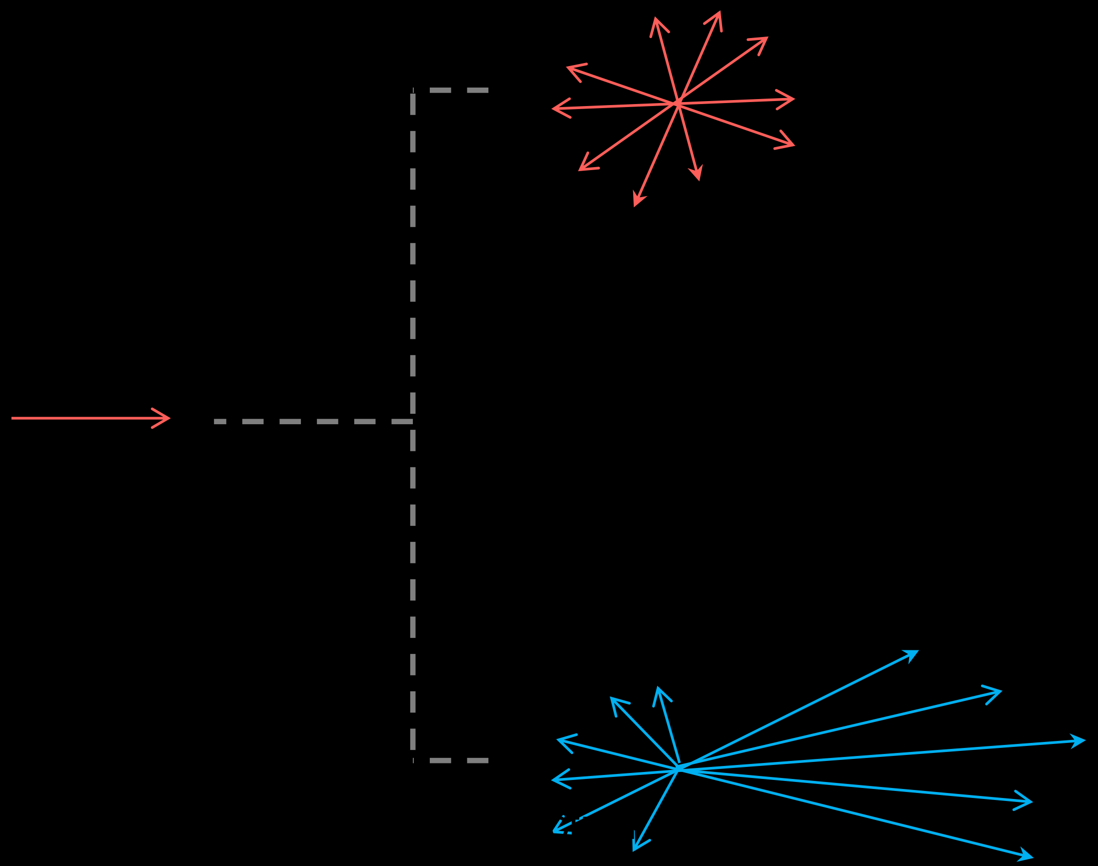
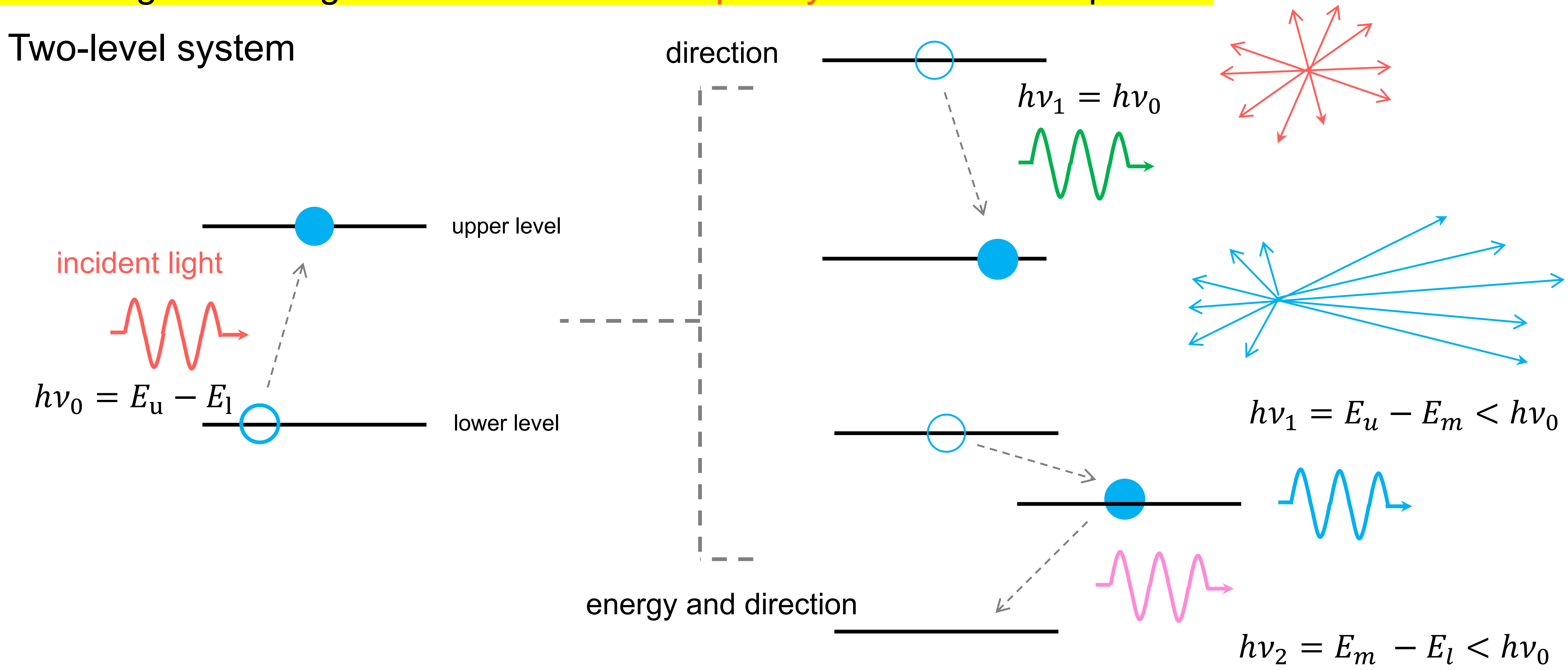


Image credit: Junjie Mao

Scattering

Scattering significantly complicates the radiative transfer.

Scattering can change the direction and frequency of the incident photon.



Pure scattering

Consider a medium has only isotropic scattering (no emission or absorption), which only changes the direction of the incident photon

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{sca} I_\nu + \sigma_\nu^{sca} S_\nu^{sca}$$

scattered outside the line of sight

scattered back into the line of sight

scattering coefficient

$$\frac{dI_\nu}{d\tau_\nu^{sca}} = -I_\nu + S_\nu^{sca} \qquad d\tau_\nu^{sca} \equiv \sigma_\nu^{sca} dl$$

scattering optical depth

Source function for scattering

Random walk

The mean displacement traveled by photons is (order-of-magnitude estimation)

see Sect. 1.7 of the REF book (p35) by Rybicki & Lightman

$$\bar{l}_\nu^{\mathrm{RM}} \sim \sqrt{N}\,\bar{l}_\nu$$

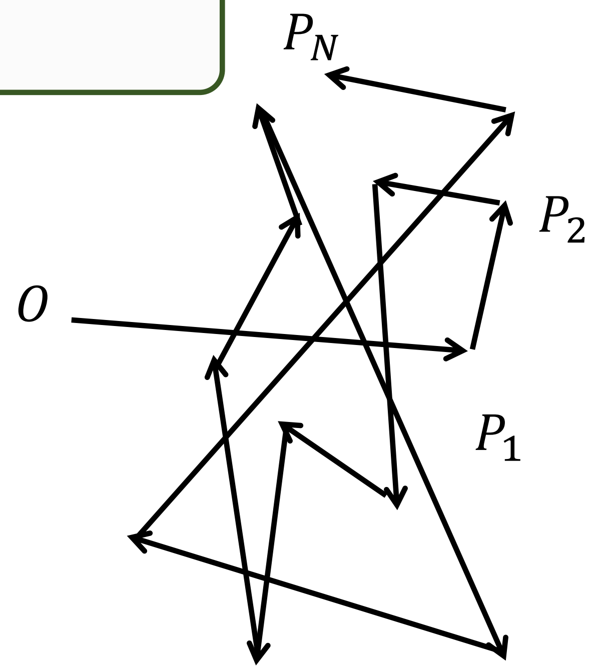
$\bar{l}_\nu = \frac{1}{\sigma_\nu^{\mathrm{sca}}}$: scattering mean free path

Number of scattering

size scale of the medium

for $\tau_\nu^{\mathrm{sca}} \gg 1$ $\quad N \sim \frac{L^2}{\bar{l}_\nu^2} = (\sigma_\nu^{\mathrm{sca}} L)^2 \sim (\tau_\nu^{\mathrm{sca}})^2$

for $\tau_\nu^{\mathrm{sca}} \ll 1$ $\quad N \sim \sigma_\nu^{\mathrm{sca}} L \sim \tau_\nu^{\mathrm{sca}}$



Emission + absorption + scattering

Considering scattering + absorption + emission for a medium in TE

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + j_\nu - \alpha_\nu I_\nu$$

$$j_\nu = \alpha_\nu B_\nu$$

$$\frac{dI_\nu}{dl} = -(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) I_\nu + \ (\sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + \alpha_\nu B_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{AS}}} = -I_\nu + S_\nu^{\mathrm{EAS}}$$

$$S_\nu^{\mathrm{EAS}} \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Source function for emission, absorption, and scattering

$$(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) dl = d\tau_\nu^{\mathrm{AS}}$$

Absorption and scattering probability

An alternative way to define the source function

$$S_\nu^{\mathrm{EAS}} = \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Introducing absorption and scattering probabilities

absorption probability

$$\epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

scattering probability

$$1 - \epsilon_\nu = \frac{\sigma_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

$$S_\nu^{\mathrm{EAS}} = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) S_\nu^{\mathrm{sca}}$$

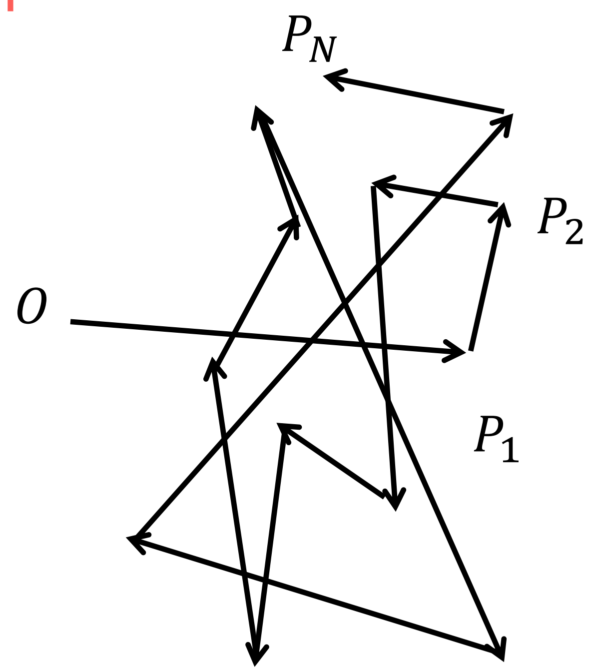
Effective mean free path

prev. sl. $\epsilon_\nu = \dfrac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$

After $N = \epsilon_\nu^{-1}$ steps, a photon is **truly** absorbed

$\bar{l}_\nu^{\mathrm{AS}} = \frac{1}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$: mean free path

$$\left(\bar{l}_\nu^{\mathrm{RM}}\right)^2 = N\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2 = \frac{\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2}{\epsilon_\nu} = \frac{1}{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}$$



$$l_\nu^{\mathrm{eff}} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}}$$

effective mean free path
diffusion length
thermalization length

for $\alpha_\nu \ll \sigma_\nu^{\mathrm{sca}}$ $\quad l_\nu^{\mathrm{eff}} = \dfrac{1}{\sqrt{\alpha_\nu \sigma_\nu^{\mathrm{sca}}}}$

Chpt.2 Fundamentals of radiation

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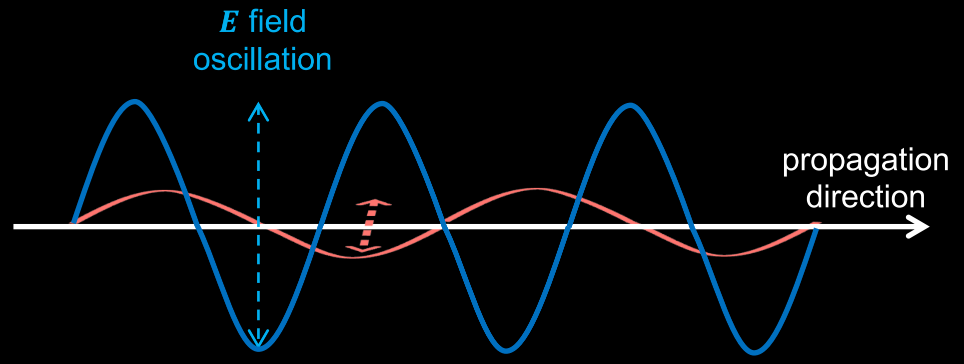
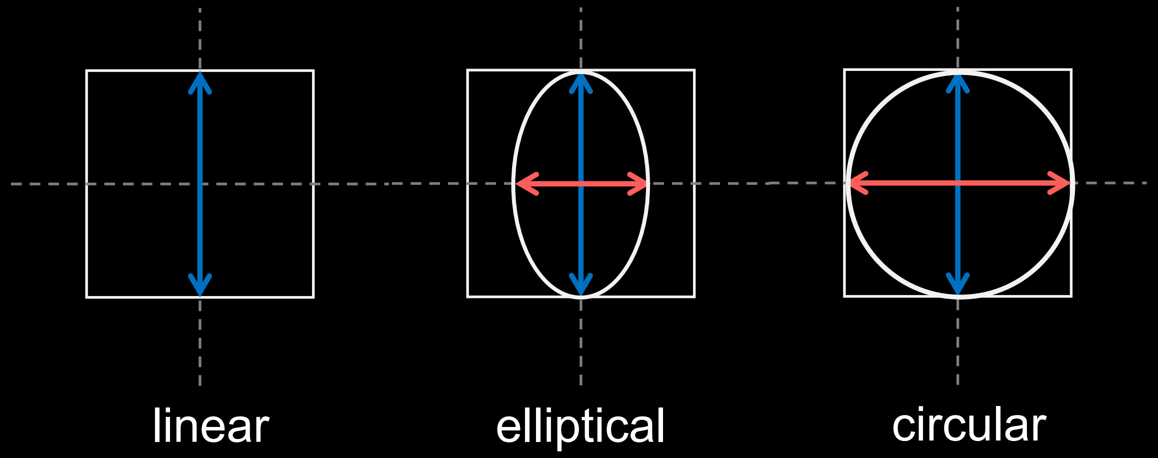
2.5 Scattering

2.6 Polarization
2.6.1 Stokes parameters
2.6.2 Polarization fraction

2.7 Dispersion and rotation measures

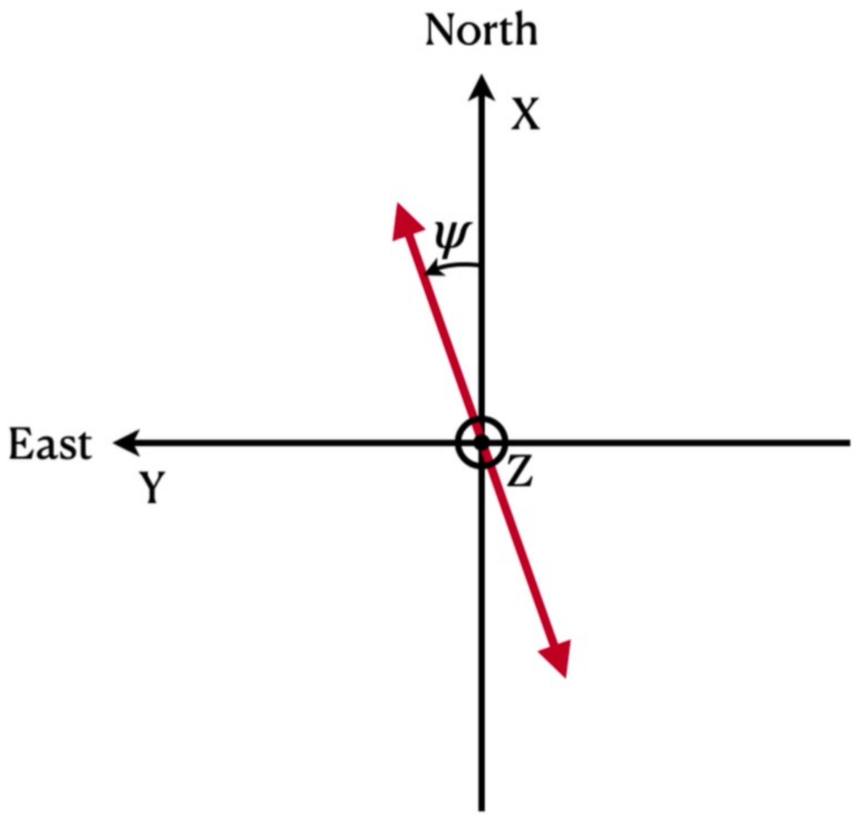
Image credit: Junjie Mao

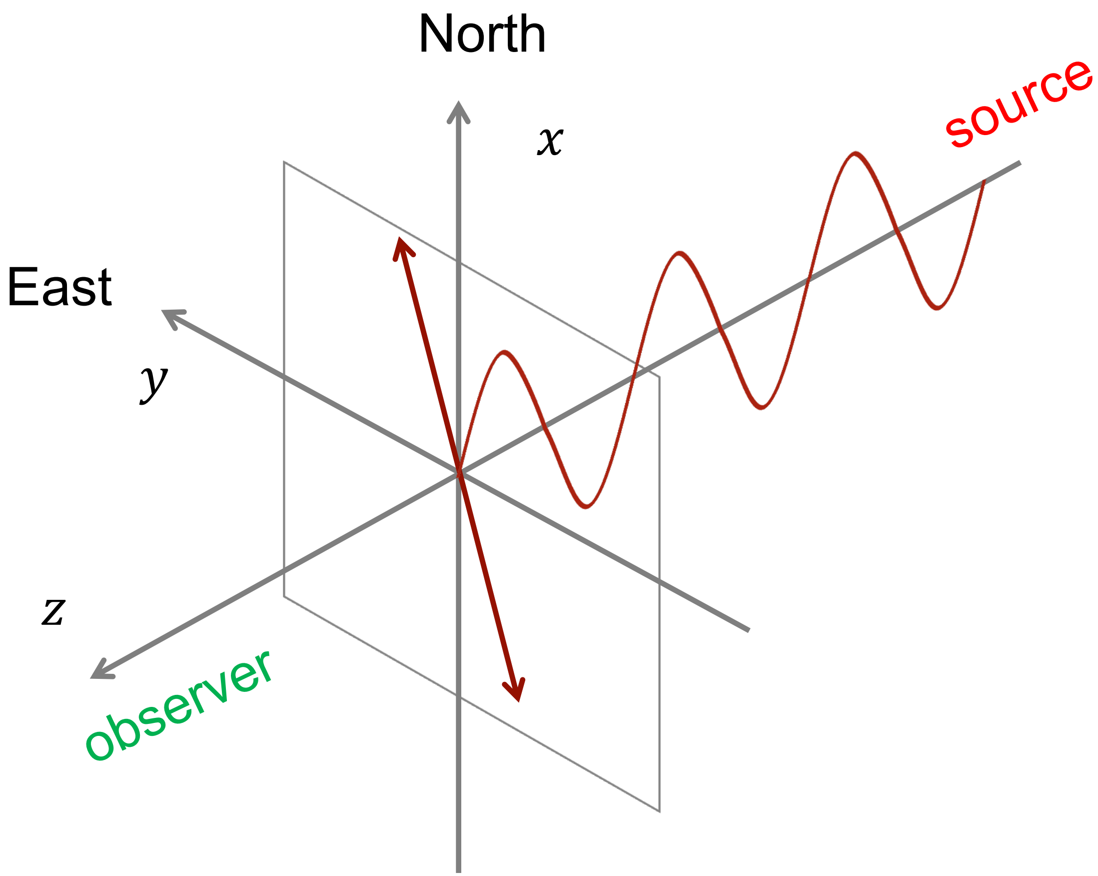




IAU standard of polarization

IAU reference frame for polarization
(e.g., Ferriere et al. 2021)





IAU definition:

- ✓ X and Y are the Cartesian coordinates of the plane of the sky
- ✓ Z is along the line of sight (+Z towards the observer)
- ✓ ψ: linear polarization angle (measured counterclockwise from the North)

Stokes parameters

Stokes parameters, all related to intensity, are **observable**

- ❑ I: Intensity
- ❑ $\sqrt{Q^2 + U^2}$: Intensity of linear polarization
- ❑ V: Intensity of circular polarization

$$\mathrm{S}_0 \equiv I$$
$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$
$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2 + U^2}}, -\frac{\pi}{4} \leq \chi \leq \frac{\pi}{4}$$
$$\tan 2\psi = U/Q, 0 \leq \psi \leq \pi$$

linear polarization angle

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

polarization fraction

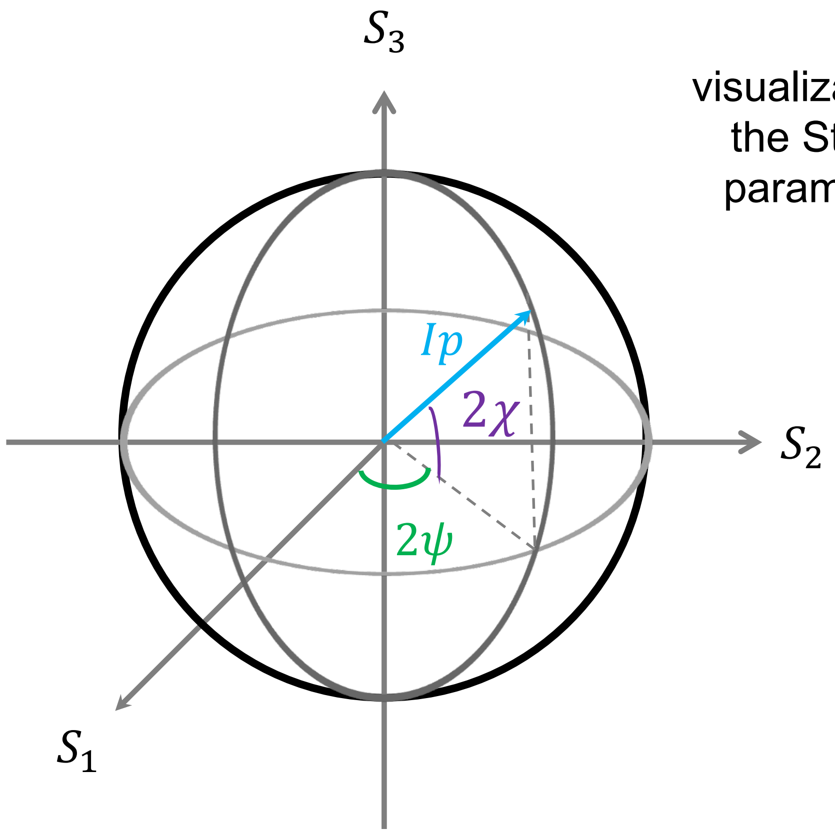
$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized

Poincaré sphere

prev. sl.

$$S_0 \equiv I$$
$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$
$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2+U^2}}, -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$$
$$\tan 2\psi = U/Q\,, 0 \le \psi \le \pi$$



visualization of the Stokes parameters

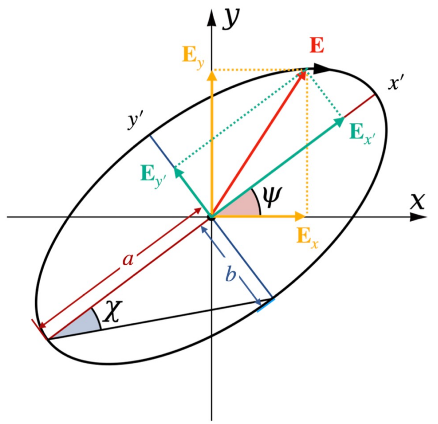


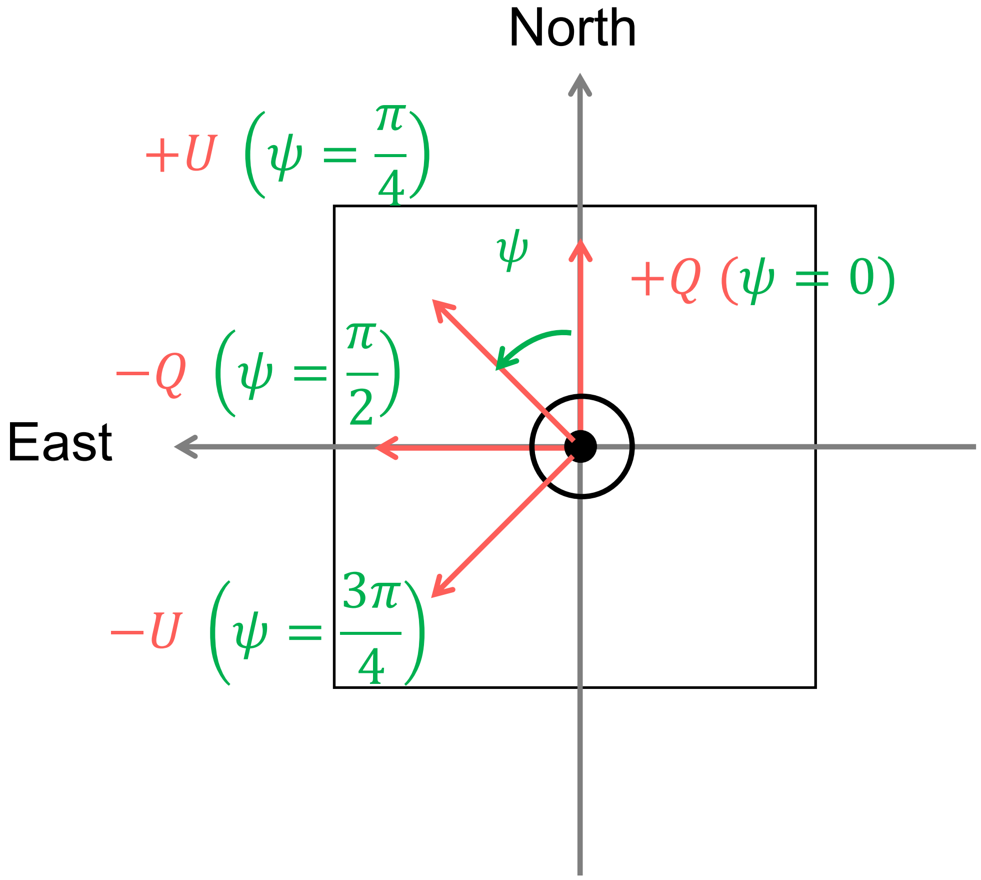
Image credit: Jiejia Liu

Linear polarization

For $V = 0$ ($\chi = 0$), we have the linear polarization

$$S_0 = I$$
$$S_1 = Q = Ip\cos 2\psi$$
$$S_2 = U = Ip\sin 2\psi$$
$$S_3 = V = 0$$

$$\cos\pi = -1$$
$$\sin\left(\frac{3\pi}{2}\right) = -1$$



e.g., Hamaker & Bregman (1996)

prev. sl.

$$S_3 \equiv V = Ip\sin 2\chi$$
$$\tan 2\psi = U/Q$$

linear polarization angle

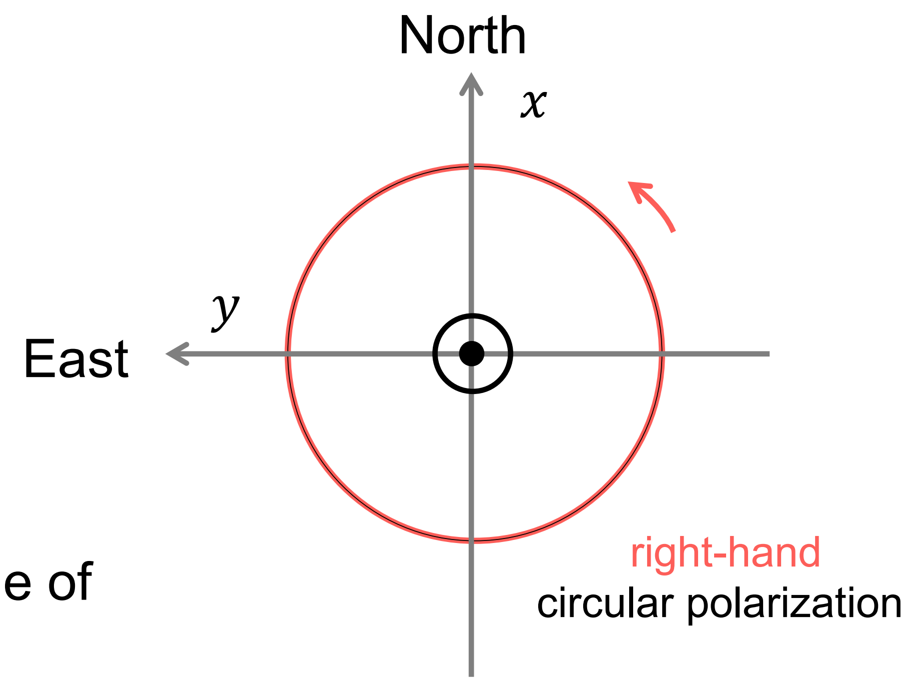
Circular polarization

prev. sl.

$$S_1 \equiv Q = Ip \cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip \cos 2\chi \sin 2\psi$$

For $Q = U = 0$ ($\chi = \pm\pi/4$), we have the circular polarization

$$S_0 = I$$
$$S_1 = Q = 0$$
$$S_2 = U = 0$$
$$S_3 = V = Ip \sin 2\chi$$



IAU definition:

- ✓ $+V$ for the right-handed circular polarization
- ✓ For the right-handed circular polarization, the position angle of $\vec{E}$ at any point **increases** with time
- ✓ Along the $+z$ direction, $\vec{E}$ traces a left-handed circle at any time

e.g., Hamaker & Bregman (1996)

Linear/circular polarization fraction

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - pI \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} pI \\ Q \\ U \\ V \end{pmatrix}$$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - p_L I - p_C I \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} p_L I \\ Q \\ U \\ 0 \end{pmatrix} + \begin{pmatrix} p_C I \\ 0 \\ 0 \\ V \end{pmatrix}$$

polarization fraction

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

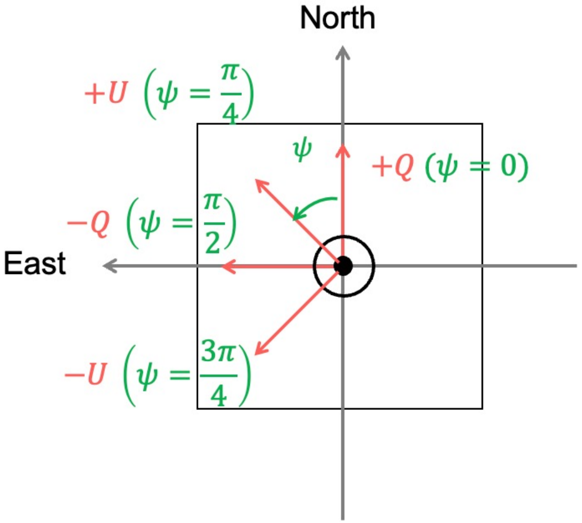
linear or circular polarization fraction

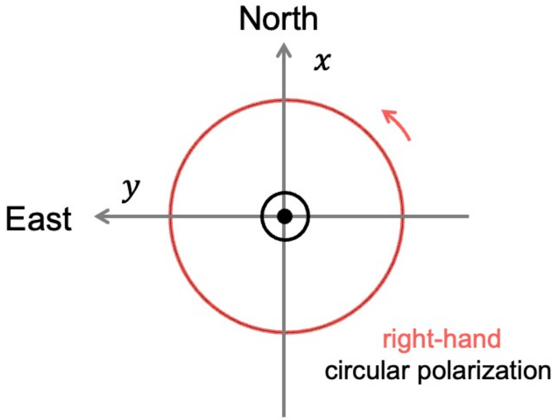
$$p_L = \sqrt{Q^2 + U^2}/I$$

$$p_C = V/I$$

prev. sl.

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized





Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
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2.4 Escape probability
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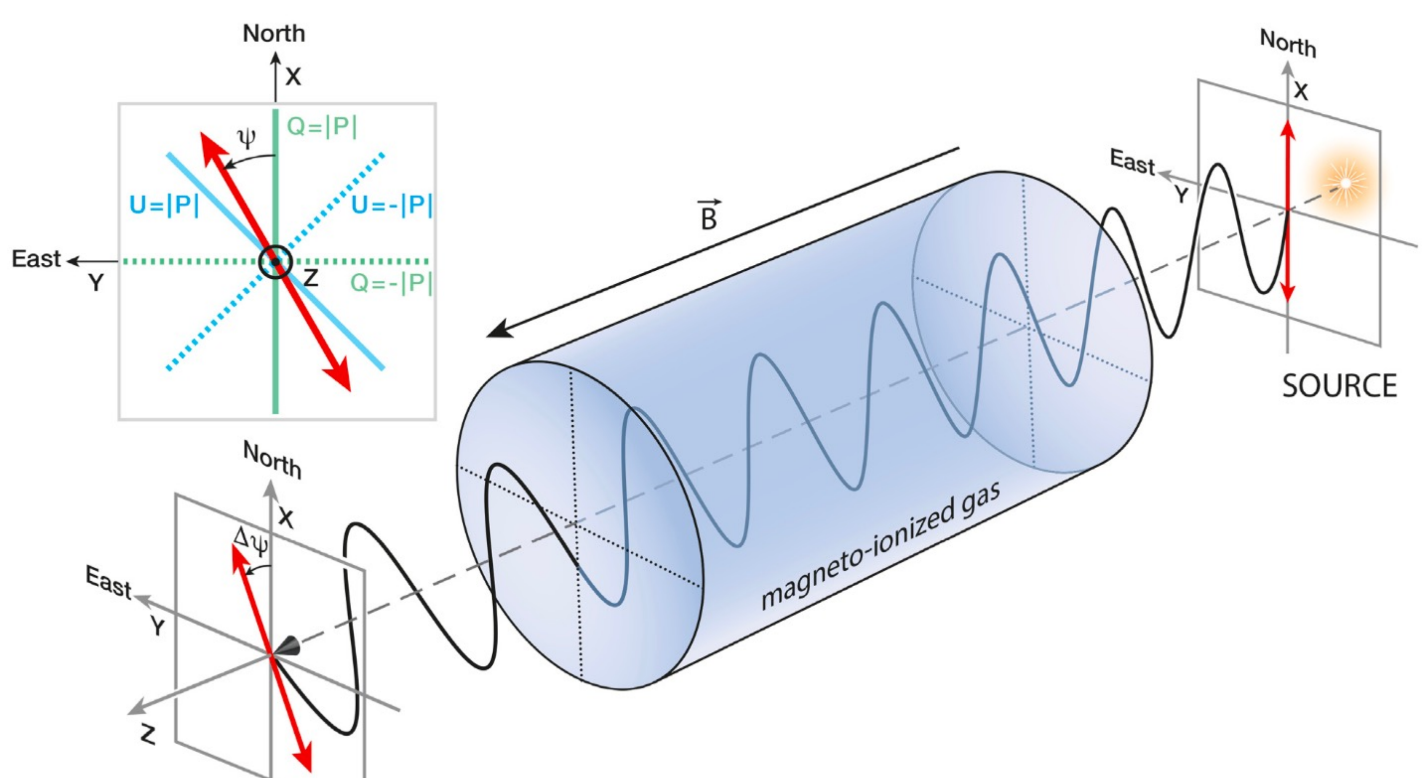
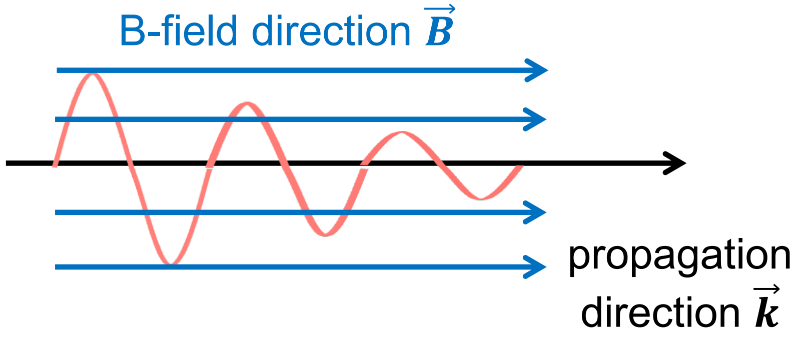
2.7 Dispersion and rotation measures
2.7.1 Dispersion relation
2.7.2 Refractive index
2.7.3 Time of arrival difference
2.7.4 Dispersion measure
2.7.5 Fast Radio Bursts
2.7.6 Phase difference
2.7.7 Rotation measure
2.7.8 Faraday rotation
2.7.9 CMB polarization



Ferriere et al. 2021

Dispersion relation

Consider a pair of left- and right-handed circularly polarized waves propagate along the B-field with $\vec{k} \parallel \vec{B}$, the dispersion relation is



Eq. 1 of Ferriere et al. 2021

$$\omega^2 = c^2 k^2 + \frac{\omega_\mathrm{p}^2}{1 \mp \frac{\omega_\mathrm{L}}{\omega}}$$

angular frequency $\omega = 2\pi\nu$

wave number $k = \frac{2\pi}{\lambda}$

- Right circular polarized (RCP) mode: −
- Light circular polarized (LCP) mode: +

plasma frequency

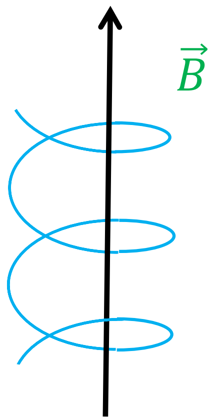
$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

Larmor (or gyro) frequency

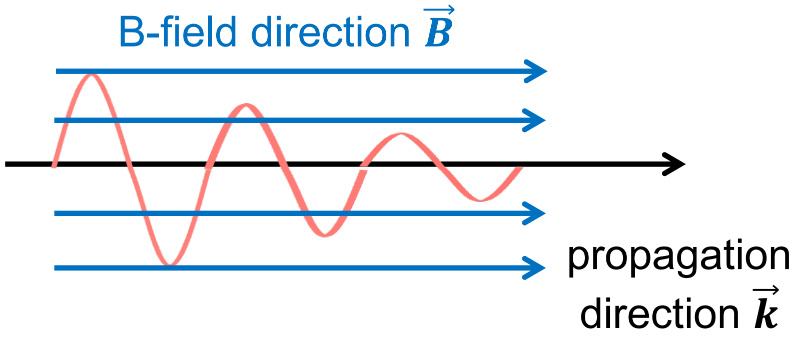
$$\omega_L \equiv \frac{eB}{m_e c}$$

《原子物理学》杨福家
3rd edition, p153 - 155

gyro motion



Dispersion relation (cont.)



For astrophysical waves of interest, the following always apply (Assignment #3: Plasma frequency)

$$\omega_\mathrm{p} \ll \omega, \qquad \omega_L \ll \omega$$

prev. sl.

$$\omega_\mathrm{p} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

prev. sl.

$$\omega^2 = c^2 k^2 + \frac{\omega_\mathrm{p}^2}{1 \mp \frac{\omega_\mathrm{L}}{\omega}}$$

$$\frac{1}{1+x} = 1 - x + \ \ldots$$

$$= c^2 k^2 + \omega_\mathrm{p}^2 \pm \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega}$$

EM wave in free space | electric force | magnetic force

$$c^2 k^2 = \omega^2 - \omega_\mathrm{p}^2 \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega} \simeq \omega^2$$

$$k(\omega) = \frac{\omega}{c} \sqrt{1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega^3}}$$

Refractive index

Consider the propagation of EM waves through an unmagnetized plasma (mainly free electrons), the medium refractive index is

$$n = \sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}}$$

see Sect. 8.1 of the REF book (p227) by Rybicki & Lightman

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Only waves with $\omega > \omega_{\mathrm{p}}$ can propagate, which is not an issue in most cases

$$v_\phi = \frac{\omega}{k} = \frac{c}{n}$$

phase velocity

Time of arrival difference

Consider a pulse propagates in a non-magnetized medium ($B = 0, \omega_{\rm L} = 0$), the group velocity is

prev. sl.

$$c^2k^2 = \omega^2 - \omega_{\rm p}^2 \mp \frac{\omega_{\rm p}^2\omega_{\rm L}}{\omega} \simeq \omega^2$$

$$v_{\rm grp}(\omega) = \frac{d\omega}{dk} = \frac{c^2k}{\omega} = c\sqrt{1 - \frac{\omega_{\rm p}^2}{\omega^2}}$$

prev. sl.

$$\omega_{\rm p} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

This leads to a frequency-dependent time of arrival

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \ ...$$

$$t_{\rm arv}(\omega) = \int_0^l \frac{1}{v_{\rm grp}(\omega)} dl = \int_0^l \frac{1}{c}\left(1 + \frac{\omega_{\rm p}^2}{2\omega^2}\right) dl$$

Dispersion measure

For $\omega \gg \omega_p$ (Assignment #3: Plasma frequency), the time of arrival for a specific frequency is

traveling distance in the medium

prev. sl.

$$t_{\mathrm{arv}}(\omega) = \int_0^l \frac{1}{c}\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\right) dl$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

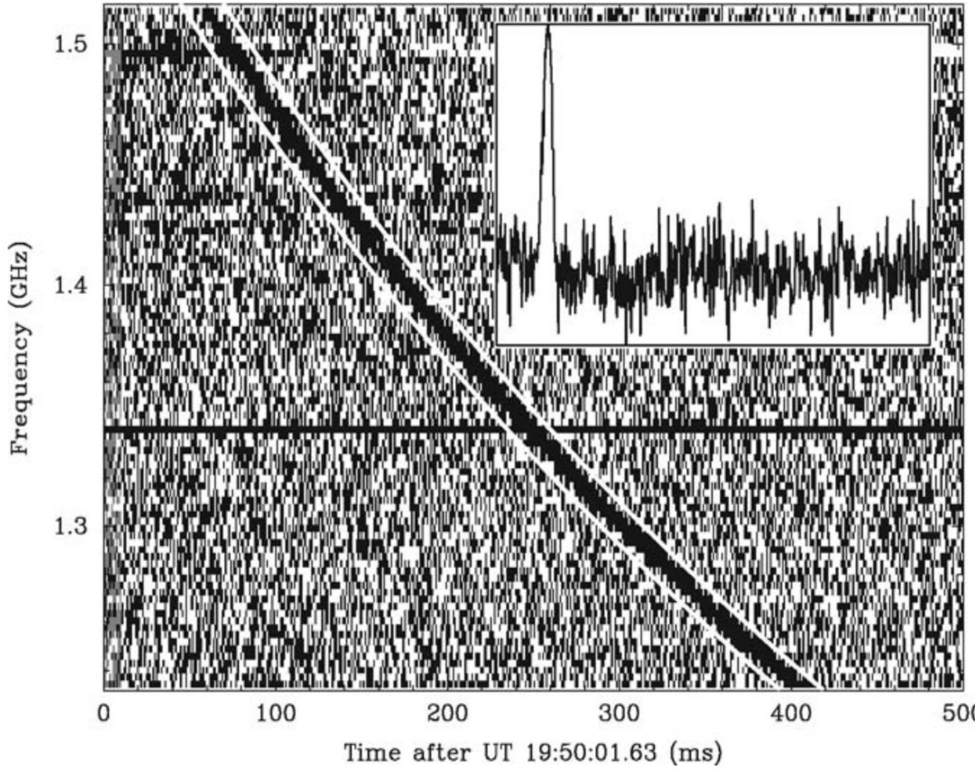
Waves with different frequencies arrive at different times

$$\Delta t_{\mathrm{arv}} = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\mathrm{low}}^2} - \frac{1}{\omega_{\mathrm{high}}^2}\right)\int_0^l n_e dl$$

High-frequency waves arrive earlier than the low-frequency ones

Fast Radio Bursts (FRBs)

Waterfall plot
measured DM= $375 \pm 1 \mathrm{pc\ cm}^{-3}$



FRBs are radio bursts with extremely short duration (a few milliseconds) yet rather energetic ($\gtrsim 10\ L_\odot$). The first FRB was discovered in 2001 (Lorimer et al. 2007)

prev. sl.

$$\Delta t_{\mathrm{arv}} = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\mathrm{low}}^2} - \frac{1}{\omega_{\mathrm{high}}^2}\right)\int_0^l n_e dl$$

Dispersion Measure (DM)

$$= \frac{4.149}{\mathrm{ms}}\left(\left(\frac{1\ \mathrm{GHz}}{\nu_{\mathrm{low}}}\right)^2 - \left(\frac{1\ \mathrm{GHz}}{\nu_{\mathrm{high}}}\right)^2\right)\left(\frac{\mathrm{DM}}{\mathrm{pc\ cm}^{-3}}\right)$$

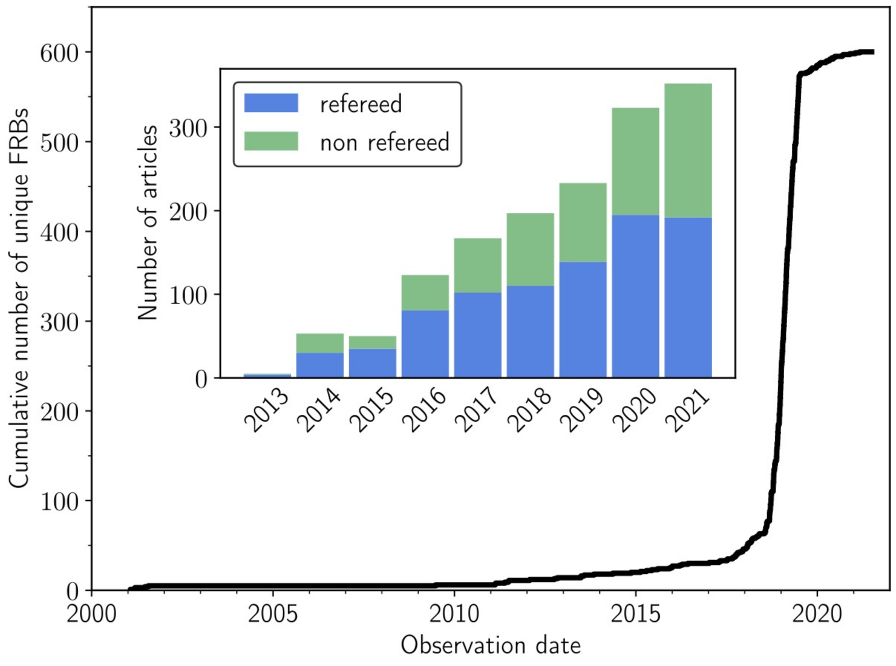
Lorimer et al. 2007

Fast Radio Bursts (FRBs)

FRBs have been put in the spot light in the past decade:

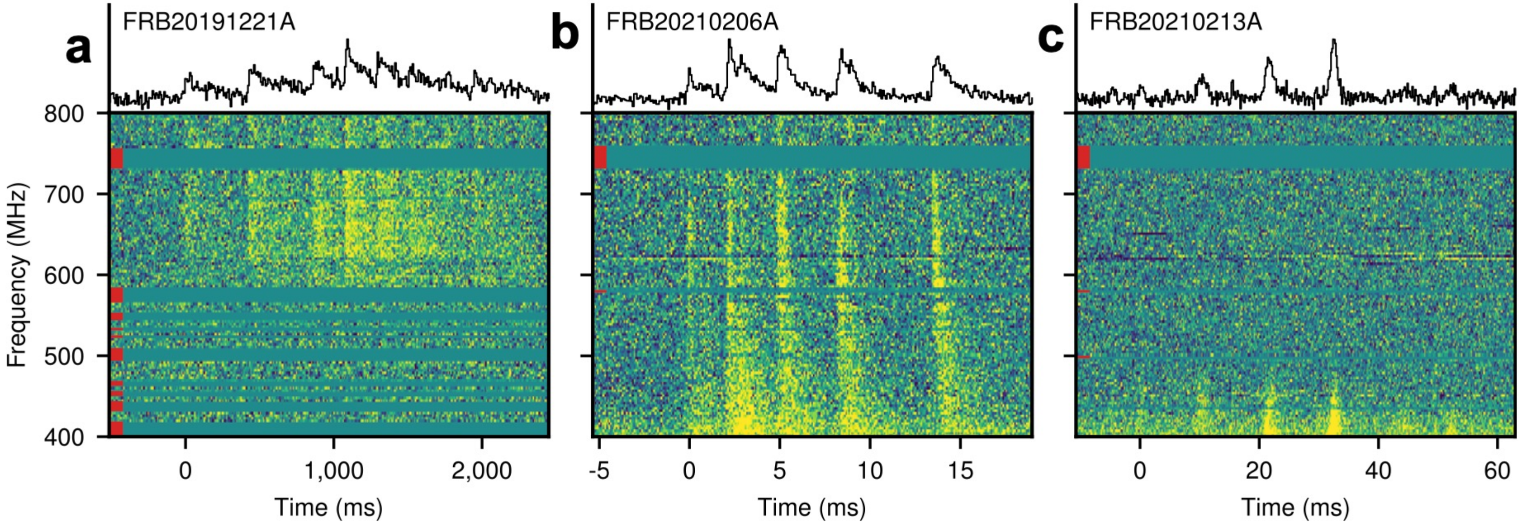
- Repeating FRBs
- FRBs with known hosts
- FRBs with relatively long durations (slow)

But, we still don't know much about FRBs …



Publications on FRBs from ADS

repeating FRBs (a small yet growing population) Petroff et al. 2022

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About Latest Research

Some "not so fast" fast radio bursts

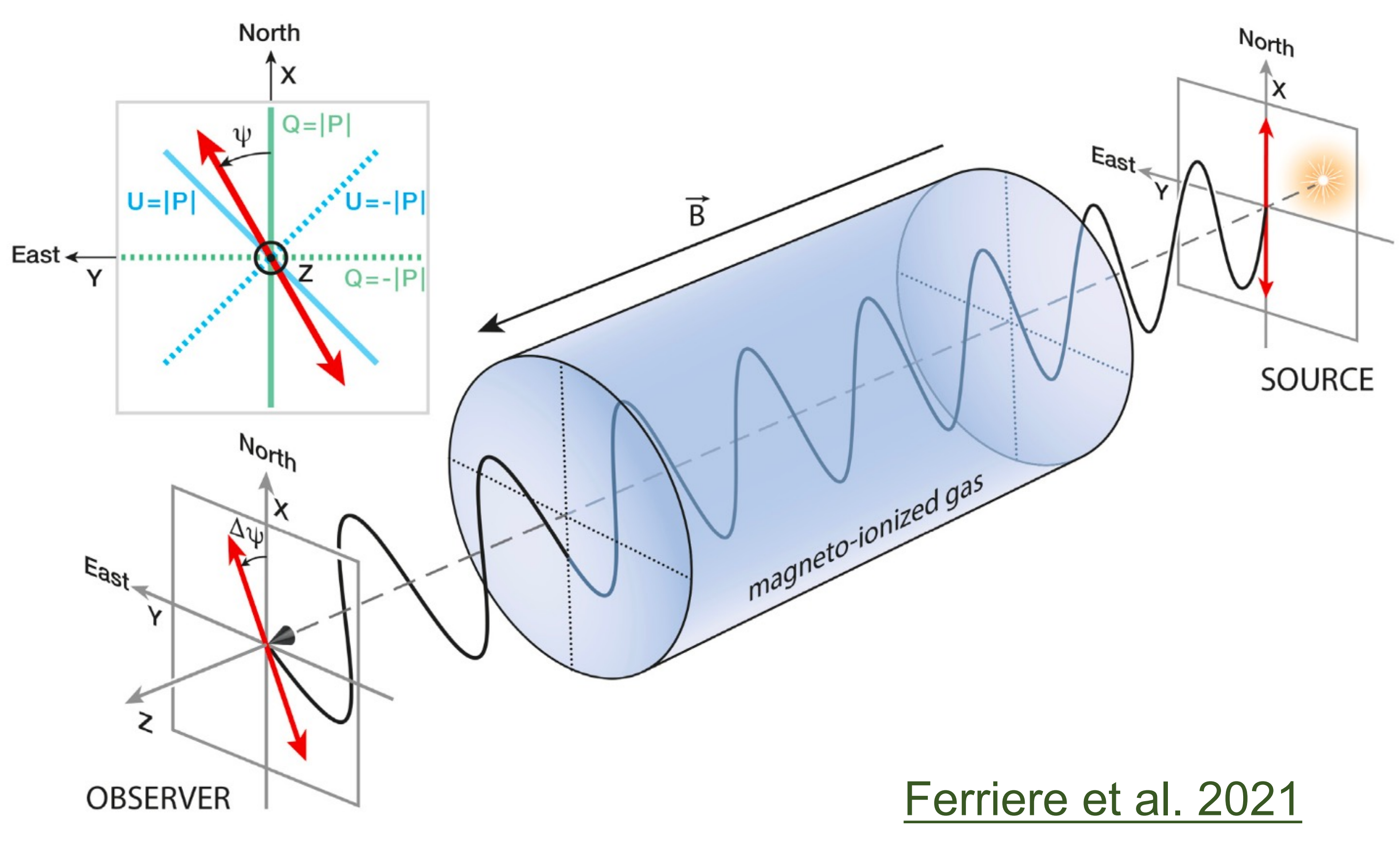
by Alice Curtin | Nov 22, 2022 | Daily Paper Summaries | 1 comment

Title: Four New Fast Radio Bursts Discovered in the Parkes 70-cm Pulsar Survey Archive

Authors: F. Crawford, S. Hisano, M. Golden, T. Kikunaga, A. Laity, D. Zoeller

First Author's Institution: Franklin & Marshall College

Chpt.2 Fundamentals of radiation



Ferriere et al. 2021

Phase difference

In a homogeneous plasma, the phase can be defined as one of the following

Eq. 4 of Ferriere et al. 2021

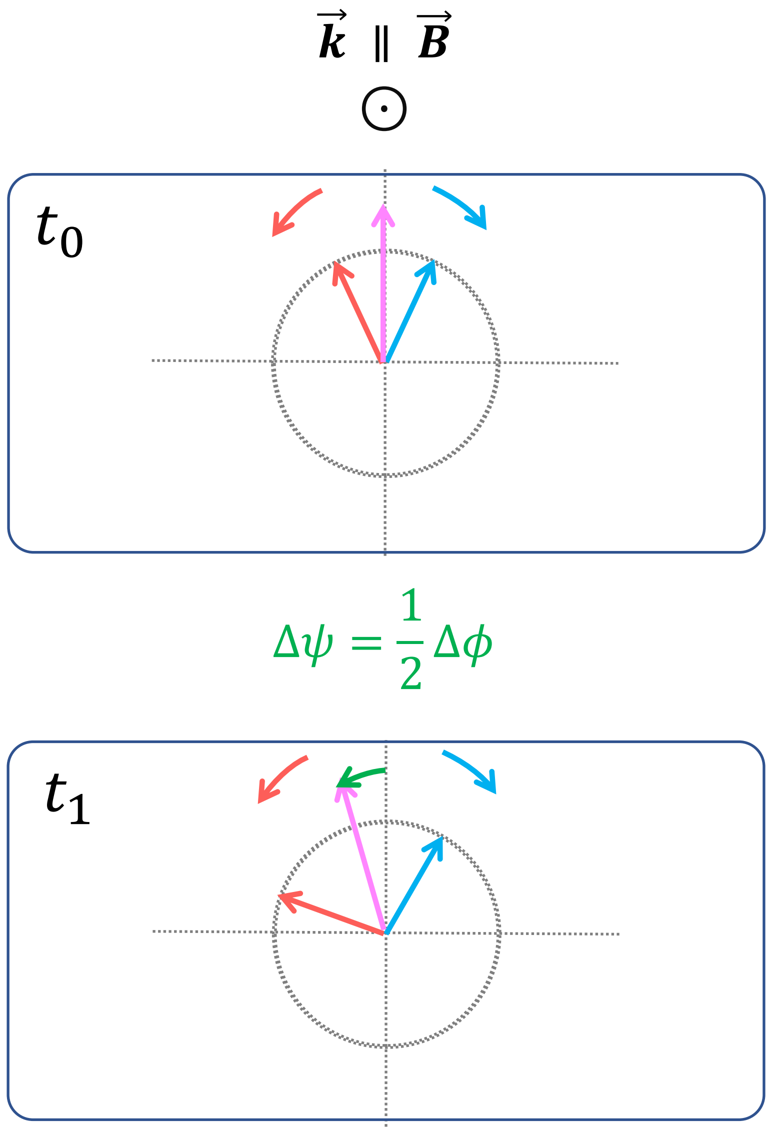
$$\phi \equiv \omega t - \int_0^l k \, dl$$

Decomposing a linearly polarized wave into a pair of left- and right-hand circular polarized waves. At the source ($l = 0$), the two waves have the same phase, as the waves propagates (with $\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$) in a magnetized medium, phase difference will occur

prev. sl.

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega^3}}$$

$$\phi_\mathrm{RCP} > \phi_\mathrm{LCP}$$



- Right circular polarized (RCP) mode: $-$
- Light circular polarized (LCP) mode: $+$

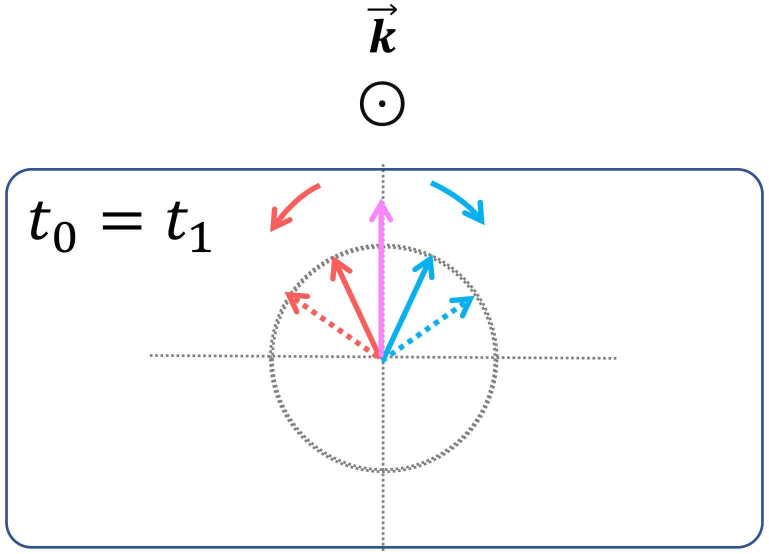
Phase difference introduced by B-field

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k\, dl$$

prev. sl.

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{\rm p}^2}{\omega^2} \mp \frac{\omega_{\rm p}^2 \omega_{\rm L}}{\omega^3}}$$



- Right circular polarized (RCP) mode: –
- Light circular polarized (LCP) mode: +

When $B = 0$, $\omega_L = \frac{eB}{m_0 c} = 0$, there is no phase difference

$$k_{\rm RCP} = k_{\rm LCP}$$
$$\Delta\phi \equiv \phi_{\rm RCP} - \phi_{\rm LCP} = 0$$

Phase difference in parallel B-field

$\vec{k} \parallel \vec{B}$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: – $\otimes$

Eq. 4 of Ferriere et al. 2021

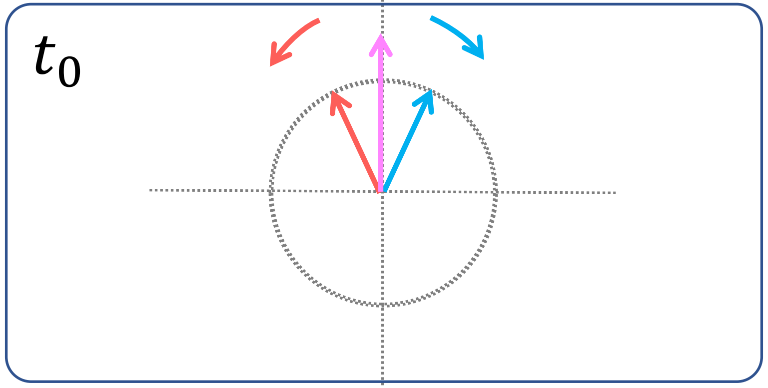
$$\phi \equiv \omega t - \int_0^l k \, dl$$

prev. sl.

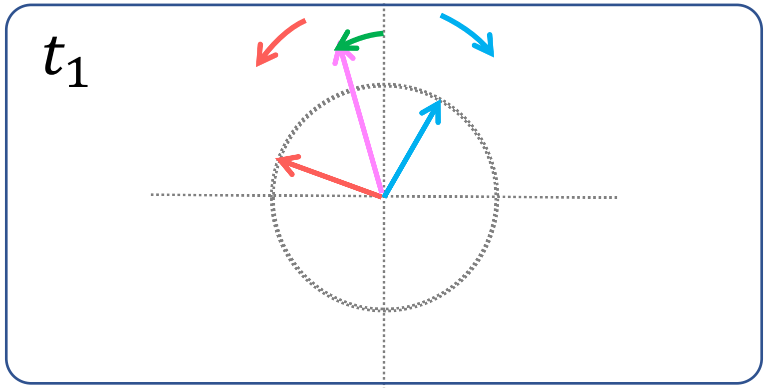
$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega^3}}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \ \dots$$

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_\mathrm{p}^2}{2\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{2\omega^3}\right)$$



$$\Delta\psi = \frac{1}{2}\Delta\phi$$



$$\Delta\psi = \frac{1}{2}\Delta\phi = -\frac{1}{2}\int_0^l (k_\mathrm{RCP} - k_\mathrm{LCP}) dl$$

$$= \frac{1}{2}\int_0^l \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{c\omega^2} \, dl$$

- Right circular polarized (RCP) mode: –
- Light circular polarized (LCP) mode: +

Rotation measure

prev. sl.

$$\Delta\psi = \frac{1}{2}\int_0^l \frac{\omega_\mathrm{p}^2\omega_\mathrm{L}}{c\omega^2}\, dl$$

prev. sl.

$$\omega_\mathrm{p} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency — Larmor frequency

Eq. 9 of Ferriere et al. 2021

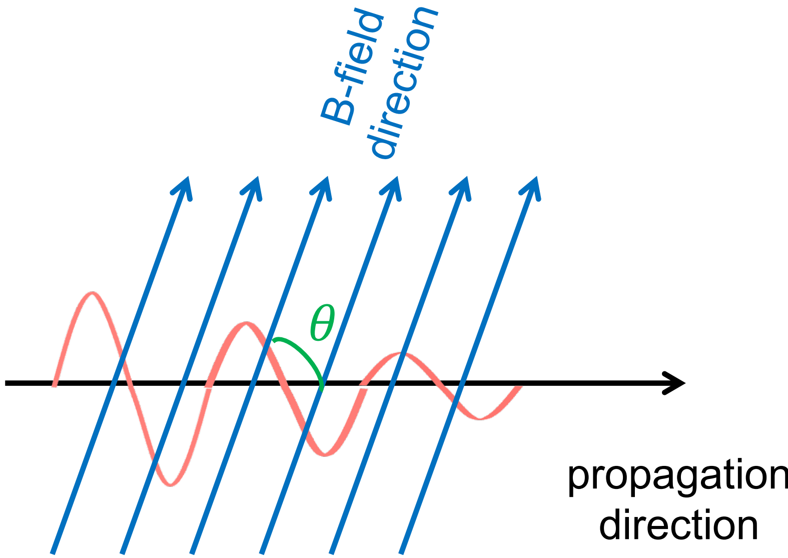
$$\Delta\psi = \frac{e^3}{2\pi\, m_e^2\, c^4}\lambda^2 \int_0^l n_e(\pm B_\parallel)dl$$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: – $\otimes$

Rotation Measure (RM) $\longrightarrow$

$$\mathrm{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4}\int_0^l n_e(\pm B_\parallel)dl$$

$$\mathrm{RM} \;= 0.812\ \mathrm{rad\ m^{-2}} \int_0^l \left(\frac{n_e}{\mathrm{cm^{-3}}}\right)\left(\pm\frac{B_\parallel}{\mu\mathrm{G}}\right) d\left(\frac{l}{\mathrm{pc}}\right)$$



Faraday rotation

When a linearly polarized wave travels through a magnetized medium, its polarization angle will rotate (Burn 1966)

Eq. 9 of Ferriere et al. 2021

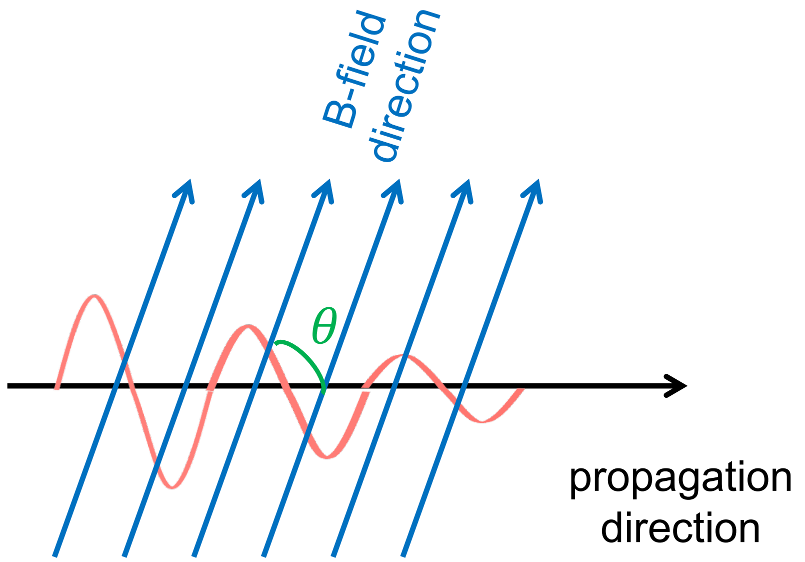
$$\Delta\psi = \frac{e^3}{2\pi\, m_e^2\, c^4}\lambda^2 \int_0^l n_e(\pm B_\parallel)dl$$

prev. sl.

$$\mathrm{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4}\int_0^l n_e(\pm B_\parallel)dl$$

$$\psi_{\mathrm{obs}} = \psi_{\mathrm{src}} + \mathrm{RM}\cos\theta\ \lambda^2$$

- $\vec{B}$ points towards the observer: + ⊙
- $\vec{B}$ points away from the observer: – ⊗



Cosmic Microwave Background

CMB is the leftover radiation from the Big Bang. It was firstly predicted by Gamow (1948).

In1965, CMB was observed by Arno Penzias and Robert Wilson using the 6-meter Holmdel Horn Antenna.

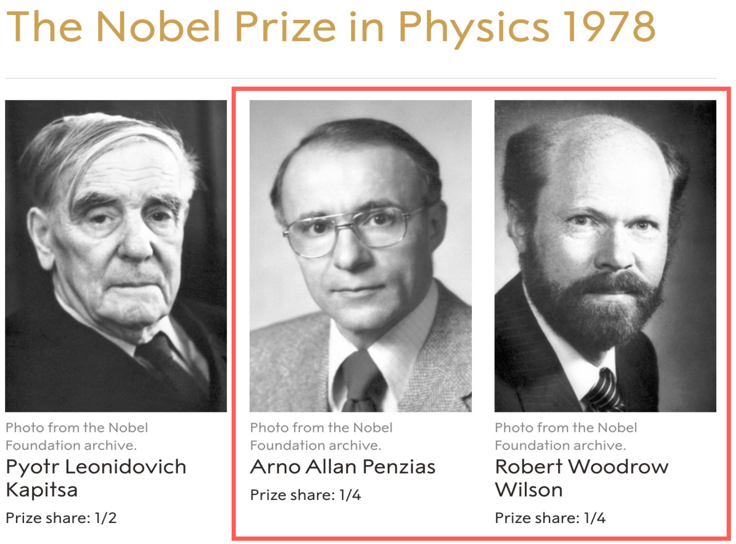


Image credit: nobelprize.org

Wrong data excluded

Image credit: wikipedia

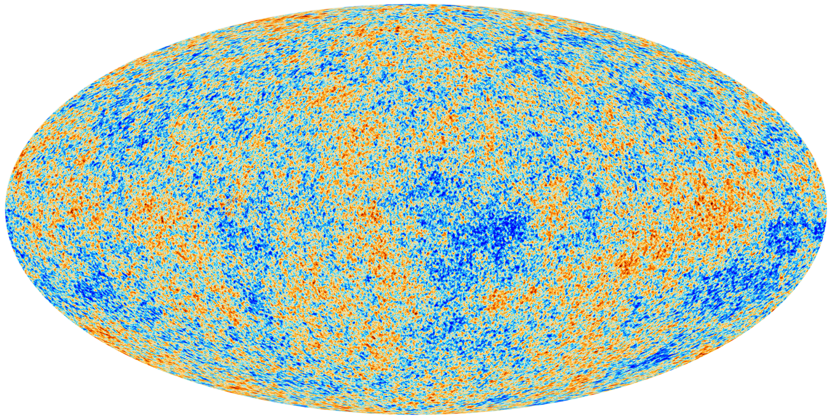


Image credit: ESA

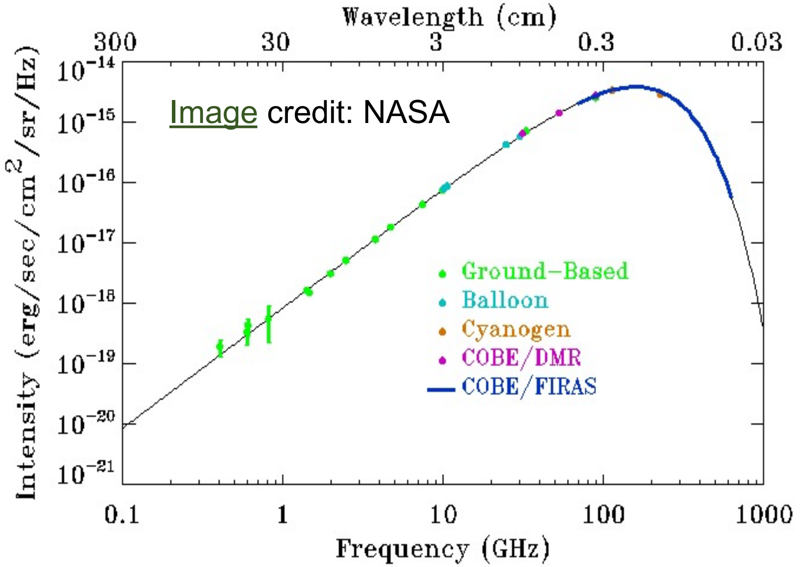
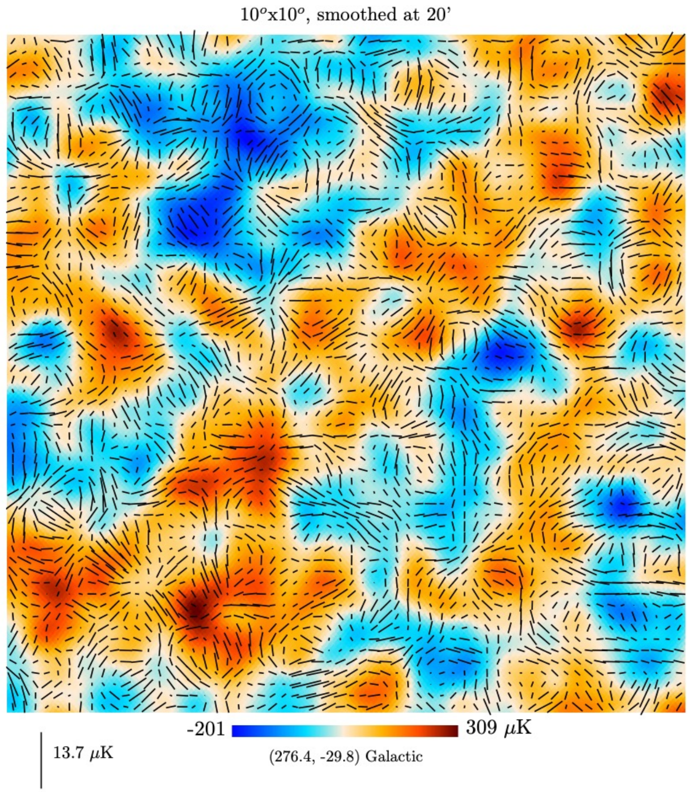
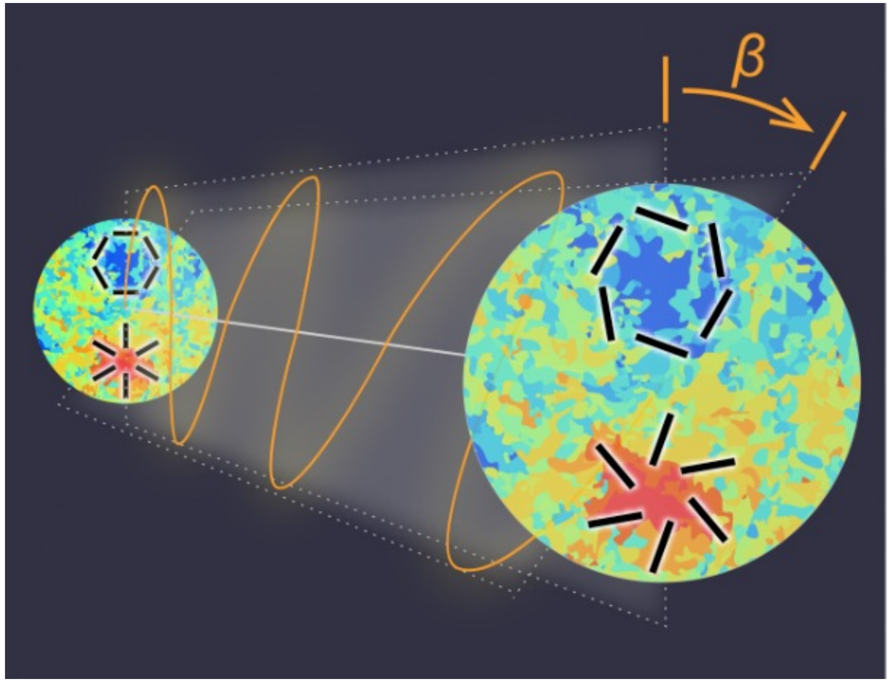


Image credit: NASA

Cosmic Microwave Background polarization





Komatsu (2022, review): β is the cosmic-birefringence-induced rotation angle

- ❑ B-mode (curl-like)
  - ➢ $\perp \vec{k}$ or $\parallel \vec{k}$
  - ➢ dominated by density perturbation
- ❑ E-mode (gradient-like)
  - ➢ $45°$ w.r.t. $\vec{k}$
  - ➢ fingerprints of GW background (too faint to detect directly nowadays) from cosmic inflation

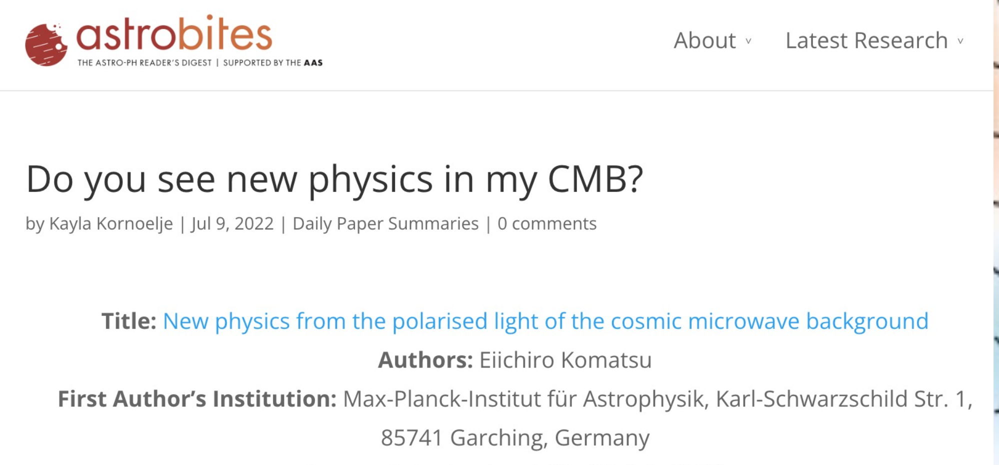
CMB is linearly polarized (E-mode and B-mode), firstly detected in 2002 (Kovac et al. 2002).

Birefringence: the refractive index depends on the polarization direction. Typically, double refraction (ordinary and extraordinary indices)

CMB B-mode

The detection of B-mode polarization will provide strong constraints for cosmic inflationary models.

The B-mode power is $\propto r^2$, where, $r < 0.07$ is the tensor-to-scalar ratio (BICEP2 collaboration 2016).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Do you see new physics in my CMB?

by Kayla Kornoelje | Jul 9, 2022 | Daily Paper Summaries | 0 comments

Title: New physics from the polarised light of the cosmic microwave background

Authors: Eiichiro Komatsu

First Author's Institution: Max-Planck-Institut für Astrophysik, Karl-Schwarzschild Str. 1, 85741 Garching, Germany

Komatsu (2022, review)

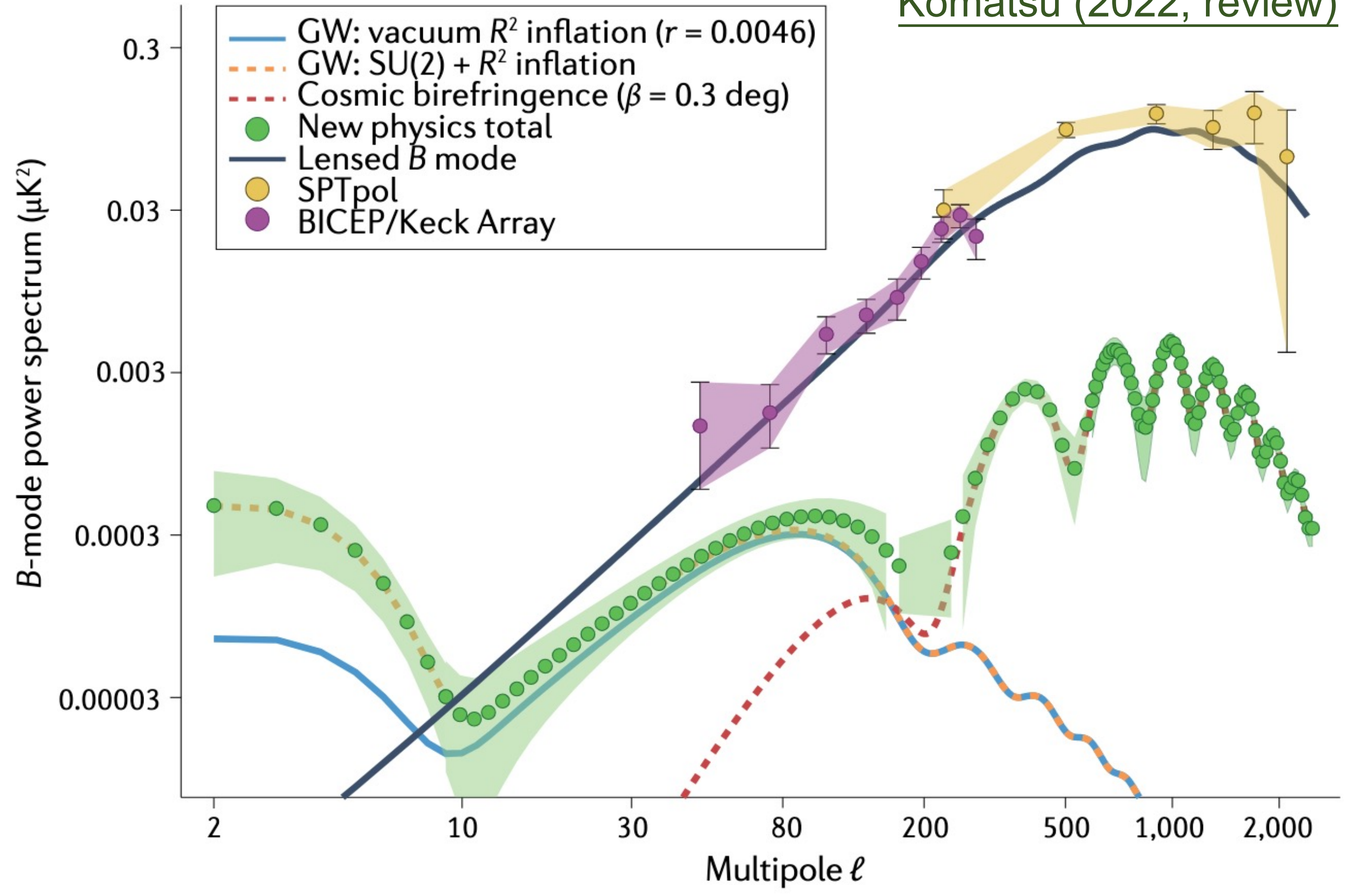


Image credit: bicepkeck.org

AY2024F

Junjie Mao

Radiative Processes in Astrophysics

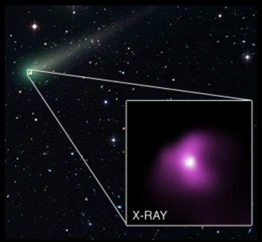
Observation

Up to cosmic size scale

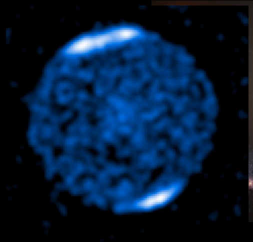
Theory

Down to atomic size scale

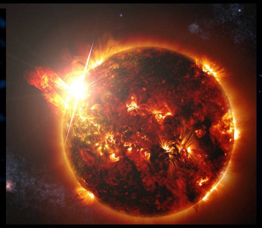
C/2012S1 (comet)



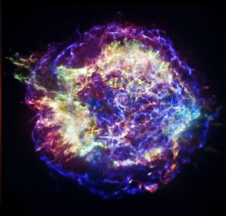
Jupiter (planet)



Sun (star)

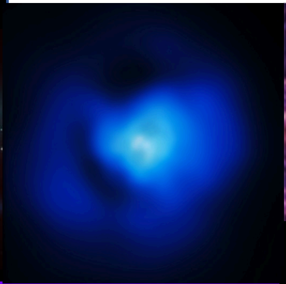


Cas A (SNR)



M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

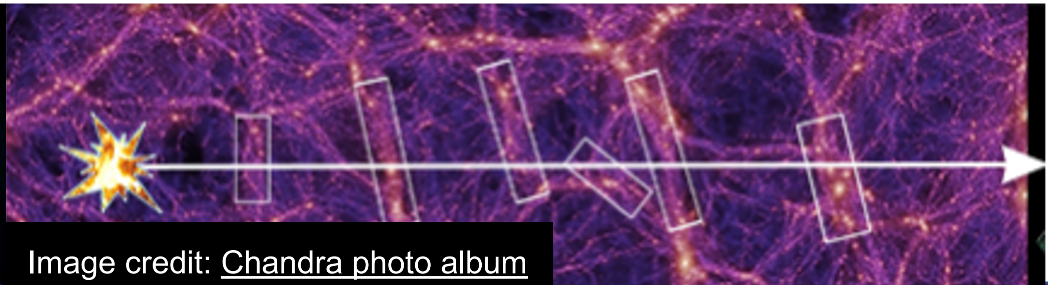


Image credit: Chandra photo album

Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge
3.1.1 Electric and magnetic field
3.1.2 Radiative power angular distribution
3.1.3 Asymptotic behavior (non-relativistic)
3.1.4 Asymptotic behavior (relativistic)
3.1.5 Pulsar spin-down

3.2 Thomson scattering
3.3 Cyclotron radiation
3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

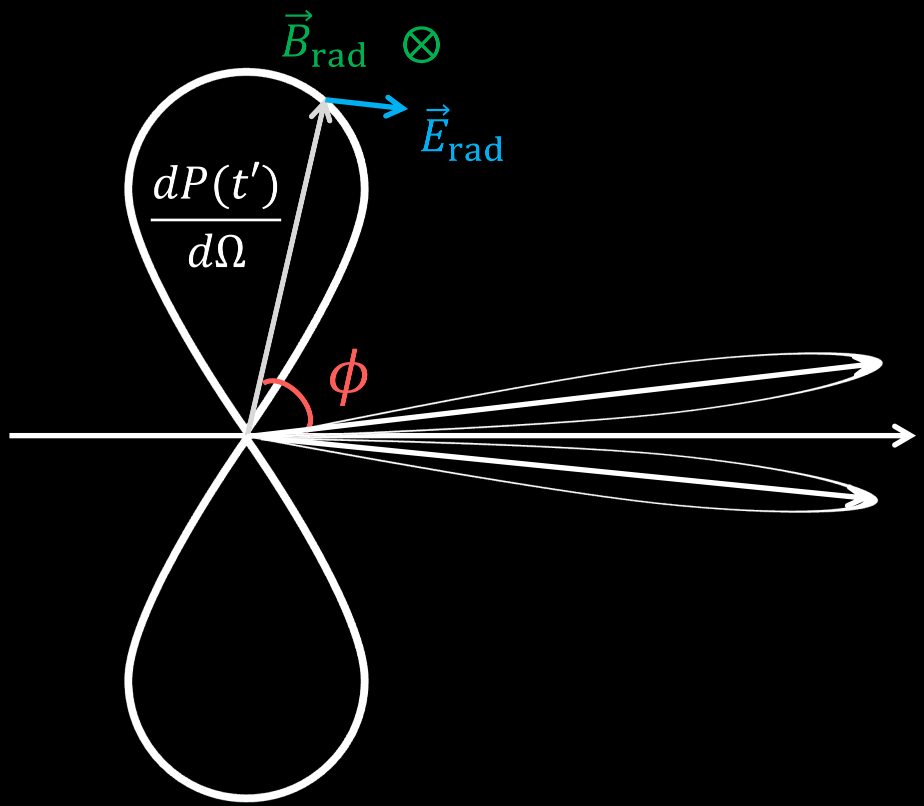


Image credit: Junjie Mao

Electric and magnetic field of an accelerated charge

Consider a moving point charge, its electric and magnetic fields are

charge

$$\vec{E}(r,t) = \frac{q}{\kappa^3 R^2}\left(\vec{e}_{\mathrm{R}} - \vec{\beta}\right)\left(1-\beta^2\right) + \frac{q}{c\kappa^3 R}\vec{e}_{\mathrm{R}} \times \left(\left(\vec{e}_{\mathrm{R}} - \vec{\beta}\right) \times \frac{\partial \vec{\beta}}{\partial t}\right)$$

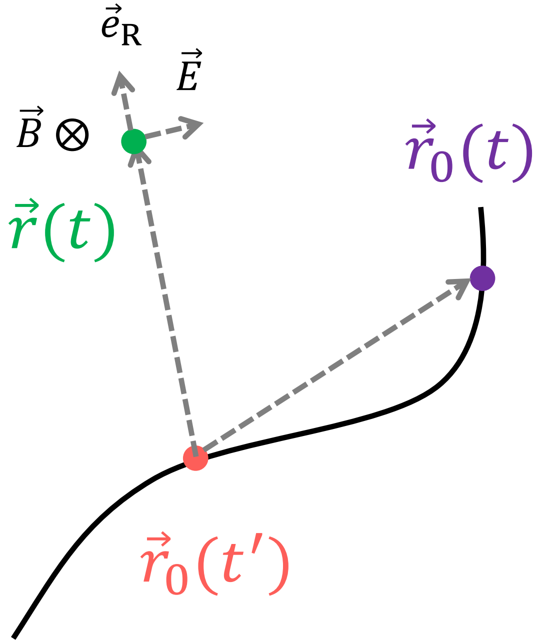
$$\vec{B}(r,t) = \vec{e}_{\mathrm{R}} \times \vec{E}(r,t)$$

$$R = |\vec{r}(t) - \vec{r}_0(t')|, \vec{e}_R = \frac{\vec{R}}{R}$$

see Chpt. 3 of the REF book (p77 - 80) by Rybicki & Lightman

$$\vec{\beta} = \frac{\vec{v}(t')}{c}, \beta = \frac{v(t')}{c}, v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\overrightarrow{e_R} \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

Retarded time

prev. sl.

Consider a moving point charge, its electric and magnetic fields are

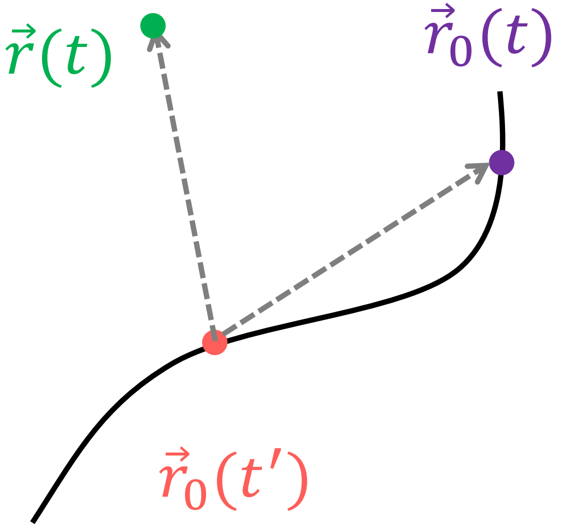
$$\vec{E}(r,t) = \cdots$$

$$\vec{B}(r,t) = \vec{e}_{\mathrm{R}} \times \vec{E}(r,t)$$

$$R = |\vec{r}(t) - \vec{r}_0(t')|, \vec{e}_R = \frac{\vec{R}}{R}$$

$$\vec{\beta} = \frac{\vec{v}(t')}{c}, \beta = \frac{v(t')}{c}, v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\vec{e}_{\mathrm{R}} \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

The propagation of electromagnetic waves has a finite speed. Accordingly, the radiation properties we observed at time t was emitted at an earlier time t'

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} = t - \frac{|\vec{r}_0(t) - \vec{r}_0(t')|}{c}$$

retarded time

Velocity field of an accelerated charge

prev. sl.

$$\vec{E}(r,t) = \frac{q}{\kappa^3\, R^2}(\vec{e}_R - \vec{\beta})(1-\beta^2) + \frac{q}{c\kappa^3 R}\vec{e}_R \times \left((\vec{e}_R - \vec{\beta}) \times \frac{\partial \vec{\beta}}{\partial t} \right)$$

velocity field $\vec{E}_{\text{vel}}$

$$\vec{e}_R = \frac{\vec{R}}{R}$$

$$\kappa = 1 - \frac{\vec{e}_R\, \vec{v}(t')}{c}$$

$$\beta = \frac{v(t')}{c}$$

For a constantly $(\partial\beta/\partial t = 0)$ moving charge, $\vec{E} = \vec{E}_{\text{vel}}$

$$\vec{E}_{\text{vel}} \propto \frac{1}{R^2}$$

little contribution to radiation at large distance (R)

$$\rightarrow \frac{q}{R^2}\vec{e}_R \text{ for } \beta \ll 1$$

Coulomb's law

Radiation field of an accelerated charge

prev. sl.

$$\vec{E}(r,t) = \frac{q}{\kappa^3\, R^2}(\vec{e}_R - \vec{\beta})(1-\beta^2) + \frac{q}{c\kappa^3 R}\vec{e}_R \times \left((\vec{e}_R - \vec{\beta}) \times \frac{\partial \vec{\beta}}{\partial t} \right)$$

$$\vec{e}_R = \frac{\vec{R}}{R}$$

$$\kappa = 1 - \frac{\vec{e}_R\, \vec{v}(t')}{c}$$

$$\beta = \frac{v(t')}{c}$$

acceleration field $\vec{E}_{\text{acc}}$

$\vec{B}_{\text{acc}} = \overrightarrow{e_R} \times \vec{E}_{\text{acc}}$

radiation field

$\vec{E}$ refers to $\vec{E}_{\text{acc}}$ hereafter

Only accelerated $(\partial\beta/\partial t \neq 0)$ particles can give rise to radiation!

Acceleration can be caused by

(1) external B-fields (e.g., cyclotron and synchrotron);

(2a) collision with photons (e.g., Compton);

(2b) collision among electrons (bremsstrahlung caused by Coulomb force)

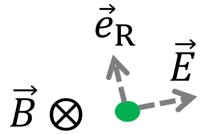
Radiative power

Poynting vector (radiative power vector per unit area)

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\vec{E}\times\left(\vec{e}_R\times\vec{E}\right) = \frac{c}{4\pi}\vec{e}_R\left|\vec{E}\right|^2$$

Sect. 1.3.1 of 《天体物理中的辐射机制》by 尤峻汉 (p10)

prev. sl. $\vec{B}(r,t) = \vec{e}_R\times\vec{E}(r,t)$



$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = (\vec{a}\cdot\vec{c})\,\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$

Along the direction of the Poynting vector $\vec{e}_R$, the radiative power received by an observer at the distance R with a collecting area $d\vec{A} = \vec{e}_R\, R^2 d\Omega$

$$dP(t)dt = \vec{S}\cdot d\vec{A} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2\, d\Omega$$

Radiative power received by an observer per unit solid angle

$$\frac{dP(t)}{d\Omega} = \frac{dE(t)}{dt\, d\Omega} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2$$

Radiative power (cont.)

prev. sl.

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} = t - \frac{|\vec{r}_0(t) - \vec{r}_0(t')|}{c}$$

$$\kappa = 1 - \frac{\vec{e}_R \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$

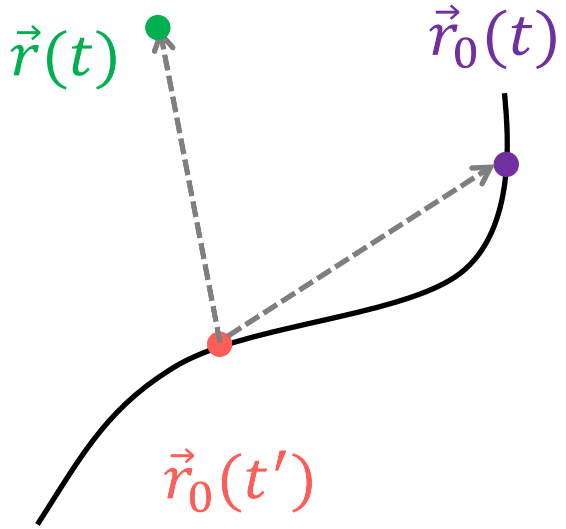
Conservation of energy: Energy received by an observer equals energy emitted by the emitter

$$dP(t)dt = dP(t')dt'$$

$$\vec{R} = |\vec{r}(t) - \vec{r}_0(t')| \approx |\vec{r}(t)| - \vec{e}_R \cdot \vec{r}_0(t') \text{ for } |\vec{r}_0(t')| \ll |\vec{r}|$$

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} \approx t - \frac{1}{c}\left(|\vec{r}(t)| - \vec{e}_R \cdot \vec{r}_0(t')\right)$$

$$dt' = dt + \frac{1}{c}\vec{e}_R \cdot d\vec{r}_0(t')$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

Radiative power angular distribution

prev. sl.

$$dt = dt' - \frac{1}{c}\vec{e}_R \cdot d\vec{r}_0(t')$$

$$\frac{dt}{dt'} = 1 - \frac{1}{c}\,\vec{e}_R \cdot \frac{d\vec{r}_0(t')}{dt'} = \kappa$$

$$\frac{dP(t')}{d\Omega} = \frac{dP(t)}{d\Omega}\frac{dt}{dt'} = \frac{dP(t)}{d\Omega}\kappa$$

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c\kappa^5}\left|\vec{e}_R \times \left((\vec{e}_R - \vec{\beta}) \times \dot{\vec{\beta}}\right)\right|^2$$

prev. sl.

$$v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\vec{e}_R \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$

prev. sl.

$$dP(t)dt = dP(t')dt'$$

prev. sl.

$$\frac{dP(t)}{d\Omega} = \frac{dE(t)}{dt\,d\Omega} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2$$

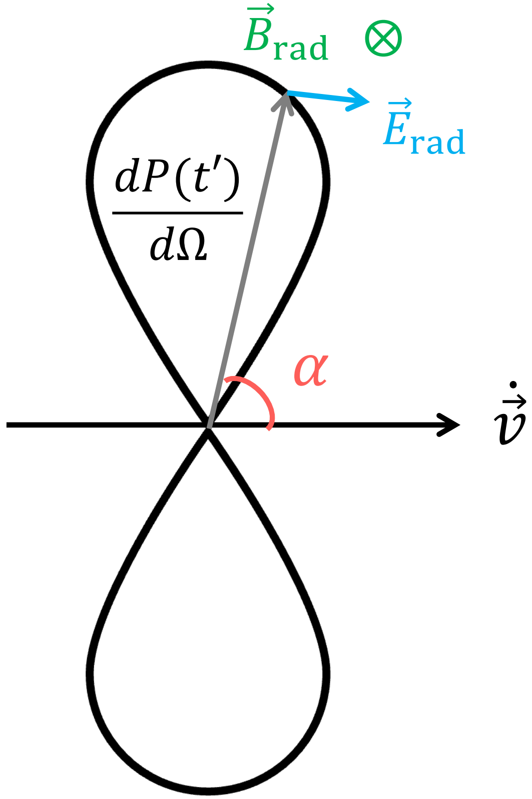
prev. sl.

$$\vec{E} = \frac{q}{c\kappa^3 R}\vec{e}_R \times \left((\vec{e}_R - \vec{\beta}) \times \dot{\vec{\beta}}\right)$$

Asymptotic behavior (non-relativistic)

prev. sl.

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c\kappa^5}\left|\vec{e}_R\times\left((\vec{e}_R - \vec{\beta})\times\dot{\vec{\beta}}\right)\right|^2$$



For non-relativistic particles with $v \ll c$ $(t \approx t')$, and α the angle between $\vec{e}_R$ and $\dot{\vec{v}}$

$$\kappa = 1 - \frac{\vec{e}_R\cdot\vec{v}}{c} \to 1$$

$$\beta = \frac{v}{c} \to 0$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\,\dot{v}^2\sin^2\alpha$$

$$\vec{e}_R\times(\vec{e}_R\times\dot{\vec{v}}) = (\vec{e}_R\cdot\dot{\vec{v}})\vec{e}_R - (\vec{e}_R\cdot\vec{e}_R)\dot{\vec{v}} = (\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}$$

$$\left|\vec{e}_R\times\vec{e}_R\times\dot{\vec{v}}\right|^2 = \left((\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}\right)\left((\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}\right)$$

$$= (\dot{v}\cos\alpha)^2(\vec{e}_R\cdot\vec{e}_R) - 2(\dot{v}\cos\alpha)(\dot{\vec{v}}\cdot\vec{e}_R) + \dot{v}^2$$

$$= \dot{v}^2(\cos^2\alpha - 2\cos^2\alpha + 1) = \dot{v}^2\sin^2\alpha$$

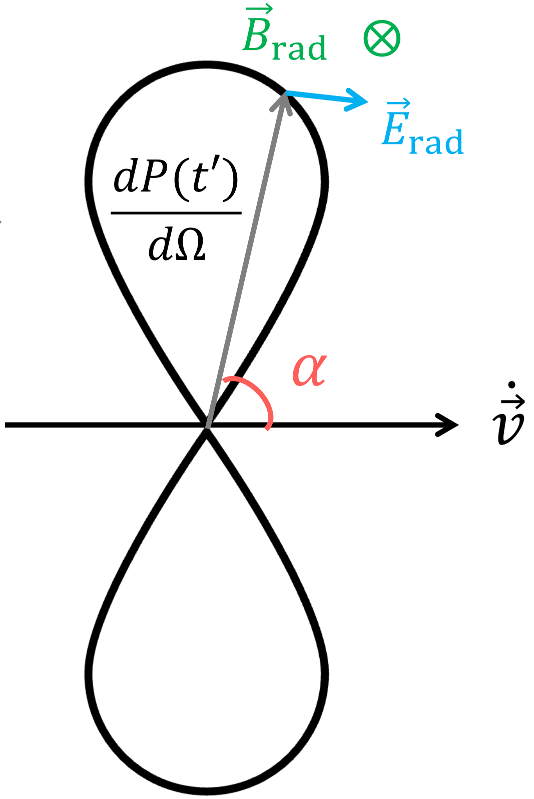
Larmor's formula (non-relativistic)

prev. sl.

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\dot{v}^2 \sin^2\alpha$$

$$d\Omega = \int_0^{2\pi} d\phi \int_{-\pi/2}^{\pi/2} \sin\alpha \, d\alpha = 2\pi \int_{-1}^{1} d\cos\alpha$$

$$\int_{-1}^{1} (1-\cos^2\theta) d\cos\theta = \frac{4}{3}$$



$$P = \frac{q^2}{4\pi c^3}\dot{v}^2 \int \sin^2\alpha \, d\Omega = \frac{q^2}{2c^3}\dot{v}^2 \int_{-1}^{1} (1-\cos^2\alpha) d\cos\alpha$$

$$\boxed{P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}} \longleftarrow \ddot{d} = q\,\ddot{r} = q\,\dot{v}$$

see Chpt. 3 of the REF book (p85 - 88) by Rybicki & Lightman for more discussions on the dipole approximation

✓ This dipole angular distribution is symmetrically perpendicular to the acceleration direction $\dot{\vec{v}}$

✓ The maximum radiation is perpendicular to $\dot{\vec{v}}$ and there is no radiation along $\dot{\vec{v}}$

Asymptotic behavior (relativistic)

For parallel acceleration,

see Chpt. 4 of Ribicki & Lightman (p138-145)

$$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16q^2\,\dot{v}_{\parallel}^2}{\pi c^3}\,\gamma^{10}\,\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6}$$

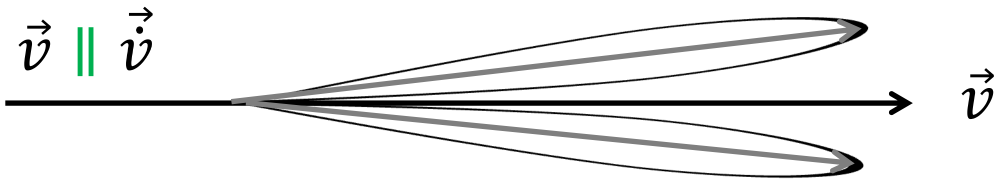
For perpendicular acceleration,

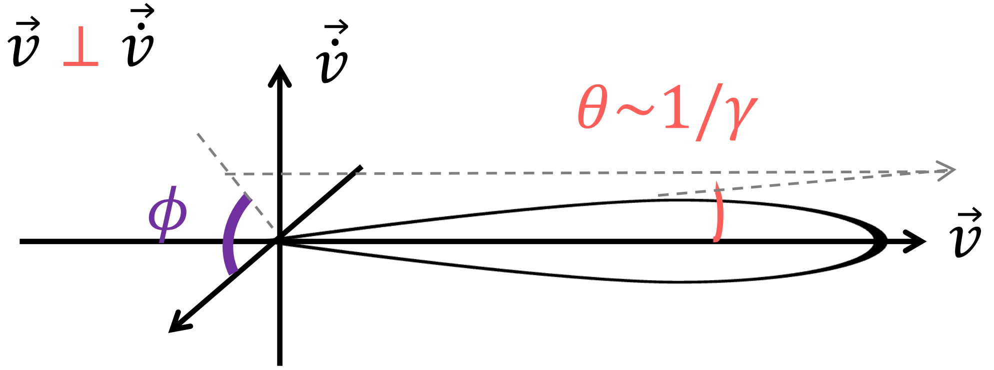
$$\frac{dP_{\perp}}{d\Omega} \approx \frac{4q^2\,\dot{v}_{\perp}^2}{\pi c^3}\,\gamma^{8}\,\frac{1-2\gamma^2\theta^2\cos 2\phi+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^6}$$

Lorentz factor

$$\gamma = \frac{mc^2}{m_0c^2} = \frac{1}{\sqrt{1-\beta^2}}$$

rest mass of the particle





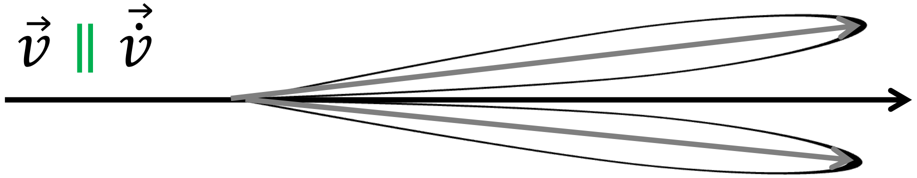
the larger the Lorentz factor,
the larger the radiative power

Parallel acceleration (relativistic)

prev. sl.

$$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16q^2\,\dot{v}_{\parallel}^2}{\pi c^3}\,\gamma^{10}\,\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6}$$

$\vec{v} \parallel \dot{\vec{v}}$

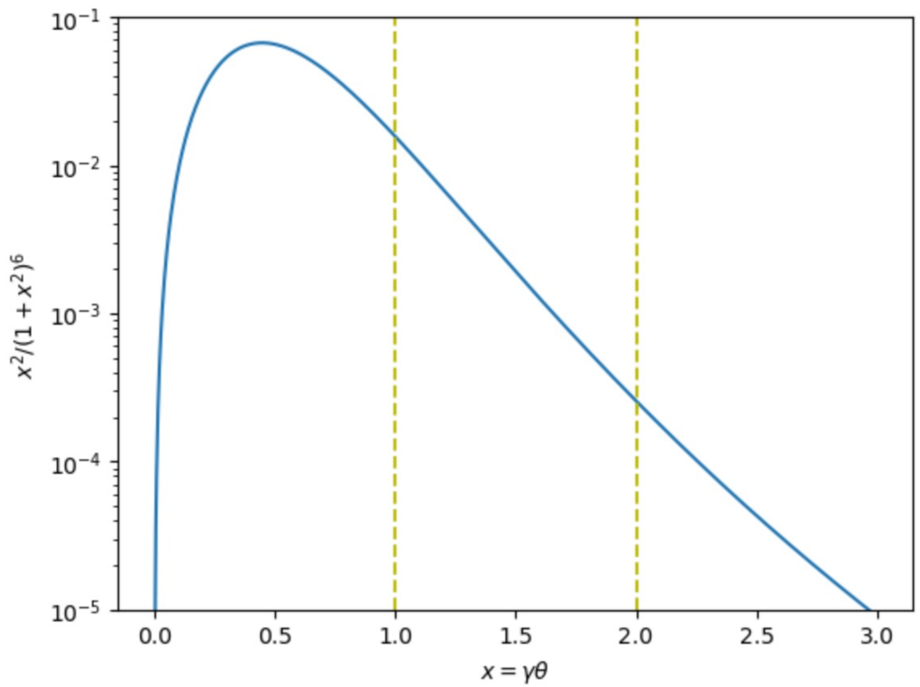


For $\theta = 0$ $$\frac{dP_{\parallel}}{d\Omega} = 0$$

For $\theta = 1/\gamma$ $$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16q^2\,\dot{v}_{\parallel}^2}{\pi c^3}\,\gamma^{10}\,\frac{1}{2^6}$$

$$\frac{1/2^6}{2^2/(1+2^2)^6} \sim 60$$

For $\theta = 2/\gamma$ $$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16q^2\,\dot{v}_{\parallel}^2}{\pi c^3}\,\gamma^{10}\,\frac{2^2}{(1+2^2)^6}$$



Perpendicular acceleration (relativistic)

prev. sl.

$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\,\gamma^8\,\frac{1-2\gamma^2\theta^2\cos 2\phi+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^6}$$

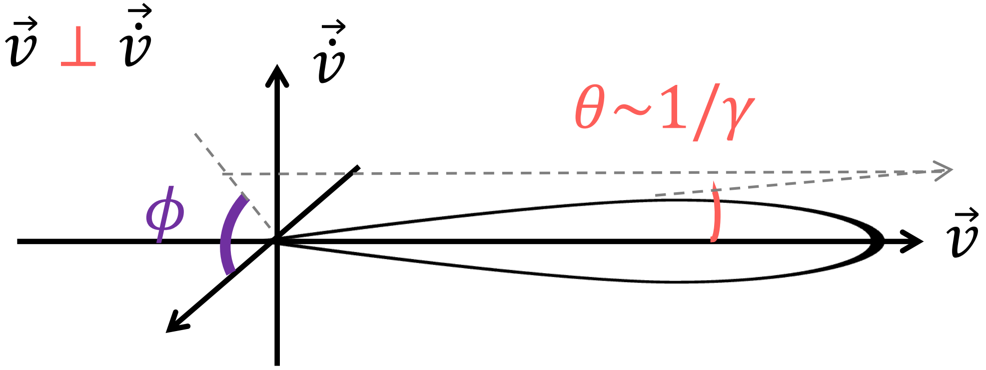
For $\theta = 0$

$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\,\gamma^8$$

For $\theta = 1/\gamma$

$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\gamma^8\frac{(1-\cos 2\phi)}{2^5}$$

$\vec{v} \perp \dot{\vec{v}}$



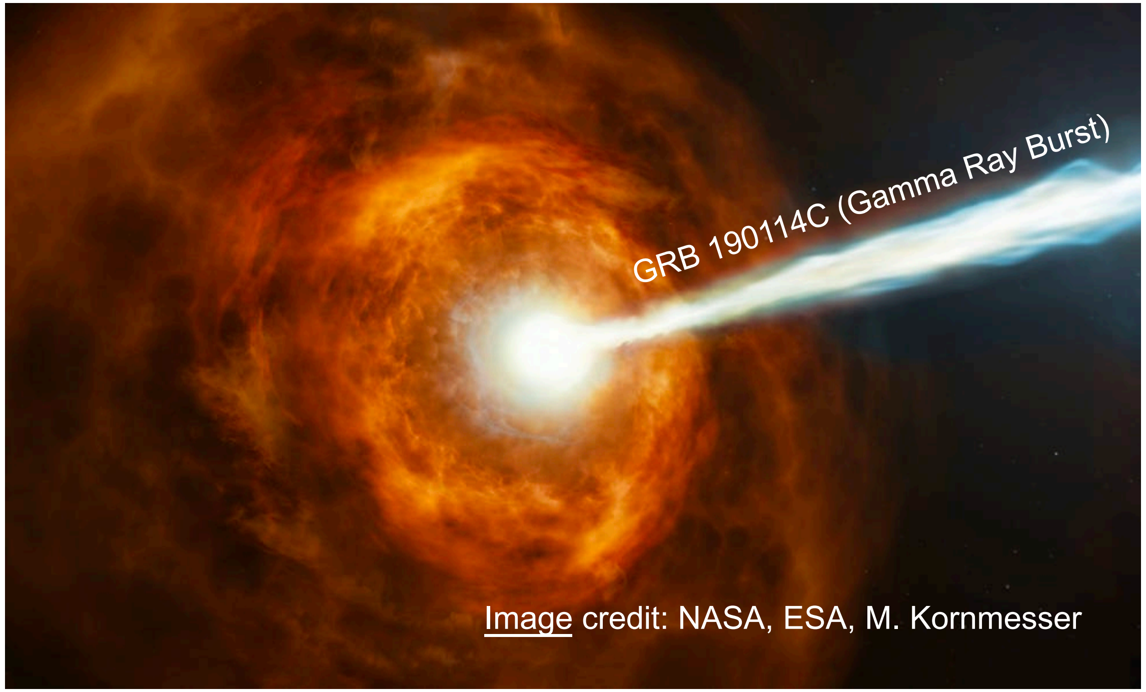
The radiation is smaller by a factor of $\sim 2^5 = 32$ at the half opening angle of $\phi = \gamma^{-1}$
The larger the Lorentz factor, the smaller the opening angle

Gamma-Ray bursts (GRBs)

GRBs are the most luminous explosions in the Universe (Meszaro 2006) with a typical energy of 10^{51} erg s^{-1} (Frail et al. 2001).

cf. $L(\mathrm{FRB}) \sim 10\ L_{\odot} \sim 10 \times 3.8 \times 10^{33}$ erg s^{-1}

GRBs are discovered serendipitously in the 1967 by Vela satellites (Klebesadel et al. 1973)



The Lorentz factor of GRBs can be $\gtrsim 10^2$ (Lithwick & Sari 2001, Ghirlanda et al. 2018)

Total radiative power (relativistic)

the radiative power is significantly larger ($\propto \gamma^n$) than the non-relativistic one

Chpt. 4 of the REF book (p137-140) by Ribicki & Lightman

$$P = \frac{2q^2}{3c^3}\gamma^4\left(\dot{v}_\perp^2 + \gamma^2 \dot{v}_\parallel^2\right)$$

Larmor's formula

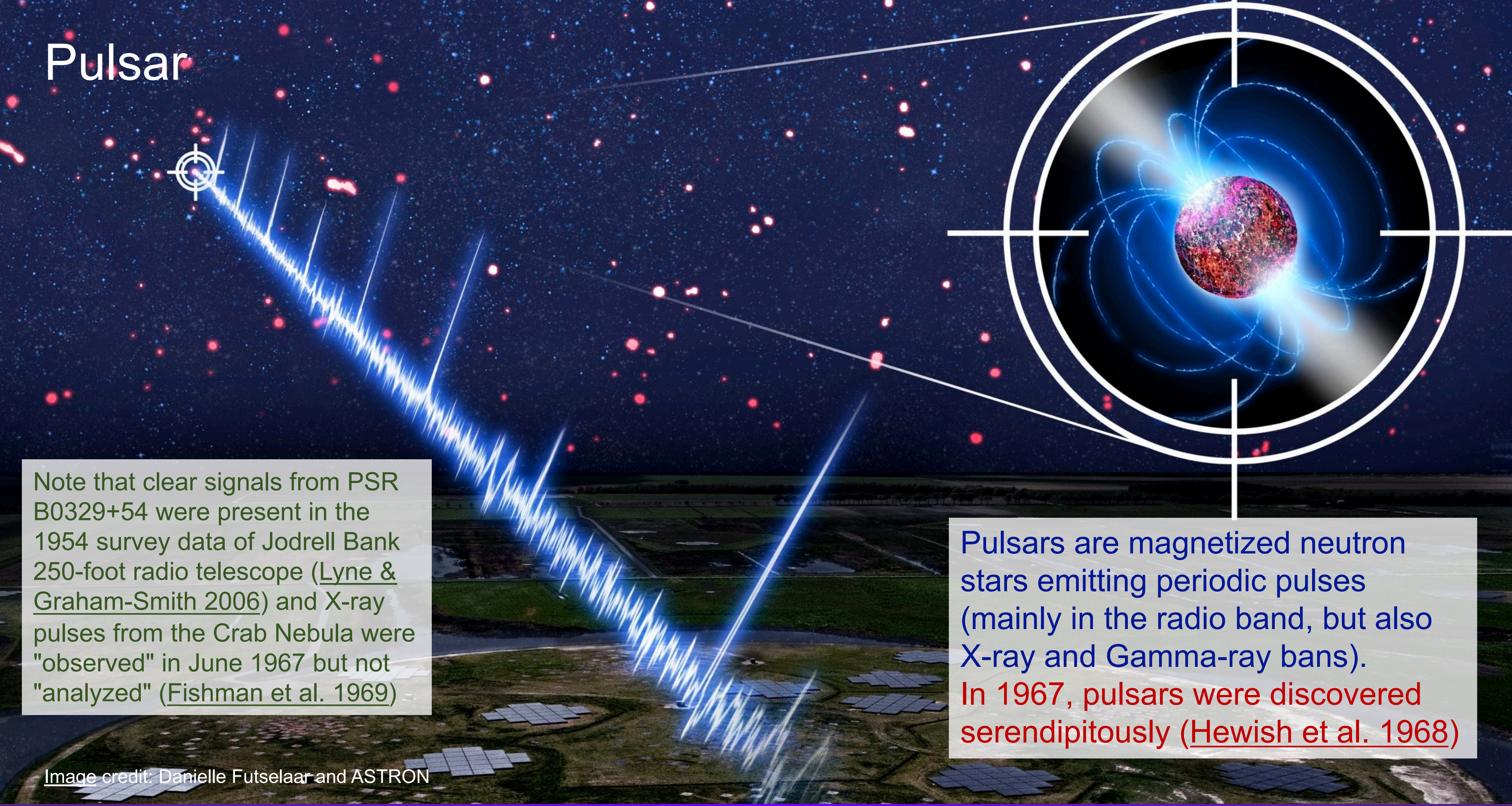
$$P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}$$

When $\beta \to 0$ $(\gamma \to 1)$, the radiative power is identical to that of the Larmor's formula

$$\dot{v} \propto \frac{1}{m_0}$$

The smaller the particle mass, the larger the radiative power
→ Radiation of electrons are much larger than that of protons

Pulsar



Note that clear signals from PSR B0329+54 were present in the 1954 survey data of Jodrell Bank 250-foot radio telescope (Lyne & Graham-Smith 2006) and X-ray pulses from the Crab Nebula were "observed" in June 1967 but not "analyzed" (Fishman et al. 1969)

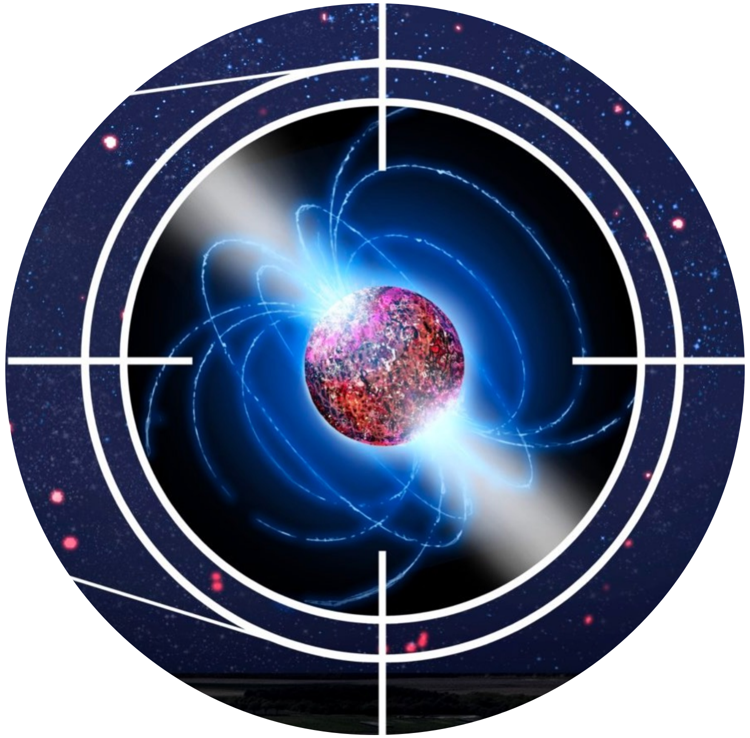
Pulsars are magnetized neutron stars emitting periodic pulses (mainly in the radio band, but also X-ray and Gamma-ray bans).
In 1967, pulsars were discovered serendipitously (Hewish et al. 1968)

Image credit: Danielle Futselaar and ASTRON

Neutron star (NS)

A feeling of fear or anxiety about something that may happen

The concept of neutron stars was envisioned with trepidation by Baade & Zwicky (1934): "With all reserve we advance the view that a supernova represents the transition of an ordinary star into a new form of star, the neutron star, which would be the endpoint of stellar evolution. Such a star may possess a very small radius and an extremely high density."



PSR J0250+5854

$P_{\text{spin}} = 23.5\ \text{s}$

Tan et al. 2018

Normal star

$B \sim 1\ \text{G}$

$R \sim 10^6\ \text{km}$

$P_{\text{spin}} \sim 1\ \text{yr}$

Conservation of angular momentum

$L \propto m\, r^2\, \omega$

collapse

Conservation of magnetic flux

$\Phi = \int \vec{B} \cdot \vec{e}_A dA$

Neutron star

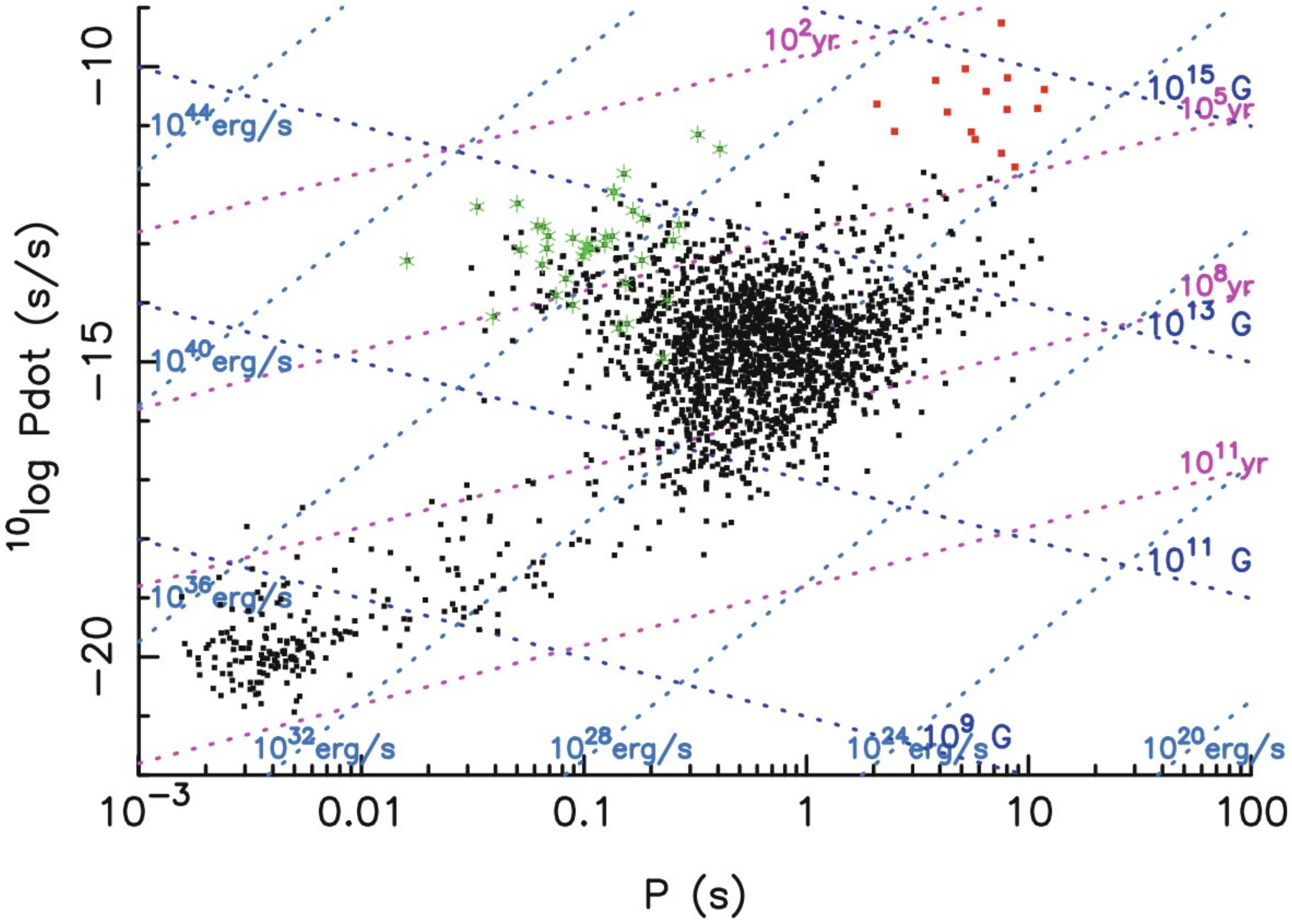
$R \sim 10\ \text{km}$

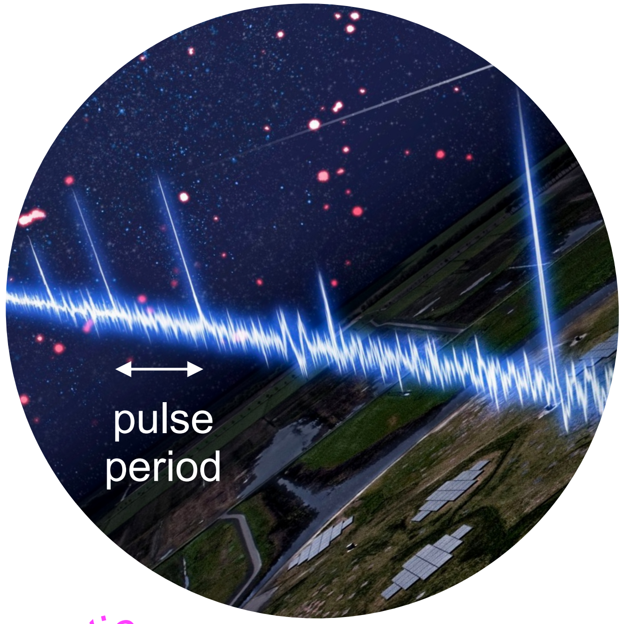
$B \sim 10^{10}\ \text{G}$

$P_{\text{spin}} \sim 10^{-3}\ \text{s}$

Why is PSR J0250+5854 so slow?

P-Pdot diagram for pulsars





pulse period derivative

pulse period

characteristic pulsar age

surface B-field strength

red squares: magnetars
green stars: pulsars in pulsar wind nebulae

Vink (2021)

Magnetic dipole

A young neutron star can be viewed as a rapidly rotating dipole

The power of magnetic dipole rotation is similar to that of electric dipole

$$P = \frac{2\,(\ddot{\mu}\sin\alpha)^2}{3c^3}$$

$$= \frac{2}{3}\frac{\mu^2\omega^4}{c^3}\sin^2\alpha$$

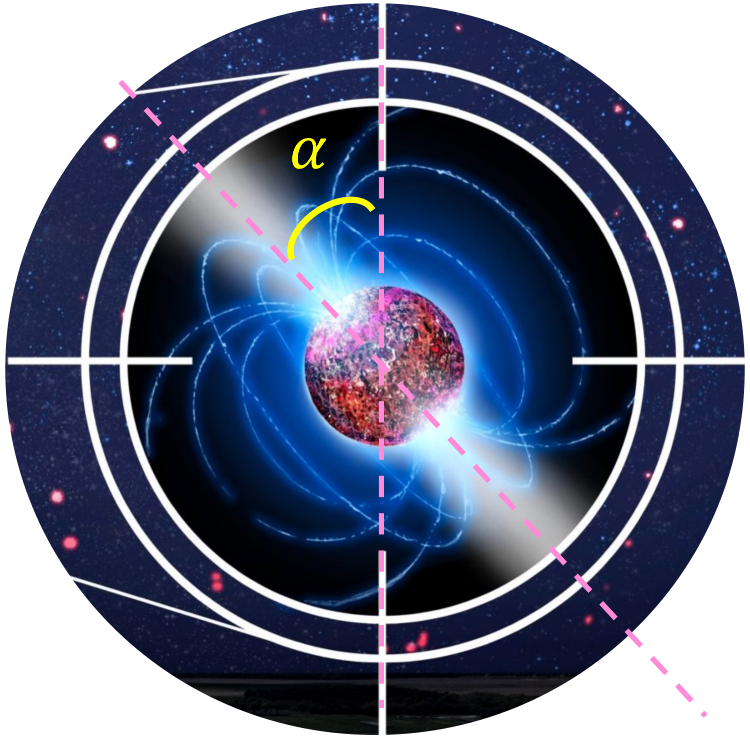
magnetic dipole momentum $\longrightarrow$ $\mu = \frac{1}{2}BR_{\mathrm{NS}}^3$

$$\frac{d\vec{\mu}}{dt} = \vec{\omega}\times\vec{\mu}$$

$$\ddot{\mu} = \mu\,\omega^2$$

$$\omega = \frac{2\pi}{P}$$

pulse period



Why are some pulsars so slow?

Larmor's formula

$$P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}$$

Pulsar spin down and B-field strength

Lattimer & Schutz, 2005

$$\mu = \frac{1}{2} B R_{\mathrm{NS}}^3$$

$$\omega = \frac{2\pi}{P}$$

$$I \simeq \frac{1}{5} M_{\mathrm{NS}} R_{\mathrm{NS}}^2$$

Assuming the spin-down of pulsars is due to the energy and momentum loss of the magnetic dipole radiation,

$$\frac{d}{dt}\left(-\frac{1}{2} I \omega^2\right) = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \alpha$$

$$\frac{d}{dt}\left(-\frac{1}{2} I \omega^2\right) = \frac{4\pi^2 \; M_{\mathrm{NS}} R_{\mathrm{NS}}^2}{2\times 5} \frac{2\dot{P}}{P^3}$$

$$\frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \alpha = \frac{1}{6} \frac{B^2 R_{\mathrm{NS}}^6}{c^3} \frac{(4\pi^2)^2}{P^4} \sin^2 \alpha$$

$$B = \left(\frac{3c^3}{10\pi^2 \sin^2 \alpha} P\dot{P} \frac{M_{\mathrm{NS}}}{R_{\mathrm{NS}}^4}\right)^{1/2} = \frac{2.4\times 10^{19}}{\sin \alpha} (P\dot{P})^{\frac{1}{2}} \left(\frac{M_{\mathrm{NS}}}{1.4\; M_{\odot}}\right)^{\frac{1}{2}} \left(\frac{10\ \mathrm{km}}{R_{\mathrm{NS}}}\right)^2 \ \mathrm{G}$$

Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge

3.2 Thomson scattering
3.2.1 Scattered power and angular distribution
3.2.2 Differential cross section
3.2.3 Total cross section
3.2.4 Thomson scattering of unpolarized radiation
3.2.5 Eddington limit
3.2.6 Compton-thick AGN

3.3 Cyclotron radiation
3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

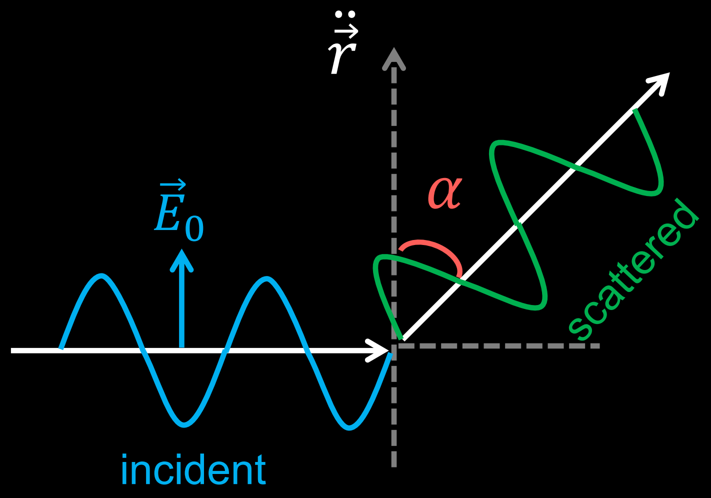
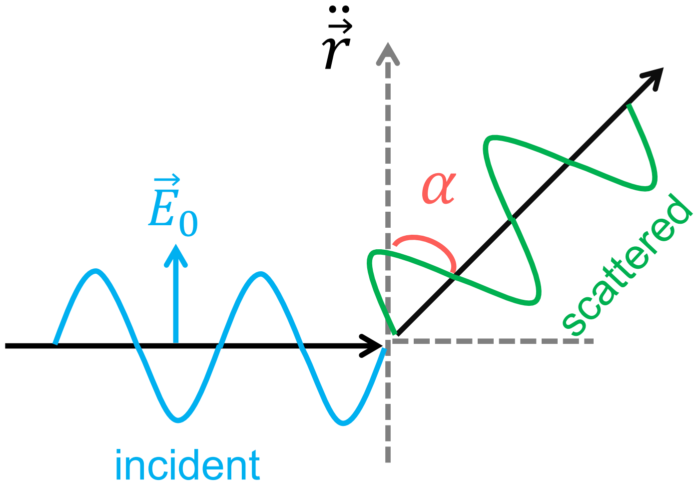


Image credit: Junjie Mao

Scattered power and angular distribution

Consider the response of a free electron to a linearly polarized incident EM wave, the time-averaged acceleration is



$$\langle \ddot{r}^2 \rangle = \frac{1}{2}\left(\frac{eE}{m_e}\right)^2$$

see Chpt. 3 of the REF book (p90-91) by Rybicki & Lightman

prev. sl.

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\dot{v}^2 \sin^2\alpha$$

$$\langle \frac{dP}{d\Omega} \rangle = \frac{e^2}{4\pi c^3}\langle \ddot{r}^2 \rangle \sin^2\alpha = \frac{e^4 E^2}{8\pi m_e^2 c^3}\sin^2\alpha$$

$$\langle P \rangle = \int \langle \frac{dP}{d\Omega} \rangle \, d\Omega = \frac{e^4 E^2}{8\pi m_e^2 c^3}\int_0^{2\pi} d\phi \int_{-1}^{1} (1-\cos^2\alpha)\, d\cos\alpha = \frac{e^4 E^2}{3\, m_e^2\, c^3}$$

$$\int_{-1}^{1} (1-\cos^2\theta) d\cos\theta = \frac{4}{3}$$

Differential cross section

The time-averaged incident flux is

$$\langle S \rangle = \frac{cE^2}{8\pi}$$

see Chpt. 3 of the REF book (p90) by Rybicki & Lightman

prev. sl.

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2}{4\pi c^3} \langle \ddot{r}^2 \rangle \sin^2\alpha = \frac{e^4 E^2}{8\pi m_e^2 c^3} \sin^2\alpha$$

The angular distribution of scattered power relates to the time-averaged incident flux and the differential cross section ($d\sigma$) for scattering into solid angle $d\Omega$ as

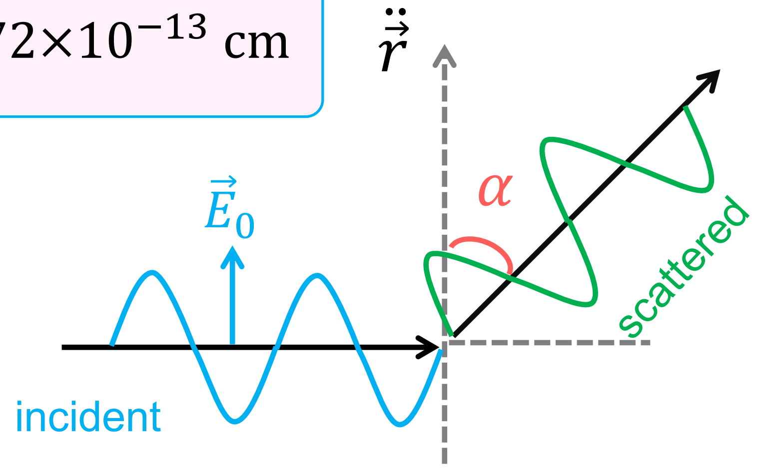
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{cE^2}{8\pi} \frac{d\sigma}{d\Omega}$$

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} = 2.72\times10^{-13}\ \mathrm{cm}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2\alpha = r_0^2 \sin^2\alpha$$

The scattered radiation is linearly polarized



Total cross section

prev. sl.

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2\alpha = r_0^2 \sin^2\alpha$$

$$r_0 = \frac{e^2}{m_e c^2} = 2.72\times10^{-13}\ \text{cm}$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = r_0^2 \int_0^{2\pi} d\phi \int_{-1}^{1} (1-\cos^2\alpha)\, d\cos\alpha$$

$$\int_{-1}^{1} (1-\cos^2\theta) d\cos\theta = \frac{4}{3}$$

Thomson scattering cross section

$$\sigma_\mathrm{T} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \text{cm}^2$$

Alternative approach to get the total cross section

prev. sl.

$$\langle P\rangle = \frac{e^4 E^2}{3\, m_e^2\, c^3}$$

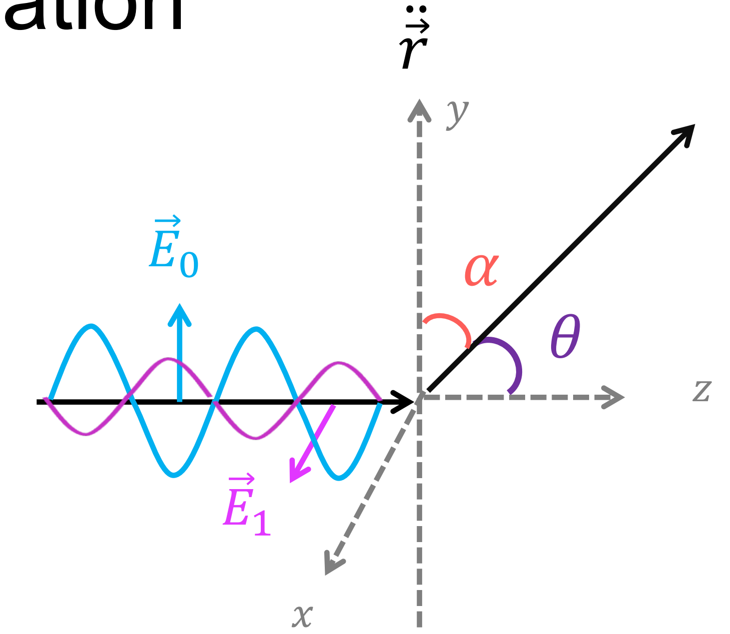
prev. sl.

$$\langle S\rangle = \frac{cE^2}{8\pi}$$

$$\sigma = \frac{\langle P\rangle}{\langle S\rangle}$$

Differential cross section of unpolarized radiation

Thomson scattering of unpolarized radiation can be obtained by decomposing the radiation into a pair of two orthogonally linear-polarized radiations ($\vec{E}_0 \perp \vec{E}_1$), the differential cross section of unpolarized radiation



$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}^{\text{total}} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}(\pi/2)\right)_{\text{pol}} + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}(\alpha)\right)_{\text{pol}}$$

$$= \frac{1}{2} r_0^2 (1 + \sin^2\alpha)$$

$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta)$$

scattering angle $\theta = \pi/2 - \alpha$

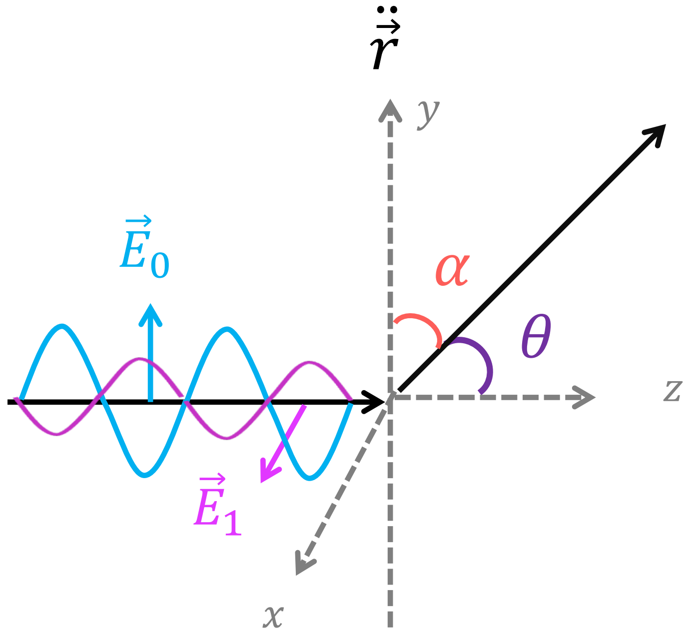
prev. sl.

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2\alpha = r_0^2 \sin^2\alpha$$

The differential cross section is symmetric for $\pm\theta$

Total cross section of unpolarized radiation

prev. sl. $$\left(\frac{d\sigma}{d\Omega}\right)^{\text{total}}_{\text{unpol}} = \frac{1}{2} r_0^2 (1+\cos^2\theta)$$



$$\sigma^{\text{total}}_{\text{unpol}} = \int \left(\frac{d\sigma}{d\Omega}\right)^{\text{total}}_{\text{unpol}} d\Omega$$

$$\int_{-1}^{1} (1+\cos^2\theta) d\cos\theta = \frac{8}{3}$$

$$= \frac{r_0^2}{2} \int_0^{2\pi} d\phi \int_{-1}^{1} (1+\cos^2\theta)\, d\cos\theta$$

$$= \frac{8}{3} \pi r_0^2$$

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652\times 10^{-25}\ \mathrm{cm}^2$$

While the differential cross sections differ for polarized and unpolarized radiations, the total cross section is identical.

✓ Only valid for $h\nu \ll m_e c^2$ (i.e. $\ll 511$ keV). Below this threshold, the (differential and total) Thomson scattering cross section is **independent** of the incident photon energy.

Gravity vs Radiation

Consider a test particle with mass m at the distance r with respect to a black hole with mass M_{BH} and luminosity L

Gravitational force

$$f_{\mathrm{grav}} = \frac{G M_{\mathrm{BH}} m}{r^2}$$

Radiation pressure

$$p_{\mathrm{rad}} = \frac{1}{c}\frac{L}{4\pi r^2}$$

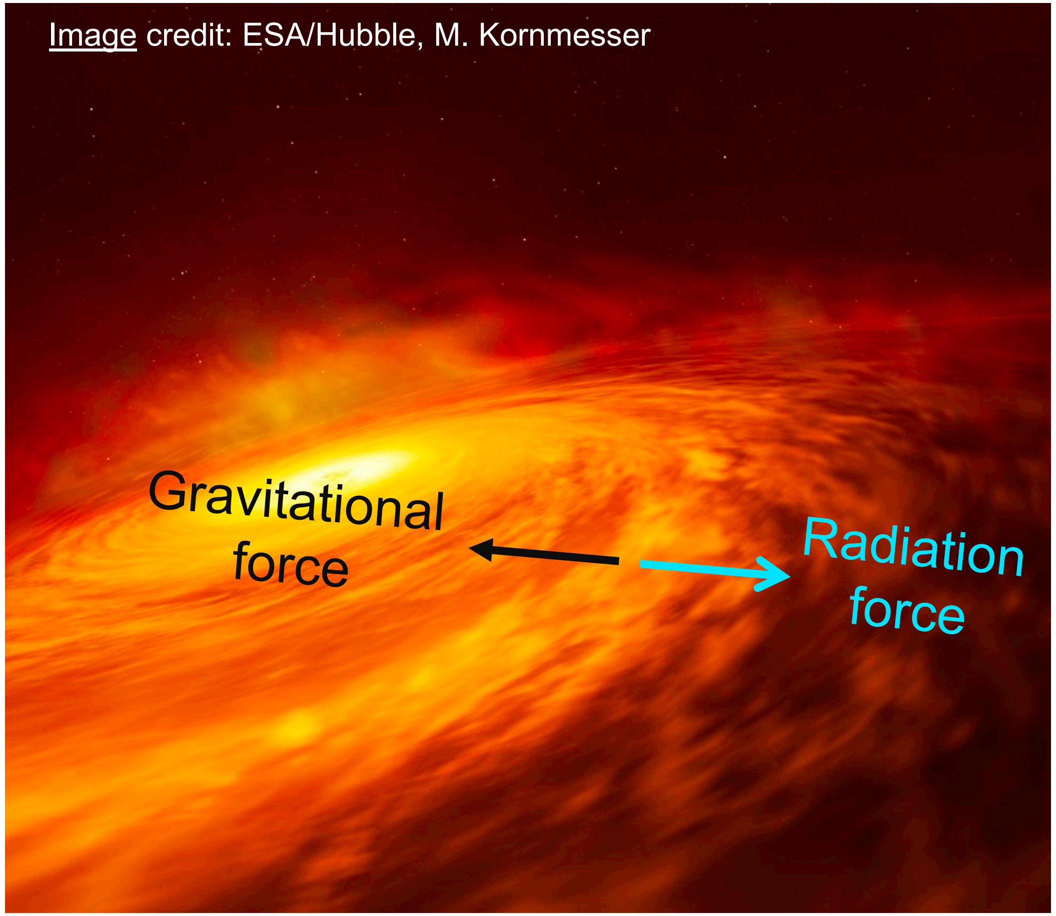
slide Sect. 2.1.4

$$p = \frac{F}{c}$$

Radiation force

opacity in $\mathrm{g}^{-1}\,\mathrm{cm}^2$

$$f_{\mathrm{rad}} = p_{\mathrm{rad}}\,\kappa\, m = \frac{\kappa\, m}{c}\frac{L}{4\pi r^2}$$



slide Sect. 2.2.2

$$\alpha = \rho\,\kappa$$

Eddington luminosity

prev. sl.

$$f_{\mathrm{grav}} = \frac{G M_{\mathrm{BH}} m}{r^2} \quad ? \quad f_{\mathrm{rad}} = \frac{\kappa\, m}{c} \frac{L}{4\pi r^2}$$

$$L_{\mathrm{balance}} = \frac{4\pi c\, G M_{\mathrm{BH}}}{\kappa}$$

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652\times 10^{-25}\ \mathrm{cm}^2$$

For a fully ionized plasma, Thomson scattering (scattering of photons by free electrons) provides an opacity of $\kappa_{\mathrm{T}} = \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{H}}} = 0.398\ \mathrm{g}^{-1}\ \mathrm{cm}^2$

$$L_{\mathrm{Edd}} = \frac{4\pi c\, G M_{\mathrm{BH}}}{\kappa_{\mathrm{T}}} = 1.26\times 10^{38} \left(\frac{M_{\mathrm{BH}}}{M_{\odot}}\right) \mathrm{erg\ s^{-1}}$$

Thomson scattering optical depth

HI4PI collaboration (2016)

slides Sect. 2.4.1

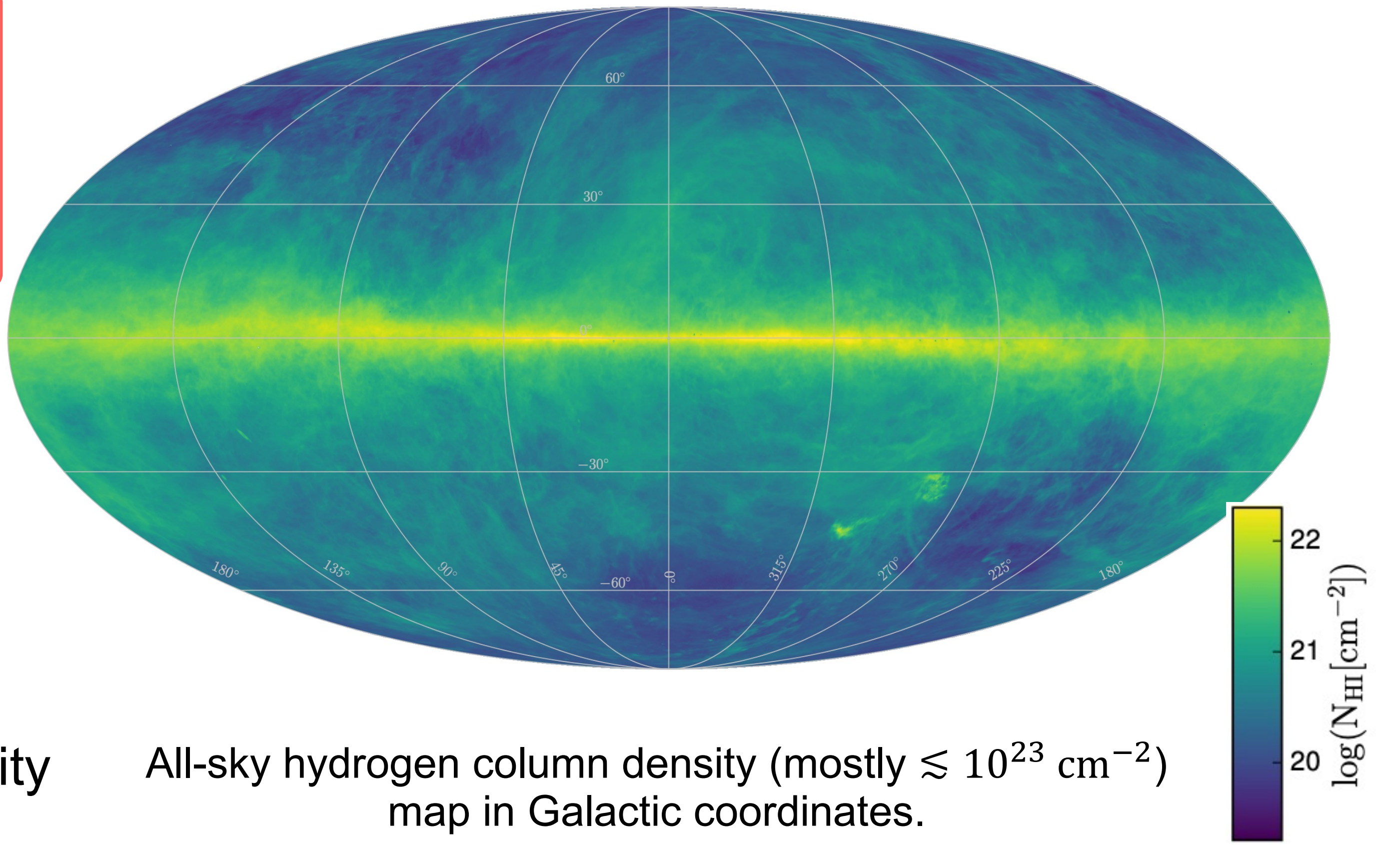
scattering coefficient

$$\tau_\nu^{\mathrm{sca}} \equiv \int \sigma_\nu^{\mathrm{sca}} dl$$

Thomson scattering cross section (ν independent)

$$\sigma_\mathrm{T} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

When the line of sight hydrogen column density (N_H) exceeds $1.5\times10^{24}\ \mathrm{cm}^{-2}$, the Thomson scattering optical depth $\tau = N_\mathrm{H}\,\sigma_\mathrm{T} > 1$



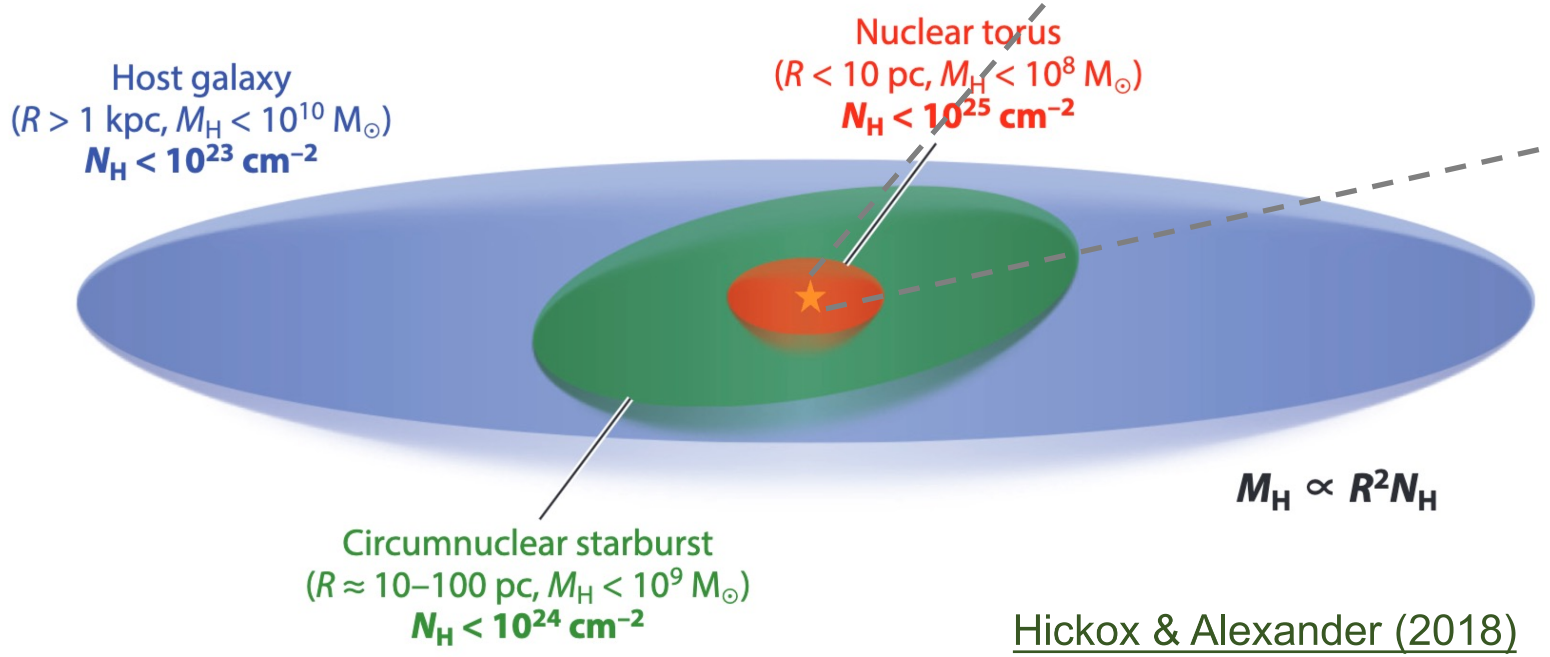
All-sky hydrogen column density (mostly $\lesssim 10^{23}\ \mathrm{cm}^{-2}$) map in Galactic coordinates.

Active Galactic Nuclei

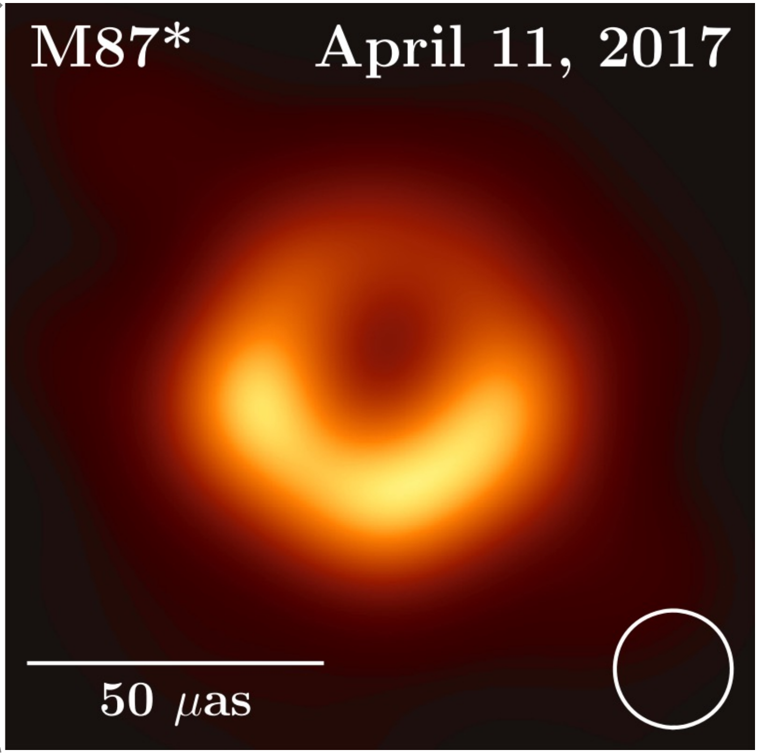
At the centre of almost all massive galaxies sit a supermassive black hole (SMBH)

Active: above 0.005% Eddington limit.

Netzer (2015, ARAA)

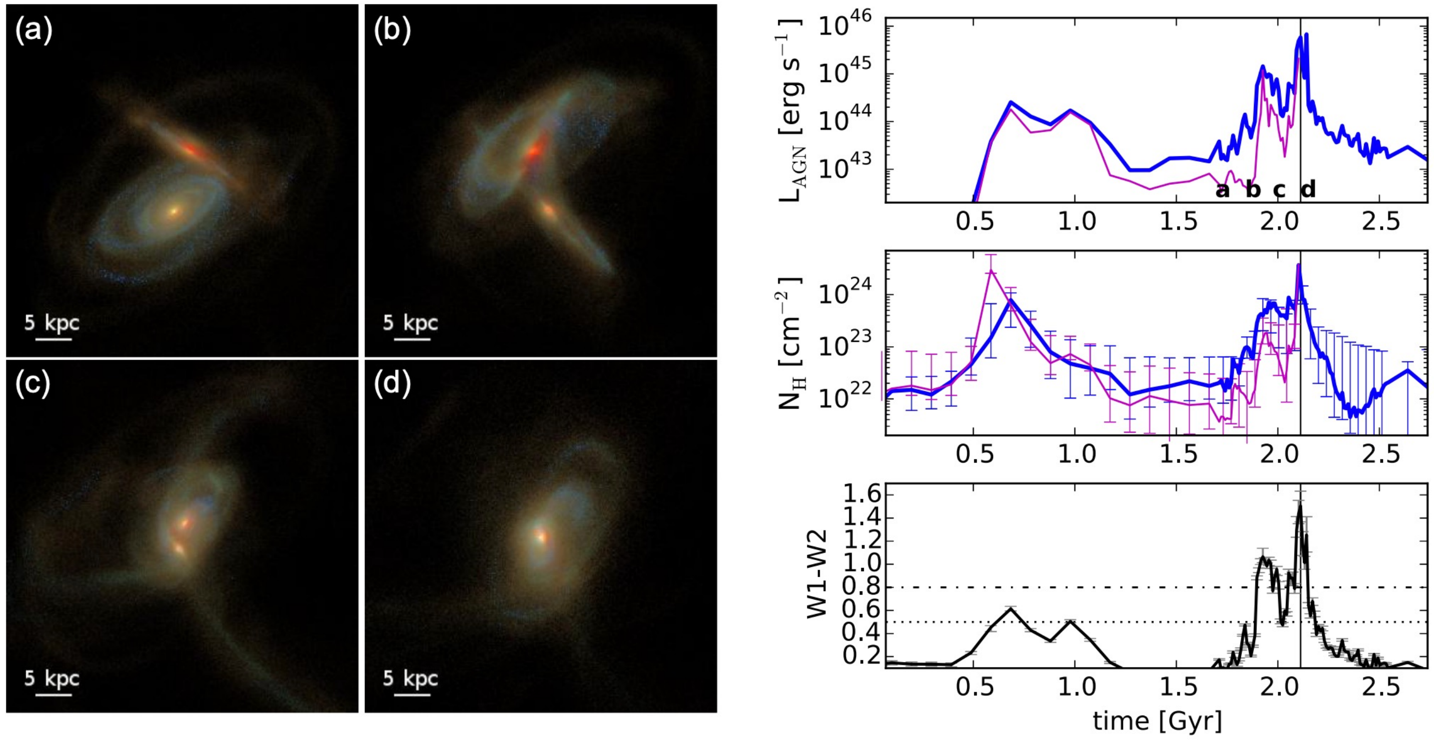


Hickox & Alexander (2018)

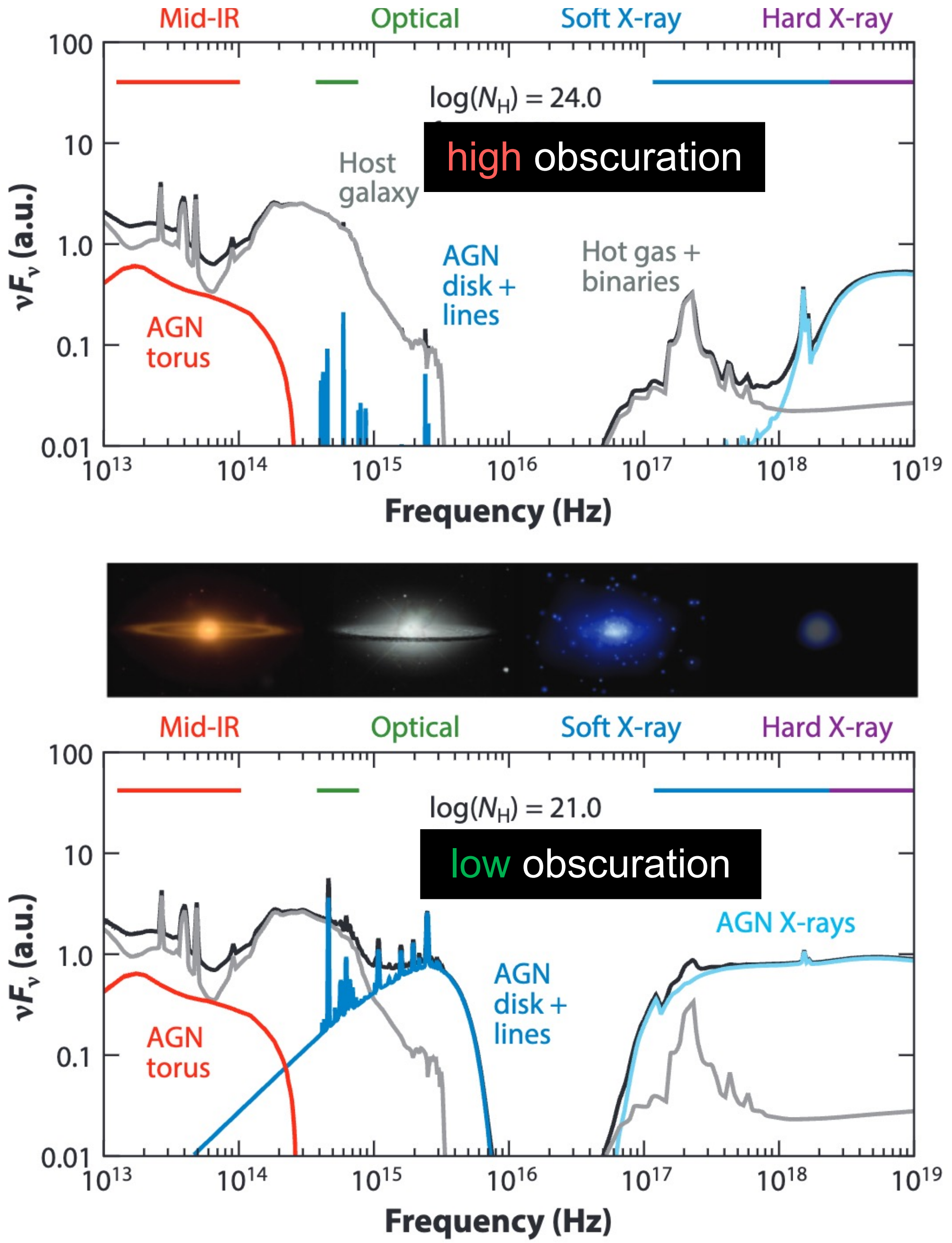


The Event Horizon Telescope network has obtained the first image of a SMBH — at the centre of the galaxy M87 (EHT Collaboration 2019)

Compton-thick AGN



An illustration of supermassive black hole (SMBH) fuelling and obscuration during a galaxy merger (Blecha et al. 2018)



Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge
3.2 Thomson scattering

3.3 Cyclotron radiation
3.3.1 Helical motion of non-relativistic electron
3.3.2 Radiative power of cyclotron radiation
3.3.3 Angular distribution of cyclotron radiation
3.3.4 Polarization of cyclotron radiation

3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

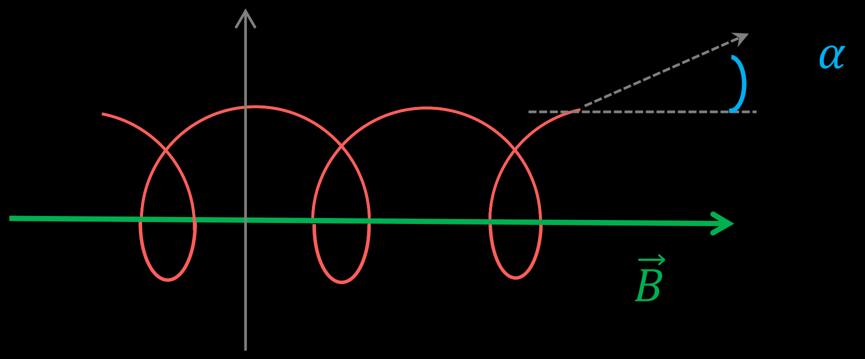
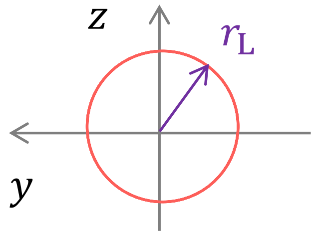
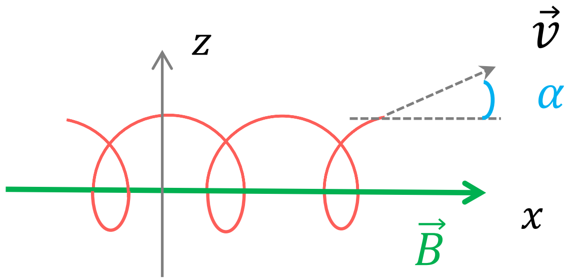


Image credit: Junjie Mao

Helical motion of non-relativistic electrons



For non-relativistic electrons, when moving in a homogenous B-field with α the angle between v and B, they will spiral along the field line.



The projected motion in the (y, z) plane is a circle with

$$r_{\mathrm{L}} = \frac{v \sin\alpha}{\omega_{\mathrm{L}}}$$

Larmor frequency

$$\omega_{\mathrm{L}} = \frac{eB}{m_e c}$$

- If non-relativistic electrons do not move along the field line ($v \sin\alpha = 0$), the radiation has only one frequency (ω_{L}).
- Otherwise, harmonic frequencies $2\omega_{\mathrm{L}}, 3\omega_{\mathrm{L}}$, ... can also be observed.
- The radiative power of cyclotron radiation declines rapidly for higher-order harmonic frequencies

Sect. 4.1.3 of 《天体物理中的辐射机制》 by 尤峻汉 (p161)

$$\frac{P_{n+1}}{P_n} \sim \beta^2, n = 1, 2, 3, \ldots$$

Radiative power of cyclotron radiation

For non-relativistic electrons, the radiative power (consider only the base frequency radiation) simply follows Larmor's formula

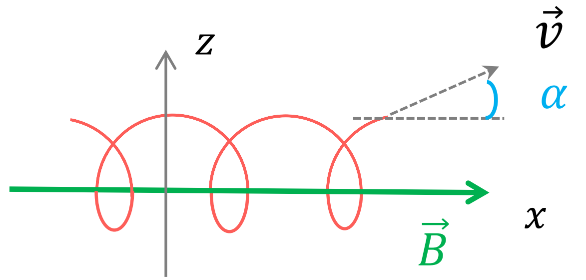
Larmor's formula

$$P = \frac{2e^2\dot{v}^2}{3c^3}$$

Often is the case that we want to know the radiative power given the B-field strength and the velocity of electrons

$$P(\alpha) = \frac{2}{3}\frac{e^4}{m_e^2 c^5}\, v^2 B^2 \sin^2\alpha = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2\alpha$$

$$= 1.59\times10^{-15}\,\beta^2\left(\frac{B}{\mathrm{G}}\right)^2 \sin^2\alpha\ \ \mathrm{erg\,s^{-1}}$$



Sect. 4.1.2 of 《天体物理中的辐射机制》by 尤峻汉 (p156)

$$m_e\dot{v} = -\frac{e}{c}\, v\, B \sin\alpha$$

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} = 2.72\times10^{-13}\ \mathrm{cm}$$

Mean radiative power of cyclotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2\alpha$$

$$\int_0^{\pi} \sin^3\alpha \; d\alpha = \frac{4}{3}$$

If the velocity distribution of the electrons is isotropic, we have

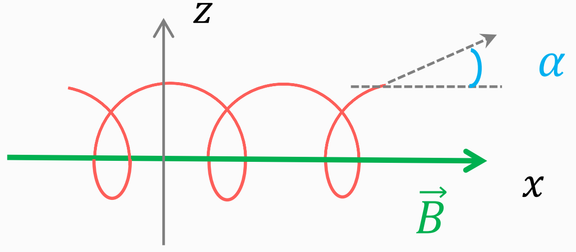
$$\langle \sin^2\alpha \rangle = \frac{\int \sin^2\alpha \; d\Omega}{\int d\Omega} = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\alpha \; d\alpha = \frac{2}{3}$$

$$\langle P \rangle = \frac{4}{9} c\, r_0^2 \beta^2 B^2 = 1.06\times 10^{-15}\, \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \; \mathrm{erg\,s^{-1}}$$

Angular distribution of cyclotron radiation

The angular distribution of cyclotron radiation (again, referring to the base frequency radiation) received by a distant observer is

$$\frac{dP_1}{d\Omega} \sim \frac{\pi e^2 \nu_L^2 \beta^2}{2c}(1+\cos^2\alpha)$$



- Along the direction of the B-field ($\alpha = 0$), the radiative power reaches its maximum.
- Perpendicular to the B-field ($\alpha = \pi/2$), the radiative power reaches its minimum, yet only smaller by a factor of 2 compared to the maximum value.
- At other directions, radiative power has intermediate values

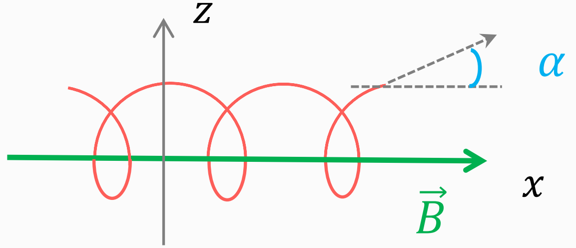
see Sect. 4.1.4 of 《天体物理中的辐射机制》 by 尤峻汉(p164-165)

Polarization of cyclotron radiation

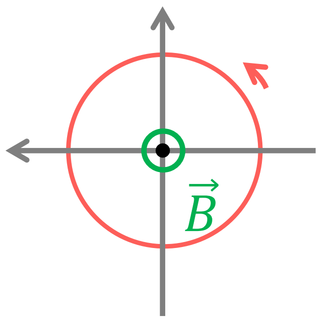
The polarization of cyclotron radiation (again, referring to the base-frequency radiation) is

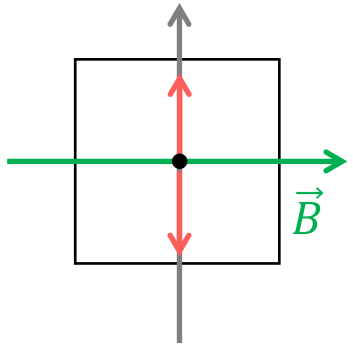
see Sect. 4.1.4 of 《天体物理中的辐射机制》 by 尤峻汉(p164-165)

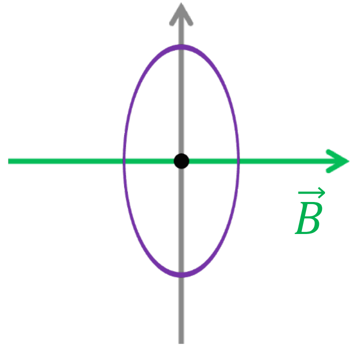
$$\vec{E}_1 = \left(-\frac{1}{2\nu_0}\beta\cos^2\alpha\right)\vec{x} + \left(-\frac{-i\,\beta}{2\nu_0}\right)\vec{y} + \left(\frac{\beta}{2\nu_0}\cos\alpha\sin\alpha\right)\vec{z}$$



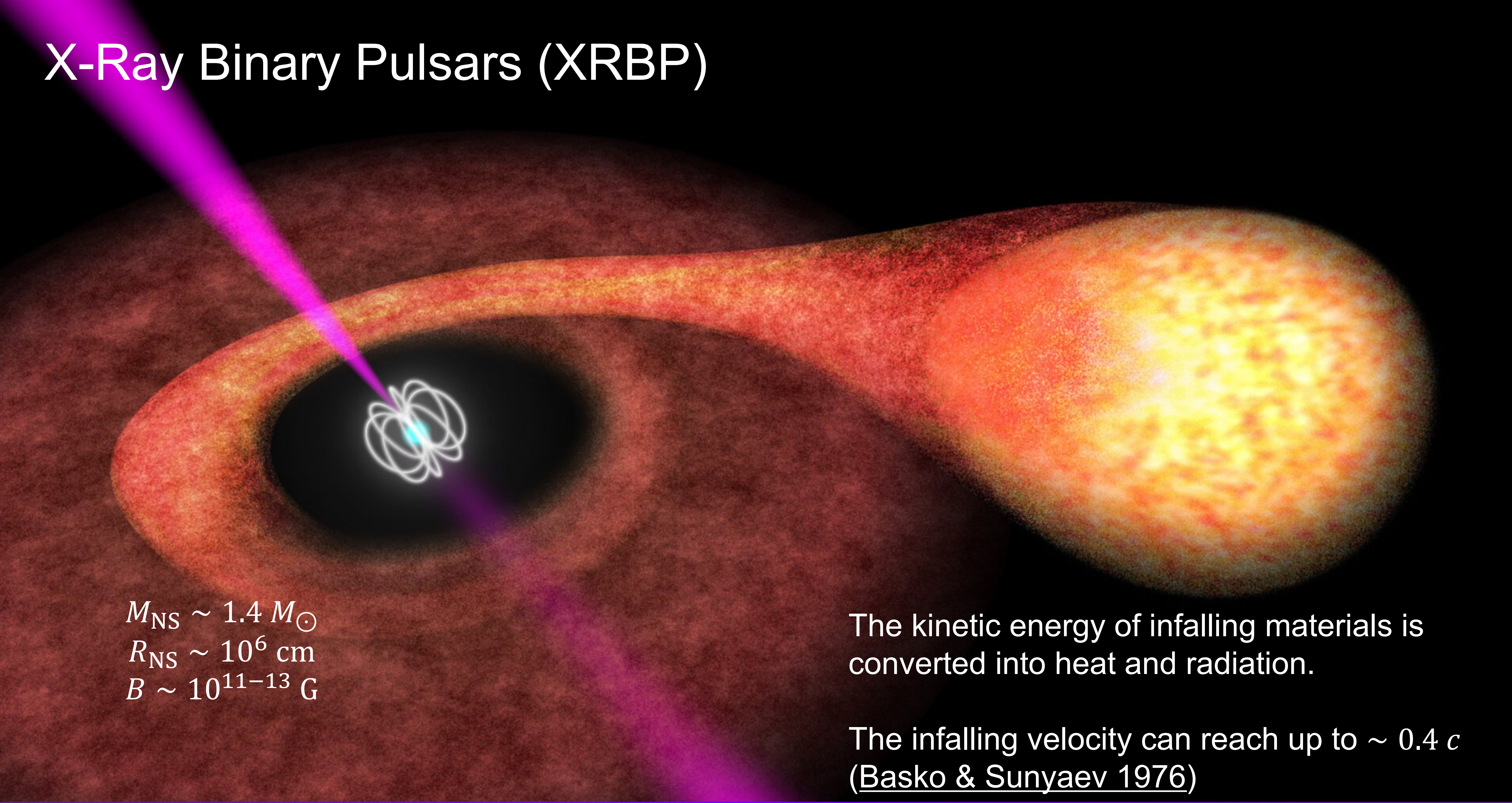
- ❑ Along the direction of the B-field ($\alpha = 0$), cyclotron radiation is circularly polarized
- ❑ Perpendicular to the B-field ($\alpha = \pi/2$), cyclotron radiation is linearly polarized
- ❑ At other directions, cyclotron radiation is elliptically polarized







X-Ray Binary Pulsars (XRBP)

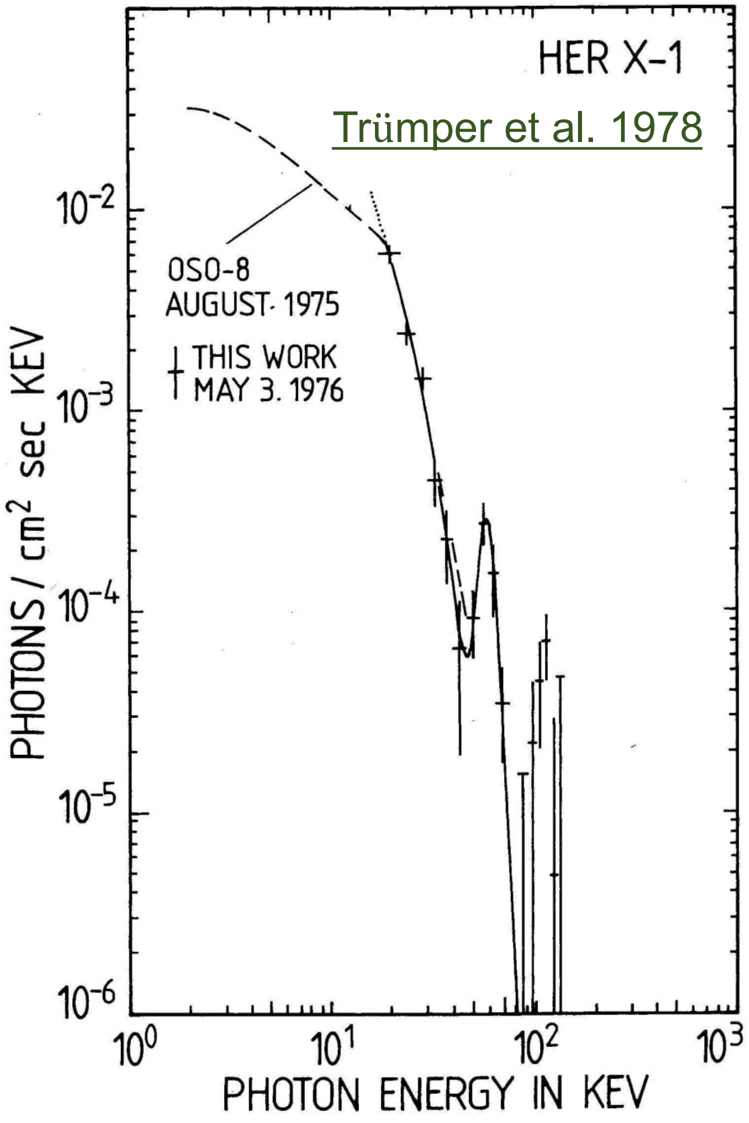


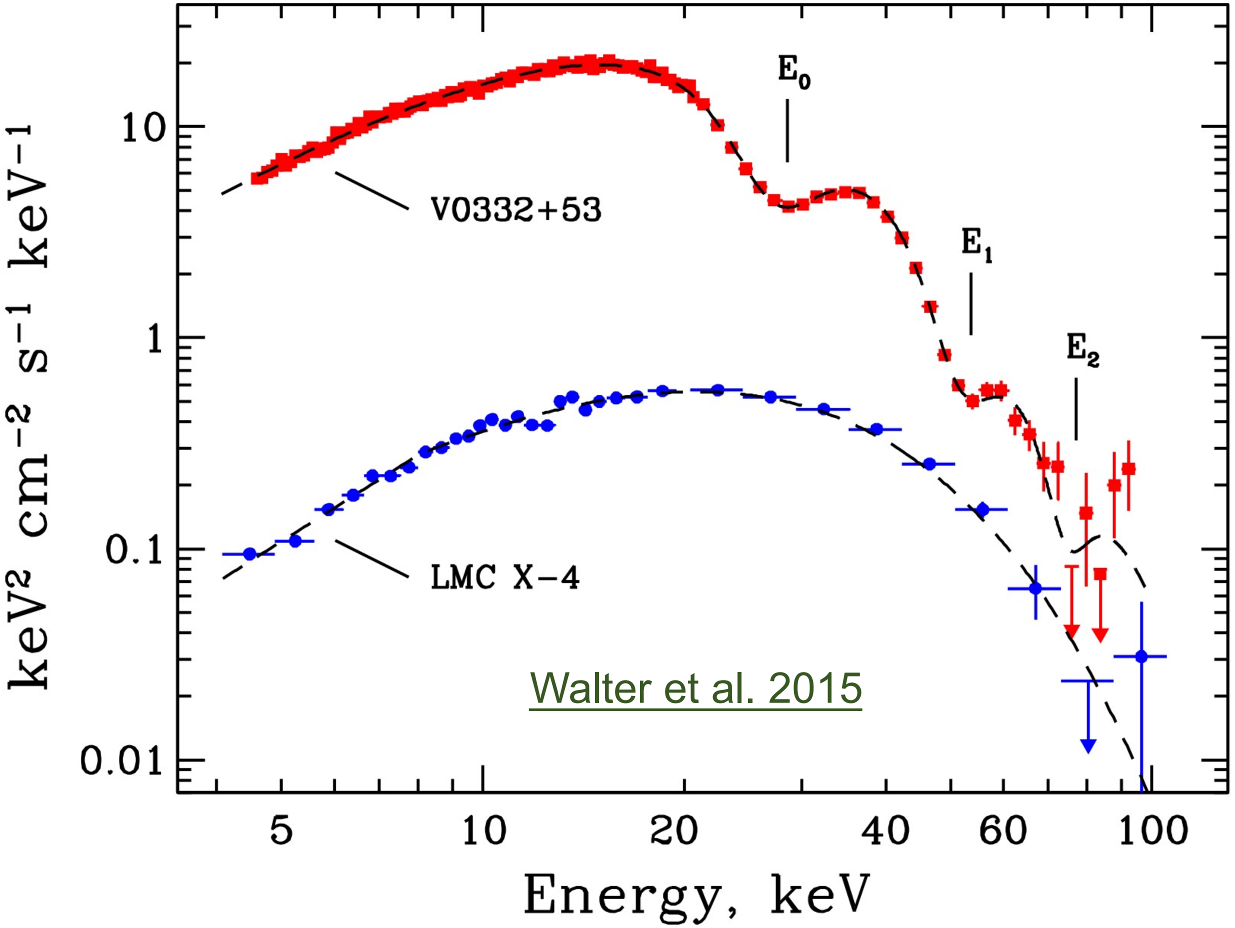
$M_{\mathrm{NS}} \sim 1.4\ M_{\odot}$
$R_{\mathrm{NS}} \sim 10^{6}\ \mathrm{cm}$
$B \sim 10^{11-13}\ \mathrm{G}$

The kinetic energy of infalling materials is converted into heat and radiation.

The infalling velocity can reach up to $\sim 0.4\ c$ (Basko & Sunyaev 1976)

X-ray absorption features

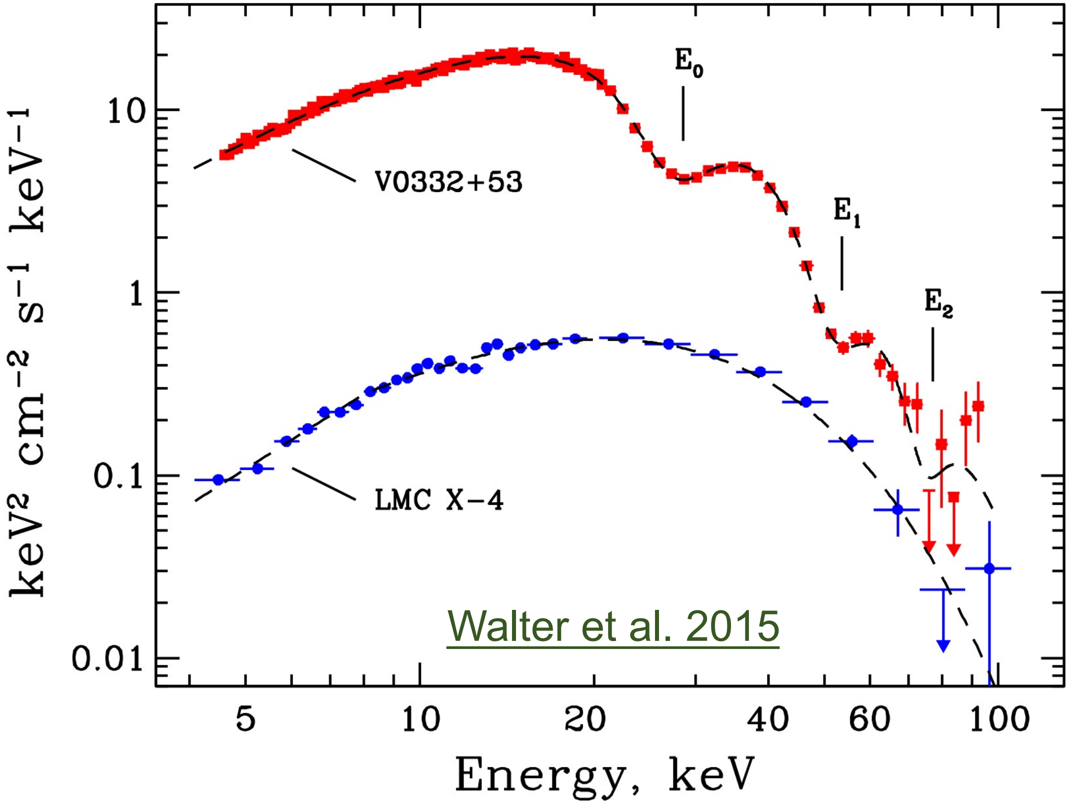




Before leaving the accretion column, these photons undergo scattering with (relativistic) electrons. The cross section is resonant at discrete energies of the so-called Landau levels (Schönherr et al. 2007)

Cyclotron resonance scattering features (CRSFs)

The scattering process gives rise to resonance absorption features in the X-ray spectrum (e.g., Staubert et al. 2019)



Walter et al. 2015

Larmor frequency

$$\omega_{\rm L} = \frac{eB}{m_e c}$$

$n = 1, 2, 3, \ldots$ Landau level

12-B-12 rule

$$E_{\rm CRSF} = \frac{n}{1+z}\frac{h}{2\pi}\omega_{\rm L} \sim \frac{n}{1+z}\frac{11.6}{\rm keV}\frac{B}{10^{12}\ {\rm G}}$$

Gravitational redshift

$$z = \left(1 - \frac{2GM_{\rm NS}}{R_{\rm NS}\, c^2}\right)^{-1/2} - 1$$

inhomogeneous B-field broadening

$$\Delta\nu = \frac{e}{2\pi m_e c}\,\Delta B$$

There are other broadening mechanisms at play

Chpt.3 Radiation from accelerating charges

Helical motion of synchrotron radiation

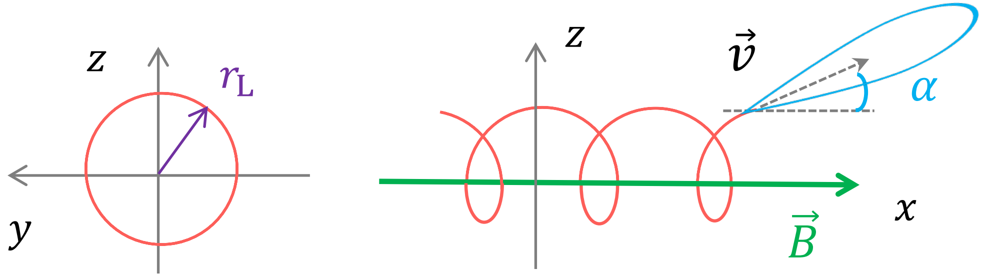
The base frequency of synchrotron radiation is a factor of γ (Lorentz factor) lower than Larmor's frequency

Larmor frequency

$$\omega_{\mathrm{L}} = \frac{eB}{m_e c}$$

$$\omega_0 = \frac{1}{\gamma}\,\omega_{\mathrm{L}}$$

Sect. 6.1 of the REF book (p167-168) by Rybicki & Lightman



The reduced base frequency leads to a large radius of the helical motion ($r \sim c/\nu_0$)

- Considering a relativistic electron with $\gamma = 100$ travelling in a normal spiral galaxy with a typical interstellar B-field strength of $10\ \mu\mathrm{G}$ (Condon & Ransom 2016), we have $\nu_{\mathrm{L}} \sim 30\ \mathrm{Hz}$ and $\nu_0 = \frac{\nu_{\mathrm{L}}}{\gamma} \sim 0.3\ \mathrm{Hz}$. The radius of the helical motion is of the order of $10^{11}\ \mathrm{cm}$.
- Almost linear at small scales ($r \ll c/\nu_0$).
- Harmonic frequencies $2\omega_0, 3\omega_0, \ldots$ can also be observed.
- The base and harmonic frequencies are much closer to each other in the frequency domain

Time duration of synchrotron radiation

- The radiation is strongly beamed with the half-angle of the radiation cone equals $1/\gamma$.
- We only see a short pulse of radiation in each orbit when the beam is pointing towards us.
- The time duration (Δt) of the radiation emitted between P_1 and P_2 is

$$\gamma m_e \frac{\Delta v}{\Delta t} = \frac{e}{c}\, v\, B \sin\alpha \qquad \frac{\Delta\theta}{2} = \frac{1}{\gamma}$$

$$\Delta v = v\,\Delta\theta = \frac{2v}{\gamma}$$

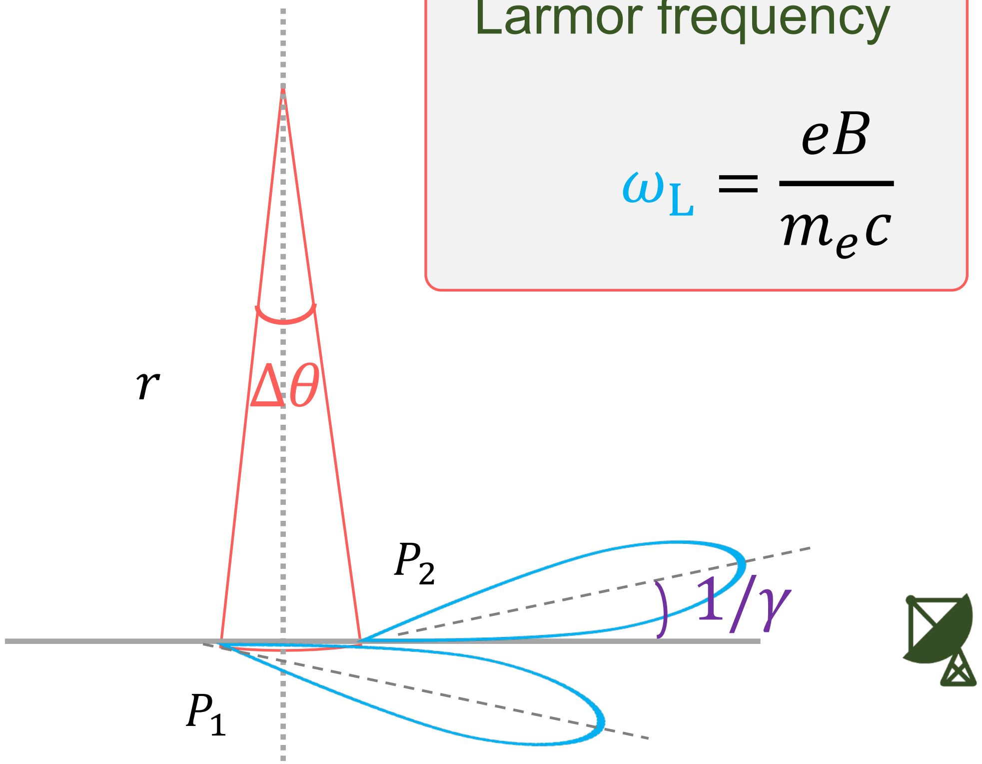
$$\Delta t = \frac{\gamma m_e c\,\Delta\theta}{e\, B \sin\alpha} = \frac{\gamma\,\Delta\theta}{\omega_\mathrm{L} \sin\alpha} = \frac{2}{\omega_\mathrm{L} \sin\alpha}$$

Sect. 4.1.2 of 《天体物理中的辐射机制》by 尤峻汉 (p156)

$$m_e \dot{v} = -\frac{e}{c}\, v\, B \sin\alpha$$

Larmor frequency

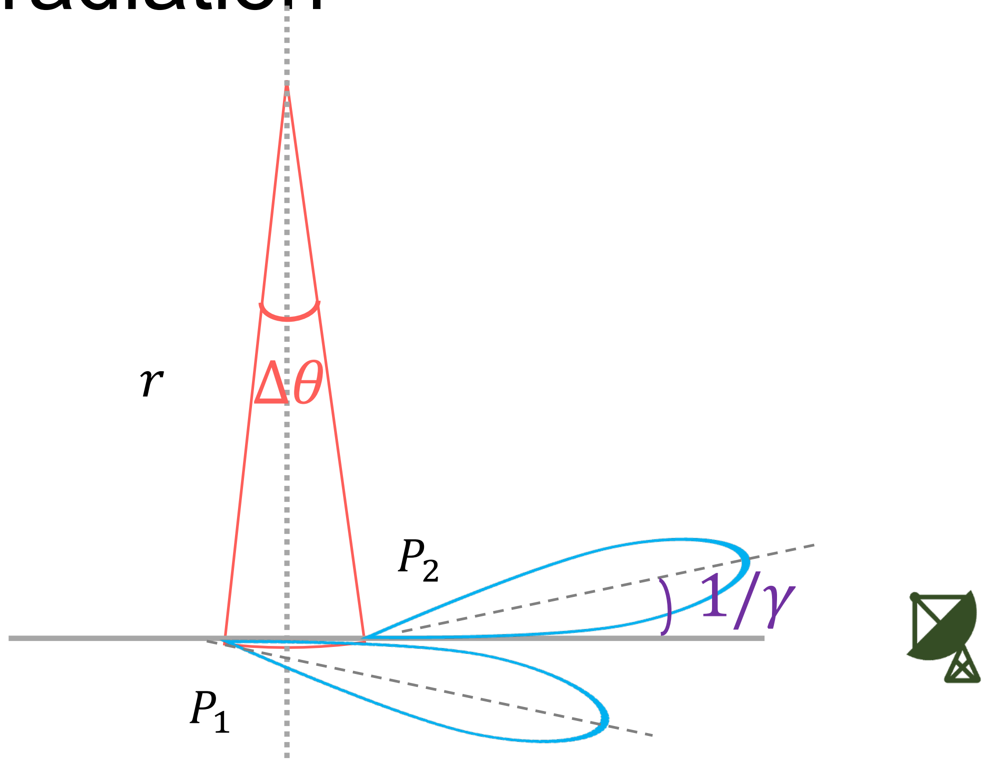
$$\omega_\mathrm{L} = \frac{eB}{m_e c}$$



Observed time duration of synchrotron radiation

prev. sl.

$$\Delta t = \frac{\gamma mc\,\Delta\theta}{e\,B\sin\alpha} = \frac{\gamma\,\Delta\theta}{\omega_{\mathrm{L}}\sin\alpha} = \frac{2}{\omega_{\mathrm{L}}\sin\alpha}$$



The observed time duration is a factor of $(1-\beta)$ smaller

$$\gamma^2 = \frac{1}{(1-\beta)(1+\beta)} \sim \frac{1}{2(1-\beta)}, \gamma \gg 1$$

$$\Delta t_{\mathrm{obs}} = \Delta t(1-\beta) \sim \frac{\Delta t}{2\gamma^2} = \frac{1}{\gamma^2\,\omega_{\mathrm{L}}\sin\alpha}$$

Lorentz factor

$$\gamma = \frac{mc^2}{m_0c^2} = \frac{1}{\sqrt{1-\beta^2}}$$

Radiative power of synchrotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2 \alpha$$

Compared to the radiative power of cyclotron, the radiative power of synchrotron is increased by a factor of γ^2

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \gamma^2 \beta^2 B^2 \sin^2 \alpha$$

$$= 1.587 \times 10^{-15}\, \gamma^2 \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \sin^2 \alpha \ \ \mathrm{erg\, s^{-1}}$$

Sect. 6.1 of the REF book (p167-169) by Rybicki & Lightman

Mean radiative power of synchrotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 \gamma^2 B^2 \sin^2\alpha$$

prev. sl.

$$\langle \sin^2\alpha \rangle = = \frac{2}{3}$$

If the velocity distribution of the electrons is isotropic, we have

$$\langle P \rangle = \frac{4}{9} c\, r_0^2 \gamma^2 \beta^2 B^2 = \frac{4}{3}\, \sigma_{\mathrm{T}}\, c \beta^2 \gamma^2 U_B$$

$$= 1.058 \times 10^{-15}\, \gamma^2 \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \ \mathrm{erg\,s^{-1}}$$

Magnetic energy density

$$U_B = \frac{B^2}{8\pi}$$

Sect. 2.1 of the REF book (p51-53) by Rybicki & Lightman

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652 \times 10^{-25}\ \mathrm{cm}^{-2}$$

Spectral shape of synchrotron radiation

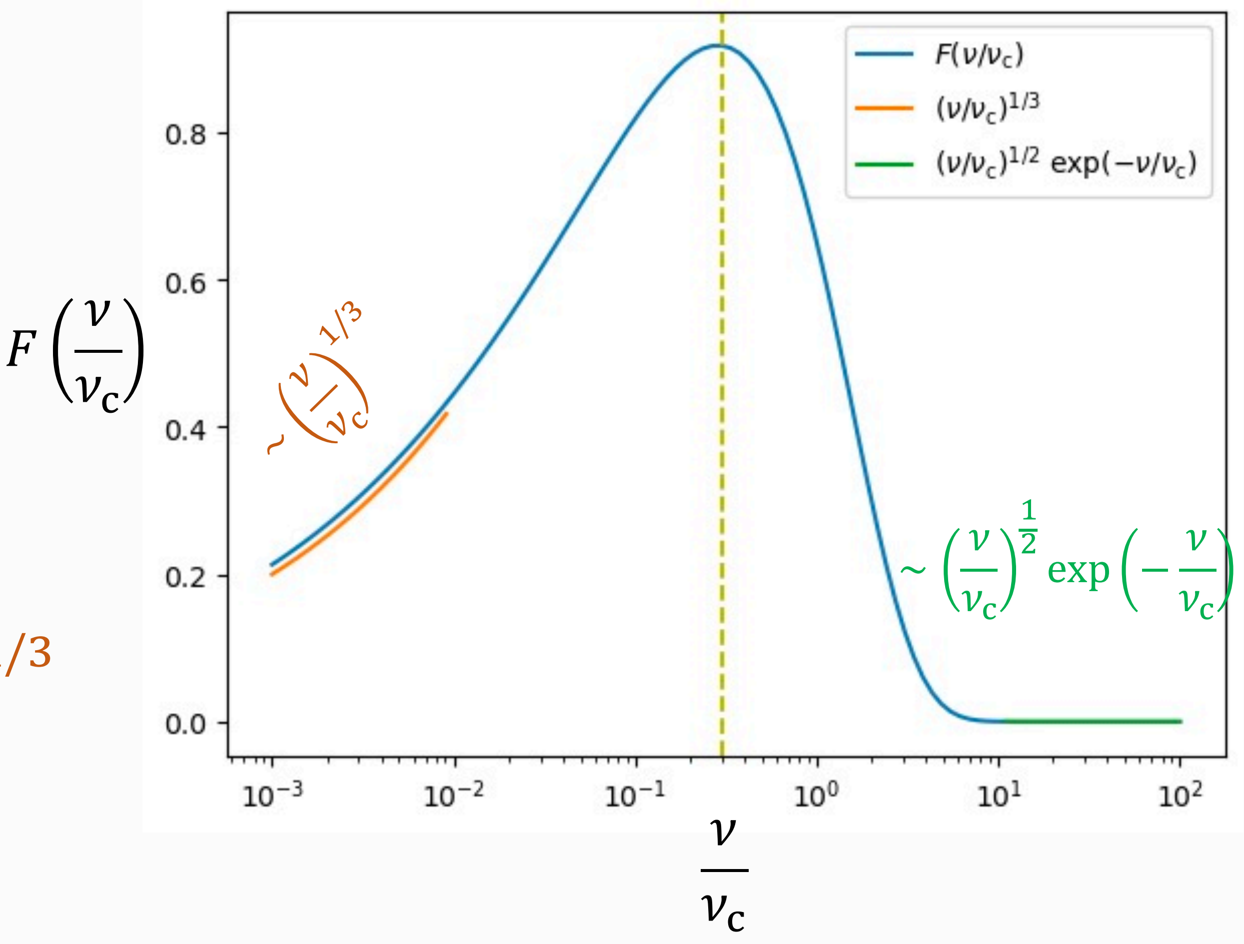
The observed spectrum of synchrotron radiation will spread over a wide (frequency) range

$$\frac{dP(\nu)}{d\nu} = \frac{2\pi\sqrt{3}e^2\nu_\mathrm{L}}{c} F\left(\frac{\nu}{\nu_c}\right) \ \ \mathrm{erg\,s^{-1}Hz^{-1}}$$

$$\nu_c = \frac{3}{2}\gamma^2\nu_\mathrm{L}$$

$$F\left(\frac{\nu}{\nu_c}\right) = \left(\frac{\nu}{\nu_c}\right)\int_{\nu/\nu_c}^{\infty} K_{5/3}(t)dt \begin{cases} \propto \left(\frac{\nu}{\nu_c}\right)^{1/3} \\ \propto \left(\frac{\nu}{\nu_c}\right)^{\frac{1}{2}} \exp\left(-\frac{\nu}{\nu_c}\right) \end{cases}$$

modified Bessel function of the 5/3 order



see Chpt. 4.2.4 of 《天体物理中的辐射机制》 by 尤峻汉(p184-188)

Spectrum of synchrotron radiation (cont.)

The observed spectrum of synchrotron radiation will spread over a wide (frequency) range

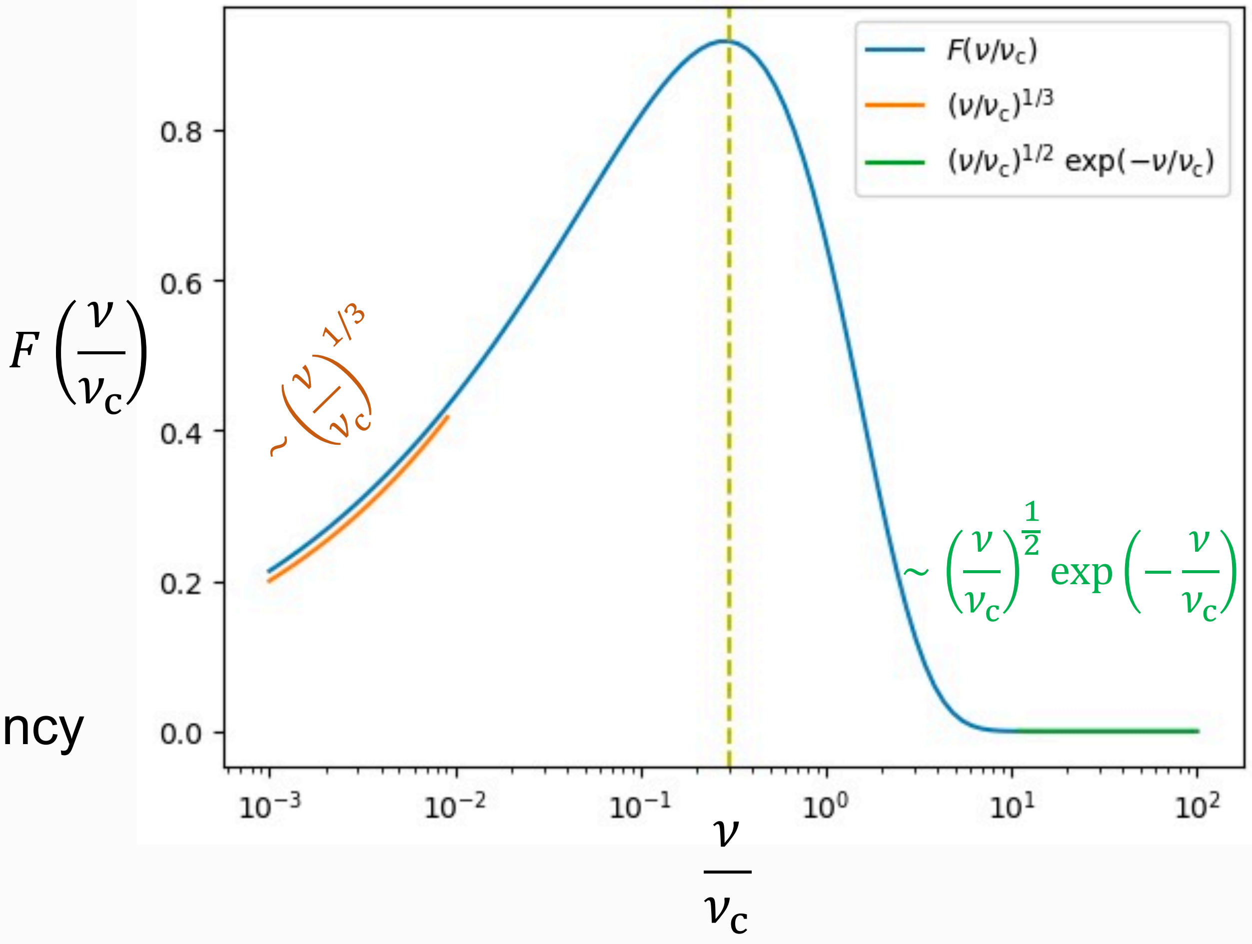
$$\frac{dP(\nu)}{d\nu} = \frac{2\pi\sqrt{3}e^2\nu_{\rm L}}{c} F\left(\frac{\nu}{\nu_c}\right) \ \ \mathrm{erg\,s^{-1}Hz^{-1}}$$

$$\nu_c = \frac{3}{2}\gamma^2\nu_{\rm L}$$

Most of the energy is radiated away at the frequency

$$\nu_{\rm peak} \sim 0.3\,\nu_c \sim 0.45\,\gamma^2\nu_{\rm L}$$

$$\sim 1.256\times10^6\,\gamma^2\left(\frac{B}{\rm G}\right)\ \mathrm{Hz}$$



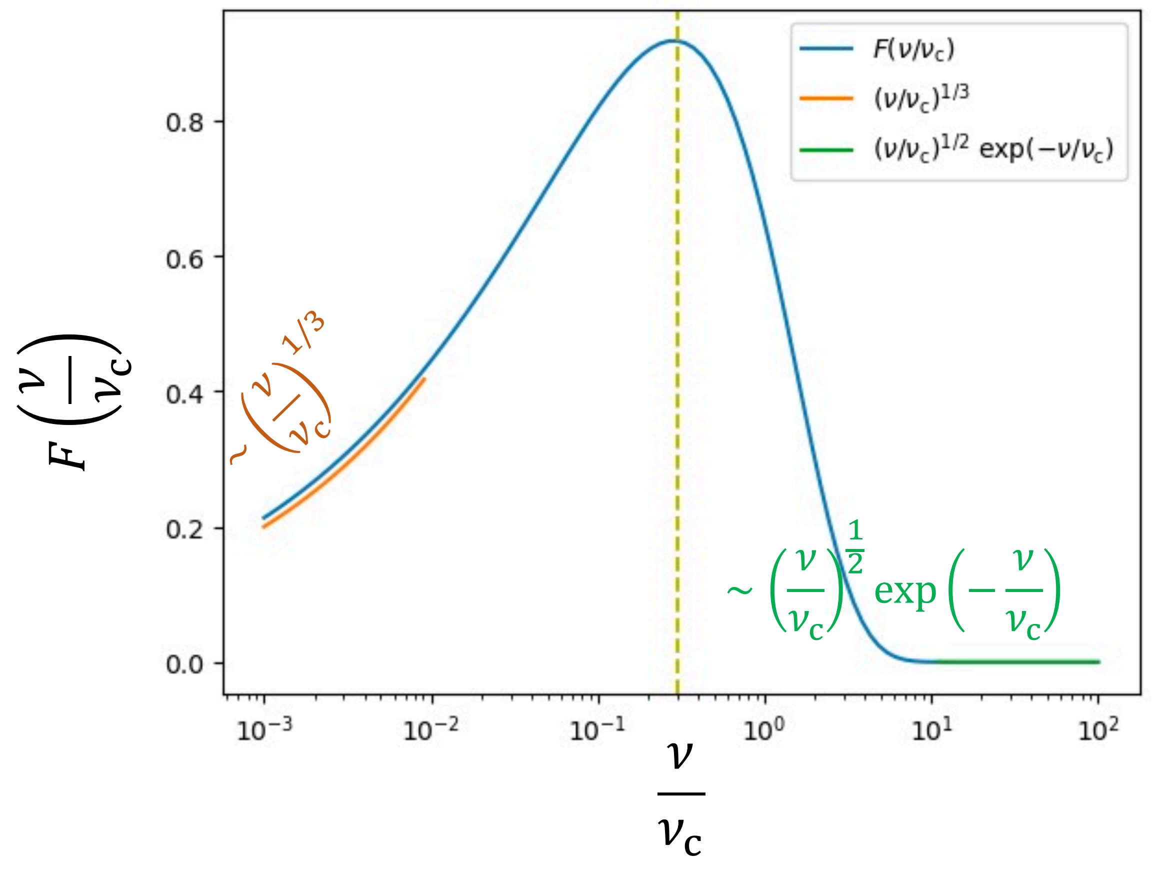
see Chpt. 4.2.4 of 《天体物理中的辐射机制》 by 尤峻汉(p184-188)

Synchrotron spectrum of power law electron distribution

Previous slides are limited to properties of a single relativistic electron. The observed spectral properties come from a large population of relativistic electrons with a power law energy distribution

$$N(\gamma)d\gamma \propto \gamma^{-p}d\gamma, \gamma_{\min} < \gamma < \gamma_{\max}$$

$$P(\nu) \propto P(\nu)\gamma^{-p}d\gamma \propto \int_{\gamma_{\min}}^{\gamma_{\max}} F\left(\frac{\nu}{\nu_c}\right)\gamma^{-p}d\gamma$$



Each single relativistic electron with γ gives rise to a broad emission "bump"

Synchrotron spectrum (cont.)

prev. sl.

$$P(\nu) \propto P(\nu)\gamma^{-p}d\gamma \propto \int_{\gamma_{\min}}^{\gamma_{\max}} F\left(\frac{\nu}{\nu_c}\right)\gamma^{-p}d\gamma$$

prev. sl.

$$\frac{dP(\nu)}{d\nu} \propto F\left(\frac{\nu}{\nu_c}\right)$$

$$\nu_c = \frac{3}{2}\gamma^2\nu_{\mathrm{L}}$$

$$P(\nu) \propto \nu^{-(p-1)/2}\int_{x_{\min}}^{x_{\max}} F(x)x^{(p-3)/2}dx$$

$$x = \frac{\nu}{\nu_c} \propto \frac{\nu}{\gamma^2}, \gamma \propto \left(\frac{\nu}{x}\right)^{\frac{1}{2}}$$

$$d\gamma \propto -\frac{1}{2}\frac{\nu^{1/2}}{x^{3/2}}dx$$

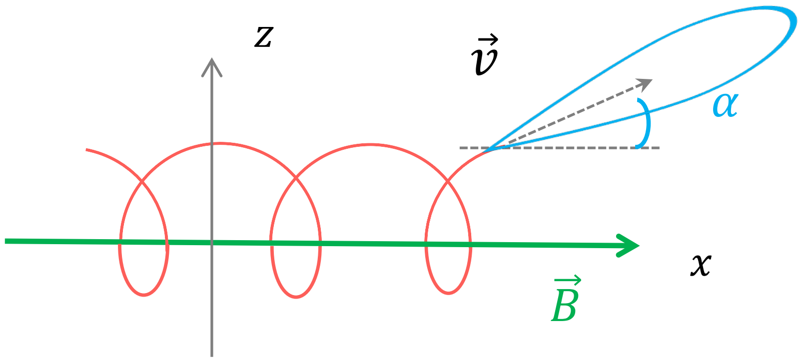
If $x_{\min} \to 0$ and $x_{\max} \to \infty$

$$\int_0^\infty F(x)x^{(p-3)/2}dx \sim \text{const.}$$

$$P(\nu) \propto \nu^{-(p-1)/2}$$

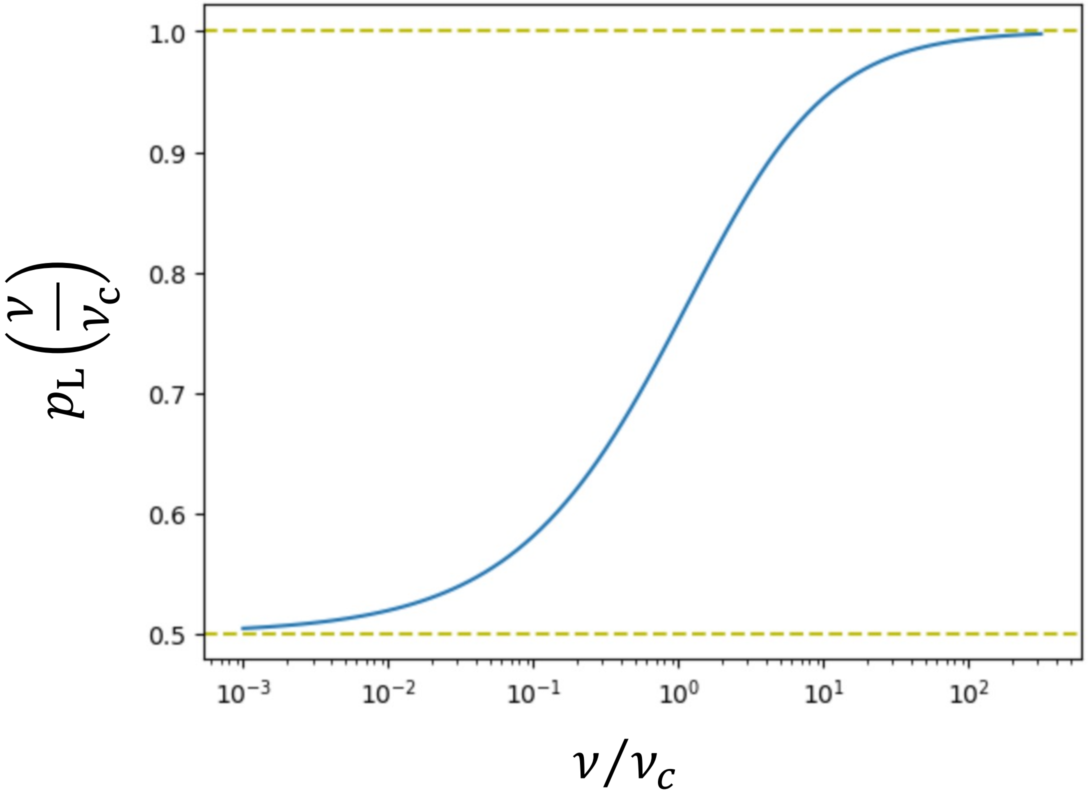
Polarization of synchrotron radiation

For a population of relativistic electrons with the same γ factor but different radiation angle $\theta \simeq \alpha \pm 1/\gamma$, the synchrotron radiation is linearly polarized



$$p_L = \frac{K_{2/3}(\nu/\nu_c)}{\int_{\nu/\nu_c}^{\infty} K_{5/3}(t)dt} \begin{cases} 0.5 \text{ for } \nu \ll \nu_c \\ 1.0 \text{ for } \nu \gg \nu_c \end{cases}$$

Westfold (1959)



prev. sl.

$$\nu_c = \frac{3}{2}\gamma^2\nu_L$$

Synchrotron polarization of power law electron distribution

The observed average linear polarization fraction come from a large population of relativistic electrons with a power law energy distribution

$$N(\gamma)d\gamma \propto \gamma^{-p} d\gamma, \gamma_{\min} < \gamma < \gamma_{\max}$$

$$\langle p_L \rangle = \frac{p+1}{p+\frac{7}{3}}$$

See Chpt. 6 of the REF book (p180 - 181) by Rybicki & Lightman

- ✓ The average linear polarization fraction depends only on the power-law index of the relativistic electron population.
- ✓ The average linear polarization fraction is **<u>frequency-independent</u>**.

Synchrotron self-absorption

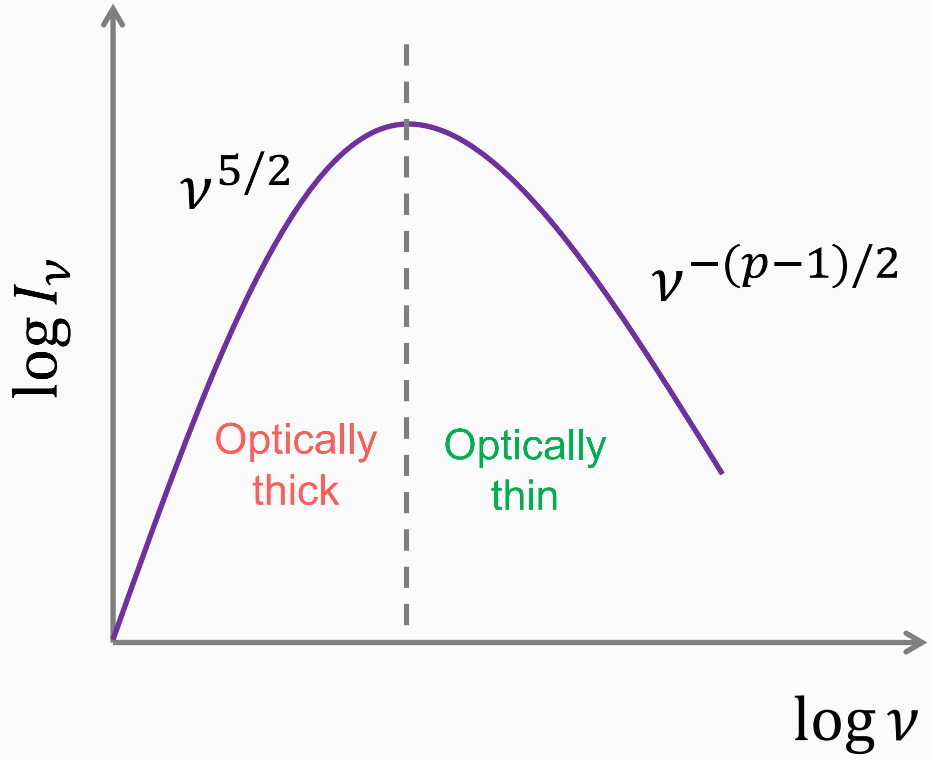
Consider a power-law distribution of electrons with $N(\gamma)d\gamma \propto \gamma^{-p}d\gamma$, for $h\nu \ll \gamma m_0 c^2$, the absorption coefficient and source function of synchrotron radiation

$$j_\nu = P_\nu$$

$$\alpha_\nu \propto \nu^{-(p+4)/2}$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P_\nu}{4\pi\,\alpha_\nu} \propto \nu^{5/2}$$

See Chpt. 6 of the REF book (p186-190) by Rybicki & Lightman



$$I_\nu \propto \begin{cases} P_\nu\,, \text{optically thin} \\ S_\nu, \text{optically thick} \end{cases}$$

prev. sl.

$$P(\nu) \propto \nu^{-(p-1)/2}$$

Lifetime of synchrotron radiation

prev. sl.

$$P_{\text{syn}}(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 \gamma^2 B^2 \sin^2 \alpha$$

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$, it can lose energy via synchrotron radiation

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2}$$

$$\frac{d}{dt}(\gamma m_e c^2) = -P_{\text{syn}} = -\frac{2}{3} \frac{e^4}{m_e^2 c^3} \frac{v^2}{c^2} B^2 \gamma^2 \sin^2 \alpha$$

$$= -A_{\text{syn}}\, m_e c^2 (1 - \gamma^{-2}) \gamma^2$$

$$\frac{v^2}{c^2} = 1 - \gamma^{-2}$$

$$\frac{d\gamma}{dt} = -A_{\text{syn}}\, (\gamma^2 - 1) \sim -A_{\text{syn}}\, \gamma^2 \text{ for } \gamma \gg 1$$

$$A_{\text{syn}} = \frac{2}{3} \frac{e^4}{m_e^3 c^5} B^2 \sin^2 \alpha$$

$$-\gamma^2\, d\gamma \sim A_{\text{syn}}\, dt$$

$$\frac{1}{\gamma(t)} - \frac{1}{\gamma_0} = A_{\text{syn}}\, t$$

$$\gamma(t) = \frac{\gamma_0}{1 + A_{\text{syn}}\, \gamma_0 t}$$

Half-energy lifetime due to synchrotron radiation

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$ and emits synchrotron radiation, the half-energy lifetime is

$$\frac{\gamma_0}{2} = \frac{\gamma_0}{1 + A\,\gamma_0 t_{\mathrm{syn},1/2}}$$

$$t_{\mathrm{syn},1/2} = \frac{1}{\gamma_0 A} = \frac{3 m_e^3 c^5}{2e^4}\frac{1}{\gamma_0} B^{-2} (\sin\alpha)^{-2}$$

$$\sim 5.16\times 10^8 \frac{1}{\gamma_0}\left(\frac{B}{\mathrm{G}}\right)^{-2} (\sin\alpha)^{-2}\ \mathrm{s}$$

$$t_{\mathrm{syn},1/2} = 5.78\times 10^{11} \left(\frac{\nu_{\mathrm{peak}}}{\mathrm{Hz}}\right)^{-\frac{1}{2}} \left(\frac{B}{\mathrm{G}}\right)^{-3/2} (\sin\alpha)^{-2}\ \mathrm{s}$$

prev. sl.

$$\gamma(t) = \frac{\gamma_0}{1 + A\,\gamma_0 t}$$

$$A = \frac{2e^4 B^2 \sin^2\alpha}{3\, m_e^3 c^5}$$

prev. sl.

$$\nu_{\mathrm{peak}} \sim 1.256\times 10^6\, \gamma^2 \left(\frac{B}{\mathrm{G}}\right)\ \mathrm{Hz}$$

Requirement of synchrotron radiation

To ensure the energy loss is negligible in a specific cycle, the following requirement needs to be met on $t_{\mathrm{syn},1/2}$ and the period of circular motion $T = 2\pi/\omega_0$

prev. sl.

$$t_{\mathrm{syn},1/2} = \frac{1}{\gamma_0 A} = \frac{3m_e^3c^5}{2e^4}\frac{1}{\gamma_0}B^{-2}(\sin\alpha)^{-2}$$

$$\frac{2\pi}{\omega_0 t_{\mathrm{syn},1/2}} = \frac{2\pi\gamma_0}{\omega_\mathrm{L}}\frac{2e^4\gamma_0 B^2\sin^2\alpha}{3m_e^3c^5} = \frac{4\pi e^3}{3m_e^2c^4}\gamma_0^2 B\sin^2\alpha$$

$$\sim 6.93\times10^{-16}\,\gamma_0^2\left(\frac{B}{\mathrm{G}}\right)\sin^2\alpha \ll 1$$

prev. sl.

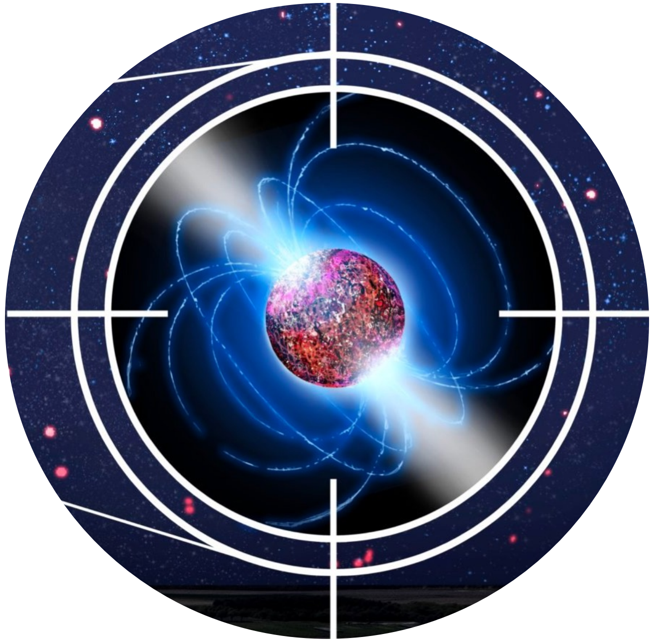
$$\nu_\mathrm{peak} \sim 1.256\times10^{6}\,\gamma^2\left(\frac{B}{\mathrm{G}}\right)\ \mathrm{Hz}$$

If $\nu_\mathrm{peak} \ll 10^{21}$ Hz, the above requirement can be met

Synchrotron radiation in strong B-fields

prev. sl.

$$t_{\text{syn},1/2} \sim 5.78{\times}10^{8}\, \gamma_0^{-1} \left(\frac{B}{\text{G}}\right)^{-2} (\sin\alpha)^{-2}\ \text{s}$$



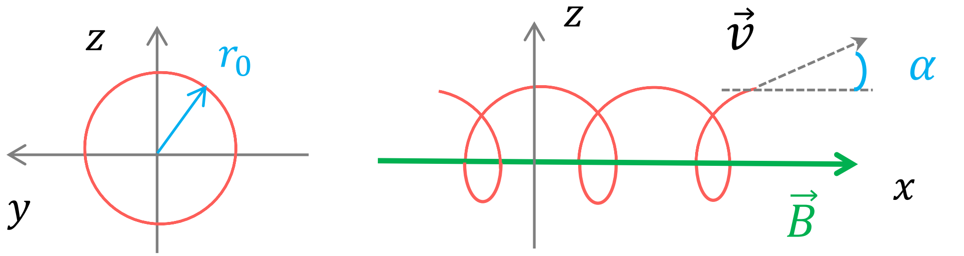
For a neutron star with a B-field strength of 10^{12} G, if it emits synchrotron radiation with $\gamma \sim 100$, $t_{\text{syn},1/2}$ is as short as $6{\times}10^{-18}$ s!

$$\frac{1}{\nu_0} \sim \frac{\gamma}{\nu_\text{L}} \sim \frac{\gamma}{2.8{\times}10^{6}\left(\frac{B}{\text{G}}\right)}\ \text{s}$$

For comparison, the gyration time is $\sim 4{\times}10^{-16}$ s.

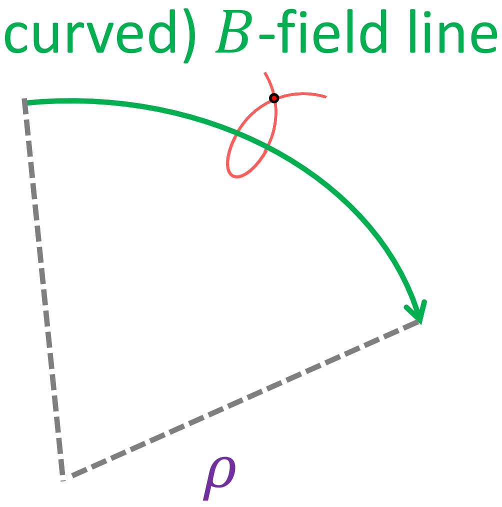
Curvature process

For curved B-field lines, such a process can give rise to curvature radiation. The instantaneous behavior is similar to synchrotron radiation.



$$\nu_0^{\mathrm{crv}} \simeq \frac{c}{2\pi\rho}$$

(instantaneous)
curvature radius



$$\nu_0^{\mathrm{syn}} = \frac{c}{2\pi r_0}$$

radius of the
helical motion

$$\nu_{\mathrm{peak}}^{\mathrm{crv}} \sim 0.45\,\gamma^2 \nu_0^{\mathrm{crv}}$$

prev. sl.

$$\nu_{\mathrm{peak}} \sim 0.45\,\gamma^2 \nu_{\mathrm{L}}$$

AY2024F

Junjie Mao@THU

Radiative Processes in Astrophysics

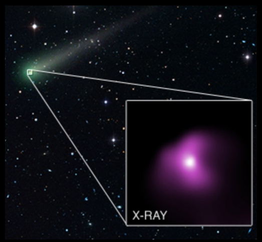
Observation

Up to cosmic size scale

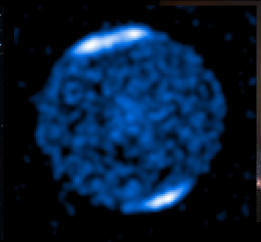
Theory

Down to atomic size scale

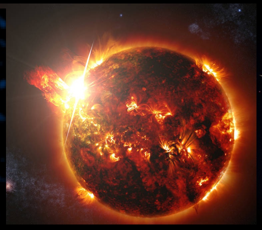
C/2012S1 (comet)



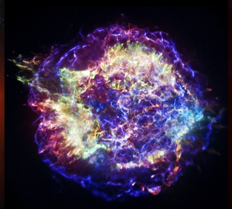
Jupiter (planet)



Sun (star)

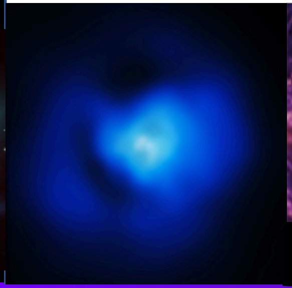


Cas A (SNR)



M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

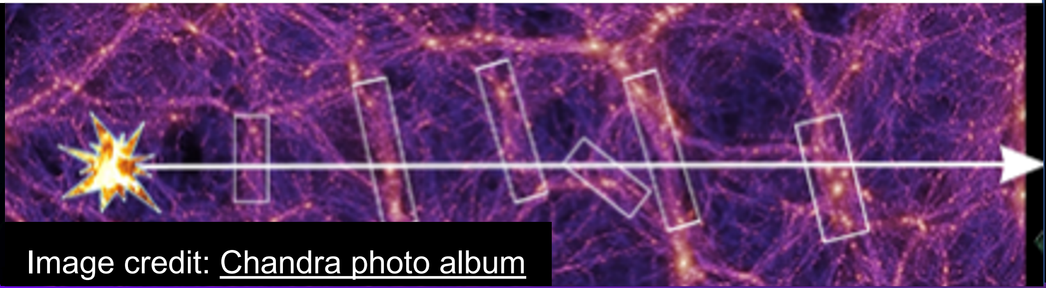


Image credit: Chandra photo album

Chpt.4 Compton and inverse Compton scattering

4.1 Compton scattering
4.1.1 Compton scattering
4.1.2 Differential Compton scattering cross section
4.1.3 Total Compton scattering cross section

4.2 Inverse Compton scattering
4.3 Sunyaev-Zel'dovich effect
4.4 Comptonization

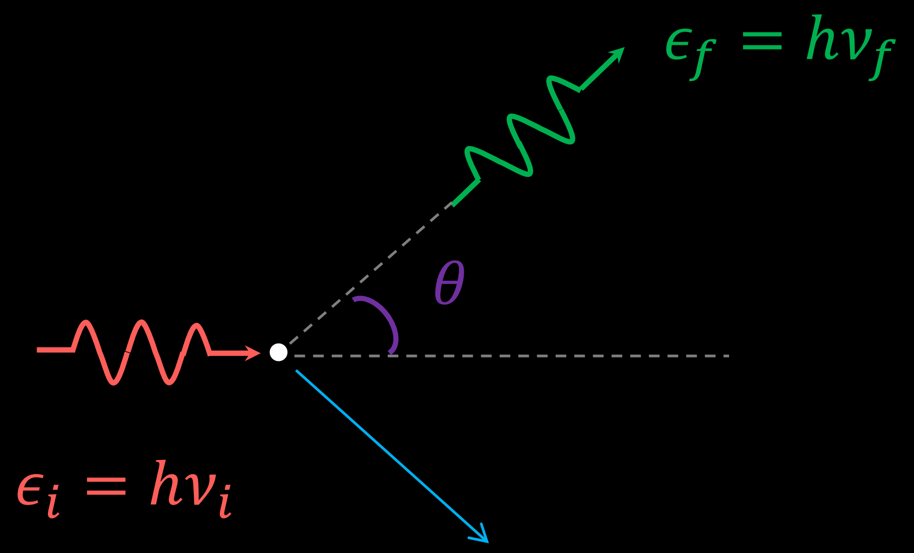


Image credit: Junjie Mao

Compton scattering (frequency form)

Thomson scattering cross section

$$\sigma = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

Thomson scattering

- ✓ Only valid for $h\nu \ll m_e c^2$ (i.e. $\ll 511$ keV)
- ✓ Elastic (i.e., incident energy = emergent energy)

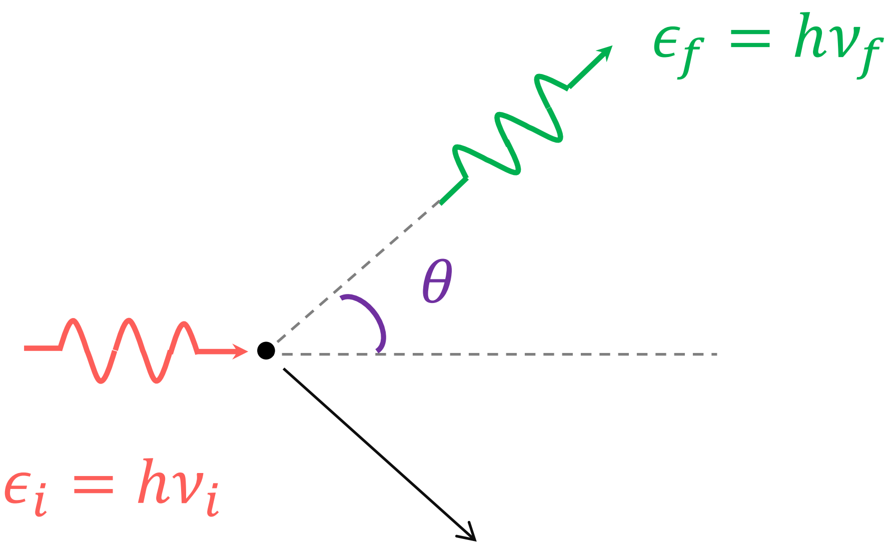
Compton scattering

Still the scattering of a photon by an electron at rest

With $h\nu \gtrsim m_e c^2$, quantum effects cannot be neglected and the scattering is inelastic

$$\epsilon_f = \frac{\epsilon_i}{1 + \frac{\epsilon_i}{m_e c^2}(1 - \cos\theta)}$$

see Chpt. 7 of the REF book (p196) by Rybicki & Lightman



Compton scattering (wavelength form)

prev. sl.

$$\epsilon_f = \frac{\epsilon_i}{1 + \frac{\epsilon_i}{m_e c^2}(1 - \cos\theta)}$$

$$\frac{hc}{\lambda_i} = \frac{hc}{\lambda_f}\left(1 + \frac{h}{\lambda_i m_e c}(1 - \cos\theta)\right)$$

$$\lambda_f = \lambda_i\left(1 + \frac{h}{\lambda_i m_e c}(1 - \cos\theta)\right)$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda_f - \lambda_i = \lambda_{\text{comp}}(1 - \cos\theta)$$

$$\lambda_{\text{comp}} = \frac{h}{m_e c} = 0.02426\ \text{Å}$$

Compton wavelength

With $\lambda_f - \lambda_i > 0$, the scattered photon always loses energy after scattering

Differential Compton scattering cross section

The differential Compton scattering cross section of unpolarized radiation is given by Heitler (1954)

Klein-Nishina formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{\epsilon_f^2}{\epsilon_i^2} \left(\frac{\epsilon_i}{\epsilon_f} + \frac{\epsilon_f}{\epsilon_i} - \sin^2 \theta \right)$$

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} = 2.72 \times 10^{-13}\ \mathrm{cm}$$

Thomson scattering (unpolarized)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{unpol}}^{\mathrm{total}} = \frac{1}{2} r_0^2 (1 + \cos^2 \theta)$$

With $\epsilon_f \to \epsilon_i$, Klein-Nishina formula reduces to the Thomson scattering differential cross section of unpolarized radiation

Total Compton scattering cross section

The total Compton scattering cross section is

$$x = \frac{h\nu}{m_e c^2}$$

$$\sigma = \sigma_{\mathrm{T}} \frac{3}{4} \left(\frac{1+x}{x^3} \left(\frac{2x(1+x)}{1+2x} - \ln(1+2x) \right) + \frac{1}{2x} \ln(1+2x) - \frac{1+3x}{(1+2x)^2} \right)$$

In the non-relativistic regime ($x \ll 1$)

$$\ln(1+2x) = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4 \ldots \text{for } x \ll 1$$

$$\sigma = \sigma_{\mathrm{T}} \left(1 - 2x + \frac{26x^2}{5} + \ldots \right) \rightarrow \sigma_{\mathrm{T}}$$

In the relativistic regime ($x \gg 1$)

$$\ln(1+2x) = \ln 2x \ \text{ for } x \gg 1$$

$$\sigma = \frac{3\sigma_{\mathrm{T}}}{8x} \left(\ln 2x + \frac{1}{2} \right)$$

As the energy of the incident photon increases, the total Compton scattering cross section decreases.

Chpt.4 Compton and inverse Compton scattering

4.1 Compton scattering

4.2 Inverse Compton scattering
4.2.1 Inverse Compton scattering
4.2.2 Power of ICS by a single electron
4.2.3 Lifetime of ICS radiation
4.2.4 Spectrum of ICS radiation

4.3 Sunyaev-Zel'dovich effect
4.4 Comptonization

Inverse Compton scattering (ICS)

If the electron is moving with velocity v, energy can be transferred from the electron to the photon.

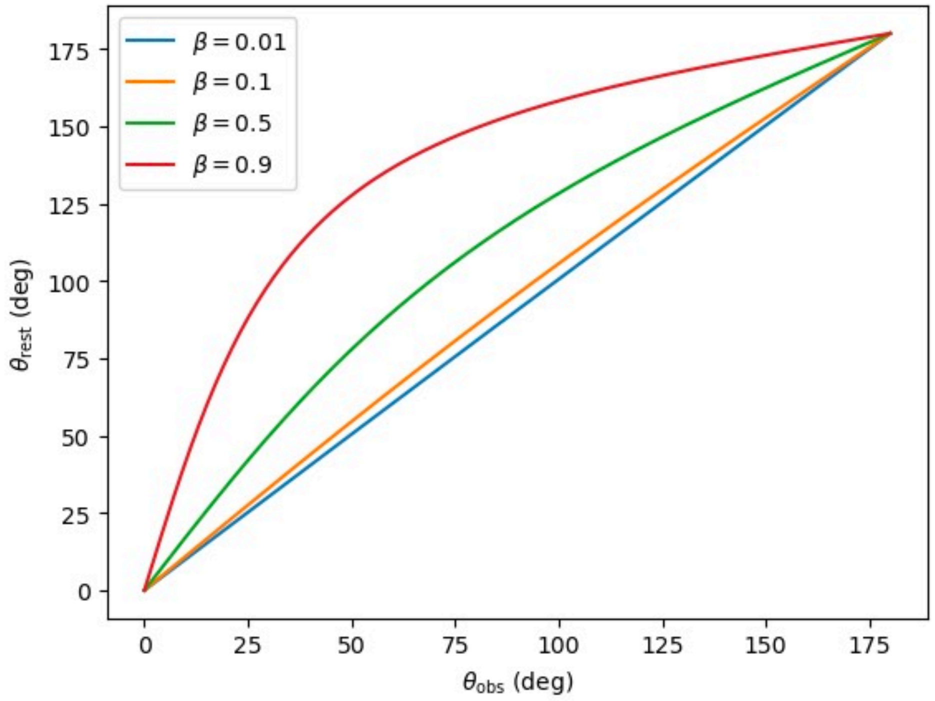
While photons lose energy (to electrons, heating) in the Compton scattering process,
photons gain energy (from electrons, cooling) in the inverse Compton scattering process.

In the rest frame (K') of the electron with $\gamma\epsilon_i \ll m_e c^2$,

$$\epsilon_f' = \frac{\epsilon_i{}'}{1+\frac{\epsilon_i{}'}{m_e c^2}(1-\cos\theta')}, \theta' = \theta_f' - \theta_i'$$

$$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$$

see Sect. 5.4.2 of 《天体物理中的辐射机制》(p232) by 尤峻汉

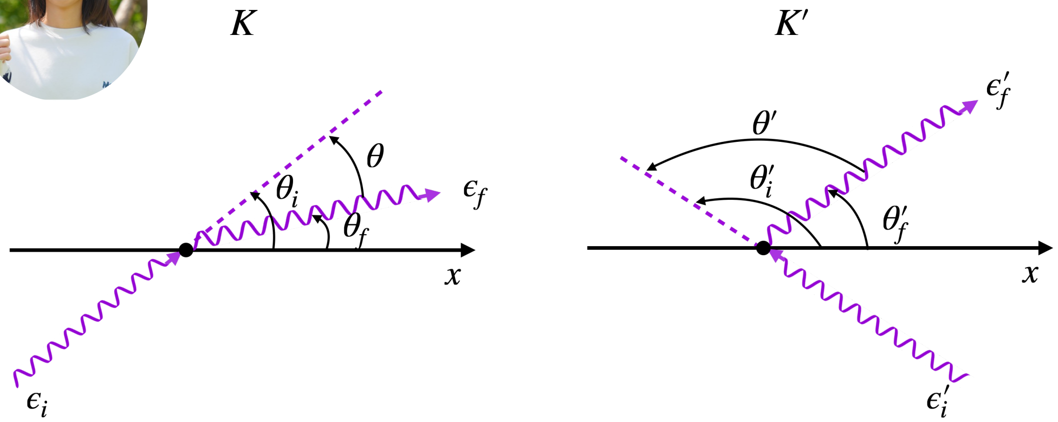


Initial energies

$$\epsilon_i' = \epsilon_i \gamma (1 - \beta \cos \theta_i)$$

$$\epsilon_f = \epsilon_f' \gamma (1 + \beta \cos \theta_f')$$

see Chpt. 4 of the REF book (p127-128) by Rybicki & Lightman



For a head-on collision with $\theta_i \sim \pi$ and large γ (i.e. $1 + \beta \to 2$),

$$\epsilon_i' = \epsilon_i \gamma (1 + \beta) \to 2\gamma\epsilon_i$$

For the photon approaches from the rear ($\theta_i \sim 0$) and large γ ($1 + \beta \to 2$),

$$\epsilon_i' = \epsilon_i \gamma (1 - \beta) = \frac{\epsilon_i}{\gamma(1+\beta)} \to \frac{\epsilon_i}{2\gamma}$$

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{1}{(1-\beta)(1+\beta)}$$

Final energies

$$\epsilon_i' = \epsilon_i \gamma (1 - \beta \cos\theta_i)$$

$$\epsilon_f = \epsilon_f' \gamma (1 + \beta \cos\theta_f')$$

see Chpt. 4 of the REF book (p127-128) by Rybicki & Lightman

If the initial photon energy in the rest frame satisfies $\epsilon_i \ll m_e^2 c$, (i.e., $\epsilon_f' = \epsilon_i{}'$)

$$\epsilon_f = \epsilon_i \gamma^2 (1 + \beta \cos\theta_f')(1 - \beta \cos\theta_i)$$

prev. sl.

$$\epsilon_f' = \frac{\epsilon_i'}{1 + \frac{\epsilon_i'}{m_e c^2}(1 - \cos\theta)}$$

For a relativistic electron ($\beta \to 1$), a head on collision with $\theta_i \sim \pi$ and $\theta_f' \sim 0$ (i.e., the photon turns around after scattering)

$$\left(\frac{\epsilon_f}{\epsilon_i}\right)_{\max} = 4\gamma^2$$

The photon gains energy in an efficient way (by a maximum factor of $4\gamma^2$)

Power of ICS by a single electron

The previous section considers Compton scattering between a single photon and a single electron.

Now we consider Compton scattering between an isotropic distribution of photons and a single electron. If the energy transfer in the electron rest frame is negligible, as shown by Blumenthal & Gould (1970), the total radiative power of ICS is

$$P_{\rm comp} = \frac{4}{3}\sigma_{\rm T} c \beta^2 \gamma^2 U_{\rm ph} = \frac{32\pi}{9} r_0^2 c\, \beta^2 \gamma^2 U_{\rm ph}$$

$$= 2.66\times 10^{-14} \beta^2 \gamma^2 \left(\frac{U_{\rm ph}}{\rm erg\ cm^{-3}}\right)\ {\rm erg\ s^{-1}}$$

see Sect. 7.2 of the REF book (p199-201) by Rybicki & Lightman

(initial) photon energy density
(cf. Sect. 2.1.5 of lecture slides)

$$U_{\rm ph} = \frac{4\pi I}{c}$$

cf. ICS and synchrotron

prev. sl.

$$P_{\mathrm{comp}} = \frac{4}{3}\sigma_{\mathrm{T}} c \beta^2 \gamma^2 U_{\mathrm{ph}}$$

Sect. 3.5.2 of lecture slides

$$P_{\mathrm{syn}} = \frac{4}{3}\sigma_{\mathrm{T}} c \beta^2 \gamma^2 U_{\mathrm{B}}$$

By comparing the ratio of magnetic energy density to photon energy density, we can estimate whether synchrotron or ICS is the leading radiative process of each relativistic electron

$$\frac{P_{\mathrm{syn}}}{P_{\mathrm{comp}}} = \frac{U_{\mathrm{B}}}{U_{\mathrm{ph}}}$$

$$U_B = \frac{B^2}{8\pi}$$

$$U_{\mathrm{ph}} = \frac{4\pi I}{c}$$

Lifetime of ICS radiation

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$, it can lose energy via ICS radiation

$$\frac{d}{dt}(\gamma m_e c^2) = -P_{\text{comp}} = -\frac{32\pi}{9}\frac{e^4}{m_e^2 c^3}\frac{v^2}{c^2}\gamma^2 U_{\text{ph}}$$

$$\frac{d}{dt}(\gamma m_e c^2) = -A_{\text{comp}}\, m_e c^2 (1-\gamma^{-2})\gamma^2$$

$$\frac{d\gamma}{dt} = -A_{\text{comp}}\,(\gamma^2 - 1) = -A\,\gamma^2 \text{ for } \gamma \gg 1$$

$$-\gamma^2 d\gamma = A_{\text{comp}} dt$$

$$\gamma(t) = \frac{\gamma_0}{1 + A_{\text{comp}}\,\gamma_0 t}$$

prev. sl.

$$P_{\text{comp}} = \frac{32\pi}{9} r_0^2 c\, \beta^2 \gamma^2 U_{\text{ph}}$$

$$\frac{v^2}{c^2} = 1 - \gamma^{-2}$$

$$A_{\text{comp}} = \frac{32\pi}{9}\frac{e^4}{m_e^3 c^5} U_{\text{ph}}$$

$$\frac{1}{\gamma(t)} - \frac{1}{\gamma_0} = A_{\text{comp}}\, t$$

Half-energy lifetime due to ICS radiation

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$ and emits synchrotron radiation, the half-energy lifetime is

$$\frac{\gamma_0}{2} = \frac{\gamma_0}{1 + A_{\rm comp}\, \gamma_0 t_{\rm ICS,1/2}}$$

$$t_{\rm ICS,1/2} = \frac{1}{\gamma_0 A_{\rm comp}} = \frac{9}{32\pi}\frac{m_e^3 c^5}{e^4}\frac{1}{\gamma_0 U_{\rm ph}}$$

$$\sim 3.08 \times 10^7 \frac{1}{\gamma_0}\left(\frac{U_{\rm ph}}{\rm erg\ cm^{-3}}\right)^{-1} \ {\rm s}$$

prev. sl.

$$\gamma(t) = \frac{\gamma_0}{1 + A_{\rm comp}\, \gamma_0 t}$$

$$A_{\rm comp} = \frac{32\pi}{9}\frac{e^4}{m_e^3 c^5} U_{\rm ph}$$

Spectrum of ICS (single electron)

Even for a monotonic incident photon (ν_i), after the inverse Compton scattering, the scattered photon frequency ranges from 0 to $4\gamma^2\nu_i$

prev. sl.

$$\left(\frac{\epsilon_f}{\epsilon_i}\right)_{\max} = 4\gamma^2$$

The spectrum of ICS between an isotropic distribution of photons and a single electron is Blumenthal & Gould (1970)

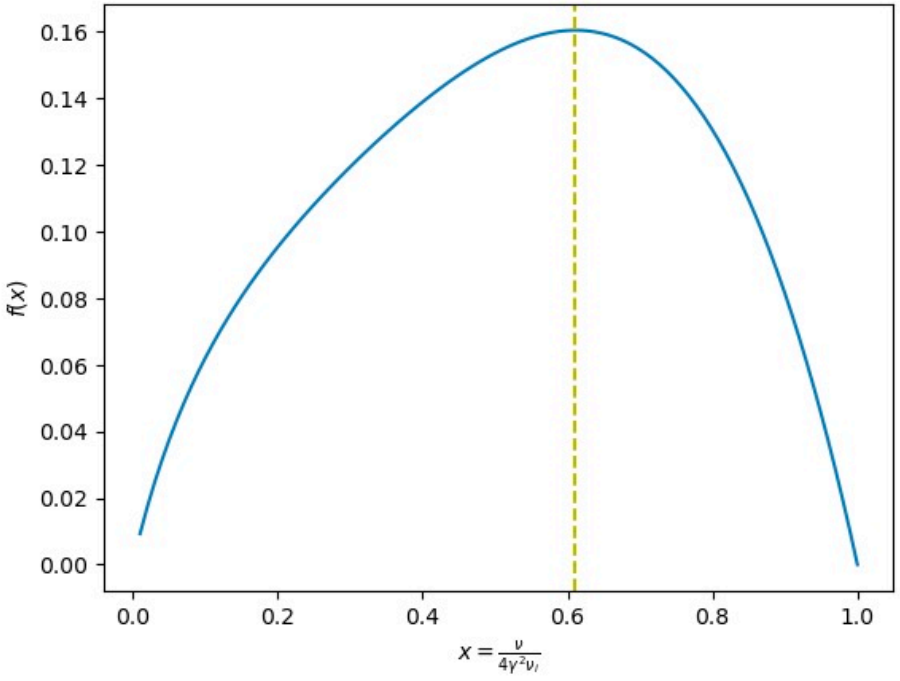
$$P(\nu) = 8\pi r_0^2 ch \int f(x)\, n_{\rm ph}(\nu_i) d\nu_i$$

volume density of photons with a specific frequency ν_i

$$x = \frac{\nu}{4\gamma^2\nu_i}$$

$$f(x) = x + 2x^2 \ln x + x^2 - 2x^3, \text{for } 0 < x < 1$$

$f(x) \sim 0.16@x \sim 0.61$



Power density of ICS by a population of electrons

For a population of relativistic electrons following a power law energy distribution, the total radiative power density of ICS is

$$N(\gamma)d\gamma \propto \gamma^{-p}d\gamma, \gamma_{\min} < \gamma < \gamma_{\max}$$

$$U_{\text{comp}} = \int_{\gamma_{\min}}^{\gamma_{\max}} P_{\text{comp}} N(\gamma) d\gamma$$

$$\propto \frac{4}{3}\sigma_{\text{T}} c \beta^2 U_{\text{ph}} \frac{\gamma_{\max}^{3-p} - \gamma_{\min}^{3-p}}{3-p}$$

prev. sl.

$$P_{\text{comp}} = \frac{4}{3}\sigma_{\text{T}} c \beta^2 \gamma^2 U_{\text{ph}}$$

P_{comp} in erg s^{-1} and U_{comp} in erg s^{-1} cm^{-3}

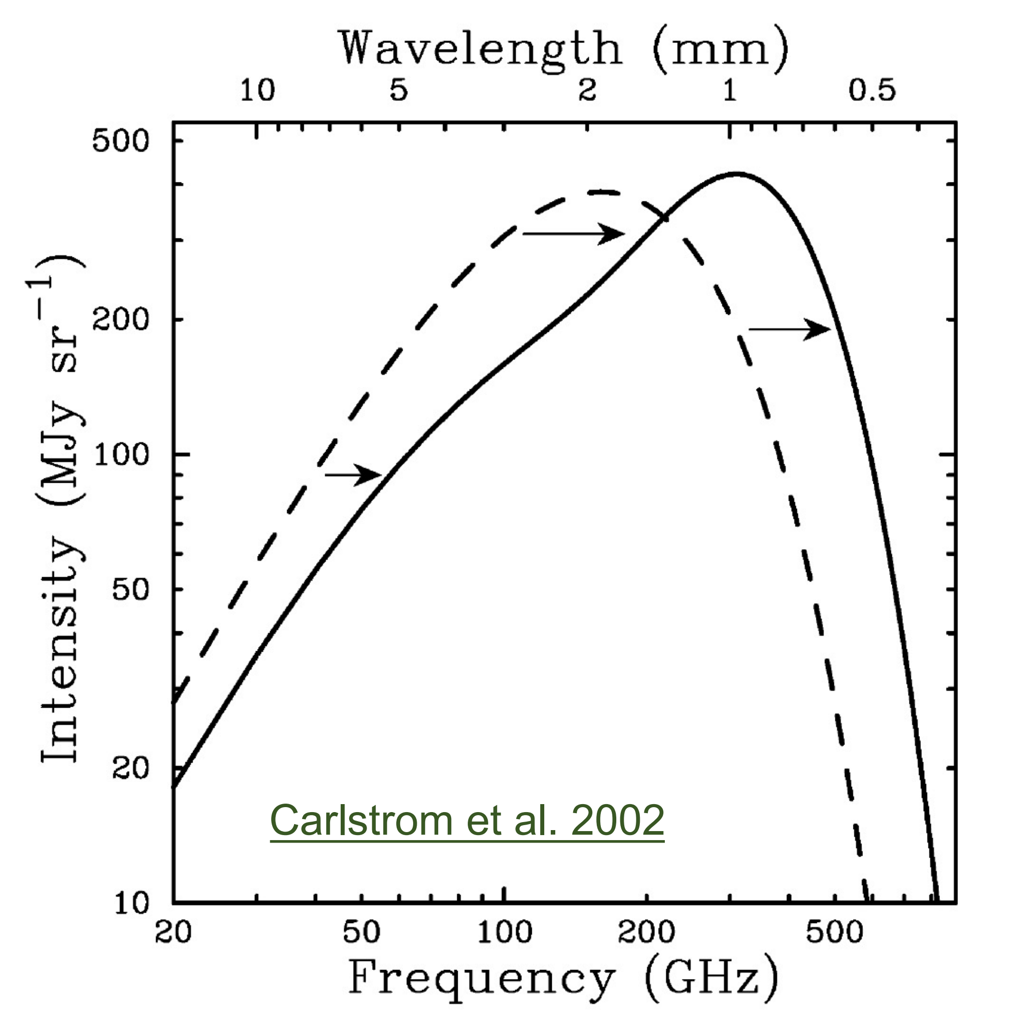
The spectral shape of Compton radiation generally follows a power law, but the power law index can vary depending on the volume density of photons

Chpt.4 Compton and inverse Compton scattering

4.1 Compton scattering
4.2 Inverse Compton scattering

4.3 Sunyaev-Zel'dovich effect
4.3.1 Clusters of galaxies
4.3.2 Thermal SZ effect
4.3.3 tSZ spectral distortion on CMB
4.3.4 Kinetic SZ effect
4.3.5 Advantages of the SZ effect

4.4 Comptonization



Clusters of galaxies (ClG)

Clusters of galaxies (or galaxy clusters) are the largest gravitational bounded structures in the Universe.

The hot (X-ray emitting) intracluster medium (ICM) is a key component of ClG.

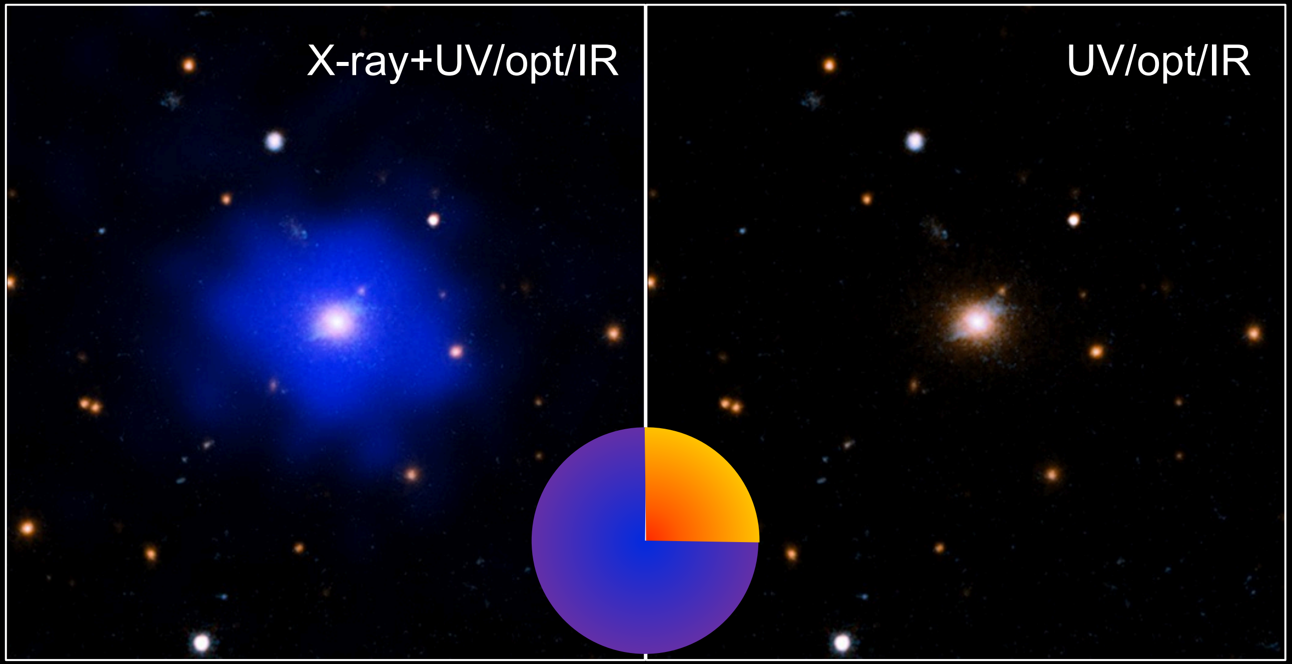


Image credit: X-ray: NASA/CXC/MIT/M. Calzadilla; UV/Optical/Near-IR/IR: NASA/STScI/HST; Image processing: N. Wolk

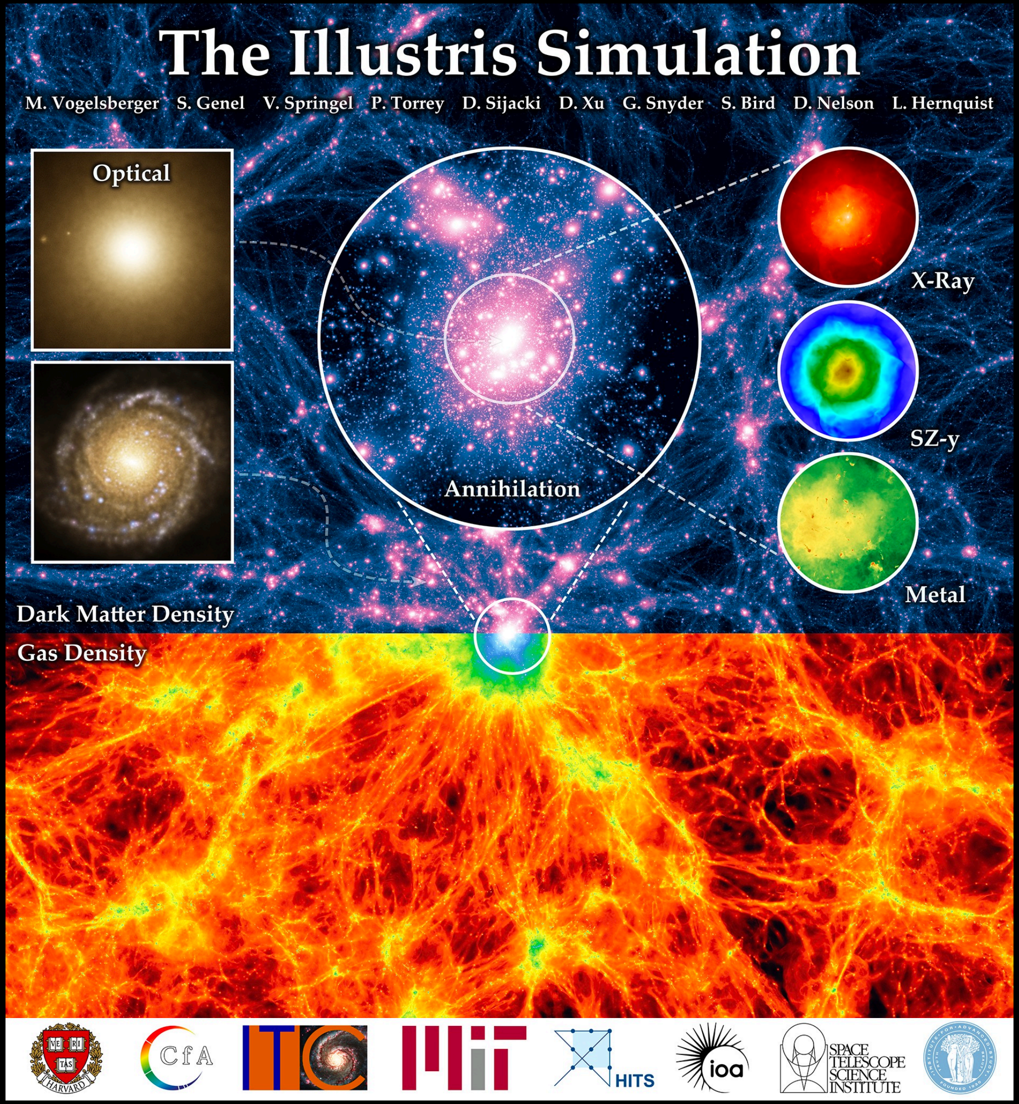
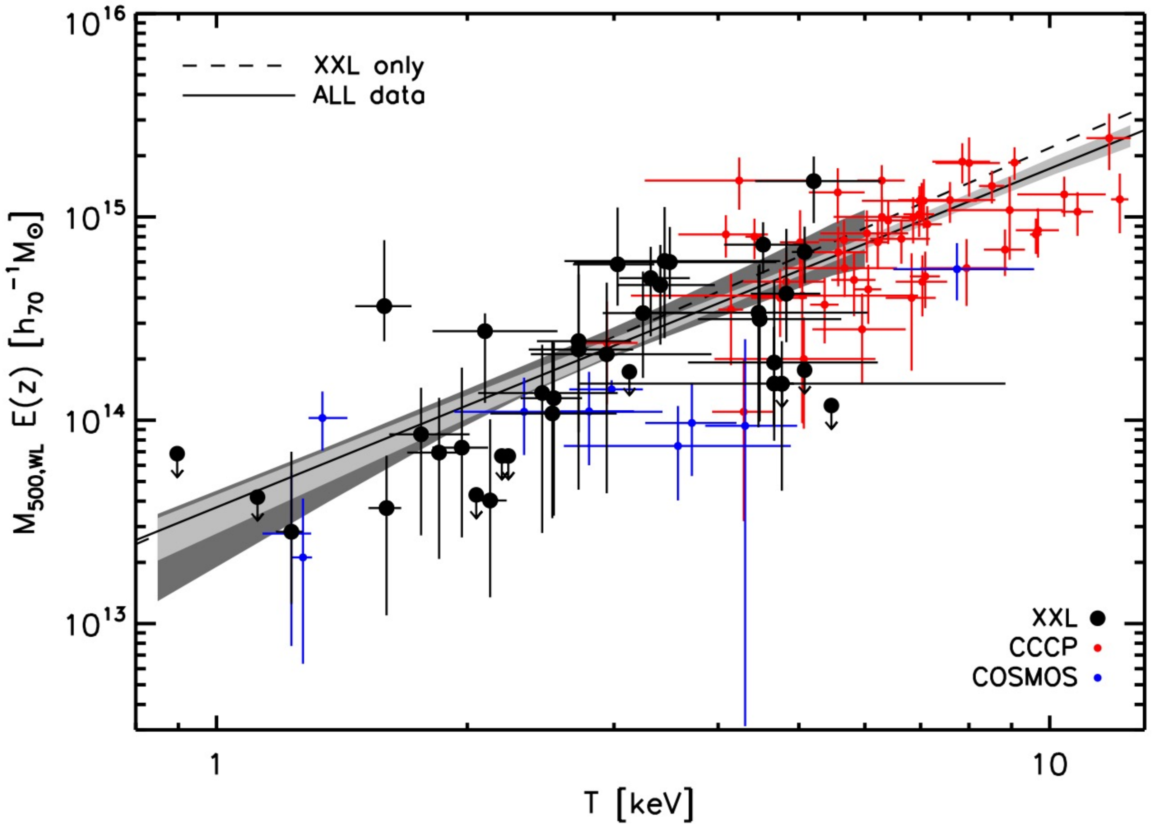


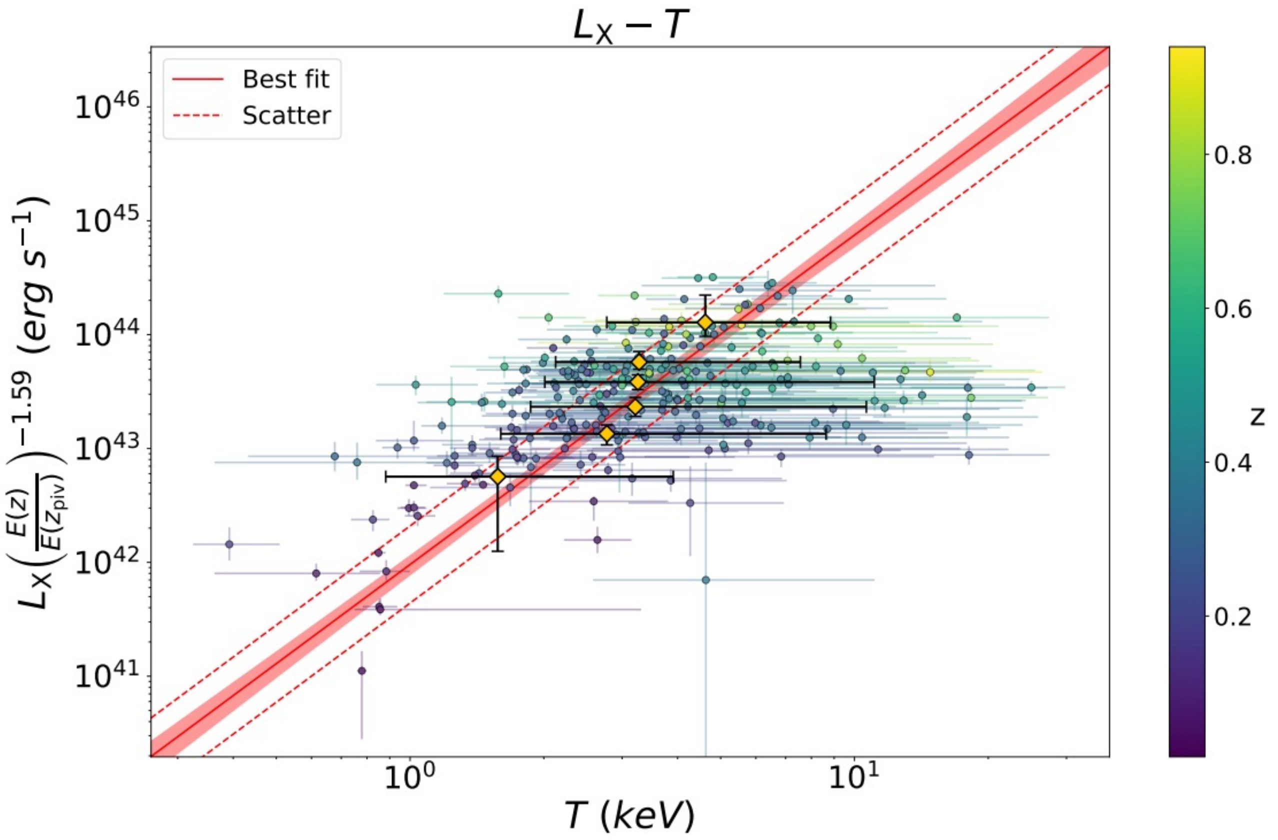
Image credit: The Illustris Collaboration

Scaling relations for CIGs

The hotter the ICM, the more massive and more luminous (X-ray) the CIG



Lieu et al. 2016

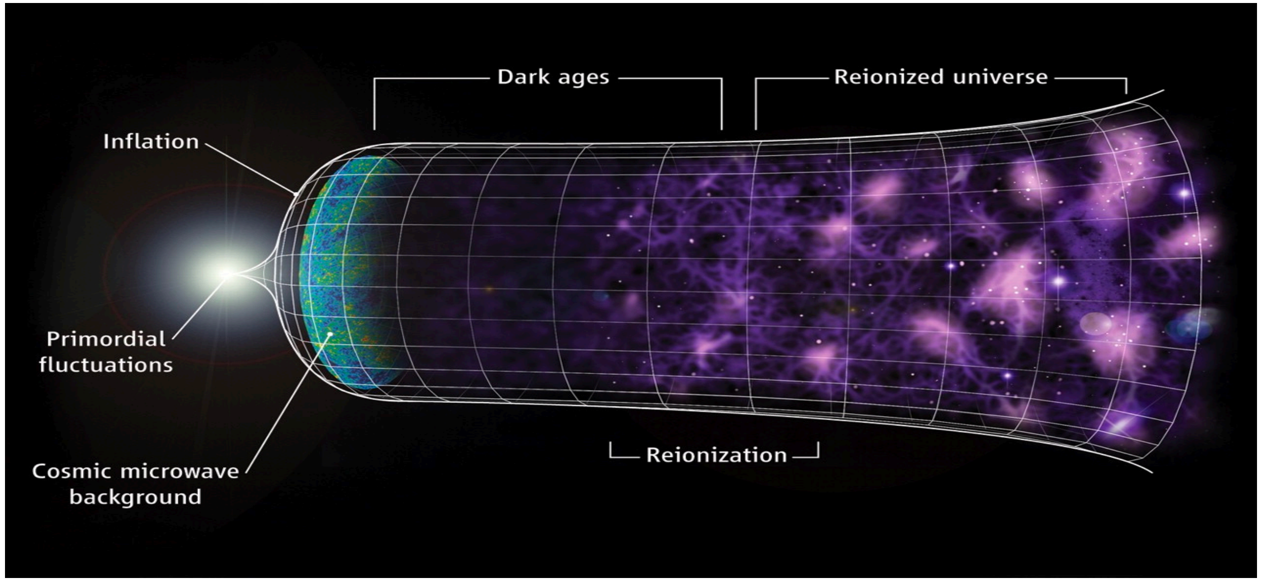


Bahar et al. 2022

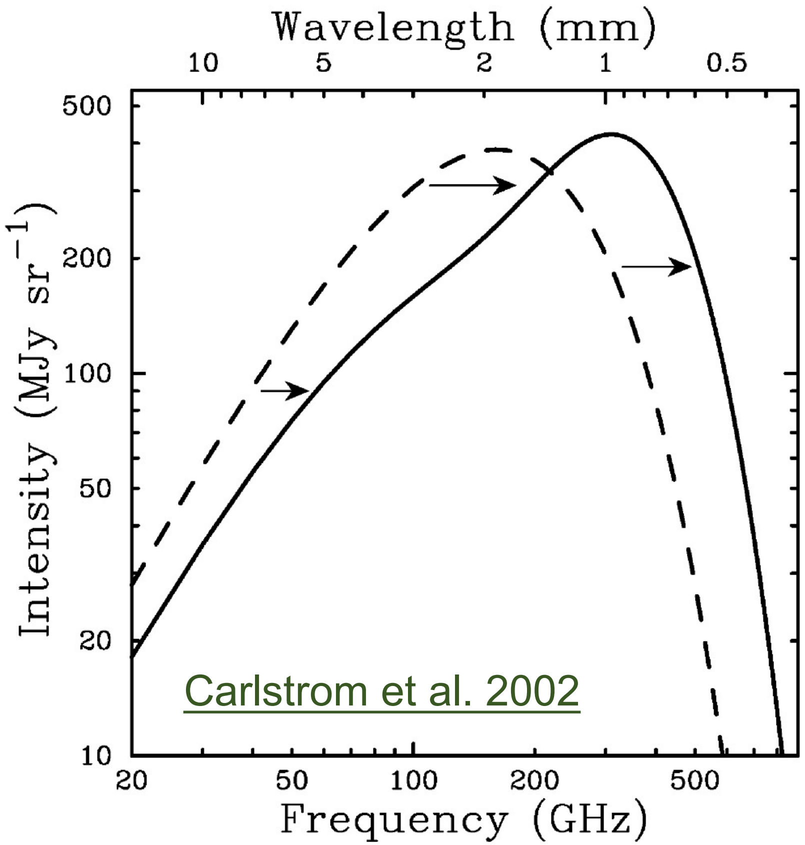
Sunyaev-Zel'dovich effect

The Sunyaev-Zel'dovich (SZ) effective is the inverse Compton scattering of radiation from Cosmic Microwave Background (CMB) off hot electrons in galaxy clusters. This changes the energy of CMB photons and produce observable deviations from the Planck spectrum.

The SZ effect was first proposed in the 1970s (Sunyaev & Zel'dovich 1970, 1972)



Faucher-Giguère et al. 2008



Carlstrom et al. 2002

Thermal SZ effect

The typical change of energy of a photon due to ICS with a single electron is

$$\Delta E_{\mathrm{ph}} = \frac{P_{\mathrm{comp}}}{R_{\mathrm{col}}}$$

collision rate

$$R_{\mathrm{col}} = \frac{dN}{dt} = n_{\mathrm{ph}} \sigma_{\mathrm{T}} c$$

photon volume density

Thomson scattering cross section

prev. sl.

$$P_{\mathrm{comp}} = \frac{4}{3} \sigma_{\mathrm{T}} c \beta^2 \gamma^2 U_{\mathrm{ph}}$$

average photon energy

$$\Delta E_{\mathrm{ph}} = \frac{P_{\mathrm{comp}}}{R_{\mathrm{col}}} = \frac{\frac{4}{3}\sigma_{\mathrm{T}}\, c\beta^2\gamma^2 U_{\mathrm{ph}}}{n_{\mathrm{ph}}\sigma_{\mathrm{T}} c} = \frac{4}{3}\beta^2\gamma^2 \frac{U_{\mathrm{ph}}}{n_{\mathrm{ph}}} = \frac{4}{3}\beta^2\gamma^2 E_{\mathrm{ph}}$$

Thermal SZ effect (cont.)

prev. sl.

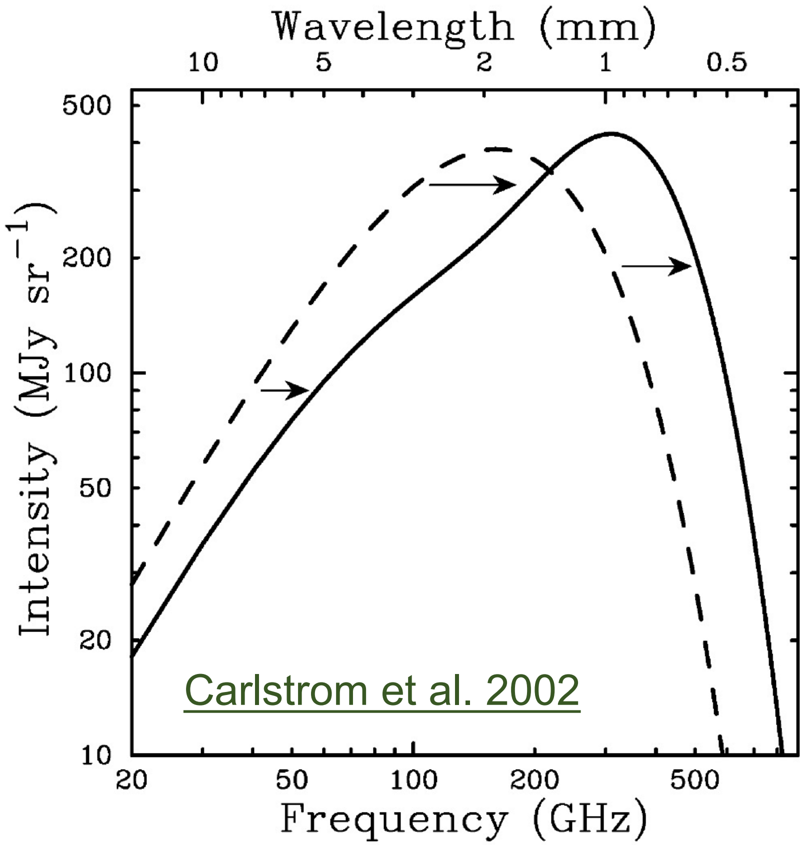
$$\Delta E_{\mathrm{ph}} = \frac{4}{3}\beta^2\gamma^2 E_{\mathrm{ph}}$$

For non-relativistic electrons, $\gamma \to 1$

$$\frac{1}{2}m_e v^2 = \frac{3}{2}kT_e \leftrightarrow \beta^2 = \frac{v^2}{c^2} = \frac{3\,kT_e}{m_e c^2}$$

The typical energy gain of a photon due to ICS with a single electron is

$$\Delta E_{\mathrm{ph}} \simeq \frac{4}{3}\beta^2 E_{\mathrm{ph}} = 4\left(\frac{kT_e}{m_e c^2}\right)E_{\mathrm{ph}}$$



Carlstrom et al. 2002

Compton optical depth

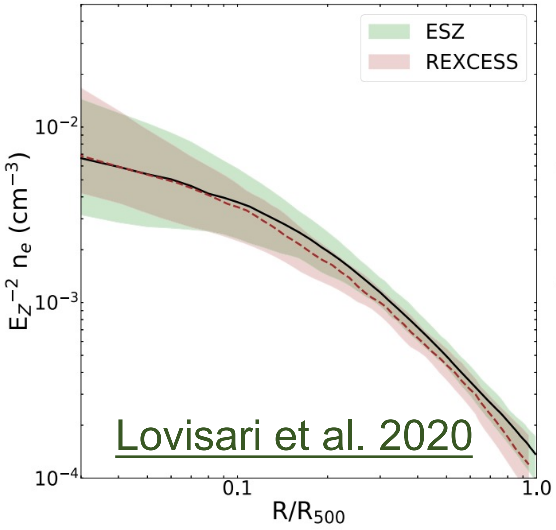
The scattering optical depth is

$$\tau = n_e \sigma_{\mathrm{T}} l$$

$$= 0.021 \left(\frac{n_e}{10^{-2}\ \mathrm{cm}^{-3}}\right) \left(\frac{l}{\mathrm{Mpc}}\right)$$

Hence, the number of scattering is

$$N \sim \tau = 0.021 \left(\frac{n_e}{10^{-2}\ \mathrm{cm}^{-3}}\right) \left(\frac{l}{\mathrm{Mpc}}\right)$$



Lovisari et al. 2020

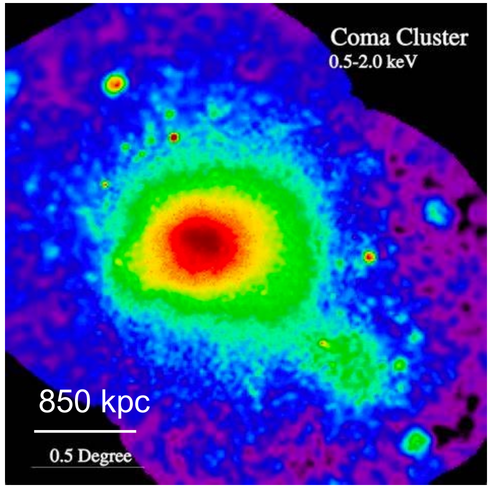


Image credit: ROSAT/MPE/S. L. Snowden

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

Sect. 2.5.1.1 Pure scattering (Random walk)

For $\tau^{\mathrm{sca}} \ll 1, N^{\mathrm{sca}} \sim \tau^{\mathrm{sca}}$

Compton y parameter

prev. sl.

$$\Delta E_{\rm ph} \simeq \frac{4}{3}\beta^2 E_{\rm ph} = 4\left(\frac{kT_e}{m_e c^2}\right) E_{\rm ph}$$

prev. sl.

$$N \sim \tau = 0.0205\left(\frac{n_e}{10^{-2}\ {\rm cm}^{-3}}\right)\left(\frac{l}{\rm Mpc}\right)$$

In the limit of N times of collisions, we have

$$E_{\rm ph}^N = E_{\rm ph}\left(1 + 4\frac{kT_e}{m_e c^2}\right)^N$$

$$\sim E_{\rm ph}\exp\left(N\frac{4kT_e}{m_e c^2}\right) = E_{\rm ph}\exp(y)$$

$$y = N\frac{4kT_e}{m_e c^2}$$

mean number of scattering or optical depth $\times$ the fractional energy gain per scattering

- ❑ Most of the CMB photons pass through the Universe freely.
- ❑ Merely a few percent $(N \sim \tau)$ of the CMB photons will increase their energy by a factor of $\exp(y){\sim}y$

Compton y parameter (cont.)

prev. sl.

$$y = N\frac{4kT_e}{m_e c^2}$$

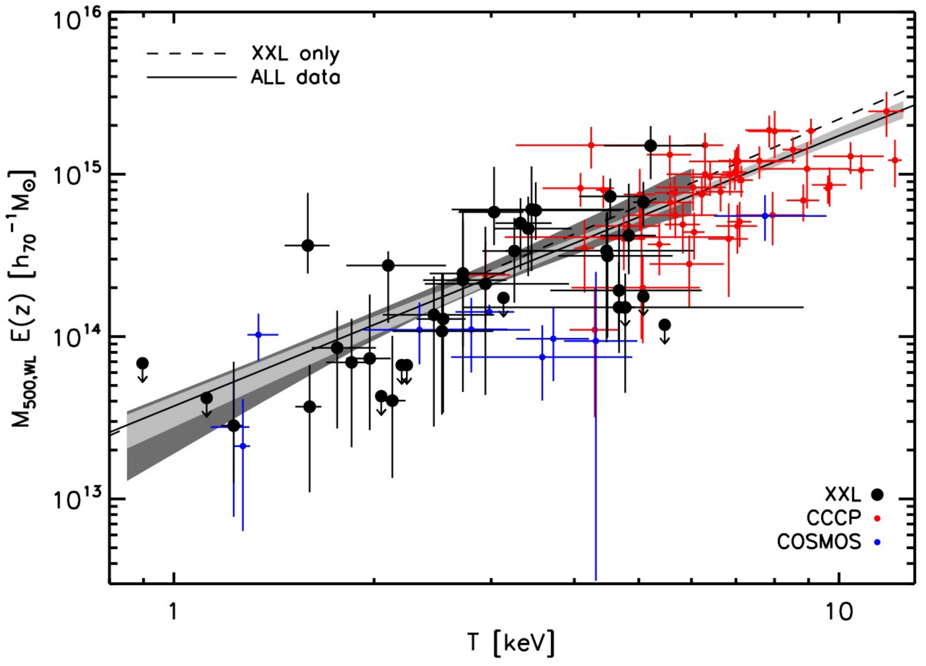
prev. sl.

$$N \sim \tau = 0.0205\left(\frac{n_e}{10^{-2}\ \mathrm{cm}^{-3}}\right)\left(\frac{l}{\mathrm{Mpc}}\right)$$

$$y = N \times 6.75 \times 10^{-3}\left(\frac{T_e}{10^7\ \mathrm{K}}\right)$$

$$= 1.39 \times 10^{-4}\left(\frac{T_e}{10^7\ \mathrm{K}}\right)\left(\frac{n_e}{10^{-2}\ \mathrm{cm}^{-3}}\right)\left(\frac{l}{\mathrm{Mpc}}\right)$$

- Most of the CMB photons pass through the Universe freely.
- Merely a few percent ($N \sim \tau$) of the CMB photons will increase their energy by a factor of $\exp(y) \sim y$



Lieu et al. 2016

tSZ spectral distortion of CMB

Previous slides give some rough estimation of the tSZ effect.
To be more precise, the temperature change ΔT_{tSZ} is

$$\frac{\Delta T_{\text{tSZ}}}{T_{\text{CMB}}} = f(x)y = f(x)\int \frac{kT_e}{m_e c^2} n_e \sigma_T dl$$

$$x = \frac{h\nu}{kT_{CMB}}$$

relativistic correction

$$f(x) = \left(x\frac{e^x+1}{e^x-1} - 4\right)\left(1 + \delta_{\text{R}}(x, T_e)\right)$$

for $x \ll 1$, $f(x) \to -2$

$\exp x = 1 + x + \ \ldots$

Compton y parameter

$$y = \int \frac{kT_e}{m_e c^2} n_e \sigma_{\text{T}}\, dl$$

Sunyaev & Zel'dovich (1980)

tSZ spectral distortion of CMB (cont.)

In terms of specific intensity, common in millimeter observations,

$$\Delta I_{\mathrm{tSZ}} = g(x) I_0 y \qquad x = \frac{h\nu}{kT_{\mathrm{CMB}}}$$

$$I_0 = 2\frac{(kT_{\mathrm{CMB}})^3}{(hc)^2}$$

Compton y parameter

$$y = \int \frac{kT_e}{m_e c^2} n_e \sigma_{\mathrm{T}}\, dl$$

relativistic correction

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2}\left(x\frac{e^x + 1}{e^x - 1} - 4\right)\left(1 + \delta_{\mathrm{R}}(x, T_e)\right)$$

Carlstrom et al. 2002

for $x \ll 1$, $g(x) \to 0$

Kinetic SZ effect

If the galaxy cluster is moving with respect to the CMB rest frame, there will be an additional spectral distortion due to the Doppler effect.

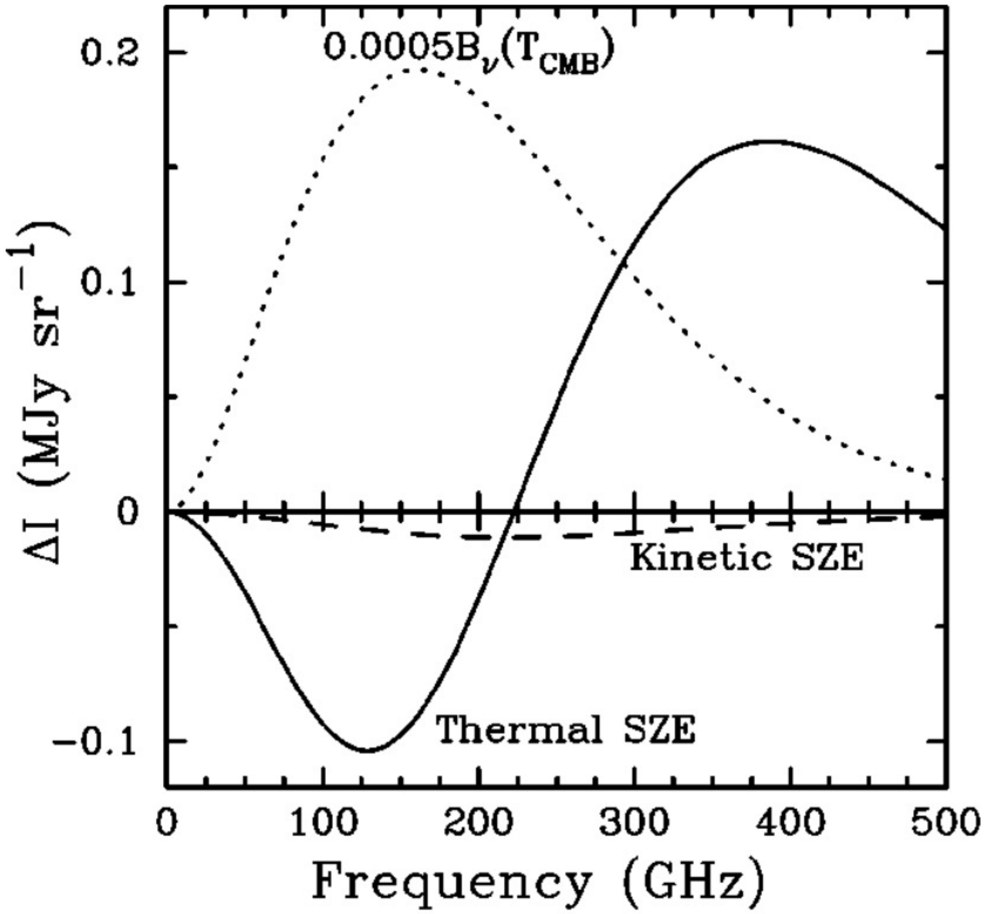
Consider the galaxy cluster has a line of sight velocity $v_{\mathrm{ClG,LOS}}$, in the non-relativistic limit, the spectral signature of the kinetic SZ (kSZ) is a pure thermal distortion of magnitude

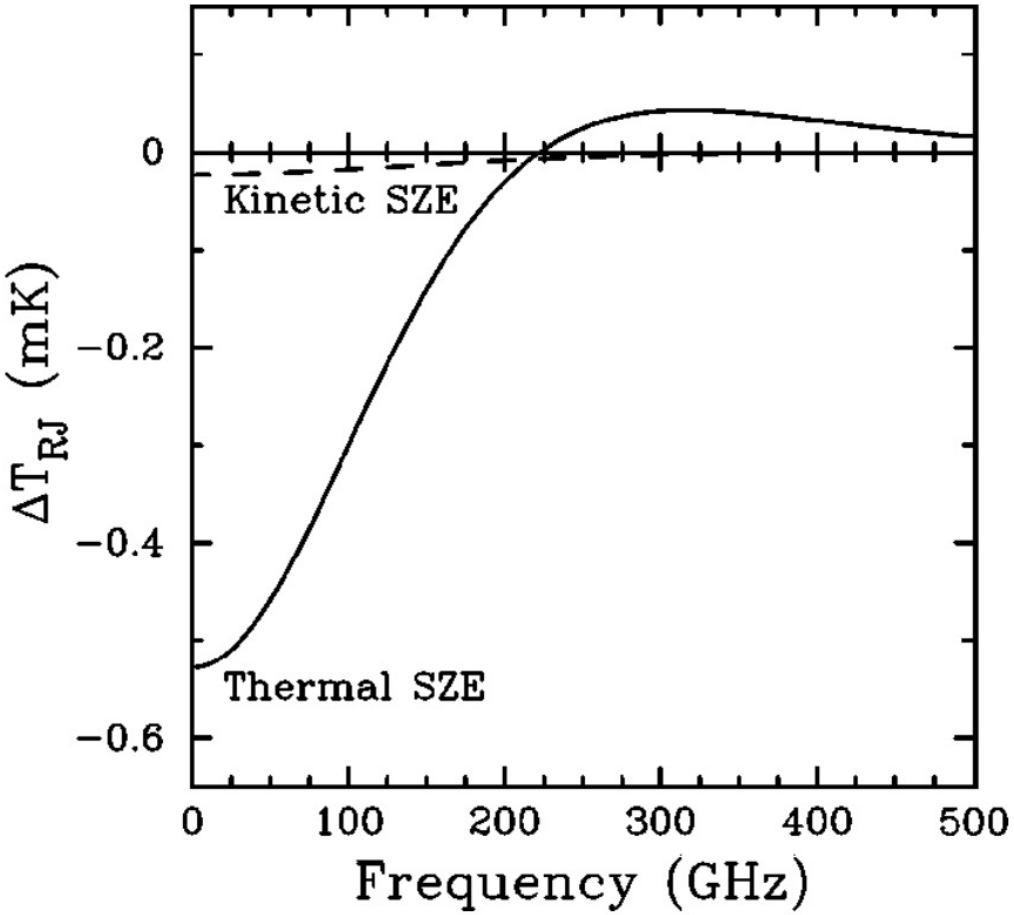
$$\frac{\Delta T_{\mathrm{kSZ}}}{T_{\mathrm{CMB}}} = -\tau \frac{v_{\mathrm{ClG,LOS}}}{c} = \frac{v_{\mathrm{ClG,LOS}}}{c} \int n_e \sigma_{\mathrm{T}} dl$$

Sunyaev & Zel'dovitch (1972)

Relativistic corrections is of the order of a few percent level (see Carlstrom et al. 2002 and reference therein).

cf. tSZ vs. kSZ





Carlstrom et al. 2002

Advantages of the SZ effect

The SZ effect is redshift-independent, thus, galaxy clusters at all redshifts can be found.

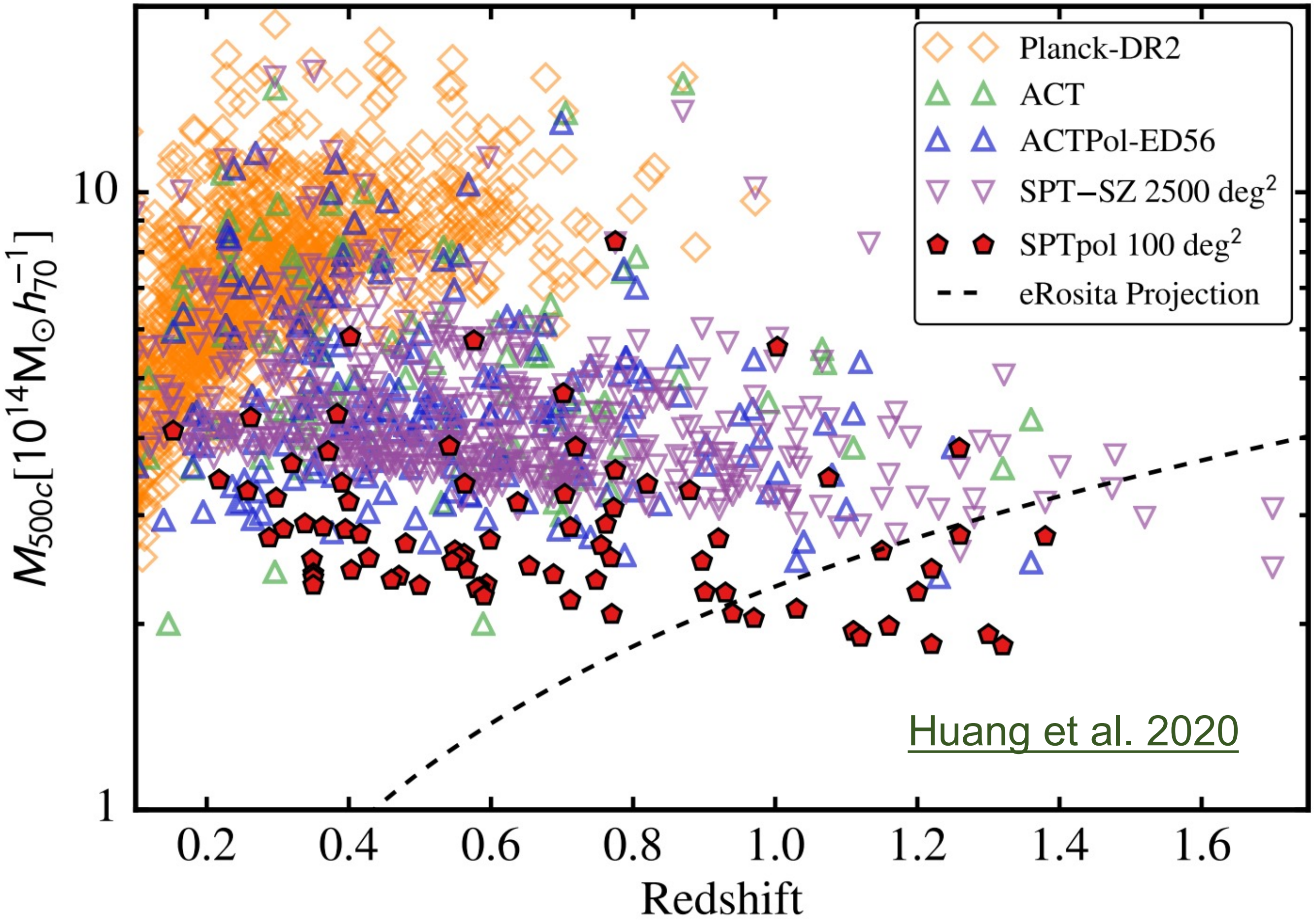
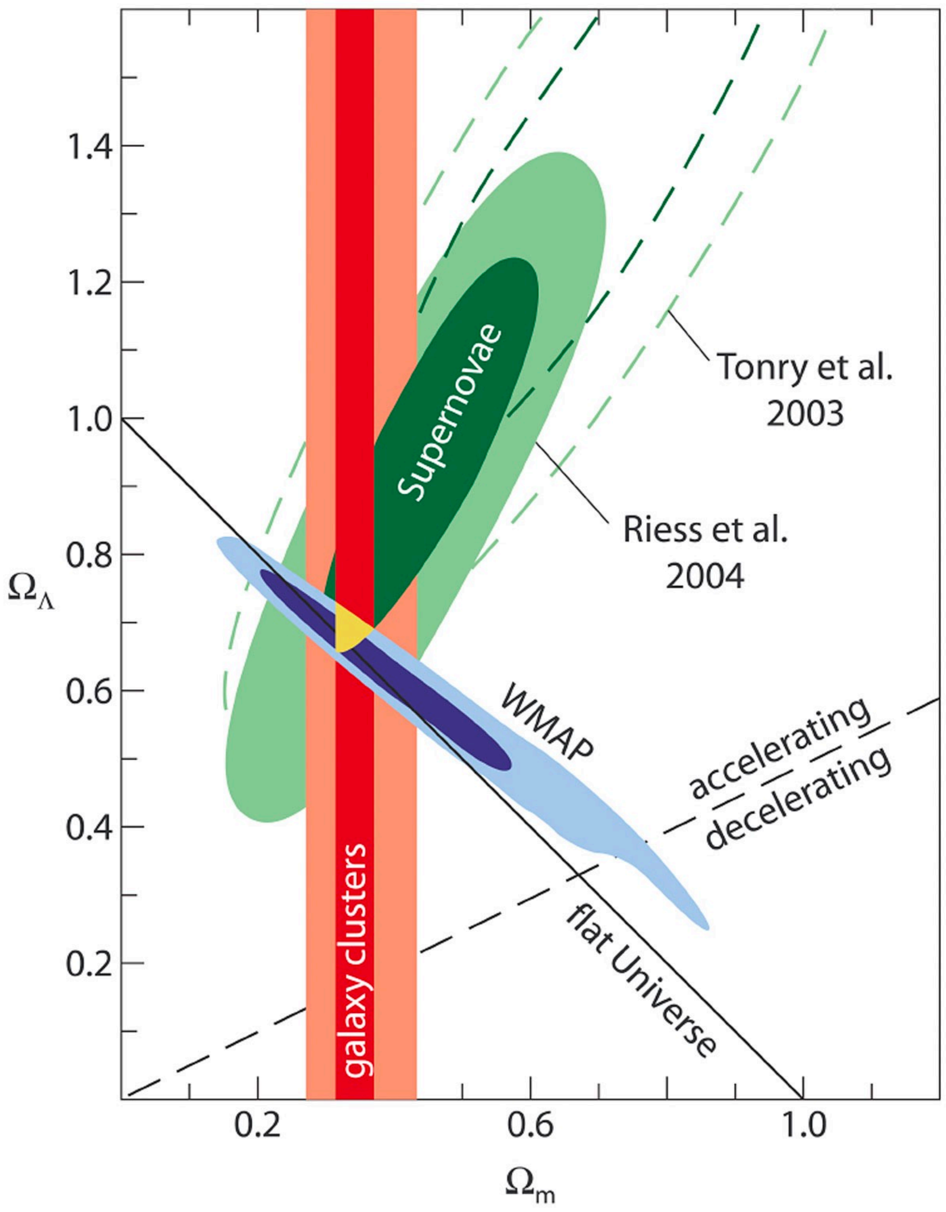


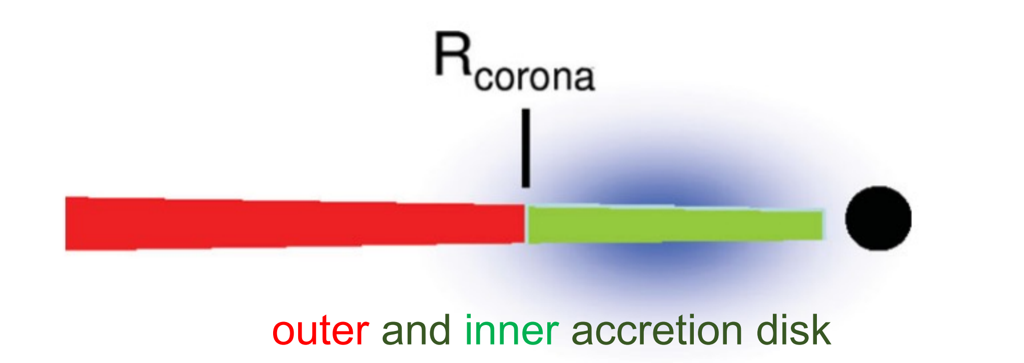
Image credit: ESO

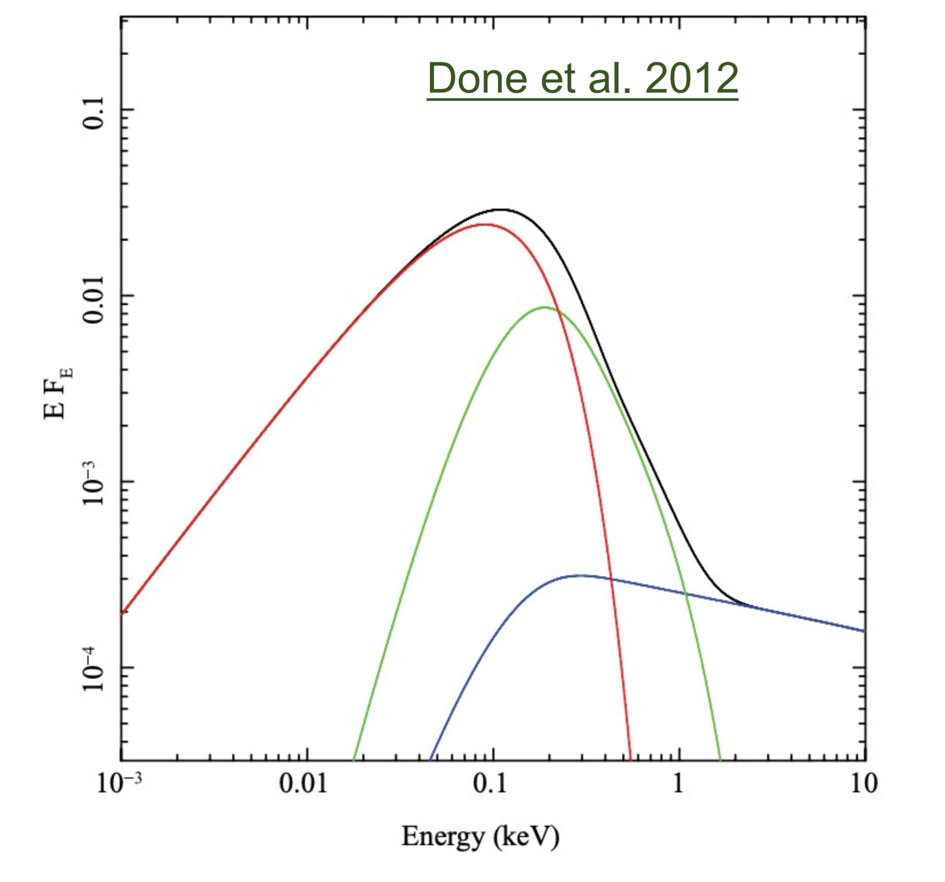


Chpt.4 Compton and inverse Compton scattering

4.1 Compton scattering
4.2 Inverse Compton scattering
4.3 Sunyaev-Zel'dovich effect

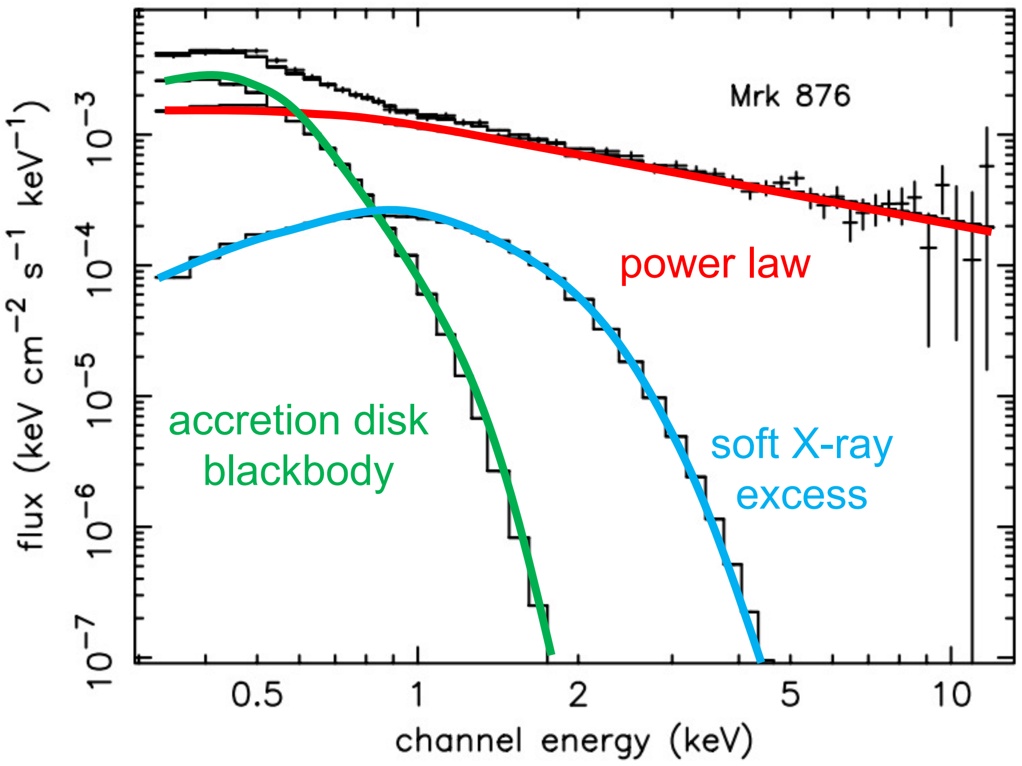
4.4 Comptonization
4.4.1 Comptonization equilibrium
4.4.2 AGN warm coronae





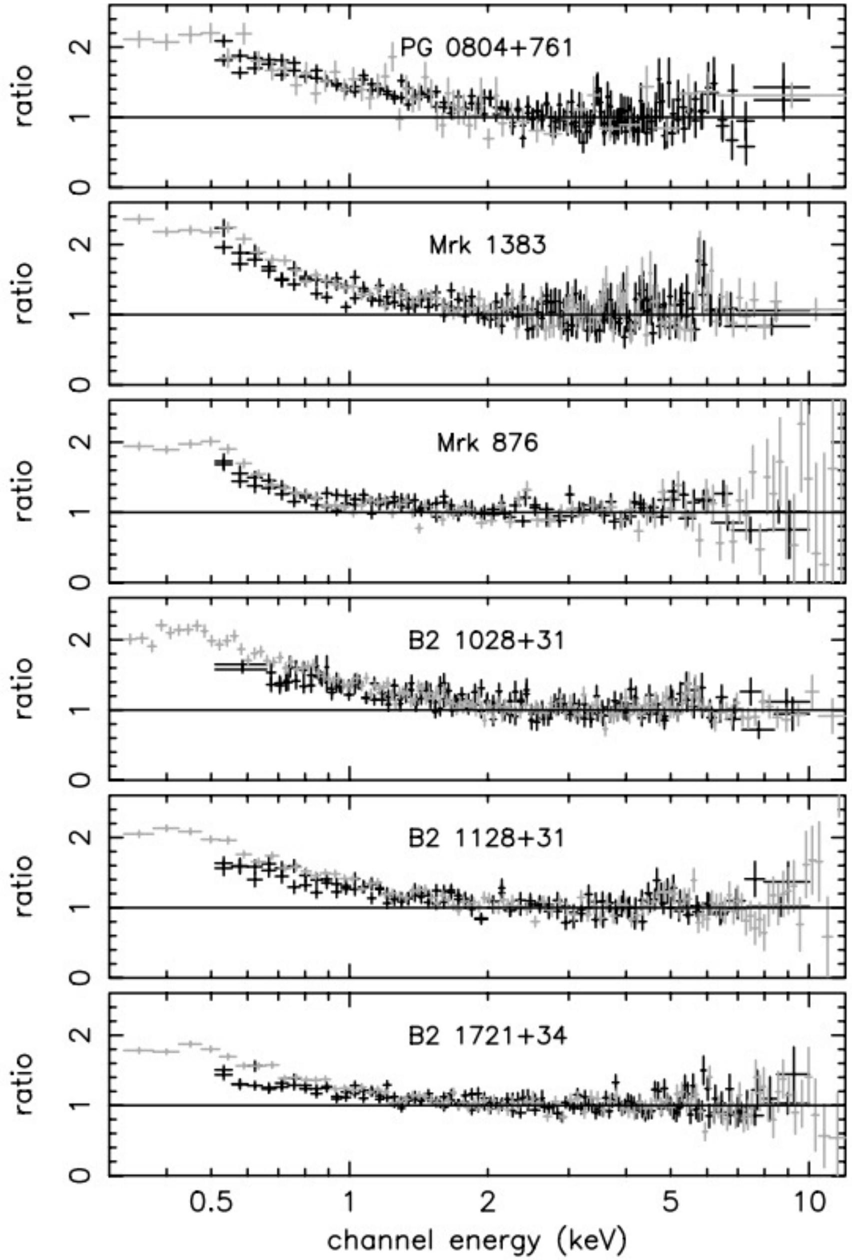
AGN soft X-ray excess problem

Page et al. 2004



What is the soft X-ray excess?

$\frac{\text{obs.}}{\text{power law}}$



Comptonization

Due to multiple inverse Compton scatterings, the spectral energy distribution can be significantly impacted.

In the observers' frame and nonrelativistic limit, the energy gain of a photon due to ICS is

unknown coefficient to be determined

$$\frac{\Delta E}{\epsilon_i} = \frac{\alpha k T_e}{m_e c^2} - \frac{\epsilon_i}{m_e c^2}$$

see Sect. 7.4 of the REF book (p209) by Rybicki & Lightman

$$\langle \Delta E \rangle = \frac{\alpha k T_e}{m_e c^2} \langle \epsilon_i \rangle - \frac{\langle \epsilon_i \rangle^2}{m_e c^2} = 0 \rightarrow \alpha = 4$$

Assuming photons and electrons are in equilibrium ($\Delta E = 0$) and assuming photons are not created or destroyed via scattering, then photons follow a power law distribution

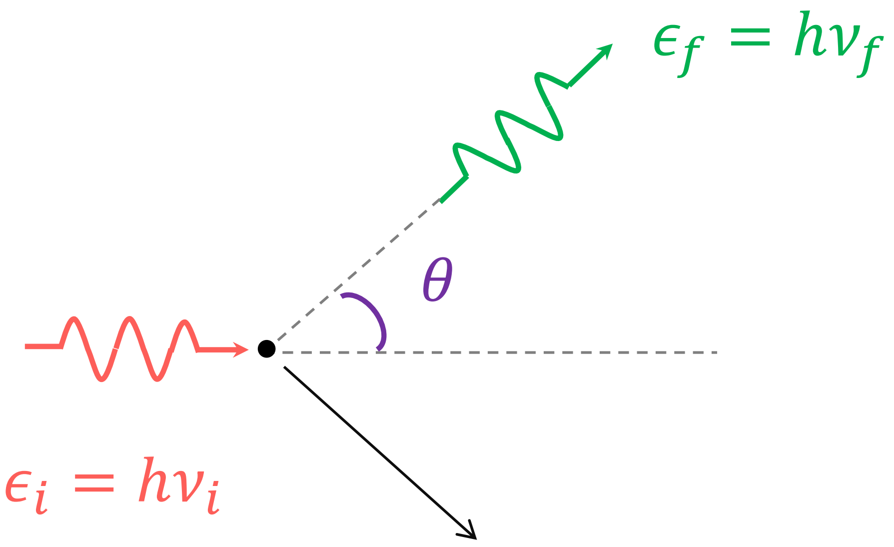
see Sect. 7.4 of the REF book (p209) by Rybicki & Lightman

$$\frac{dN}{d\epsilon} \propto \epsilon^2 \exp\left(-\frac{\epsilon}{kT}\right)$$

$$\langle \epsilon \rangle = \frac{\int \epsilon \frac{dN}{d\epsilon} d\epsilon}{\int \frac{dN}{d\epsilon} d\epsilon} = 3kT$$

$$\langle \epsilon^2 \rangle = \frac{\int \epsilon^2 \frac{dN}{d\epsilon} d\epsilon}{\int \frac{dN}{d\epsilon} d\epsilon} = 12\,(kT)^2$$

Comptonization equilibrium



prev. sl.

$$\frac{\Delta E}{\epsilon_i} = \frac{4kT_e}{m_e c^2} - \frac{\epsilon_i}{m_e c^2}$$

The equilibrium is reached when

$$\epsilon_i = 4kT_e$$

This can be achieved after multiple scatterings, where the optical depth is

prev. sl.

$$\tau = n_e \sigma_\mathrm{T} l$$

$$\tau = 6.65 \left(\frac{n_e}{10^{10}\ \mathrm{cm}^{-3}}\right)\left(\frac{l}{10^{15}\ \mathrm{cm}}\right)$$

Sect. 2.5.1.1 Pure scattering (Random walk)

For $\tau^{\mathrm{sca}} \gg 1, N^{\mathrm{sca}} \sim (\tau^{\mathrm{sca}})^2$

Thomson scattering cross section

$$\sigma_\mathrm{T} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

Comptonization equilibrium (cont.)

prev. sl.

$$E_{\rm ph}^N \sim E_{\rm ph} \exp\left(4N \frac{kT_e}{m_e c^2}\right)$$

Sect. 2.5.1.1 Pure scattering (Random walk)

For $\tau^{\rm sca} \gg 1, N^{\rm sca} \sim (\tau^{\rm sca})^2$

prev. sl.

$$\epsilon_i = 4kT_e$$

$$\epsilon_i = E_{\rm ph}$$

$$4kT_e = E_{\rm ph} \exp\left(4\tau^2 \frac{kT_e}{m_e c^2}\right)$$

The Comptonization equilibrium optical depth is

$$\tau = \left(\frac{m_e c^2}{4\,kT_e} \ln\left(\frac{4\,kT_e}{E_{\rm ph}}\right)\right)^{1/2}$$

AGN warm corona

Photons from the (inner) accretion disc gains energy via ICS from the warm coronae, giving rise to the soft X-ray excess.

prev. sl.

$$\tau = 6.65 \left(\frac{n_e}{10^{15}\ \mathrm{cm}^{-3}}\right) \left(\frac{l}{10^{10}\ \mathrm{cm}}\right)$$

Typical warm coronae have optical depth $\tau \sim 10 - 40$ and a temperature of $0.1 - 1\ \mathrm{keV}$ (Petrucci et al. 2018)