Physical Cosmology - a “Minimum in a nutshell” for GUT magnetic monopole and Inflation

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2024, Oct

1 Symmetry-breaking phase transition, topological defects and the Kibble mechanism

Phase transition: the field potential $V(\phi)$ of a scalar field $\phi(x, t)$ varies with temperature such that it changes the symmetry status between a high temperature state and low-temperature state, for example, from $V(\phi) \sim \phi^2$ at high temperatures to $V(\phi) \sim (\phi^2 - a^2)^2$ (a Mexican hat type) at some lower temperatures. As temperature drops, the phase transition breaks the symmetry which is present in the equilibrium state where the average $\phi = 0$ at high temperatures. After the phase transition from high to lower temperatures, the field ends up in the lowest energy state (vacuum) where ϕ is non-zero. The location of ϕ on the vacuum points (i.e., two values for 1 field, a circle for 2 fields, a sphere for 3 fields) is a random choice. The pointing from $\phi = 0$ to such a location is in the field internal space.

Various places in a 3d space can have different $\phi(x, t)$, in particular different pointings of ϕ in the internal space after phase transition (at vacuum). But a region within a horizon distance is expected to have sufficient time for frequent interaction, so that the fields therein would follow the same trend as that of the very first randomly orientated internal-space pointing that randomly started at a some location in that region in real space. However, between regions that can “talk”, chaos emerge. In such places, the real-space spatial arrangement of ϕ (in its internal space) would not be well configured initially during the phase transition. But with time, as temperature contines going down, all will settle to global lowest energy. The remaining pattern will be a central point having a very high energy $V(\phi) \sim a^4$ at $\phi = 0$ (tip of the Mexican hat), and gradually lowered energy $V(\phi) \sim (\phi^2 - a^2)^2$ with non-zero ϕ as the real-space location radially going outwards. These configurations are topological defects. They are remnants of phase transition that happens at different places separated beyond the horizon distance (i.e., not connected by causality) and thus not yet having time to sufficiently interact. This mechanism to generate topological defects is called the Kibble mechanism (1973, after Tom Kibble). Symmetry-breaking phase transitions that happen in the early Universe may produce topological defects, such as domain-wall in the case of 1 field, cosmic string when 2 scalar fields involved, and monopole in the case of 3 fields.

2 EW phase transition, Higgs mechanism and stardard model Higgs boson

The Higgs bosons are excitations/oscillations of a scalar field named the Higgs field. They are in charge of giving mass to standard-model particles. This is done through the so-called Higgs mechanism (after Peter Higgs). It was introduced initially to explain the imbalance between massive Z^0, $W^\pm$ bosons that carry the weak interaction and massless photons that transmit the electromagnetic (EM) interaction. This happens during the so-called EW (electro-weak) phase transition of the EW Higgs field, which causes spontaneous symmetry breaking in the following way. At higher energies, a symmetry exists between the weak and EM gauge bosons in the sense that the interactions between the EW Higgs field at equilibrium (with average $\phi = 0$) and both kinds of (weak and EM) gauge bosons are identical and indistinguishable. After the phase transition of the Higgs field to the lower state (i.e., the vacuum), such a symmetry no longer exists. This is because on the vacuum circle the non-zero Higgs ϕ would interact differently with the weak and EM gauge bosons, giving them different masses. As a result, EM force travels at speed of light, but weak force only acts within the atomic nucleus distance. This is the Higgs mechanism that happens during the EW phase transition at an energy scale about 100GeV. Thus the EW Higgs (already discovered by CERN experiment) has a mass of $\sim$ 100GeV. Note that the EW phase transition that once happened in the early Universe may also generate topological defects, which have gravitational effects and may leave detectable imprints.

3 GUT Higgs, GUT phase transition and the Magnetic monopole

Grand unified theory (GUT) phase transition is said to happen at an energy scale of 10^{16} GeV. The involved Higgs mechanism is responsible for spontaneously breaking the supersymmetry between all three forces (strong, weak, EM) that exists at above such energy scale. The GUT Higgs bosons thus have mass around such an energy scale (much heavier than EW Higgs). The simplest GUT model involves 24 Higgs fields (each interact with a field corresponding to a standard model particle species). GUT was popular for many charming features (apart from unifying three forces at GUT energy scale) including its prediction on equal charge of electron and proton, a channel to break symmetry between matter and anti-matter to facilitate baryogenesis (note, there are also studies focusing on EW baryogenesis), producing magnetic monopoles. In this aspect, topological solitons in form of monopoles (involving three GUT Higgs fields) are produced via the Kibble mechanism by the end of GUT phase transition. The spatial configuration of the monopole (everywhere with gradually changing and non-zero field ϕ except for at the very central point where $V(\phi = 0)$ is the highest) causes local interactions with the EW fields to be different at different positions in real space, resulting in a magnetic field strength that corresponds to one smallest magnetic charge as predicted by Dirac 1931.

But GUT is not yet a fundamental theory. For example, QED can precisely predict electromagnetic fine-structure constant α, tested to high precision by experiments. However, GUT do not have such predictive power, as the parameters need to be put in by hand. GUT phase transition may have once happened in very early Universe at an energy level of $\sim 10^{16}$ GeV. It was also originally proposed to account for early Universe cosmic inflation by particle physicists such as Guth and Linde. Cosmic inflation can significantly dilute the abundance of predicted magnetic monopoles in our observed Universe (to be compatible with current null detection), meanwhile solve the flatness problem and the horizon problem. We shall note that in term of accounting for an inflationary universe that is consistent with modern CMB observations, GUT phase transition however has to be replaced by a phase transition associated with a new kind of scalar fields – “inflaton”.