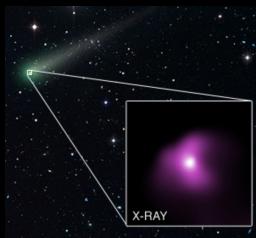


Radiative Processes in Astrophysics

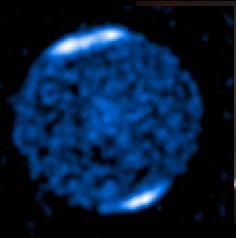
Observation

Up to cosmic size scale

C/2012S1
(comet)



Jupiter
(planet)



Sun
(star)



Cas A
(SNR)



M82
(galaxy)



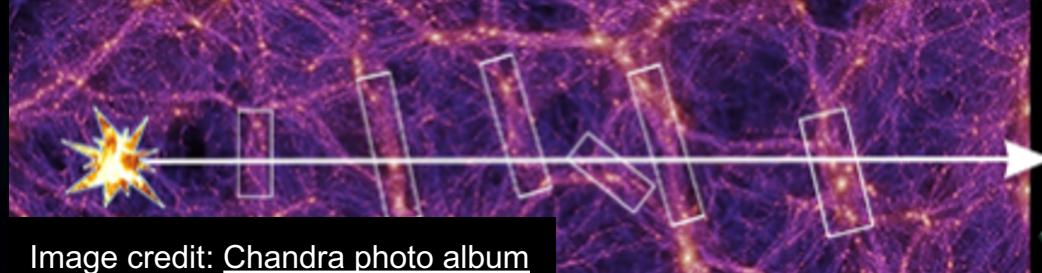
Phoenix
(gal. cluster)



Cosmic web filament

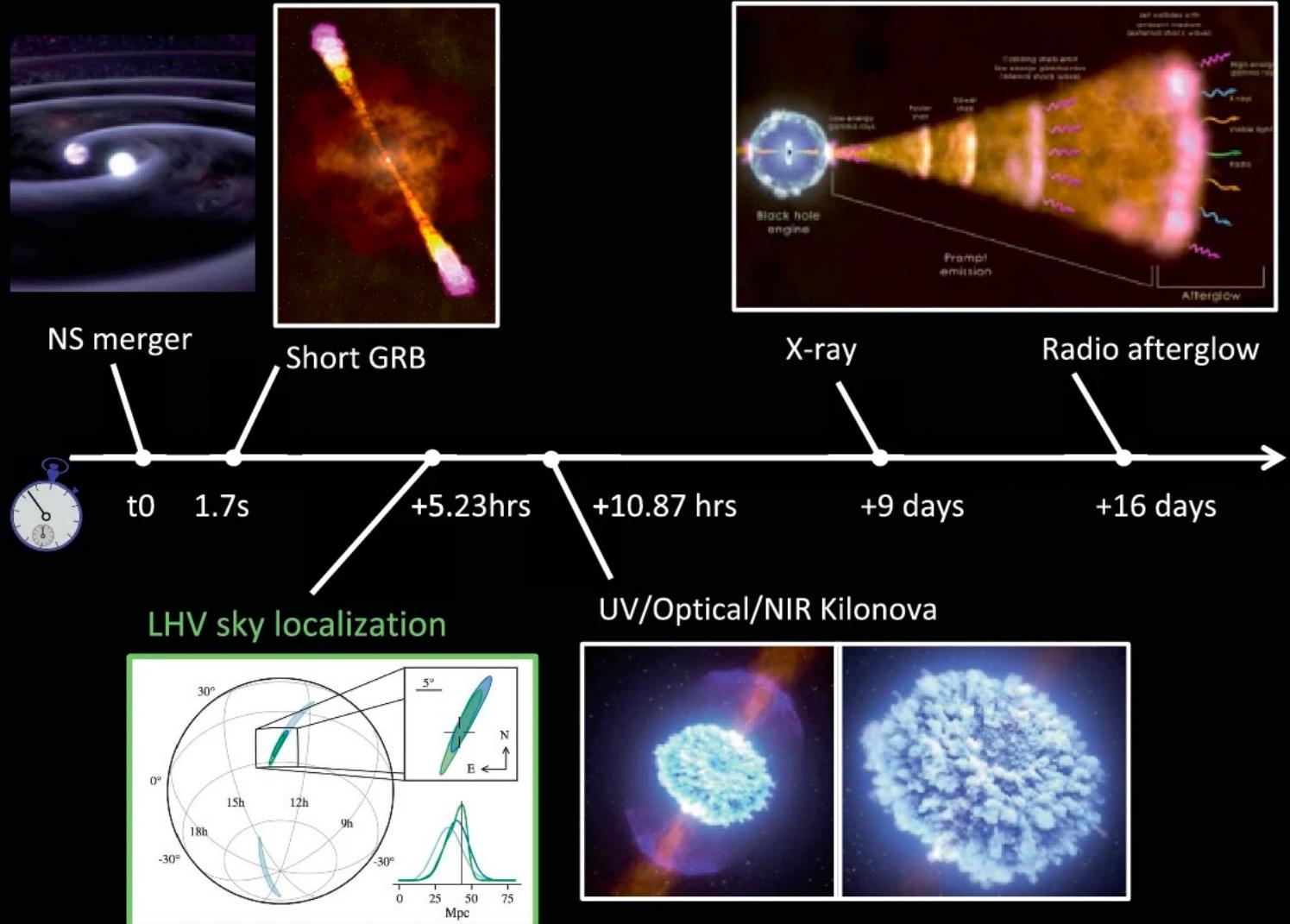
Theory

Down to atomic size scale



Chpt.1 Astrophysical messengers

- 1.1 Electromagnetic waves
- 1.2 Neutrinos
- 1.3 Cosmic rays
- 1.4 Gravitational waves

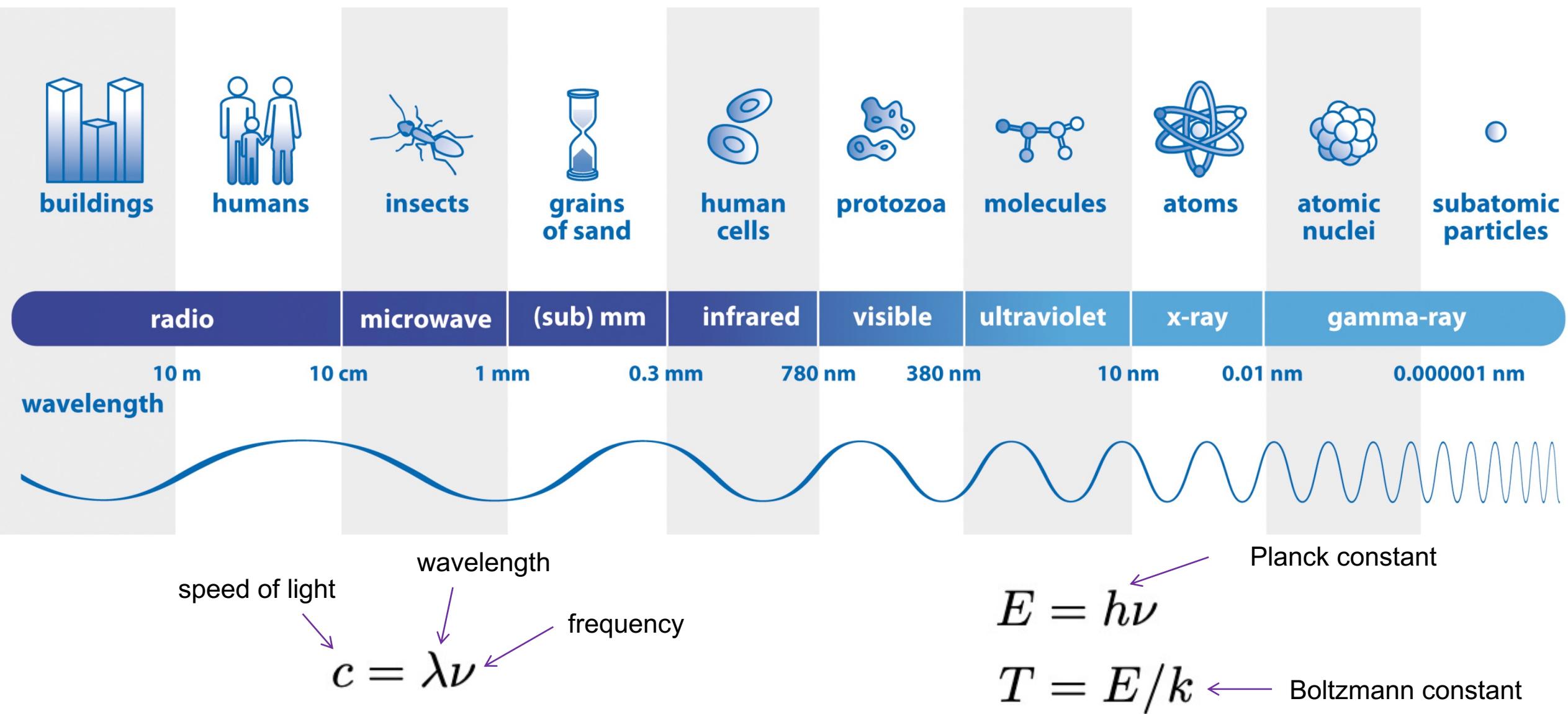


GW170817 (GW)
+GRB170817A (γ -ray)
+AT2017gfo (NIR to UV)
+ GRB afterglow (X-ray + radio)

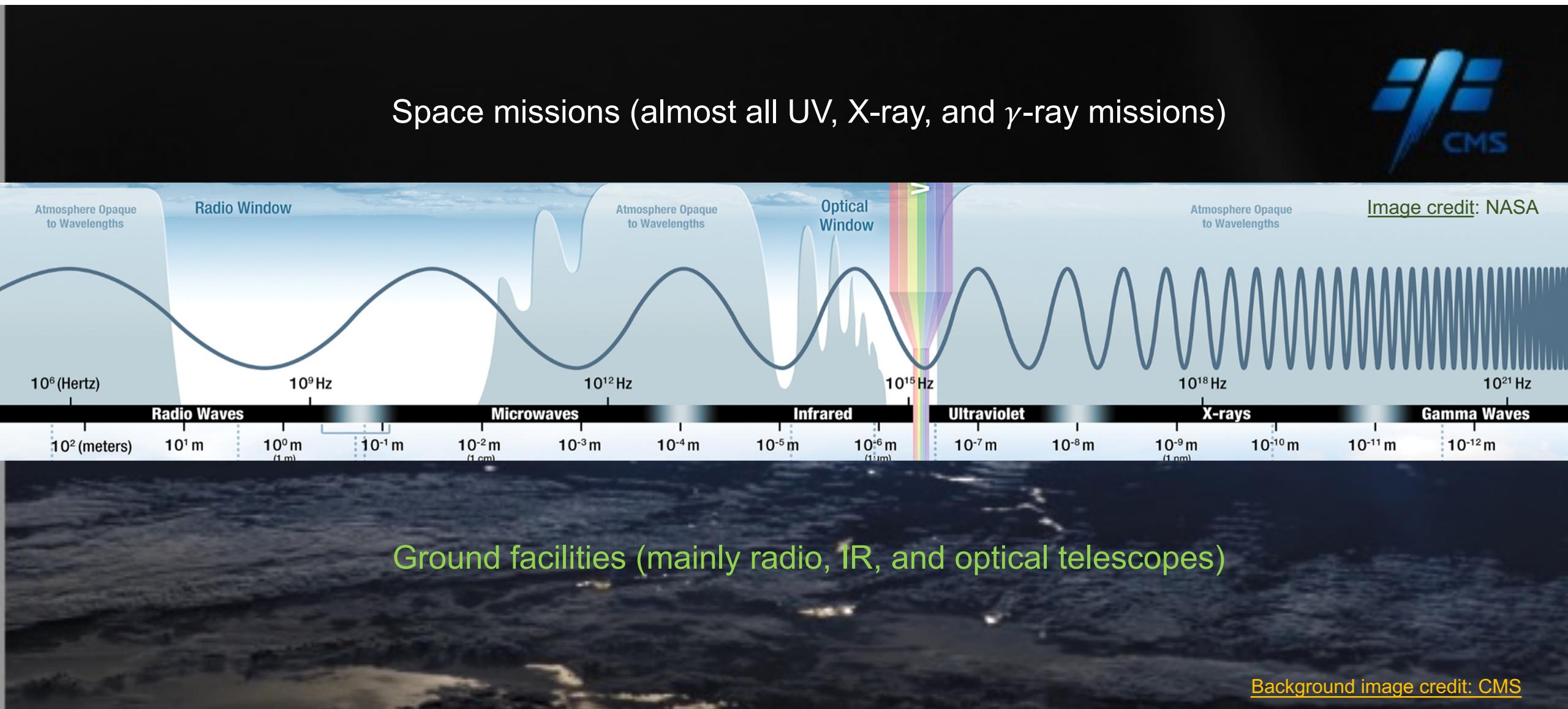
Image credit: Branchesi (2023)

Electromagnetic waves

Image credit: ESA



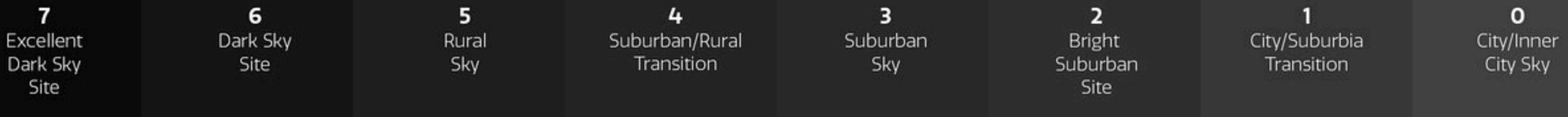
Atmospheric window



Not just atmospheric window

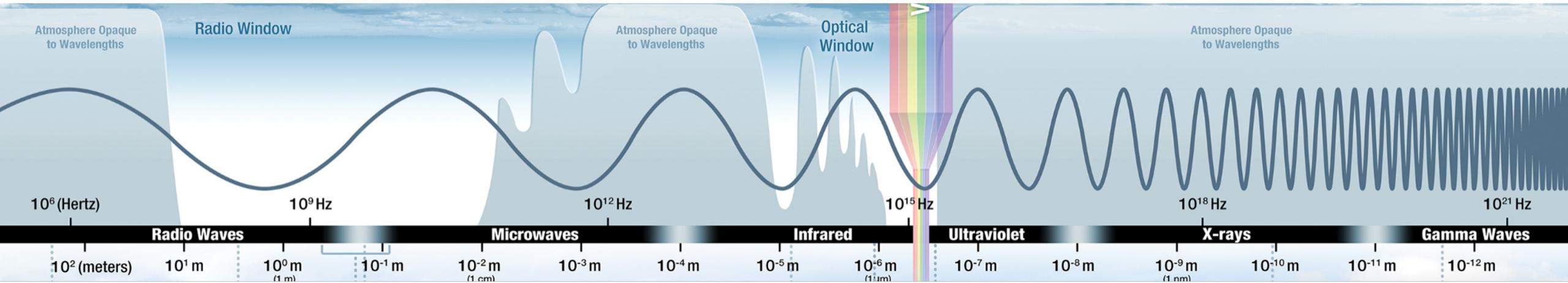


Background image credit: NOIRLab/NSF/AURA, P. Marenfeld



EM observing facilities

[Image credit: NASA](#)



LOFAR (radio)
[Image credit:](#)
ASTRON



ALMA (microwave)
[Image credit:](#) ESO

Planck (microwave)
[Image credit:](#) ESA



JWST (IR to optical)
[Image credit:](#)
NASA



HST (IR to UV)
[Image credit:](#)
HST.org



Gemini (IR to optical)
[Image credit:](#)
NOIRLab



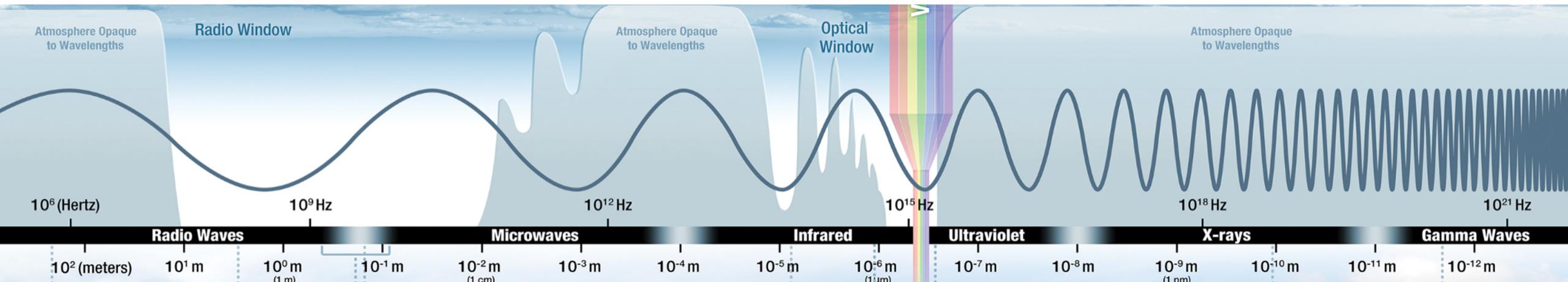
XRISM(X-ray)
[Image credit:](#)
JAXA

Fermi (γ -ray)
[Image credit:](#)
NASA



EM observing facilities (cont.)

Image credit: NASA



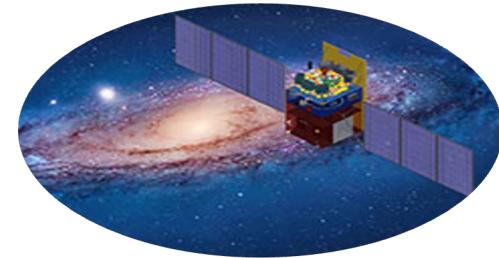
FAST (radio)
Image credit: Nature



LAMOST (optical)
Image credit: NAOC



ASO-S (UV to X-ray)
Image credit: PMO

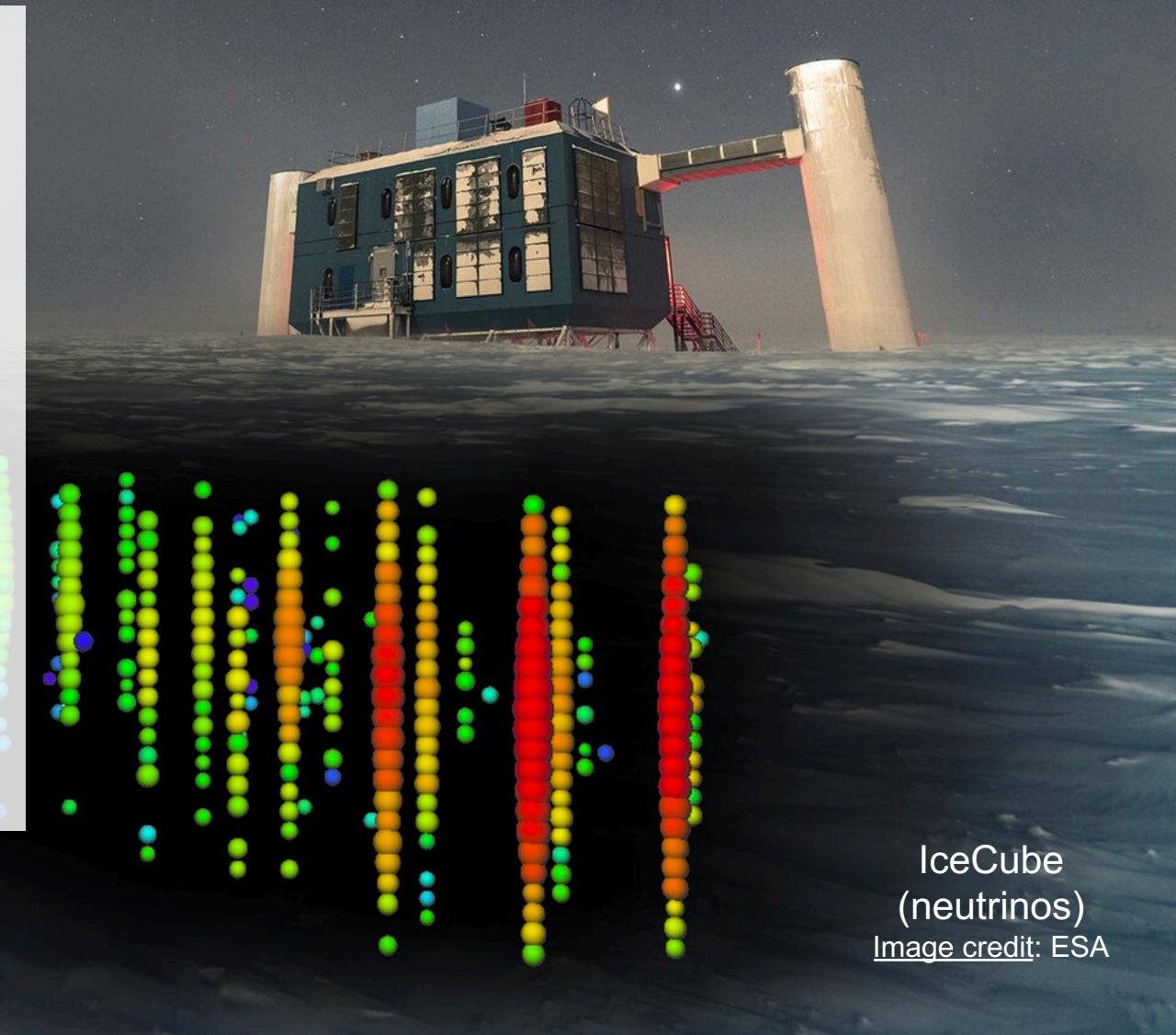


insight-HMXT
(X-ray to γ -ray)
Image credit: IHEP

Neutrinos

- Symbol is lowercase Greek letter: ν
 - Three flavors: electron ν_e , muon ν_μ , tau ν_τ
 - Oscillates between flavors
- Produced by nuclear processes
- Basic properties
 - Electrically neutral
 - Nearly massless
 - Travels close to the speed of light
- Interaction
 - Reacts to gravity and the weak nuclear force
 - Mostly passes through matter
 - Billions of neutrinos pass through our body every second

The neutrino was postulated by Wolfgang Pauli in the early 1930s but could only be detected for the first time in the 1950s



IceCube
(neutrinos)
Image credit: ESA

Neutrinos (IceCube)

High energy (~ 290 TeV) neutrino from IceCube-170922A was detected in association with a known gamma-ray blazar TXS 0506+056 ($z \sim 0.3365$). This is the third detections of individual astrophysical sources of neutrinos, after the Sun and SN1987A.

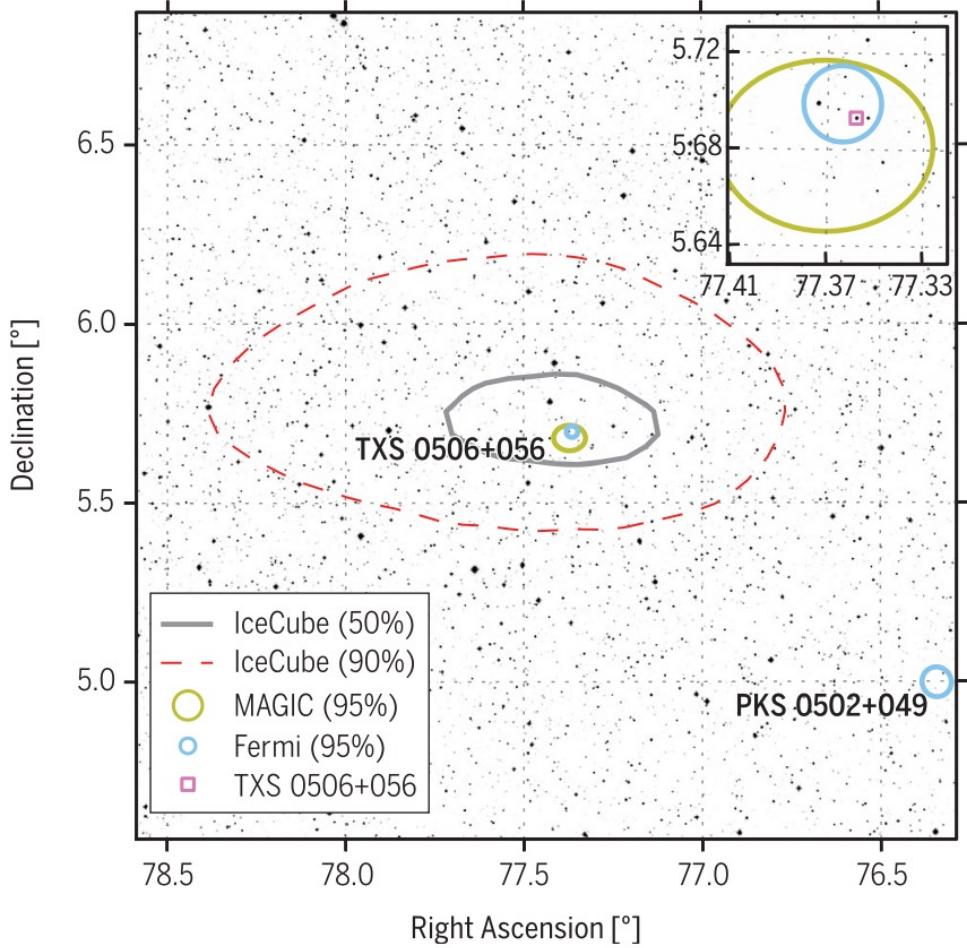
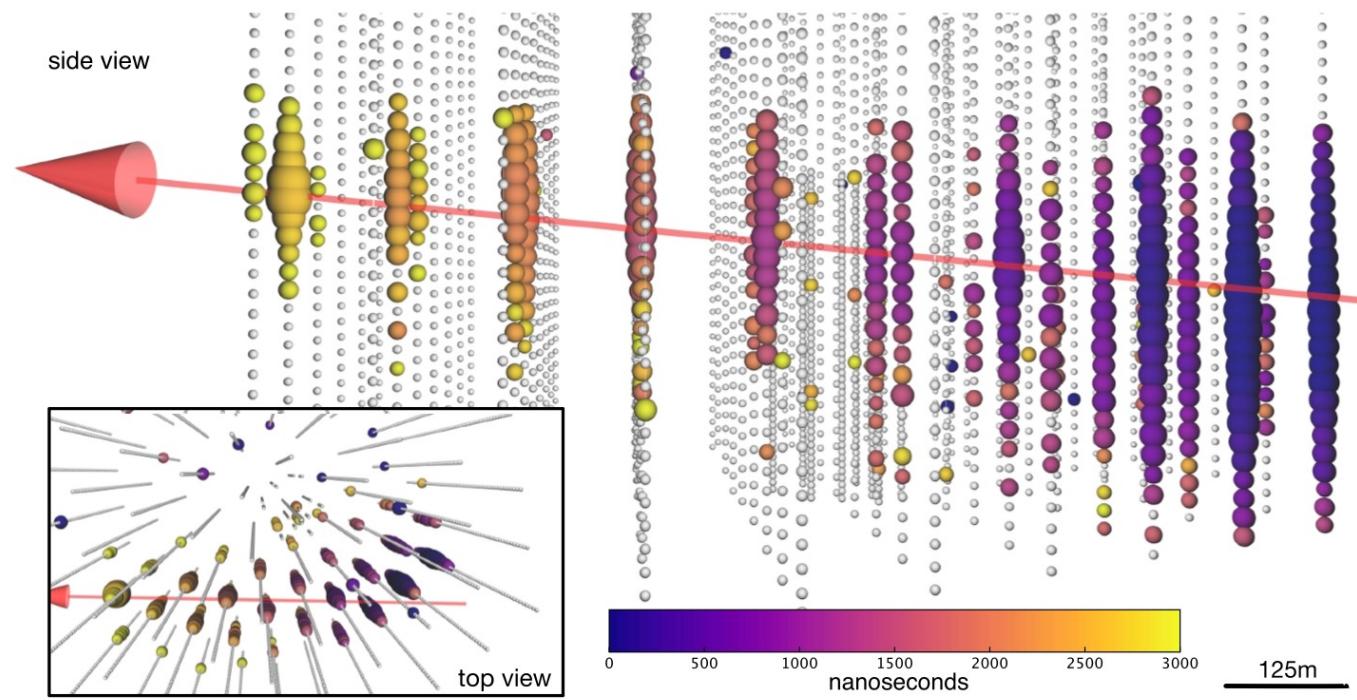


Image credit: IceCube collaboration (2023)

~ 5800 photoelectrons in ~ 3000 ns (from left to right)



Cosmic rays

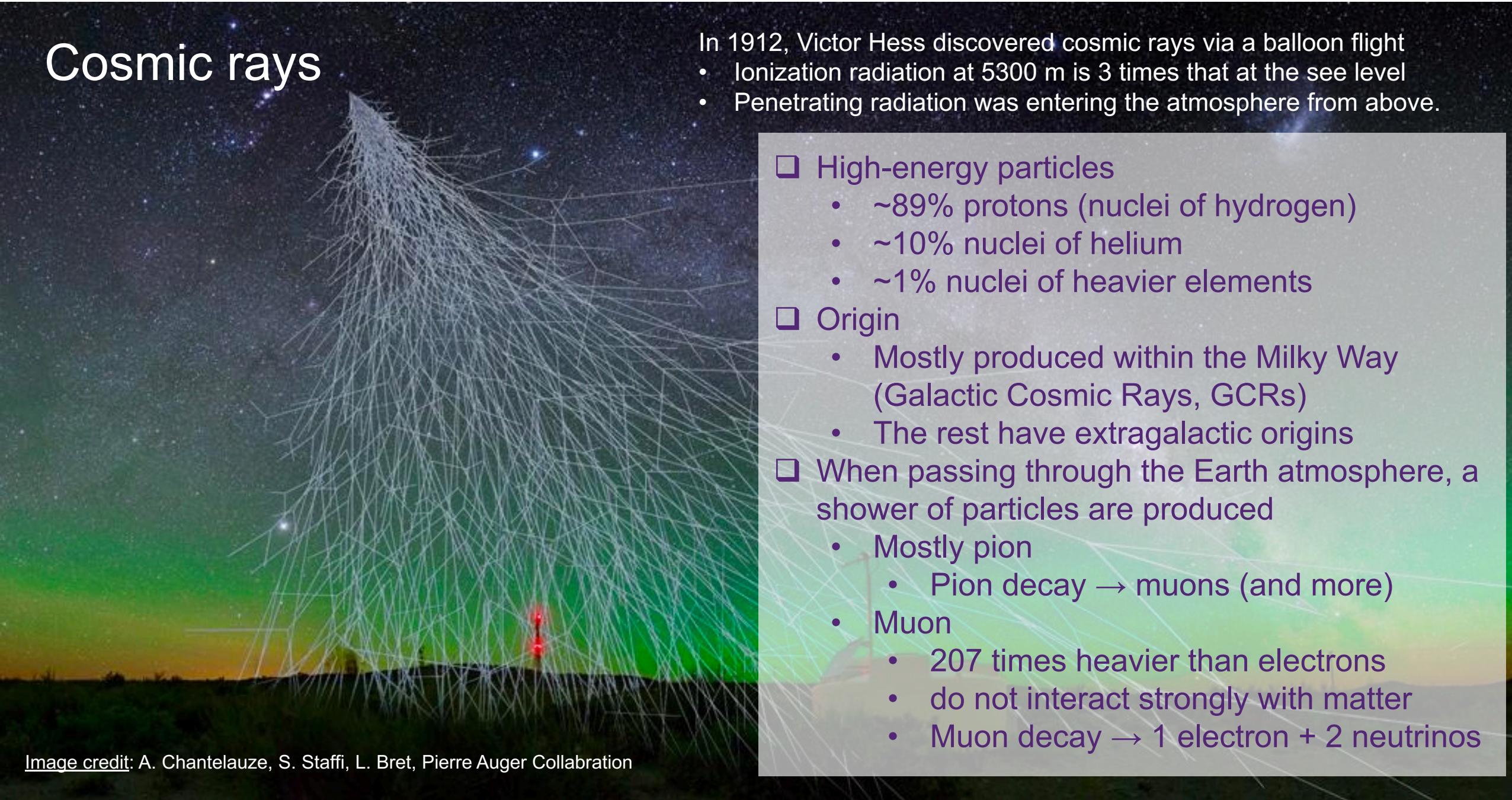
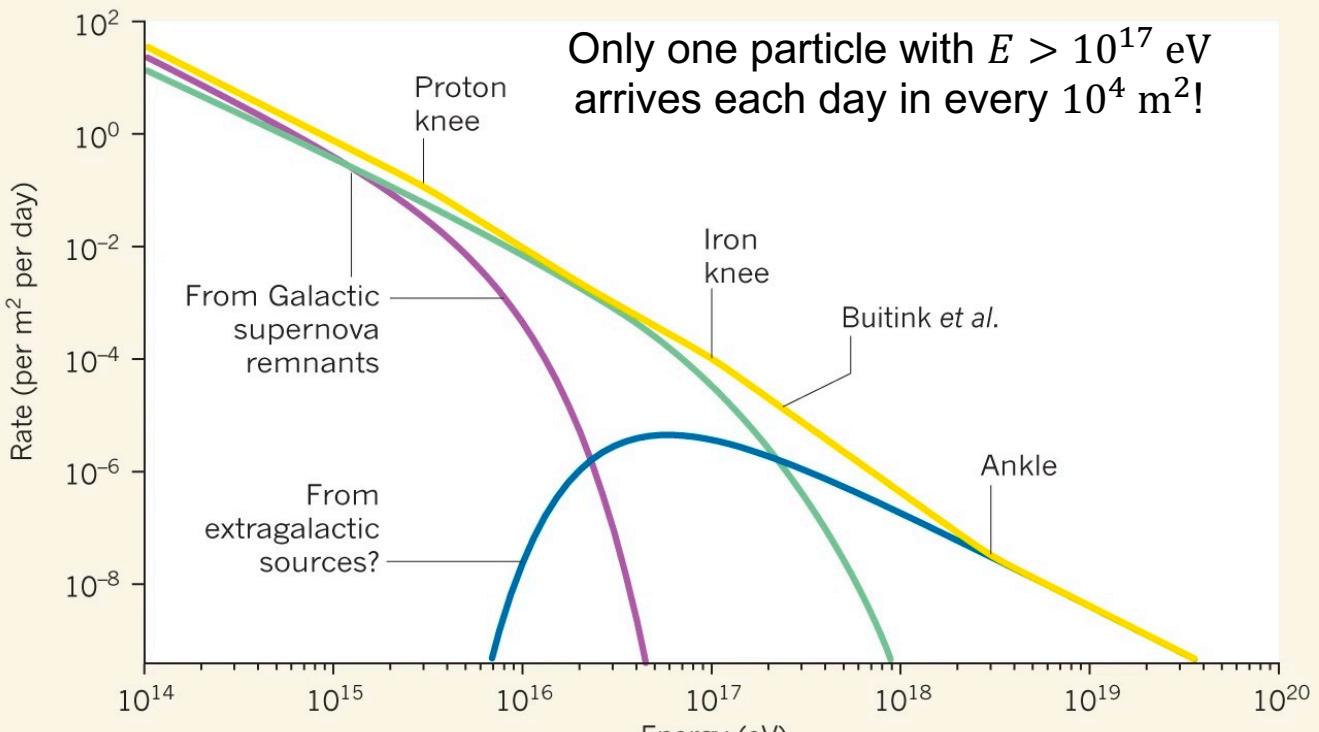


Image credit: A. Chantelauze, S. Staffi, L. Bret, Pierre Auger Collaboration

Cosmic rays (LHAASO)



Clues about the origin of cosmic rays come from both their composition and their energy spectra.

Large High Altitude Air Shower Observatory (LHAASO) (cosmic rays)
Image credit: Science

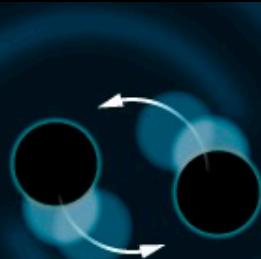


Tera (trillion): $\text{TeV} = 10^{12} \text{ eV}$
Peta (quadrillion): $\text{PeV} = 10^{15} \text{ eV}$
Exa (quintillion): $\text{EeV} = 10^{18} \text{ eV}$
Zetta (sextillion): $\text{ZeV} = 10^{21} \text{ eV}$

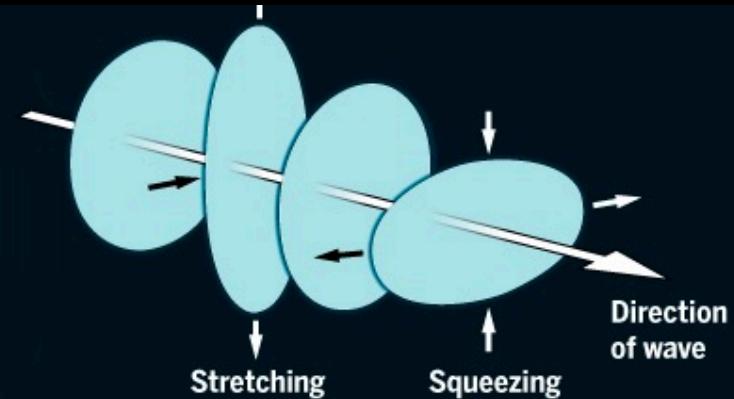
Gravitational wave

Image credit: V. Altounian/Science

As Einstein calculated, a whirling barbell-shaped mass, such as two black holes spiraling together, radiates ripples in space-time: gravitational waves.



Zipping along at light speed, a wave stretches space in one direction and squeezes in the perpendicular direction, then reverses the distortions.



THE ASTRO-PH READER'S DIGEST | SUPPORTED BY THE AAS

[link: Astrobites](#)

Guide to Gravitational Waves

by Astrobites | Nov 8, 2023 | Daily Paper Summaries, Guides | 0 comments

- Gravitational waves
 - “Ripples” in space-time caused
- Origin
 - Mostly produced by mergers of massive objects

Gravitational waves (LIGO)

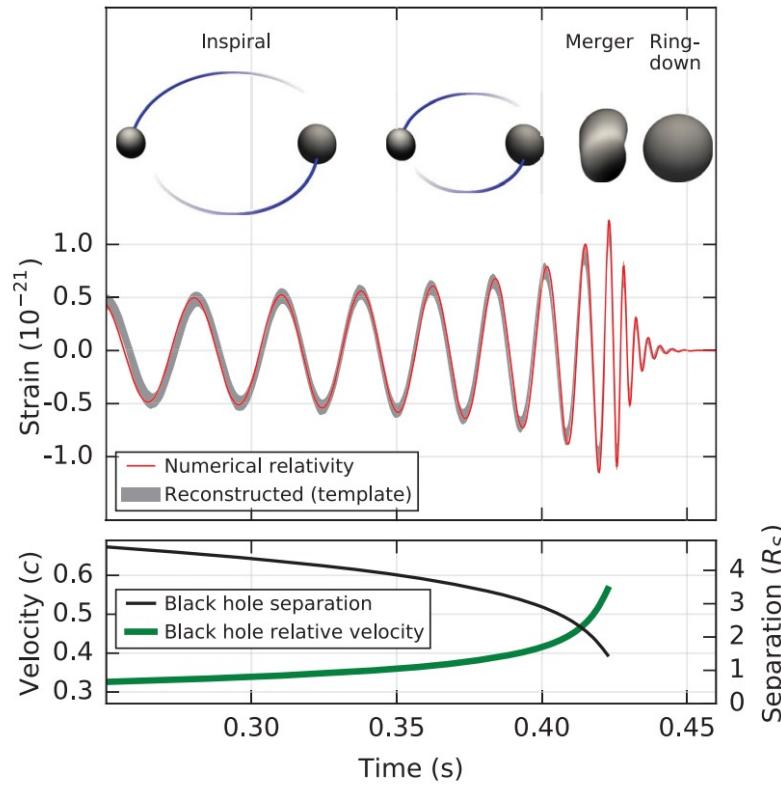


Image credit: Abbott et al. 2016

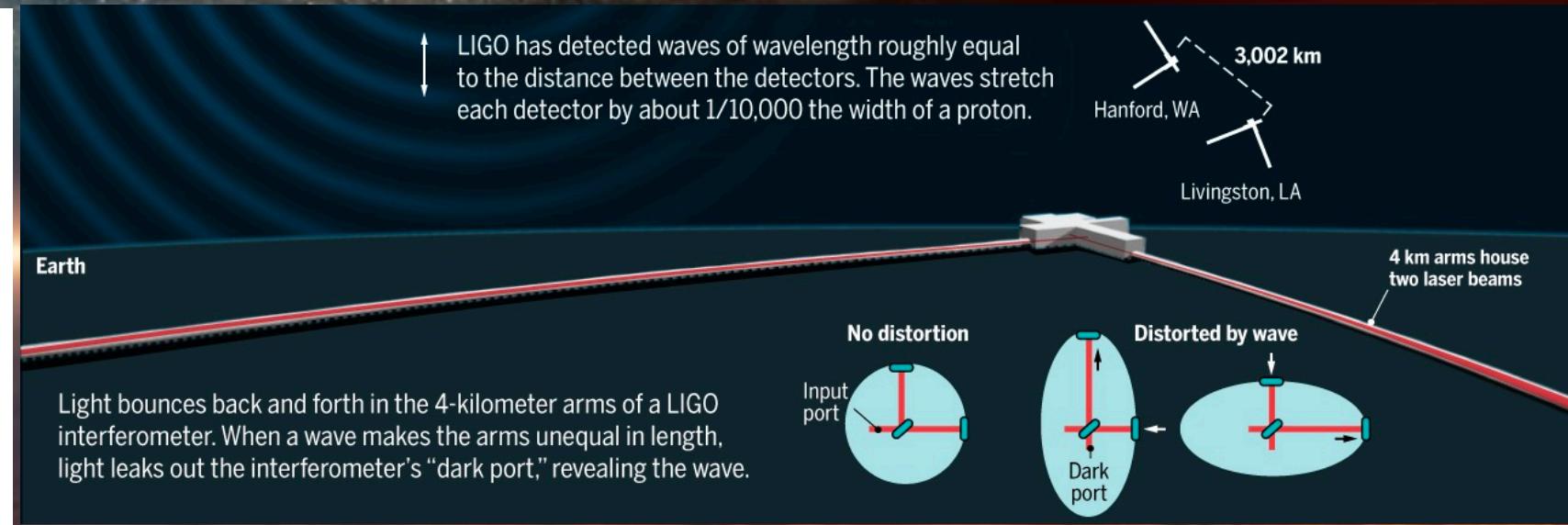


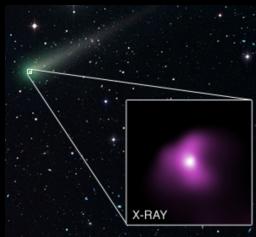
Image credit: V. Altounian/Science

Radiative Processes in Astrophysics

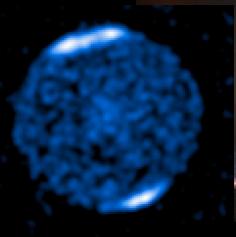
Observation

Up to cosmic size scale

C/2012S1
(comet)



Jupiter
(planet)



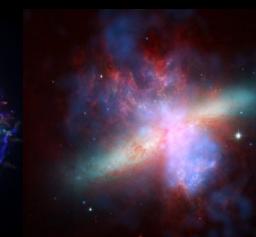
Sun
(star)



Cas A
(SNR)



M82
(galaxy)



Goal

Theory

Down to atomic size scale

Phoenix
(gal. cluster)



Cosmic web filament

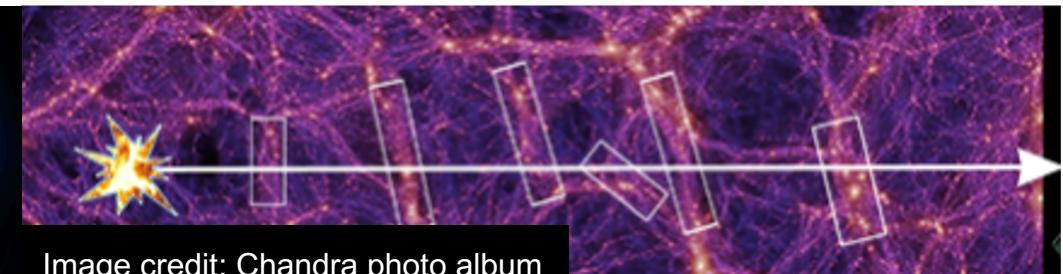


Image credit: [Chandra photo album](#)

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.1.1 Radiative flux

2.1.2 Radiative intensity

2.1.3 Specific intensity

2.1.4 Specific flux and momentum

2.1.5 Specific energy density

2.2 Radiative transfer

2.3 Thermal radiation

2.4 Scattering

2.5 Escape probability

2.6 Polarization

2.7 Dispersion and rotation measures

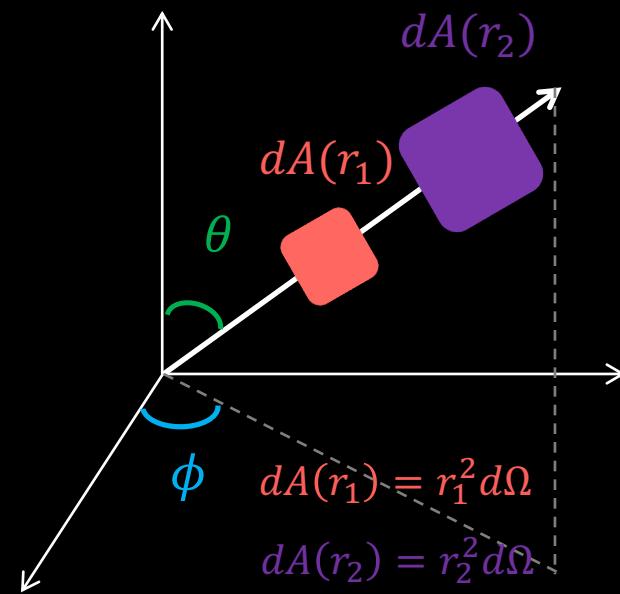


Image credit: Junjie Mao

Radiative flux

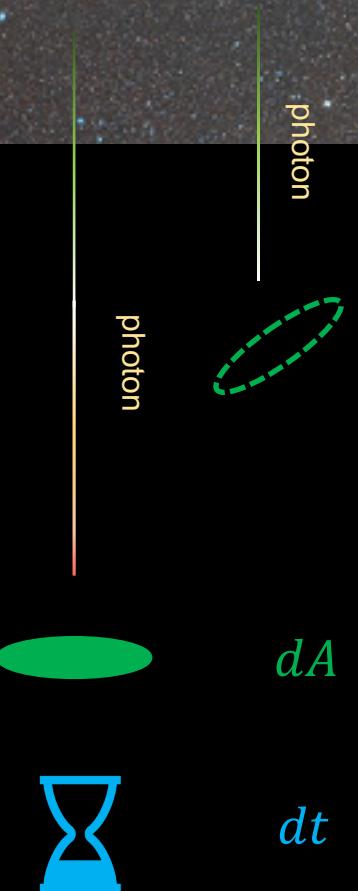
For a detector element (with a photon collecting area dA) exposed to a radiation source for a time interval dt , the number of photons or the amount of energy received is

energy or photon flux

$\text{erg cm}^{-2} \text{ s}^{-1}$
 $\text{ph cm}^{-2} \text{ s}^{-1}$

$$dF \equiv \frac{dE}{dA \ dt}$$

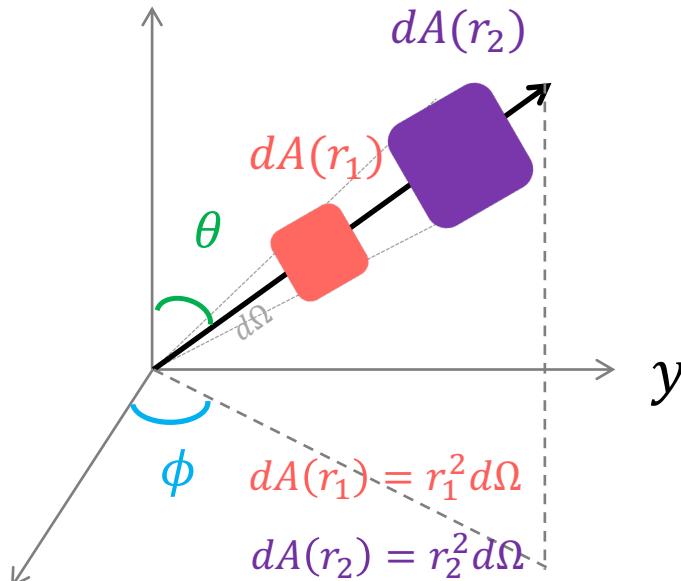
↗
 photon collecting
area (cm^2) ↗
 integration time (s)



F can vary for different orientations of the detector element

Inverse square law

Consider an isotropic source emitting photons/energy equally in all directions (e.g., a spherically symmetric and isolated star)



Conservation of energy (no loss or gain)

$$F(r) = \frac{\text{constant}}{r^2}$$

energy flux ($\text{erg cm}^{-2} \text{ s}^{-1}$)

luminosity (erg s^{-1})

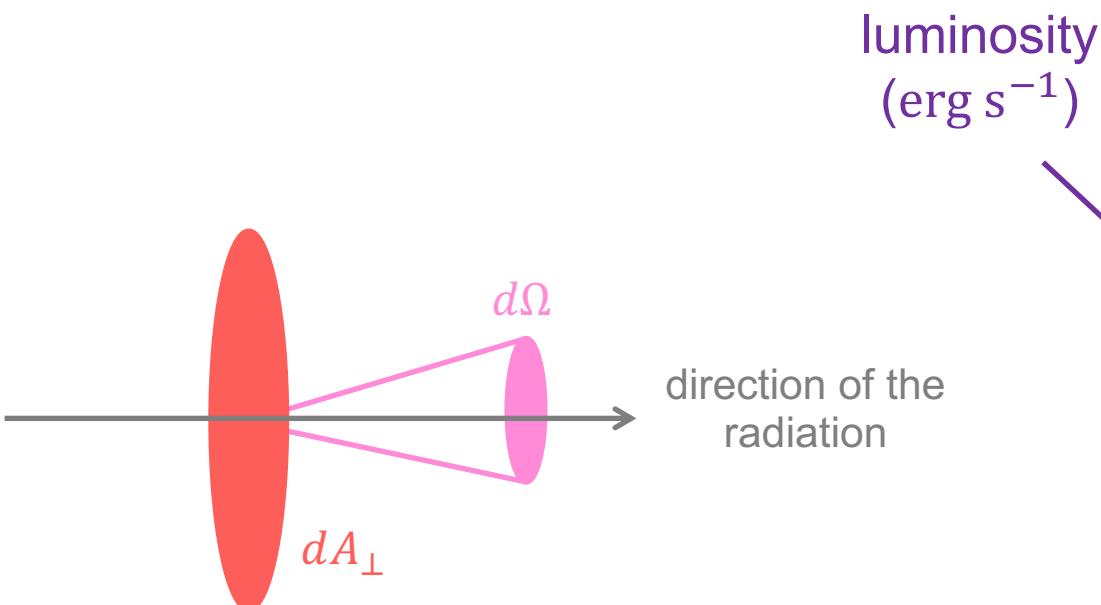
$F(r) \equiv \frac{L}{4\pi r^2}$

distance to the source (cm)

$$dF(r) = \frac{dE}{dA(r)dt} = \frac{dE}{r^2 d\Omega dt}$$
$$d\Omega = \sin \theta d\theta d\phi$$

Radiative intensity

Since flux changes with distance to the source, it is useful to a concept that is **intrinsic** to the properties of the source, e.g., luminosity (L), intensity (I)



luminosity
(erg s^{-1})

$$L \equiv \frac{dE}{dt}$$

d Ω

direction of the radiation

dA_{\perp}

$I \equiv \frac{dE}{dt \ dA_{\perp} \ d\Omega}$

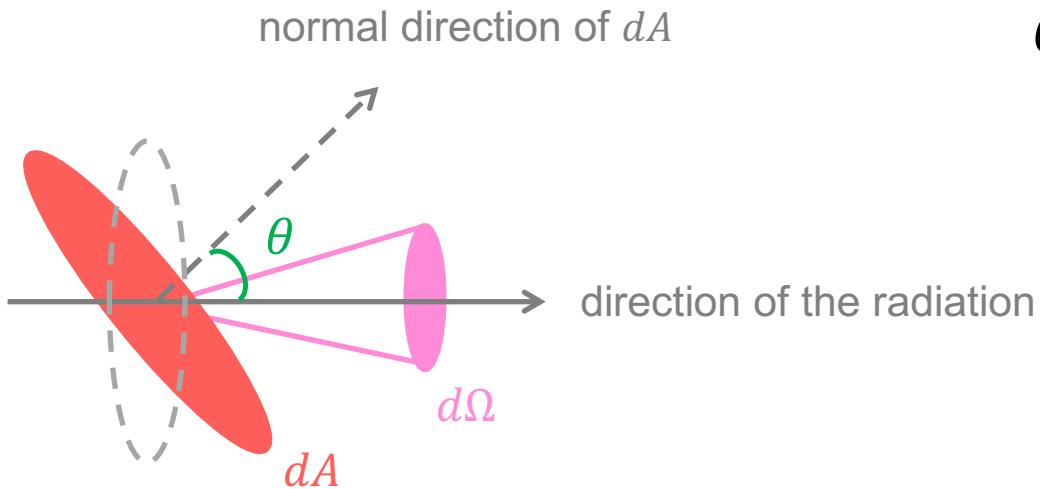
By definition, dA_{\perp} is normal to the direction of the radiation

Solar luminosity

$$L_{\odot} = 3.8 \times 10^{33} \text{ erg s}^{-1}$$

Intensity and flux

Consider a radiation field (radiation in all directions), for a detector element (with photon collecting area dA) at some arbitrary orientation, we have



$$dF = I \cos \theta \, d\Omega$$

effective area: $dA_{\perp} = dA \cos \theta$

prev. sl.

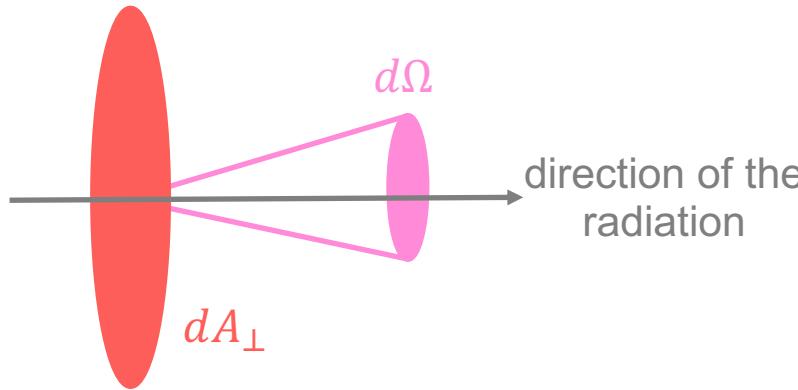
$$I = \frac{dE}{dt \, dA_{\perp} \, d\Omega}$$

prev. sl.

$$dF = \frac{dE}{dt \, dA}$$

Specific intensity

Radiative energy in a **specific frequency** crossing unit area (dA_{\perp}) in unit time in unit solid angle

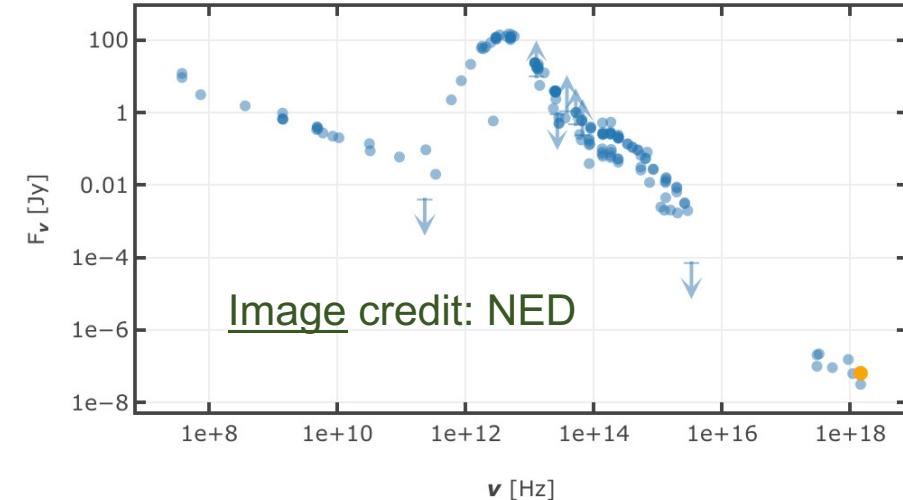


specific intensity



$$dE_{\nu} = I_{\nu} dA d\nu d\Omega dt$$

$$I_{\nu} \equiv \frac{dE_{\nu}}{dA_{\perp} d\nu d\Omega dt}$$



$\text{erg Hz}^{-1} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$

prev. sl.

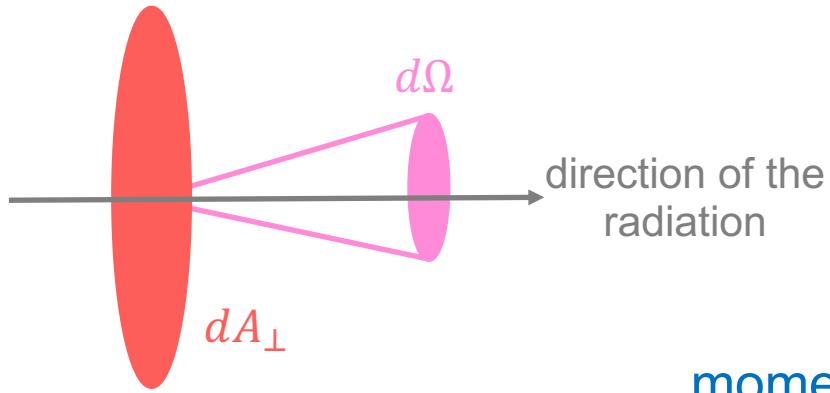
$$I = \frac{dE}{dt dA_{\perp} d\Omega}$$

$$I_{\nu} = \frac{\lambda}{\nu} I_{\lambda}$$

Specific flux and momentum

prev. sl.

$$F = \pi I$$



$$F_\nu = \pi I_\nu$$

erg Hz⁻¹cm⁻²s⁻¹

The diagram illustrates the components of the flux density formula. A green arrow points from the label "specific flux" to the term F_ν . A purple arrow points from the label "specific intensity" to the term I_ν .

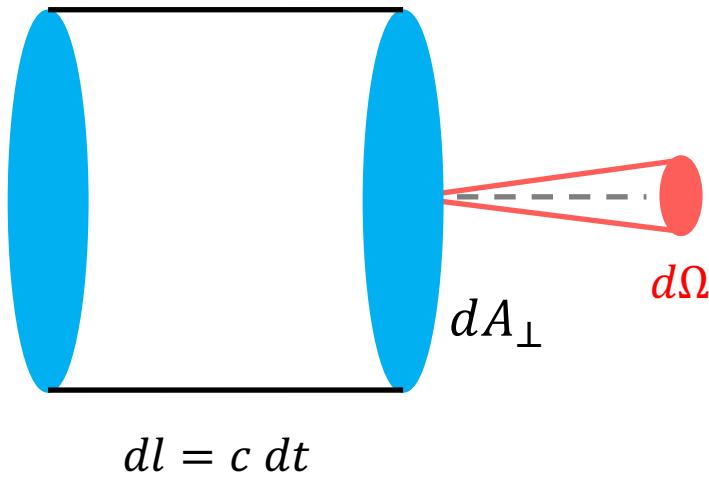
specific flux **specific intensity**

$$\text{momentum} \rightarrow p_\nu = \frac{F_\nu}{c} \quad \text{dyn Hz}^{-1}\text{cm}^{-2}$$

1 dyn = 1 g cm s⁻²

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c dt$ and its volume $dA_{\perp} c dt$



$$dE_{\nu} = u_{\nu}(\Omega) d\Omega d\nu dA_{\perp} c dt$$

$$u_{\nu}(\Omega) = \frac{I_{\nu}}{c}$$

$$u_{\nu} = \frac{1}{c} \int I_{\nu} d\Omega = \begin{cases} \frac{4\pi}{c} I_{\nu} & \text{isotropic} \\ \frac{4\pi}{c} \bar{I}_{\nu} & \text{anisotropic} \end{cases} \quad \text{erg Hz}^{-1} \text{ cm}^{-3}$$

specific energy density

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer

2.2.1 Spontaneous emission coefficient

2.2.2 Absorption coefficient

2.2.3 Optical depth and mean free path

2.2.4 Radiative transfer equation

2.3 Thermal radiation

2.4 Scattering

2.5 Escape probability

2.6 Polarization

2.7 Dispersion and rotation measures

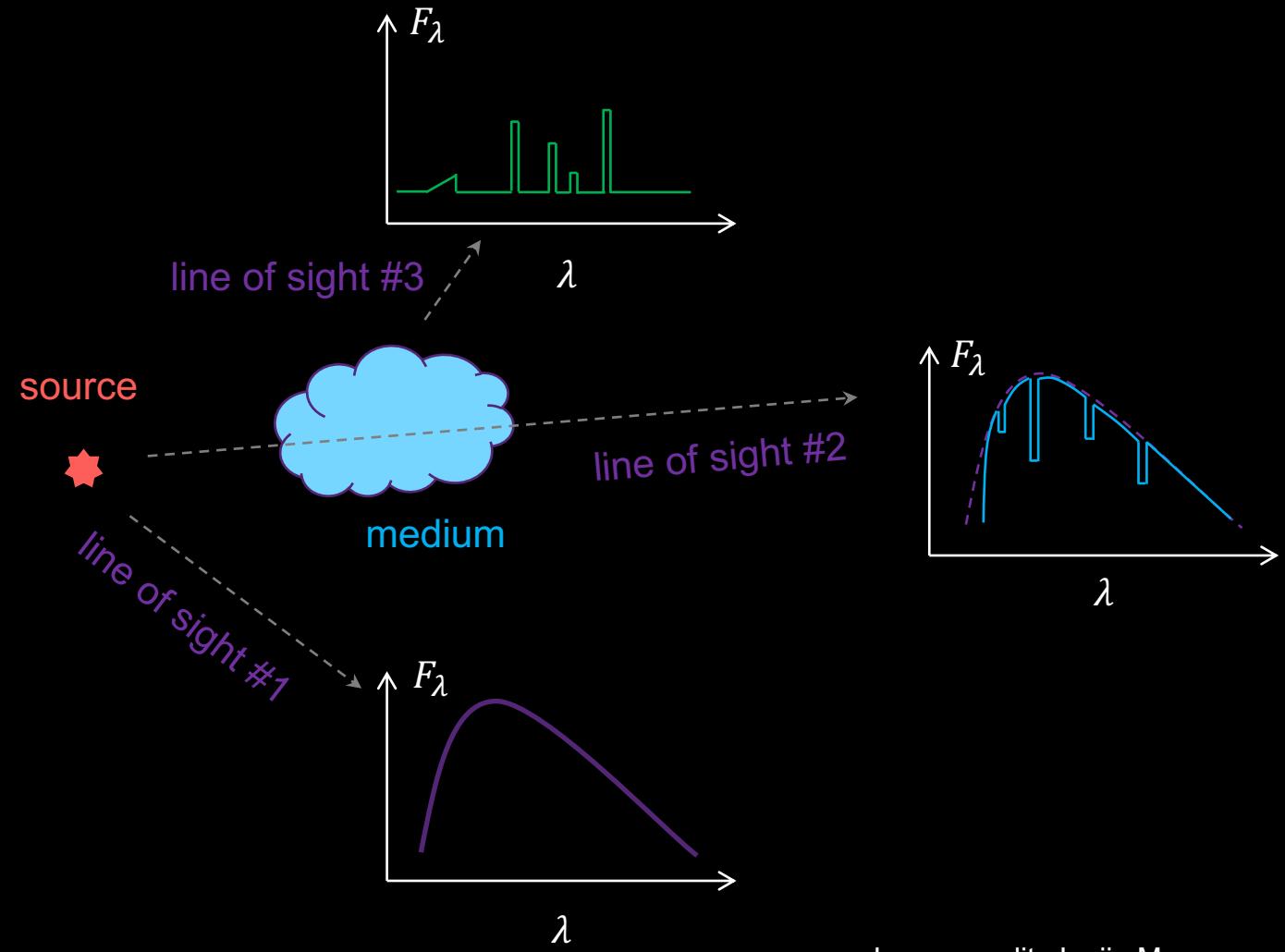


Image credit: Junjie Mao

Radiative transfer

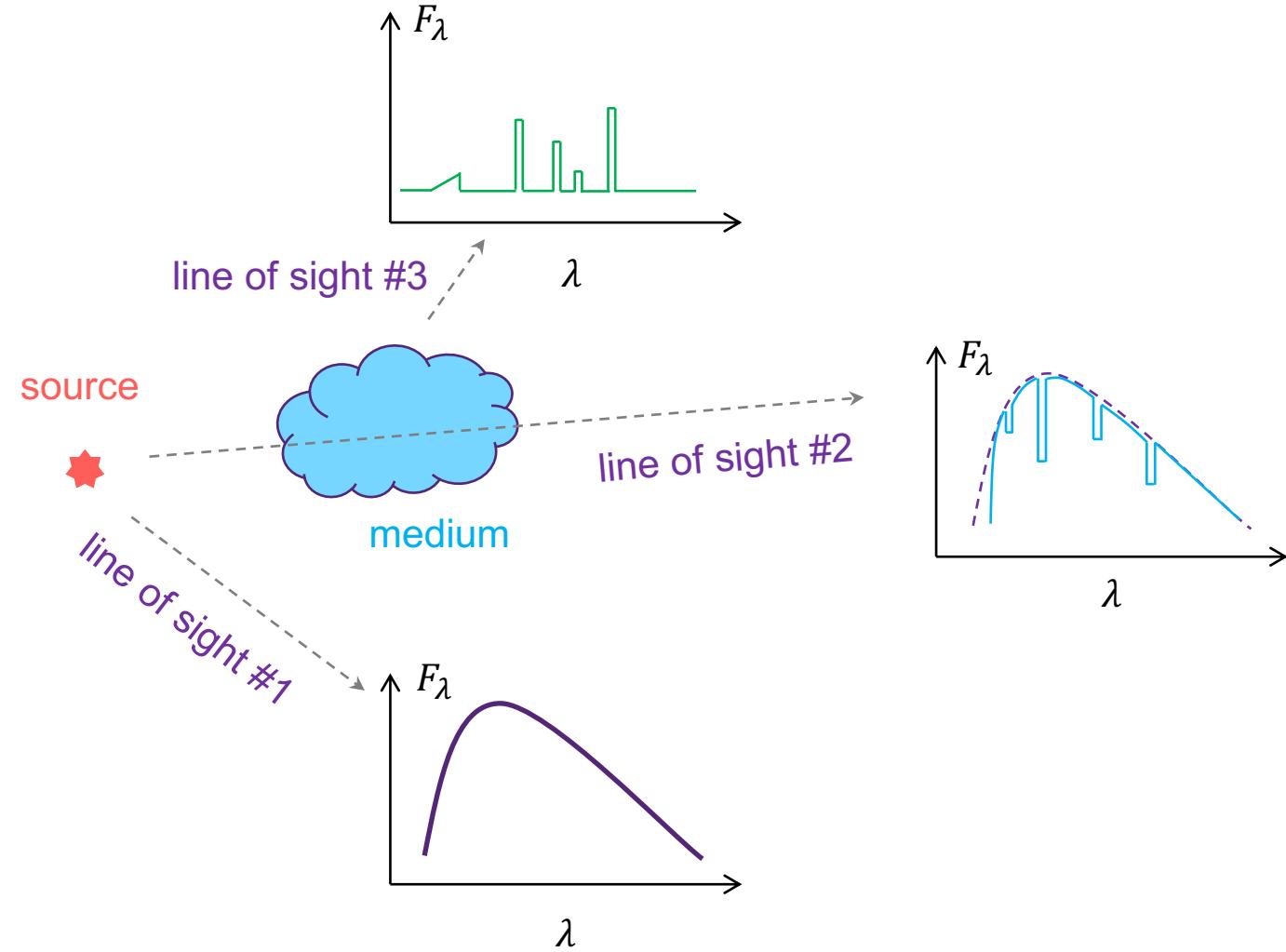
- Radiation is one of the key processes to exchange energy for most celestial bodies
- When a ray passes through matter, the energy exchange process is referred to as radiation transfer

unabsorbed flux

prev. sl.

$$F(r) \equiv \frac{L}{4\pi r^2}$$

observed flux $\longrightarrow F_{\text{obs}} \leq \frac{L}{4\pi d^2}$



Spontaneous emission coefficient

Radiative energy emitted spontaneously in a specific frequency in unit time in unit solid angle in unit volume

$$dE_\nu = j_\nu \downarrow \text{spontaneous emission coefficient} dV d\nu d\Omega dt$$

$$j_\nu \equiv \frac{dE_\nu}{dV d\nu d\Omega dt} \quad \text{erg Hz}^{-1} \text{cm}^{-3} \text{s}^{-1} \text{sr}^{-1}$$

Energy (frequency)⁻¹(volume)⁻¹(time)⁻¹(solid angle)⁻¹

Spontaneous emission coefficient (cont.)

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$$dV = dA_\perp \, dl$$

prev. sl.

$$I_\nu = \frac{dE_\nu}{dA_\perp \, d\nu \, d\Omega \, dt}$$

$$I_\nu = j_\nu \, dl$$

Emissivity

For an isotropic emitter or a distribution of randomly oriented emitters, the emission can also be characterized via emissivity ϵ_ν , which is the radiative energy (angle integrated) emitted spontaneously in a specific frequency per unit time per unit mass

$$dE_\nu = \epsilon_\nu \rho \, dV dt \, d\nu \frac{d\Omega}{4\pi} \quad \begin{matrix} \text{normalized fraction of energy} \\ \text{radiated into the solid angle } d\Omega \end{matrix}$$

$$\epsilon_\nu \equiv \frac{dE_\nu}{\rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}} \quad \text{erg Hz}^{-1} \text{g}^{-1} \text{s}^{-1}$$

↑
mass density g cm⁻³

Spontaneous emission coefficient

For an isotropic emitter or a distribution of randomly oriented emitters,

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$$\epsilon_\nu = \frac{dE_\nu}{\rho \, dVdt \, d\nu \, \frac{d\Omega}{4\pi}}$$

$$j_\nu = \frac{\epsilon_\nu \, \rho}{4\pi}$$

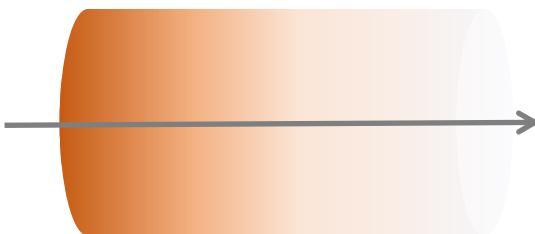
Absorption coefficient

Loss of intensity in a beam as radiation travels a distance dl

absorption coefficient (positive values for
energy taken out of the beam)

dl

$$dI_\nu \equiv -\alpha_\nu I_\nu dl$$



direction of the radiation

$$\alpha_\nu = \rho \kappa_\nu$$

mass density g cm^{-3}

mass absorption coefficient
(a. k. a. opacity coefficient)

$$\int_{I_\nu^{\text{in}}}^{I_\nu^{\text{out}}} \frac{dI_\nu}{I_\nu} = - \int \alpha_\nu dl$$

$$\ln \frac{I_\nu^{\text{out}}}{I_\nu^{\text{in}}} = - \int \alpha_\nu dl$$

$$\frac{I_\nu^{\text{out}}}{I_\nu^{\text{in}}} = \exp \left(- \int \alpha_\nu dl \right)$$

Optical depth

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu)$$

optical depth 

$$\tau_\nu \equiv \int \alpha_\nu dl$$

absorption coefficient 

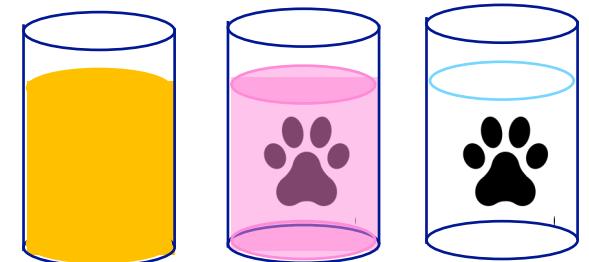
Optical thin

$$\tau_\nu \ll 1, I_\nu^{\text{out}} \simeq I_\nu^{\text{in}}$$

$$\tau_\nu = 1, I_\nu^{\text{out}} \sim 0.368 I_\nu^{\text{in}}$$

Optical thick

$$\tau_\nu \gg 1, I_\nu^{\text{out}} \sim 0$$



Mean free path

prev. sl.

$$I_{\nu}^{\text{out}} = I_{\nu}^{\text{in}} \exp(-\tau_{\nu})$$

For homogeneous medium, $\bar{\tau}_{\nu} = \alpha_{\nu} \bar{l}_{\nu}$

$$\bar{\tau}_{\nu} = \frac{\int_0^{\infty} \tau_{\nu} \exp(-\tau_{\nu}) d\tau_{\nu}}{\int_0^{\infty} \exp(-\tau_{\nu}) d\tau_{\nu}} = 1$$

$$\int_0^{\infty} x \exp(-x) dx = 1$$

$$\int_0^{\infty} \exp(-x) dx = 1$$

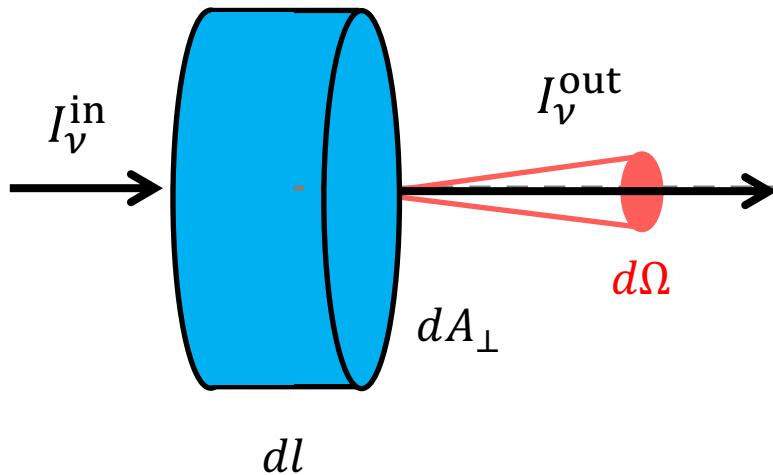
Mean free path (the average distance a photon can travel within a medium without being absorbed)

$$\bar{l}_{\nu} = \frac{1}{\alpha_{\nu}}$$

Radiative transfer equation (cont.)

Conservation of energy (no loss or gain)

$$dE^{\text{out}} = dE^{\text{in}} + dE^{\text{abs}} + dE^{\text{em}}$$



prev. sl.

$$dE^{\text{in}} = I_{\nu} dv d\Omega dA_{\perp} dt$$

$$dE^{\text{out}} = (I_{\nu} + dI_{\nu}) dv d\Omega dA_{\perp} dt$$

$$dE^{\text{em}} = j_{\nu} dv d\Omega dA_{\perp} dl dt$$

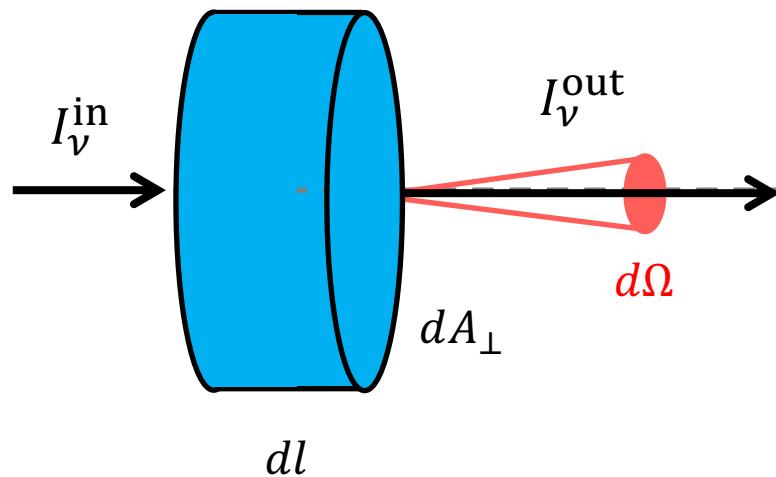
$$dE^{\text{abs}} = -\alpha_{\nu} dE^{\text{in}} dl$$

$$\frac{dI_{\nu}}{dl} = -\alpha_{\nu} I_{\nu} + j_{\nu}$$

Radiative transfer equation (cont.)

prev. sl.

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$



$$d\tau_\nu = \alpha_\nu dl$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

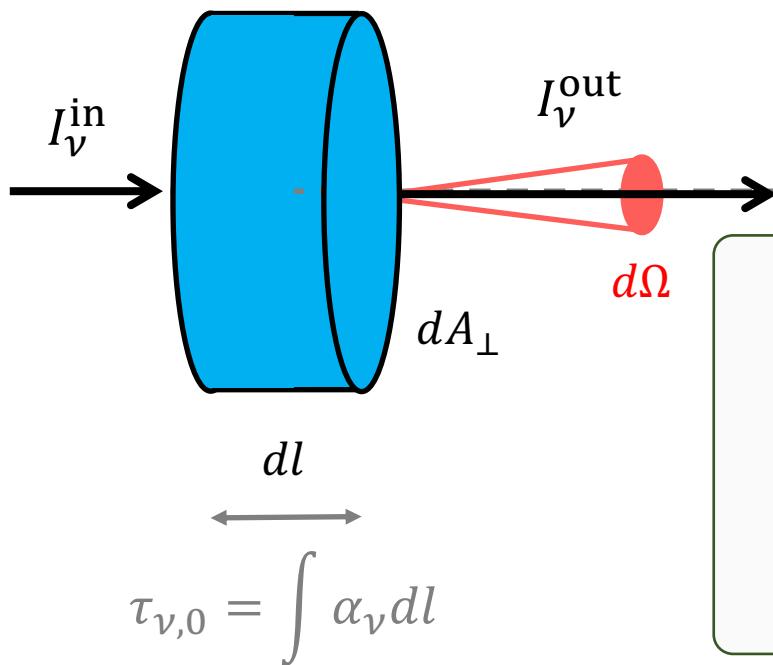
$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

Source function

Solution to the RT equation

prev. sl.

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$



$$e^{\tau_\nu} \frac{dI_\nu}{d\tau_\nu} = S_\nu e^{\tau_\nu} - I_\nu e^{\tau_\nu}$$

$$e^{\tau_\nu} \left(\frac{dI_\nu}{d\tau_\nu} + I_\nu \right) = \frac{d}{d\tau_\nu} (I_\nu e^{\tau_\nu}) = S_\nu e^{\tau_\nu}$$

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

see Sect. 3.2 of 《天体物理中的辐射机制》 by 尤峻汉 (p81–82)

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer

2.3 Thermal radiation

2.3.1 Blackbody radiation spectrum

2.3.2 Blackbody emission intensity

2.3.3 Blackbody emission flux

2.3.4 Thermal equilibrium

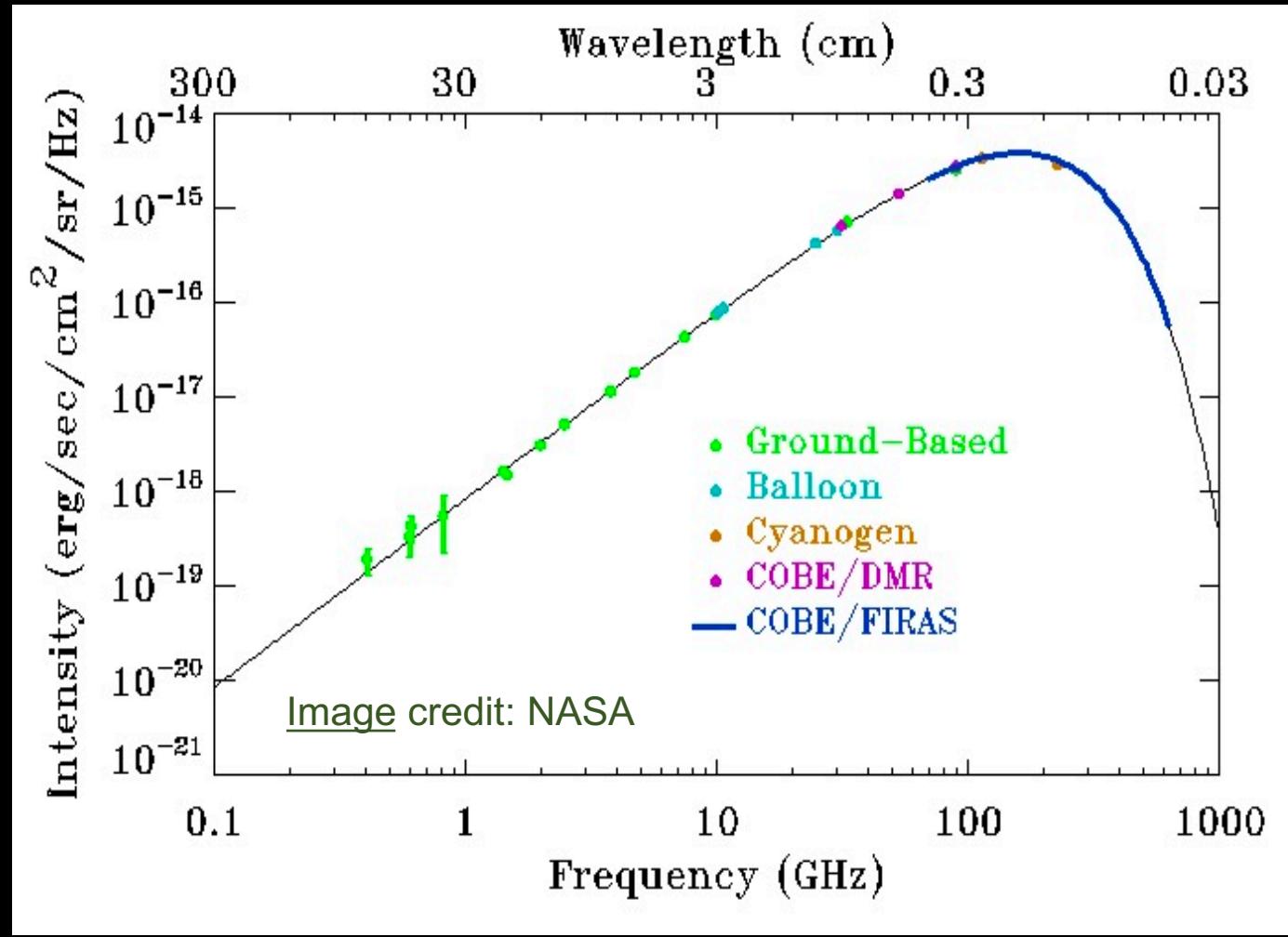
2.3.5 Local thermal equilibrium

2.4 Scattering

2.5 Escape probability

2.6 Polarization

2.7 Dispersion and rotation measures



Blackbody radiation spectrum

Thermal radiation is radiation emitted by matter in thermal equilibrium. In this case, the free electrons follows the Maxwellian distribution

$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2k T_e}\right)$$

Blackbody radiation spectrum is the Planck spectrum

$$h\nu_{\max} = 2.82 kT$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \left(\exp\left(\frac{hc}{\lambda kT}\right) - 1 \right)^{-1}$$

Caution that even if particles in the matter follow the Maxwellian distribution, radiation emitted by this matter does **not** necessarily have to be thermal radiation

Blackbody emission intensity

$$I_{\nu}^{\text{BB}} \equiv B_{\nu}(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

Blackbody emission intensity

$$\begin{aligned} I^{\text{BB}} &= \int_0^{\infty} \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1} d\nu \\ &= \frac{2k^4 T^4}{h^3 c^2} \int_0^{\infty} x^3 (e^x - 1)^{-1} dx \quad x = \frac{h\nu}{kT} \end{aligned}$$

Blackbody emission flux

prev. sl.

$$I^{\text{BB}} = \frac{2k^4 T^4}{h^3 c^2} \int_0^\infty x^3 (e^x - 1)^{-1} dx$$

Blackbody emission flux

$$\rightarrow F^{\text{BB}} = \pi I^{\text{BB}} = \sigma T^4$$

Blackbody radiation flux depends
only on the temperature

$$\int_0^\infty x^3 (e^x - 1)^{-1} dx = \frac{\pi^4}{15}$$
$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2}$$
$$= 5.670 \times 10^{-5} \text{ erg s}^{-1} \text{cm}^{-2} \text{K}^{-4}$$

↑
Stefan-Boltzmann
constant

Asymptotic behavior

prev. sl.

$$I_{\nu}^{\text{BB}} = B_{\nu}(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

$$\frac{h\nu}{kT} \ll 1 \quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \frac{h\nu}{kT} \quad I_{\nu}^{\text{RJ}} = \frac{2\nu^2}{c^2} kT \quad \text{Rayleigh-Jeans Law}$$

$$\frac{h\nu}{kT} \gg 1 \quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \exp\left(\frac{h\nu}{kT}\right) \quad I_{\nu}^{\text{W}} = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right) \quad \text{Wien Law}$$

Thermodynamic Equilibrium (TE)

prev. sl.

$$I_{\nu}^{\text{BB}} = B_{\nu}(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

For medium in thermodynamic equilibrium, Kirchoff's law links the spontaneous emission coefficient and emission source function with Planck's law

$$j_{\nu} \equiv \alpha_{\nu} B_{\nu}(T)$$

$$S_{\nu} \equiv B_{\nu}(T)$$

TE (cont.)

prev. sl.

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

$$I_\nu^{\text{out}} = B_\nu(T)(1 - \exp(-\tau_{\nu,0}))$$

$$\tau_{\nu,0} = \int \alpha_\nu dl \gg 1$$

$$I_\nu^{\text{out}} = B_\nu(T)$$

$$\tau_{\nu,0} = \int \alpha_\nu dl \ll 1$$

$$I_\nu^{\text{out}} = \tau_{\nu,0} B_\nu(T)$$

Only if the medium is in TE and it is optically thick, then its radiation follows the Planck's Law.
For non-thermal processes, the source function can only be derived from j_ν and α_ν .

Local Thermodynamic Equilibrium (LTE)

LTE differs from TE in that the radiation field does not need to follow Planck's law, i.e., $I_\nu \neq B_\nu(T)$.

The assumption of LTE is valid if either of the follow two applies:

- collisional processes among particles dominate photon-related processes
- collisional processes among particles are in equilibrium with photo-related processes

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer

2.3 Thermal radiation

2.4 Scattering

2.4.1 Pure scattering

2.4.2 Emission, absorption, and scattering

2.4.3 Absorption and scattering probabilities

2.4.4 Effective mean free path

2.5 Escape probability

2.6 Polarization

2.7 Dispersion and rotation measures

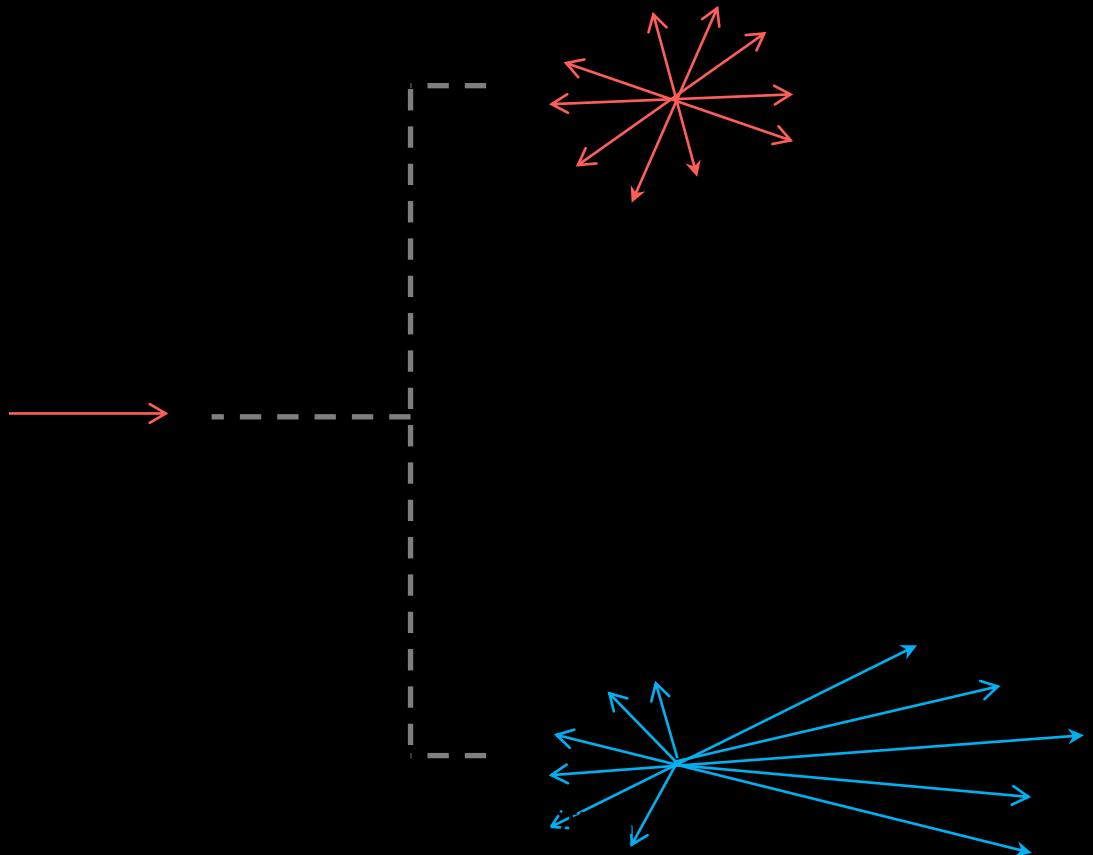


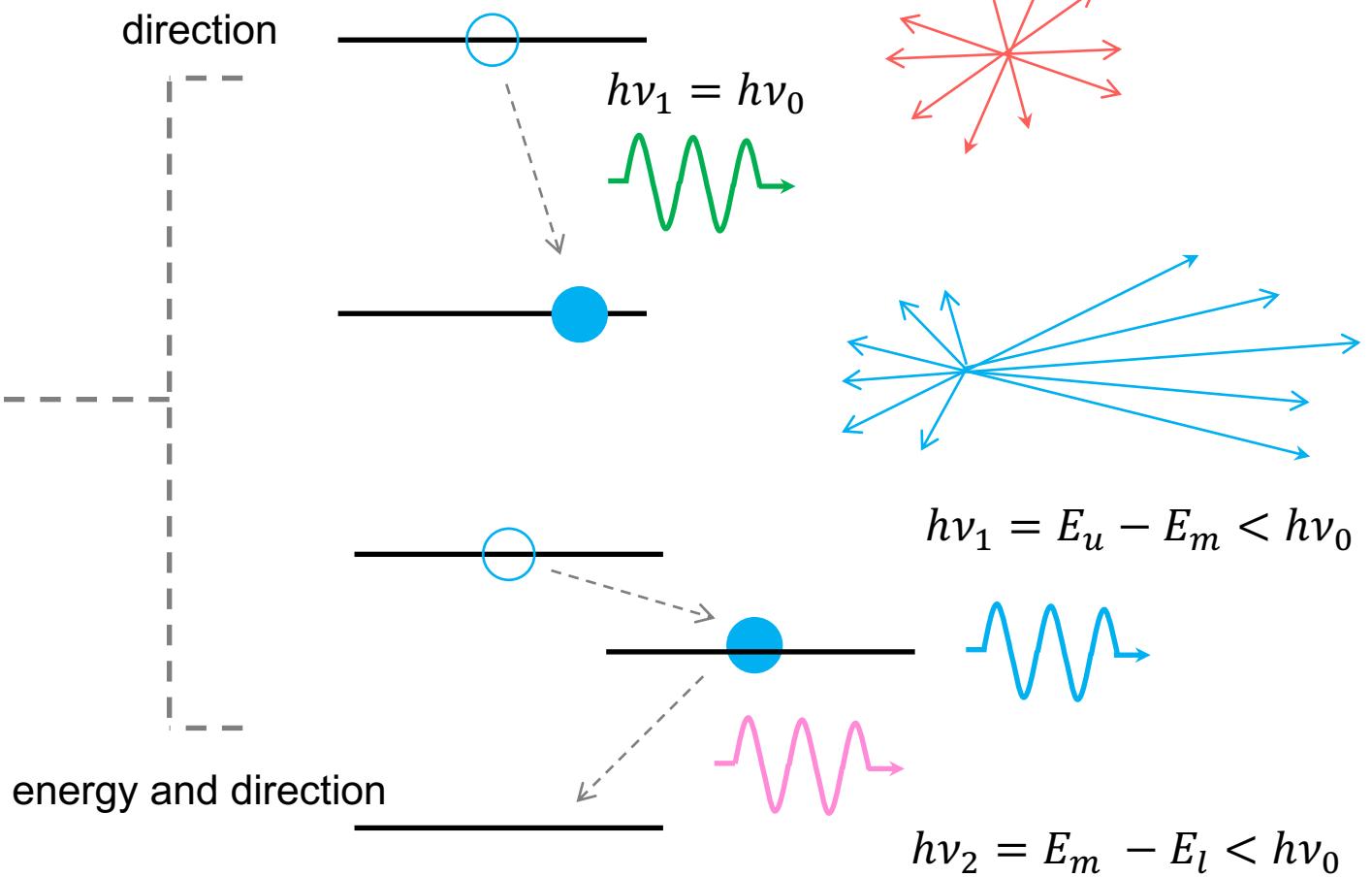
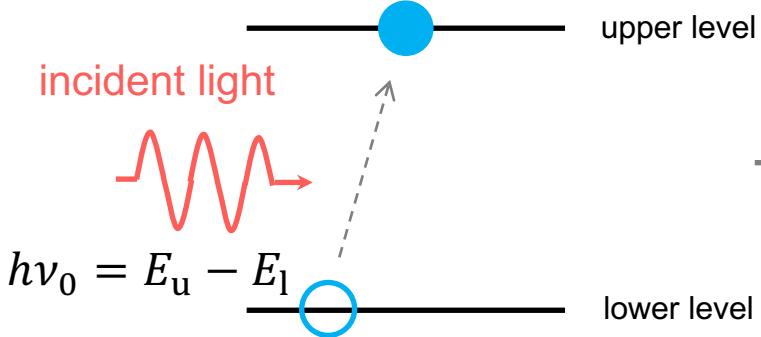
Image credit: Junjie Mao

Scattering

Scattering significantly complicates the radiative transfer.

Scattering can change the direction **and frequency** of the incident photon.

Two-level system



Pure scattering

Consider a medium has **only isotropic scattering** (no emission or absorption), which only changes the direction of the incident photon

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\text{sca}} I_\nu + \sigma_\nu^{\text{sca}} S_\nu^{\text{sca}}$$

scattered outside
the line of sight

scattered back into
the line of sight

scattering coefficient

$$\frac{dI_\nu}{d\tau_\nu^{\text{sca}}} = -I_\nu + S_\nu^{\text{sca}}$$

scattering optical depth

$$d\tau_\nu^{\text{sca}} \equiv \sigma_\nu^{\text{sca}} dl$$

Source function for scattering

Random walk

The mean displacement traveled by photons is
(order-of-magnitude estimation)

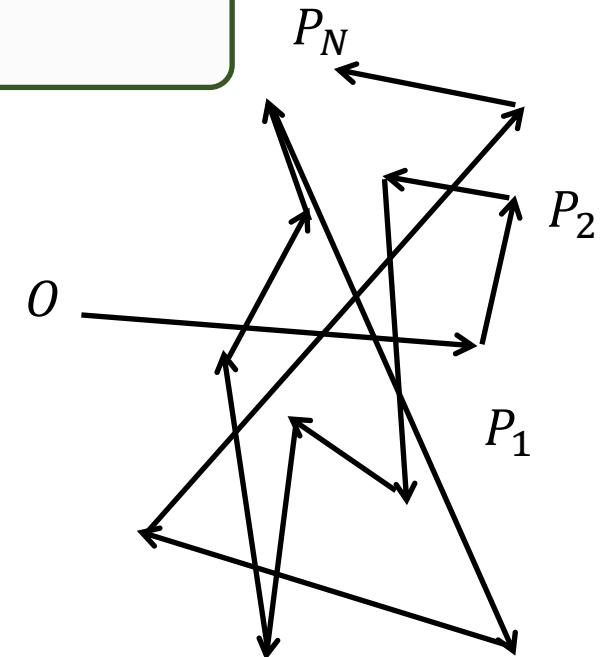
see Sect. 1.7 of the REF book
(p35) by Rybicki & Lightman

$$\bar{l}_\nu^{\text{RM}} \sim \sqrt{N} \bar{l}_\nu \quad \leftarrow \bar{l}_\nu = \frac{1}{\sigma_\nu^{\text{sca}}} : \text{scattering mean free path}$$

Number of scattering \downarrow size scale of the medium \downarrow

for $\tau_\nu^{\text{sca}} \gg 1$ $N \sim \frac{L^2}{\bar{l}_\nu^2} = (\sigma_\nu^{\text{sca}} L)^2 \sim (\tau_\nu^{\text{sca}})^2$

for $\tau_\nu^{\text{sca}} \ll 1$ $N \sim \sigma_\nu^{\text{sca}} L \sim \tau_\nu^{\text{sca}}$



Emission + absorption + scattering

Considering scattering + absorption + emission for a medium in TE

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\text{sca}} I_\nu + \sigma_\nu^{\text{sca}} S_\nu^{\text{sca}} + j_\nu - \alpha_\nu I_\nu$$
$$\frac{dI_\nu}{dl} = -(\sigma_\nu^{\text{sca}} + \alpha_\nu) I_\nu + (\sigma_\nu^{\text{sca}} S_\nu^{\text{sca}} + \alpha_\nu B_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu^{\text{AS}}} = -I_\nu + S_\nu^{\text{EAS}}$$

$$S_\nu^{\text{EAS}} \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu^{\text{sca}} S_\nu^{\text{sca}}}{\alpha_\nu + \sigma_\nu^{\text{sca}}}$$

$$(\sigma_\nu^{\text{sca}} + \alpha_\nu)dl = d\tau_\nu^{\text{AS}}$$

Source function for emission,
absorption, and scattering

Absorption and scattering probability

An alternative way to define the source function

$$S_\nu^{\text{EAS}} = \frac{\alpha_\nu B_\nu + \sigma_\nu^{\text{sca}} S_\nu^{\text{sca}}}{\alpha_\nu + \sigma_\nu^{\text{sca}}}$$

Introducing absorption and scattering probabilities

absorption probability

$$\epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\text{sca}}}$$

scattering probability

$$1 - \epsilon_\nu = \frac{\sigma_\nu^{\text{sca}}}{\alpha_\nu + \sigma_\nu^{\text{sca}}}$$

$$S_\nu^{\text{EAS}} = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) S_\nu^{\text{sca}}$$

Effective mean free path

prev. sl.

$$\epsilon_\nu = \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\text{sca}}}$$

After $N = \epsilon_\nu^{-1}$ steps, a photon is **truly** absorbed

$$\bar{l}_\nu^{\text{AS}} = \frac{1}{\alpha_\nu + \sigma_\nu^{\text{sca}}} : \text{mean free path}$$

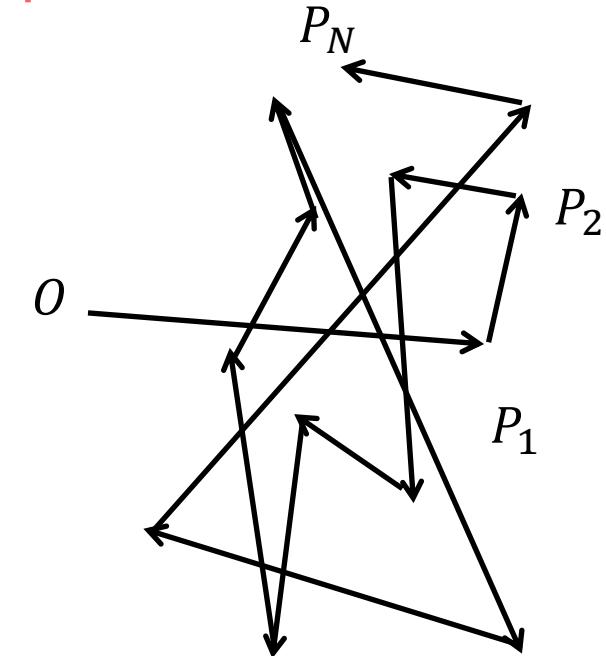
$$(\bar{l}_\nu^{\text{RM}})^2 = N (\bar{l}_\nu^{\text{AS}})^2 = \frac{(\bar{l}_\nu^{\text{AS}})^2}{\epsilon_\nu} = \frac{1}{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\text{sca}})}$$

effective mean free path
diffusion length
thermalization length

$$l_\nu^{\text{eff}} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\text{sca}})}}$$

for $\alpha_\nu \ll \sigma_\nu^{\text{sca}}$

$$l_\nu^{\text{eff}} = \frac{1}{\sqrt{\alpha_\nu \sigma_\nu^{\text{sca}}}}$$



Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

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2.5 Scattering

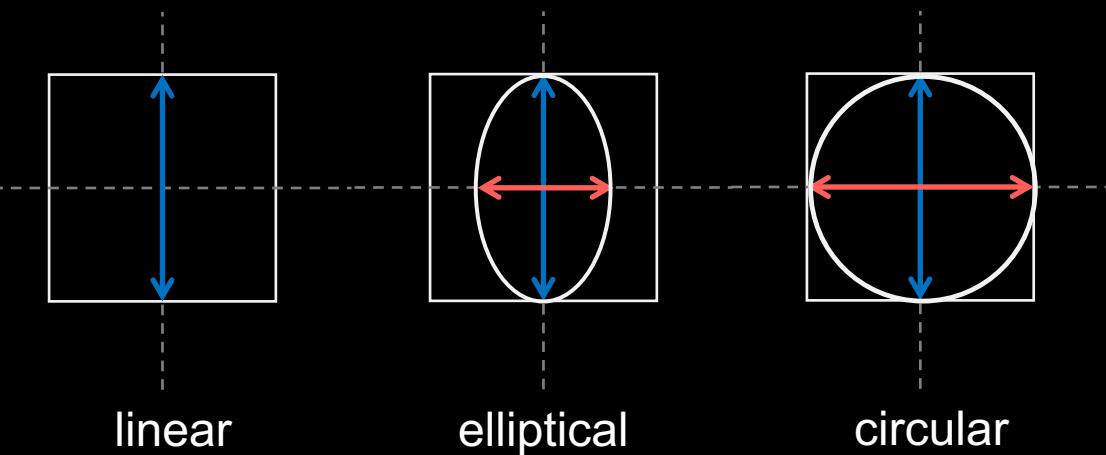
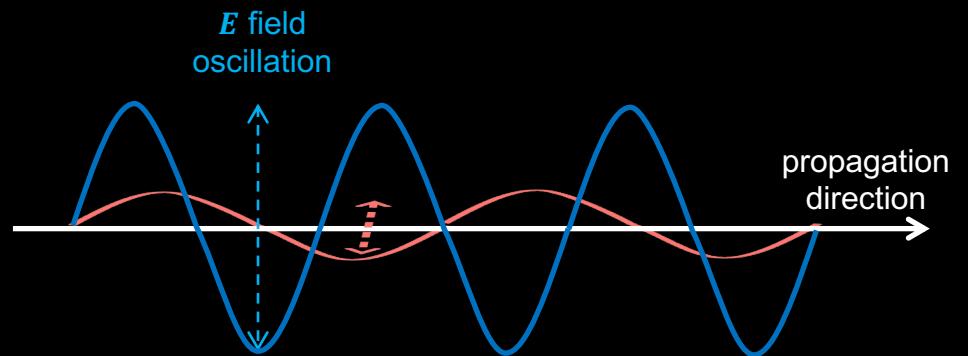
2.6 Polarization

2.6.1 Stokes parameters

2.6.2 Polarization fraction

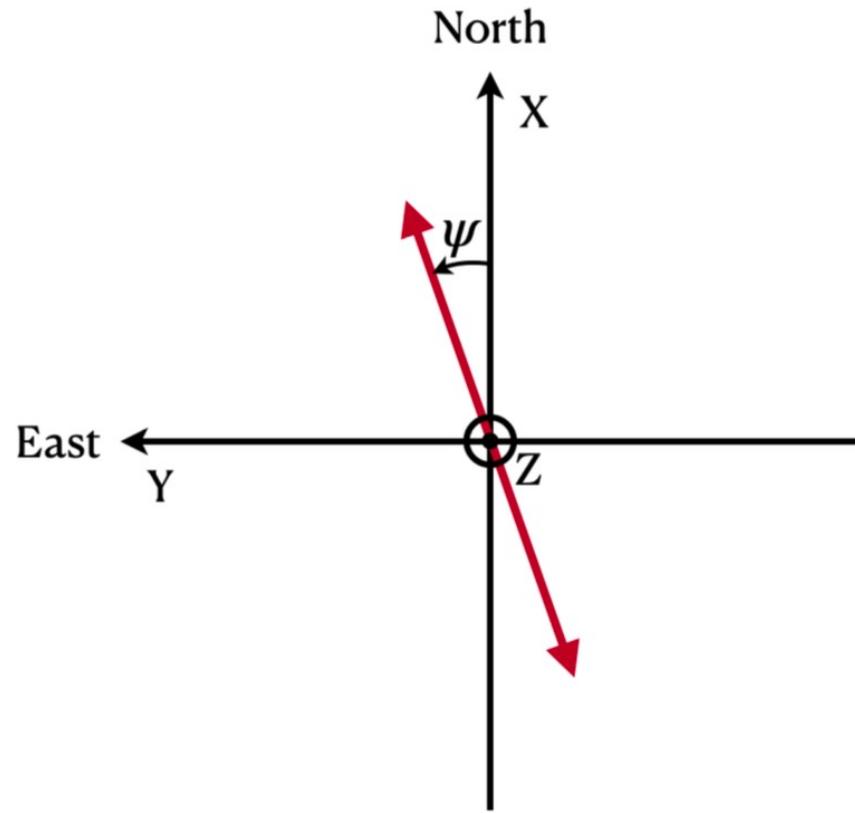
2.7 Dispersion and rotation measures

Image credit: Junjie Mao



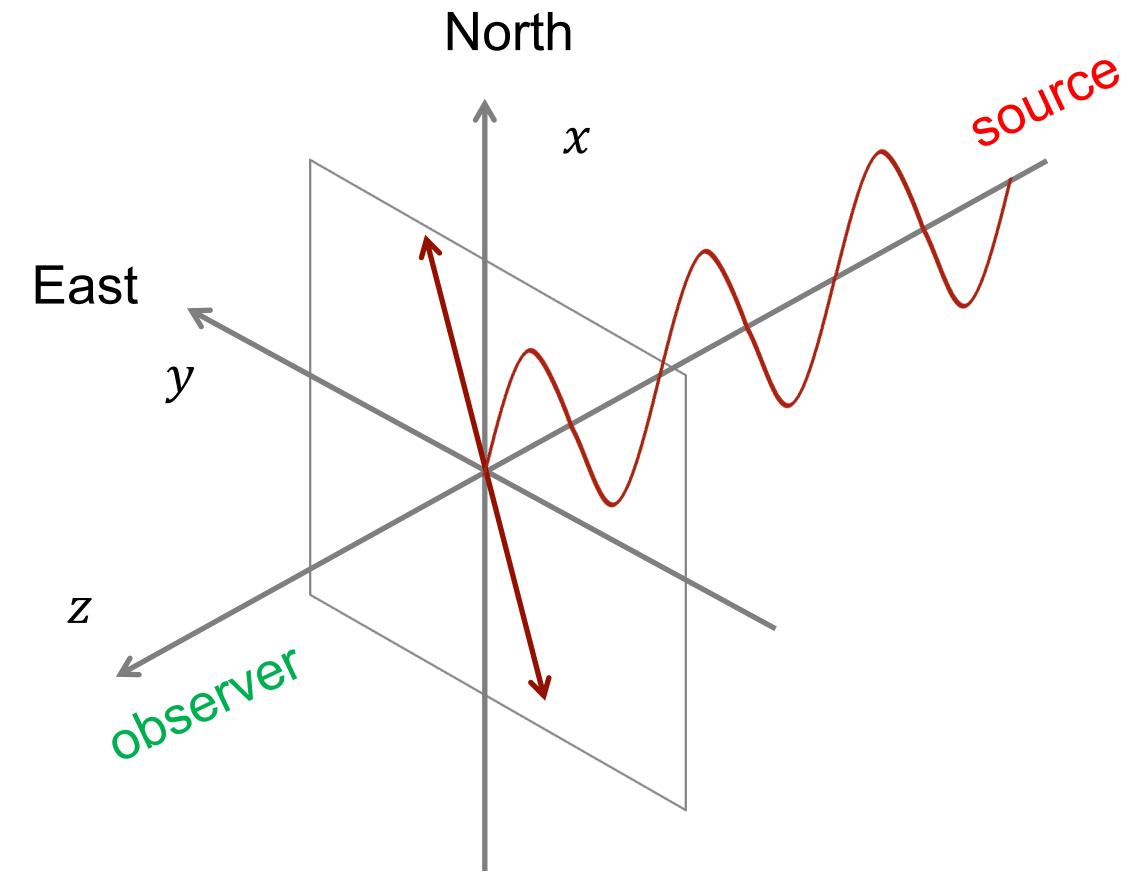
IAU standard of polarization

IAU reference frame for polarization
(e.g., [Ferriere et al. 2021](#))



IAU definition:

- ✓ X and Y are the Cartesian coordinates of the plane of the sky
- ✓ Z is along the line of sight (+Z towards the observer)
- ✓ ψ : linear polarization angle (measured counterclockwise from the North)



Stokes parameters

Stokes parameters, all related to intensity, are **observable**

- I : Intensity
- $\sqrt{Q^2 + U^2}$: Intensity of linear polarization
- V : Intensity of circular polarization

$$S_0 \equiv I$$

$$S_1 \equiv Q = Ip \cos 2\chi \cos 2\psi$$

$$S_2 \equiv U = Ip \cos 2\chi \sin 2\psi$$

$$S_3 \equiv V = Ip \sin 2\chi$$

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

polarization fraction

$$\tan 2\chi = \frac{V}{\sqrt{Q^2 + U^2}}, -\frac{\pi}{4} \leq \chi \leq \frac{\pi}{4}$$

$$\tan 2\psi = U/Q, 0 \leq \psi \leq \pi$$



linear polarization angle

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized

Poincaré sphere

[prev. sl.](#)

$$S_0 \equiv I$$

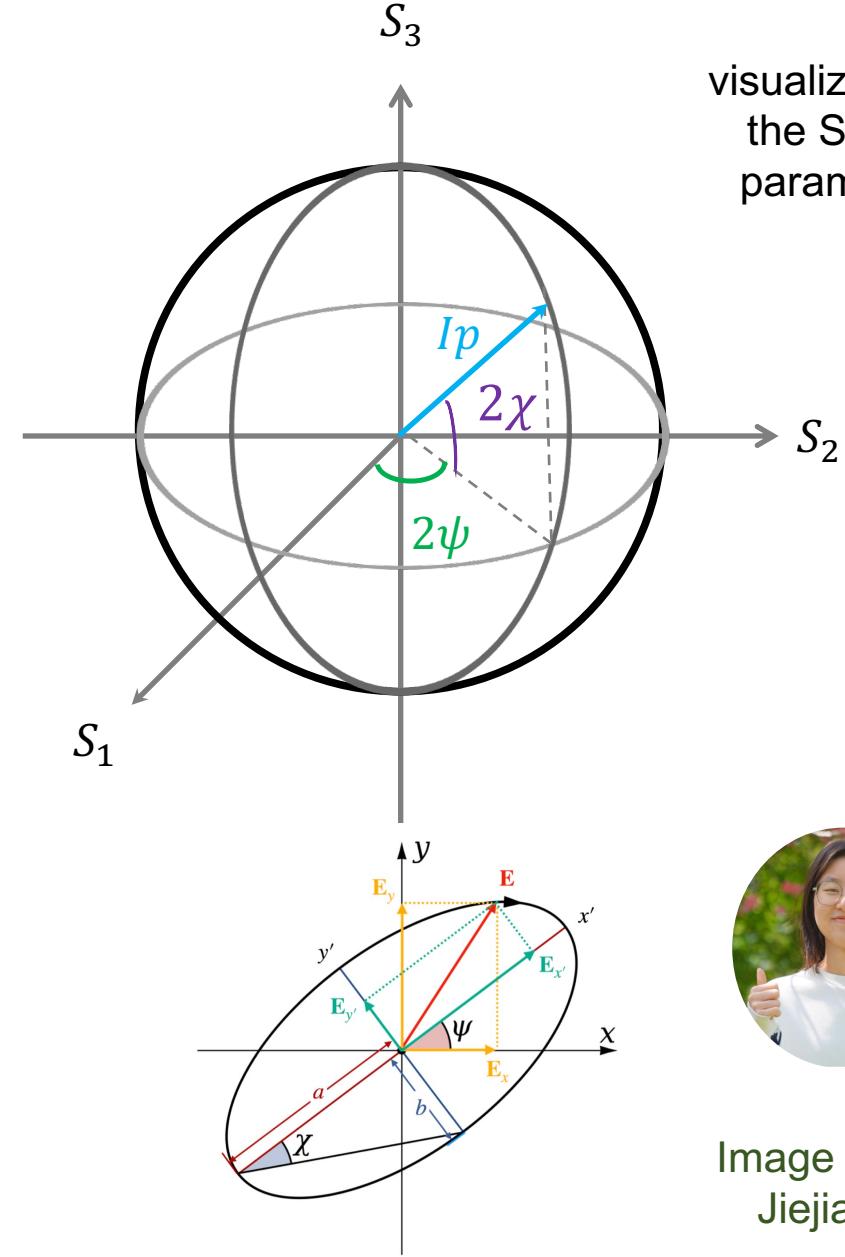
$$S_1 \equiv Q = Ip \cos 2\chi \cos 2\psi$$

$$S_2 \equiv U = Ip \cos 2\chi \sin 2\psi$$

$$S_3 \equiv V = Ip \sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2 + U^2}}, -\frac{\pi}{4} \leq \chi \leq \frac{\pi}{4}$$

$$\tan 2\psi = U/Q, 0 \leq \psi \leq \pi$$



visualization of
the Stokes
parameters



Image credit:
Jiejia Liu

Linear polarization

For $V = 0$ ($\chi = 0$), we have the linear polarization

$$S_0 = I$$

$$S_1 = Q = Ip \cos 2\psi$$

$$S_2 = U = Ip \sin 2\psi$$

$$S_3 = V = 0$$

prev. sl.

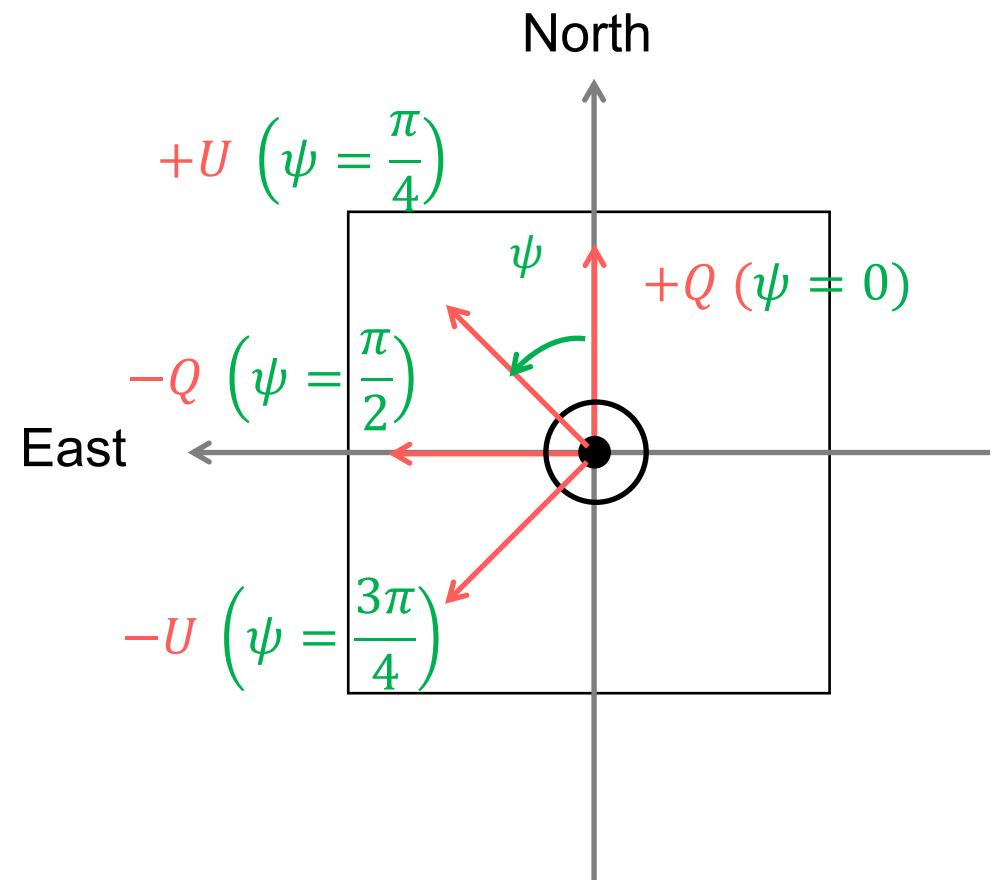
$$S_3 \equiv V = Ip \sin 2\chi$$

$$\tan 2\psi = U/Q$$

linear polarization angle

$$\cos \pi = -1$$

$$\sin \left(\frac{3\pi}{2} \right) = -1$$



e.g., [Hamaker & Bregman \(1996\)](#)

Circular polarization

For $Q = U = 0$ ($\chi = \pm\pi/4$), we have the circular polarization

$$S_0 = I$$

$$S_1 = Q = 0$$

$$S_2 = U = 0$$

$$S_3 = V = Ip \sin 2\chi$$

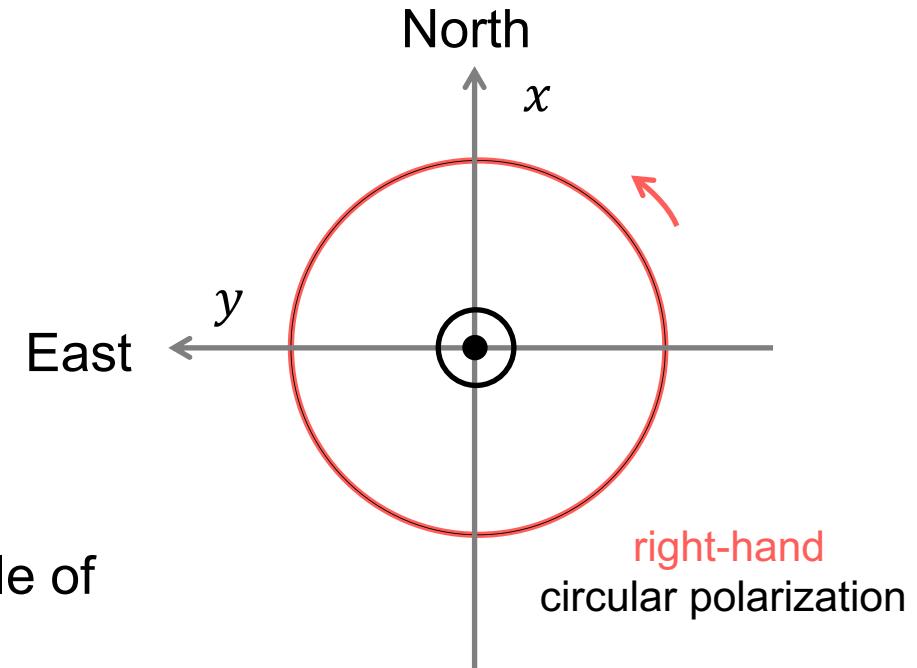
IAU definition:

- ✓ $+V$ for the **right-handed** circular polarization
- ✓ For the **right-handed** circular polarization, the position angle of \vec{E} at any point **increases** with time
- ✓ Along the $+z$ direction, \vec{E} traces a **left-handed** circle at any time

prev. sl.

$$S_1 \equiv Q = Ip \cos 2\chi \cos 2\psi$$

$$S_2 \equiv U = Ip \cos 2\chi \sin 2\psi$$



e.g., Hamaker & Bregman (1996)

Linear/circular polarization fraction

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - pI \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} pI \\ Q \\ U \\ V \end{pmatrix}$$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - p_L I - p_C I \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} p_L I \\ Q \\ U \\ 0 \end{pmatrix} + \begin{pmatrix} p_C I \\ 0 \\ 0 \\ V \end{pmatrix}$$

prev. sl.

- $p = 1$ completely polarized
- $0 < p < 1$ partially polarized
- $p = 0$ unpolarized

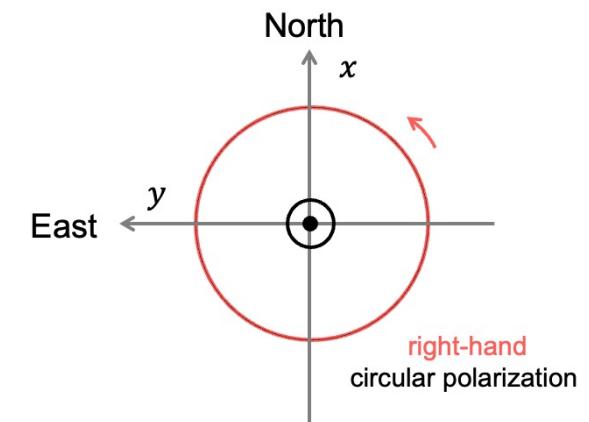
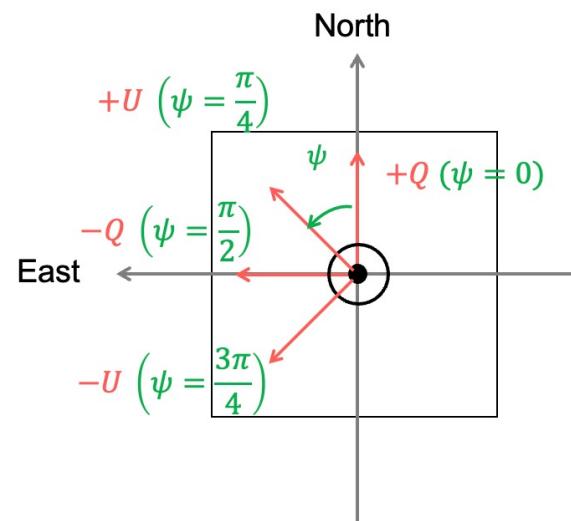
polarization fraction

linear
or
circular
polarization
fraction

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

$$p_L = \sqrt{Q^2 + U^2}/I$$

$$p_C = V/I$$



Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

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2.6 Polarization

2.7 Dispersion and rotation measures

2.7.1 Dispersion relation

2.7.2 Refractive index

2.7.3 Time of arrival difference

2.7.4 Dispersion measure

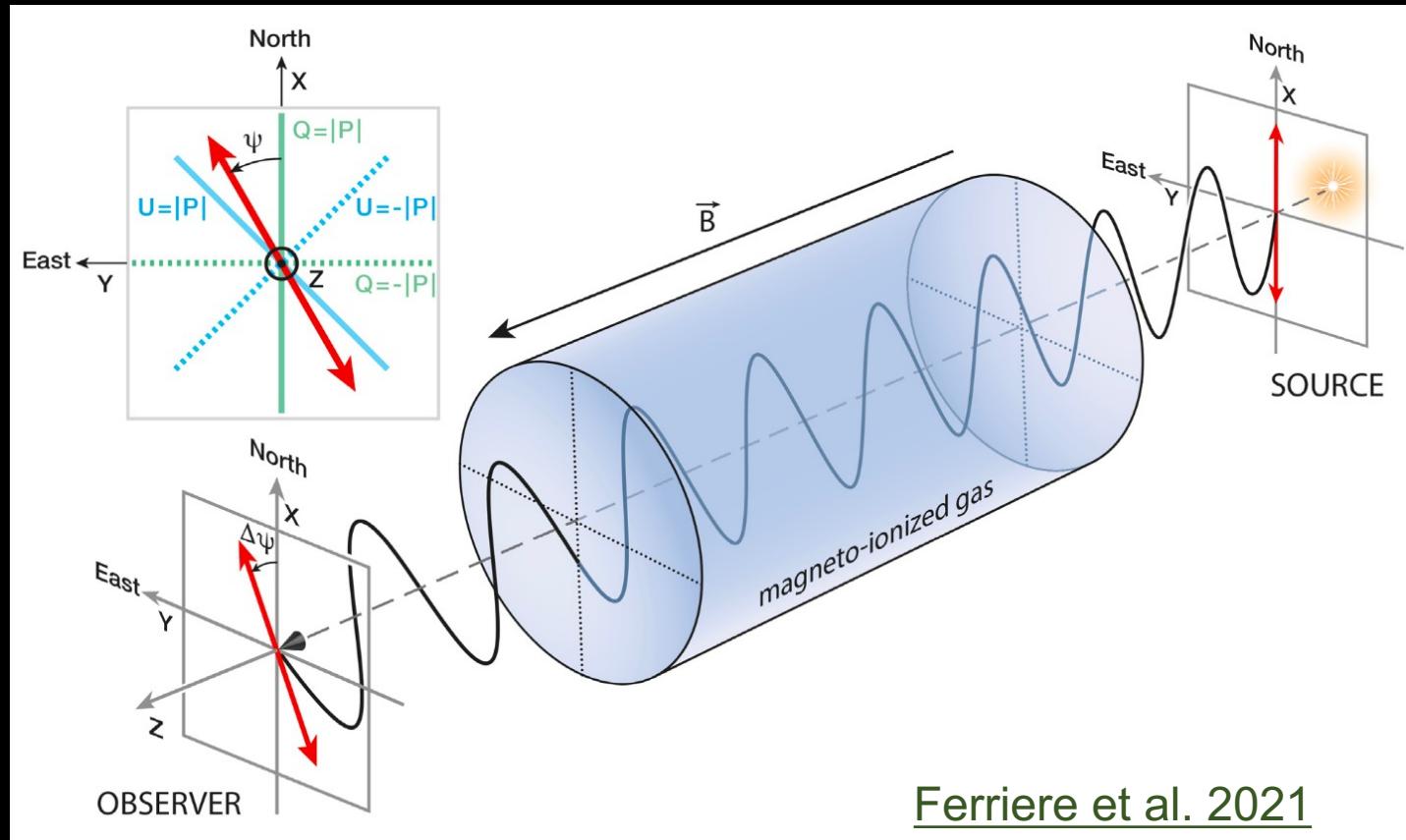
2.7.5 Fast Radio Bursts

2.7.6 Phase difference

2.7.7 Rotation measure

2.7.8 Faraday rotation

2.7.9 CMB polarization



Ferriere et al. 2021

Dispersion relation

Consider a pair of left- and right-handed circularly polarized waves propagate along the B-field with $\vec{k} \parallel \vec{B}$, the dispersion relation is

Eq. 1 of Ferriere et al. 2021

$$\omega^2 = c^2 k^2 + \frac{\omega_p^2}{1 \mp \frac{\omega_L}{\omega}}$$

angular frequency $\omega = 2\pi\nu$ wave number $k = \frac{2\pi}{\lambda}$

- Right circular polarized (RCP) mode: –
- Left circular polarized (LCP) mode: +

plasma frequency

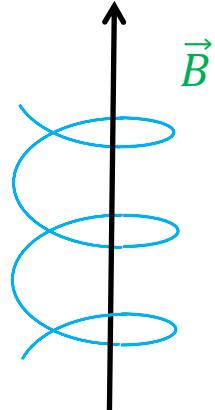
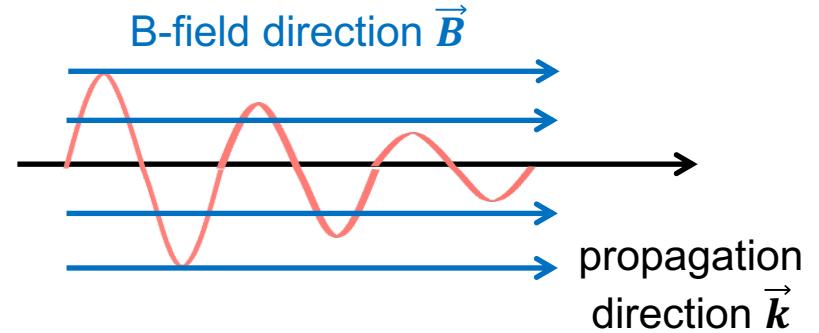
$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

gyro motion

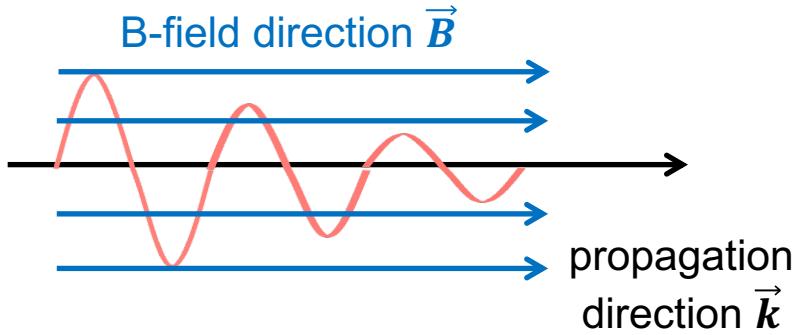
Larmor (or gyro) frequency

$$\omega_L \equiv \frac{eB}{m_e c}$$

《原子物理学》杨福家
3rd edition, p153 - 155



Dispersion relation (cont.)



For astrophysical waves of interest, the following always apply (Assignment #3: Plasma frequency)

$$\omega_p \ll \omega, \quad \omega_L \ll \omega$$

prev. sl.

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

$$\omega_L \equiv \frac{eB}{m_e c}$$

Larmor frequency

prev. sl.

$$\omega^2 = c^2 k^2 + \frac{\omega_p^2}{1 \mp \frac{\omega_L}{\omega}}$$

$$\frac{1}{1+x} = 1 - x + \dots$$

$$= c^2 k^2 + \omega_p^2 \pm \frac{\omega_p^2 \omega_L}{\omega}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{EM wave} & \text{electric} & \text{magnetic} \\ \text{in free space} & \text{force} & \text{force} \end{array}$$

$$c^2 k^2 = \omega^2 - \omega_p^2 \mp \frac{\omega_p^2 \omega_L}{\omega} \simeq \omega^2$$

$$k(\omega) = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2} \mp \frac{\omega_p^2 \omega_L}{\omega^3}}$$

Refractive index

Consider the propagation of EM waves through an unmagnetized plasma (mainly free electrons), the medium refractive index is

$$n = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

see Sect. 8.1 of the
REF book (p227) by
Rybicki & Lightman

prev. sl.

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Only waves with $\omega > \omega_p$ can propagate, which is not an issue in most cases

$$v_\phi = \frac{\omega}{k} = \frac{c}{n}$$

↑
phase velocity

Time of arrival difference

Consider a pulse propagates in a non-magnetized medium ($B = 0, \omega_L = 0$), the group velocity is

$$v_{\text{grp}}(\omega) = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

This leads to a frequency-dependent time of arrival

$$t_{\text{arv}}(\omega) = \int_0^l \frac{1}{v_{\text{grp}}(\omega)} dl = \int_0^l \frac{1}{c} \left(1 + \frac{\omega_p^2}{2\omega^2} \right) dl$$

prev. sl.

$$c^2 k^2 = \omega^2 - \omega_p^2 \mp \frac{\omega_p^2 \omega_L}{\omega} \simeq \omega^2$$

prev. sl.

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \quad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency

Larmor frequency

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \dots$$

Dispersion measure

For $\omega \gg \omega_p$ (Assignment #3: Plasma frequency), the time of arrival for a specific frequency is

traveling distance in the medium

prev. sl.

$$t_{\text{arv}}(\omega) = \int_0^l \frac{1}{c} \left(1 + \frac{\omega_p^2}{2\omega^2} \right) dl$$

prev. sl.

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Waves with different frequencies arrive at different times

$$\Delta t_{\text{arv}} = \frac{2\pi e^2}{m_e c} \left(\frac{1}{\omega_{\text{low}}^2} - \frac{1}{\omega_{\text{high}}^2} \right) \int_0^l n_e dl$$

High-frequency waves arrive earlier than the low-frequency ones

Fast Radio Bursts (FRBs)

FRBs are radio bursts with extremely short duration (a few milliseconds) yet rather energetic ($\gtrsim 10 L_\odot$). The first FRB was discovered in 2001 ([Lorimer et al. 2007](#))

prev. sl.

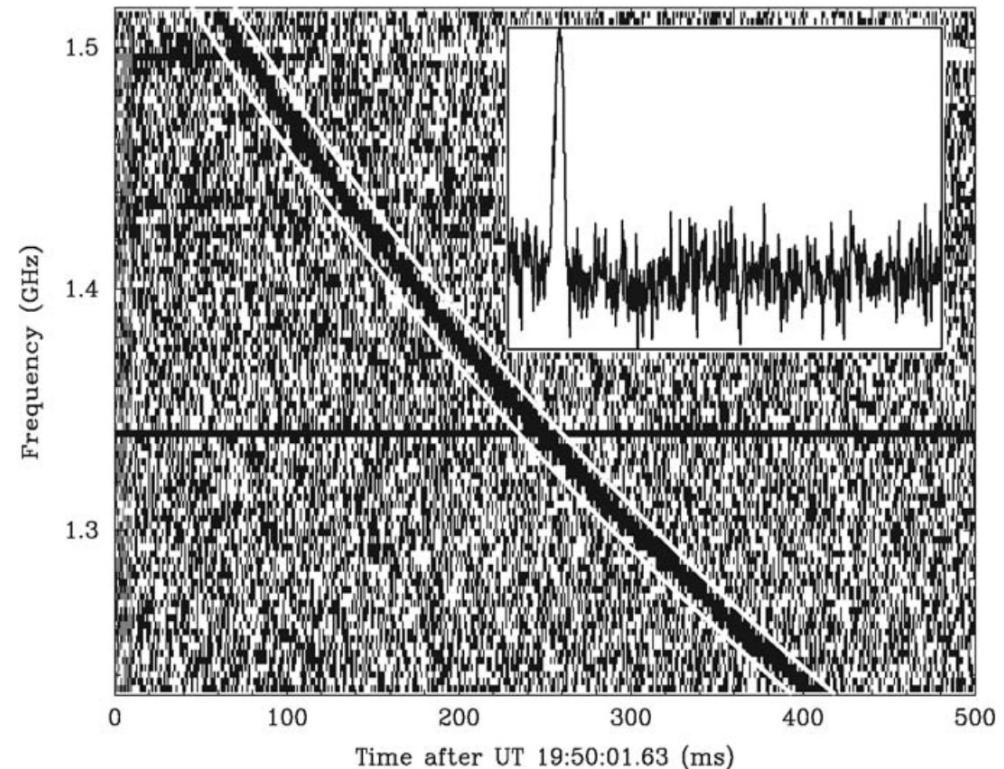
$$\Delta t_{\text{arr}} = \frac{2\pi e^2}{m_e c} \left(\frac{1}{\omega_{\text{low}}^2} - \frac{1}{\omega_{\text{high}}^2} \right) \int_0^l n_e dl$$

Dispersion Measure (DM)



$$= \frac{4.149}{\text{ms}} \left(\left(\frac{1 \text{ GHz}}{\nu_{\text{low}}} \right)^2 - \left(\frac{1 \text{ GHz}}{\nu_{\text{high}}} \right)^2 \right) \left(\frac{\text{DM}}{\text{pc cm}^{-3}} \right)$$

Waterfall plot
measured DM = $375 \pm 1 \text{ pc cm}^{-3}$



[Lorimer et al. 2007](#)

Fast Radio Bursts (FRBs)

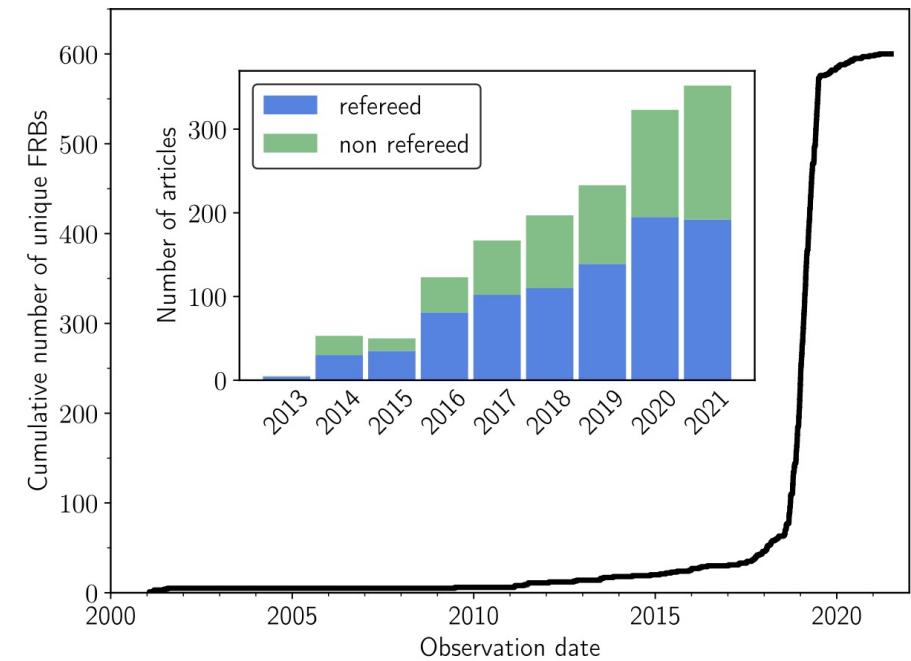
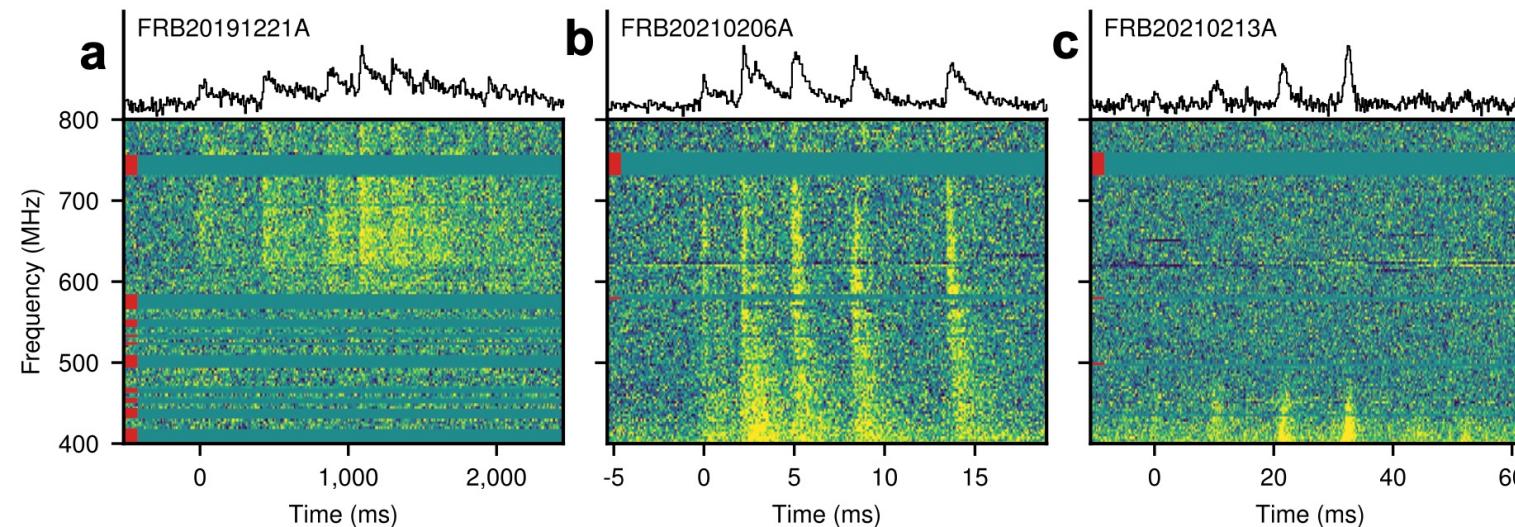
FRBs have been put in the spot light in the past decade:

- Repeating FRBs
- FRBs with known hosts
- FRBs with relatively long durations (slow)

But, we still don't know much about FRBs ...

repeating FRBs (a small yet growing population)

Petroff et al. 2022



Publications on FRBs from ADS

Some “not so fast” fast radio bursts

by Alice Curtin | Nov 22, 2022 | Daily Paper Summaries | 1 comment

Title: Four New Fast Radio Bursts Discovered in the Parkes 70-cm Pulsar Survey Archive

Authors: F. Crawford, S. Hisano, M. Golden, T. Kikunaga, A. Laity, D. Zoeller

First Author's Institution: Franklin & Marshall College

Phase difference

In a homogeneous plasma, the phase can be defined as one of the following

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k dl$$

Decomposing a linearly polarized wave into a pair of left- and right-hand circular polarized waves. At the source ($l = 0$), the two waves have the same phase, as the waves propagates (with $\vec{k} \parallel \vec{B}$) in a magnetized medium, phase difference will occur

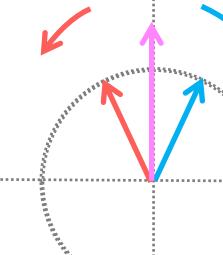
prev. sl.

$$k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2} \mp \frac{\omega_p^2 \omega_L}{\omega^3}}$$

$$\phi_{RCP} > \phi_{LCP}$$

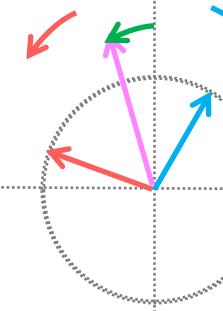
$$\vec{k} \parallel \vec{B}$$

t_0



$$\Delta\psi = \frac{1}{2} \Delta\phi$$

t_1



- Right circular polarized (RCP) mode: -
- Light circular polarized (LCP) mode: +

Phase difference introduced by B-field

Eq. 4 of [Ferriere et al. 2021](#)

$$\phi \equiv \omega t - \int_0^l k dl$$

prev. sl.

$$k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2} \mp \frac{\omega_p^2 \omega_L}{\omega^3}}$$

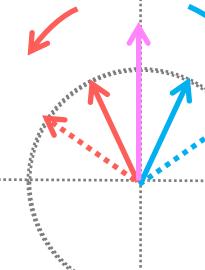
When $B = 0$, $\omega_L = \frac{eB}{m_0 c} = 0$, there is no phase difference

$$k_{\text{RCP}} = k_{\text{LCP}}$$
$$\Delta\phi \equiv \phi_{\text{RCP}} - \phi_{\text{LCP}} = 0$$

\vec{k}



$$t_0 = t_1$$



- Right circular polarized (RCP) mode: –
- Light circular polarized (LCP) mode: +

Phase difference in parallel B-field

$$\vec{k} \parallel \vec{B}$$

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k dl$$

prev. sl.

$$k = \frac{\omega}{c} \sqrt{1 - \frac{\omega_p^2}{\omega^2} \mp \frac{\omega_p^2 \omega_L}{\omega^3}}$$

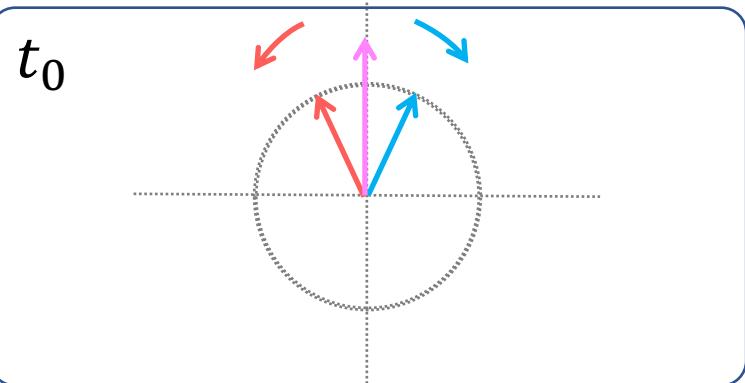
$$\sqrt{1+x} = 1 + \frac{x}{2} + \dots$$

$$k = \frac{\omega}{c} \left(1 - \frac{\omega_p^2}{2\omega^2} \mp \frac{\omega_p^2 \omega_L}{2\omega^3} \right)$$

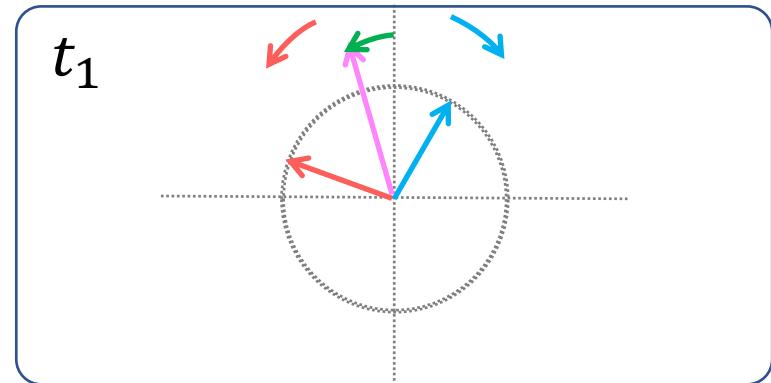
$$\Delta\psi = \frac{1}{2}\Delta\phi = -\frac{1}{2} \int_0^l (k_{\text{RCP}} - k_{\text{LCP}}) dl$$

$$= \frac{1}{2} \int_0^l \frac{\omega_p^2 \omega_L}{c \omega^2} dl$$

- \vec{B} points towards the observer: +
- \vec{B} points away from the observer: -



$$\Delta\psi = \frac{1}{2}\Delta\phi$$



- Right circular polarized (RCP) mode: -
- Light circular polarized (LCP) mode: +

Rotation measure

prev. sl.

$$\Delta\psi = \frac{1}{2} \int_0^l \frac{\omega_p^2 \omega_L}{c \omega^2} dl$$

prev. sl.

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \quad \text{plasma frequency}$$

$$\omega_L \equiv \frac{eB}{m_e c} \quad \text{Larmor frequency}$$

Eq. 9 of Ferriere et al. 2021

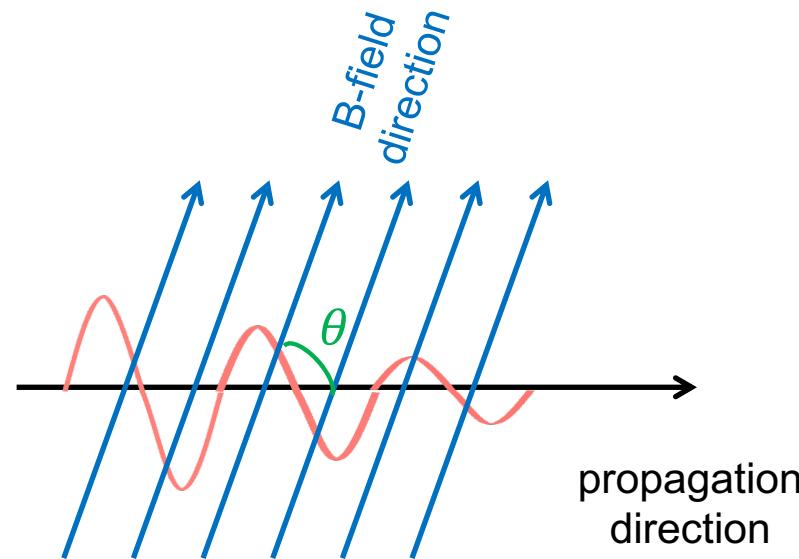
$$\Delta\psi = \frac{e^3}{2\pi m_e^2 c^4} \lambda^2 \int_0^l n_e (\pm B_{||}) dl$$

Rotation
Measure
(RM)

$$\rightarrow \text{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4} \int_0^l n_e (\pm B_{||}) dl$$

$$\text{RM} = 0.812 \text{ rad m}^{-2} \int_0^l \left(\frac{n_e}{\text{cm}^{-3}} \right) \left(\pm \frac{B_{||}}{\mu\text{G}} \right) d \left(\frac{l}{\text{pc}} \right)$$

- \vec{B} points towards the observer: +
- \vec{B} points away from the observer: -



Faraday rotation

When a linearly polarized wave travels through a magnetized medium, its polarization angle will rotate ([Burn 1966](#))

Eq. 9 of [Ferriere et al. 2021](#)

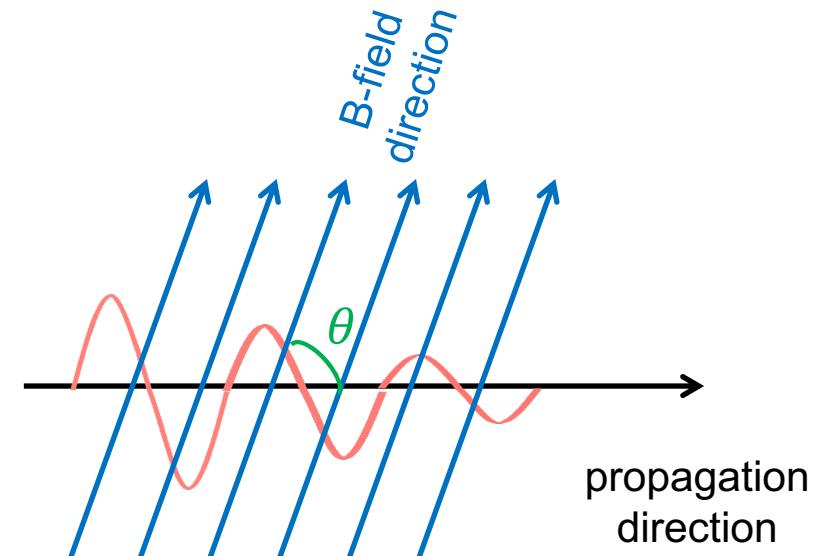
$$\Delta\psi = \frac{e^3}{2\pi m_e^2 c^4} \lambda^2 \int_0^l n_e (\pm B_{\parallel}) dl$$

prev. sl.

$$RM \equiv \frac{e^3}{2\pi m_e^2 c^4} \int_0^l n_e (\pm B_{\parallel}) dl$$

$$\psi_{\text{obs}} = \psi_{\text{src}} + RM \cos \theta \lambda^2$$

- \vec{B} points towards the observer: + 
- \vec{B} points away from the observer: - 



Cosmic Microwave Background

CMB is the leftover radiation from the Big Bang. It was firstly predicted by Gamow (1948).

In 1965, CMB was observed by Arno Penzias and Robert Wilson using the 6-meter Holmdel Horn Antenna.



Image credit: wikipedia

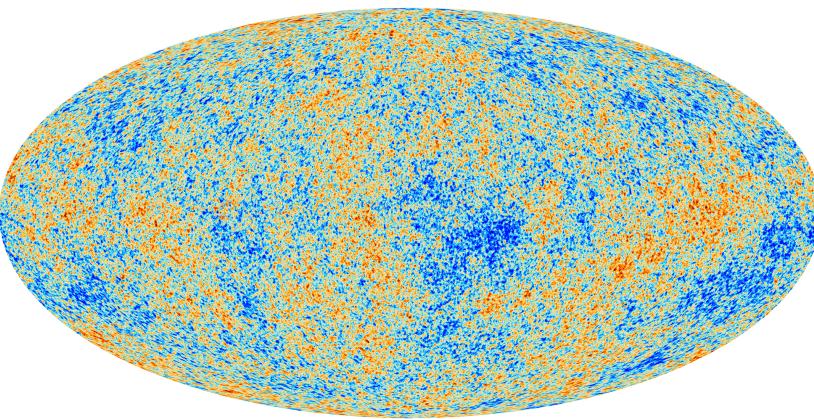


Image credit: ESA

The Nobel Prize in Physics 1978

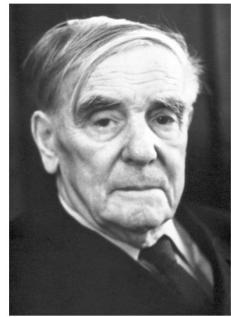


Photo from the Nobel Foundation archive.
Pyotr Leonidovich Kapitsa
Prize share: 1/2



Photo from the Nobel Foundation archive.
Arno Allan Penzias
Prize share: 1/4

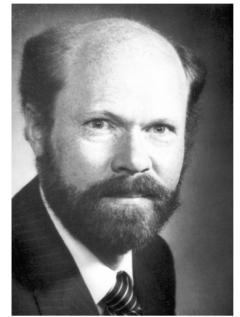


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Robert Woodrow Wilson
Prize share: 1/4

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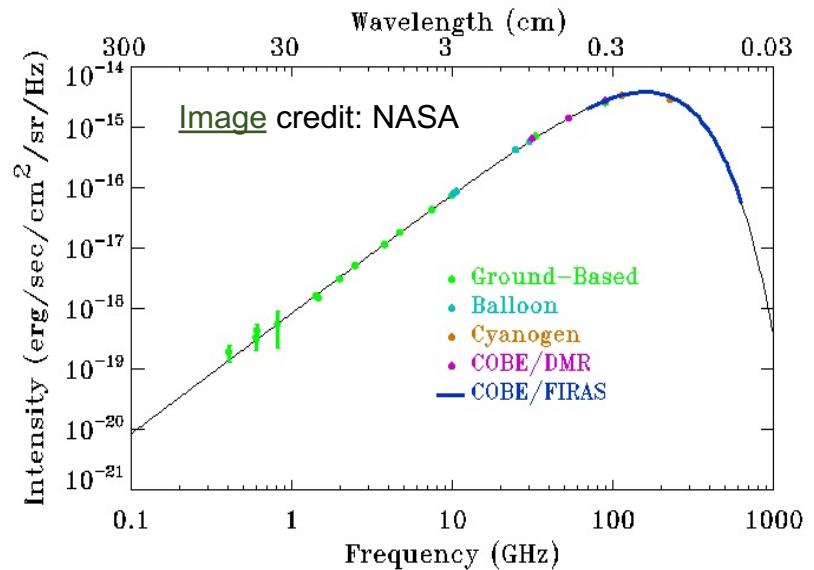
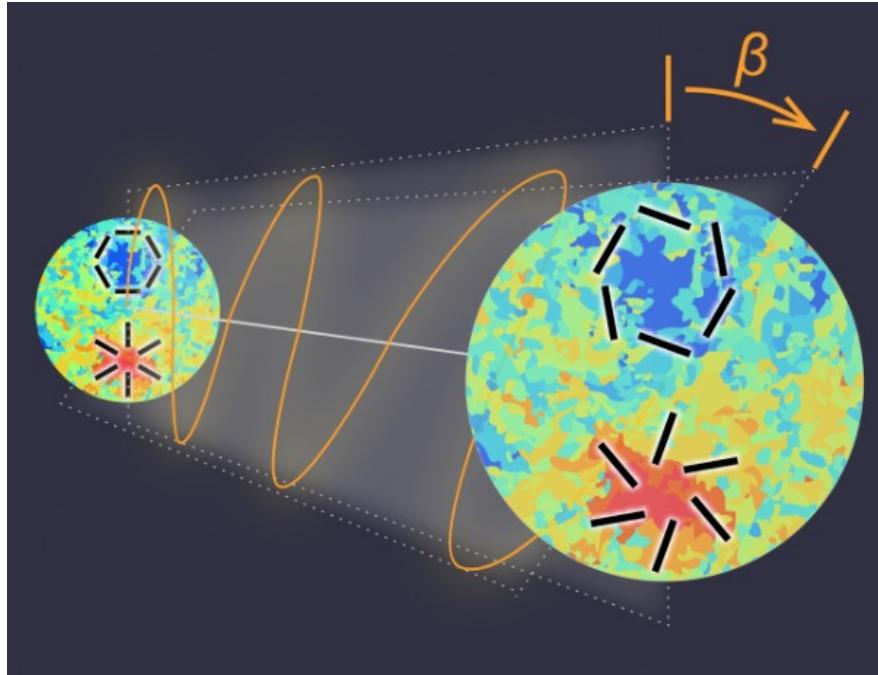
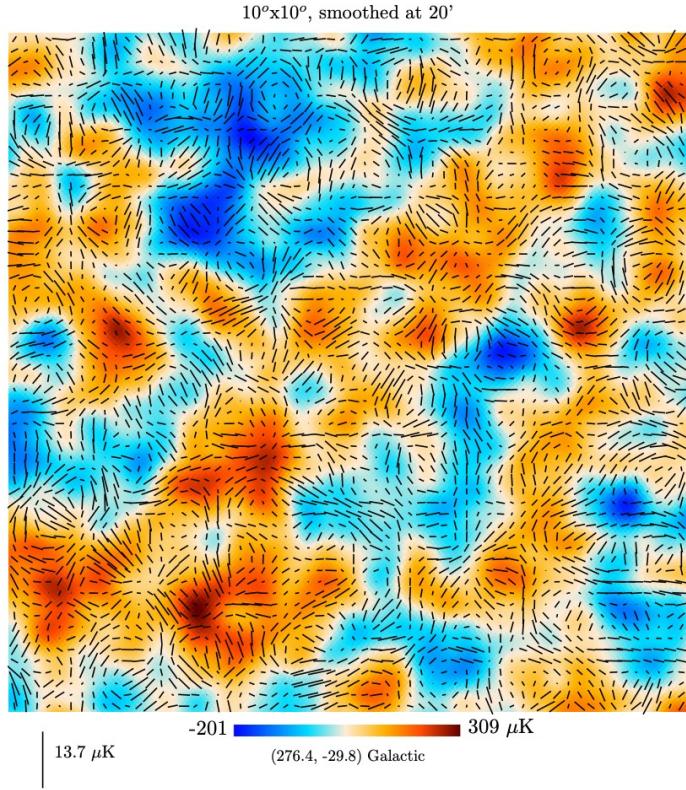


Image credit: NASA

Cosmic Microwave Background polarization



CMB is linearly polarized (E-mode and B-mode), firstly detected in 2002 ([Kovac et al. 2002](#)).

Birefringence: the refractive index depends on the polarization direction. Typically, double refraction (ordinary and extraordinary indices)

[Komatsu \(2022, review\)](#): β is the cosmic-birefringence-induced rotation angle

- B-mode (curl-like)
 - $\perp \vec{k}$ or $\parallel \vec{k}$
 - dominated by density perturbation
- E-mode (gradient-like)
 - 45° w. r. t. \vec{k}
 - fingerprints of GW background (too faint to detect directly nowadays) from cosmic inflation

CMB B-mode

The detection of B-mode polarization will provide strong constraints for cosmic inflationary models.

The B-mode power is $\propto r^2$, where, $r < 0.07$ is the tensor-to-scalar ratio (BICEP2 collaboration 2016).



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Do you see new physics in my CMB?

by Kayla Kornelje | Jul 9, 2022 | Daily Paper Summaries | 0 comments

Title: New physics from the polarised light of the cosmic microwave background

Authors: Eiichiro Komatsu

First Author's Institution: Max-Planck-Institut für Astrophysik, Karl-Schwarzschild Str. 1,
85741 Garching, Germany

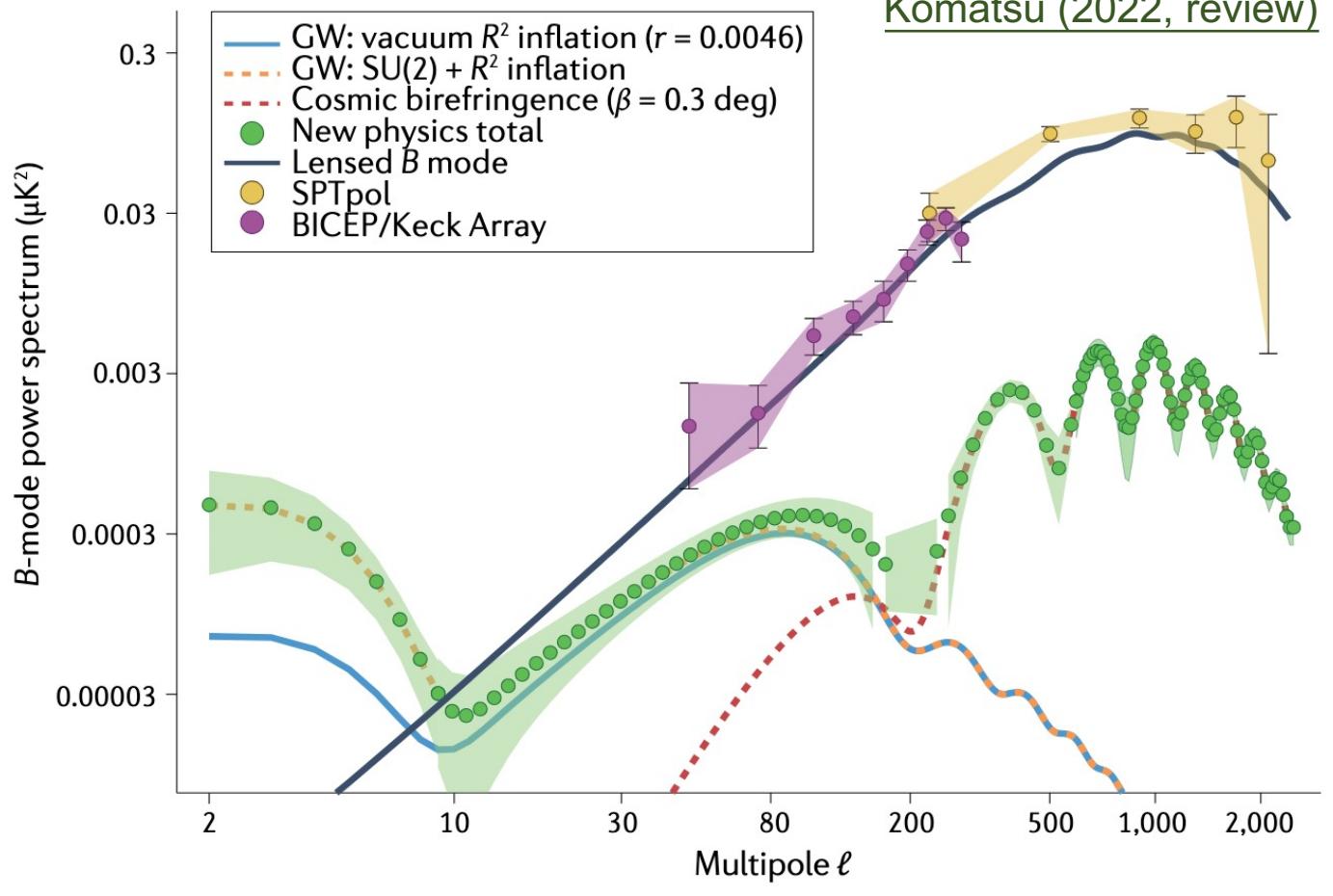


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