清華大學天文系
Department of Astronomy, Tsinghua University

5. Warm Neutral Medium

Instructor: Hui Li   TA: Chengzhe Li

Department of Astronomy

Tsinghua University

Detecting WNM (neutral hydrogen)

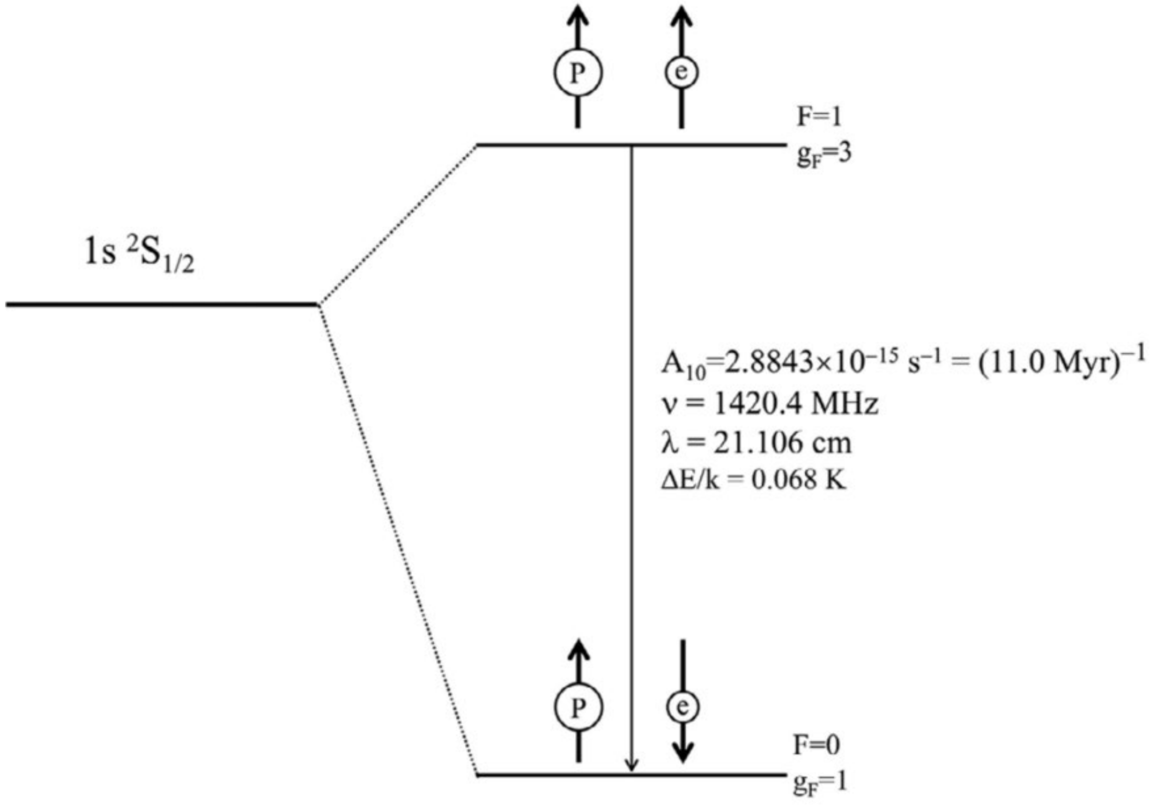
Lyman alpha absorption line

- Abundant.
- Very large Einstein A coefficient. High tau.
- The line has a wavelength of 121.6 nm, at which the Earth's atmosphere is highly opaque.
- The excitation temperature is 1.18e5 K, much higher than the kinetic temperature of the WNM. Absorption only.
- Need strong background UV sources to detect absorption along the line-of-sight. Cannot do a global survey of atomic gas.

HI 21 cm emission line

- Hyperfine structure line and therefore very tiny Einstein A coefficient. A mean lifetime of the upper-level atom is 11 Myr!!
- The line has a very long wavelength of 21 cm, can be observed in the radio band on Earth.
- Correspond to a very low excitation temperature 0.068 K, much lower than the temperature of any gas in the ISM.
- Emission lines from all line-of-sights!

Energy populations of 21 cm hyperfine levels



$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT} = 3e^{-0.0682\,\mathrm{K}/T} \approx 3$$

- Excitation temperature (0.068 K) is much lower than the kinetic temperature of the gas.
- In excitation eq., the upper to lower level ratio is always ~3.

Optical depth of 21cm emission.

$$\left(\frac{dn_u}{dt}\right)_{\text{abs}} = n_\ell B_{\ell u} \varepsilon_\nu$$

$$\left(\frac{dn_u}{dt}\right)_{\text{se}} = -n_u B_{u\ell} \varepsilon_\nu = -n_u \left(\frac{g_\ell}{g_u} B_{\ell u}\right) \varepsilon_\nu$$

$$= -\left(\frac{n_u}{n_\ell}\frac{g_\ell}{g_u}\right)\left(\frac{dn_u}{dt}\right)_{\text{abs}} .$$

- For a system with both absorption and stimulated emission against incoming radiation, absorption coefficient in radiative transfer is:

$$\kappa_\nu = n_\ell \sigma_{\ell u}\left[1 - \frac{g_\ell}{g_u}\frac{n_u}{n_\ell}\right]$$

$$= n_\ell \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\text{exc}}}\right)\right]$$

$$\tau_\nu = \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\text{exc}}}\right)\right] N_\ell$$

Astrophysical Maser (Microwave Amplified by Stimulated Emission of Radiation)

$$\kappa_\nu = n_\ell \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right)\right]$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$$

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu}.$$

- Population Inversion: more atoms or molecules are in an upper state than in the lower state due to external energy sources (radiation or shock) exciting the gas molecules.
- Negative excitation temperature!

$$d\tau_\nu \equiv \kappa_\nu ds$$

- Negative absorption coefficient and optical depth!
- Exponential growth of radiation intensity.

Optical depth of 21cm emission.

- In typical ISM condition, $kT_{\rm exc} \gg h\nu_{u\ell}$

$$\kappa_\nu = n_\ell \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right)\right] \approx n_\ell \sigma_{\ell u}\left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right) \ll n_\ell \sigma_{\ell u}.$$

- For 21 cm lines, the attenuation coefficient and thus the optical depth is smaller by a factor of $h\nu_{u\ell}/kT_{\rm exc}$ than it would be in the absence of stimulated emission.
- The optical depth in the limit of doppler broadening (why?) is:

$$\tau_0 = \frac{1}{(4\pi)^{3/2}}\left[\frac{g_u}{g_\ell}\frac{c^2}{\nu_{u\ell}^2}A_{u\ell}\right]\frac{c}{b}N_\ell\left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right) \approx 0.31\left(\frac{N_{\rm HI}}{10^{21}\,{\rm cm}^{-2}}\right)\left(\frac{10\,{\rm km\,s}^{-1}}{b}\right)\left(\frac{100\,{\rm K}}{T_{\rm exc}}\right)$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$$

Radiative Transfer of Line Emission

- The emissivity and attenuation coefficients are related to the Einstein A coefficient.

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \Phi_\nu \qquad \kappa_\nu \approx n_\ell \sigma_{\ell u} \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}} \right) \approx n_\ell \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} \Phi_\nu \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}} \right)$$

- The source function can be simplified because $n_u/n_\ell \approx g_u/g_\ell$:

$$S_\nu \approx \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc} \approx \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc}.$$

- Radiative transfer equation:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc}.$$

Antenna temperature as a convention in radio astronomy

- In the Rayleigh-Jeans tail of the Planck function B(Tb), the intensity is $I_\nu = \frac{2\nu^2}{c^2} kT_b.$

- There exists a linear relationship between the radiation intensity and bright temperature of the blackbody radiation. The antenna temperature is thus defined here to simplify radiative transfer calculation:

$$T_{\mathrm{A}} \equiv \frac{c^2}{2k} \frac{I_\nu}{\nu^2}.$$

- The radiative transfer equation is reduced to temperature-related equation:

$$\frac{dT_{\mathrm{A}}}{d\tau_\nu} = -T_{\mathrm{A}} + T_{\mathrm{exc}}. \quad \Longrightarrow \quad T_{\mathrm{A}} = T_{\mathrm{A}}(0)\mathrm{e}^{-\tau_\nu} + T_{\mathrm{exc}}[1 - \mathrm{e}^{-\tau_\nu}].$$

21 cm HI line emissions toward a dark background ($T_A(0)=0$)

- In optically thin regime with tau<<1,

$$T_{\mathrm{A}} = T_{\mathrm{A}}(0)\mathrm{e}^{-\tau_\nu} + T_{\mathrm{exc}}[1 - \mathrm{e}^{-\tau_\nu}] \approx \tau_\nu T_{\mathrm{exc}}.$$

- Tau is inverse propotional to excitation temperature

$$\tau_0 = \frac{1}{(4\pi)^{3/2}} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^2} A_{u\ell} \right] \frac{c}{b} N_\ell \left(\frac{h\nu_{u\ell}}{kT_{\mathrm{exc}}} \right)$$

- Therefore, the antenna temperature is independent of the excitation temperature and is only the column density of HI along the line-of-sight!!!

$$\int T_A dv = T_{\mathrm{exc}} \int \tau_\nu dv = T_{\mathrm{exc}} W_v = \sqrt{\pi} T_{\mathrm{exc}} b \tau_0$$

$$= \frac{c}{8\pi} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^2} A_{u\ell} \right] \frac{h\nu_{u\ell}}{k} N_\ell$$

Optically-thin 21cm line as direct tracer of HI column density

$$\int T_A dv = 550\,\mathrm{K\,km\,s^{-1}} \left(\frac{N_{\mathrm{HI}}}{10^{21}\,\mathrm{cm^{-2}}} \right)$$

- The integrated line intensity of the 21 cm line is linearly proportional to the HI column density. The normalization depends solely on quantum mechanical constants without any assumptions about the temperature/excitation states of the HI gas.
- This conclusion is also true for other radio lines, like molecular lines.
- The only assumption here is that the excitation temperature Texc >> 0.068 K!

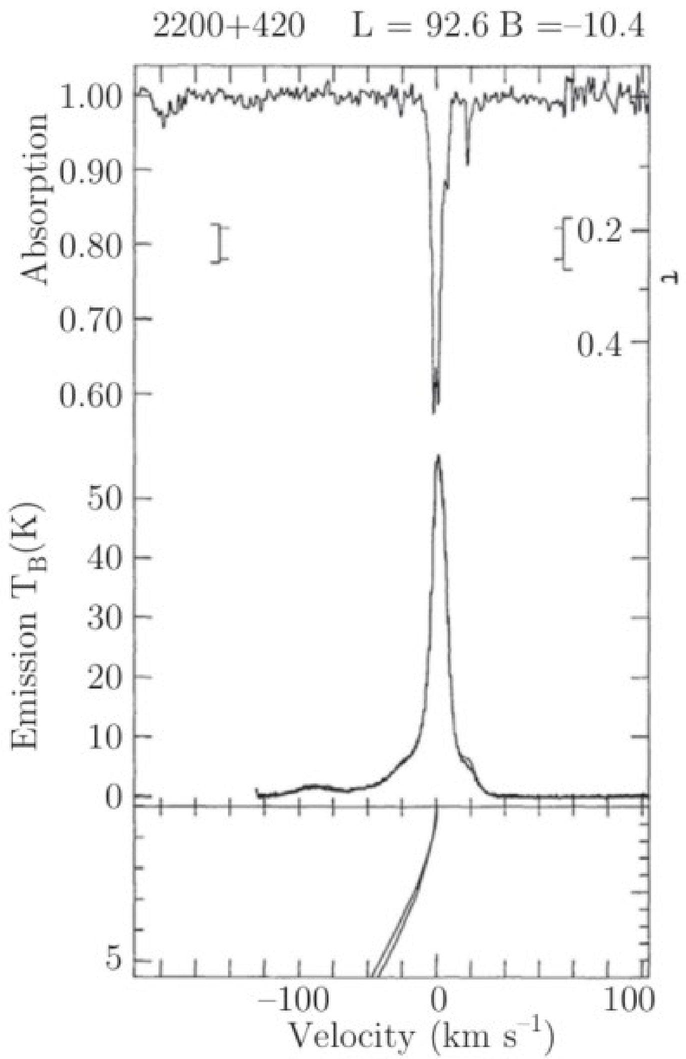
Is excitation temperature really much higher than 0.068K?

- HI absorption toward strong radio source BL Lac.

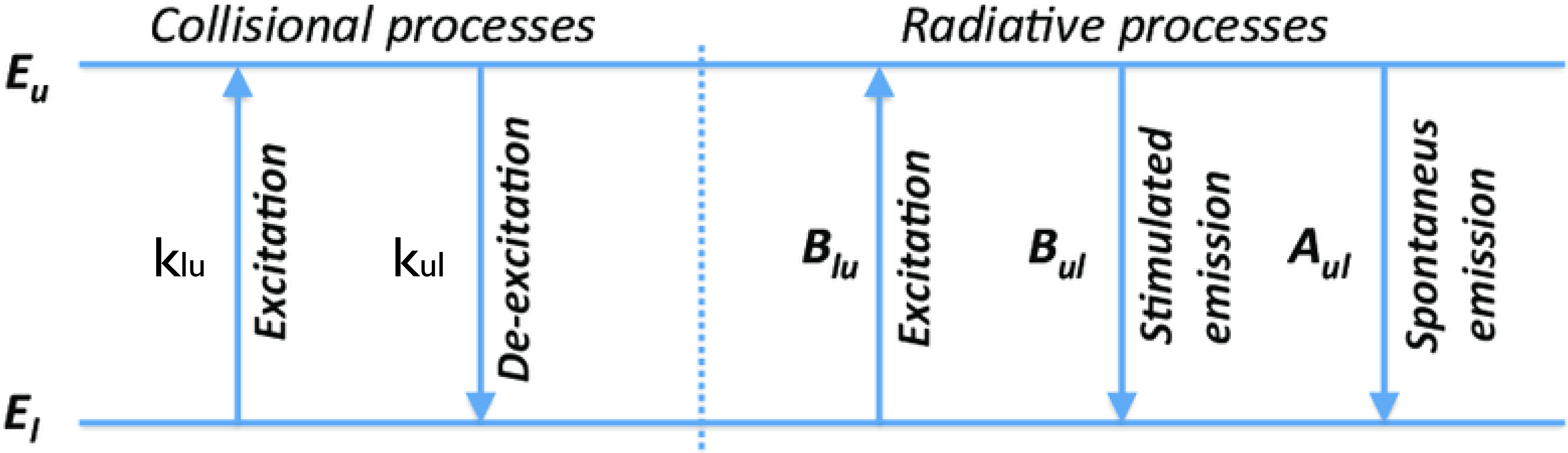
$$T_A(\nu) = T_C e^{-\tau_\nu}$$

- The EW of the absorption line is ~ 7 km/s.
- The HI emission line toward the line-of-sight that is very close to BL Lac provides the integrated line intensity of the emission.

$$T_{\rm exc} = \frac{\int T_A dv}{W_v} = \frac{930\,{\rm K\,km\,s^{-1}}}{7\,{\rm km\,s^{-1}}} \approx 130\,{\rm K}.$$



Radiative vs. Collisional excitation: is 21cm line radiatively excited?



k_{lu} and k_{ul} Rate coefficients

B_{lu}, B_{ul} et A_{ul} : Einstein coefficients

Radiative excitation recap

- The net change of upper level is:

$$\frac{dn_u}{dt} = n_\ell B_{\ell u}\varepsilon_\nu - n_u(A_{u\ell} + B_{u\ell}\varepsilon_\nu).$$

- With the relations among Einstein coefficients and the photon occupation number: $\bar{n}_\gamma \equiv \frac{c^3}{8\pi h\nu^3}\varepsilon_\nu$

$$\frac{dn_u}{dt} = n_\ell A_{u\ell}\frac{g_u}{g_\ell}\frac{c^3}{8\pi h\nu_{u\ell}^3}\varepsilon_\nu - n_u A_{u\ell}\left(1 + \frac{c^3}{8\pi h\nu_{u\ell}^3}\varepsilon_\nu\right) = n_\ell A_{u\ell}\frac{g_u}{g_\ell}\bar{n}_\gamma - n_u A_{u\ell}(1 + \bar{n}_\gamma).$$

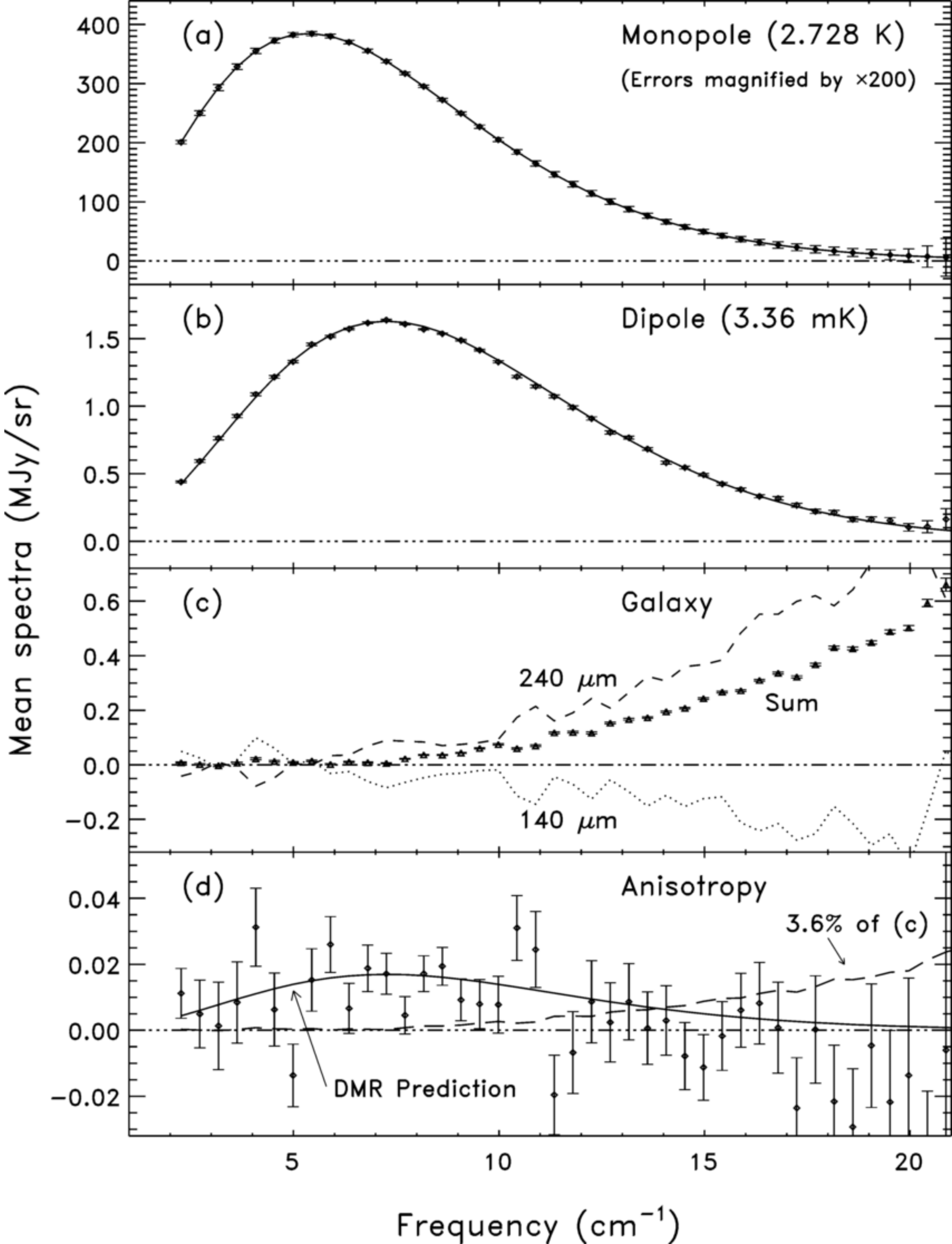
- In excitation eq, the relative population under blackbody radiation is:

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell}\frac{\bar{n}_\gamma}{1 + \bar{n}_\gamma} = \frac{g_u}{g_\ell}\exp\left(-\frac{h\nu_{u\ell}}{kT_{\rm rad}}\right).$$

Boltzmann distribution with radiation temperature!

Cosmic Microwave Background

$$u_{CMB} = aT_{CMB}^4 \approx 4.2 \times 10^{-13} erg/cm^3$$



Starlight in 1-13.6 eV

- Can be approximated by a sum of three dilute blackbodies:

$$\nu u_\nu = \sum_{j=1}^{3} \frac{8\pi h\nu^4}{c^3} \frac{W_j}{e^{h\nu/kT_j} - 1} \qquad \text{for } \lambda > 2450\,\text{Å}$$

- Habing (1968) convention:

$$\chi \equiv \frac{(\nu u_\nu)_{1000\,\text{Å}}}{4 \times 10^{-14}\,\text{erg cm}^{-3}} \qquad G_0 \equiv \frac{u(6 - 13.6\,\text{eV})}{5.29 \times 10^{-14}\,\text{erg cm}^{-3}}$$

Collisional (de-)excitation

- Inelastic collision between the target atoms and colliders!
- The rate equation is similar to the radiative excitation ones, except for the coefficients.

$$\frac{dn_u}{dt} = n_\ell n_c k_{\ell u} - n_u(n_c k_{u\ell} + A_{u\ell})$$

- "K"s are called collisional rate coefficients.

$$k_{u\ell}(T) = \langle \sigma_{u\ell} v \rangle = \left(\frac{8kT}{\pi m_r}\right)^{1/2} \int_0^\infty \sigma_{u\ell} \frac{E}{kT} \exp\left(-\frac{E}{kT}\right) \frac{dE}{kT}$$

- $\sigma_{u\ell}$ is independent of temperature for elastic collision between "billiard balls". Therefore, the rate coefficient has a temperature dependence of $k_{u\ell} \propto T^{1/2}$.
- However, when $\sigma_{u\ell} \propto E^n$, the rate coefficient $k_{u\ell} \propto T^{n+1/2}$.

Relationship between the two collisional rate coefficients

- The same as Einstein coefficients, the two collisional rate coefficients depends on each other.

$$\frac{n_u}{n_\ell} \approx \frac{g_u}{g_\ell} \exp\left(-\frac{h\nu_{u\ell}}{kT}\right) \qquad \frac{n_u}{n_\ell} = \frac{n_c k_{\ell u}}{n_c k_{u\ell} + A_{u\ell}} = \frac{g_u}{g_\ell} \frac{n_c k_{u\ell} e^{-h\nu_{u\ell}/kT}}{n_c k_{u\ell} + A_{u\ell}}$$

$$k_{\ell u} = \frac{g_u}{g_\ell} k_{u\ell} \exp\left(-\frac{h\nu_{u\ell}}{kT}\right)$$

- This can be understood by the fact that excitation from lower to upper levels requires a energy threshold of ~ hv. Gas temperature below this energy has a collisional excitation rate that is much smaller than the collisional de-excitation rate exponentially.

Combining radiative and collisional processes

$$\frac{dn_u}{dt} = n_\ell \left[A_{u\ell} \frac{g_u}{g_\ell} \bar{n}_\gamma + n_c k_{\ell u} \right] - n_u \left[A_{u\ell} (1 + \bar{n}_\gamma) + n_c k_{u\ell} \right].$$

- In the full eq state, the population ratio is determined by both coefficients:

$$\frac{n_u}{n_\ell} = \frac{A_{u\ell}(g_u/g_\ell)\bar{n}_\gamma + n_c k_{\ell u}}{A_{u\ell}(1 + \bar{n}_\gamma) + n_c k_{u\ell}}.$$

- 3 limiting cases:
  - 1) photon occupation number -> 0: pure collisional eq.
  - 2) collider density -> 0: pure radiation eq.
  - 3) blackbody radiation with temperature Trad= Tgas: independent of gas density! Photons alone are sufficient to bring the two-level system into LTE.

Critical density

- The density at which collisional deexcitation equals radiative deexcitation.

$$n_{\rm crit} = \frac{(1+\bar{n}_\gamma)A_{u\ell}}{k_{u\ell}}$$

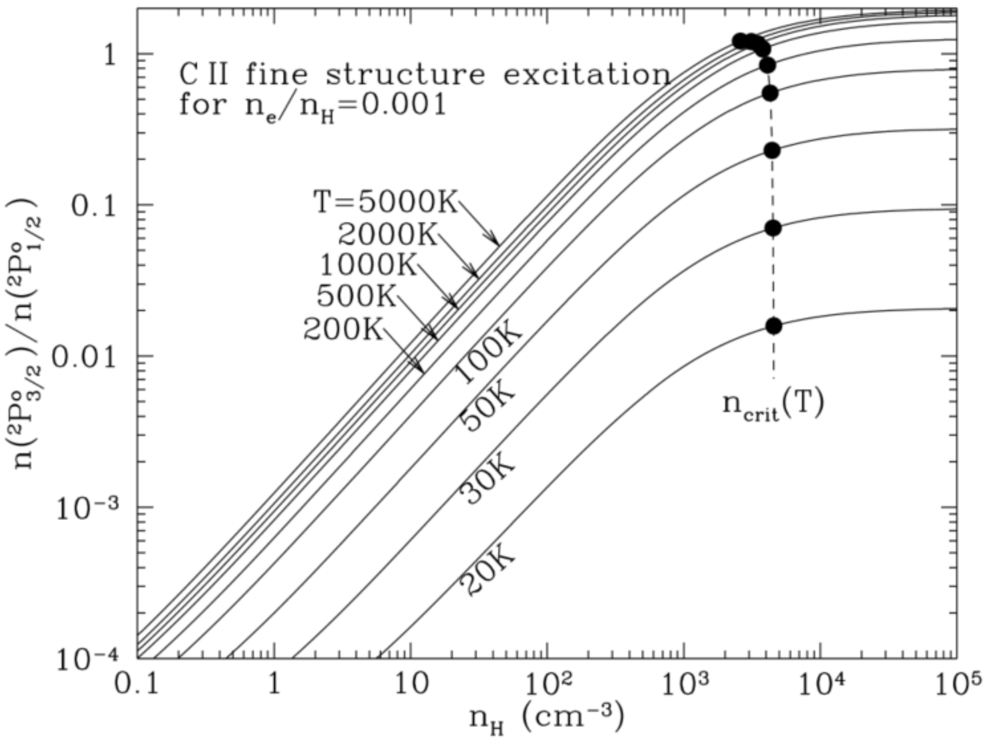
- For 21 cm levels, $k_{u\ell}(\mathrm{HH}) \approx 1.0 \times 10^{-10}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \left(\frac{T}{80\,\mathrm{K}}\right)^{0.78}$

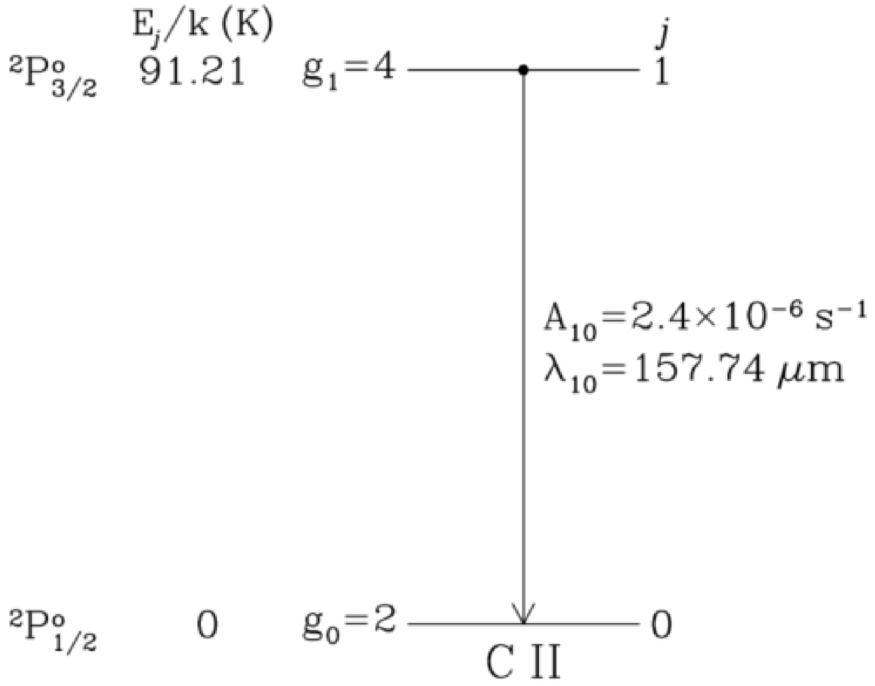
$$n_{\rm crit} = \frac{52.1(2.88 \times 10^{-15}\,\mathrm{s}^{-1})}{1.0 \times 10^{-10}\,\mathrm{cm}^3\,\mathrm{s}^{-1}} \approx 0.0015\,\mathrm{cm}^{-3}$$

[CII] 158 micro line as the main coolant of the ISM

- At 158 micro, the photon occupation number << 1 and therefore the stimulated emission can be safely ignored.

$$n_{\rm crit}({\rm H}) \approx 3.2 \times 10^3 T_2^{-0.1281-0.0087\ln T_2}\ {\rm cm}^{-3}$$





$$n_{\rm crit} = \frac{(1+\bar{n}_\gamma)A_{u\ell}}{k_{u\ell}}$$

Critical density of in HI regions and molecular gas

Ion	ℓ	u	E_ℓ/k (K)	E_u/k (K)	$\lambda_{u\ell}$ (μm)	$n_{\mathrm{H,crit}}(u)$ $T = 100\,\mathrm{K}$ (cm^{-3})	$n_{\mathrm{H,crit}}(u)$ $T = 5000\,\mathrm{K}$ (cm^{-3})
C II	$^2\mathrm{P}^\circ_{1/2}$	$^2\mathrm{P}^\circ_{3/2}$	0	91.21	157.74	2.0×10^3	1.5×10^3
C I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	23.60	609.7	620	160
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	23.60	62.44	370.37	720	150
O I	$^3\mathrm{P}_2$	$^3\mathrm{P}_1$	0	227.71	63.185	2.5×10^5	4.9×10^4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_0$	227.71	326.57	145.53	2.3×10^4	8.4×10^3
Si II	$^2\mathrm{P}^\circ_{1/2}$	$^2\mathrm{P}^\circ_{3/2}$	0	413.28	34.814	1.0×10^5	1.1×10^4
Si I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	110.95	129.68	4.8×10^4	2.7×10^4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	110.95	321.07	68.473	9.9×10^4	3.5×10^4

Molecule	Transition	E_u [K]	$n_{\mathrm{crit.}}$ [cm^{-3}]
CO	$(1 \rightarrow 0)$	5.5	$3 \cdot 10^3$
	$(2 \rightarrow 1)$	16.6	$1 \cdot 10^4$
CS	$(1 \rightarrow 0)$	2.4	$1 \cdot 10^5$
	$(2 \rightarrow 1)$	7.1	$7 \cdot 10^5$
$\mathrm{HCO^+}$	$(1 \rightarrow 0)$	4.3	$1.5 \cdot 10^5$
	$(3 \rightarrow 2)$	25.7	$3 \cdot 10^6$
HCN	$(1 \rightarrow 0)$	4.3	$4 \cdot 10^6$
	$(3 \rightarrow 2)$	25.7	$1 \cdot 10^7$
HNC	$(1 \rightarrow 0)$	4.3	$4 \cdot 10^6$
	$(3 \rightarrow 2)$	26.1	$1 \cdot 10^7$