AY2024F

Junjie Mao

Radiative Processes in Astrophysics

Observation

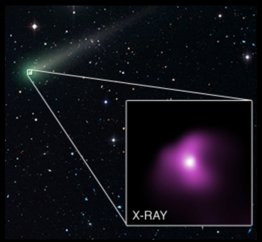
Up to cosmic size scale

Goal

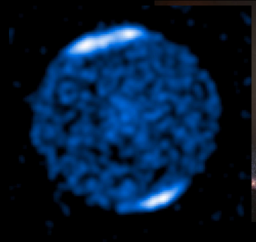
Theory

Down to atomic size scale

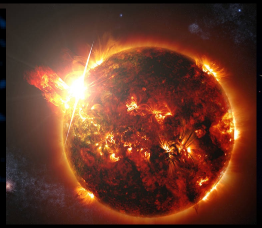
C/2012S1 (comet)



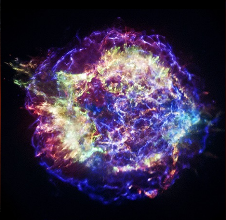
Jupiter (planet)



Sun (star)

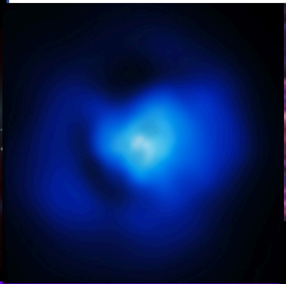


Cas A (SNR)



M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

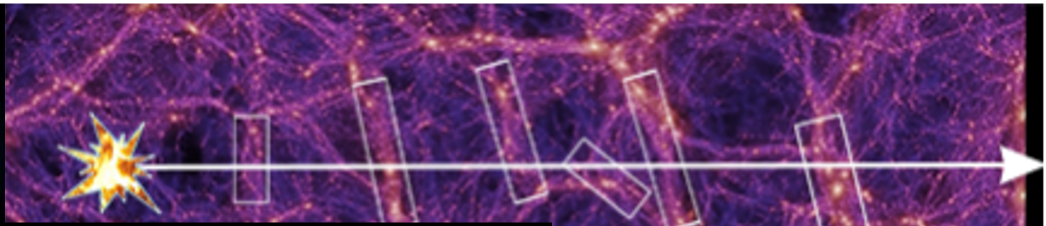


Image credit: Chandra photo album

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.1.1 Radiative flux
2.1.2 Radiative intensity
2.1.3 Specific intensity
2.1.4 Specific flux and momentum
2.1.5 Specific energy density

2.2 Radiative transfer
2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

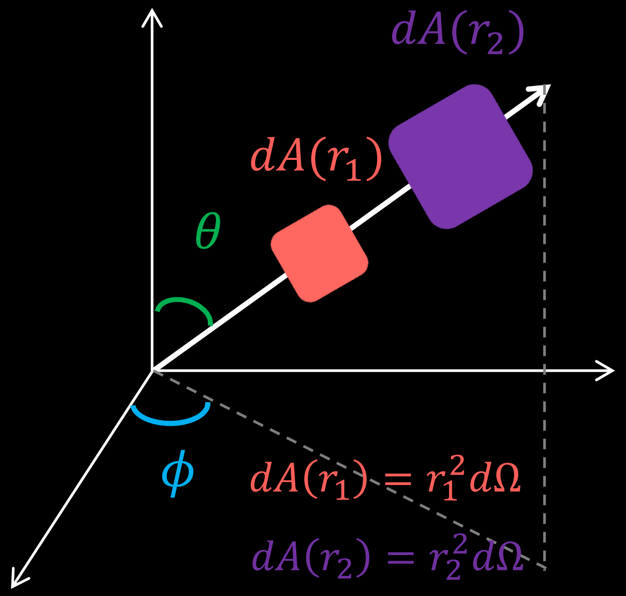


Image credit: Junjie Mao

Image credit: APOD

Radiative flux

For a detector element (with a photon collecting area dA) exposed to a radiation source for a time interval dt, the number of photons or the amount of energy received is

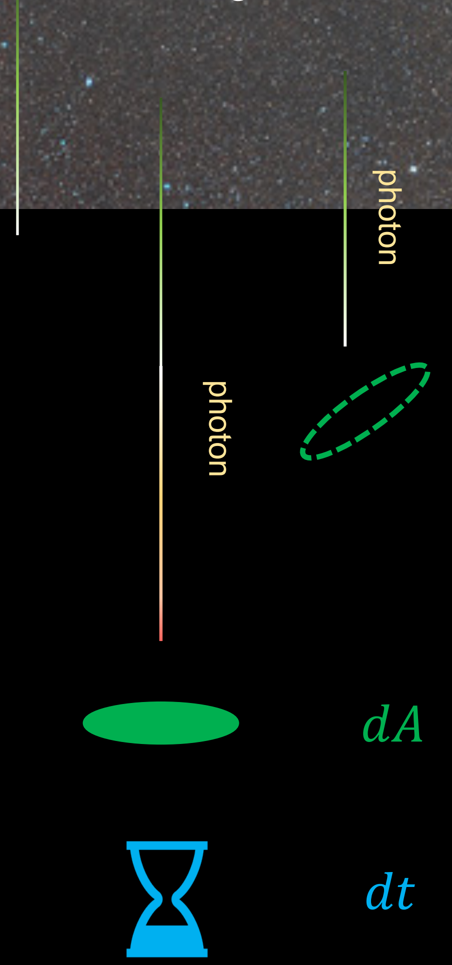
energy or photon flux

$$dF \equiv \frac{dE}{dA\,dt}$$

$\mathrm{erg\ cm^{-2}\ s^{-1}}$
$\mathrm{ph\ cm^{-2}\ s^{-1}}$

photon collecting area ($\mathrm{cm^2}$)

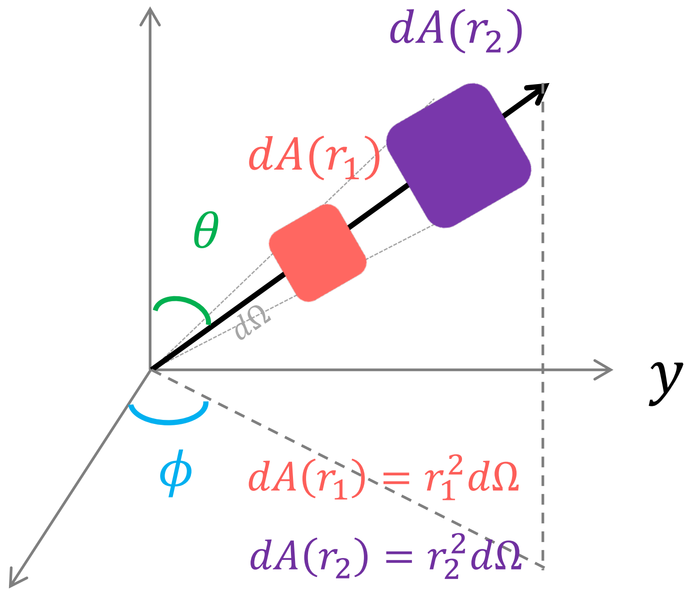
integration time (s)



F can vary for different orientations of the detector element

Inverse square law

Consider an **isotropic** source emitting photons/energy equally in all directions (e.g., a spherically symmetric and isolated star)



energy flux $(\mathrm{erg\ cm^{-2}\ s^{-1}})$

luminosity $(\mathrm{erg\ s^{-1}})$

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

distance to the source (cm)

$$dF(r) = \frac{dE}{dA(r)dt} = \frac{dE}{r^2 d\Omega dt}$$

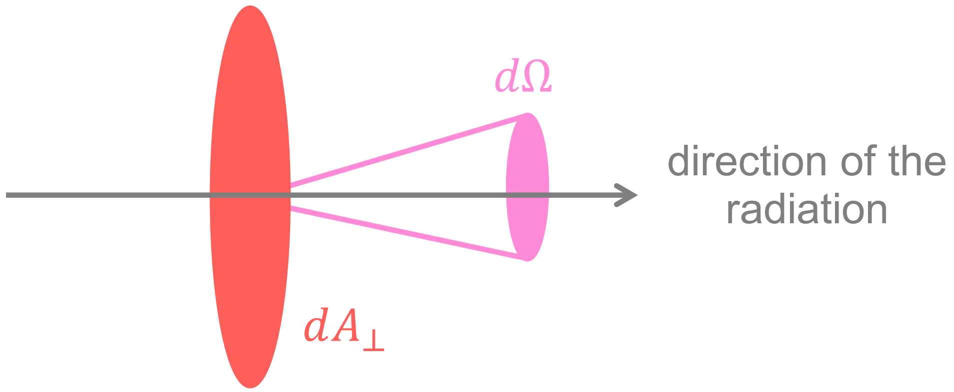
$$d\Omega = \sin\theta\ d\theta\ d\phi$$

Conservation of energy (no loss or gain)

$$F(r) = \frac{\text{constant}}{r^2}$$

Radiative intensity

Since flux changes with distance to the source, it is useful to a concept that is <u>**intrinsic**</u> to the properties of the source, e.g., luminosity (L), intensity (I)



Solar luminosity

$$L_\odot = 3.8\times10^{33}\ \mathrm{erg\ s^{-1}}$$

luminosity
($\mathrm{erg\ s^{-1}}$)

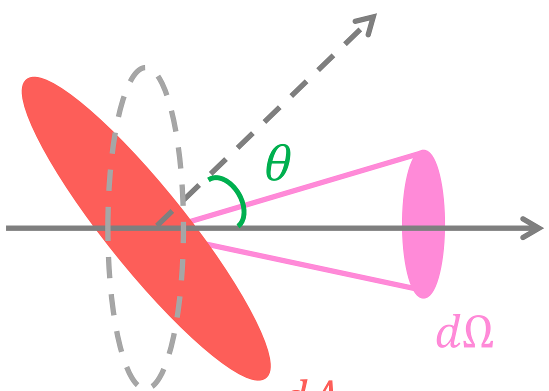
$$L \equiv \frac{dE}{dt}$$

$$I \equiv \frac{dE}{dt\, dA_\perp\, d\Omega}$$

By definition, $dA_\perp$ is normal to the direction of the radiation

Intensity and flux

Consider a radiation field (radiation in all directions), for a detector element (with photon collecting area dA) at some arbitrary orientation, we have



$$dF = I\cos\theta\, d\Omega$$

prev. sl.

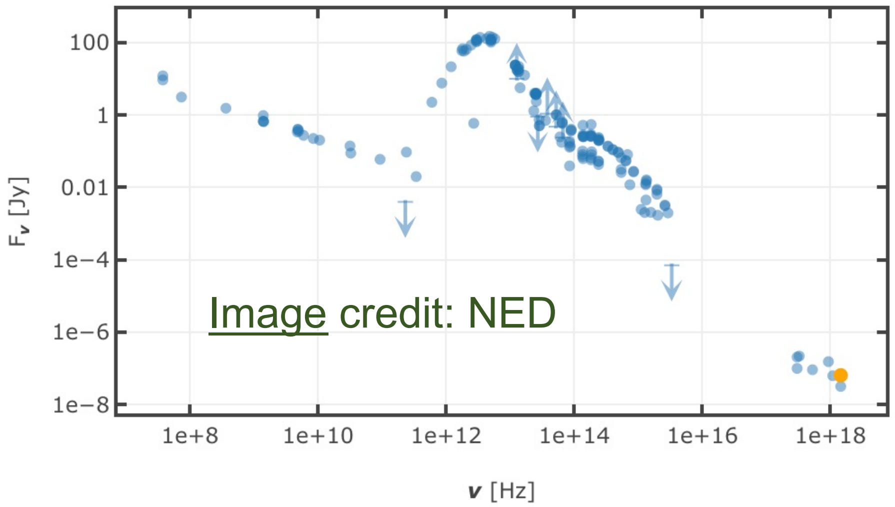
$$I = \frac{dE}{dt\, dA_\perp\, d\Omega}$$

prev. sl.

$$dF = \frac{dE}{dt\, dA}$$

effective area: $dA_\perp = dA\cos\theta$

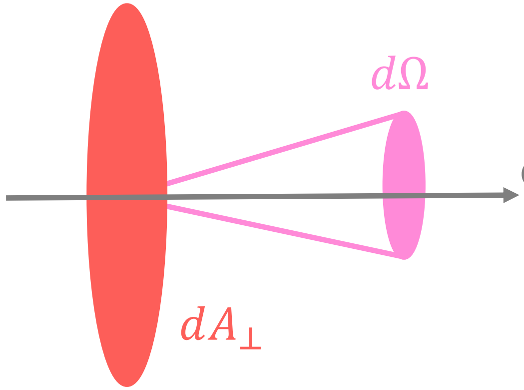
Specific intensity



Radiative energy in a specific frequency crossing unit area ($dA_\perp$) in unit time in unit solid angle

specific intensity

$$dE_\nu = I_\nu \, dA \, d\nu \, d\Omega \, dt$$



$$I_\nu \equiv \frac{dE_\nu}{dA_\perp \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1} sr^{-1}}$

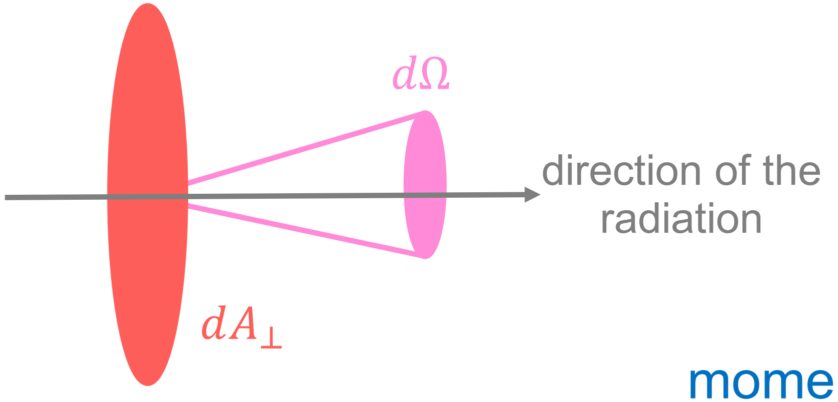
$$I_\nu = \frac{\lambda}{\nu} I_\lambda$$

prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

Specific flux and momentum

prev. sl. $F = \pi I$



$$F_\nu = \pi I_\nu$$

specific flux

specific intensity

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1}}$

$1\ \mathrm{erg} = 1\ \mathrm{g\ cm^2\ s^{-2}}$

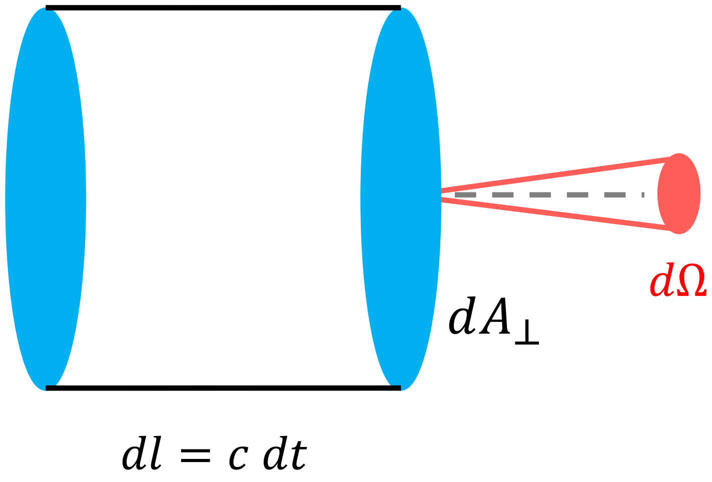
momentum $\longrightarrow$ $p_\nu = \dfrac{F_\nu}{c}$

$\mathrm{dyn\ Hz^{-1} cm^{-2}}$

$1\ \mathrm{dyn} = 1\ \mathrm{g\ cm\ s^{-2}}$

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\,dt$ and its volume $dA_\perp\,c\,dt$



$$dE_\nu = u_\nu(\Omega)\,d\Omega\,d\nu\,dA_\perp\,c\,dt$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$u_\nu = \frac{1}{c}\int I_\nu\,d\Omega = \begin{cases} \dfrac{4\pi}{c} I_\nu & \text{isotropic} \\ \dfrac{4\pi}{c} \bar{I}_\nu & \text{anisotropic} \end{cases}$$

$\mathrm{erg\ Hz^{-1}\ cm^{-3}}$

specific energy density

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer
2.2.1 Spontaneous emission coefficient
2.2.2 Absorption coefficient
2.2.3 Optical depth and mean free path
2.2.4 Radiative transfer equation

2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

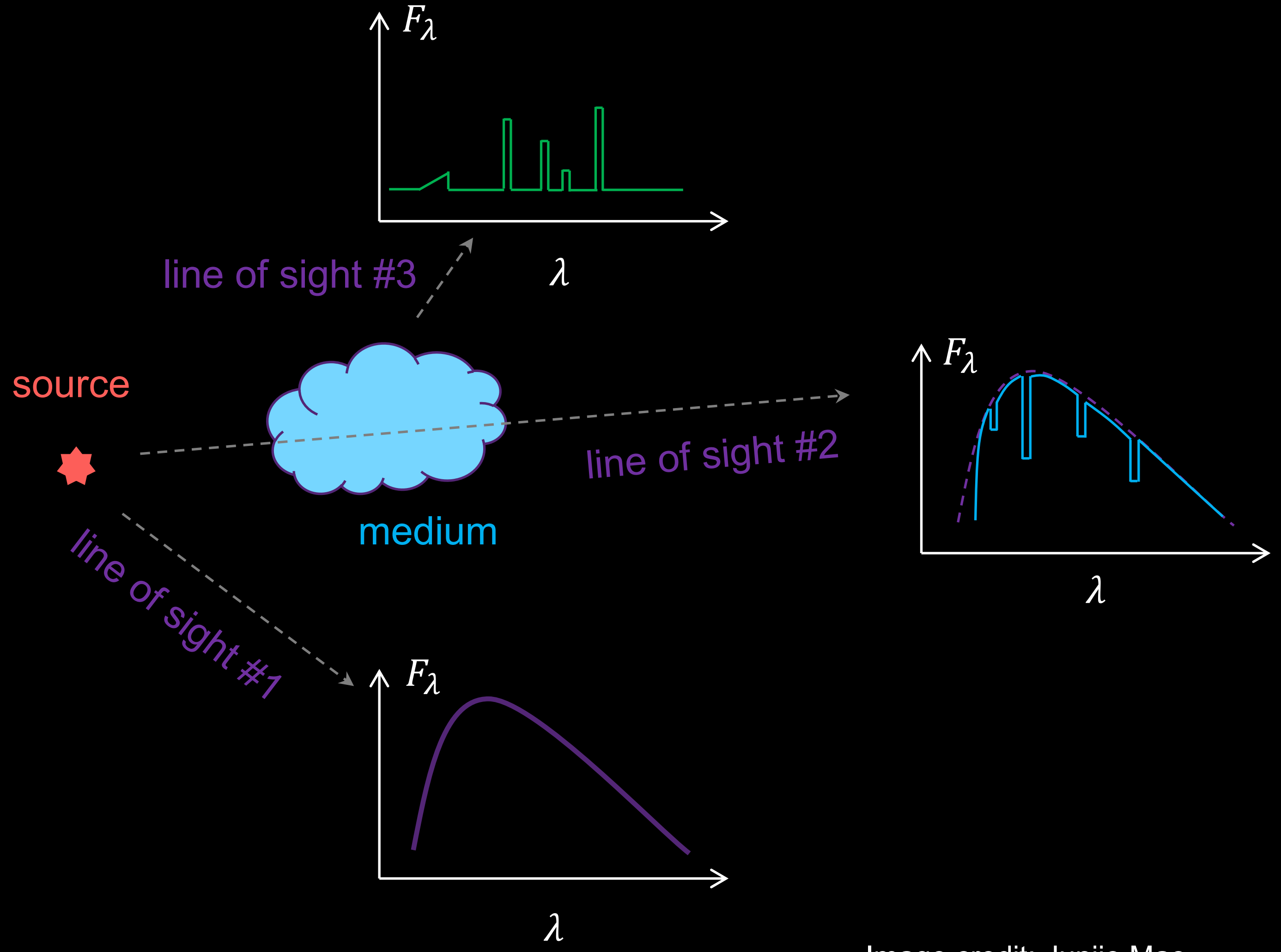


Image credit: Junjie Mao

Radiative transfer

- Radiation is one of the key processes to exchange energy for most celestial bodies

- When a ray passes through matter, the energy exchange process is referred to as radiation transfer

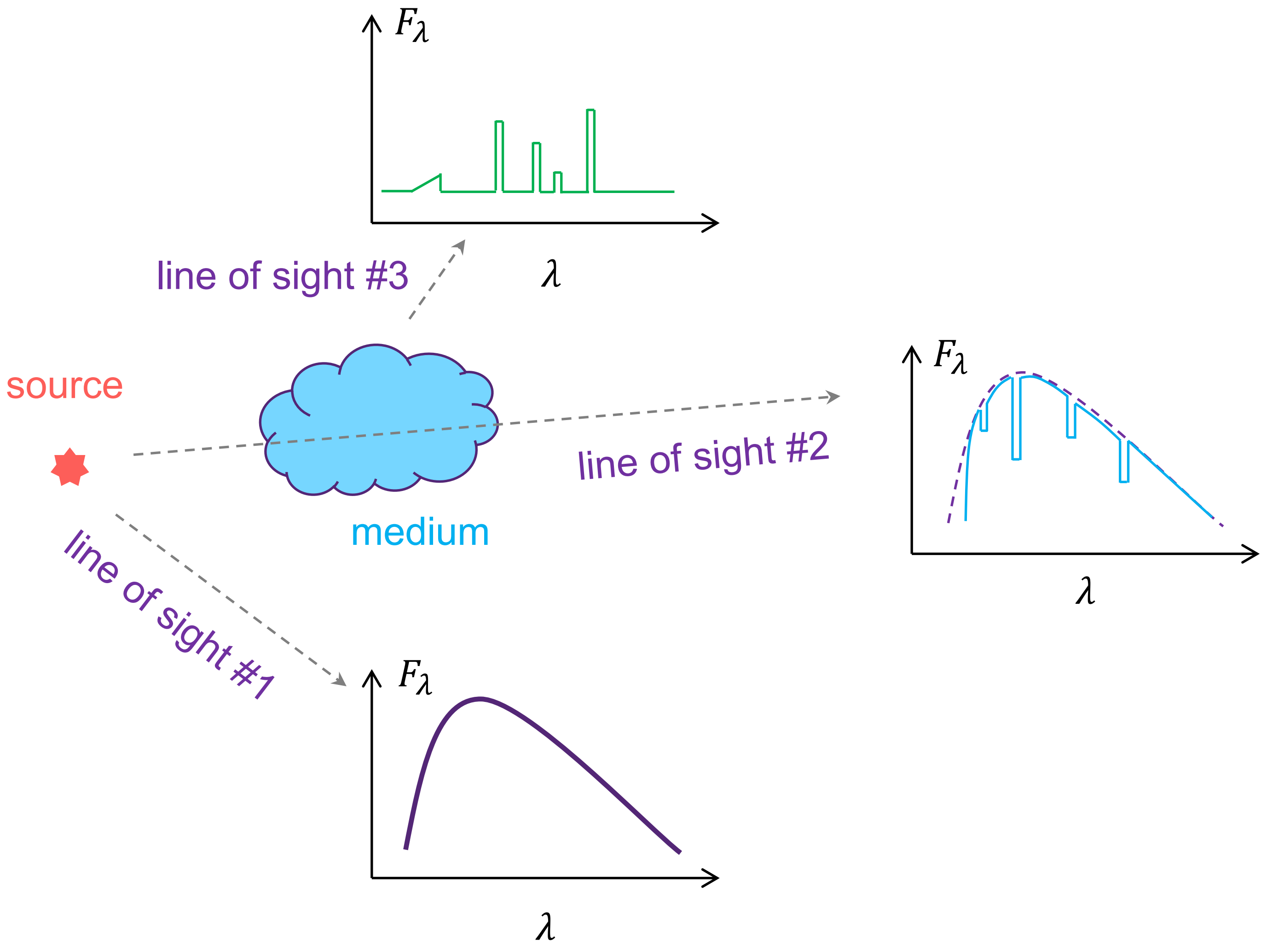
prev. sl.

unabsorbed flux

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

observed flux

$$F_{\mathrm{obs}} \leq \frac{L}{4\,\pi\, d^2}$$



Spontaneous emission coefficient

Radiative energy emitted spontaneously in a specific frequency in unit time in unit solid angle in unit volume

spontaneous emission coefficient

$$dE_\nu = j_\nu \, dV \, d\nu \, d\Omega \, dt$$

$$j_\nu \equiv \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-3} s^{-1} sr^{-1}}$

$\mathrm{Energy\ (frequency)^{-1} (volume)^{-1} (time)^{-1} (solid\ angle)^{-1}}$

Spontaneous emission coefficient (cont.)

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$dV = dA_\perp\, dl$

prev. sl.

$$I_\nu = \frac{dE_\nu}{dA_\perp\, d\nu\, d\Omega\, dt}$$

$$I_\nu = j_\nu\, dl$$

Emissivity

For an isotropic emitter or a distribution of randomly oriented emitters, the emission can also be characterized via emissivity ϵ_ν, which is the radiative energy (angle integrated) emitted spontaneously in a specific frequency per unit time per unit mass

$$dE_\nu = \epsilon_\nu \rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}$$

normalized fraction of energy radiated into the solid angle $d\Omega$

$$\epsilon_\nu \equiv \frac{dE_\nu}{\rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}}$$

$\mathrm{erg\,Hz^{-1} g^{-1} s^{-1}}$

mass density $\mathrm{g\,cm^{-3}}$

Spontaneous emission coefficient

For an isotropic emitter or a distribution of randomly oriented emitters,

prev. sl.

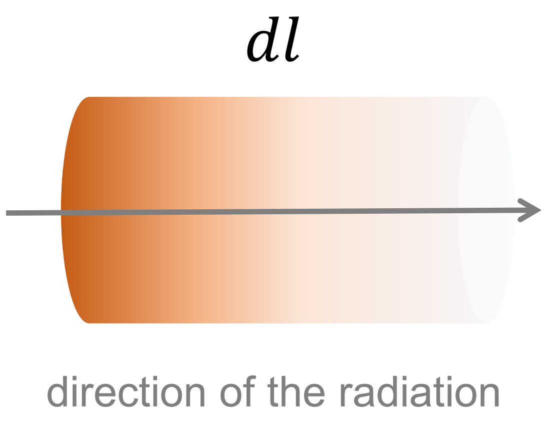
$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$$\epsilon_\nu = \frac{dE_\nu}{\rho\, dV dt\, d\nu \frac{d\Omega}{4\pi}}$$

$$j_\nu = \frac{\epsilon_\nu\, \rho}{4\pi}$$

Absorption coefficient

Loss of intensity in a beam as radiation travels a distance dl



absorption coefficient (positive values for energy taken out of the beam)

$$dI_\nu \equiv -\alpha_\nu \, I_\nu \, dl$$

$$\alpha_\nu = \rho \, \kappa_\nu$$

mass density $\mathrm{g\,cm^{-3}}$

mass absorption coefficient (a. k. a. opacity coefficient)

$$\int_{I_\nu^{\text{in}}}^{I_\nu^{\text{out}}} \frac{dI_\nu}{I_\nu} = -\int \alpha_\nu \, dl$$

$$\ln \frac{I_\nu^{\text{out}}}{I_\nu^{\text{in}}} = -\int \alpha_\nu \, dl$$

$$\frac{I_\nu^{\text{out}}}{I_\nu^{\text{in}}} = \exp\left(-\int \alpha_\nu \, dl\right)$$

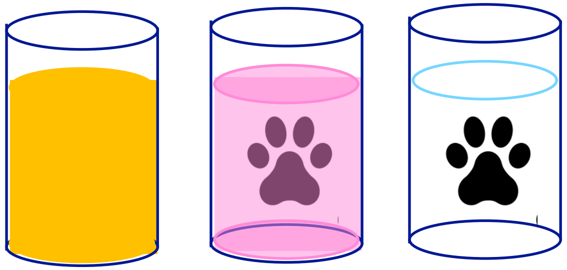
Optical depth

optical depth

absorption coefficient

$$I_\nu^{\mathrm{out}} = I_\nu^{\mathrm{in}} \exp(-\tau_\nu) \qquad \tau_\nu \equiv \int \alpha_\nu dl$$

Optical thin	$\tau_\nu \ll 1, I_\nu^{\mathrm{out}} \simeq I_\nu^{\mathrm{in}}$
	$\tau_\nu = 1, I_\nu^{\mathrm{out}} \sim 0.368\, I_\nu^{\mathrm{in}}$
Optical thick	$\tau_\nu \gg 1, I_\nu^{\mathrm{out}} \sim 0$



Mean free path

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu)$$

For homogeneous medium, $\bar{\tau}_\nu = \alpha_\nu \bar{l}_\nu$

$$\bar{\tau}_\nu = \frac{\int_0^\infty \tau_\nu \exp(-\tau_\nu)\, d\tau_\nu}{\int_0^\infty \exp(-\tau_\nu)\, d\tau_\nu} = 1$$

$$\int_0^\infty x \exp(-x)\ dx = 1$$

$$\int_0^\infty \exp(-x)\ dx = 1$$

Mean free path (the average distance a photon can travel within a medium without being absorbed)

$$\bar{l}_\nu = \frac{1}{\alpha_\nu}$$

Radiative transfer equation (cont.)

Conservation of energy (no loss or gain)

$$dE^{\text{out}} = dE^{\text{in}} + dE^{\text{abs}} + dE^{\text{em}}$$

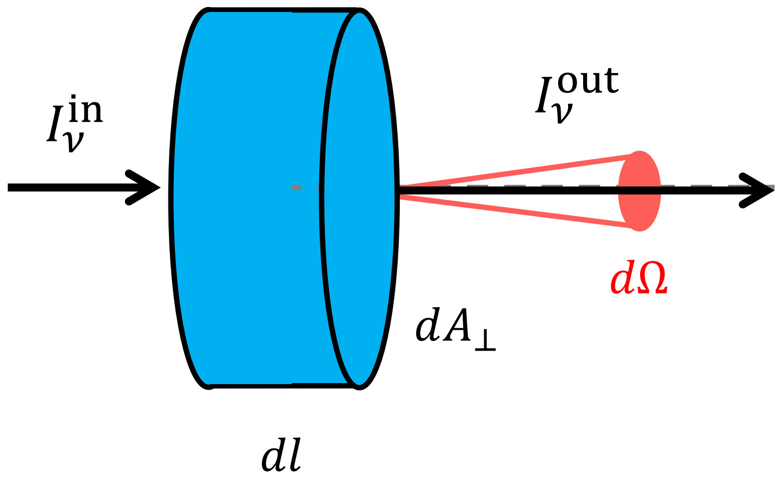
prev. sl.

$$dE^{\text{in}} = I_\nu \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{out}} = (I_\nu + dI_\nu) \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{em}} = j_\nu \, d\nu \, d\Omega \, dA_\perp \, dl \, dt$$

$$dE^{\text{abs}} = -\alpha_\nu dE^{in} dl$$



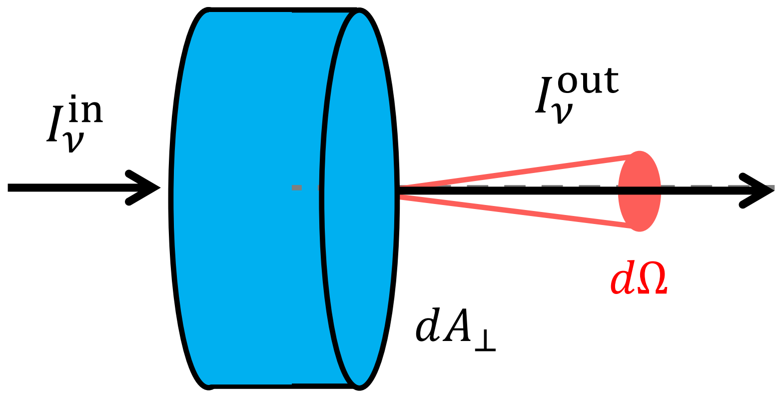
$$\frac{dI_\nu}{dl} = -\alpha_\nu \, I_\nu + j_\nu$$

Radiative transfer equation (cont.)

prev. sl.

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$d\tau_\nu = \alpha_\nu dl$



$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

Source function

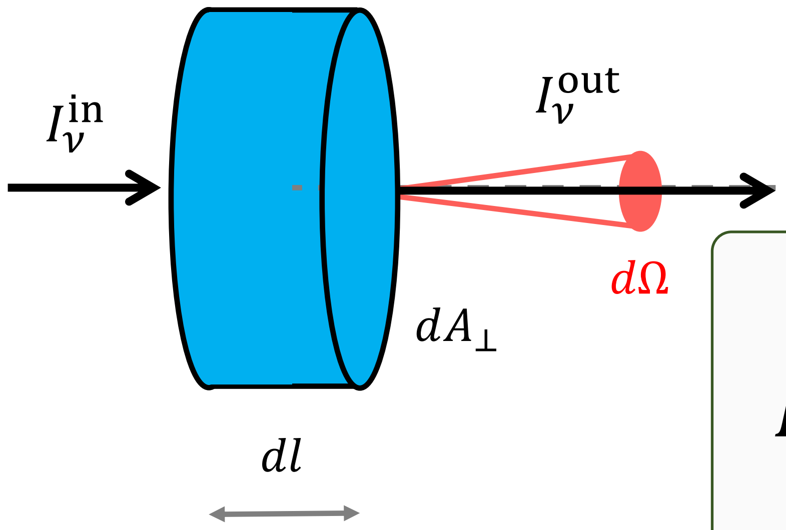
Solution to the RT equation

prev. sl.

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} = S_\nu e^{\tau_\nu} - I_\nu e^{\tau_\nu}$$

$$e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = \frac{d}{d\tau_\nu}\left(I_\nu e^{\tau_\nu}\right) = S_\nu e^{\tau_\nu}$$



$$\tau_{\nu,0} = \int \alpha_\nu dl$$

$$I_\nu^{\mathrm{out}} = I_\nu^{\mathrm{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

see Sect. 3.2 of 《天体物理中的辐射机制》by 尤峻汉 (p81–82)

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer

2.3 Thermal radiation
2.3.1 Blackbody radiation spectrum
2.3.2 Blackbody emission intensity
2.3.3 Blackbody emission flux
2.3.4 Thermal equilibrium
2.3.5 Local thermal equilibrium

2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

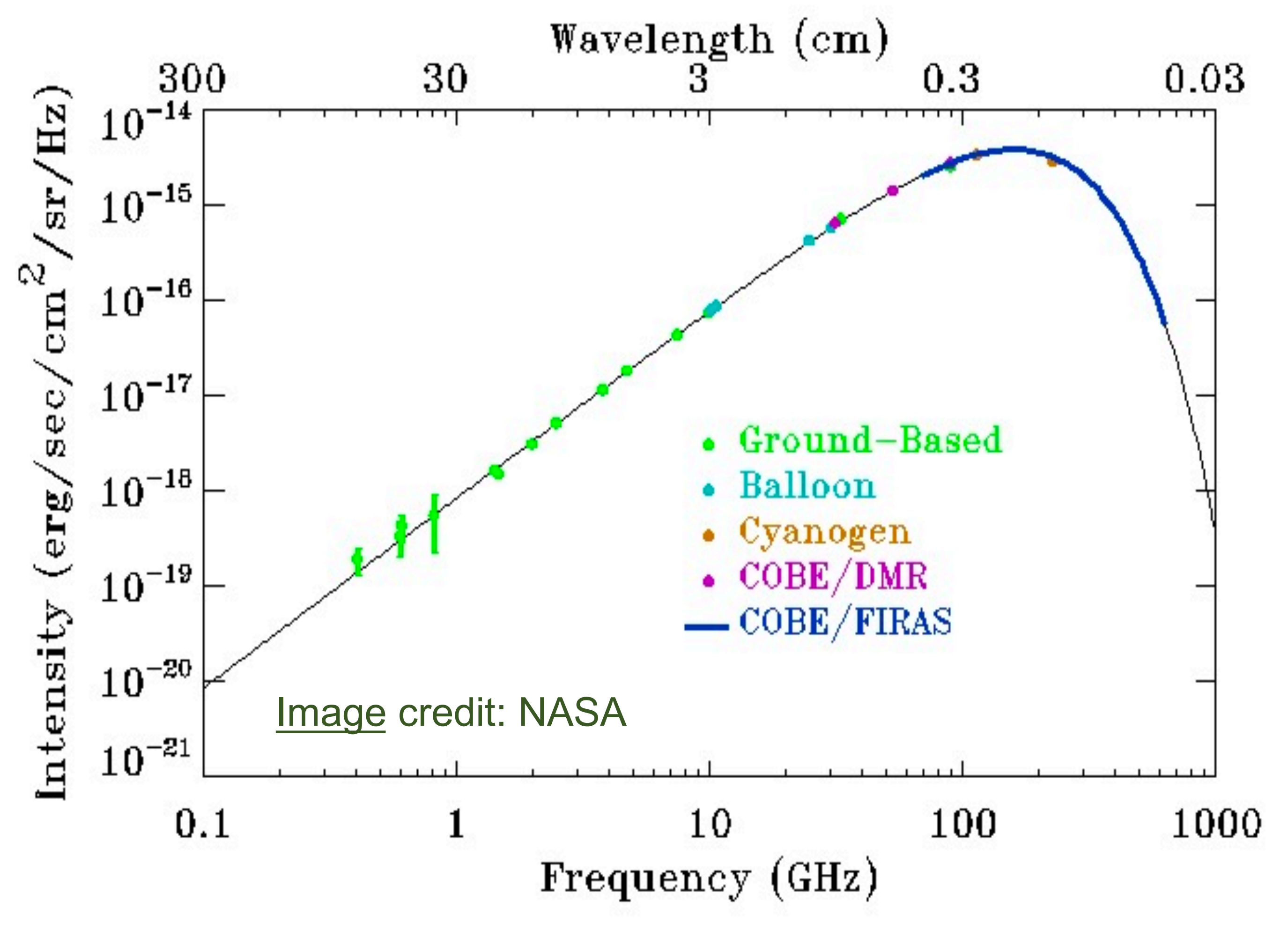


Image credit: NASA

Blackbody radiation spectrum

Thermal radiation is radiation emitted by matter in thermal equilibrium. In this case, the free electrons follows the Maxwellian distribution

$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT_e}\right)$$

Blackbody radiation spectrum is the Planck spectrum

$$h\nu_{\max} = 2.82\ kT$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)^{-1}$$

Caution that even if particles in the matter follow the Maxwellian distribution, radiation emitted by this matter does **not** necessarily have to be thermal radiation

Blackbody emission intensity

$$I_\nu^{\mathrm{BB}} \equiv B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

Blackbody emission intensity

$$I^{\mathrm{BB}} = \int_0^\infty \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} d\nu$$

$$= \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x - 1)^{-1}dx$$

$$x = \frac{h\nu}{kT}$$

Blackbody emission flux

prev. sl.

$$I^{\mathrm{BB}} = \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x-1)^{-1}dx$$

Blackbody emission flux

$$F^{\mathrm{BB}} = \pi I^{\mathrm{BB}} = \sigma T^4$$

$$\int_0^\infty x^3(e^x-1)^{-1}dx = \frac{\pi^4}{15}$$

$$\sigma = \frac{2\pi^5k^4}{15\,h^3c^2}$$

$$= 5.670\times 10^{-5}\ \mathrm{erg\,s^{-1}cm^{-2}K^{-4}}$$

Stefan-Boltzmann constant

Blackbody radiation flux depends **only** on the temperature

Asymptotic behavior

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

$$\frac{h\nu}{kT} \ll 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \to \frac{h\nu}{kT} \qquad I_\nu^{\mathrm{RJ}} = \frac{2\nu^2}{c^2}kT$$

Rayleigh-Jeans Law

$$\frac{h\nu}{kT} \gg 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \to \exp\left(\frac{h\nu}{kT}\right) \qquad I_\nu^{\mathrm{W}} = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$$

Wien Law

Thermodynamic Equilibrium (TE)

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

For medium in thermodynamic equilibrium, Kirchoff's law links the spontaneous emission coefficient and emission source function with Planck's law

$$j_\nu \equiv \alpha_\nu B_\nu(T)$$
$$S_\nu \equiv B_\nu\,(T)$$

TE (cont.)

prev. sl.

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

$$I_\nu^{\text{out}} = B_\nu(T)\left(1 - \exp\left(-\tau_{\nu,0}\right)\right)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \gg 1 \qquad I_\nu^{\text{out}} = B_\nu(T)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \ll 1 \qquad I_\nu^{\text{out}} = \tau_{\nu,0} B_\nu(T)$$

Only if the medium is in TE and it is optically thick, then its radiation follows the Planck‘s Law.
For non-thermal processes, the source function can only be derived from j_ν and α_ν.

Local Thermodynamic Equilibrium (LTE)

LTE differs from TE in that the radiation field does not need to follow Planck's law, i.e., $I_\nu \neq B_\nu(T)$.

The assumption of LTE is valid if either of the follow two applies:

- ❑ collisional processes among particles dominate photon-related processes
- ❑ collisional processes among particles are in equilibrium with photo-related processes

Chpt.2 Fundamentals of radiation

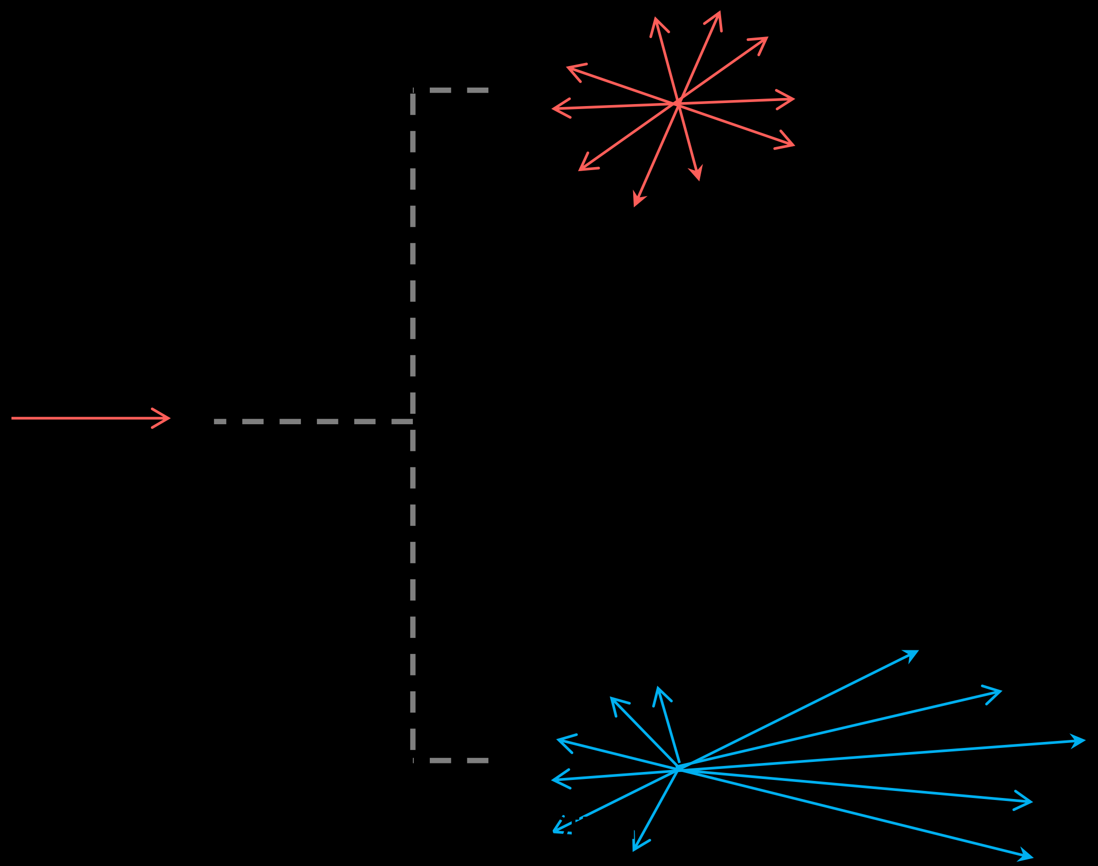
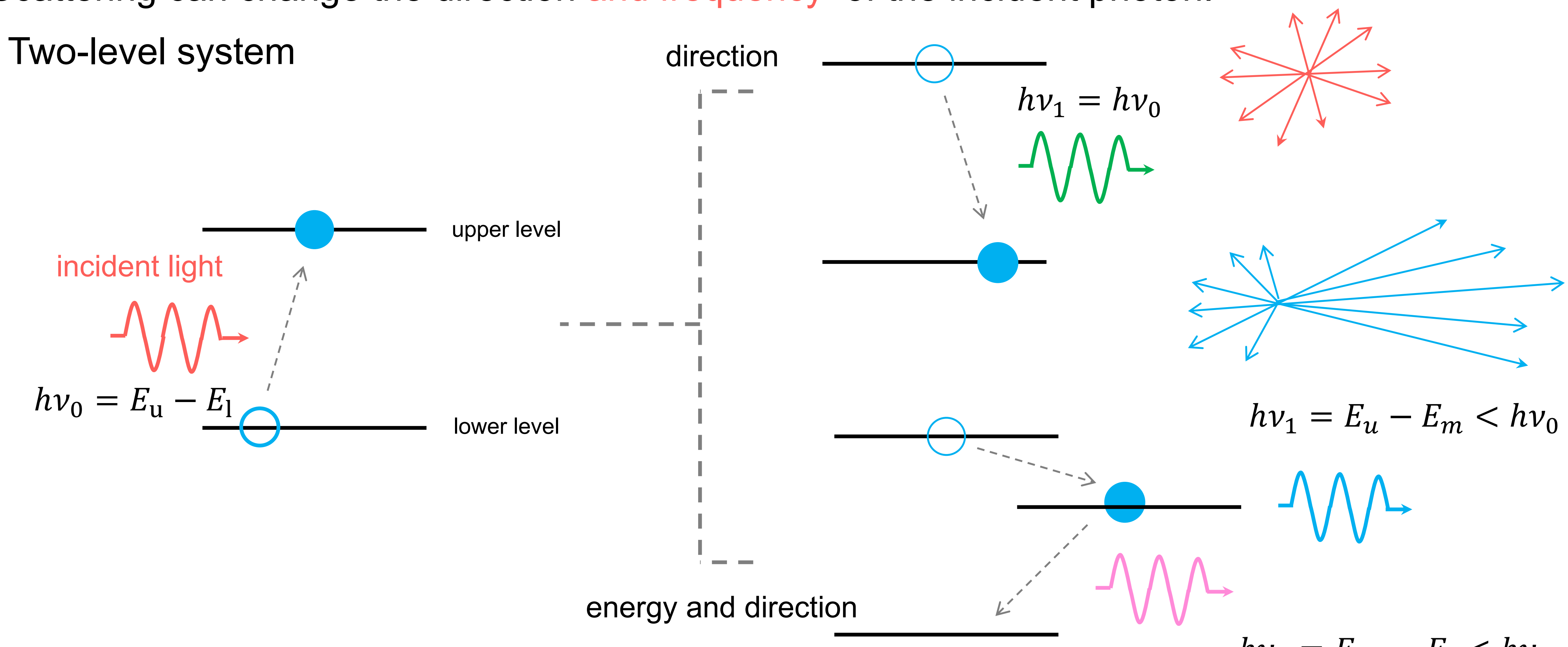


Image credit: Junjie Mao

Scattering

Scattering significantly complicates the radiative transfer.
Scattering can change the direction and frequency of the incident photon.



Pure scattering

Consider a medium has only isotropic scattering (no emission or absorption), which only changes the direction of the incident photon

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}$$

scattered outside the line of sight

scattered back into the line of sight

scattering coefficient

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{sca}}} = -I_\nu + S_\nu^{\mathrm{sca}} \qquad d\tau_\nu^{\mathrm{sca}} \equiv \sigma_\nu^{\mathrm{sca}} dl$$

scattering optical depth

Source function for scattering

Random walk

The mean displacement traveled by photons is (order-of-magnitude estimation)

see Sect. 1.7 of the REF book (p35) by Rybicki & Lightman

$$\bar{l}_\nu^{\mathrm{RM}} \sim \sqrt{N}\, \bar{l}_\nu$$

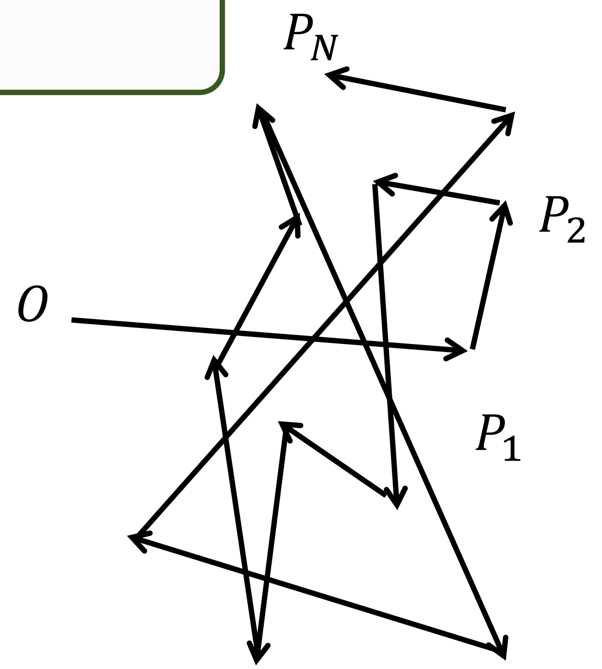
$\bar{l}_\nu = \frac{1}{\sigma_\nu^{\mathrm{sca}}}$: scattering mean free path

Number of scattering

size scale of the medium

for $\tau_\nu^{\mathrm{sca}} \gg 1$ $\quad N \sim \frac{L^2}{\bar{l}_\nu^2} = (\sigma_\nu^{\mathrm{sca}} L)^2 \sim (\tau_\nu^{\mathrm{sca}})^2$

for $\tau_\nu^{\mathrm{sca}} \ll 1$ $\quad N \sim \sigma_\nu^{\mathrm{sca}} L \sim \tau_\nu^{\mathrm{sca}}$



Emission + absorption + scattering

Considering scattering + absorption + emission for a medium in TE

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + j_\nu - \alpha_\nu I_\nu$$

$$j_\nu = \alpha_\nu B_\nu$$

$$\frac{dI_\nu}{dl} = -(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) I_\nu + (\sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + \alpha_\nu B_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{AS}}} = -I_\nu + S_\nu^{\mathrm{EAS}}$$

$$S_\nu^{\mathrm{EAS}} \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Source function for emission, absorption, and scattering

$$(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) dl = d\tau_\nu^{\mathrm{AS}}$$

Absorption and scattering probability

An alternative way to define the source function

$$S_\nu^{\mathrm{EAS}} = \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Introducing absorption and scattering probabilities

absorption probability

$$\epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

scattering probability

$$1 - \epsilon_\nu = \frac{\sigma_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

$$S_\nu^{\mathrm{EAS}} = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) S_\nu^{\mathrm{sca}}$$

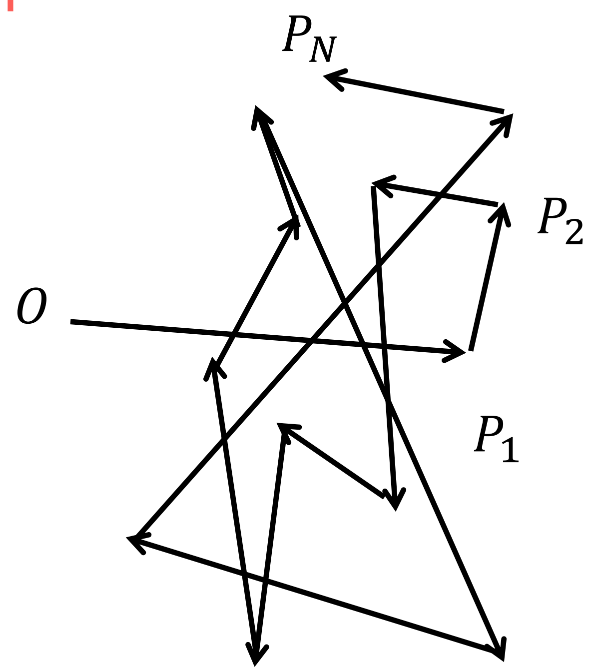
Effective mean free path

prev. sl. $\epsilon_\nu = \dfrac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$

After $N = \epsilon_\nu^{-1}$ steps, a photon is **truly** absorbed

$\bar{l}_\nu^{\mathrm{AS}} = \frac{1}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$: mean free path

$$\left(\bar{l}_\nu^{\mathrm{RM}}\right)^2 = N\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2 = \frac{\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2}{\epsilon_\nu} = \frac{1}{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}$$



$$l_\nu^{\mathrm{eff}} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}}$$

effective mean free path
diffusion length
thermalization length

for $\alpha_\nu \ll \sigma_\nu^{\mathrm{sca}}$ $\quad l_\nu^{\mathrm{eff}} = \dfrac{1}{\sqrt{\alpha_\nu \sigma_\nu^{\mathrm{sca}}}}$