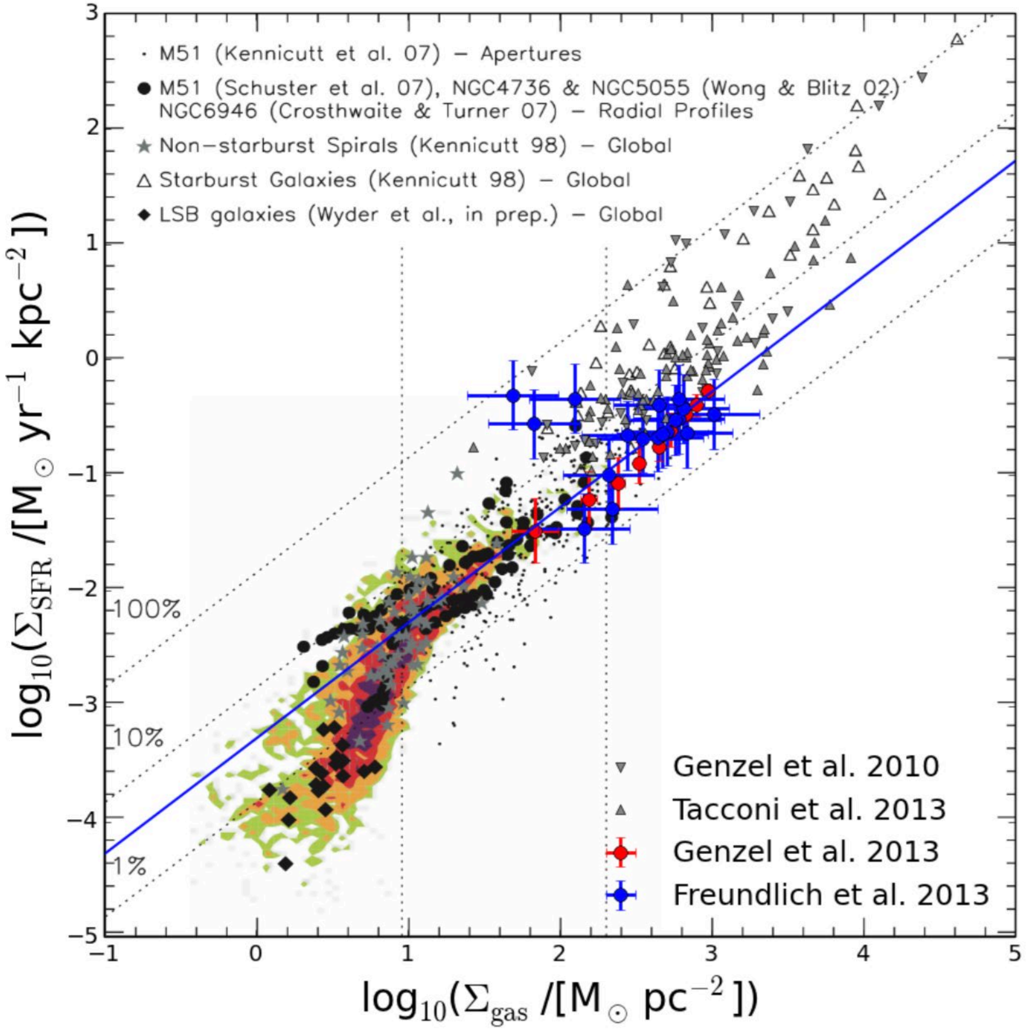
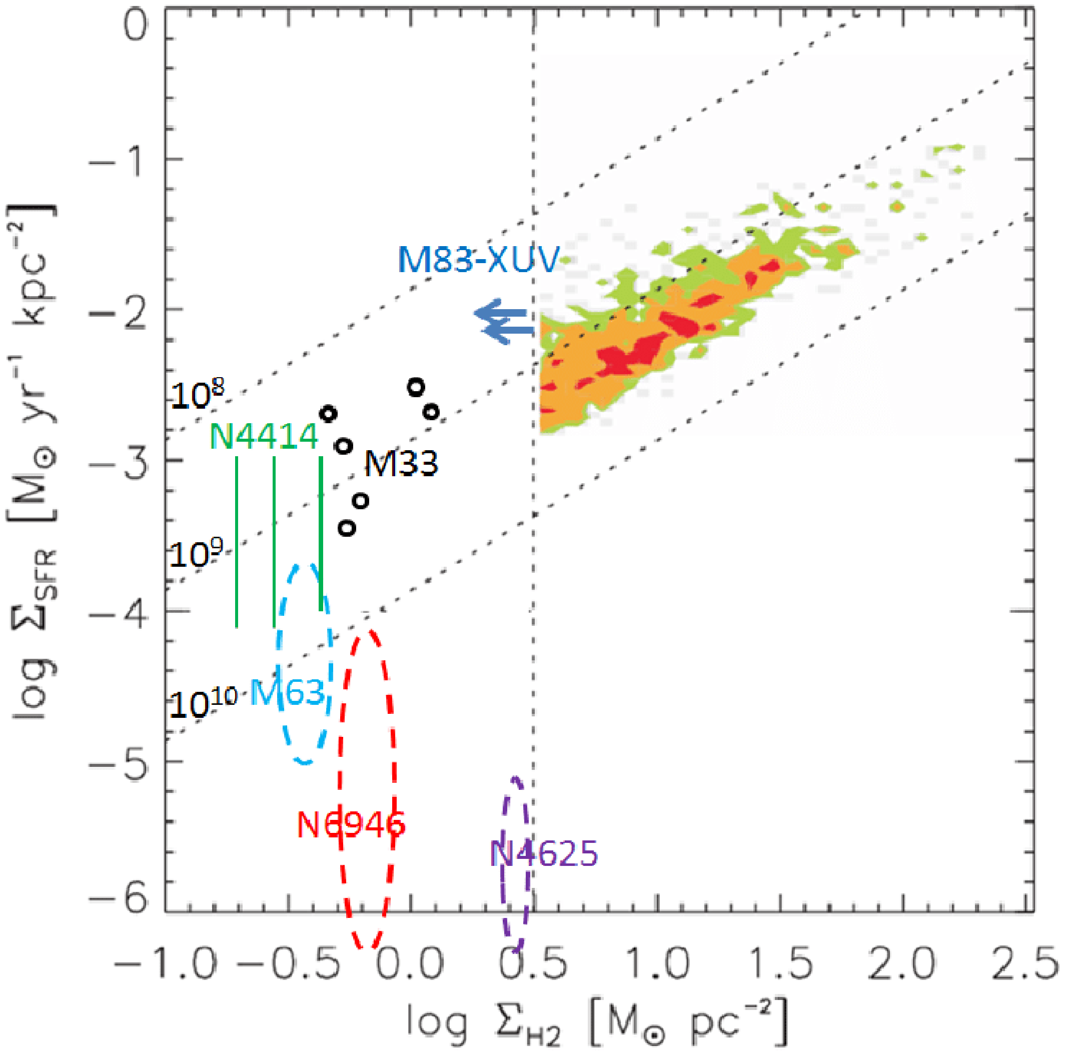
10. Star Formation

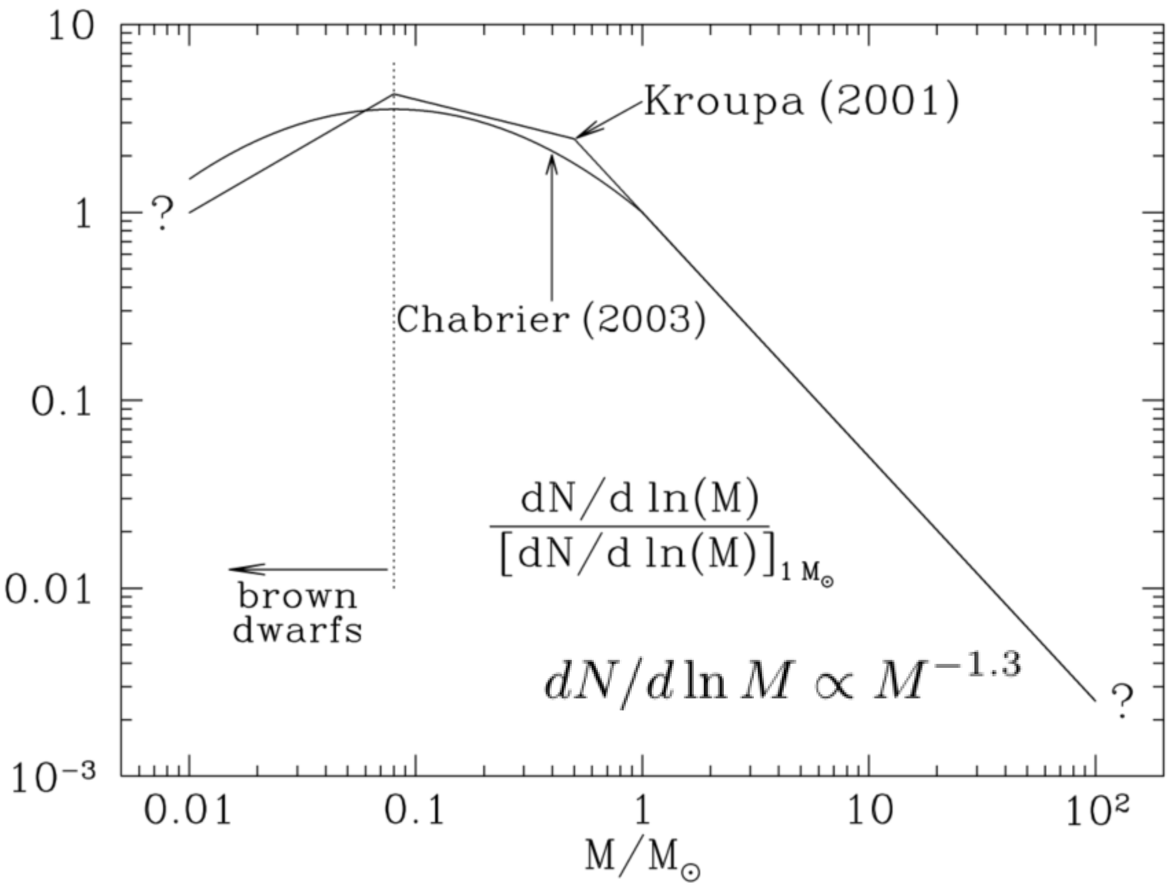
Instructor: Hui Li TA: Chengzhe Li
Department of Astronomy
Tsinghua University

Kennicutt-Schmidt Relation



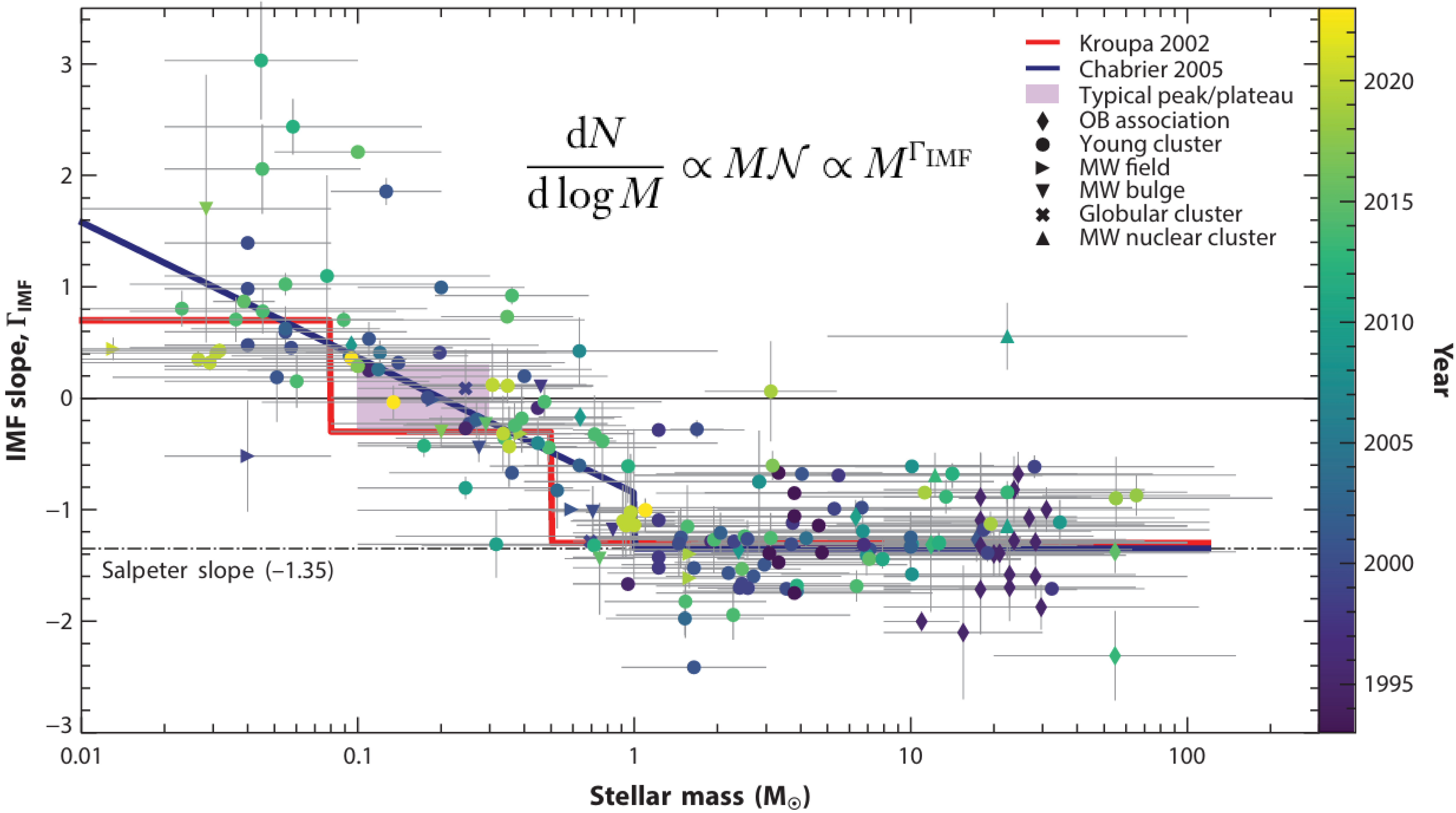


Initial mass function: a unique laboratory to test theories



Mass range ($M_\odot$)	mass/total mass	$\langle M \rangle / M_\odot$
0.01–0.08	0.0482	0.0379
0.08–1	0.3950	0.2830
1–8	0.3452	2.156
8–16	0.0749	10.96
16–100	0.1369	32.31
8–100	0.2118	19.14
0.01–100	1.0000	0.3521

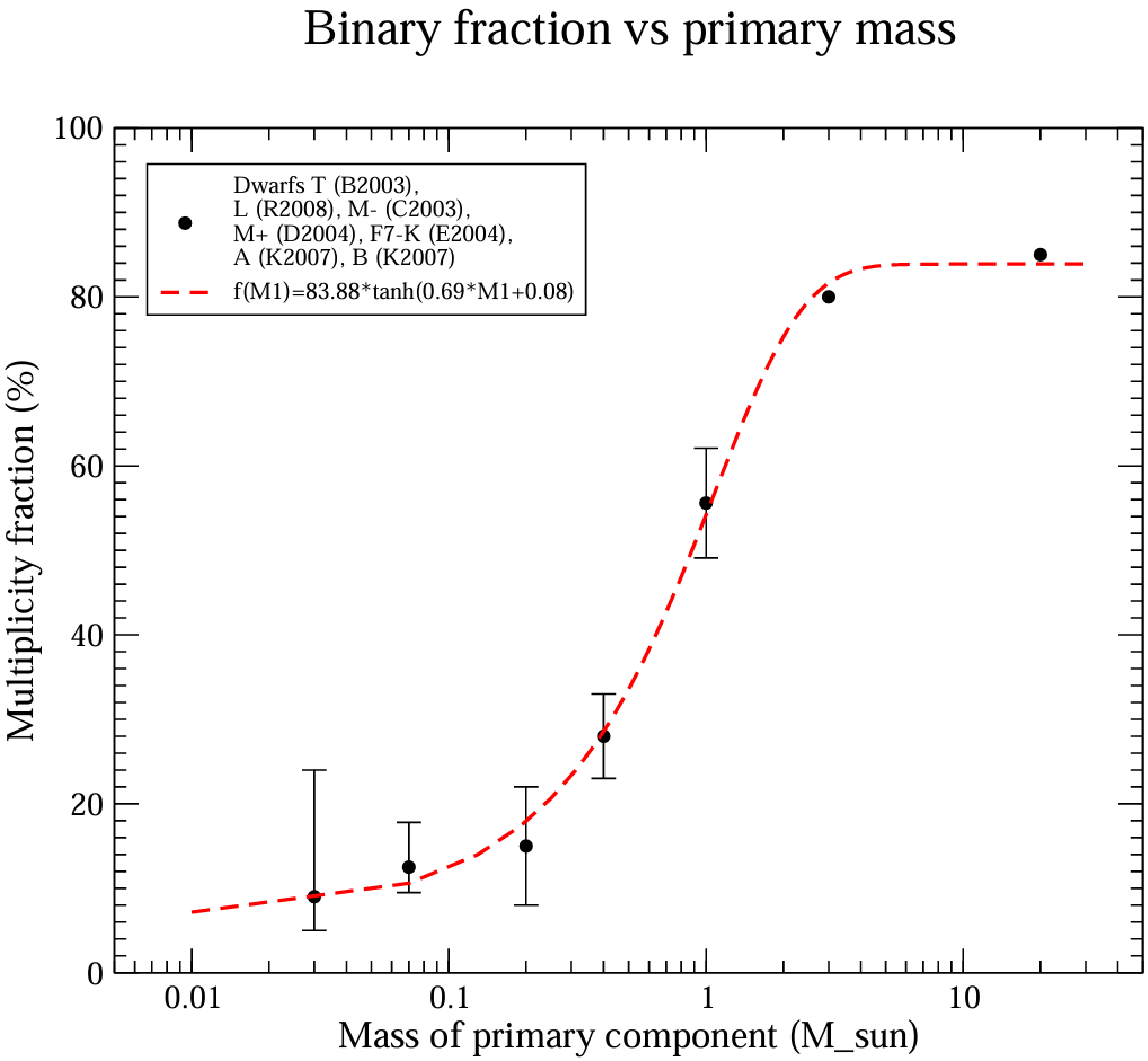
[a] For lower and upper cutoffs of 0.01 and 100 $M_\odot$.



Kroupa 2002
Chabrier 2005
Typical peak/plateau
OB association
Young cluster
MW field
MW bulge
Globular cluster
MW nuclear cluster
$\frac{dN}{d\log M} \propto MN \propto M^{\Gamma_{\rm IMF}}$
Salpeter slope (–1.35)
IMF slope, $\Gamma_{\rm IMF}$
Stellar mass ($M_\odot$)
Year
3
2
1
0
−1
−2
−3
0.01
0.1
1
10
100
2020
2015
2010
2005
2000
1995

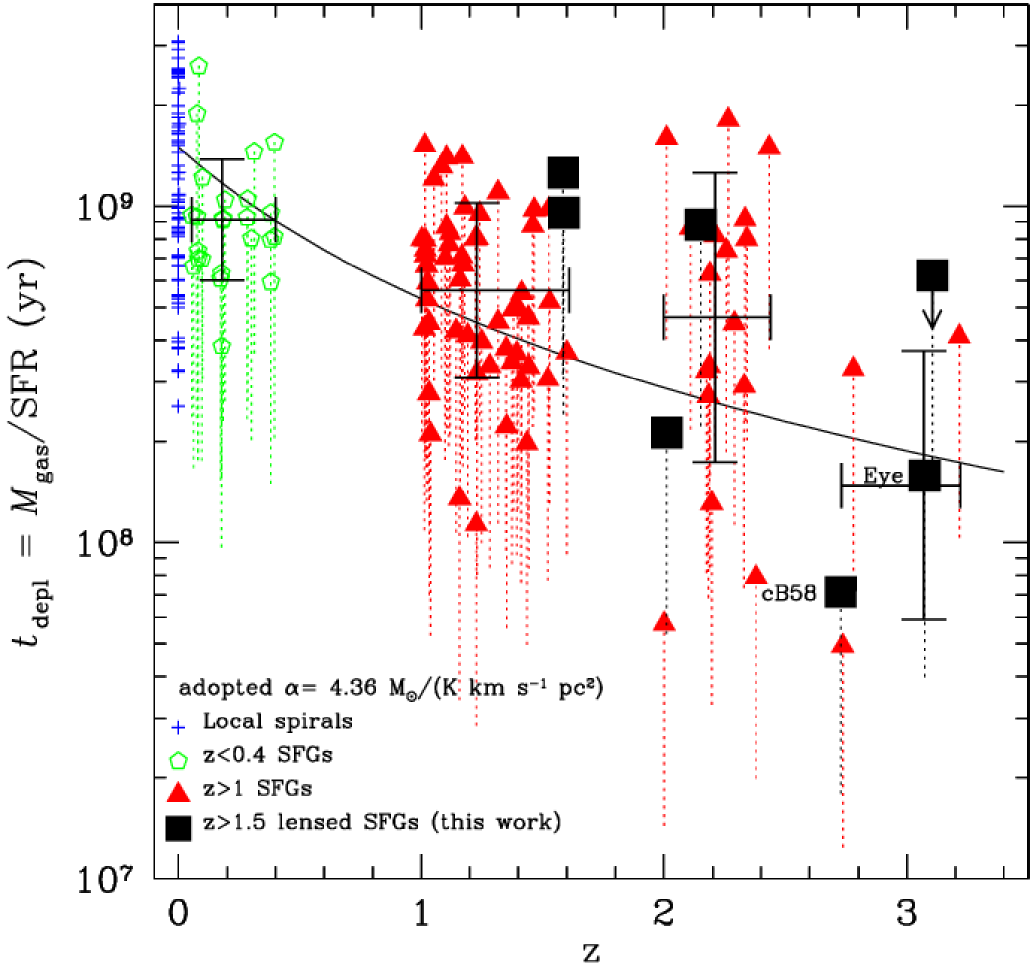
Binaries!

- Binary (multiplicity) fraction is a strong function of the primary mass.
- Almost all massive stars are formed in multiple systems.
- Primordial vs. dynamical-captured binaries?



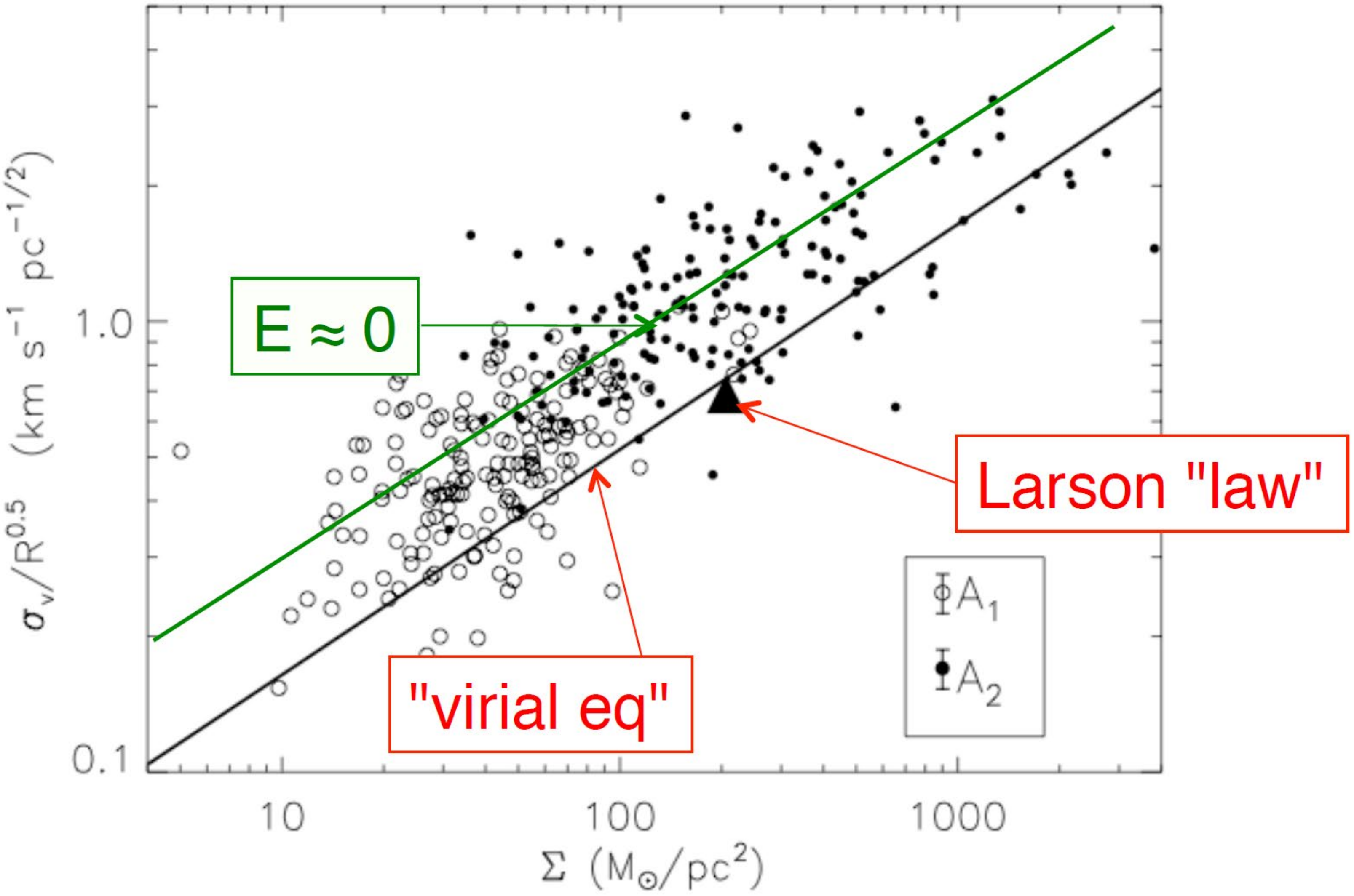
Comparing relevant timescales

- Gas depletion timescale Mg/SFR is around 5-10 Gyr. For example, MW has gas mass ~1e10 Msun and SFR ~ 1-2 Msun/yr.
- Dynamical time (i.e. orbital time) of star-forming galaxies is ~ 200 Myr. It takes 20-50 orbits to consume all gas.
- Turbulent (sigma~10km/s) crossing time for disk scale height (100-300 pc) is ~10-30 Myr.
- Free-fall time of dense molecular gas is ~ 10 Myr, much longer than the global crossing time or gas depletion timescale.
- Why is star formation so inefficient?



$$\sigma_v \approx (G\,\Sigma\,R)^{1/2} \approx (GM/R)^{1/2}$$

Larson's relation or Heyer's relation



Gravitational Instabilities: Jeans instability

- Jeans 1928 considered a perturbation of an initially uniform, stationary, non-rotating, unmagnetized gas.
- There exists a critical Jeans length, above which the region becomes unstable.

$$\lambda_J = \left(\frac{\pi c_s^2}{G\rho_\circ}\right)^{1/2}$$

- Correspondingly, there are Jeans mass and Jeans time.

$$M_{\mathrm{J}} \equiv \frac{4\pi}{3}\rho_0\left(\frac{\lambda_{\mathrm{J}}}{2}\right)^3 = \frac{1}{8}\left(\frac{\pi k T}{G\mu}\right)^{3/2}\frac{1}{\rho_0^{1/2}}$$

$$= 0.32\,M_\odot\left(\frac{T}{10\,\mathrm{K}}\right)^{3/2}\left(\frac{m_{\mathrm{H}}}{\mu}\right)^{3/2}\left(\frac{10^6\,\mathrm{cm}^{-3}}{n_{\mathrm{H}}}\right)^{1/2}$$

- It is interesting to see that the Jeans mass for the typical density and temperature of dark clouds is similar to the typical mass of stars!

Free-fall timescale

- Defined as the time to collapse from a uniform density sphere to a point without pressure support.

$$\frac{d^2r}{dt^2} = -\frac{4\pi G r_0^3 \rho_0}{3r^2}$$

- By integrating the above equation from the starting radius r0 to 0, we can obtain the total timescale (homework).

$$\tau_{\rm ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} = \frac{4.4\times10^4\,{\rm yr}}{\sqrt{n_{\rm H}/10^6\,{\rm cm}^{-3}}}$$

- The Jeans criterion can be interpreted as meaning that sound cannot traverse the region (and hence pressure cannot operate) in time to prevent the collapse.

More realistic cloud model: Bonnor-Ebert sphere

- The most important configuration for spherically-symmetric hydrostatic equilibrium.

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2}$$

- The equation can be simplified to the classical Lane-Emden equation:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\xi^2\frac{du}{d\xi} = e^{-u} \qquad \ln(\rho/\rho_c) \equiv -u$$

$$\xi = r/(c_s^2/4\pi G\rho_c)^{1/2}$$

- The pressure-bounded isothermal Bonnor-Ebert mass for a given pressure P_0 is:

$$M_{\rm BE}(p_0) = \frac{225}{32\sqrt{5\pi}}\frac{c_s^4}{(aG)^{3/2}}\frac{1}{\sqrt{p_0}}$$

$$= 0.26\left(\frac{T}{10\,\mathrm{K}}\right)^2\left(\frac{10^6\,\mathrm{cm}^{-3}\,\mathrm{K}}{p_0/k}\right)^{1/2} M_\odot$$

Can self-gravitation clumps/cores collapse?

- Magnetic flux problem: magnetic field in the ISM often acts as though the field lines are "frozen" into the fluid. If flux-freezing continues to hold in a collapsing clump, flux will be conserved.

$$\frac{\Phi}{M} > \left(\frac{\Phi}{M}\right)_{\rm crit} = 3\pi\sqrt{\frac{2aG}{5b}} = 1.54 \times 10^{-3}\sqrt{\frac{a}{b}}\ \mathrm{gauss\,cm^2\,g^{-1}}$$

- Angular momentum problem: Clumps/cores always have some sort of angular momentum. Core collapse conserves angular momentum. From the simplest energy argument, when rotational energy exceeds gravitational energy, the core has to stop collapsing at a radius:

$$R_{\rm min} \approx \frac{(J/M)^2}{GM} = 8 \times 10^{15} \left(\frac{J/M}{10^{21}\ \mathrm{cm^2\,s^{-1}}}\right)^2 \frac{M_\odot}{M}\ \mathrm{cm}$$

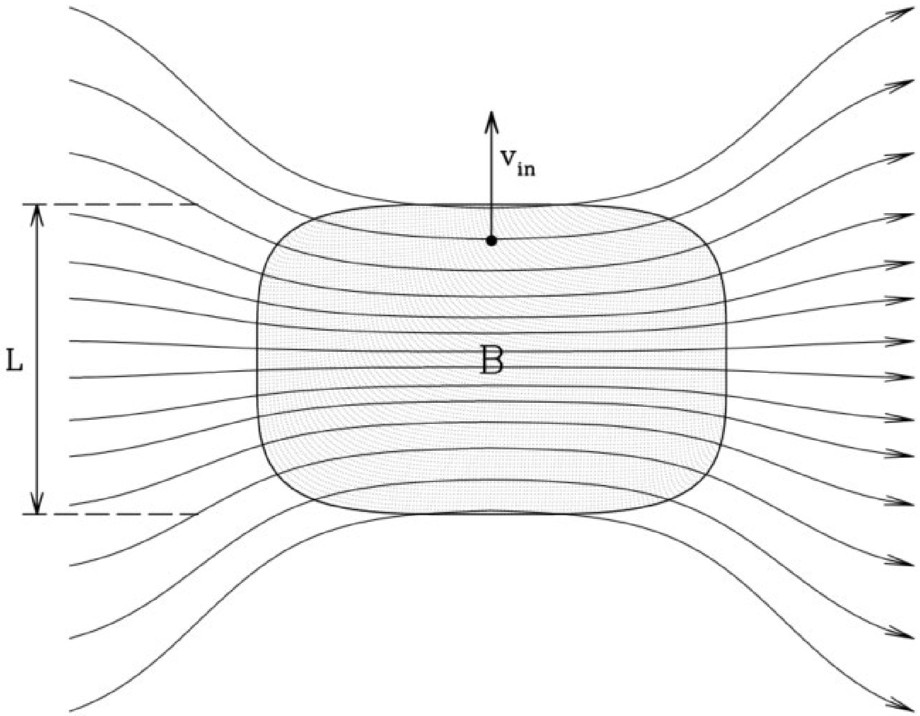
Magnetic flux problem: Ambipolar diffusion

- Typical T Tauri stars have surface magnetic fields strength 2 kG, corresponding to a flux-to-mass ratio ~ 3e-8 gauss cm2/g. This is 6 orders of magnitude below the ratio for clumps and the critical threshold ratio!!!

$$\frac{\Phi}{M} = 5.8 \times 10^{-3} n_4^{-0.02} \left(\frac{M}{M_\odot}\right)^{-1/3} \text{gauss cm}^2\,\text{g}^{-1}$$

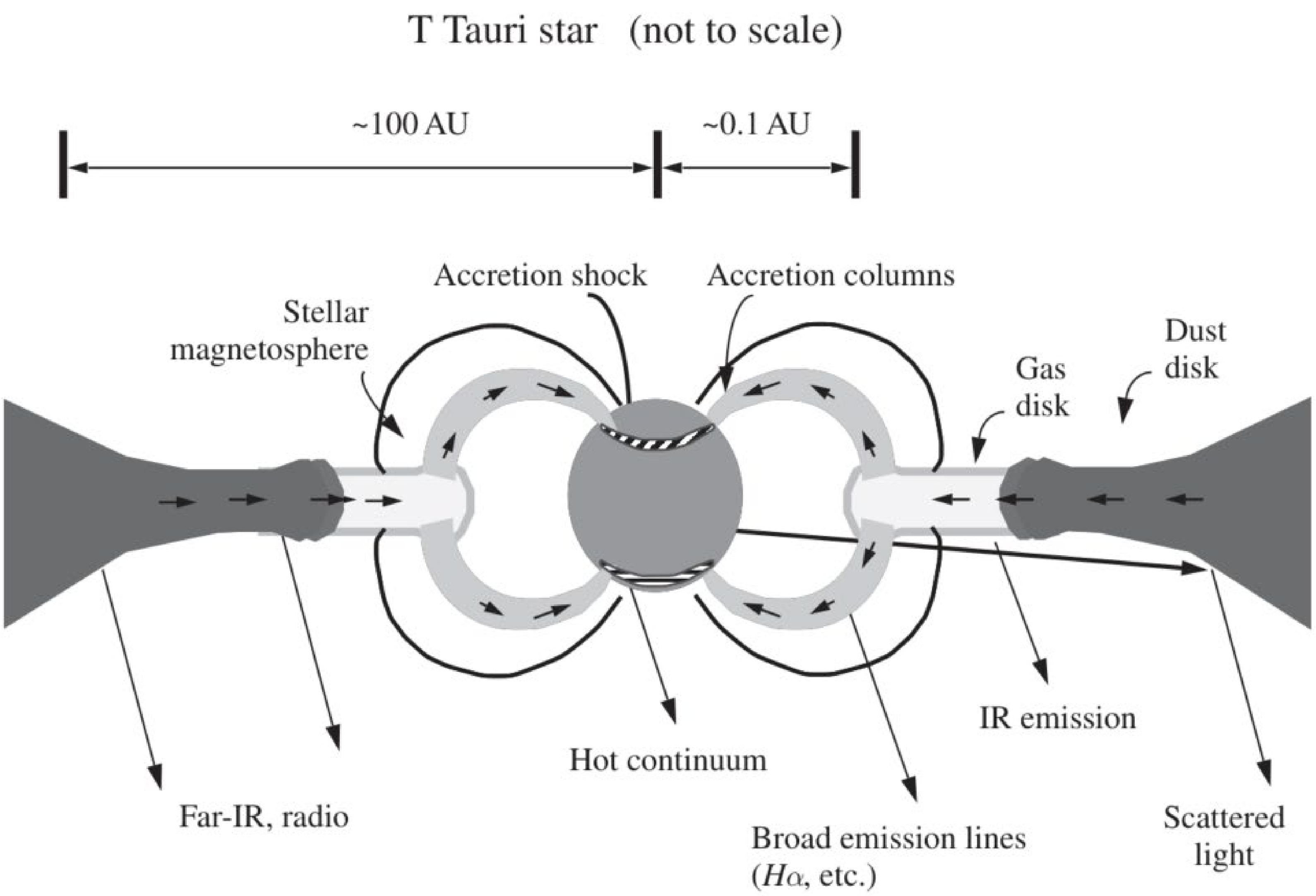
- Almost all magnetic flux that was initially present in the core escape before forming stars. How?
- Ambipolar diffusion may be important here.
- Magnetic fields only couple ionized plasma, not the neutral.
- Magnetic field lines slip out of the clumps with a rate that

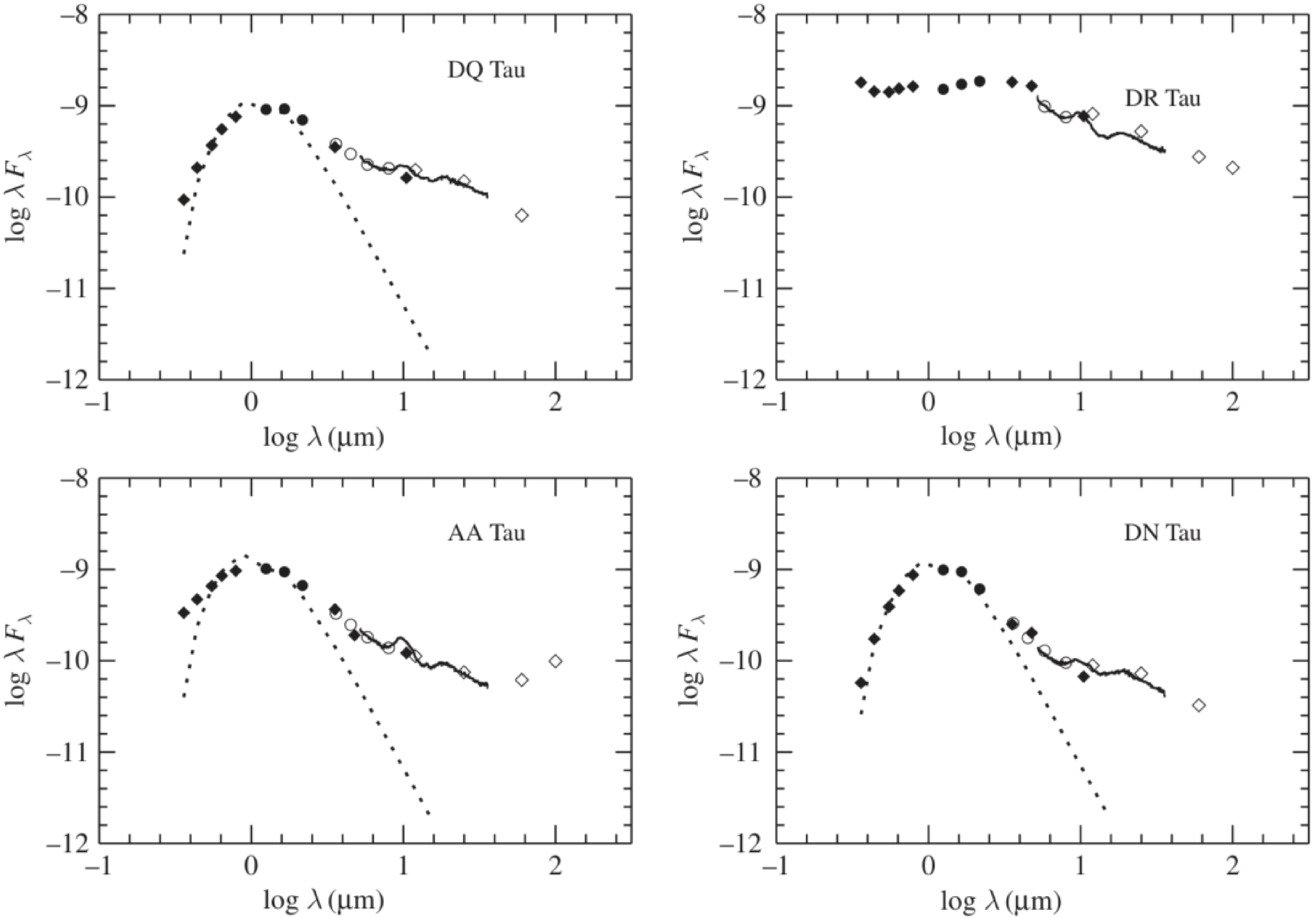
depends on the balance between the Lorentz force provided by the magnetic fields and ion-neutral collision.



$$\tau_{\rm slip} = \frac{L/2}{|\mathbf{v}_i - \mathbf{v}_n|} = \frac{8\pi(L/2)^2}{B^2} n_i n_n \langle\sigma v\rangle \frac{m_i m_n}{(m_n+m_i)} \approx 7 \times 10^7 n_4^{-1.42}\,\text{yr}$$

Angular momentum problem: accretion disk




DQ Tau
DR Tau
AA Tau
DN Tau
log λ F_λ
log λ (μm)
−8
−9
−10
−11
−12
−1
0
1
2

Thermal structure of the accretion disks

- For a steady accretion flow, accreted material dissipates their potential energy locally to heat up the disk. The energy is then radiated away by the disk surface via blackbody radiation.

$$\frac{GM_* \dot{M}}{2R} \frac{\Delta R}{R} \sim 2 \times 2\pi R \,\Delta R\, \sigma T_{\mathrm{d}}^4 \longrightarrow T_{\mathrm{d}} \sim \left(\frac{GM_* \dot{M}}{8\pi \sigma R^3} \right)^{1/4}$$

- While accretion disks around blackholes don't have central sources, accretion disks around proto-stars are shined by the radiation from the proto-stars powered by infalling material hitting the surface of the stars and the deuterium nuclear reaction.

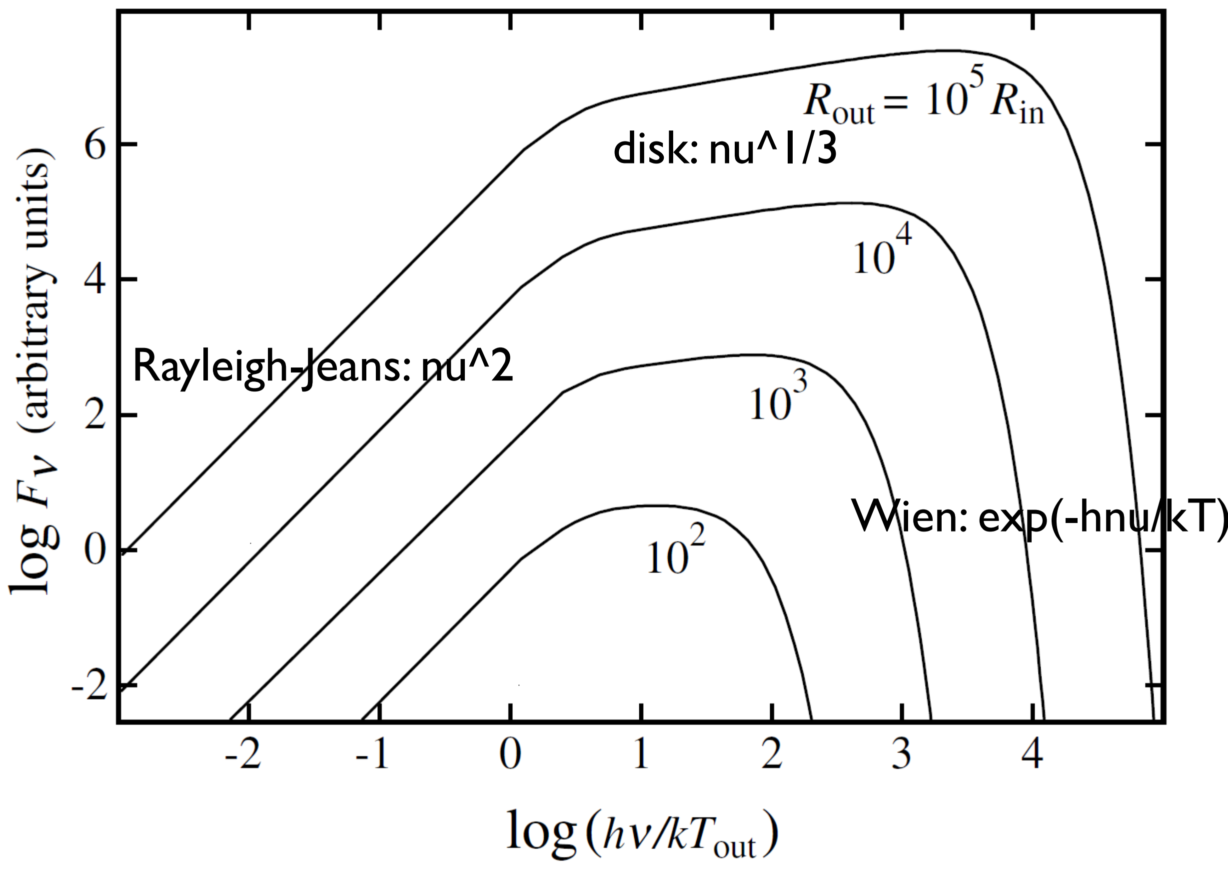
$$\frac{L_*}{4\pi R^2} < \cos\gamma > \sim \sigma T_{\mathrm{d}}^4 \longrightarrow T_d \sim \left(\frac{L_* R_*}{4\pi \sigma R^3} \right)^{1/4}$$

Thermal emission from accretion disks

- The luminosity of the hot disk is the sum of blackbody radiation from different annular of disk with radius-dependent temperature!
- Remember, T~R^-3/4!

$$L_\nu = \int_{R_{\rm in}}^{R_{\rm out}} \pi\, B_\nu[T_{\rm d}(R)]\, 2\pi R\, dR$$

$$= \frac{16\pi^2 h R_\circ^2}{3c^2} \left(\frac{kT_\circ}{h}\right)^{8/3} \nu^{1/3} \int_{x_{\rm in}}^{x_{\rm out}} \frac{x^{5/3}\, dx}{(e^x - 1)}$$

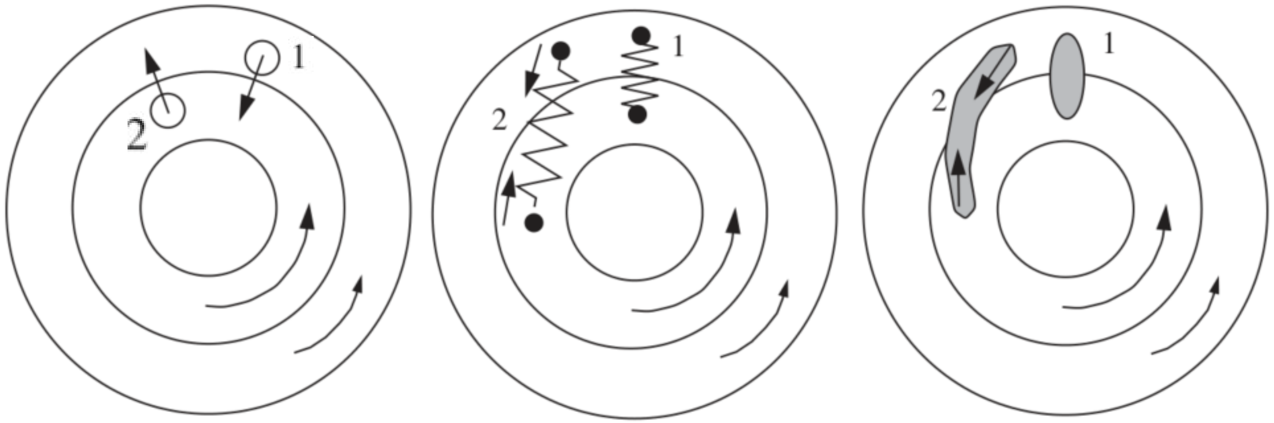


But how to get rid of angular momentum to accrete?

- Consider an idealized situation involving only two bodies (m1 and m2) orbiting around a central mass of M.
- The total energy and angular momentum:

$$E = -\frac{GM}{2}\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$$

$$J = (GM)^{1/2}(m_1 r_1^{1/2} + m_2 r_2^{1/2})$$



- Perturb the system while conserving total angular momentum by nagging r1 with dr1

$$m_1 r_1^{-1/2}\Delta r_1 = -m_2 r_2^{-1/2}\Delta r_2$$

$$\Delta E = -\frac{GMm_1\Delta r_1}{2r_1^2}\left[\left(\frac{r_1}{r_2}\right)^{3/2} - 1\right]$$