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6. Warm and Hot Ionized Medium

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Ionization processes

- **Photoelectric absorption:** $X + h\nu \rightarrow X^+ + e^-$.
- **Photoelectric absorption followed by the Auger effect:** $X + h\nu \rightarrow (X^+)^* + e^- \rightarrow X^{+n} + ne^-$ $(n \geq 2)$.
- **Collisional ionization:** $X + e^- \rightarrow X^+ + 2e^-$.
- **Cosmic ray ionization:** $X + CR \rightarrow X^+ + e^- + CR$.
- **Charge exchange:** $X + Y^+ \rightarrow X^+ + Y$.

Spherical cow model of HII regions

- Assumptions: uniform density; fully ionized; static-state solution; hydrogen only
- Stromgren (1939) sphere!

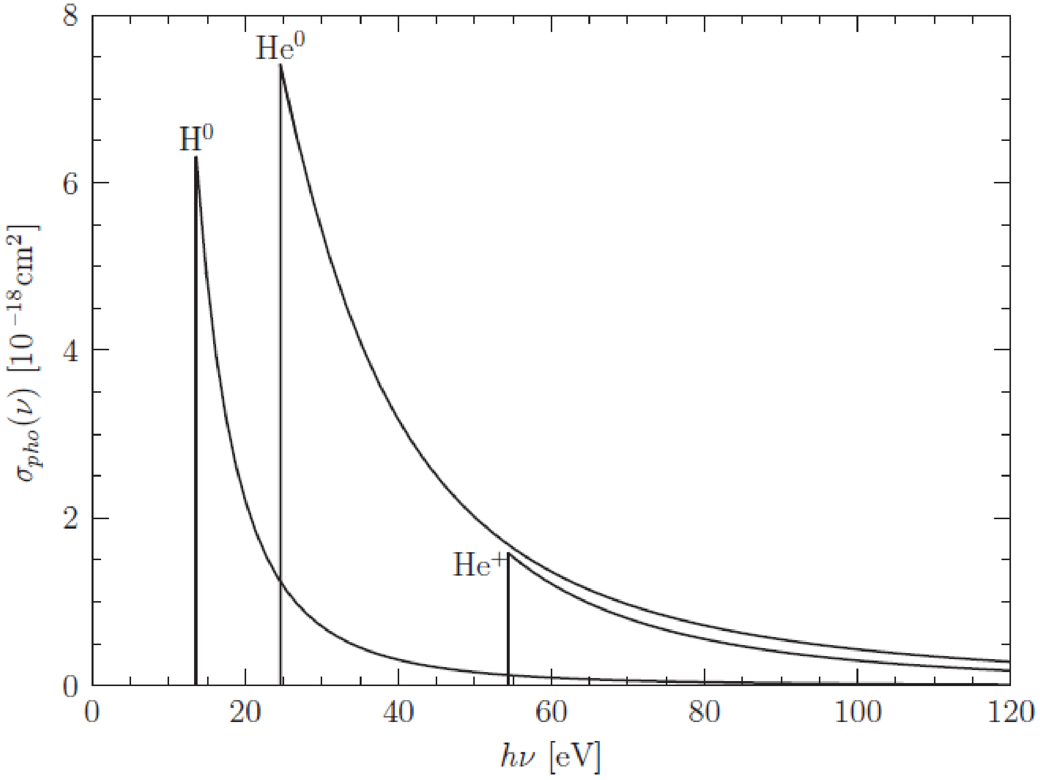
Three processes govern the physics of HII regions:

- **Photoionization Equilibrium**, the balance between photoionization and recombination.
- **Thermal Balance** between heating and cooling.
- **Hydrodynamical response** of the overpressured HII regions surrounded by cold atomic or molecular gas.

The Rosette Nebula

- Photo-ionization

$$X + h\nu \rightarrow X^+ + e^-$$



$$\sigma_{\rm pe}(\nu) \approx \sigma_0 \left(\frac{h\nu}{Z^2 I_{\rm H}}\right)^{-3} \qquad \sigma_0 = \frac{6.30 \times 10^{-18}\,{\rm cm}^2}{Z^2} = \frac{0.225 a_0^2}{Z^2}$$

- Radiative Recombination (RR)

$$X^+({\rm core}) + e^- \rightarrow X({\rm core} + n\ell) + h\nu$$

Recombination coefficient for RR to an arbitrary nl energy level

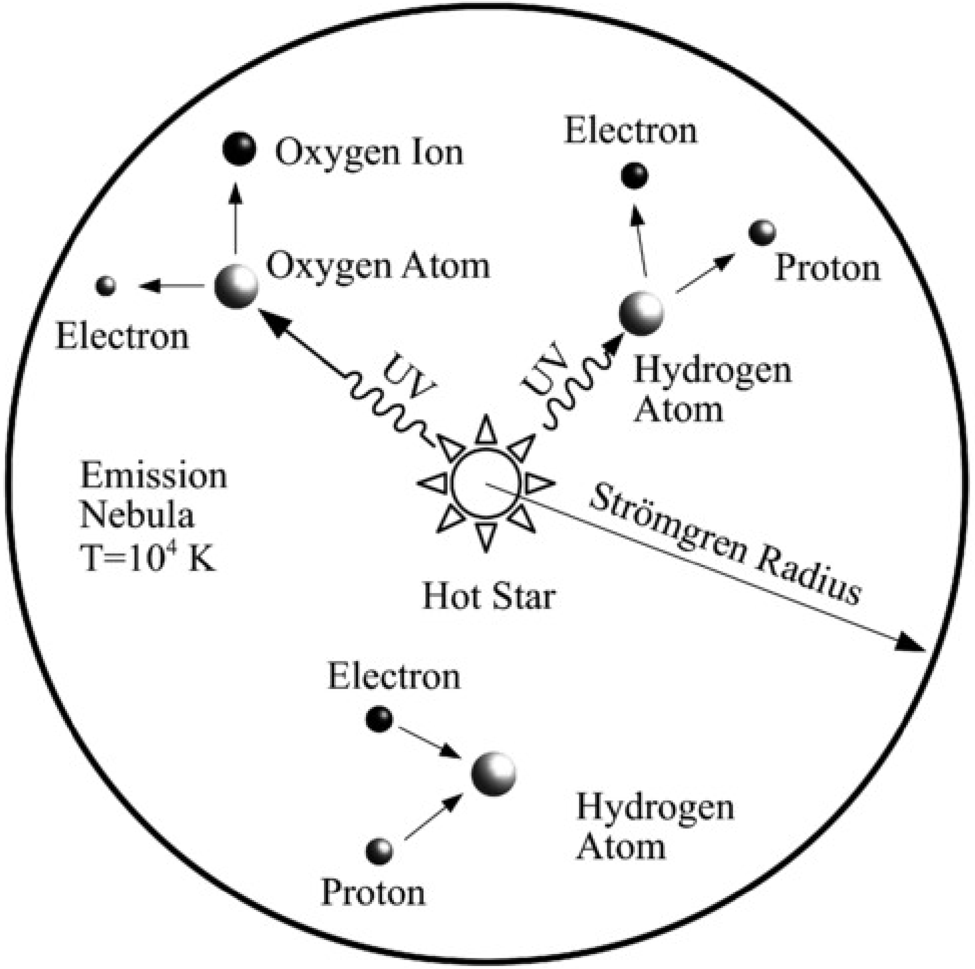
$$\alpha_{n\ell}(T) = \left(\frac{8kT}{\pi m_e}\right)^{1/2} \int_0^\infty \sigma_{{\rm rr},n\ell}(E) \frac{E}{kT} {\rm e}^{-E/kT} \frac{dE}{kT}$$

$$\alpha_A(T) \equiv \sum_{n=1}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) \quad ,$$

On The Spot Approximation:

$$\alpha_B(T) \equiv \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_A(T) - \alpha_{1s}(T)$$

I. Photoionization Equilibrium



Case B recombination coefficient:
~ 2.6e-13 cm3/s at T=1e4 K

rate of emission of hydrogen-ionizing photons, i.e., hv>13.6eV

$$Q_0 = \frac{4\pi}{3} {R_{S0}}^3 \, \alpha_B \, n(\mathrm{H}^+) n_e$$

Recombination is a two-body process!

Fully ionized, uniform density: $n(\mathrm{H}^+) = n_e = n_{\mathrm{H}}$

$$R_{S0} \equiv \left(\frac{3\, Q_0}{4\pi {n_{\mathrm{H}}}^2 \alpha_B} \right)^{1/3} = 9.77 \times 10^{18} \, Q_{0,49}^{1/3} \, {n_2}^{-2/3} \, T_4^{0.28} \,\mathrm{cm}$$

assuming the temperature of HII region is around 1e4 K.

Assumption 1: static state?

$$R_{S0} \equiv \left(\frac{3\, Q_0}{4\pi {n_{\rm H}}^2 \alpha_B} \right)^{1/3}$$

- Ionization timescale for the HII region

$$\tau_{\rm ioniz.} \equiv \frac{(4/3)\pi {R_{S0}}^3 n_{\rm H}}{Q_0} = \frac{1}{\alpha_B n_{\rm H}} = \frac{1.22 \times 10^3\,{\rm yr}}{n_2}$$

- Recombination timescale

$$\tau_{\rm rec} = \frac{1}{\alpha_B n_{\rm H}} = \frac{1.22 \times 10^3\,{\rm yr}}{n_2}$$

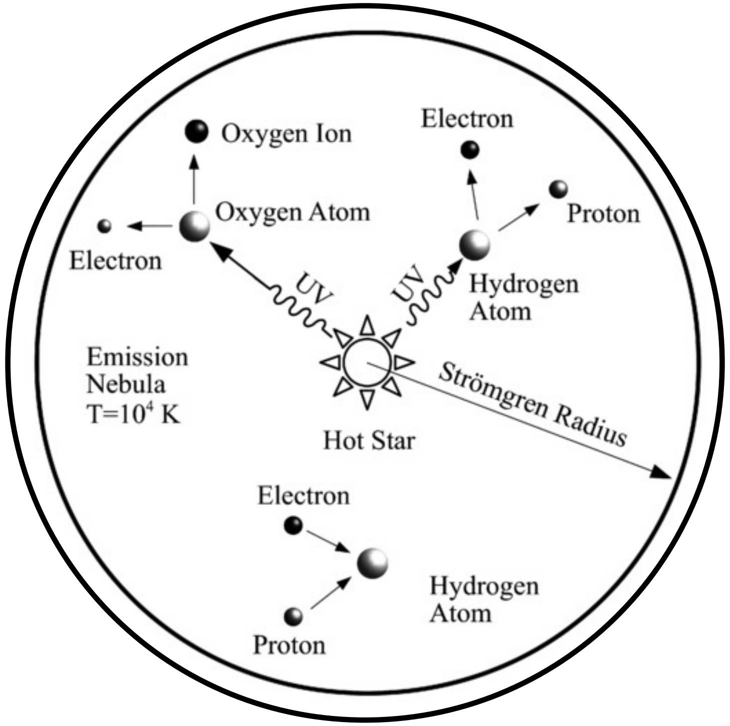
- Recombination timescale is identical to the ionization timescale!
- For density n>0.03/cc, the timescale is shorter than the stellar lifetime (5Myr) for a massive star.

Assumption 2: fully ionized?

- Whether the transition from ionized gas to neutral gas at the boundary of the HII region is shape or not?
- Mean-free path for ionizing photons

$$\mathrm{mfp} = \frac{1}{n(\mathrm{H}^0)\sigma_{\mathrm{p.i.}}} = 3.39 \times 10^{17} \left(\frac{\mathrm{cm}^{-3}}{n(\mathrm{H}^0)} \right) \mathrm{cm}$$

photo-ionization
cross section

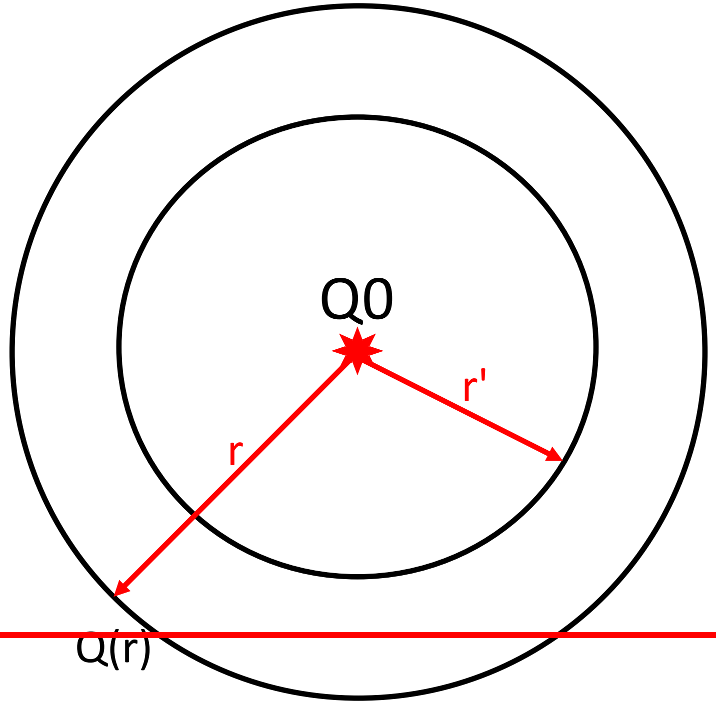


Better treatment on the ionization structure of HII

Photon number conservation

$$Q(r) = Q_0 - \int_0^r {n_{\rm H}}^2 \alpha_B x^2 4\pi (r')^2 dr'$$

$$= Q_0 \left[1 - 3 \int_0^{r/R_{\rm S0}} x^2 y^2 dy \right]$$



Ionization fraction as a function of radius

$$x \equiv n({\rm H}^+)/n_{\rm H} = n_e/n_{\rm H},\ y \equiv r/R_{\rm S0}$$

Detailed balance between ionization and recombination at each point within the HII region:

$${n_{\rm H}}^2 \alpha_B x^2 = \frac{Q(r)}{4\pi r^2} n_{\rm H}(1-x)\sigma_{\rm p.i.}$$

Homework:
Calculate the ionization structure of HII region numerically
Evaluate the radius where the ionization fraction is 50%.

II. Thermal Balance

Energy equation represents conservation of energy:

$$\frac{\mathrm{D}\epsilon}{\mathrm{D}t} = -\frac{P}{\rho}\vec{\nabla}\cdot\vec{u} - \frac{1}{\rho}\vec{\nabla}\cdot\vec{F} + \frac{1}{\rho}\Psi + \boxed{\frac{\Gamma - \Lambda}{\rho}}$$

$$\Gamma \equiv \text{volumetric radiative heating rate} \quad [\mathrm{erg\,cm^{-3}\,s^{-1}}]$$
$$\Lambda \equiv \text{volumetric radiative cooling rate} \quad [\mathrm{erg\,cm^{-3}\,s^{-1}}]$$

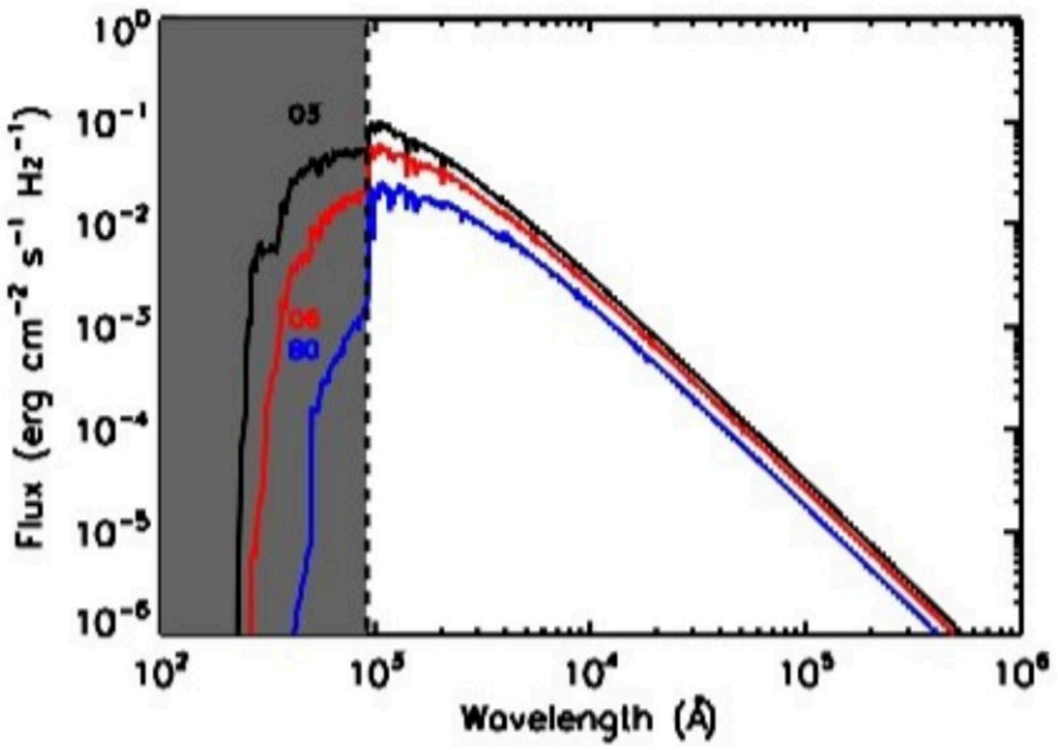
If hydrostatic equilibrium is achieved and external energies such as gravity are ignored, heating and cooling of hot plasma dominate the thermal-dynamical state of the HII region.

Heating

- **Heating by photoionization**

$$X^{+r} + h\nu \rightarrow X^{+r+1} + e^- + KE$$

- Photoelectric heating from dust
- Cosmic rays
- X-rays
- Damping of MHD waves



Photon with wavelength shorter than 912 A can generate photoelectrons that heat up HII region

$$\Gamma_{\rm pe} = n(X^{+r}) \int_{\nu_0}^{\infty} \sigma_{\rm pe}(\nu) c \left[\frac{u_\nu}{h\nu}\right] (h\nu - h\nu_0)\, d\nu$$

$$\langle E \rangle = \langle h\nu \rangle - I_{\rm H}.$$

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Photoionization heating by nearby ionizing sources

- For typical O/B stars, the required 13.6 eV energy lies on the high-energy spectrum, the Wien tail.

$$\varepsilon_\nu \propto \nu^3 \exp\left(-\frac{h\nu}{kT_c}\right)$$

- The average energy for the effects of photoionization heating must be weighted by pi cross section.

$$\langle h\nu \rangle = \frac{\int_{\nu_0}^{\infty} (\varepsilon_\nu / h\nu)(h\nu)\sigma_{\mathrm{pho}} d\nu}{\int_{\nu_0}^{\infty} (\varepsilon_\nu / h\nu)\sigma_{\mathrm{pho}} d\nu} = \frac{h \int_{\nu_0}^{\infty} (\nu^2 \mathrm{e}^{-h\nu/kT_c})\nu \cdot \nu^{-3} d\nu}{\int_{\nu_0}^{\infty} (\nu^2 \mathrm{e}^{-h\nu/kT_c})\nu^{-3} d\nu} = kT_c \frac{\int_{x_0}^{\infty} \mathrm{e}^{-x} dx}{\int_{x_0}^{\infty} \mathrm{e}^{-x} x^{-1} dx} = \frac{kT_c \mathrm{e}^{-x_0}}{\int_{x_0}^{\infty} \mathrm{e}^{-x} x^{-1} dx}.$$

$$x \equiv h\nu / kT_c$$

$$\langle E \rangle = \langle h\nu \rangle - I_{\mathrm{H}}.$$

Photoionization heating by nearby ionizing sources

- The average energy can be derived by the approximation of the first exponential integral:

$$E_1(x_0) \approx \frac{e^{-x_0}}{x_0}\left[1 - \frac{1}{x_0} + O(x_0^{-2})\right]$$

- The average energy is, to the zeroth order, just the ionization threshold plus the photon temperature:

$$\langle h\nu \rangle \approx h\nu_0 + kT_c \approx I_{\mathrm{H}} + kT_c.$$

- Surprisingly simple, the electrons liberated by photoionization will go off with the mean kinetic energy $\langle E \rangle = \langle h\nu \rangle - I_{\mathrm{H}} \approx kT_c.$

$$\langle E \rangle = \langle h\nu \rangle - I_{\mathrm{H}} \approx kT_c.$$

Photoionization heating by nearby ionizing sources

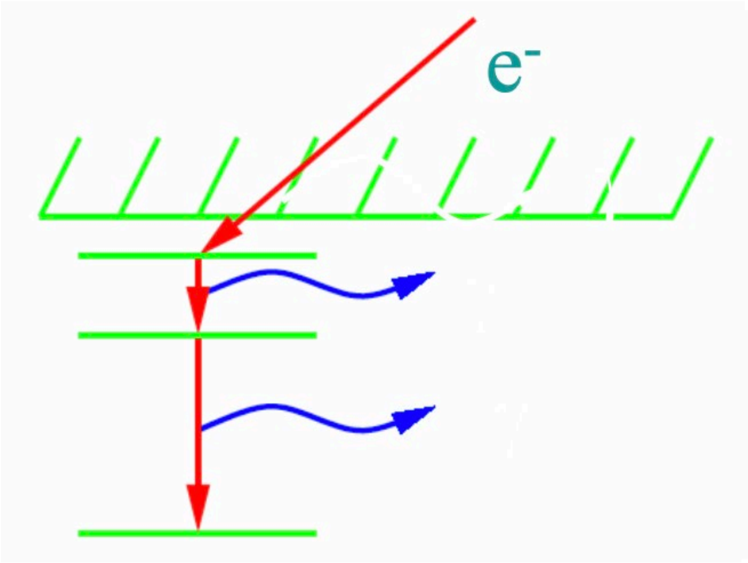
- In ionization equilibrium, the ionization is balanced by recombination:

$$n_{\mathrm{H\,I}}\zeta_{\mathrm{pho}} = n_e n_{\mathrm{H\,II}}\alpha_{\mathrm{B,H}}.$$

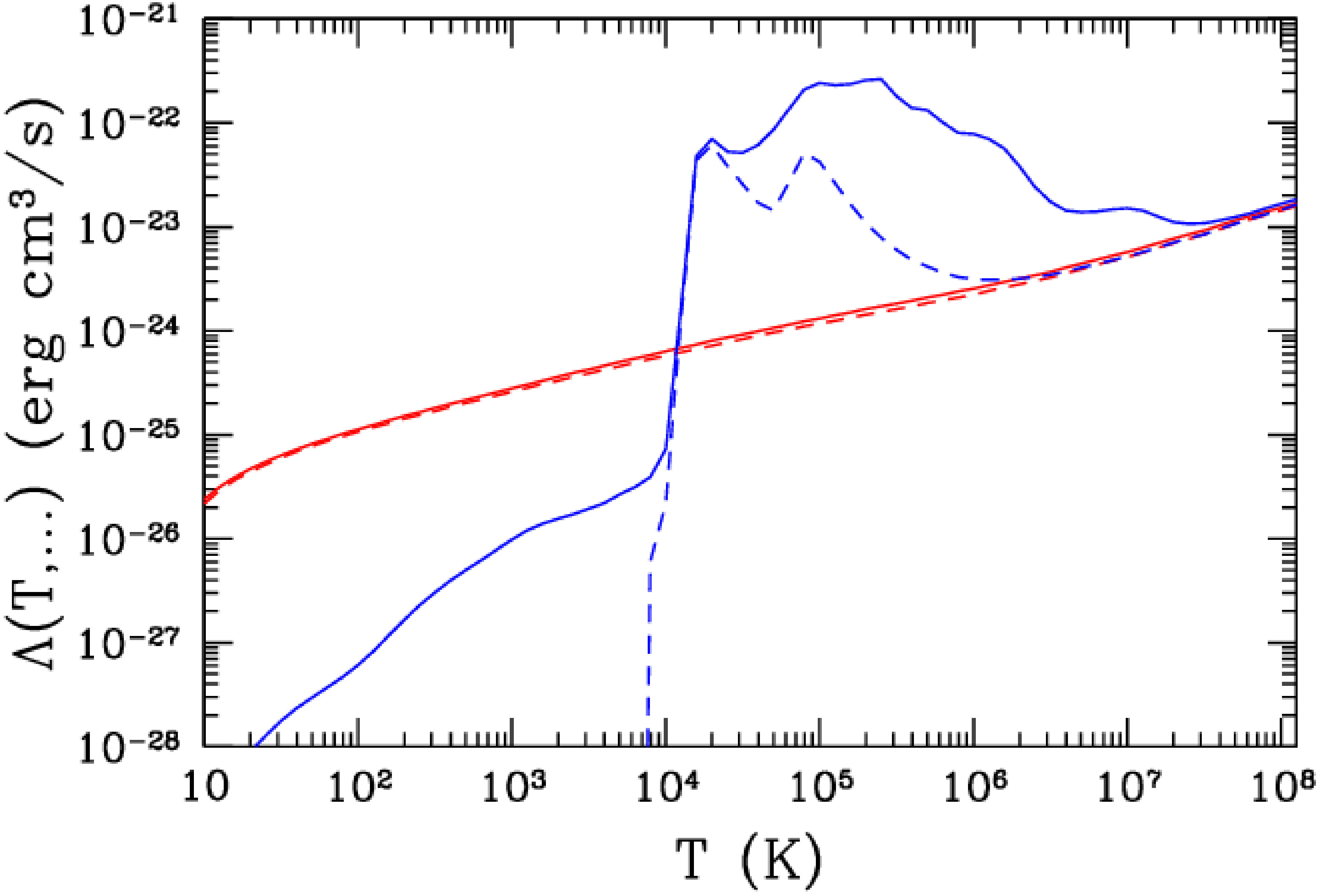
- The volumetric heating rate is the number rate times the average electron energy:

$$g_{\mathrm{pho}} = n_{\mathrm{H\,I}}\zeta_{\mathrm{pho}}\langle E \rangle \approx n_e n_{\mathrm{H\,II}}\alpha_{\mathrm{B,H}}kT_{\mathrm{eff}}.$$

Cooling



- Recombination radiation
- Free-free emission
- **Collisionally excited line radiation from heavy elements (metals)**



Gnedin & Hollon 2012

Table 1
Physical Conditions and Oxygen Abundances for the Sample Objects

Object	n_e (cm^{-3})	T_e ([N II]) (K)	T_e ([O III]) (K)	$12 + \log(O/H)$	$\log(O^+/O^{++})$	ADF (O^{++})
M8	1550 ± 150	8500 ± 120	8020 ± 100	8.44 ± 0.03	$+0.40 \pm 0.05$	2.2 ± 0.2
M16	980 ± 120	8500 ± 150	7580 ± 150	8.52 ± 0.03	$+0.45 \pm 0.06$	2.4 ± 0.6
M17	420 ± 80	9150 ± 250	7950 ± 100	8.53 ± 0.02	-0.76 ± 0.06	1.8 ± 0.2
M20	220 ± 40	8500 ± 150	7750 ± 250	8.48 ± 0.03	$+0.64 \pm 0.07$	1.9 ± 1.2
M42	6800 ± 600	10100 ± 250	8250 ± 60	8.53 ± 0.02	-0.68 ± 0.05	1.50 ± 0.05
NGC 2467	260 ± 50	9600 ± 200	8900 ± 100	8.35 ± 0.03	$+0.35 \pm 0.05$	1.8 ± 0.2
NGC 3576	1400 ± 200	8950 ± 200	8400 ± 80	8.53 ± 0.02	-0.40 ± 0.06	2.0 ± 0.3
NGC 3603	2350 ± 400	11650 ± 570	9000 ± 150	8.47 ± 0.03	-1.21 ± 0.09	1.9 ± 0.3
30 Doradus	380 ± 50	10800 ± 300	9850 ± 100	8.36 ± 0.02	-0.77 ± 0.06	1.5 ± 0.1

e.g. Rodriguez & Garcia-Rojas 2010

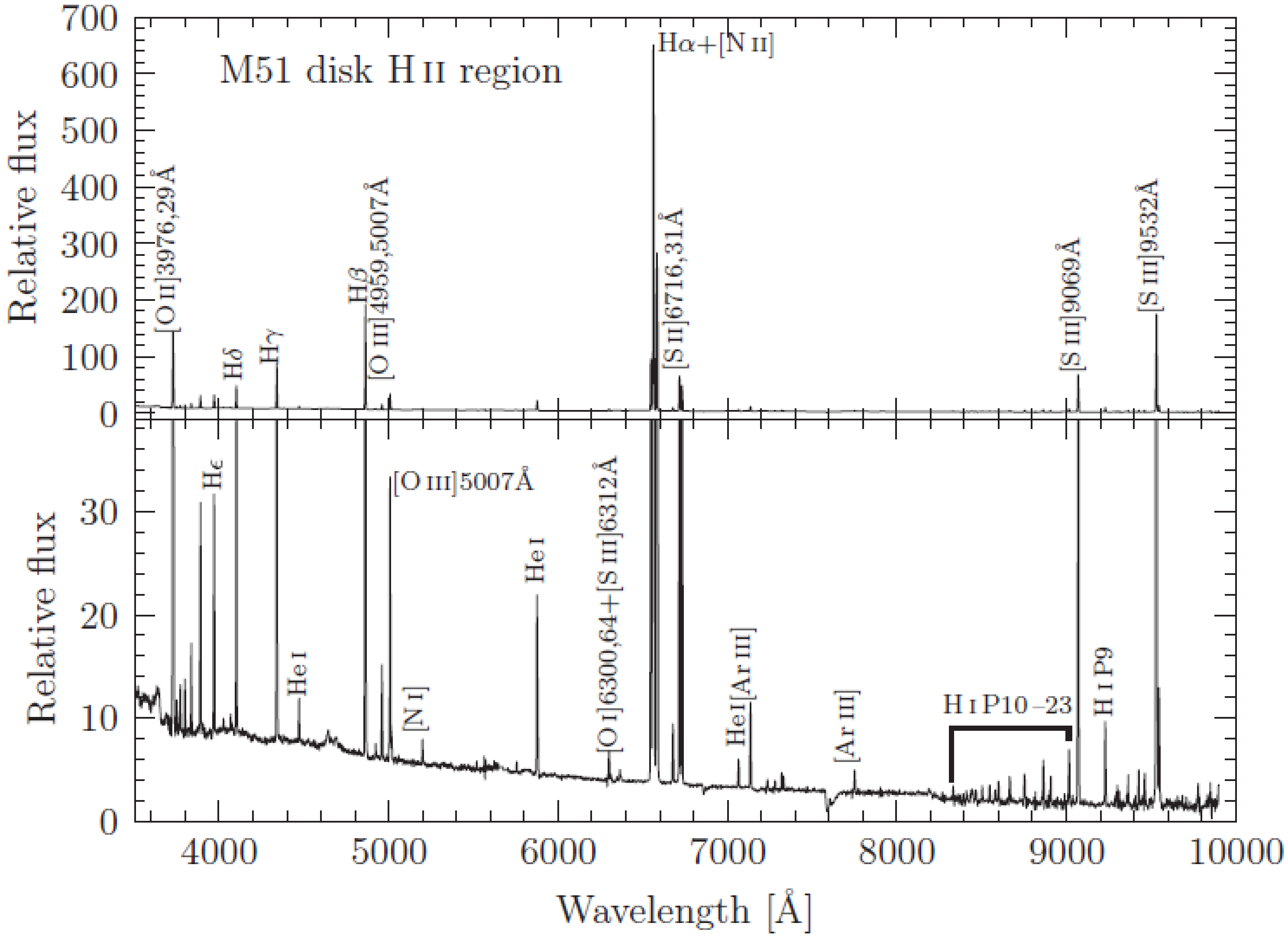
Summary of the physical processes in HII regions

- **Photoionization Equilibrium**, the balance between photoionization and recombination. This determines the structure of the nebula and the rough spatial distribution of ionic states of the elements in the ionized zone.

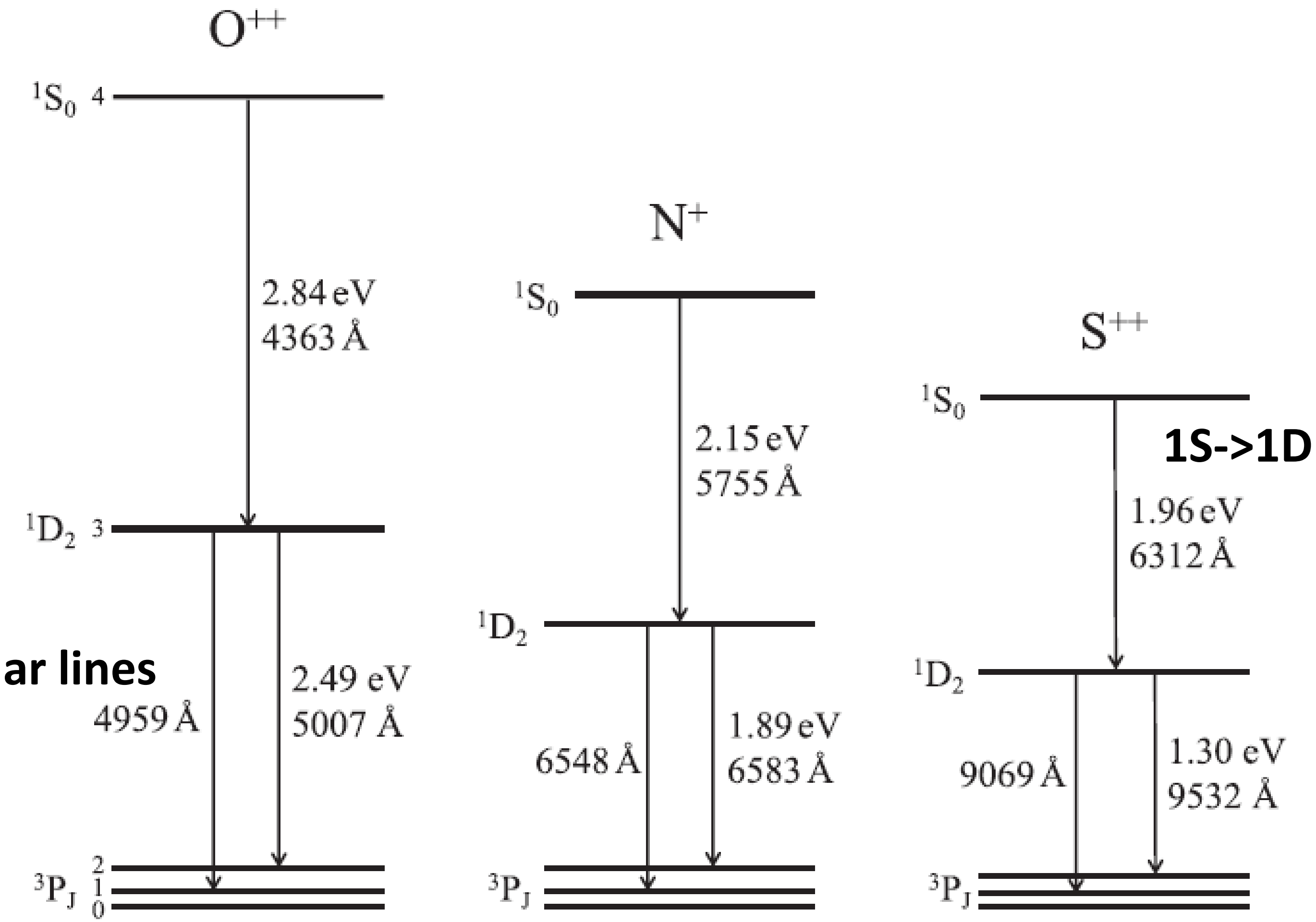
- **Thermal Balance** between heating and cooling. Heating is dominated by photoelectrons ejected from Hydrogen and Helium with thermal energies of a few eV. Cooling is dominated in most HII regions by electron-ion impact excitation of metal ions followed by emission of "forbidden" lines from low-lying fine structure levels. It is these cooling lines that give HII regions their characteristic spectra.

- **Hydrodynamical response**, including hydro shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.

Line diagnostics: how to determine the temperature and density of the HII regions or planetary nebulae?



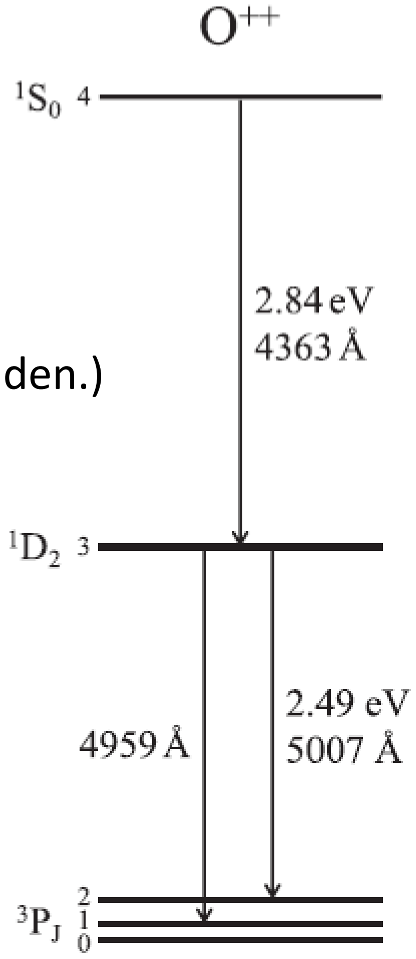
Line diagnostics in structured energy level system



1S->1D: auroral lines

1D->3P: nebular lines

Temperature diagnostics in structured energy level system



Emissivity from 4->3 level: $$j(4 \to 3) = n_4 \frac{A_{43}}{4\pi} h\nu_{43}.$$

Excitation eq for level 4: $$n_0 n_e k_{04} = n_4 (A_{43} + A_{41}).$$ (4->2 and 4->0 are strongly forbidden.)

$$j(4 \to 3) = \frac{n_0 n_e}{4\pi} k_{04} \frac{A_{43}}{A_{43} + A_{41}} h\nu_{43}.$$

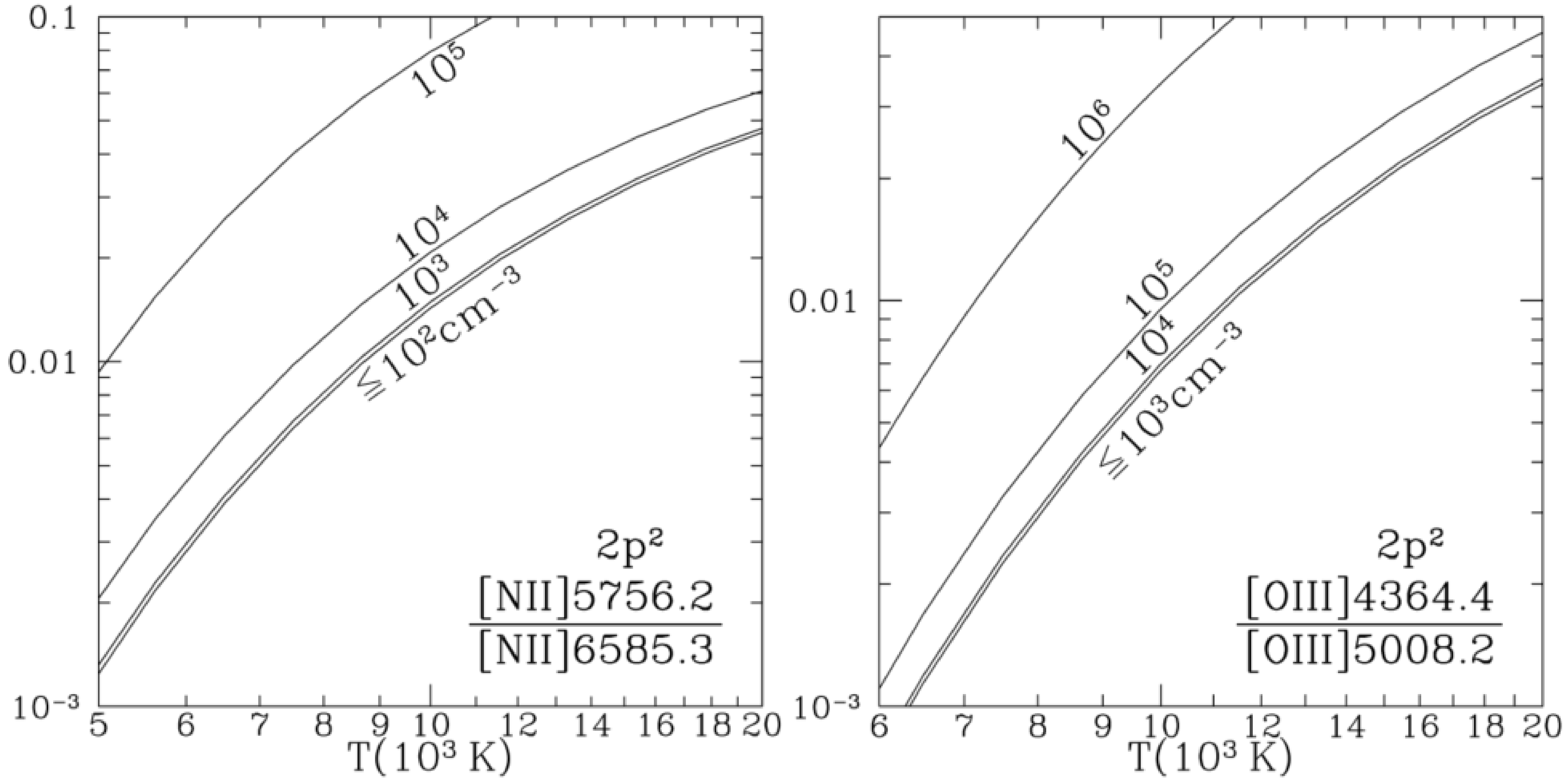
Emissivity from 3->2 level:

$$j(3 \to 2) = \frac{n_0 n_e}{4\pi} \left[k_{03} + k_{04} \frac{A_{43}}{A_{43} + A_{41}} \right] \frac{A_{32}}{A_{32} + A_{31}} h\nu_{32}.$$

Line ratio between 4->3 auroral line and 3->2 nebula line:

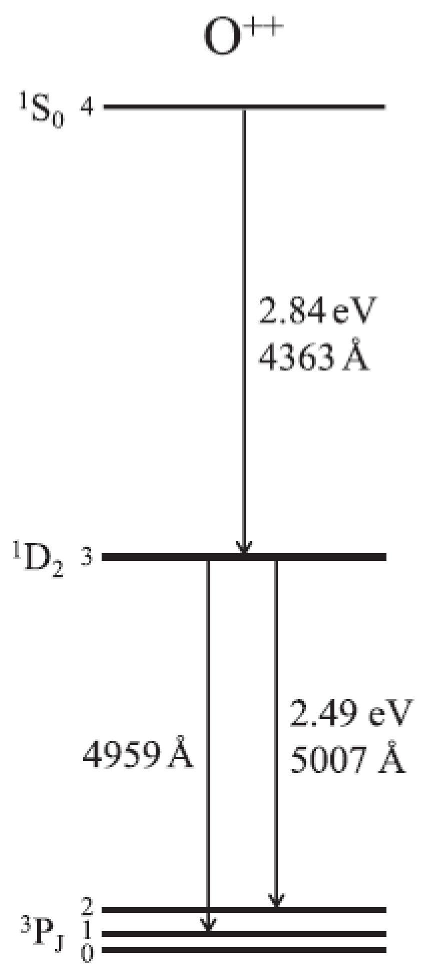
$$\frac{j(4 \to 3)}{j(3 \to 2)} = \frac{A_{43}\nu_{43}}{A_{32}\nu_{32}} \frac{(A_{32} + A_{31}) k_{04}}{(A_{43} + A_{41}) k_{03} + A_{43} k_{04}} = \frac{A_{43}\nu_{43}}{A_{32}\nu_{32}} \frac{(A_{32} + A_{31}) \Omega_{40} \exp(-h\nu_{43}/kT)}{(A_{43} + A_{41}) \Omega_{30} + A_{43} \Omega_{40} \exp(-h\nu_{43}/kT)}.$$

$$k_{0u} = k_{u0} \frac{g_u}{g_0} \exp\left(-\frac{h\nu_{u0}}{kT}\right). \quad k_{u0} = \frac{\beta}{T^{1/2}} \frac{\Omega_{u0}}{g_u}$$



0.1
0.01
10⁻³
10⁵
10⁴
10³
≦10²cm⁻³
2p²
[NII]5756.2
[NII]6585.3
5 6 7 8 9 10 12 14 16 18 20
T(10³ K)
0.01
10⁻³
10⁶
10⁵
10⁴
≦10³cm⁻³
2p²
[OIII]4364.4
[OIII]5008.2
6 7 8 9 10 12 14 16 18 20
T(10³ K)

Density diagnostics in structured energy level system



Emissivity from 1->0 level: $$j(1 \to 0) = n_1 \frac{A_{10}}{4\pi} h\nu_{10}.$$

Excitation eq (both collisional and radiative de-excitation is considered) for level 1:

$$n_0 n_e k_{01} = n_1 (A_{10} + n_e k_{10})$$

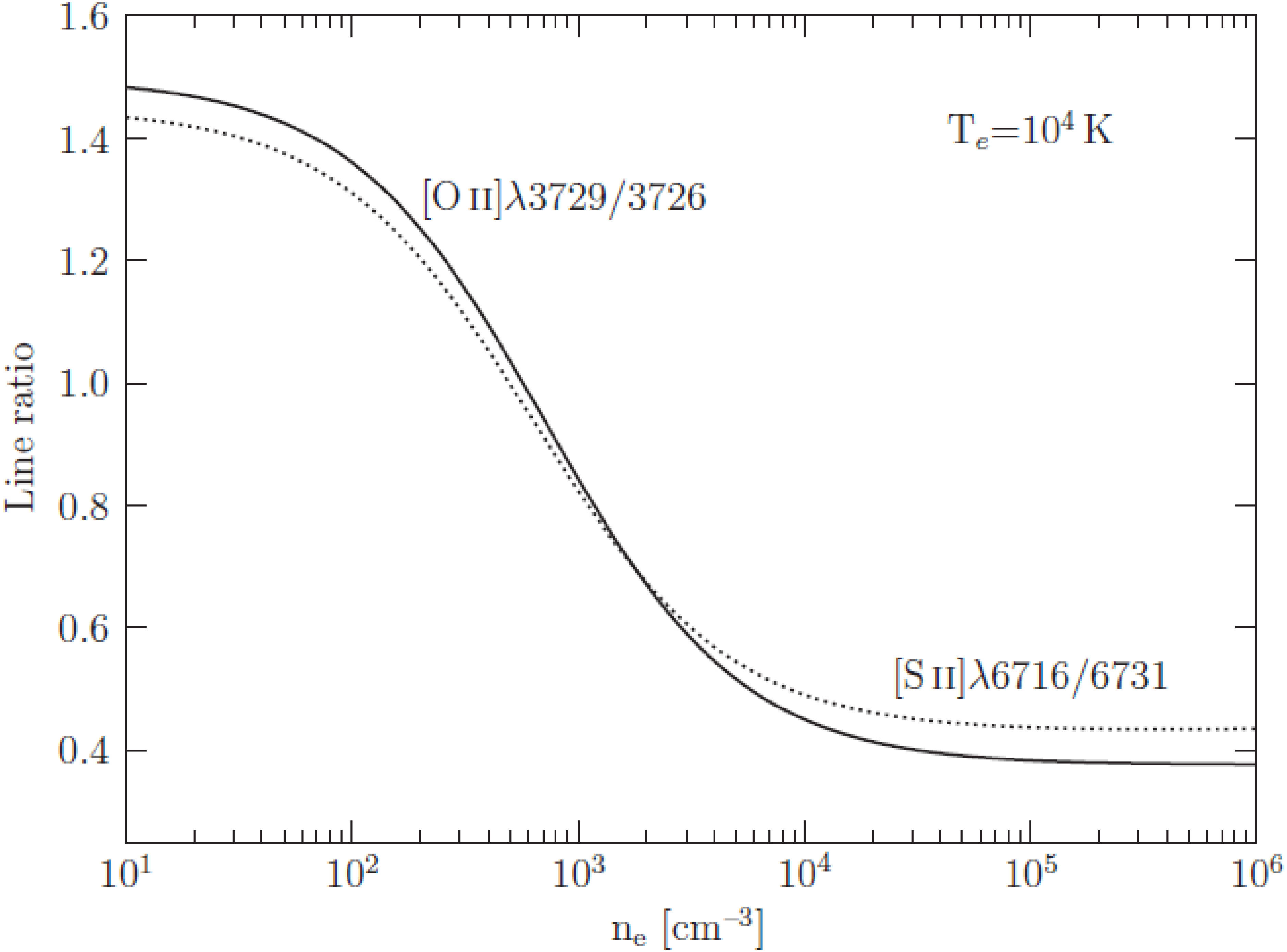
$$j(1 \to 0) = n_0 n_e \frac{k_{01}}{A_{10} + n_e k_{10}} \frac{A_{10}}{4\pi} h\nu_{10}$$

$$= n_0 n_e \frac{k_{01}}{1 + n_e / n_{\mathrm{crit},1}} \frac{h\nu_{10}}{4\pi}.$$

Emissivity from 2->0 level: $$j(2 \to 0) = n_0 n_e \frac{k_{02}}{1 + n_e / n_{\mathrm{crit},2}} \frac{h\nu_{20}}{4\pi}.$$

$$\frac{j(2 \to 0)}{j(1 \to 0)} = \frac{\nu_{20}}{\nu_{10}} \frac{k_{02}}{k_{01}} \frac{1 + n_e/n_{\mathrm{crit},1}}{1 + n_e/n_{\mathrm{crit},2}} \approx \frac{\Omega_{20}}{\Omega_{10}} \frac{1 + n_e/n_{\mathrm{crit},1}}{1 + n_e/n_{\mathrm{crit},2}}$$

$$\frac{k_{02}}{k_{01}} = \frac{\Omega_{20}}{\Omega_{10}} \exp(-h\nu_{21}/kT) \approx \frac{\Omega_{20}}{\Omega_{10}}$$


1.6
1.4
1.2
1.0
0.8
0.6
0.4
Line ratio
$T_e = 10^4$ K
[O II]λ3729/3726
[S II]λ6716/6731
10^1
10^2
10^3
10^4
10^5
10^6
n_e [cm^{-3}]

Hot ionize medium (HIM): SNR, CGM and IGM

$$X^i + e^- \rightarrow X^{i+1} + e^- + e^-$$

Volumetric ci rate: $$\frac{dn(X^i)}{dt} = -n_e n(X^i) k_{ci}$$

$$k_{ci} = 2.32 \times 10^{-8}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \left(\frac{kT}{I_H}\right)^{1/2} e^{-I_H/kT}.$$ for hydrogen

$$I_H/k = 1.58 \times 10^5\,\mathrm{K}$$

$$t_{ci} = \frac{1}{n_e k_{ci}}$$

$$\approx 160\,\mathrm{yr} \left(\frac{T}{10^6\,\mathrm{K}}\right)^{-0.5} \exp\left[0.158 \left(\frac{10^6\,\mathrm{K}}{T}\right)\right] \left(\frac{n_e}{0.004\,\mathrm{cm}^{-3}}\right)^{-1}$$

Collisional ionization equilibrium (CIE): collisional ionization is balanced by radiative recombination.

$$\frac{dn_{\mathrm{HI}}}{dt} = -n_e n_{\mathrm{HI}} k_{ci} + n_e n_{\mathrm{HII}} \alpha_{A,H}.$$

Why Case A?

$$n_e n_{\mathrm{HI}} k_{ci} = n_e n_{\mathrm{HII}} \alpha_{A,H}. \quad \Longrightarrow \quad \frac{n_{\mathrm{HII}}}{n_{\mathrm{HI}}} = \frac{k_{ci}(T)}{\alpha_{A,H}(T)}$$

$$\frac{n_{\mathrm{HII}}}{n_{\mathrm{HI}}} \approx 4.0 \times 10^5 \left(\frac{kT}{I_H}\right)^{1.21} e^{-I_H/kT}$$