2. Collisional Processes

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Collisional Rate Coefficients

- The rate per unit volume of a two-body collision process (A -> B) is

$$n_A n_B \langle \sigma v \rangle_{AB}$$

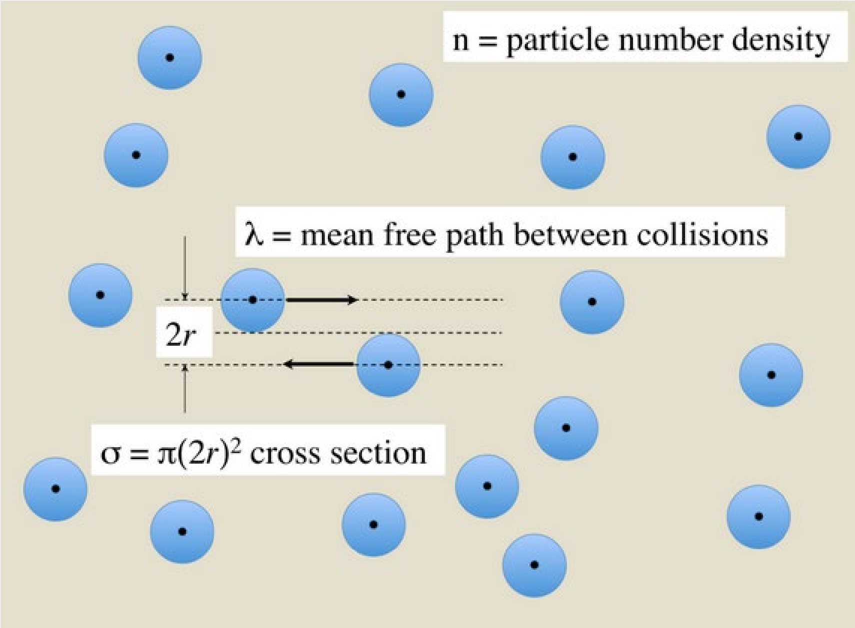
- The two-body collisional rate coefficient is

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv$$

- The mean free path is $\lambda_{\rm mfp} \sim 1/(n\sigma)$

- The collisional timescale is

$$t_{\rm coll} \sim \frac{\lambda_{\rm mfp}}{v_{\rm rms}} \sim \frac{1}{n\sigma}\left(\frac{m}{2\langle E \rangle}\right)^{1/2} .$$



Kinetic Equilibrium

- Maxwellian distribution $$f_v\,dv = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} \mathrm{e}^{-\mu v^2/2kT}\, v^2\,dv$$

$$\langle E\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT = 1.293\,\mathrm{eV}\left(\frac{T}{10^4\,\mathrm{K}}\right)$$

$$\langle\sigma v\rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv \quad = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E)\,\frac{E}{kT}\,\mathrm{e}^{-E/kT}\,\frac{dE}{kT} \quad .$$

Close vs. Distance Encounters

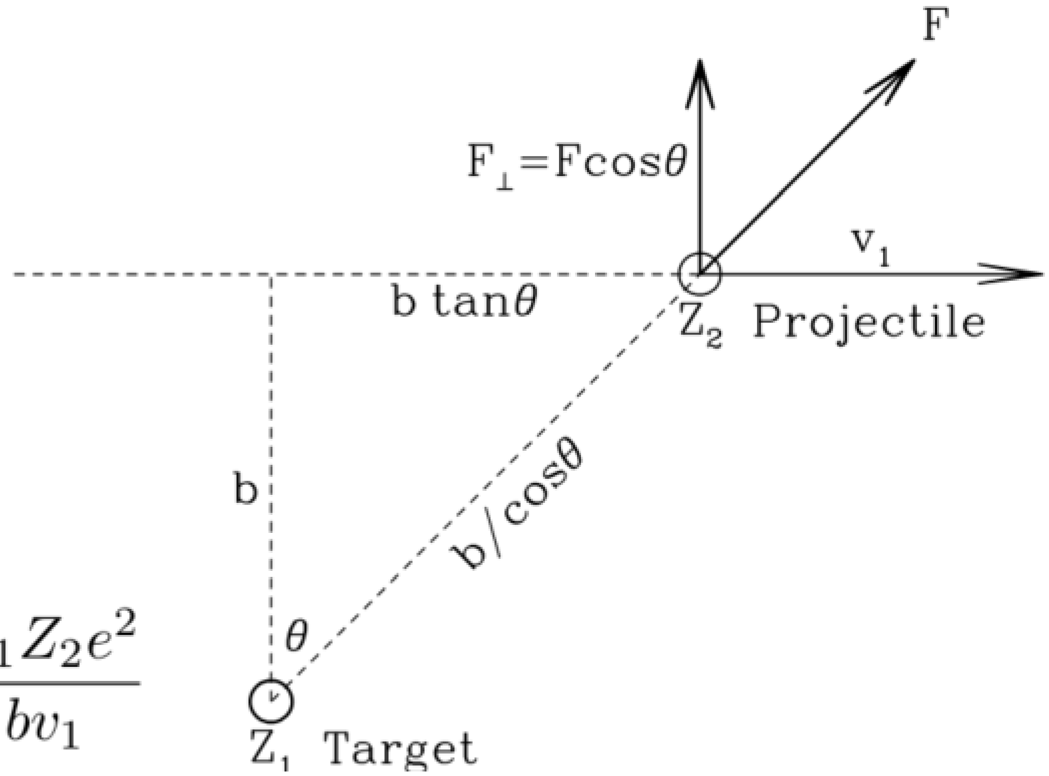
- A close encounter between two charged particles produces relatively large changes in velocity comparable with the initial velocities.
- In a distance encounter these changes are much smaller with negligible deflection angle -> impact approximation.
- For inverse-square forces between particles (gravity, Coulomb), the velocities changes decreases relatively slowly with increasing impact parameters. In consequence, the numerous small velocity changes produced by distance encounters outweigh the effect of rare close encounters.
- This is not true for neutral atom collision!

Inverse-Square Law Forces: Elastic scattering

- The impact approximation

$$F_\perp = \frac{Z_1 Z_2 e^2}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$

$$\Delta p_\perp = \int_{-\infty}^{\infty} F_\perp dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = 2\frac{Z_1 Z_2 e^2}{b v_1}$$

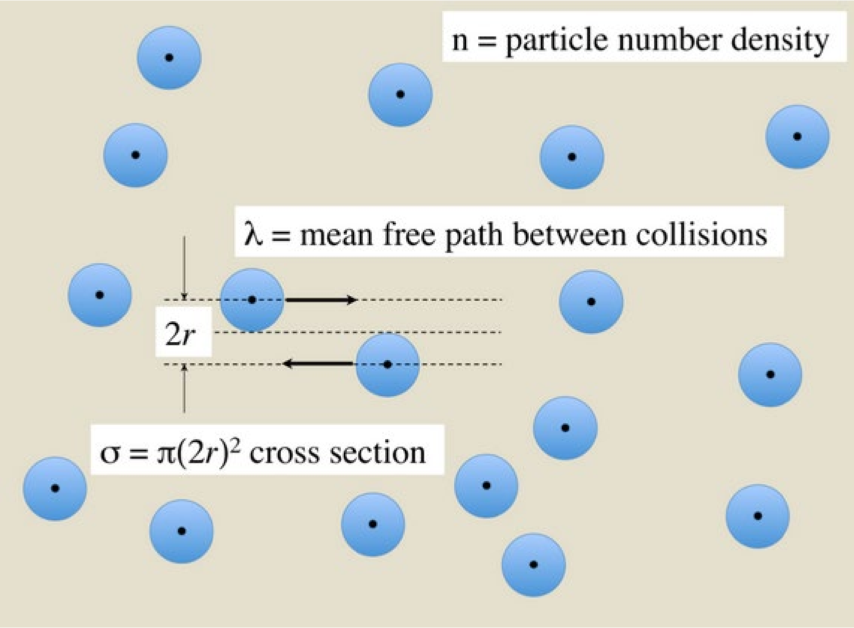


Deflection / Collisional Timescale

- In the impact approximation, each interaction gives an impulse in the direction that is perpendicular to the direction of motion of the projectile.
- The orientation, though, is randomly distributed in this plane. Thus the net vector momentum transferred to the projectile follows a random walk process (dp^2).

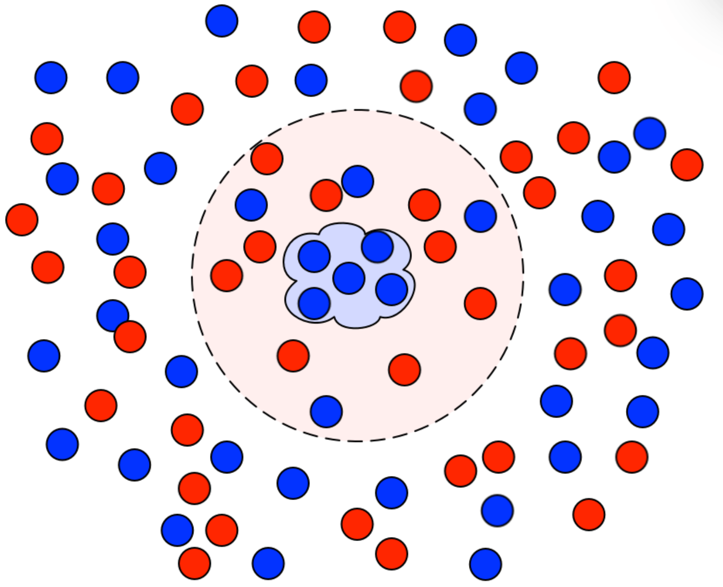
$$\langle \frac{d}{dt}[(\Delta p)_\perp]^2 \rangle = \int_{b_{\min}}^{b_{\max}} \underbrace{[2\pi b db \, n_2 v_1]}_{d(\text{event rate})} \times \underbrace{\left[\frac{2Z_1 Z_2 e^2}{b v_1}\right]^2}_{(\Delta p_\perp)^2}$$

$$= \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} \quad .$$



The min and max distance in the logarithmic.

- The integral is logarithmically divergent at both the min and max impact parameter b, so we have to find some physical limits for the lower and upper cutoffs, b_min and b_max.
- b_min can be estimated at the separation where the Coulomb energy is the same as the kinetic energy of the particle. (The impact approximation fails). b_min~Z1Z2e^2/E
- b_max is the Debye radius, beyond which the plasma maintain electrical neutrality.



$$L_{\rm D} \equiv \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\,\mathrm{cm}\left(\frac{T}{10^4\,\mathrm{K}}\right)^{1/2}\left(\frac{\mathrm{cm}^{-3}}{n_e}\right)^{1/2}$$

Values of the logarithmic term

- The typical values of Lambda is around 20-40.
- The value much larger than one suggests that weak distant frequent encounters dominate the momentum changes.
- The impact approximation is quite accurate for normal ISM conditions.

$$\Lambda \equiv \frac{b_{\rm max}}{b_{\rm min}} = \frac{E}{kT}\frac{(kT)^{3/2}}{(4\pi n_e)^{1/2} Z_1 Z_2 e^3}$$

$$= 4.13 \times 10^9 \left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)^{1/2}$$

$$\ln\Lambda = 22.1 + \ln\left[\left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\right] \quad .$$

Timescale estimates: deflection and energy loss timescales

$$t_{\rm defl} = \frac{(m_1 v_1)^2}{\langle (d/dt)[(\Delta p)_\perp]^2 \rangle} = \frac{m_1^2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} = 7.6\times 10^3\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

$$\begin{aligned}{\rm mfp} = v_1 t_{\rm defl} &= \frac{m_1^2 v_1^4}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} \\ &= 5\times 10^{17}\,{\rm cm}\left(\frac{T}{10^6\,{\rm K}}\right)^2\left(\frac{0.01\,{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

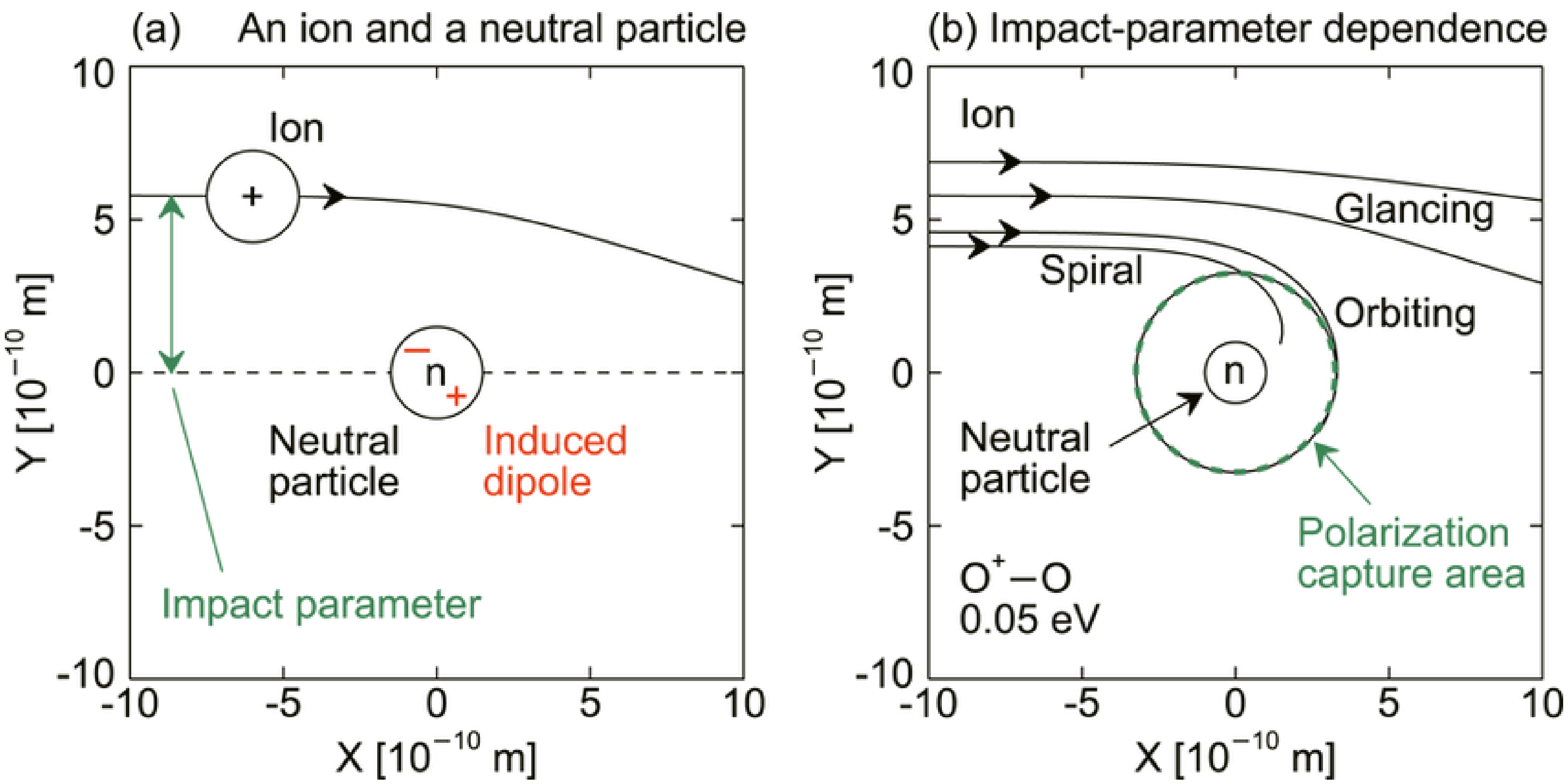
$$\begin{aligned}t_{\rm loss} \equiv \frac{E}{\langle (dE/dt)_{\rm loss}\rangle} &= \frac{m_1 v_1^2}{\langle (d/dt)[(\Delta p)_\perp]^2\rangle / m_2} = \frac{m_1 m_2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} \\ &= 1.4\times 10^7\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

Ion-Neutral Collision: short range interaction!

- The 1/r Coulomb potential means that the potential drop relatively slow for longer distant.
- The physics of Ion-Neutral collision: polarization of the neutral caused by the electric field of the ion.

$$U(r) = -\frac{1}{2}\frac{\alpha_N Z^2 e^2}{r^4}$$

- The cross-section only depends on the short-range impact!



Neutral-Neutral Collision

- If we ignore the very weak van der Waals attraction, we can assume the simplest "billiard balls" model.
- R~ 2*a0, where a0 is the Bohr radius. $a_0 = 5.292 \times 10^{-9}\,\mathrm{cm}$

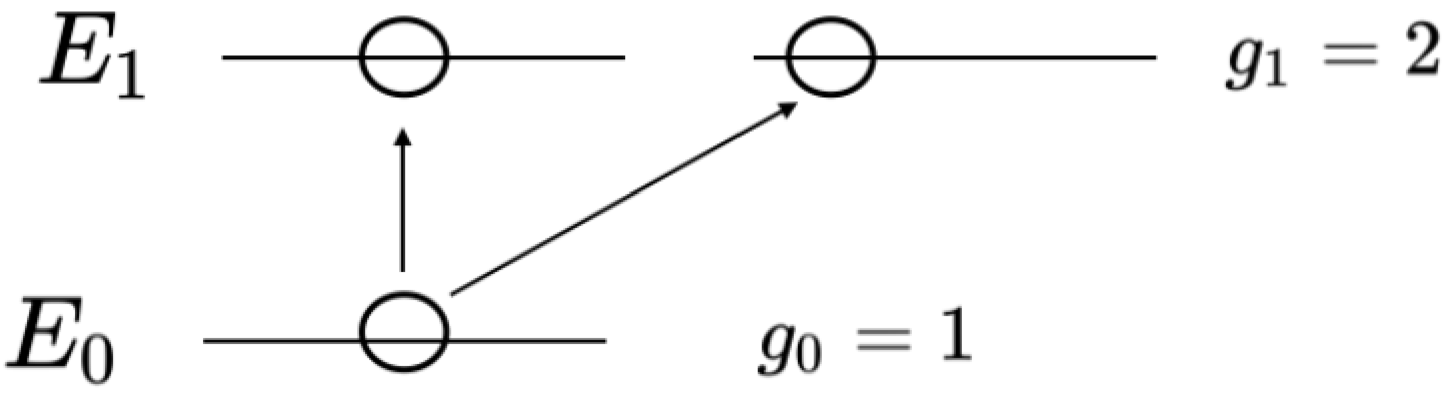
$$\langle \sigma v \rangle = \left(\frac{8kT}{\pi\mu} \right)^{1/2} \pi (R_1 + R_2)^2$$

$$= 1.81 \times 10^{-10} \left(\frac{T}{10^2\,\mathrm{K}} \right)^{1/2} \left(\frac{m_\mathrm{H}}{\mu} \right)^{1/2} \left(\frac{R_1 + R_2}{2\,\mathrm{\mathring{A}}} \right)^2 \mathrm{cm}^3\,\mathrm{s}^{-1}$$

$$t_\mathrm{coll} \sim \frac{\lambda_\mathrm{mfp}}{v_\mathrm{rms}} \sim \frac{1}{n\sigma} \left(\frac{m}{2\langle E \rangle} \right)^{1/2} \sim 2 \times 10^8\,\mathrm{s} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm}^{-3}} \right)^{-1} \left(\frac{\langle E \rangle}{1\,\mathrm{eV}} \right)^{-1/2}$$

Minimalists' Statistical Mechanism: Boltzmann distribution!

- Example of a two-level quantum system:



$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-(E_1 - E_0)/kT}$$

Excitation Temperature

- Excitation temperature is simply a convenient way to parameterize the excitation state of a two-level system, not a measure of kinetic temperature of the gas.
- In excitation equilibrium, excitation temperature is the same as the kinetic temperature.
- In general, however, Texc is far from Tkin.
- One extreme case is astrophysical maser, where a population is over-excited by shocks/strong radiation. The excitation temperature is negative in this case.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT_{\rm exc}}\right),$$

$$kT_{\rm exc} \equiv \frac{E_{u\ell}}{\ln[(g_u/g_\ell)(n_\ell/n_u)]}.$$

Ionization Equilibrium

- The first ionization energy, I, of important elements is ~ 10 eV, which corresponds to ~1e5 K.

Element	ppm by number	percentage by mass	atomic number	1st ionization energy [eV]
hydrogen (H)	910 630	71.10%	1	13.60
helium (He)	88 250	27.36%	2	24.59
oxygen (O)	550	0.68%	8	13.62
carbon (C)	250	0.24%	6	11.26
neon (Ne)	120	0.18%	10	21.56
nitrogen (N)	75	0.08%	7	14.53
magnesium (Mg)	36	0.07%	12	7.65
silicon (Si)	35	0.08%	14	8.15
iron (Fe)	30	0.13%	26	7.90
sulfur (S)	15	0.04%	16	10.36

[a] *Data from Lodders 2010*

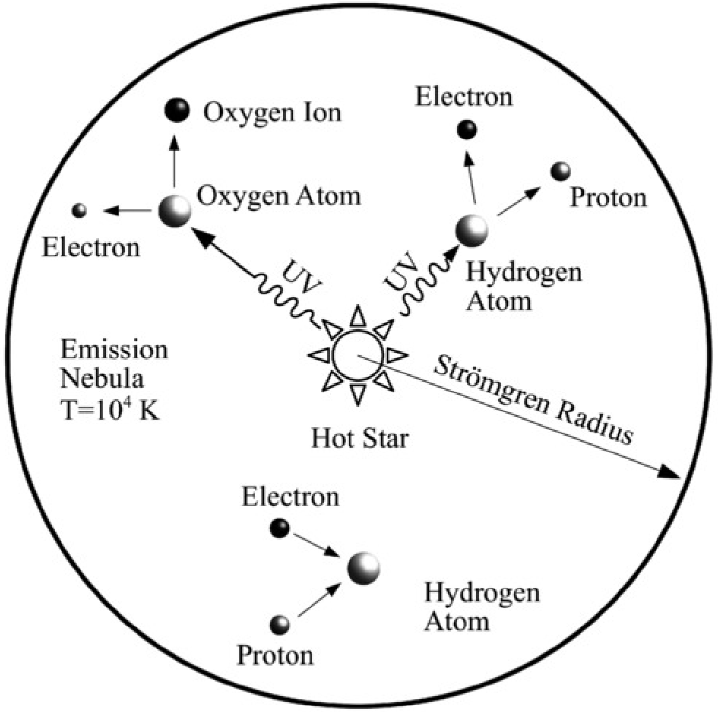
- Only regions with temperature > 1e5K (rare) are effective collisional ionization of neutral atoms.
- In cooler regions, most ionization comes from photo-ionization from UV photons.

Photo-ionization vs. Radiative Recombination

$$X^i + \gamma \rightarrow X^{i+1} + e^-, \qquad X^{i+1} + e^- \rightarrow X^i + \gamma .$$

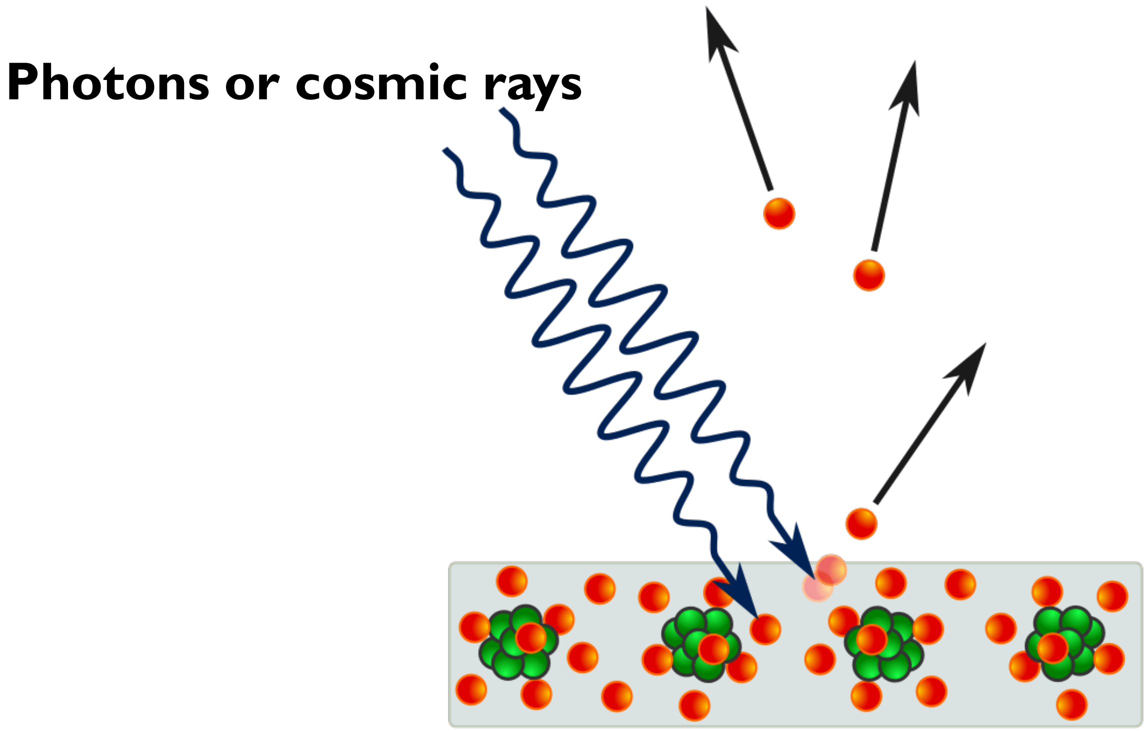
- Ionization equilibrium means the balance between the two processes.

$$n(X^i) n_\gamma \sigma_{\rm pho} c = n(X^{i+1}) n_e \sigma_{\rm rr} v$$



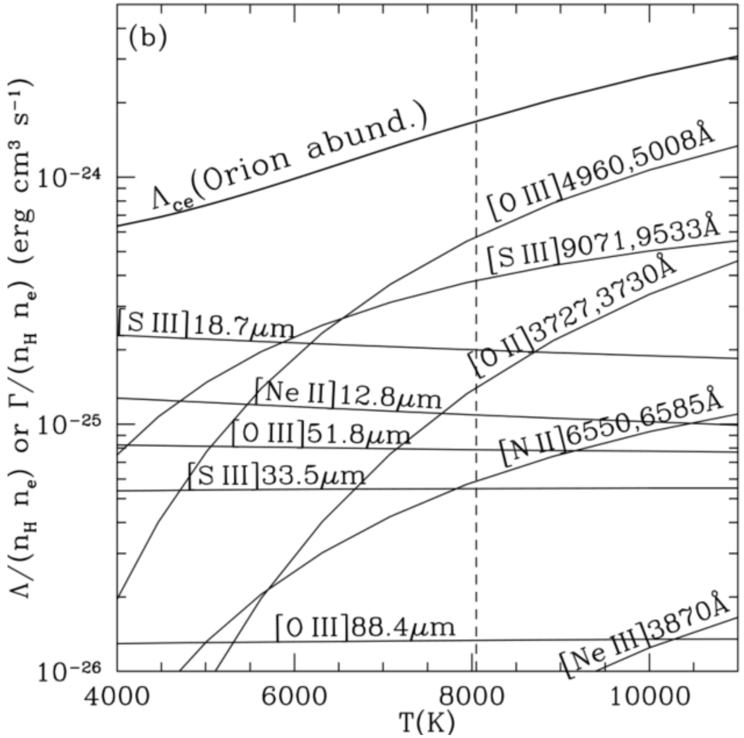
Heating of the ISM

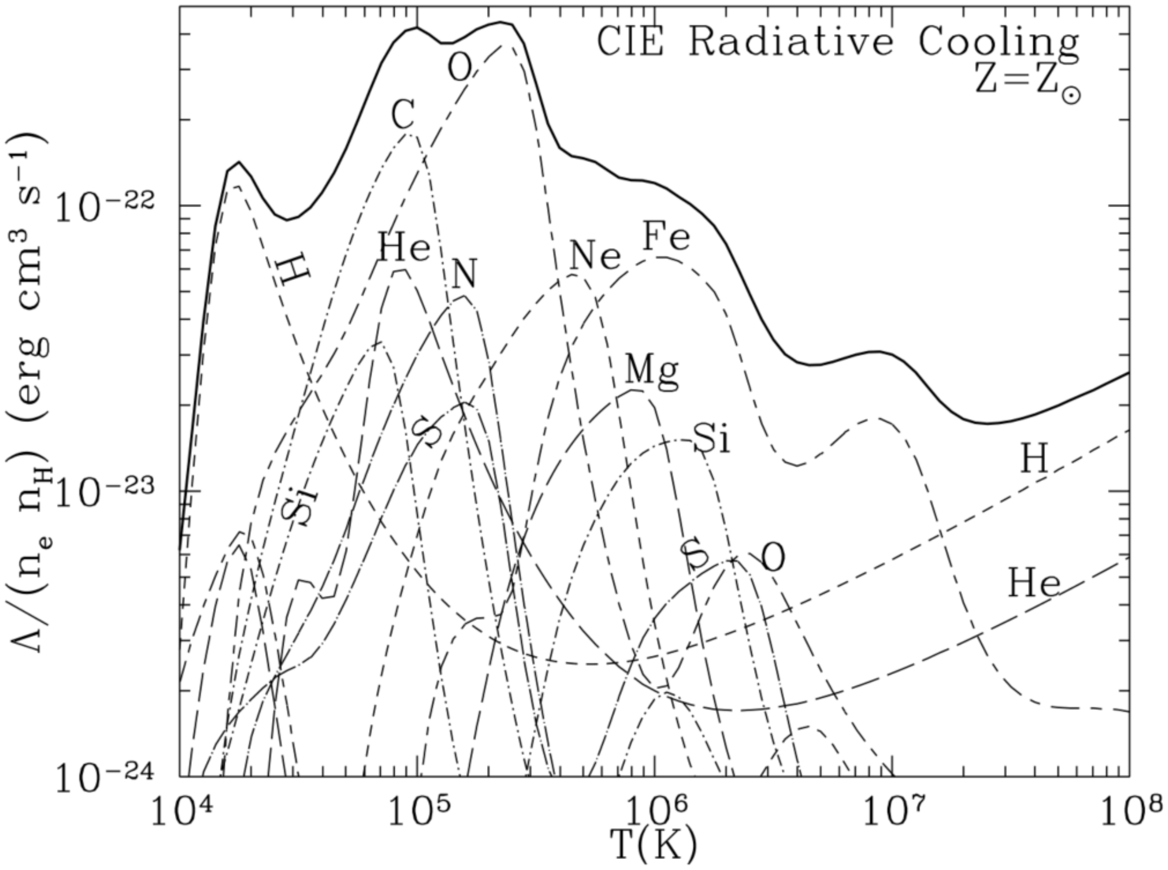
- Heating is the process of converting energy of other forms into thermal energy of the gas.
- Heating is dominated by suprathermal electrons produced via photo-ionization, photoelectric effect on dust grains, and cosmic rays. Hydrodynamical shocks can also convert kinetic energy into thermal energy by the thermalization process at the shock front.



Cooling of the ISM

- Collisional excited line emissions (H, O, C, S for warm medium, Fe for hot gas).
- Recombination radiation (mostly H+ cascade).
- Free-free emission of hot plasma.





Stable and Unstable Equilibrium