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Physics of the Interstellar Medium

Homework 1

Question 1. (20 points) Let's work on some microscopic properties of particles in the ISM.

1) Show that the mean square velocity for a Maxwellian distribution is

$$< v^2 >= \frac{3kT}{m}.$$

2) Show that for gas particle A and B in thermal and kinetic equilibrium at temperature T, the distribution function for the relative speed $w = |\vec{v}_A - \vec{v}_B|$ in encounters between A and B is given by a Maxwellian velocity distribution

$$f(w)dw = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \exp\left(-\mu w^2/2kT\right) w^2 dw,$$

where $\mu \equiv m_A m_B/(m_A + m_B)$ is the reduced mass.

3) Compute the total thermal velocities of a proton and an electron at 10^4 K.

Question 2. (20 points) The cooling for the low-temperature ISM is dominated by the following processes (assuming the C and O abundances measured in the local ISM):

a) [C II] $\lambda 158\mu$m cooling with free electrons:

$$\frac{\Lambda^e_{\rm CII}}{10^{-27}{\rm erg\,cm^3/s}} \approx 3.1 \left(\frac{x}{10^{-3}}\right) \left(\frac{T}{100{\rm K}}\right)^{-0.5} \exp\left(-\frac{91.2{\rm K}}{T}\right)$$

b) [C II] $\lambda 158\mu$m cooling with hydrogen atoms:

$$\frac{\Lambda^{\rm H}_{\rm CII}}{10^{-27}{\rm erg\,cm^3/s}} \approx 5.2 \left(\frac{T}{100{\rm K}}\right)^{0.13} \exp\left(-\frac{91.2{\rm K}}{T}\right)$$

c) [O I] $\lambda 63\mu$m cooling with hydrogen atoms:

$$\frac{\Lambda^{\rm H}_{\rm OI}}{10^{-27}{\rm erg\,cm^3/s}} \approx 4.1 \left(\frac{T}{100{\rm K}}\right)^{0.42} \exp\left(-\frac{228{\rm K}}{T}\right)$$

d) Lyα cooling with free electrons:

$$\frac{\Lambda^e_{{\rm Ly}\alpha}}{10^{-27}{\rm erg\,cm^3/s}} \approx 6 \times 10^5 \left(\frac{x}{10^{-3}}\right) \left(\frac{T}{10^4{\rm K}}\right)^{-0.5} \exp\left(-\frac{1.18 \times 10^5{\rm K}}{T}\right)$$

1) Reproduce the net cooling function for fractional ionization $x = 10^{-3}$ in the temperature range [10 K, 20000 K].

2) Plot the net cooling function for the C and O abundances that are 10% of those in the local ISM (roughly those in the Small Magellanic Cloud).

3) Now consider the heating and cooling balance in this low-metallicity ISM by assuming the dominant heating process is photoelectric heating by dust $\Gamma_{\rm pe} \approx 2 \times 10^{-26}$ erg/s. Plot the curve of the equilibrium density $n_{\rm eq}(T)$ in the density-temperature plane.

4) Plot and find the F-G-H points in the two-phase ISM equilibrium diagram in the low-metallicity ISM. Locate of possible temperature equilibrium points for pressure $P = 2 \times 10^{-13}{\rm dyn\,cm^{-2}}$. Comment on the role of gas metallicity in determining the properties of the stable phases of the ISM in a galaxy.

Question 3. (20 points) List all possible spectroscopic terms for electron configuration ...np^2 and ...np^3 and describe why.

Question 4. (20 points) Numerically integrate the Voigt function

$$H(a,u) = \frac{a}{\pi}\int_{-\infty}^{\infty} \frac{\exp(-y^2)dy}{(u-y)^2+a^2}.$$

1) Plot $H(a,u)$ as a function of u for different a from 0 to 10.

2) Show that $H(a,u)$ has a Gaussian-like core and Lorentz-like wing and propose an approximate function form that simplifies the calculation of H and covers both regimes in the limit of $a \ll 1$.

3) Overplot your proposed approximate function to the accurate numerical result and calculate the relative error of your function over u for $a = 0.01$.

Question 5. (20 points) A distant quasar at a redshift $z_Q = 2.5$ is observed on a line-of-sight which passes through the disk of an intervening galaxy. A strong absorption feature is observed in the continuum spectrum of the quasar at an observed wavelength of 3647 Å. This absorption feature is interpreted as Lyα absorption in the intervening galaxy, implying that the galaxy is at a redshift $z_G = 2.0$.

1) The absorption feature at 3647 Å has an observed equivalent width $W_{\lambda,\mathrm{obs}} = 6.0$ Å. The equivalent width that would be observed by an observer in the rest-frame of the absorbing galaxy would be $W_{\lambda,\mathrm{G}} = 6.0\text{Å}/(1+z_G) = 2.0$ Å. Estimate the HI optical depth at line-center of the Lyα line which is required to produce this equivalent width. Assume the one-dimensional velocity dispersion of the HI to be 20 km/s. [hint: by trial-and-error determine which part of the curve-of-growth you are on.]

2) What is the column density of HI in the $n = 1$ level in the intervening galaxy? Remark on the similarity/difference between the ISM of this galaxy vs. the local ISM in our Galaxy.