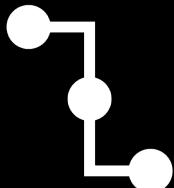


Radiative Processes in Astrophysics

Observation

Up to cosmic size scale



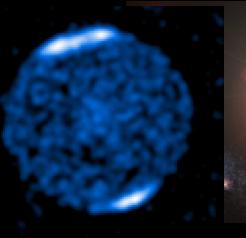
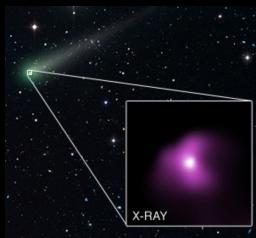
C/2012S1
(comet)

Jupiter
(planet)

Sun
(star)

Cas A
(SNR)

M82
(galaxy)



Theory

Down to atomic size scale

Phoenix
(gal. cluster)

Cosmic web filament

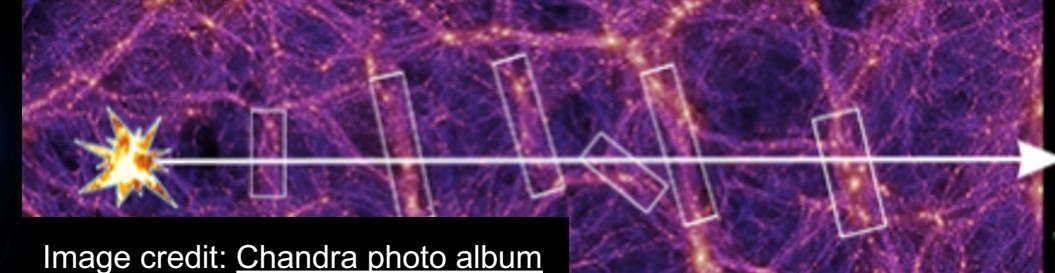


Image credit: [Chandra photo album](#)

Chpt.5 Atomic processes

- 5.1 Atomic data for astrophysics
 - 5.1.1 Astro atomic spectra
 - 5.1.2 Cosmic elements
 - 5.1.3 Nomenclature of atomic structure
 - 5.1.4 Energy levels
 - 5.1.5 NIST energy levels and lines
- 5.2 Two-level systems
- 5.3 Bremsstrahlung
- 5.4 Recombination and photoionization
- 5.5 Collisional excitation and de-excitation
- 5.6 Other atomic processes
- 5.7 Astrophysical plasma models

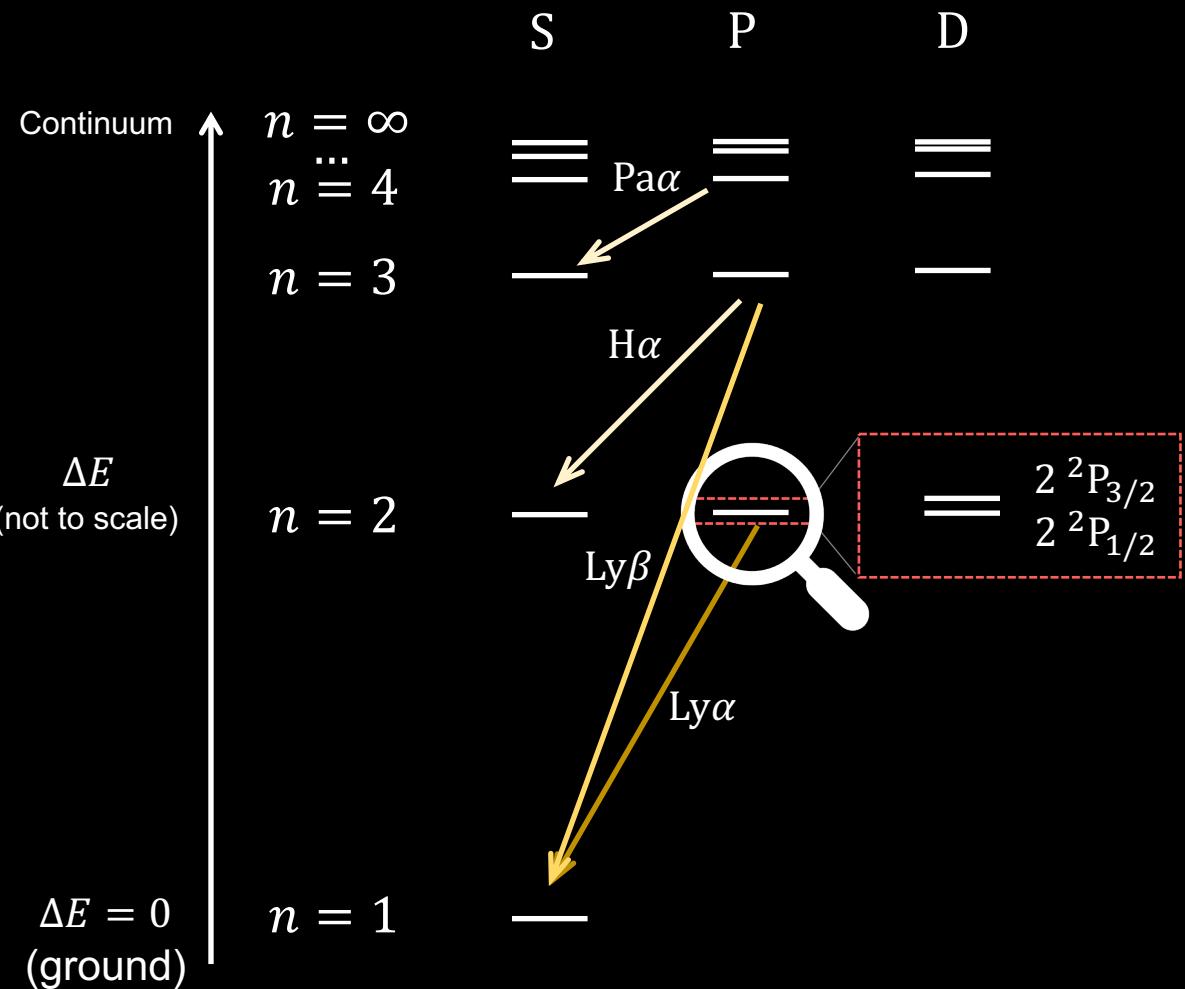


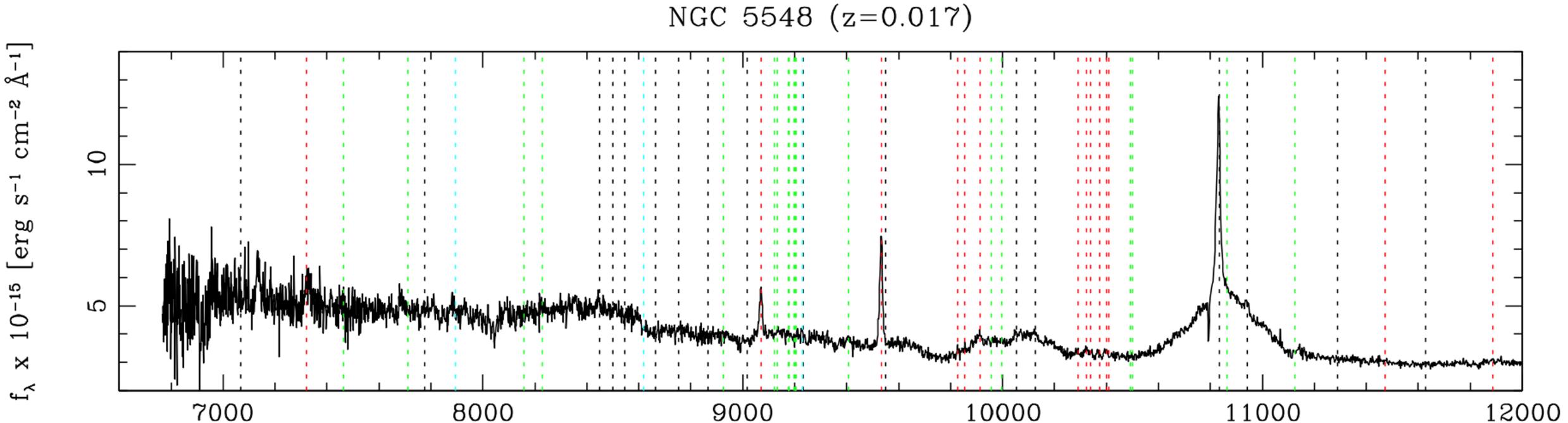
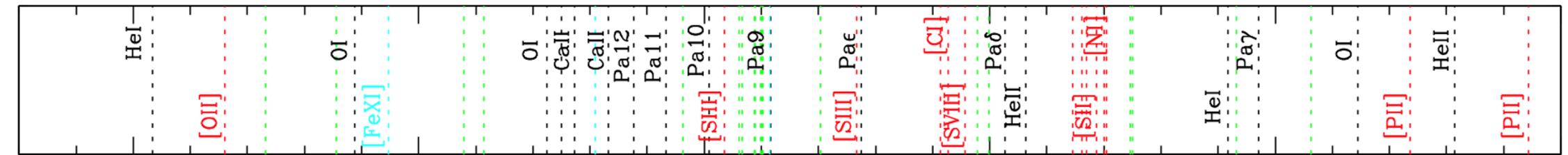
Image credit: Junjie Mao

Exemplary NIR spectra

Landt et al. 2019

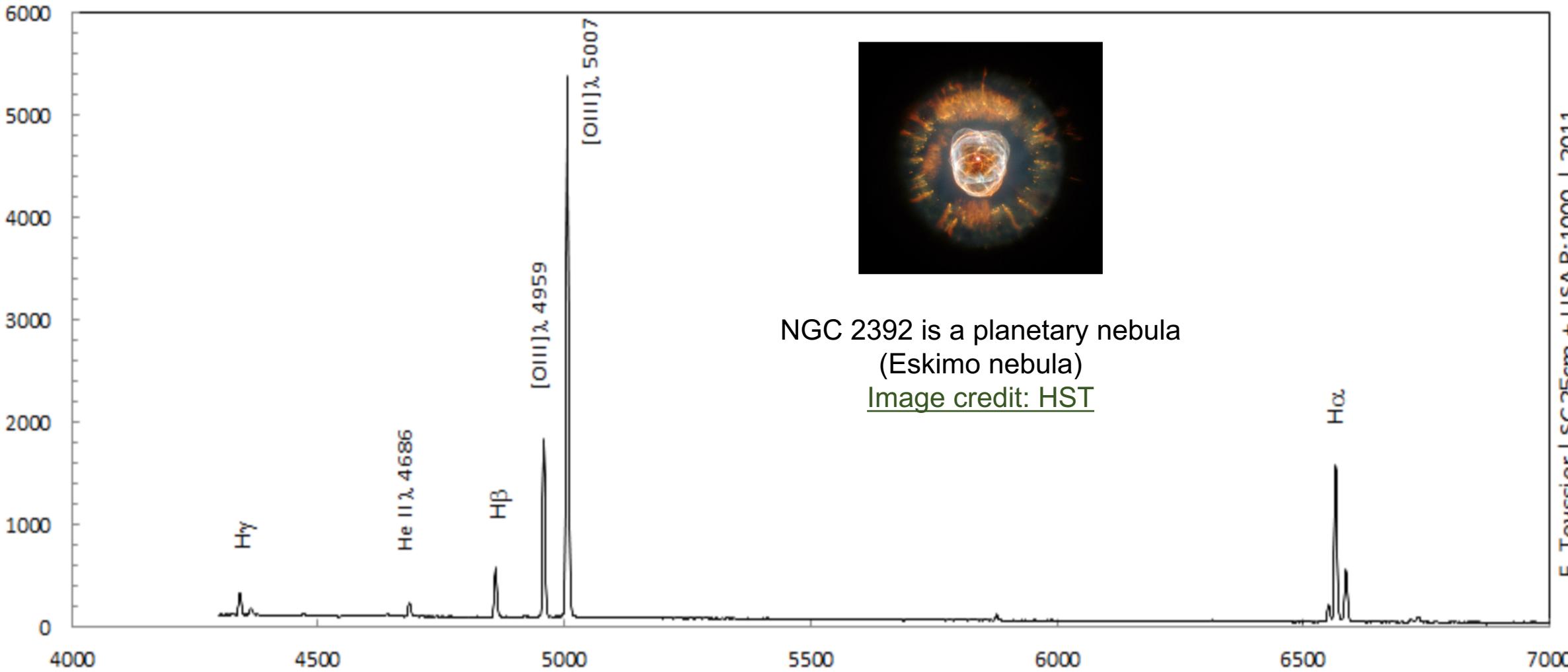
NGC 5548 is a nearby galaxy host a Seyfert 1 AGN.

HST Image credit: ESA,
NASA, Davide Martin

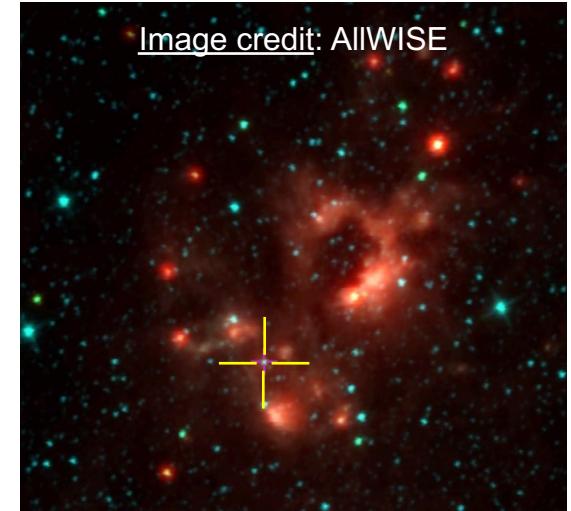
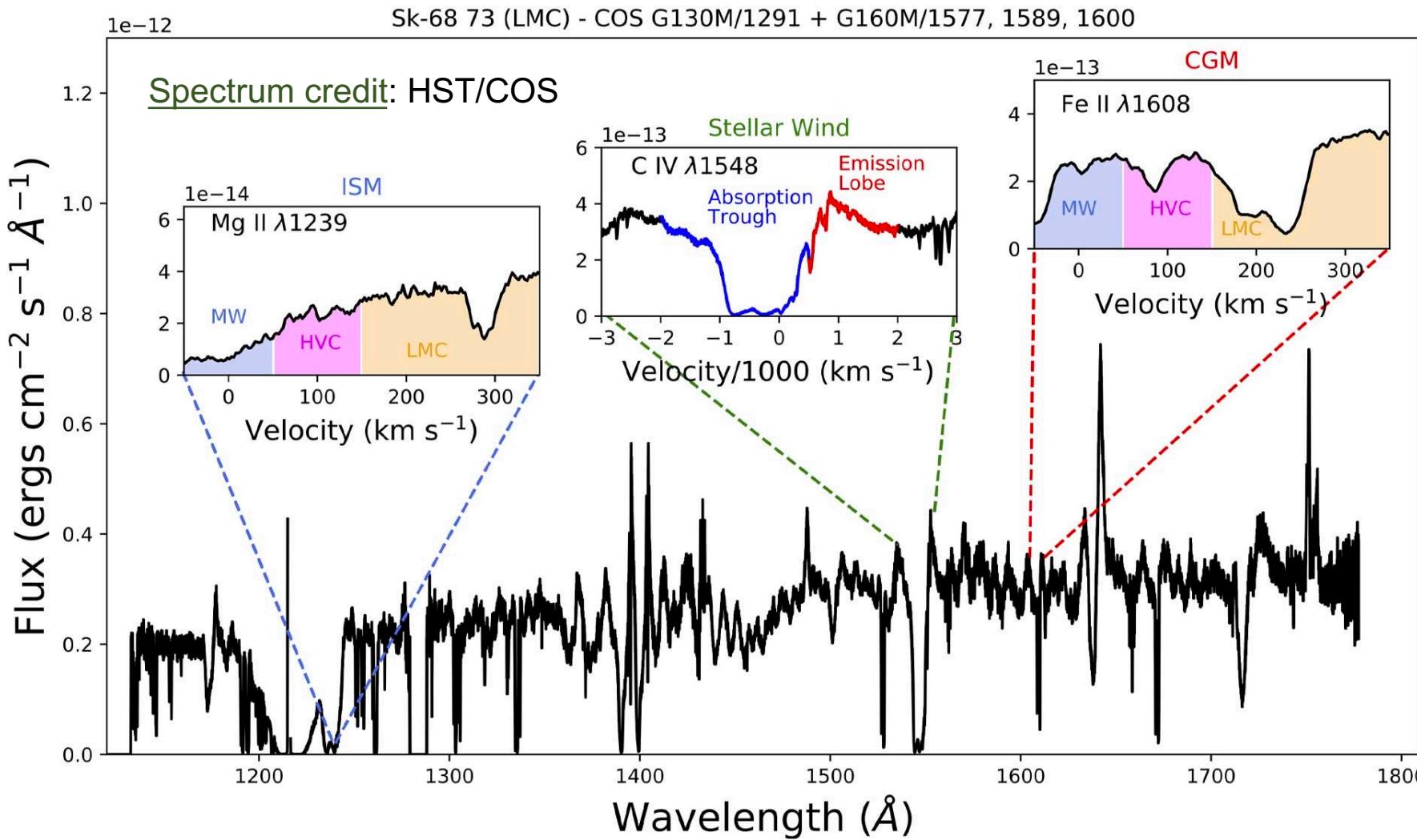


Exemplary optical spectrum

Spectrum credit: F. Teyssier

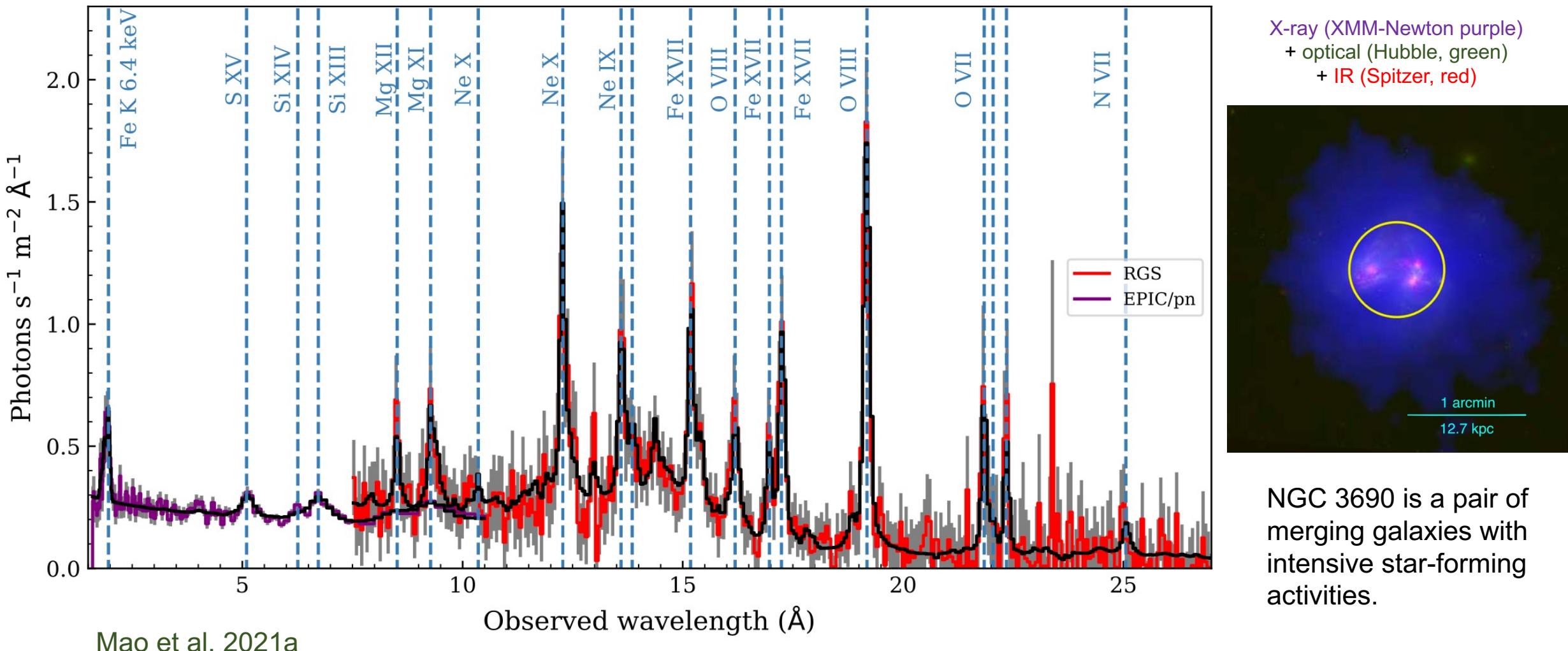


Exemplary UV spectrum

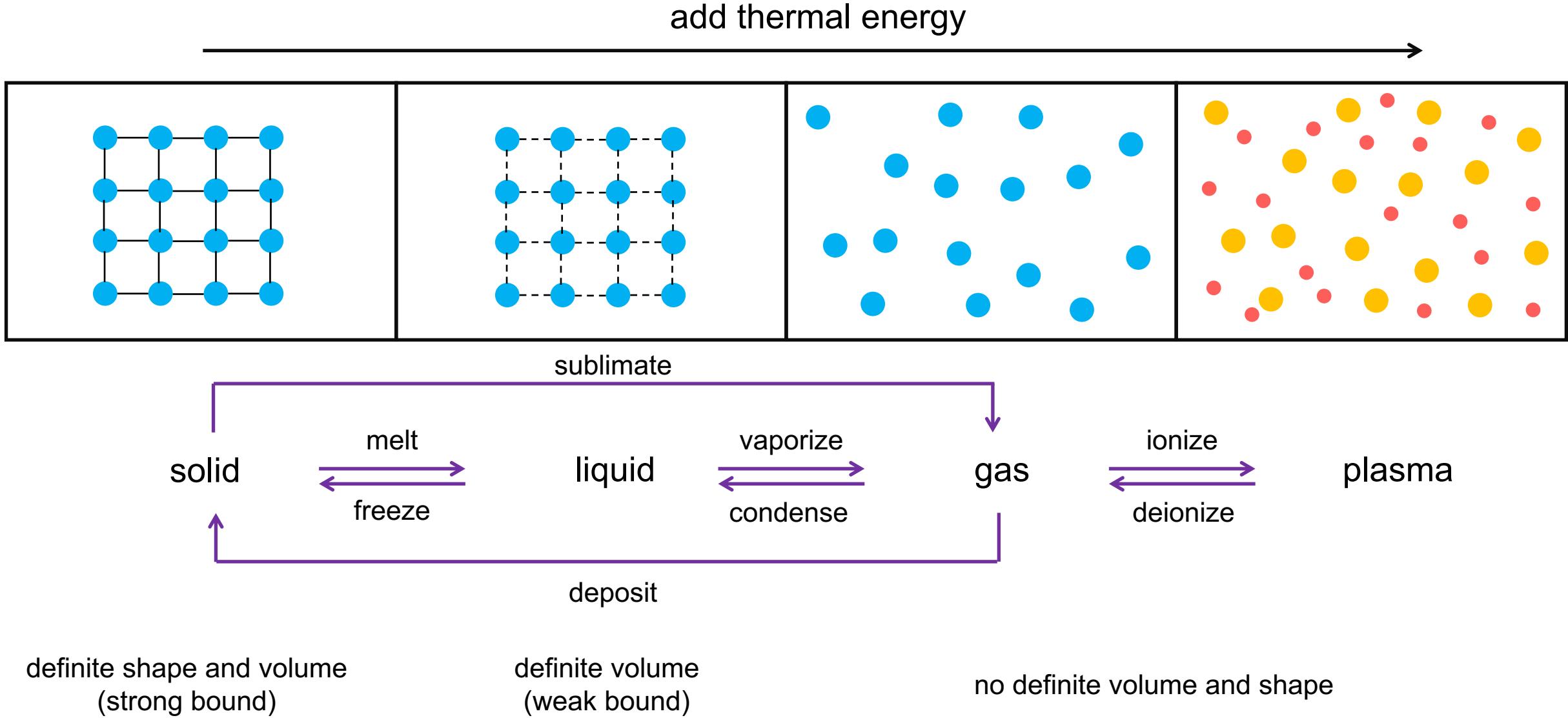


Sk-68 73 is a massive Wolf-Rayet star in the LMC

Exemplary X-ray spectrum



Four states of matter

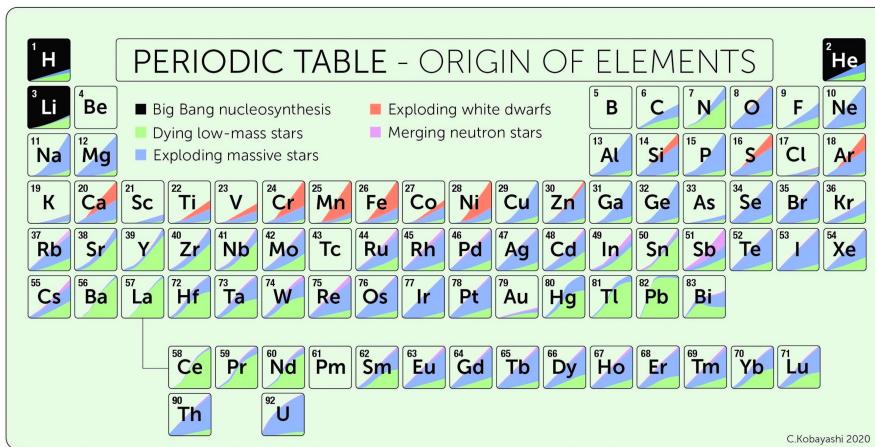


Astrophysical plasmas (cont.)

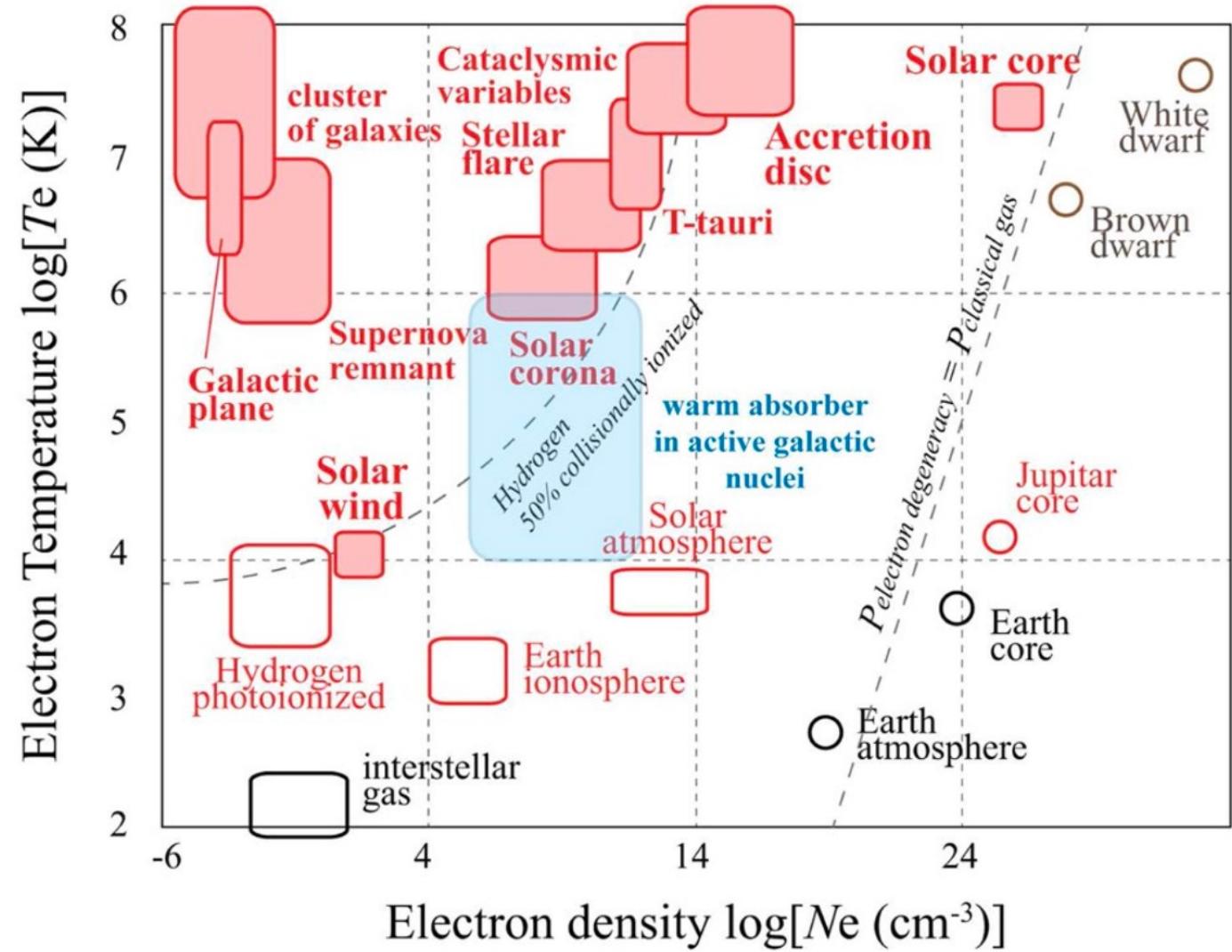
Ezoe et al. 2021

Most of the **baryons** in the Universe are in the form of ionized plasmas with a temperature of $\sim 10^{4-9}$ K (e.g., [Bregman et al. 2023](#))

All natural elements in the periodic table are baryons



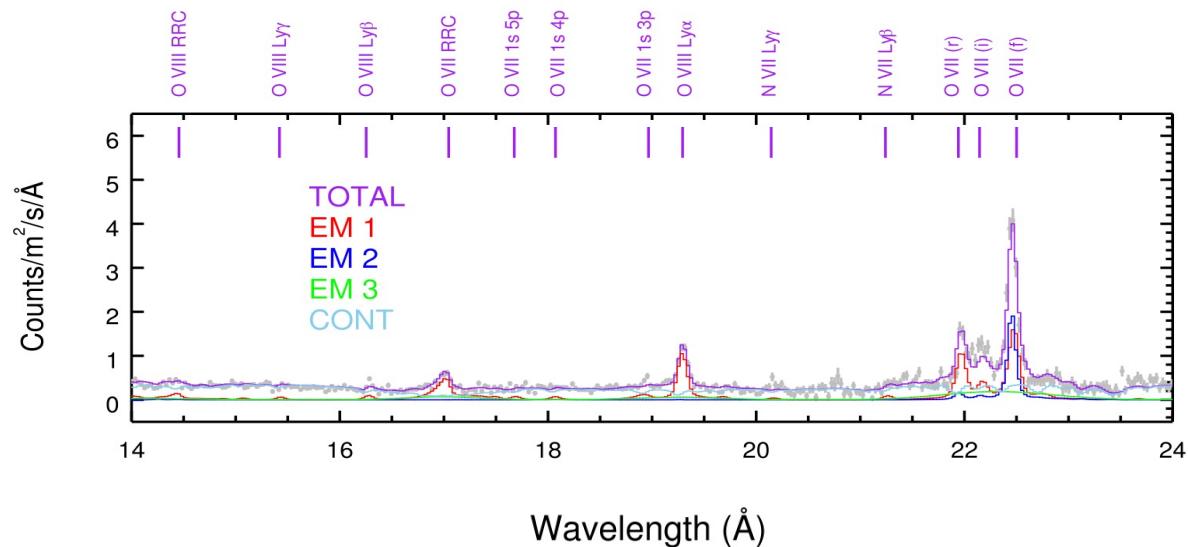
[Image Credit:](#) Chiaki Kobayashi et al. [Artwork:](#) Sahm Keily



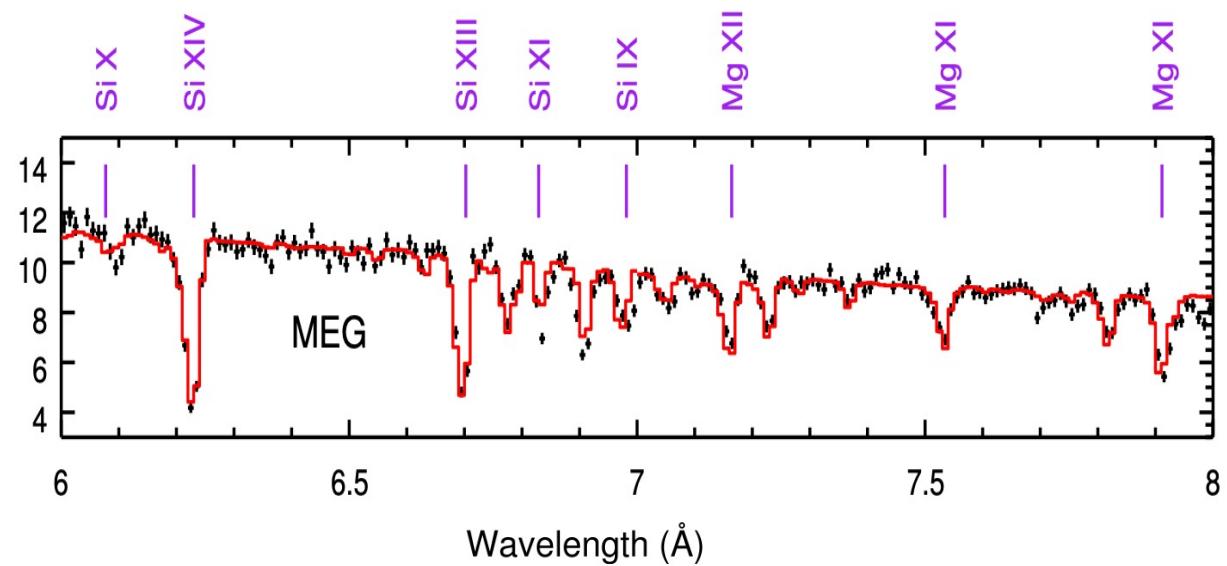
Astro plasma spectra

lines and continua

Spectral diagnostics of characteristic **emission** and **absorption features** in the observed atomic spectra can quantify the physical parameters of the observing targets (e.g., elemental abundances, density, temperature, kinematics).

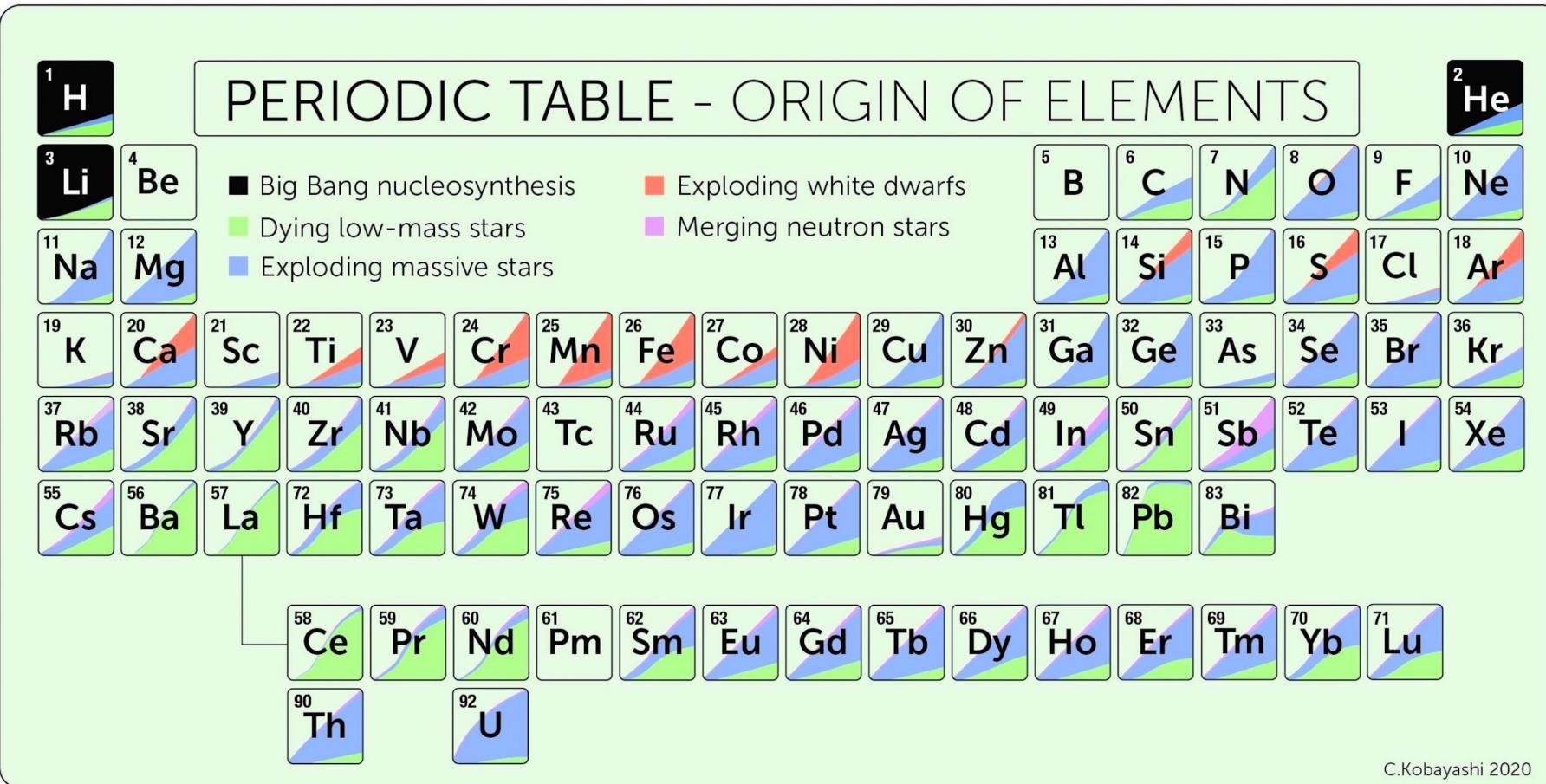


Mao et al. 2018



Mao et al. 2019b

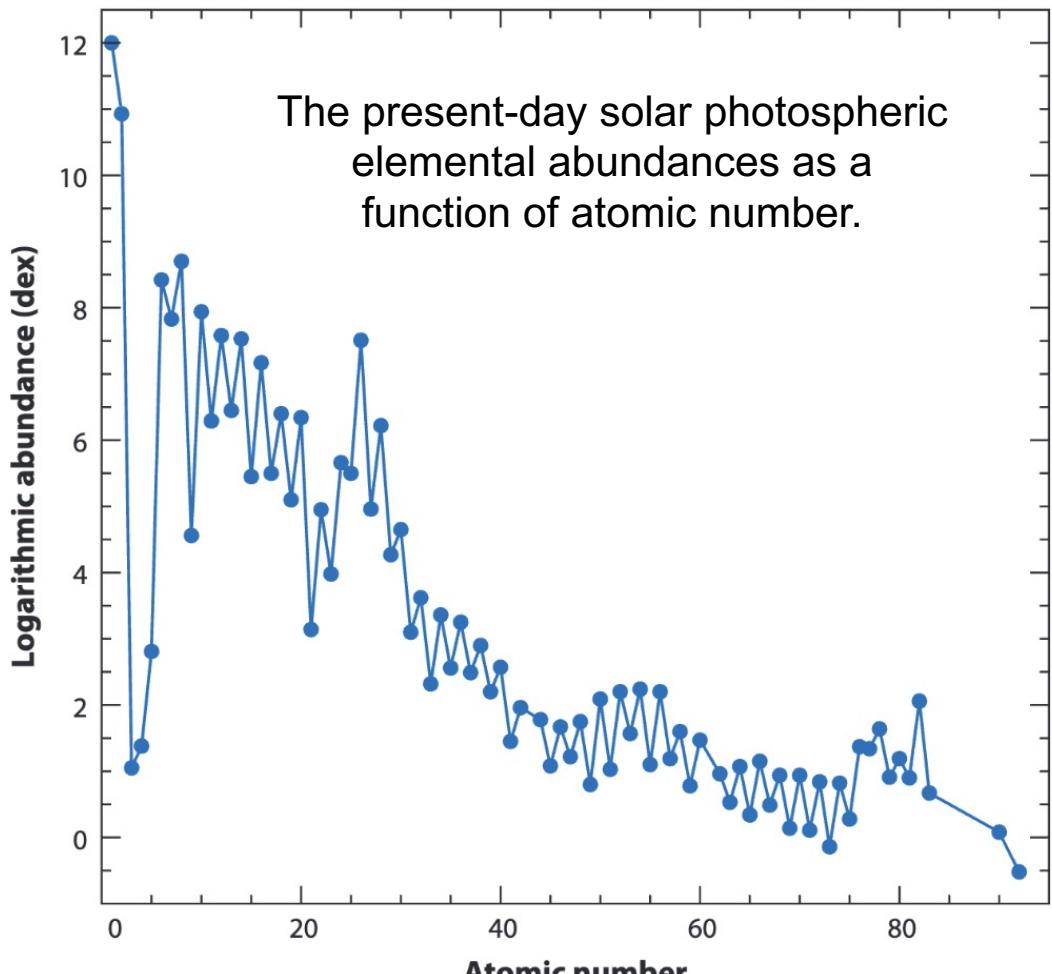
Periodic table – Origin of elements



Ideally, we would like to study as many (naturally occurring) elements as possible.

Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily

Cosmic elemental abundances



Asplund et al. 2009

Solar mass-fractions (Basu & Antia, 2004):

- ◻ $X = 0.7389$ for H
- ◻ $Y = 0.2463$ for He
- ◻ $Z = 0.0148$ for metals (Li to U)
- ◻ $Z/X = 0.020$

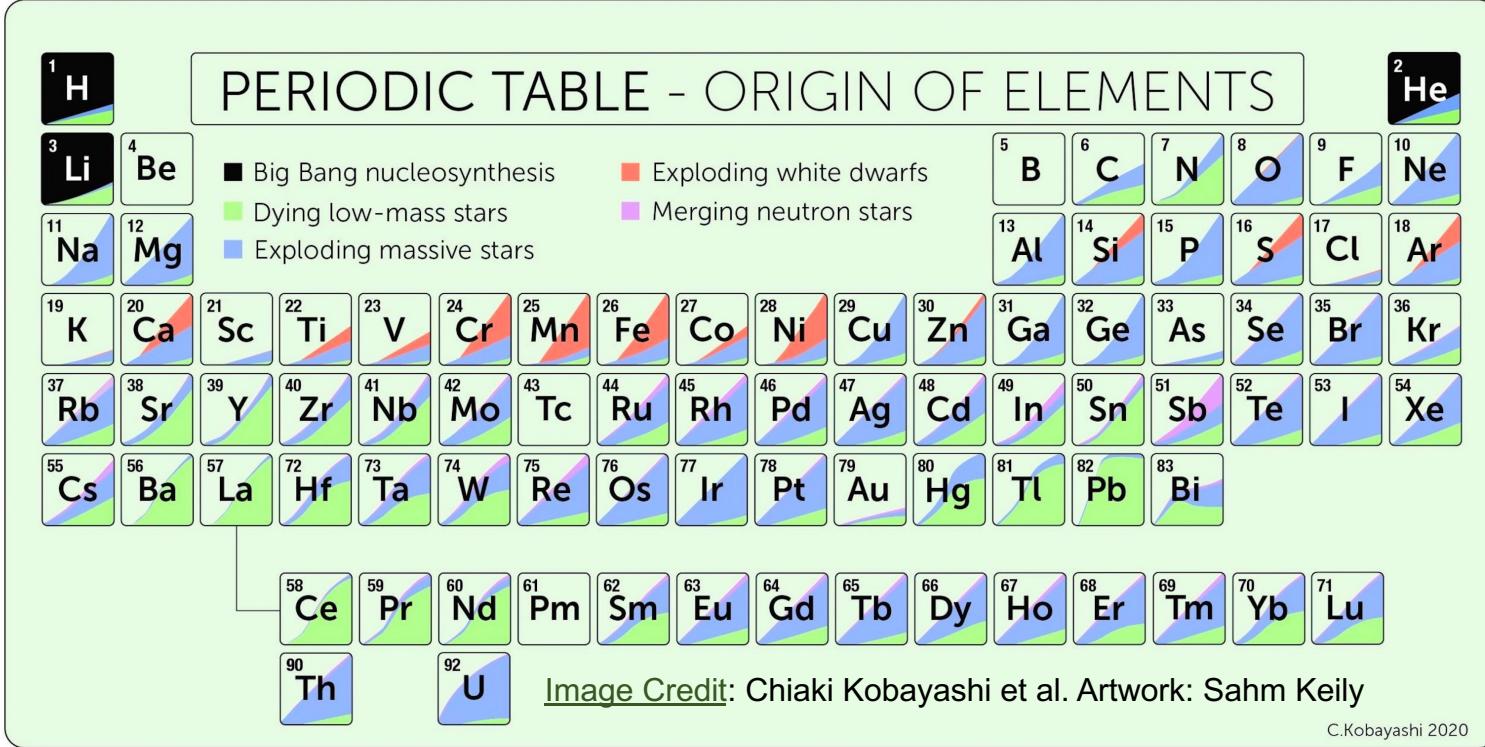
Astro abundance scales

$$A(\epsilon_i) = 12 + \log_{10} \frac{n(\epsilon_i)}{n(H)}$$

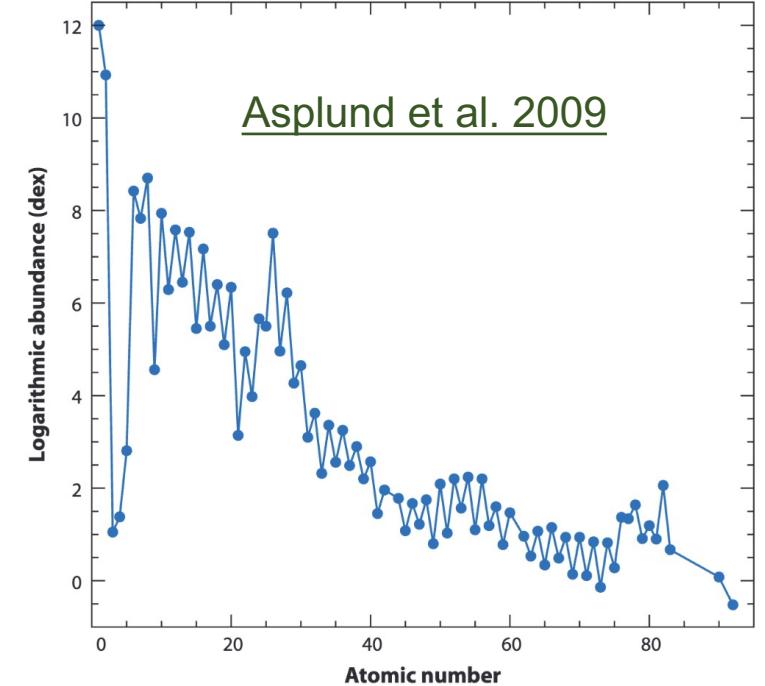
- ❖ By definition, $A(H) = 12$ for H.
- ❖ $A(O) = 8.69$ means that O is a factor of ~ 2000 less than H.

dex refers to the decadic logarithmic units where 1 dex stands for a factor of 10 and 0.1 dex stands for a factor of $10^{0.1} \sim 1.259$

Astro periodic table



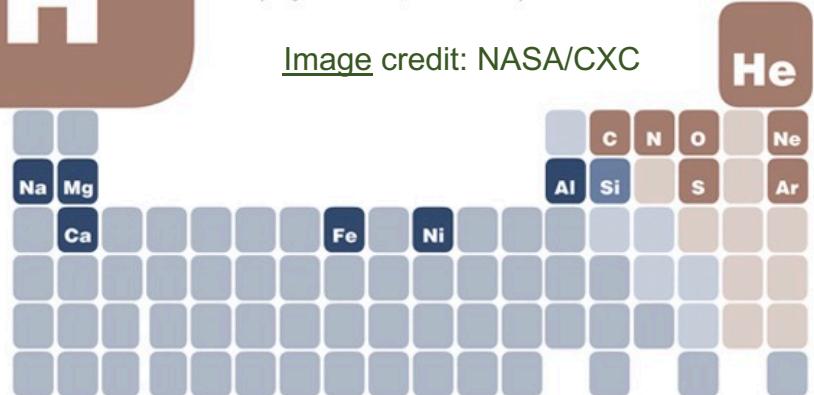
Ideally, we would like to study as many (naturally occurring) elements as possible. But we might pick and chose in practice.



The Periodic Table for Astronomy

A graphic representation of the abundances of the elements is shown in this "astronomers" version of the periodic table. What leaps out of this table is that the simplest elements, hydrogen and helium, are far and away the most abundant.

Image credit: NASA/CXC

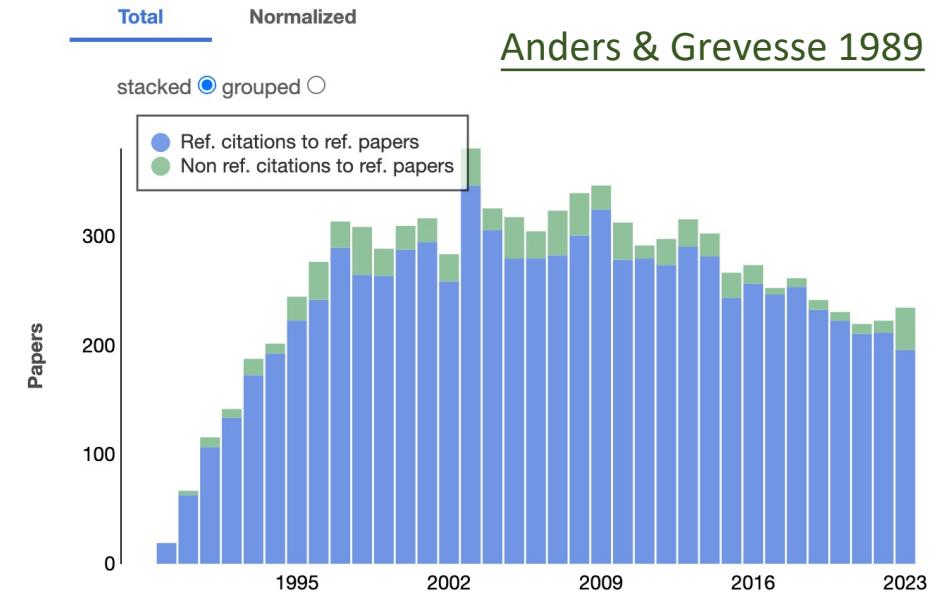


Solar abundance tables

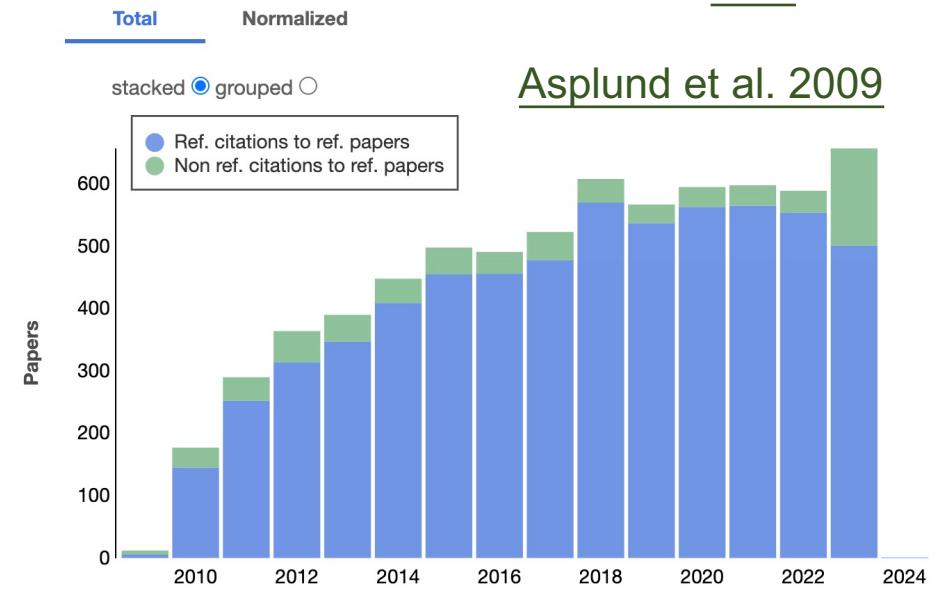
Sometimes, “solar abundance” is used without reference, but technically, various solar abundance tables exist and they do differ.

Source	X	Y	Z	Z/X
Present-day photosphere:				
Anders & Grevesse (1989) ^a	0.7314	0.2485	0.0201	0.0274
Grevesse & Noels (1993) ^a	0.7336	0.2485	0.0179	0.0244
Grevesse & Sauval (1998)	0.7345	0.2485	0.0169	0.0231
Lodders (2003)	0.7491	0.2377	0.0133	0.0177
Asplund, Grevesse & Sauval (2005)	0.7392	0.2485	0.0122	0.0165
Lodders, Palme & Gail (2009)	0.7390	0.2469	0.0141	0.0191
Present work	0.7381	0.2485	0.0134	0.0181
Protosolar:				
Anders & Grevesse (1989)	0.7096	0.2691	0.0213	0.0301
Grevesse & Noels (1993)	0.7112	0.2697	0.0190	0.0268
Grevesse & Sauval (1998)	0.7120	0.2701	0.0180	0.0253
Lodders (2003)	0.7111	0.2741	0.0149	0.0210
Asplund, Grevesse & Sauval (2005)	0.7166	0.2704	0.0130	0.0181
Lodders, Palme & Gail (2009)	0.7112	0.2735	0.0153	0.0215
Present work	0.7154	0.2703	0.0142	0.0199

Asplund et al. 2009

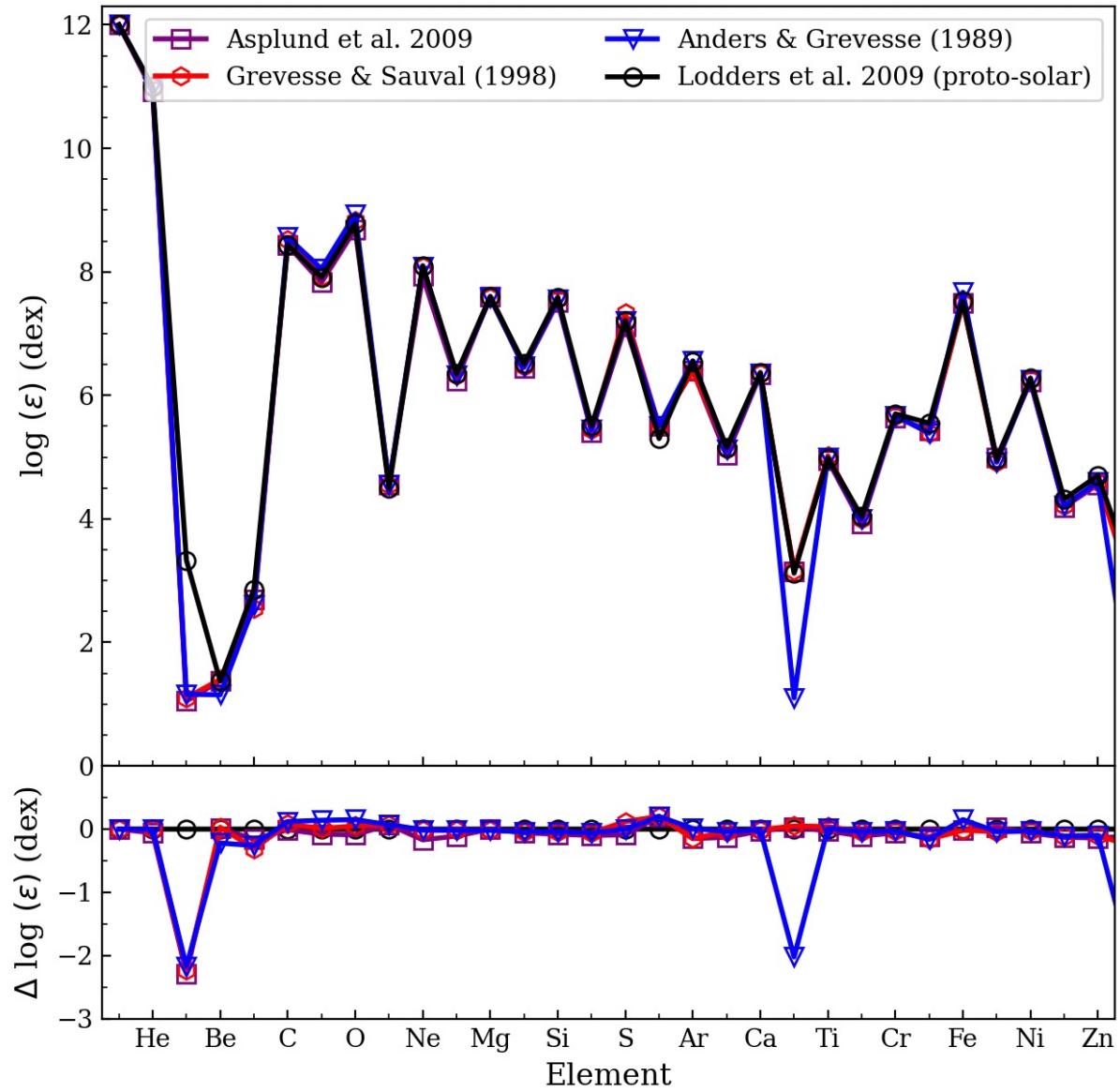
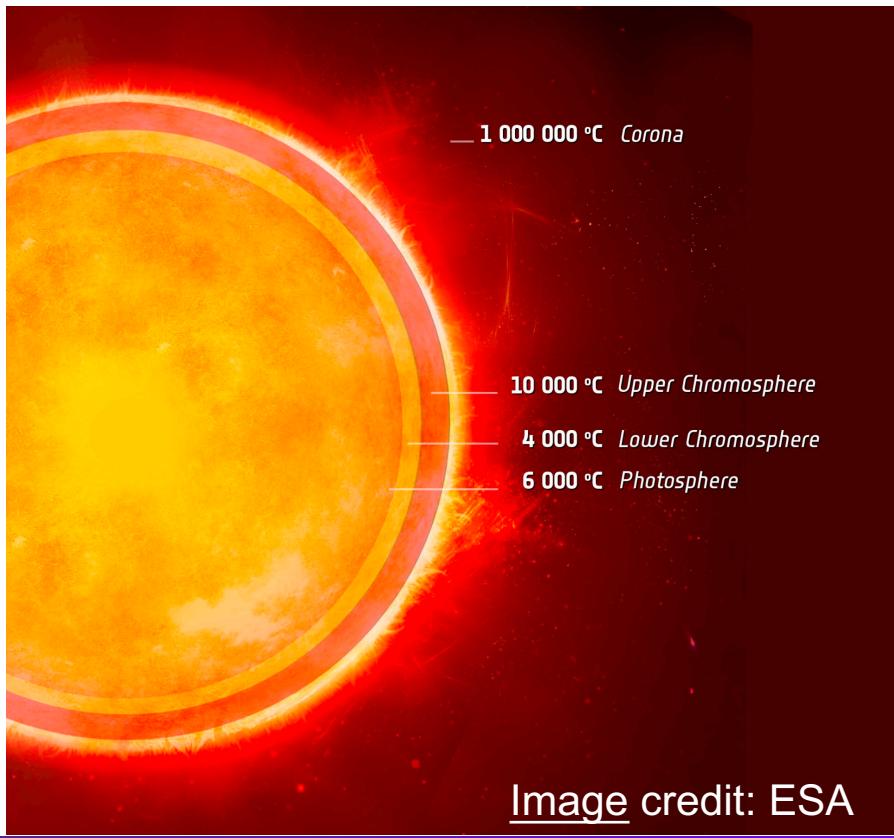


Credit: [ADS](#)



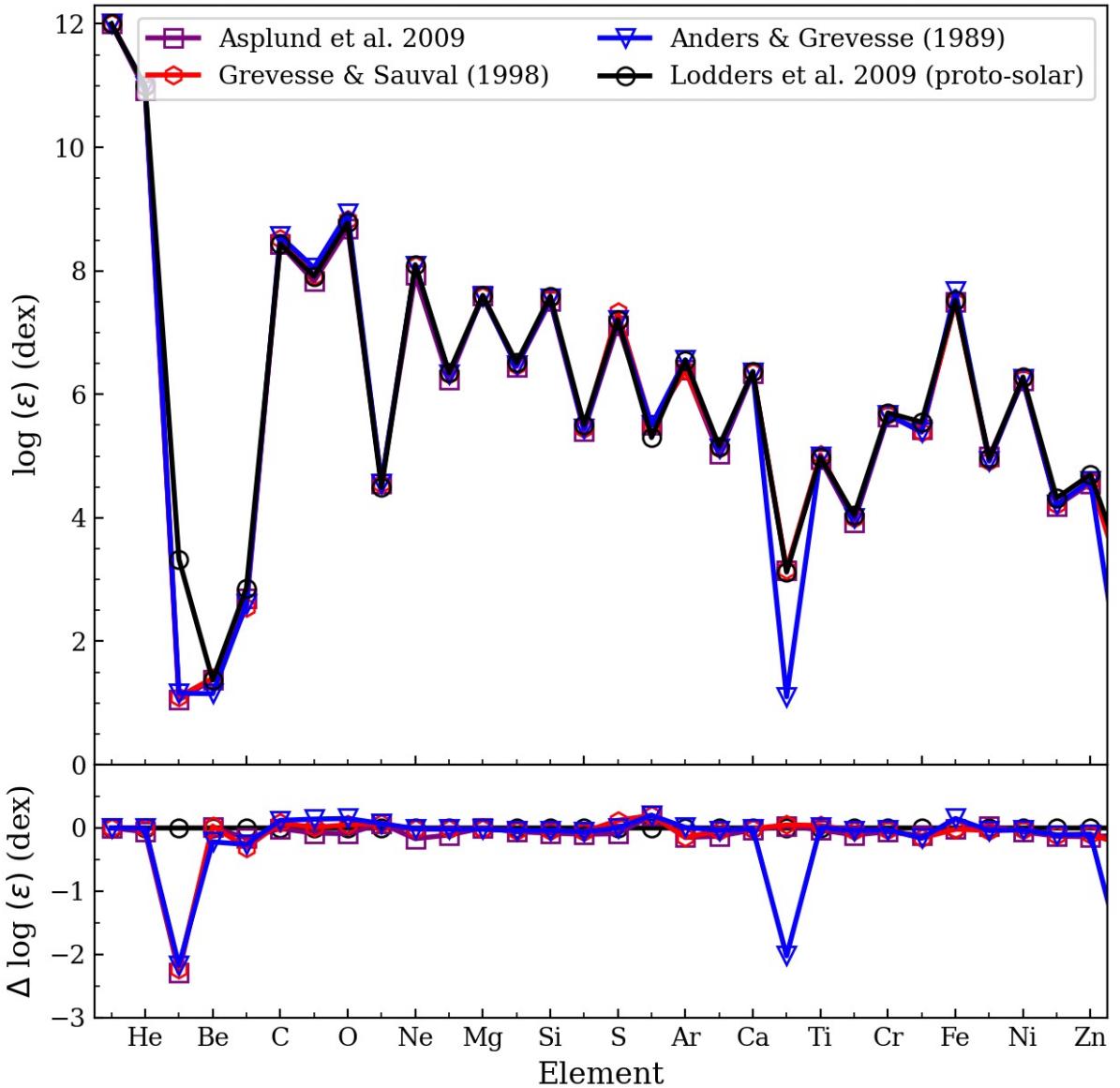
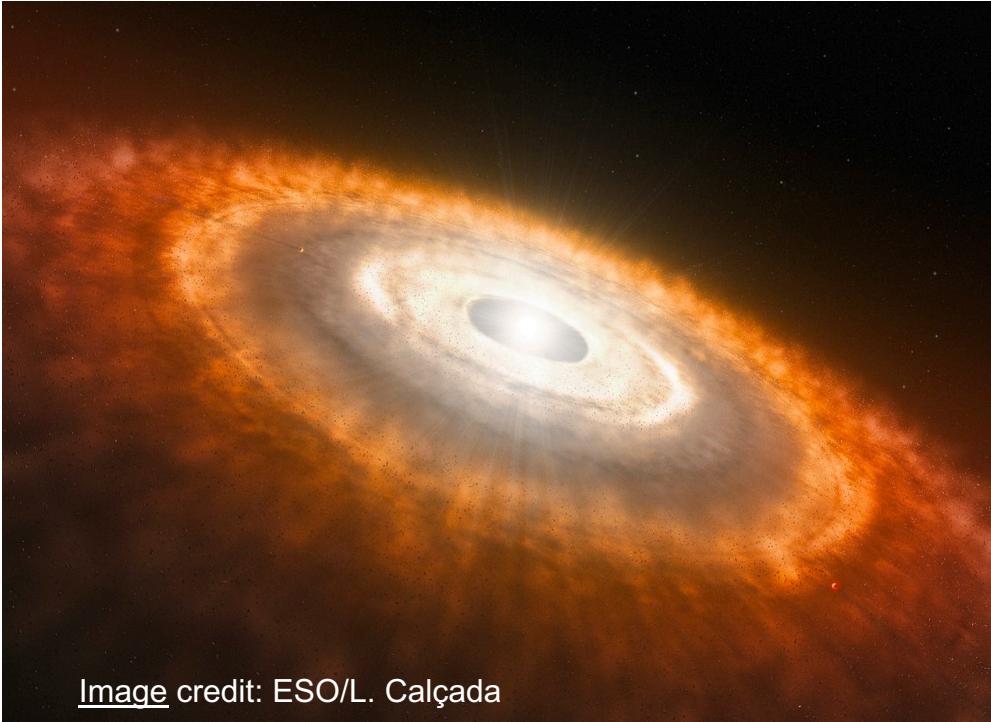
Photosphere solar abundance

Photosphere abundance: the composition of the **present day** Sun's outer convection zone (~2% of the Sun's total mass with ongoing diffusion and gravitational settling into the Sun's interior over the solar lifetime).



Solar abundance tables

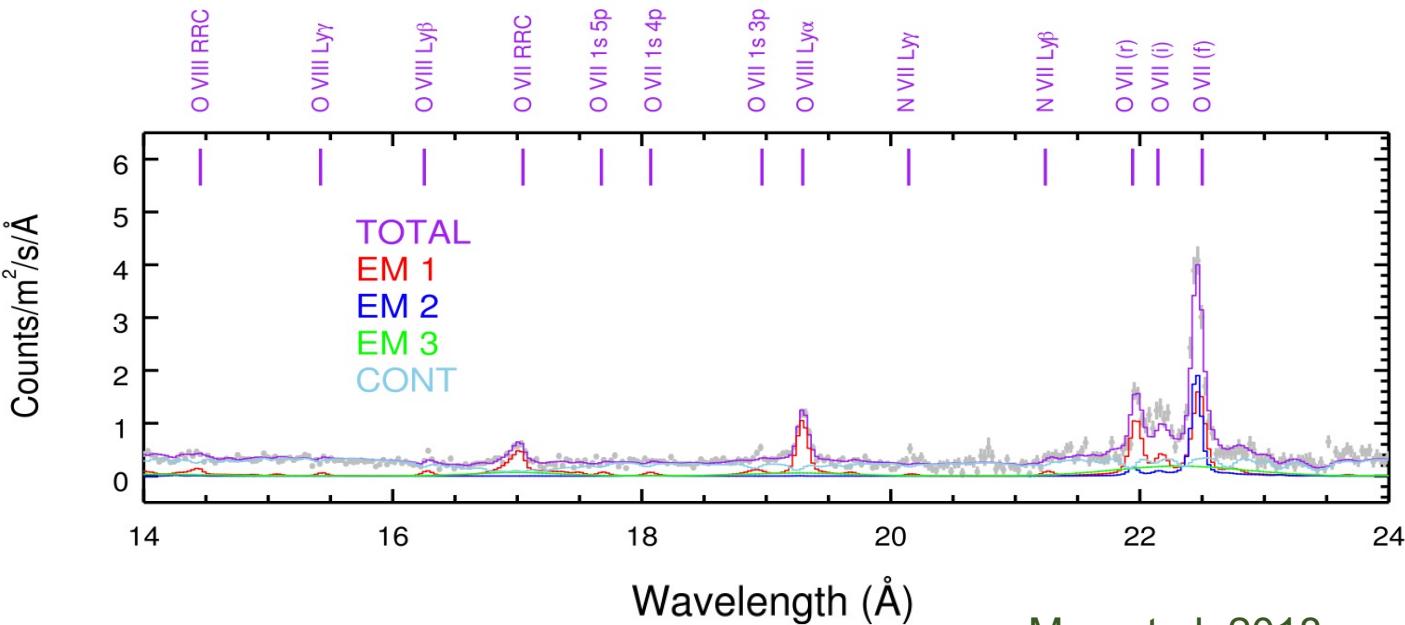
Proto-solar abundance: the overall composition of the molecular cloud within the ISM from which the solar system formed
4.567 Gyr ago.



Ionization stage

The nomenclature for atoms and ions in the astronomy community is slightly different from that in the atomic physics community. We use the combination of an element symbol (e.g., H, C, O) and a roman numeral (e.g., I, IV, XVII) to indicate the charge/ionization stage of the element.

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X
11	12	14	15	16	19	20	30	40	50
XI	XII	XIV	XV	XVI	XIX	XX	XXX	XL	L



[Mao et al. 2018](#)

Latex trick (case sensitive!)

O {\sc vii}

For A&A only

\ion{Si}{VII}

Ionization stage (cont.)

1	2	3	4	5
I	II	III	IV	V

6	7	8	9	10
VI	VII	VIII	IX	X

11	12	14	15	16
XI	XII	XIV	XV	XVI

19	20	30	40	50
XIX	XX	XXX	XL	L

Atomic number + 1 – Roman numeral
= Number of electrons

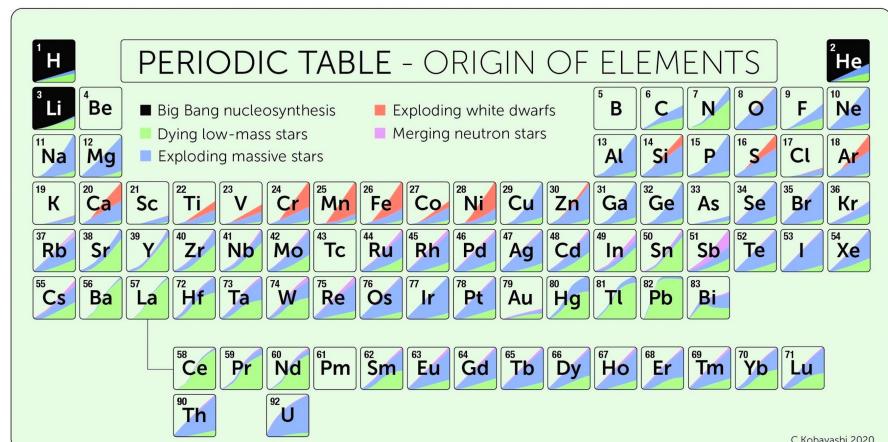


Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily

Examples:

Neutral hydrogen ($Z = 1$): H I = H

Isoelectronic sequence: H-like (i.e., one bounded electron)

Ionic charge (used by the atomic physics community): 0

Triply ionized carbon ($Z = 6$): C IV = C³⁺

Isoelectronic sequence: Li-like (i.e., three bounded electrons)

Ionic charge (used by the atomic physics community): 3

Triply ionized iron ($Z = 26$): Fe XVII = Fe¹⁶⁺

Isoelectronic sequence: Ne-like (i.e., ten bounded electrons)

Ionic charge (used by the atomic physics community): 16

Fully ionized oxygen ($Z = 8$): O IX = O⁸⁺

Isoelectronic sequence: N/A (no bounded electrons)

Ionic charge (used by the atomic physics community): 8

Atomic state

Examples:

Neutral hydrogen ($Z = 1$): H I = H

Isoelectronic sequence: H-like (i.e., one bounded electron)

Ionic charge (used by the atomic physics community): 0

Where can this electron stay?

An one-electron atomic state is defined by either one of the following two combinations of quantum numbers

principle quantum number n $n \ l \ m_l \ m_s$
 $n \ l \ j \ m_j$

Lyman	Balmer	Paschen
$\text{Ly}\alpha n = 2 \rightarrow 1$	$\text{H}\alpha n = 3 \rightarrow 2$	$\text{Pa}\alpha n = 4 \rightarrow 3$
$\text{Ly}\beta n = 3 \rightarrow 1$	$\text{H}\beta n = 4 \rightarrow 2$	$\text{Pa}\beta n = 5 \rightarrow 3$
$\text{Ly}\gamma n = 4 \rightarrow 1$	$\text{H}\gamma n = 5 \rightarrow 2$	$\text{Pa}\gamma n = 6 \rightarrow 3$
...

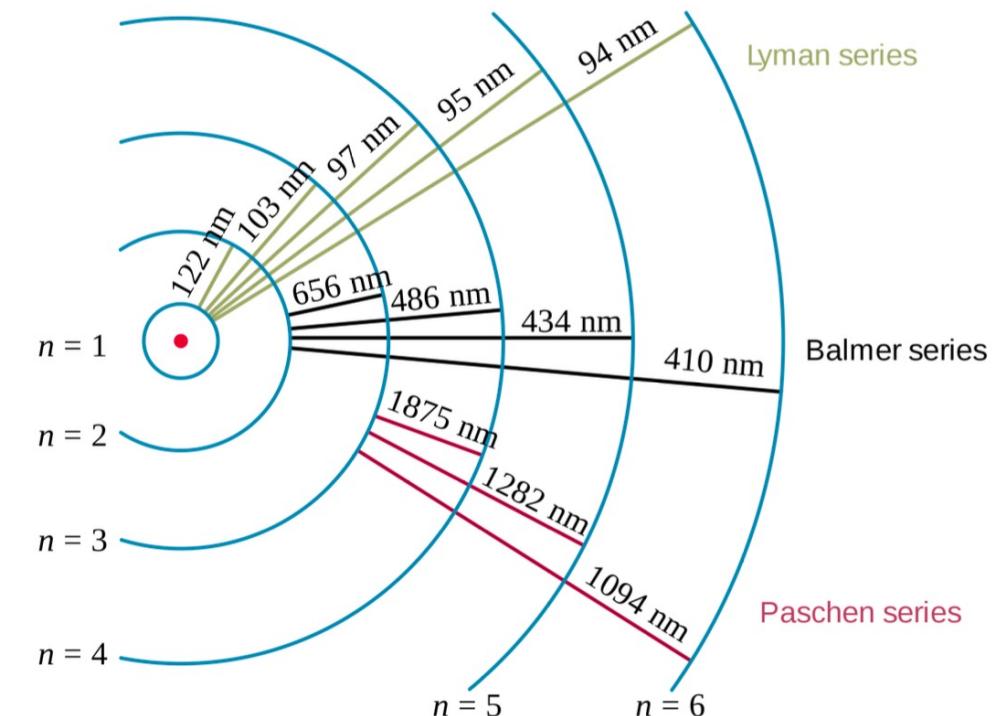


Image credit: wiki

Shell and subshell

principle quantum number
 $n = 1, \dots$

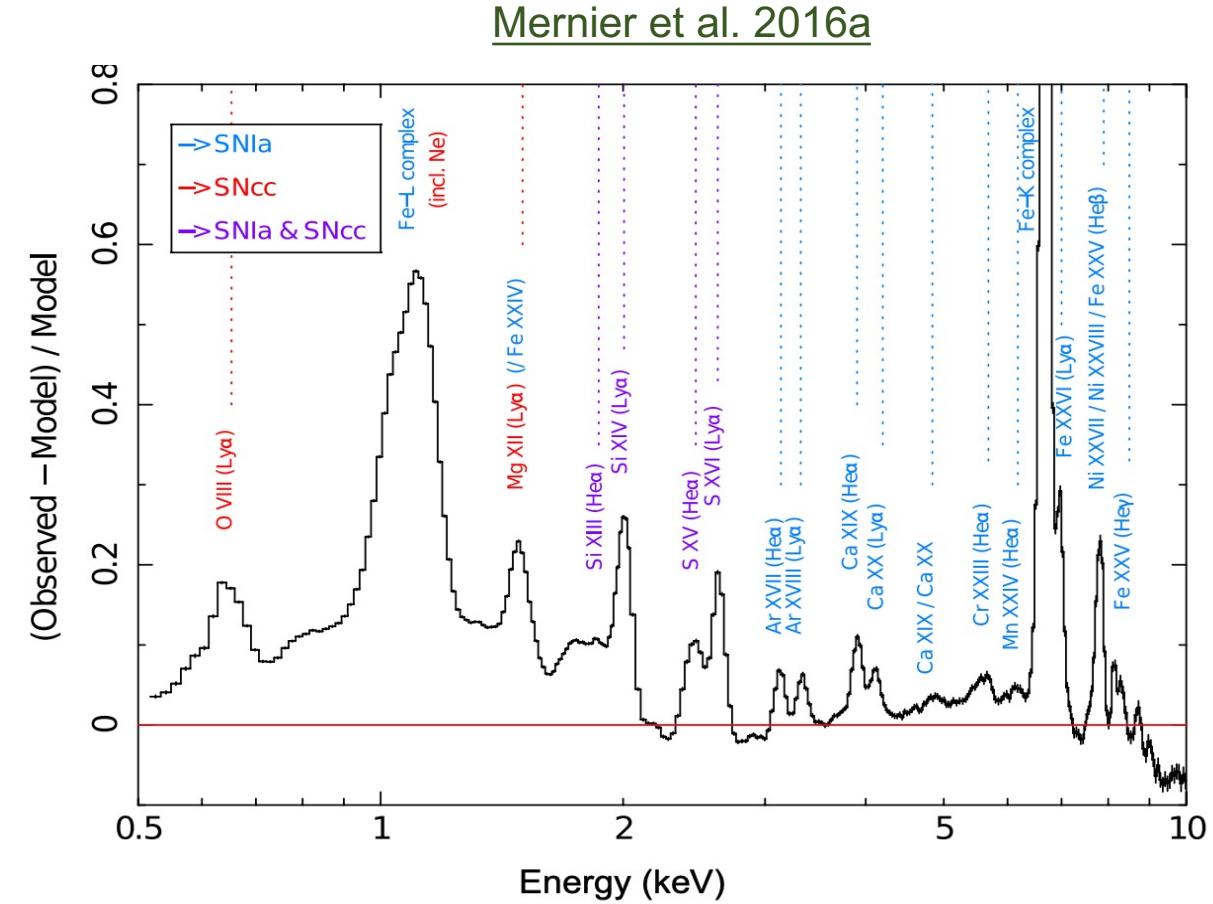
→ subshell

n

orbital angular momentum quantum number
 $l = 0, \dots, n - 1$

n	shell
1	K
2	L
3	M
4	N
5	O
6	P
7	Q

l	orbit
0	s
1	p
2	d
3	f
4	g
5	h
6	i
7	k



Caution that Fe K and L lines are not just one line ...
(e.g., Kallman et al. 2004, Gu et al. 2019)

Quantum numbers

An one-electron atomic state is defined by either one of the following two combinations of quantum numbers

$n \ l \ m_l \ m_s$

$n \ l \ j \ m_j$

principle quantum number
 $n = 1, \dots$

orbital angular momentum quantum number
 $l = 0, 1, \dots, n - 1$

spin quantum number
 $s = 1/2$

orbital magnetic quantum number
 $m_l = 0, 1, \dots, \pm(l - 1), \pm l$

spin magnetic quantum number
 $m_s = \pm s$

total angular momentum quantum number
 $j = l - s, l - s + 1, \dots, l + s - 1, l + s$

total angular momentum magnetic quantum number
 $m_j = -j, -j + 1, \dots, j - 1, j$

Shell and subshell degeneracy

principle quantum number
 $n = 1, \dots$

subshell

$\longrightarrow n \ l$

orbital angular momentum quantum number
 $l = 0, \dots, n - 1$

subshell	max. # of e^-
1s	2
2s	2
2p	6
3s	2
3p	6
3d	10
...	...

The maximum number of electrons in each shell is

$$\sum_{l=0}^{n-1} (2s + 1)(2l + 1) = \sum_{j=|l-s|}^{|l+s|} 2j + 1$$
$$= \sum_{l=0}^{n-1} 2(2l + 1) = 2n^2$$

Pauli's exclusion principle

no two electrons in the same atom can have identical values for all of the four quantum numbers $n \ l \ m_l \ m_s$ or $n \ l \ j \ m_j$.

n	shell	max. # of e^-
1	K	2
2	L	8
3	M	18
4	N	32
5	O	50
...

Electron configurations

principle quantum number $n = 1, \dots$

conf. $\rightarrow n l^x$

orbital angular momentum quantum number $l = 0, \dots, n - 1$

number of electrons in this subshell

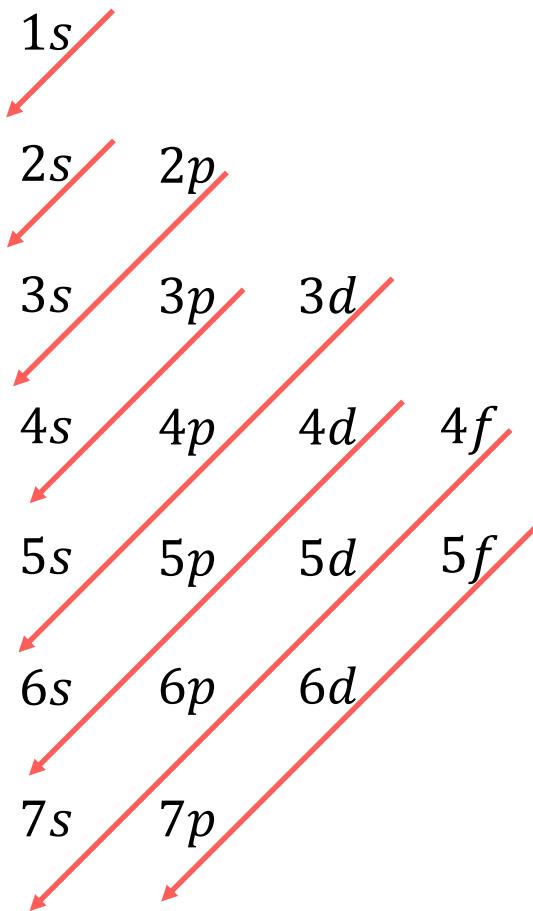
closed shell $x = 1$ (usually omitted), 2, ...
(can be omitted)

Fe atom: $[1s^2 2s^2 2p^6 3s^2 3p^6 4s^2] 3d^6$

Boron atom: $2p$ ← open shell ↑

B-like Fe XXII: $2p$

Order of subshell filling



n	shell	max. # of e^-
1	K	2
2	L	8
3	M	18
4	N	32
5	O	50
...

Coupling scheme

For elements with $Z \leq 30$, the spin-orbit coupling is weak. In this case, the orbital (spin) angular momenta of individual electrons form a total orbital (spin) angular momentum $L = \sum_i l_i$ ($S = \sum_i s_i$). This is the LS (Russell-Saunders) coupling.

$2S + 1$	multiplicity
1	singlet
2	doublet
3	triplet
4	quartet
5	quintet
6	sextet
7	septet
8	octet

L	orbit
0	S
1	P
2	D
3	F
4	G
5	H
6	I
7	K

term

$2S+1 L$

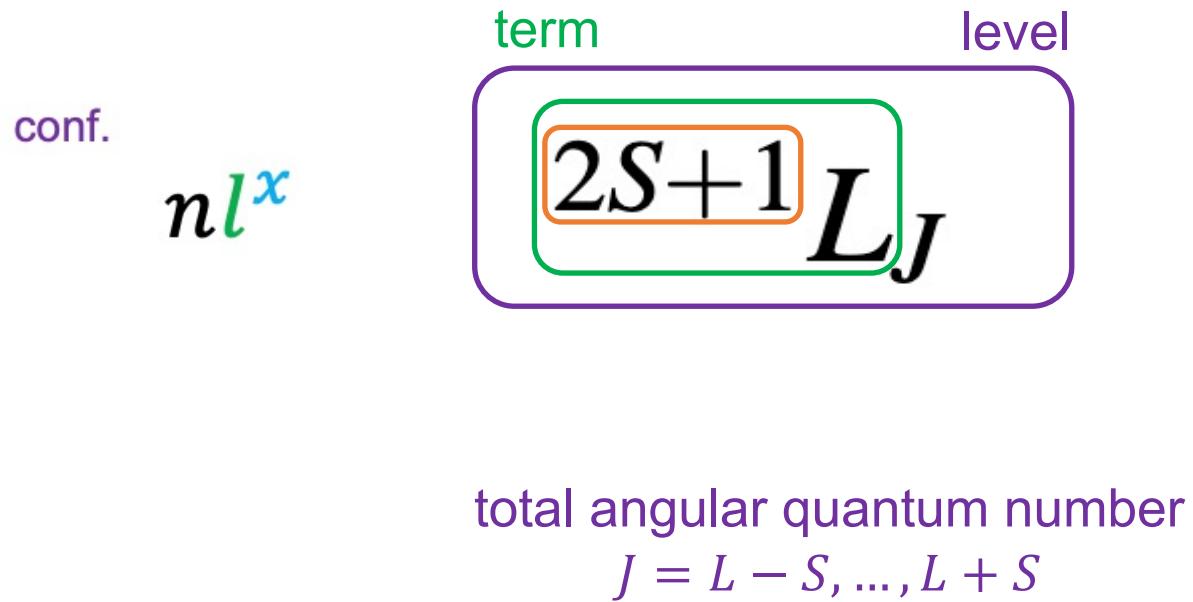
multiplicity of the term

Conf.	Term
ns	2S
ns^2	1S
np, np^5	2P
np^2, np^4	$^1S, ^1D, ^3P$
np^3	$^2P, ^2D$
np^6	1S
nd, nd^9	2D
nd^2, nd^8	$^1S, ^1D, ^1G, ^3P, ^3F$

For elements with $Z > 30$, jj coupling is needed.

Spectroscopic notation

To describe an energy level, electron configurations, spectroscopic notations (LSJ), and level energies are required.



Examples of H-like ions:

$1s\ ^2S_{1/2}$: $n = 1, S = 1/2, L = 0, J = 1/2$

$2p\ ^2P_{1/2}$: $n = 2, S = 1/2, L = 1, J = 1/2$

$2p\ ^2P_{3/2}$: $n = 2, S = 1/2, L = 1, J = 3/2$

Examples of He-like ions:

$1s^2\ ^1S_0$: $n = 1, S = 0, L = 0, J = 0$

$1s\ 2s\ ^1S_0$: $n = 2, S = 0, L = 0, J = 0$

$1s\ 2s\ ^3S_1$: $n = 2, S = 1, L = 0, J = 1$

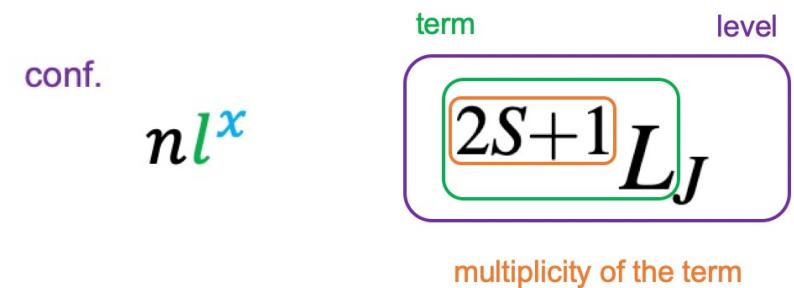
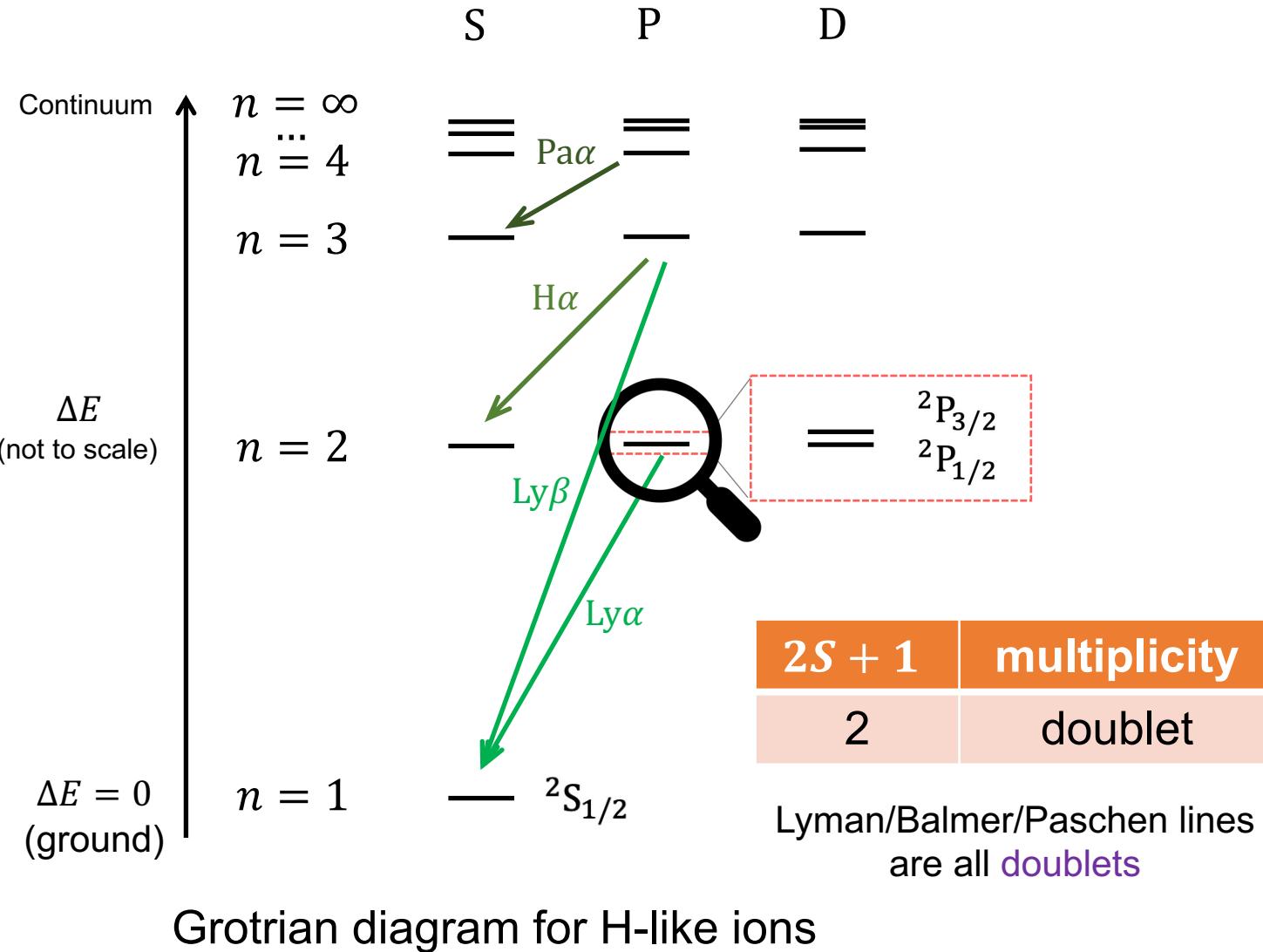
$1s\ 2p\ ^1P_1$: $n = 2, S = 0, L = 1, J = 1$

$1s\ 2p\ ^3P_0$: $n = 2, S = 1, L = 1, J = 0$

$1s\ 2p\ ^3P_1$: $n = 2, S = 1, L = 1, J = 1$

$1s\ 2p\ ^3P_2$: $n = 2, S = 1, L = 1, J = 2$

Energy levels (H-like)



Examples of H-like ions:

- $1s\ ^2S_{1/2}$: $n = 1, S = 1/2, L = 0, J = 1/2$
- $2p\ ^2P_{1/2}$: $n = 2, S = 1/2, L = 1, J = 1/2$
- $2p\ ^2P_{3/2}$: $n = 2, S = 1/2, L = 1, J = 3/2$

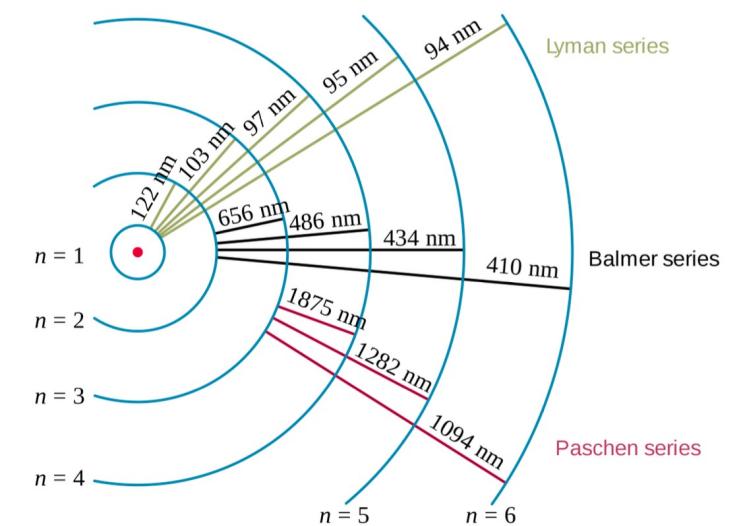
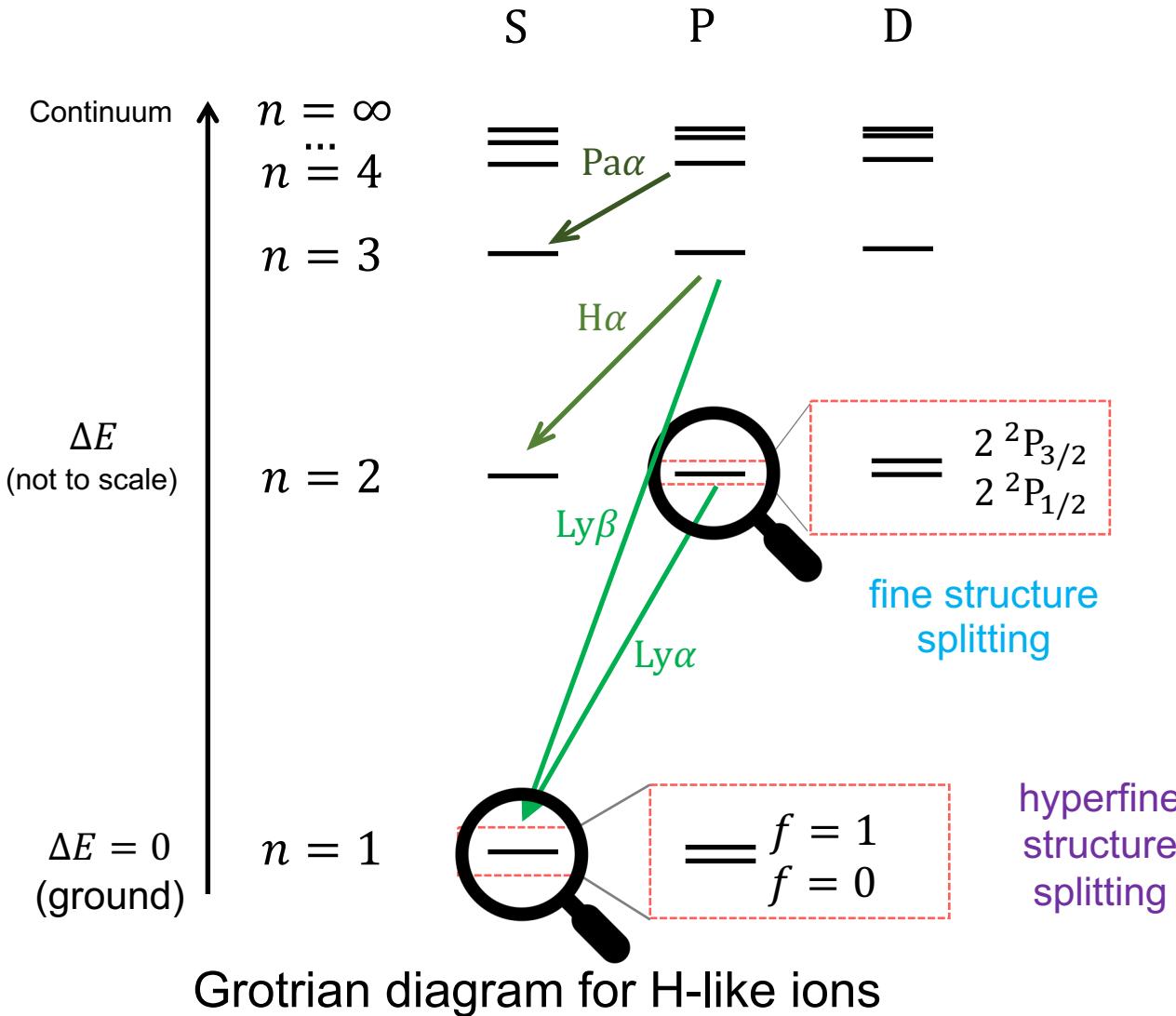


Image credit: wiki

Energy level (splitting)



For H I,

- ❖ the $2p\ ^2P$ level energy is 10.2 eV
- ❖ the $2p\ ^2P_{3/2}$ and $2p\ ^2P_{1/2}$ level energies differ by $\sim 4.5 \times 10^{-5}$ eV due to **fine structure splitting**
- ❖ the $1s\ ^2S_{1/2}$ level can be further split into two levels ($f = 0, 1$) with the level energies differ by $\sim 5.9 \times 10^{-6}$ eV (i.e., 1420 MHz = 21 cm) due to **hyperfine structure splitting**

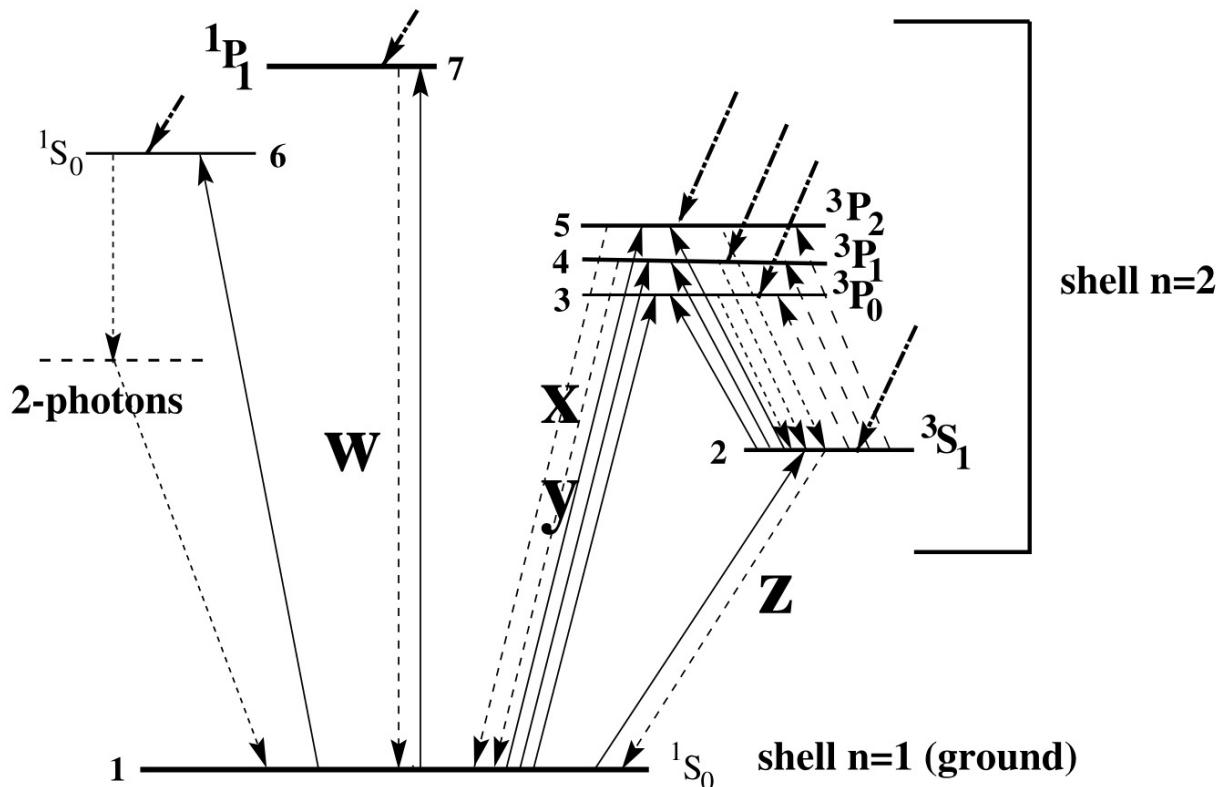
nuclear spin quantum number (for H I)
 $i = 1/2$

total spin quantum number
 $f = |j - i|, \dots, |j + i|$

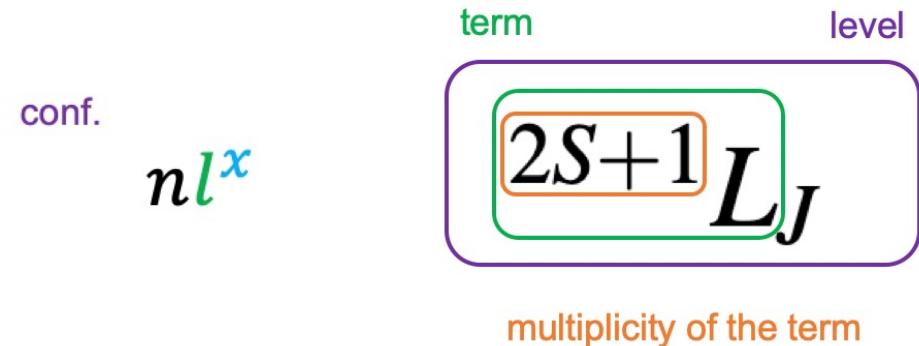
Energy levels (He-like)

$2S + 1$	multiplicity
1	singlet

$2S + 1$	multiplicity
3	triplet



Grotrian diagram for He-like ions (Porquet et al. 2010)



Examples of He-like ions:

- $1s^2 \ ^1S_0: n = 1, S = 0, L = 0, J = 0$
- $1s 2s \ ^1S_0: n = 2, S = 0, L = 0, J = 0$
- $1s 2s \ ^3S_1: n = 2, S = 1, L = 0, J = 1$
- $1s 2p \ ^1P_1: n = 2, S = 0, L = 1, J = 1$
- $1s 2p \ ^3P_0: n = 2, S = 1, L = 1, J = 0$
- $1s 2p \ ^3P_1: n = 2, S = 1, L = 1, J = 1$
- $1s 2p \ ^3P_2: n = 2, S = 1, L = 1, J = 2$

e.g., C V, N VI, O VII, Si XIII, Fe XXV

NIST energy levels



NIST Atomic Spectra Database Levels Form

Best viewed with the latest versions of Web browsers and JavaScript enabled

This form provides access to NIST critically evaluated data on atomic energy levels.

Spectrum: e.g., Fe I or Mg Li-like or Z=59 II or 198Hg I

Level Units: cm⁻¹
 eV
 Rydberg
 Hartree
 GHz

Format output: pretty printed
 XML
 JSON

Display output: text
 HTML
 PDF

Page size: 10
 20
 50
 All

Term ordered: by energy
 by frequency

Energy ordered: increasing
 decreasing

Level information: Principal configuration
 Principal term
 Level Uncertainty
 J g
 Landé-g
 Leading percentages

Bibliographic references:

Level splitting:
T_e (eV)
for partition function:

Extended Search: Set Additional Criteria for all levels searches of this spectrum

https://physics.nist.gov/PhysRefData/ASD/levels_form.html

Wavenumber (reciprocal centimeter)

$$E = h\nu = h \frac{c}{\lambda}$$

$$\frac{E}{\text{eV}} = 8065.544 \text{ cm}^{-1}$$

Rydberg

$$1 \text{ Ry} = 13.60569 \text{ eV}$$

$$1 \text{ Ry} = 1.097373 \times 10^5 \text{ cm}^{-1}$$

Hartree

$$1 \text{ Ha} = 2 \text{ Ry}$$

NIST energy levels for H I



NIST Atomic Spectra Database Levels Data

H I 106 Levels Found

Z = 1, H isoelectronic sequence

NIST Energy Levels and Wavelengths Bibliographic Reference # 15291

A Critical Compilation of Experimental Data on Spectral Lines and Energy Levels of Hydrogen, Deuterium, and Tritium,

A. E. Kramida,

At. Data Nucl. Data Tables **96**, 586–644 (2010); Erratum: **126**, 295–298 (2019)

DOI:10.1016/j.adt.2010.05.001

Configuration	Term	J	Level (eV)	Uncertainty (eV)	Leading percentages	Reference
1s	² S	¹ / ₂	0.00000000000000	0.0000000000012	100	
2p	² P°	¹ / ₂	10.19880615024	0.0000000004	100	L15291 L15291
		³ / ₂	10.19885151459	0.0000000005	100	
2s	² S	¹ / ₂	10.19881052514816	0.0000000000012	100	L15291

skipping many rows

	Limit	---	(13.598434599702)	0.00000000012		HDEL

NIST energy levels for other ions

ASD DATA ————— INFORMATION —————

Lines Levels List of Spectra Ground States & Ionization Energies Bibliography Help

NIST Atomic Spectra Database Levels Data

O VII 87 Levels Found

Z = 8, He isoelectronic sequence

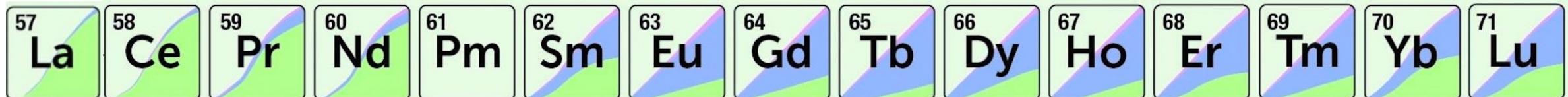
Ion	iso. seq.	# of levels
C IV	Li-like	91
N III	B-like	347
Fe XXII	B-like	61
O III	C-like	188
Ar XIII	C-like	44
O II	N-like	287
Si VII	O-like	65
S VIII	F-like	54

Configuration	Term	J	Level (eV)	Uncertainty (eV)	Reference
1s ²	¹ S	0	0.0000		L11554
1s2s	³ S	1	560.98386		
1s2p	³ P°	0	568.54442		
		1	568.55186		
		2	568.62005		
1s2s	¹ S	0	568.88662		
1s2p	¹ P°	1	573.94778		
1s3s	³ S	1	661.92932		
1s3p	³ P°	2	664.01845		
		1	664.01845		
		0	664.01845		
1s3s	¹ S	0	664.11144		
1s3d	³ D	1	665.09711		
		2	665.10455		
		3	665.10579		

NIST energy levels for other ions (cont.)

Ion	iso. seq.	# of levels
Fe XVII	Ne-like	67
Ni XIX	Ne-like	73
P V	Na-like	68
Si II	Al-like	151
Fe VII	Ca-like	210
Fe II	Mn-like	1028
La V	I-like	37
Eu VII	La-like	2

Lanthanide series ($Z = 57 - 71$)



ASD
DATA —————
INFORMATION —————

LISTS
LEVELS
List of SPECTRA
GROUND STATES & IONIZATION ENERGIES
BIBLIOGRAPHY
HELP

NIST Atomic Spectra Database Levels Data

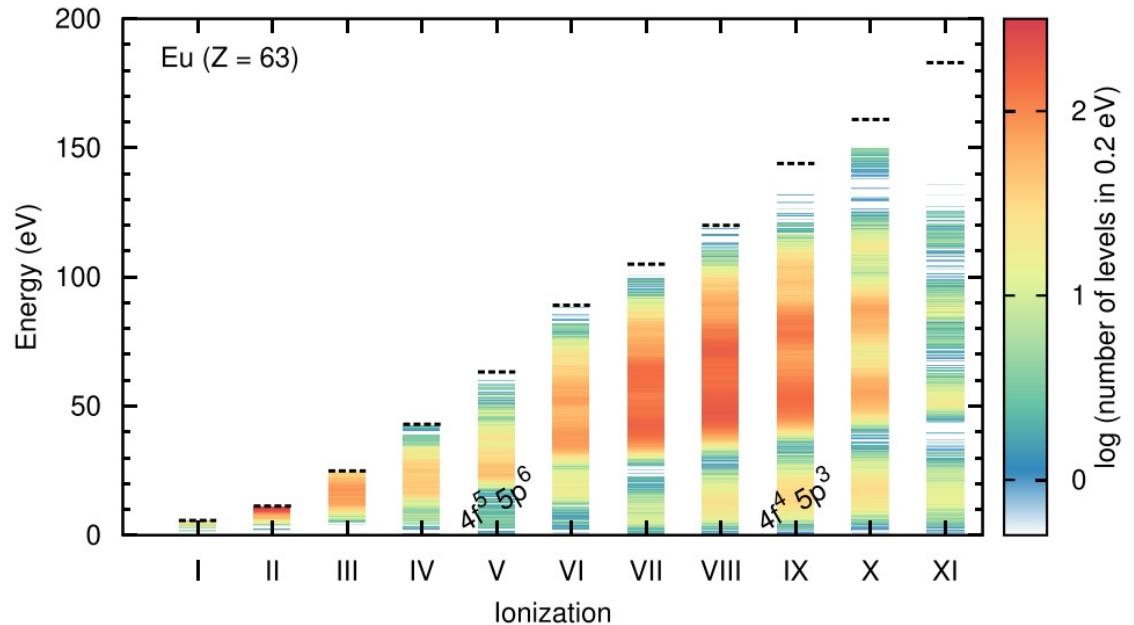
Si II 151 Levels Found
Z = 14, Al isoelectronic sequence

Configuration	Term	J	Level (eV)	Uncertainty (eV)	Leading percentages	Reference
3s ² 3p	2P°	1/2	0.000000	94	3 3p ³ 2P°	L5815
		3/2	0.035613			
3s3p ²	4P	1/2	5.309535	94	3 3p ³ 2P°	
		3/2	5.322966			
		5/2	5.344700			
3s3p ²	2D	3/2	6.857485	66	30 3s ² 3d 2D	
		5/2	6.859448			

NIST not for all

Eu VII 2 Levels Found
Z = 63, La isoelectronic sequence

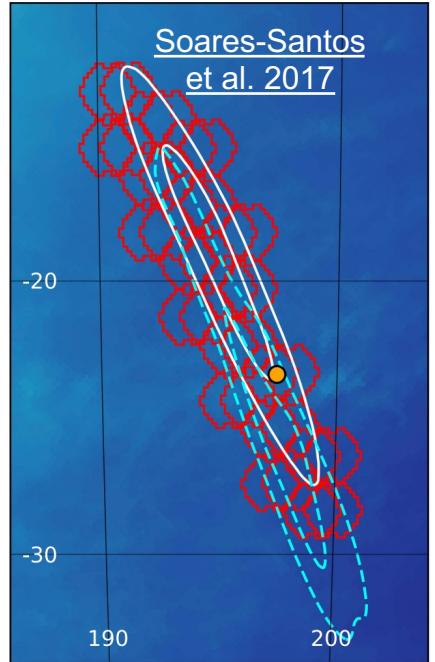
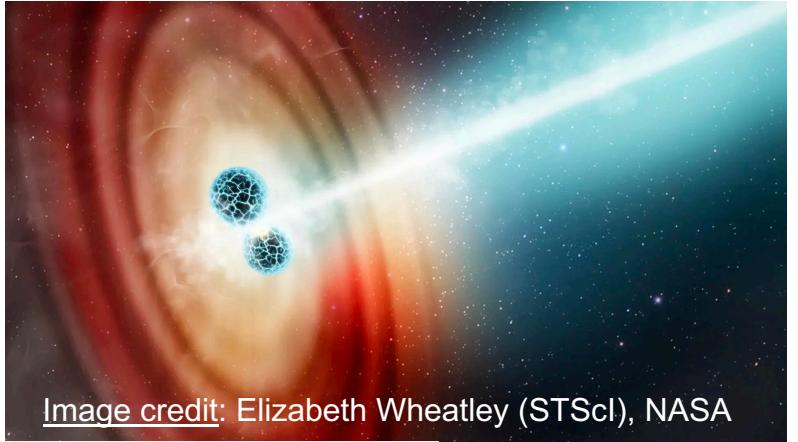
Configuration	Term	J	Level (eV)	Uncertainty (eV)	Reference
$4f^6 5s^2 5p^4$	°	--	0.0000		L582
Eu VIII ($4f^6 5s^2 5p^3$)	Limit	--	(105)	5	L582



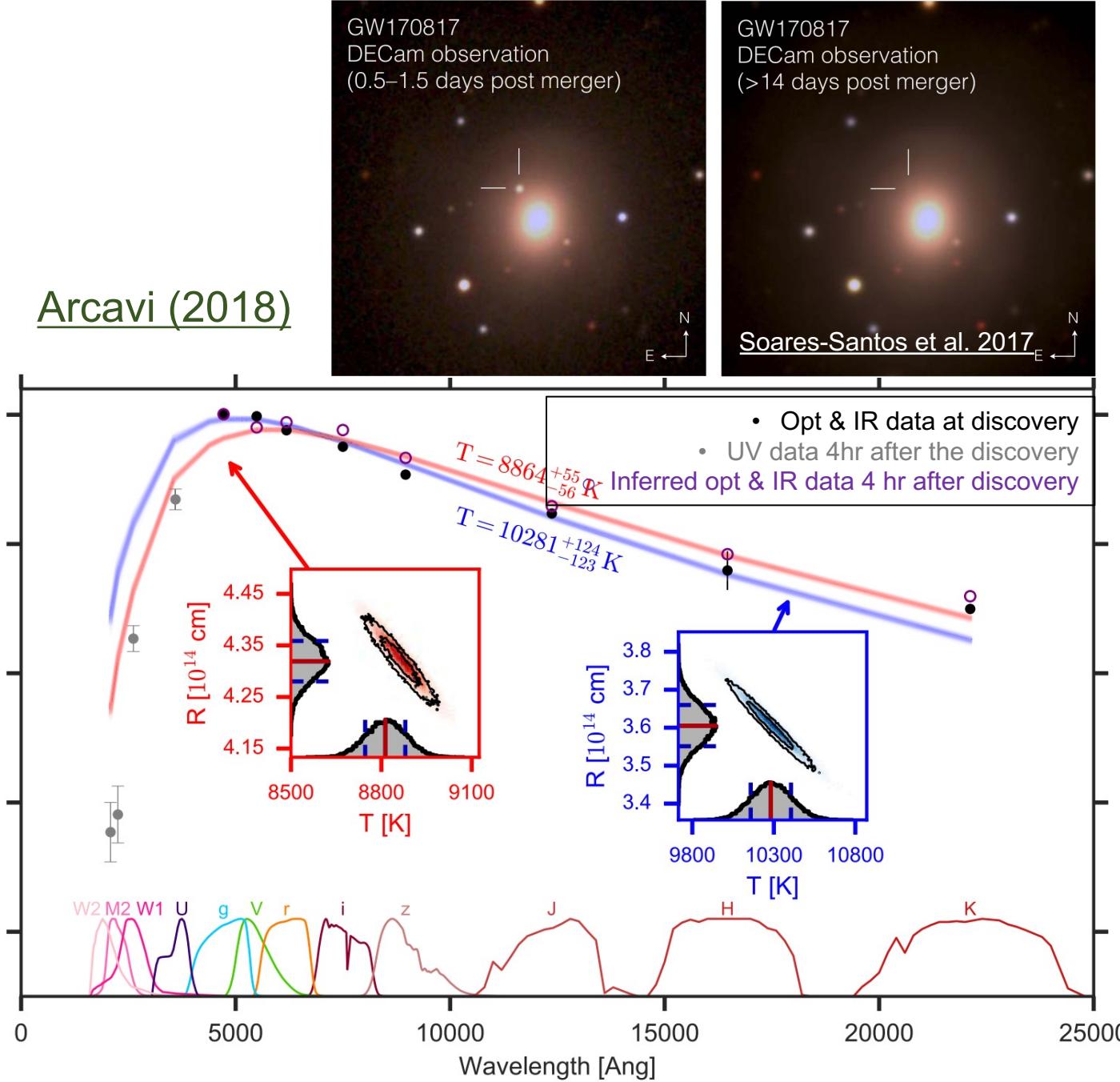
Ion	Configurations	N_{level}	N_{line}
Eu I	$4f^7 6s^2, 4f^7 5d 6s, 4f^7 6s 6p, 4f^6 5d 6s^2, 4f^7 5d 6p, 4f^7 6s 7s, 4f^6 5d^2 6s, 4f^7 5d^2, 4f^7 6s 7p, 4f^7 6s 6d, 4f^7 6s 8s, 4f^7 6s 5f,$ $4f^7 6s 8p, 4f^7 6s 7d, 4f^7 6p^2$	103229	741430825
Eu II	$4f^7 6s, 4f^7 5d, 4f^7 6p, 4f^6 5d 6s, 4f^6 5d^2$	22973	21396542
Eu III	$4f^7, 4f^6 5d, 4f^6 6s, 4f^6 6p$	5323	2073702
Eu IV	$4f^6, 4f^5 5d, 4f^5 6s, 4f^5 6p$	3737	1045697
Eu V	$4f^5 5p^6, 4f^6 5p^5, 4f^4 5p^6 6s, 4f^4 5p^6 6p, 4f^4 5p^6 5d, 4f^4 5p^6 7s$	3897	1140035
Eu VI	$4f^4 5p^6, 4f^5 5p^5, 4f^4 5p^5 6s, 4f^4 5p^5 6p, 4f^4 5p^5 5d, 4f^4 5p^5 7s$	13065	12823350
Eu VII	$4f^4 5p^5, 4f^3 5p^6, 4f^4 5p^4 6s, 4f^4 5p^4 6p, 4f^4 5p^4 5d, 4f^4 5p^4 7s$	29465	60643899
Eu VIII	$4f^4 5p^4, 4f^3 5p^5, 4f^4 5p^3 6s, 4f^4 5p^3 6p, 4f^4 5p^3 5d, 4f^4 5p^3 7s$	40241	113753012
Eu IX	$4f^4 5p^3, 4f^3 5p^4, 4f^4 5p^2 6s, 4f^4 5p^2 6p, 4f^4 5p^2 5d, 4f^4 5p^2 7s$	31393	73355941
Eu X	$4f^4 5p^2, 4f^3 5p^3, 4f^3 5p^2 6s, 4f^3 5p 6s^2, 4f^4 5p 6s, 4f^4 5p 6p, 4f^4 5p 5d, 4f^4 5p 7s$	15515	18807502

Smaranika et al. 2022

GW170817 kilonova



10.5 hr after the localization of GW 170817, Dark Energy Cam era covered 70 deg² (i and z bands) to search for optical counterparts. At 11.4 hr post-merger, a kilonova was found.



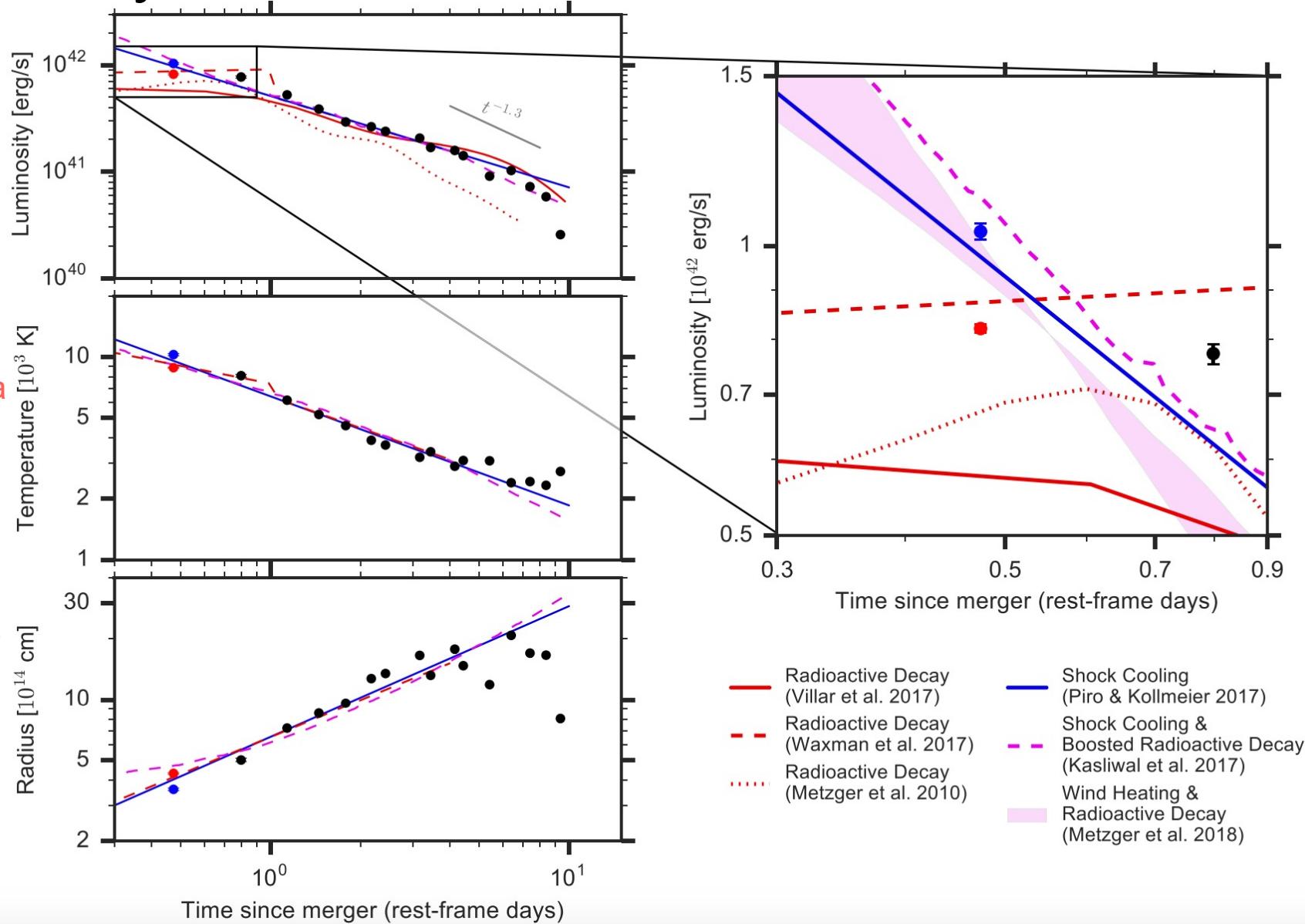
GW170817 kilonova early emission

While the late time ($t > 1$ d) lightcurve can be well explained with the kilonova, the origin and evolution of the early emission has been puzzling.

- ❖ Radiative decay in low-opacity ejecta
- ❖ Relativistic boosting of radiative decay in high-velocity ejecta
- ❖ Shock cooling
- ❖ Wind heating
- ❖ Some or all above

The opacity of the lanthanides is one of the bottlenecks.

Arcavi (2018)



NIST lines

https://physics.nist.gov/PhysRefData/ASD/lines_form.html



NIST Atomic Spectra Database Lines Form

Best viewed with the latest versions of Web browsers and JavaScript enabled

Main Parameters	Spectrum	e.g., Fe I or Na; Mg; Al or mg i-iii or 198Hg I	
Limits for	Wavelengths	Lower:	
		Upper:	
Wavelength Units: nm ▾			
<input type="button" value="Reset input"/>	<input type="button" value="Retrieve Data"/>	<input type="button" value="Show Graphical Options"/>	<input type="button" value="Show Advanced Settings"/>

Can you please provide some [feedback](#) to improve our database?

Observed Wavelength Air (Å)	Ritz Wavelength Air (Å)	Rel. Int. (?)	A _{ki} (s ⁻¹)	Acc.	E _i (cm ⁻¹)	E _k (cm ⁻¹)	Lower Level Conf., Term, J	Upper Level Conf., Term, J	Type	TP Ref.	Line Ref.
O VI	1 031.912		4.16e+08	A	0.0 -	96 907.5	1s ² 2s 2S 1/2	1s ² 2p 2P° 3/2		T5118LS	
	1 037.613		4.09e+08	A	0.0 -	96 375.0	1s ² 2s 2S 1/2	1s ² 2p 2P° 1/2		T5118LS	
	1 080.880		1.53e+09	A	863 333.8 -	955 851	1s ² 4p 2P° 1/2	1s ² 5d 2D 3/2		T5118LS	
	1 081.522		1.83e+09	A	863 397.7 -	955 860	1s ² 4p 2P° 3/2	1s ² 5d 2D 5/2		T5118LS	
	1 081.627		3.05e+08	A	863 397.7 -	955 851	1s ² 4p 2P° 3/2	1s ² 5d 2D 3/2		T5118LS	
	1 126.351		3.16e+06	B	867 077.7 -	955 860	1s ² 4f 2F° 5/2	1s ² 5d 2D 5/2		T3127LS	
	1 126.465		6.63e+07	B	867 077.7 -	955 851	1s ² 4f 2F° 5/2	1s ² 5d 2D 3/2		T3127LS	
	1 126.469		6.31e+07	B	867 087.0 -	955 860	1s ² 4f 2F° 7/2	1s ² 5d 2D 5/2		T3127LS	
	1 171.561		4.92e+08	A	863 333.8 -	948 690	1s ² 4p 2P° 1/2	1s ² 5s 2S 1/2		T5118LS	
	1 172.439		9.82e+08	A	863 397.7 -	948 690	1s ² 4p 2P° 3/2	1s ² 5s 2S 1/2		T5118LS	

O VI: 126 Lines of Data Found
Z = 8, Li isoelectronic sequence

Ion	iso. seq.	# of lines
C IV	Li-like	228
N III	B-like	1378
Fe XXII	B-like	104
O III	C-like	1005
Ar XIII	C-like	71
O II	N-like	1630
Si VII	O-like	233
S VIII	F-like	254

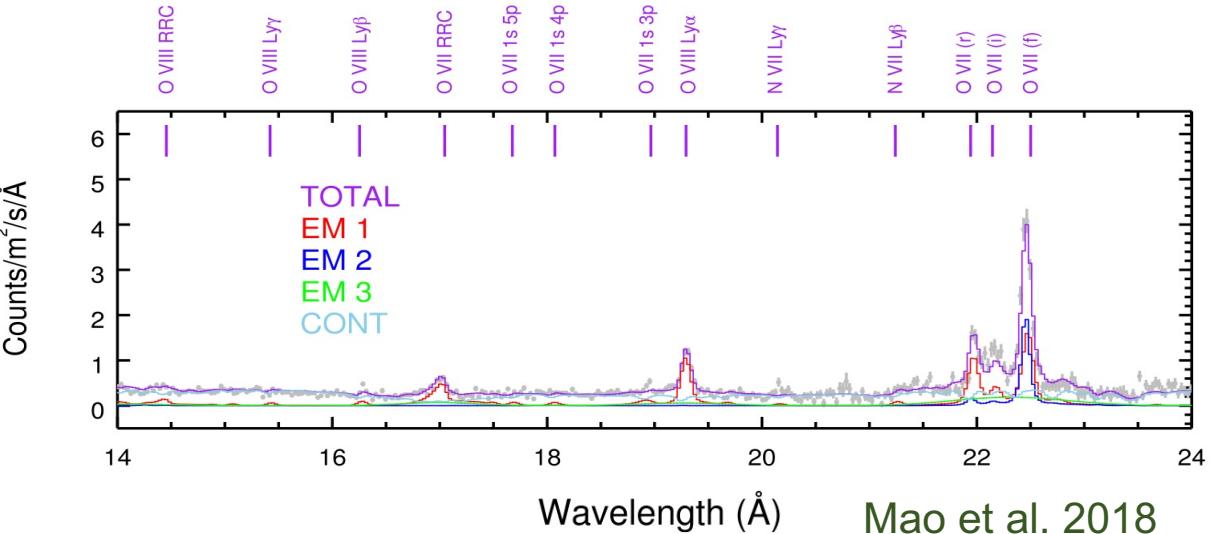
NIST lines + publications



NIST Atomic Spectra Database Lines Data

O VII: 189 Lines of Data Found

Z = 8, He isoelectronic sequence



Mao et al. 2018

Observed Wavelength Vac (Å)	Ritz Wavelength Vac (Å)	Rel. Int. (?)	A_{ki} (s⁻¹)	Acc.	E_i (cm⁻¹)	E_k (cm⁻¹)	Lower Level Conf., Term, J	Upper Level Conf., Term, J	Type	TP Ref.	Line Ref.
	17.8052		1.84e+02	C	0	- 5 616 340.0	1s²	¹S 0	1s4s ³S 1	M1	T4905
	18.6270		9.365e+11	AA	0	- 5 368 550.0	1s²	¹S 0	1s3p ¹P° 1		T5997
	18.6377		1.451e+08	AA	0	- 5 365 470.0	1s²	¹S 0	1s3d ¹D 2	E2	T3908
	18.7307		3.57e+02	C	0	- 5 338 820.0	1s²	¹S 0	1s3s ³S 1	M1	T4905
	21.6020		3.309e+12	AA	0	- 4 629 201.0	1s²	¹S 0	1s2p ¹P° 1		T5997
	21.8044		3.31e+05	A	0	- 4 586 230.0	1s²	¹S 0	1s2p ³P° 2	M2	T1494
	21.8070				0	- 4 585 680.0	1s²	¹S 0	1s2p ³P° 1		
	22.1012		1.04e+03	AA	0	- 4 524 640.0	1s²	¹S 0	1s2s ³S 1	M1	T1896
	78.8619		4.61e+09	A	4 585 620.0	- 5 853 660.0	1s2p	³P° 0	1s7d ³D 1		T5231LS
	78.8656		6.22e+09	A	4 585 680.0	- 5 853 660.0	1s2p	³P° 1	1s7d ³D 2		T5231LS

H- and He-like C to Zn
(Mao et al. 2022c)

Ion	ID	λ (Å)
O VIII	Ly α _{1/2}	18.973
O VIII	Ly α _{3/2}	18.967
O VIII	Ly β _{1/2}	16.007
O VIII	Ly β _{3/2}	16.006
O VIII	Ly γ _{1/2}	15.176
O VIII	Ly γ _{3/2}	15.176
O VIII	Ly δ _{1/2}	14.821
O VIII	Ly δ _{3/2}	14.820

AtomDB/CHIANTI lines

❖ AtomDB (esp. for the X-ray community):

<http://www.atomdb.org/Webguide/webguide.php>

ATOMDB ATOMIC DATA FOR
ASTROPHYSICISTS

❖ CHIANTI (esp. for the solar community)

<https://db.chiantidatabase.org/>

CHIANTI

An Atomic Database for Spectroscopic Diagnostics of Astrophysical Plasmas.

H I

λ (Å)	gf	A (s^{-1})	Transition
1215.674	2.780e-01	6.260e+08	1s 2S1/2 – 2p 2P1/2
1215.668	5.550e-01	6.260e+08	1s 2S1/2 – 2p 2P3/2
6564.564	2.720e-02	2.100e+06	2p 2P1/2 – 3s 2S1/2
6564.722	5.440e-02	4.210e+06	2p 2P3/2 – 3s 2S1/2

degeneracy = statistical weight = $2J+1$

O VIII

Energy level 1	
Electron configuration	1s1
Energy above ground (eV)	0.0
Quantum state	n=1, L=0, S=0.5, degeneracy=2, parity=0
Energy Level Data Source	NIST ASD 5.3
Photoionization Data Source	from XSTAR

Energy level 3	
Electron configuration	2p1
Energy above ground (eV)	653.49365
Quantum state	n=2, L=1, S=0.5, degeneracy=2, parity=1
Energy Level Data Source	NIST ASD 5.3
Photoionization Data Source	from XSTAR

Level 1 → 3 Interactions	
Electron collision rate	Nonzero (for 1 → 3)
Electron collision reference	2003JPhB...36.3707B
Wavelength (theory)*	18.972525 Å
Wavelength (lab/observed) [†]	18.972517 Å
Transition rate/Einstein A	2.57e+12 s ⁻¹
Transition type*	E1
Oscillator Strength f ₃ → 1	1.385085e-01
Wavelength (theory) reference	NIST ASD 5.3 RITZ
Wavelength (lab/observed) reference	1977JPCRD...6..831E
Transition rate reference	FAC

Chpt.5 Atomic processes

5.1 Atomic data for astrophysics

5.2 Two-level system

5.2.1 Einstein coefficients

5.2.2 Thermodynamic equilibrium

5.2.3 Einstein relations

5.2.4 Emission and absorption coefficients

5.2.5 Collisional (de-)excitation

5.2.6 Emission line regime

5.2.7 Non-thermal emission

5.3 Bremsstrahlung

5.4 Recombination and photoionization

5.5 Collisional excitation

5.6 Other atomic processes

5.7 Astrophysical plasma models

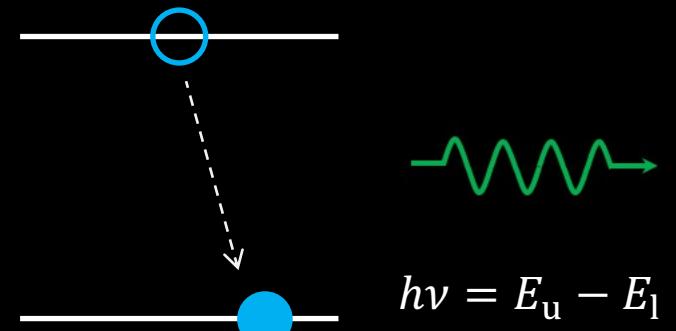
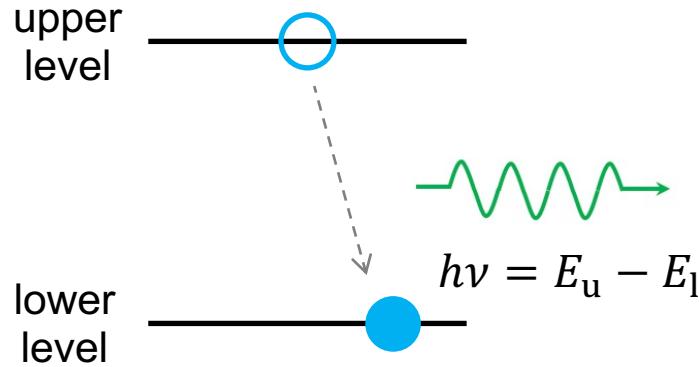


Image credit: Junjie Mao

Einstein A coefficient

Two-level system



transition from an **upper** energy level to a **lower** one → **emission** line
transition from a **lower** energy level to an **upper** one → **absorption** line

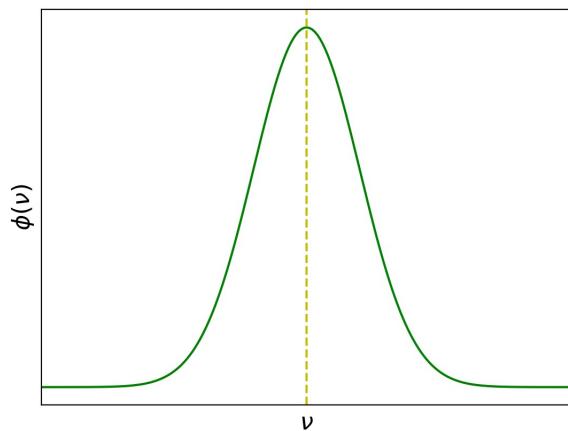
$$\frac{dn_u}{dt} = -n_u A_{ul}$$



Spontaneous emission probability
(a. k. a. Einstein A coefficient, in s^{-1})

the **larger** the A-value, the **stronger** the transition

Einstein B coefficients



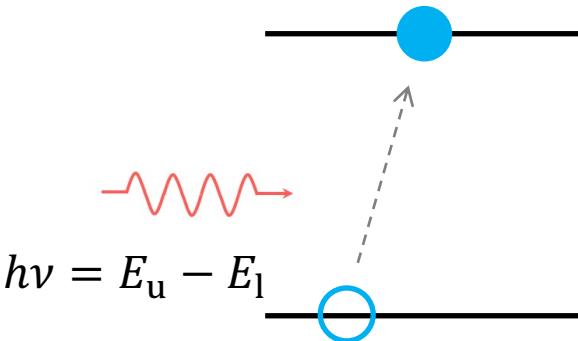
$$\langle I_\nu \rangle = \frac{1}{4\pi} \int d\Omega \int I_\nu \phi(\nu) d\nu$$

normalization
to the 4π solid
angle (isotropic
emission)

line profile (e.g., natural
broadening due to the
Heisenberg uncertainty
principle, $\Delta E \Delta t \geq \frac{\hbar}{4\pi}$)

$$1 = \int_0^\infty \phi(\nu) d\nu$$

Two-level system



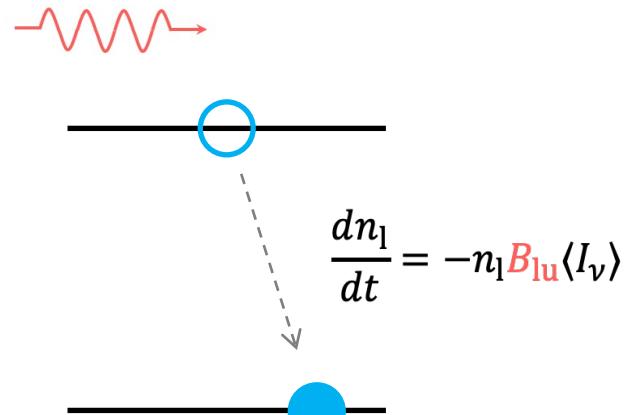
$$\frac{dn_l}{dt} = -n_l B_{lu} \langle I_\nu \rangle$$

Stimulated
absorption
coefficient

Einstein B coefficients ($\text{erg}^{-1} \text{cm}^2$)

upper
level

lower
level



$$\frac{dn_l}{dt} = n_u B_{ul} \langle I_\nu \rangle$$

Stimulated
emission
coefficient

Thermodynamic equilibrium

TE is also known as **detailed balance**, where all atomic processes are balanced by their thermodynamically inverse processes.

- ✓ radiation field is **isotropic**
- ✓ free electrons follow the **Maxwellian** distribution
- ✓ level populations follow **Maxwell-Boltzmann** equation

Accordingly, the radiation temperature **equals** the electron temperature.

Photons emitted

$$n_u(A_{ul} + B_{ul}\langle I_\nu \rangle) = n_l B_{lu} \langle I_\nu \rangle$$



upper level population

Photons absorbed



lower level population

Boltzmann equation for level populations (applies to LTE as well)

$$\frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$

- $g_{l/u}$ are statistical weights (i.e., degeneracy)
- $E_{l/u}$ are level energies

$$\begin{aligned} \langle I_\nu \rangle &= \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{A_{ul}/B_{ul}}{\frac{n_l B_{lu}}{n_u B_{ul}} - 1} \\ &= \frac{A_{ul}/B_{ul}}{\frac{g_l B_{lu}}{g_u B_{ul}} \exp\left(\frac{h\nu}{kT}\right) - 1} \end{aligned}$$

Einstein relations

prev. sl.

$$\langle I_\nu \rangle = \frac{A_{\text{ul}}/B_{\text{ul}}}{g_l B_{\text{lu}}/g_u B_{\text{ul}}} \exp\left(\frac{h\nu}{kT}\right) - 1$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

$$g_l B_{\text{lu}} = g_u B_{\text{ul}}$$

$$A_{\text{ul}} = \frac{2h\nu^3}{c^2} B_{\text{ul}}$$

- ✓ connect properties of the atom (must holds even out of TE)
- ✓ manifestation of the detailed balance relations (TE)

$$\langle I_\nu \rangle = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

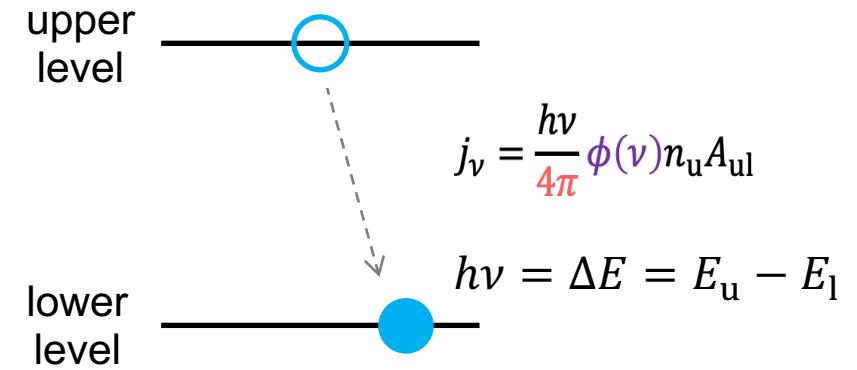
Spontaneous emission coefficient

$$j_\nu \equiv \frac{h\nu}{4\pi} \phi(\nu) n_u A_{ul}$$

line profile (e.g., natural broadening due to the Heisenberg uncertainty principle, $\Delta E \Delta t \geq \frac{\hbar}{4\pi}$)

$$dE_\nu = \frac{h\nu}{4\pi} \phi(\nu) n_u A_{ul} dV d\nu d\Omega dt$$

normalization to the
4 π solid angle
(isotropic emission)



Sect. 2.2.1

$$j_\nu = \frac{dE_\nu}{dV d\nu d\Omega dt}$$

Absorption coefficient

The total energy absorbed via **stimulated absorption** in unit time and unit volume is

$$\delta E_{SA} = -h\nu n_l B_{lu} \langle I_\nu \rangle dV dt$$

Treating the **stimulated emission** as negative absorption, total energy absorbed in unit time and unit volume is

$$\delta E_{SE} = h\nu n_u B_{ul} \langle I_\nu \rangle dV dt$$

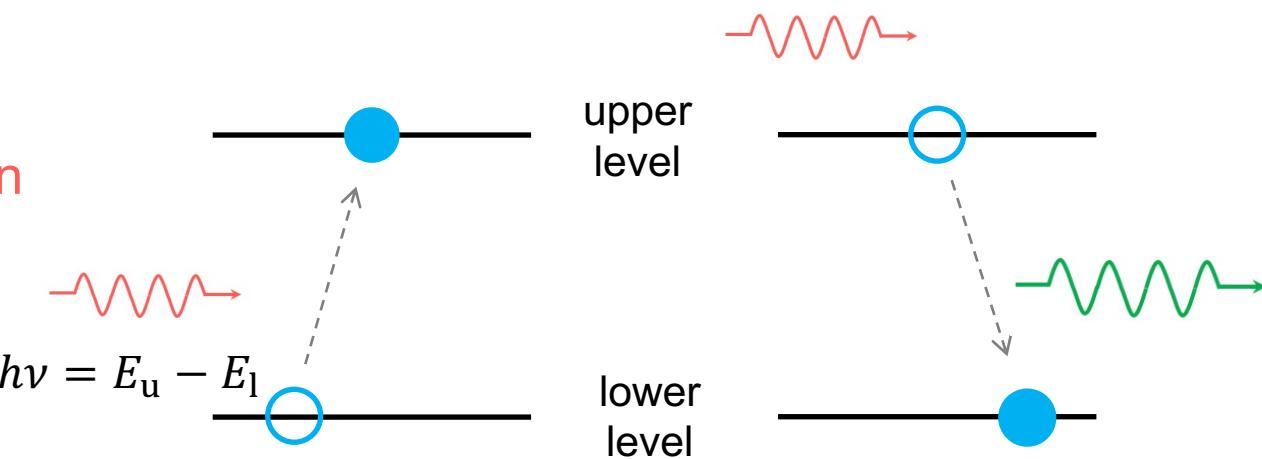
Combining the above two

$$\delta E = -\frac{h\nu}{4\pi} (n_l B_{lu} - n_u B_{ul}) I_\nu \phi(\nu) d\nu d\Omega dV dt$$

$$dI_\nu = -\frac{h\nu}{4\pi} (n_l B_{lu} - n_u B_{ul}) I_\nu \phi(\nu) ds$$

$$\alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu) (n_l B_{lu} - n_u B_{ul})$$

$$dV = d \perp ds$$



prev. sl.

$$\langle I_\nu \rangle = \frac{1}{4\pi} \int d\Omega \int I_\nu \phi(\nu) d\nu$$

Sect. 2.1.3

$$dE_\nu = I_\nu dA_\perp d\Omega dt d\nu$$

Sect. 2.2.2

$$dI_\nu = -\alpha_\nu I_\nu ds$$

Source function

prev. sl.

$$j_\nu = \frac{h\nu}{4\pi} \phi(\nu) n_u A_{ul}$$

prev. sl.

$$\alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu) (n_l B_{lu} - n_u B_{ul})$$

prev. sl.

$$g_l B_{lu} = g_u B_{ul}$$

$$A_{ul} = \frac{2h\nu^3}{c^2} B_{ul}$$

Source function

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}} = \frac{2h\nu^3}{c^2} \frac{1}{\frac{g_u n_l}{g_l n_u} - 1}$$

With the Boltzmann equation for level population (caution: not always the case)

prev. sl.

$$\frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$

Kirchoff's law

$$j_\nu = \alpha_\nu B_\nu(T)$$
$$S_\nu = B_\nu(T)$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$

Collisional (de-)excitation

$$n_u(A_{ul} + B_{ul}\langle I_\nu \rangle + n_e C_{ul}) = n_l(B_{lu}\langle I_\nu \rangle + n_e C_{lu})$$

electron volume density collisional de-excitation

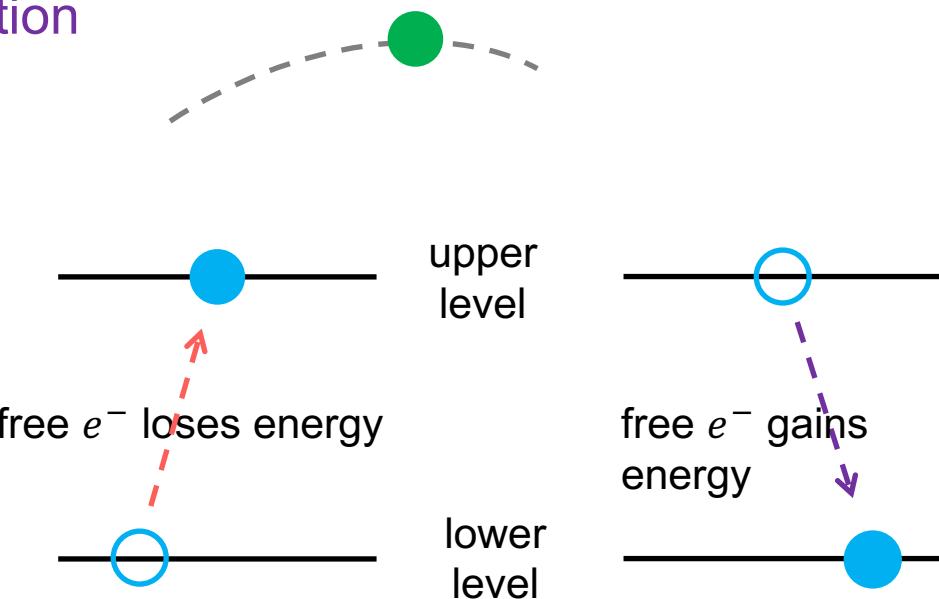
prev. sl. (under TE)

$$n_u(A_{ul} + B_{ul}\langle I_\nu \rangle) = n_l B_{lu}\langle I_\nu \rangle$$

$$n_u C_{ul} = n_l C_{lu}$$

Again, this is the manifestation of the **detailed balance**, where all atomic processes are balanced by their thermodynamically inverse processes.

collisional excitation



$$\left| \frac{1}{2} m_e v_i^2 - \frac{1}{2} m_e v_f^2 \right| = E_u - E_l$$

Collisional (de-)excitation (cont.)

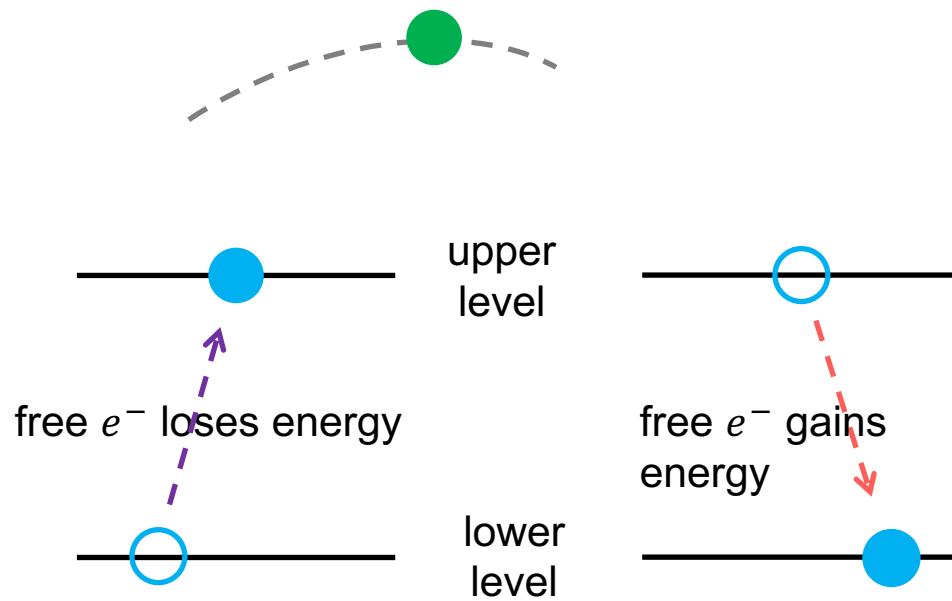
prev. sl. (Boltzmann distribution)

$$\frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$

prev. sl. (under TE)

$$n_u C_{ul} = n_l C_{lu}$$

$$\frac{C_{ul}}{C_{lu}} = \frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$



$$\left| \frac{1}{2} m_e v_i^2 - \frac{1}{2} m_e v_f^2 \right| = E_u - E_l$$

Emission line regime

NGC 2392 is a planetary nebula
(Eskimo nebula)
Image credit: HST

prev. sl.

$$n_u(A_{ul} + B_{ul}\langle I_\nu \rangle + n_e C_{ul}) = n_l(B_{lu}\langle I_\nu \rangle + n_e C_{lu})$$

In the absence of external emission (i.e., $\langle I_\nu \rangle = 0$), which is the case for planetary nebulae



$$n_u(A_{ul} + n_e C_{ul}) = n_l n_e C_{lu}$$

If $A_{ul} \gg n_e C_{ul}$, we can ignore the collisional de-excitation term

$$n_u A_{ul} \simeq n_l n_e C_{lu}$$

Critical density

$$n_e^{\text{crit}} = \frac{A_{ul}}{C_{ul}}$$

If $A_{ul} \ll n_e C_{ul}$, we can ignore the spontaneous emission term

$$n_u n_e C_{ul} \simeq n_l n_e C_{lu}$$

High-density emission line regime

prev. sl.

$$n_u(A_{ul} + n_e C_{ul}) = n_l n_e C_{lu}$$

Critical density

$$n_e^{\text{crit}} = \frac{A_{ul}}{C_{ul}}$$

Consider a high-density with $n_e \gg n_e^{\text{crit}}$, we can ignore the spontaneous emission term

$$n_u n_e C_{ul} \simeq n_l n_e C_{lu}$$

$$\frac{n_l}{n_u} \simeq \frac{C_{ul}}{C_{lu}}$$

prev. sl.

$$\frac{C_{ul}}{C_{lu}} = \frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$

- ✓ The level population follows the **Boltzmann** distribution

Low-density emission line regime

prev. sl.

$$n_u(A_{ul} + n_e C_{ul}) = n_l n_e C_{lu}$$

Consider a low-density with $n_e \ll n_e^{\text{crit}}$, we can ignore the collisional de-excitation term

$$n_u A_{ul} \simeq n_l n_e C_{lu}$$

$$\frac{n_u}{n_l} \simeq \frac{n_e C_{lu}}{A_{ul}}$$

This is also known as the **coronal approximation**, where the dominant processes are collisional excitation and spontaneous emission.

- ✓ The coronal approximation can be applied to the stellar corona (hence the name)
- ✓ The level population is **no** longer given by the Boltzmann distribution

Critical density

$$n_e^{\text{crit}} = \frac{A_{ul}}{C_{ul}}$$

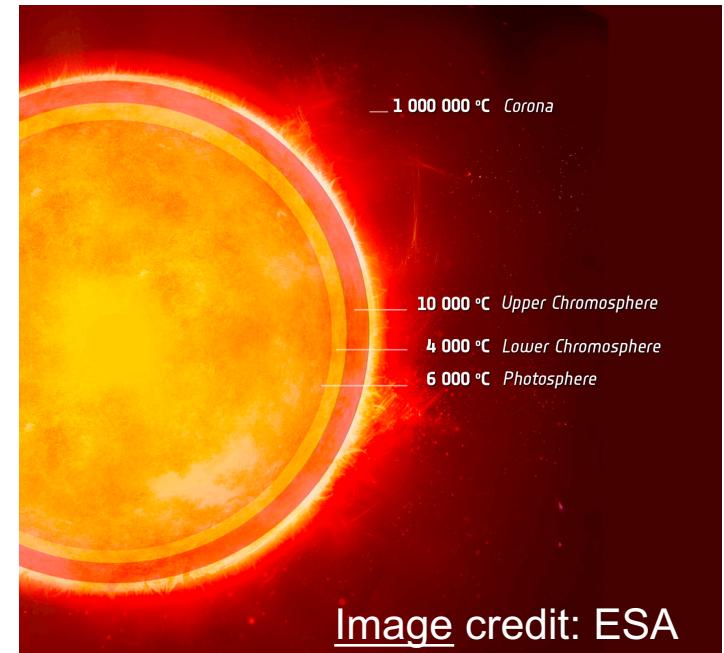


Image credit: ESA

Non-thermal emission

Boltzmann equation for level populations (applies to TE & LTE)

prev. sl.

$$\frac{n_l}{n_u} = \frac{g_l}{g_u} \exp\left(\frac{E_u - E_l}{kT}\right)$$

For non-thermal emission, the level populations do **not** follow the Boltzmann equation (e.g., astro maser), this might occur if

- ✓ radiating particles do not have a Maxwellian velocity distribution
- ✓ when scattering cannot be neglected

Chpt.5 Atomic processes

5.1 Atomic data for astrophysics

5.2 Two-level system

5.3 Bremsstrahlung

5.3.1 Bremsstrahlung for single-speed electrons

5.3.2 Thermal bremsstrahlung emission

5.3.3 Thermal bremsstrahlung absorption

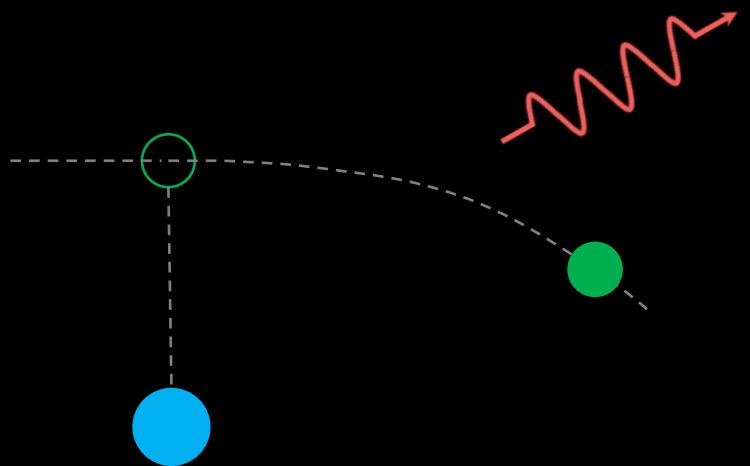
5.3.4 Spectrum of thermal gas

5.4 Recombination and photoionization

5.5 Collisional excitation

5.6 Other atomic processes

5.7 Astrophysical plasma models

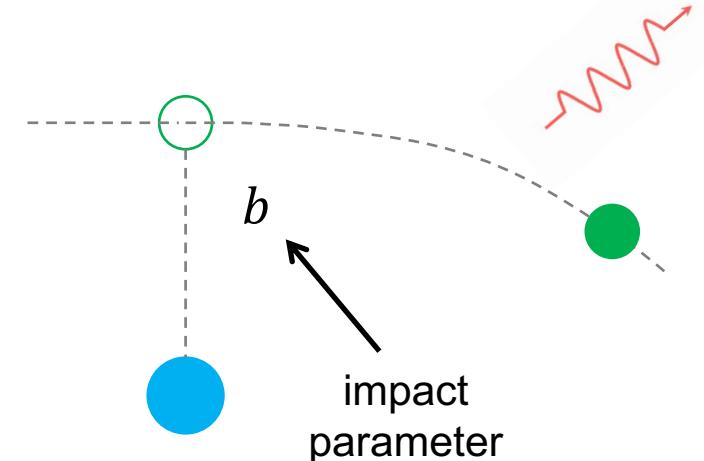


Bremsstrahlung (free-free) processes

Emission due to the **deceleration** of a charge (e.g., a **free** electron) in the Coulomb field (no strong external B-field) of another charge with a different charge-to-mass ratio (e.g., an ion)

- ✓ a **cooling** process for the electrons
- ✓ **continuum emission** spectral features

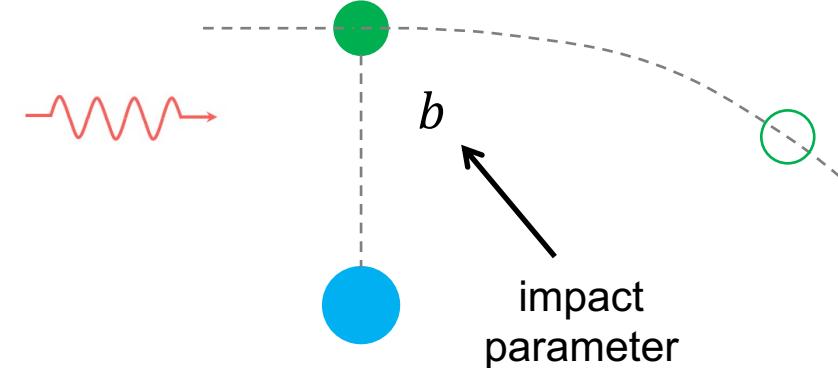
$$\Delta E = \frac{1}{2} m_e v^2 - \frac{1}{2} m_e (v - \Delta v)^2 = h\nu$$



Absorption due to the **acceleration** of a charge (e.g., a **free** electron) in the Coulomb field of another charge with a different charge-to-mass ratio (e.g., an ion)

- ✓ a **heating** process for the electrons

$$\Delta E = \frac{1}{2} m_e (v + \Delta v)^2 - \frac{1}{2} m_e v^2 = h\nu$$



Free-free power for single-speed electrons

$$P(\nu, \omega) = \frac{dE(\nu, \omega)}{d\omega dV dt} = \frac{16e^6}{3c^3m_e^2\nu} n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

$$b_{\max} = \frac{\nu}{\omega}$$

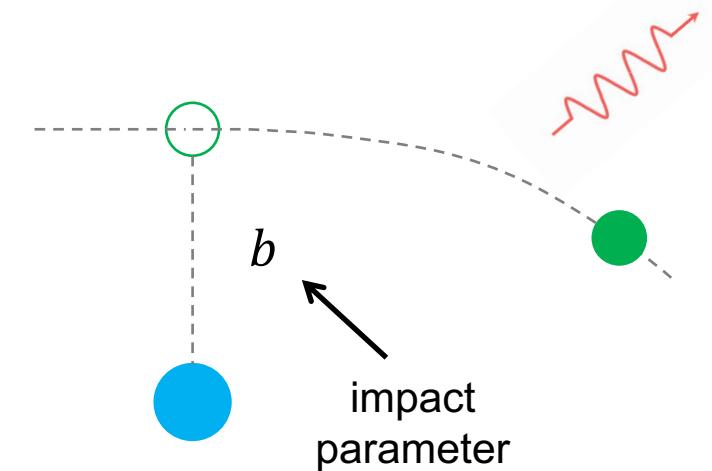
$$b_{\min} = \max\left(\frac{h/2\pi}{m_e\nu}, \frac{2Ze^2}{m_e\nu^2}\right)$$

Heisenberg uncertainty principle

$$\Delta\nu = \frac{2Ze^2}{bm_e\nu} = \nu$$

see Sect. 5.1 of the
REF book (p156-159)
by Rybicki & Lightman

Coulomb logarithm



Introducing the **Gaunt factor**

$$P(\nu, \omega) = \frac{dE}{d\omega dV dt} = \frac{16\pi e^6}{3\sqrt{3} c^3 m_e^2 \nu} n_e n_i Z^2 g_{\text{ff}}(\nu, \omega)$$

$$g_{\text{ff}}(\nu, \omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Thermal bremsstrahlung emission

Averaging the single-speed power over a thermal distribution of electron speeds

Maxwell velocity distribution

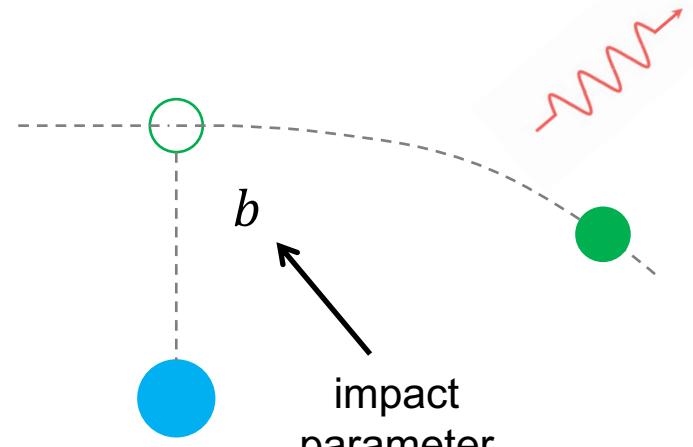
$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2k T_e}\right)$$

prev. sl.

$$P(v, \omega) = \frac{dE(v, \omega)}{d\omega dV dt} = \frac{16 \pi e^6}{3\sqrt{3} c^3 m_e^2 v} n_e n_i Z^2 g_{ff}(v, \omega)$$

$$\frac{dE(T, \omega)}{dV dt d\omega} = \frac{\int_{v_{\min}}^{\infty} \frac{dE(v, \omega)}{d\omega dV dt} v^2 \exp(-\frac{m_e v^2}{2k T_e}) dv}{\int_0^{\infty} v^2 \exp(-\frac{m_e v^2}{2k T_e}) dv}$$

$$\Delta E = \frac{1}{2} m_e v^2 - \frac{1}{2} m_e (v - \Delta v)^2 = h\nu$$



photon discreteness

$$\frac{1}{2} m_e v_{\min}^2 = h\nu$$

Spectral shape of thermal bremsstrahlung emission

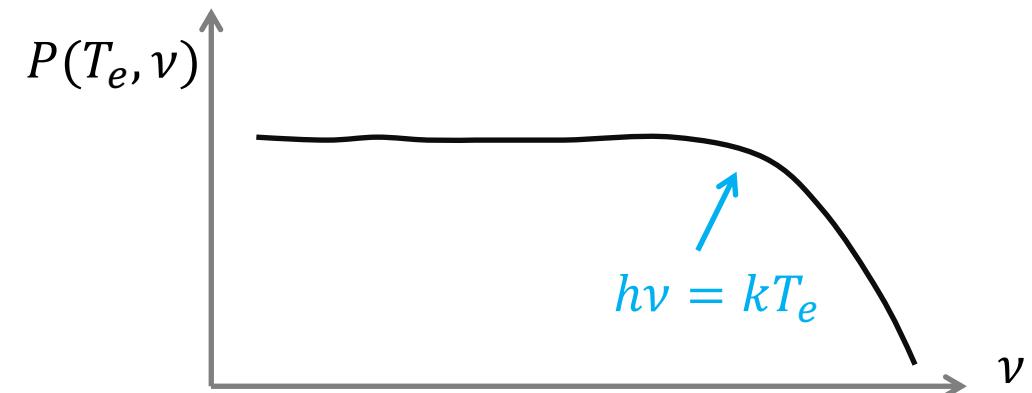
$$P(T_e, \nu) = \frac{dE(T_e, \nu)}{d\nu dV dt} = \frac{2^5 \pi e^6}{3m_e c^3} \left(\frac{2\pi}{3km_e} \right)^{1/2} T_e^{-1/2} Z^2 \langle g_{ff}(T_e, \nu) \rangle n_e n_i \exp\left(-\frac{h\nu}{kT_e}\right)$$

thermally-averaged Gaunt factor

$$\frac{P(T_e, \nu)}{\text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1}} = 6.84 \times 10^{-38} \left(\frac{\text{K}}{T_e} \right)^{1/2} Z^2 \langle g_{ff}(T_e, \nu) \rangle \left(\frac{n_e}{\text{cm}^{-3}} \right) \left(\frac{n_i}{\text{cm}^{-3}} \right) \exp\left(-\frac{h\nu}{kT_e}\right)$$

see Sect. 5.2 of the REF book
(p159-160) by Rybicki & Lightman

The thermal bremsstrahlung emission spectrum is rather **flat** at the **low-frequency** end with an **exponential decay** towards the **high-frequency** end. The **cutoff frequency** is $h\nu \sim kT_e$.



Total power of thermal bremsstrahlung emission

prev. sl.

$$\frac{dE(T_e, \nu)}{d\nu dV dt} = \frac{2^5 \pi e^6}{3m_e c^3} \left(\frac{2\pi}{3km_e} \right)^{1/2} T^{-1/2} Z^2 \langle g_{ff}(T_e, \nu) \rangle n_e n_i \exp\left(-\frac{h\nu}{kT_e}\right)$$

$$\int_0^\infty \exp(-ax) dx = \frac{1}{a}$$

$$P(T_e) = \int P(T_e, \nu) d\nu = \frac{2^5 \pi e^6}{3hm_e c^3} \left(\frac{2\pi k}{3m_e} \right)^{1/2} T_e^{1/2} Z^2 \langle g_{ff}(T_e) \rangle n_e n_i$$

$$\frac{P(T_e)}{\text{erg s}^{-1} \text{cm}^{-3}} = 1.43 \times 10^{-27} Z^2 T_e^{1/2} \langle g_{ff}(T_e) \rangle n_e n_i$$

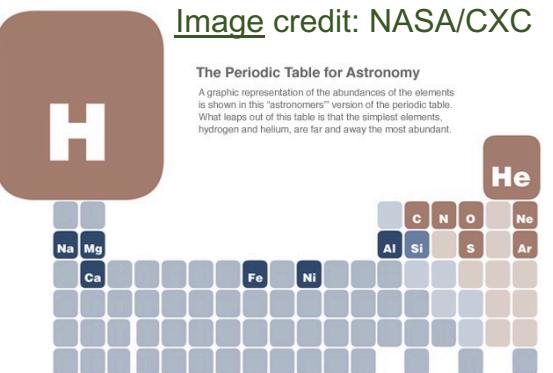


thermally- and frequency-averaged Gaunt factor, which is of the order of unity.

Thermal bremsstrahlung emission is dominated by hydrogen and helium, if the plasma has elemental abundances similar to those of the Sun.

see Sect. 5.2 of the REF book (p161-162) by Rybicki & Lightman

Image credit: NASA/CXC



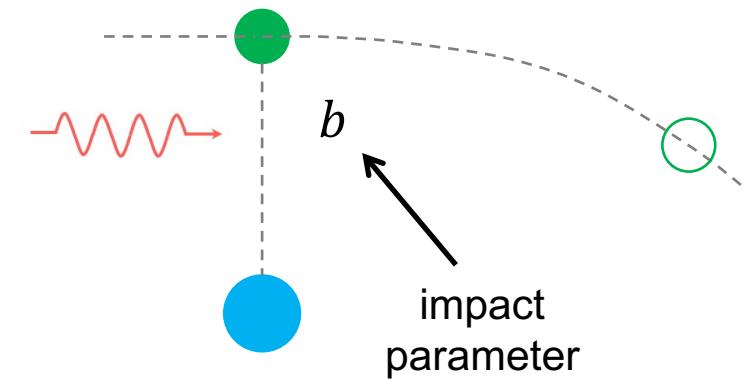
Thermal bremsstrahlung absorption

Kirchoff's law

$$j_\nu = \alpha_\nu B_\nu(T)$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \left(\exp\left(\frac{h\nu}{kT}\right) - 1 \right)^{-1}$$



prev. sl.

$$\frac{dE(T_e, \nu)}{d\nu dV dt} = \frac{2^5 \pi e^6}{3m_e c^3} \left(\frac{2\pi}{3km_e} \right)^{1/2} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \exp\left(-\frac{h\nu}{kT_e}\right)$$

$$j_\nu^{\text{ff}} = \frac{1}{4\pi} \frac{dE(T_e, \nu)}{d\nu dV dt}$$

$$\alpha_\nu^{\text{ff}} = \frac{j_\nu^{\text{ff}}}{B_\nu(T)} = \frac{4 e^6}{3m_e h c} \left(\frac{2\pi}{3km_e} \right)^{1/2} \nu^{-3} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \left(1 - \exp\left(-\frac{h\nu}{kT_e}\right) \right)$$

$$\frac{\alpha_\nu^{\text{ff}}}{\text{cm}^{-1}} = 3.69 \times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \left(1 - \exp\left(-\frac{h\nu}{kT_e}\right) \right)$$

Asymptotic behavior

prev. sl.

$$\frac{\alpha_{\nu}^{\text{ff}}}{\text{cm}^{-1}} = 3.69 \times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \left(1 - \exp \left(-\frac{h\nu}{kT_e} \right) \right)$$

For $h\nu \gg kT_e$ $\frac{\alpha_{\nu}^{\text{ff}}}{\text{cm}^{-1}} = 3.69 \times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \propto \nu^{-3}$

For $h\nu \ll kT_e$ $\frac{\alpha_{\nu}^{\text{ff}}}{\text{cm}^{-1}} = 1.77 \times 10^{-2} \nu^{-2} T^{-3/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \propto \nu^{-2}$

$$1 - e^{-x} = x + \dots$$

the **lower** the frequency, the **larger** the thermal bremsstrahlung absorption coefficient

cf. Thomson scattering

prev. sl.

$$\frac{\alpha_{\nu}^{\text{ff}}}{\text{cm}^{-1}} = 3.69 \times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_i \left(1 - \exp \left(-\frac{h\nu}{kT_e} \right) \right)$$

consider hydrogen only

$$\frac{\alpha_{\nu}^{\text{ff}}(\nu)}{n_e \sigma_T} = \frac{3.69 \times 10^8 \nu^{-3} T^{-1/2} \langle g_{\text{ff}}(T_e, \nu) \rangle n_e n_H \left(1 - \exp \left(-\frac{h\nu}{kT_e} \right) \right)}{n_e 6.65 \times 10^{-25}}$$

$$= 5.55 \times 10^{32} x^{-3} (1 - \exp(-x)) \left(\frac{h}{k} \right)^3 \left(\frac{T}{K} \right)^{\frac{5}{2}} \langle g_{\text{ff}}(T_e, \nu) \rangle n_H \quad x = \frac{h\nu}{kT}$$

$$= 61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{K} \right)^{\frac{5}{2}} \langle g_{\text{ff}} \rangle n_H$$

Thomson scattering
cross section

$$\sigma_T = 6.652 \times 10^{-25} \text{ cm}^2$$

cf. Thomson scattering (low frequency)

prev. sl.

$$\frac{\alpha_{\nu}^{\text{ff}}(\nu)}{n_e \sigma_T} = 61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{\text{K}}\right)^{\frac{5}{2}} \langle g_{\text{ff}} \rangle n_{\text{H}}$$

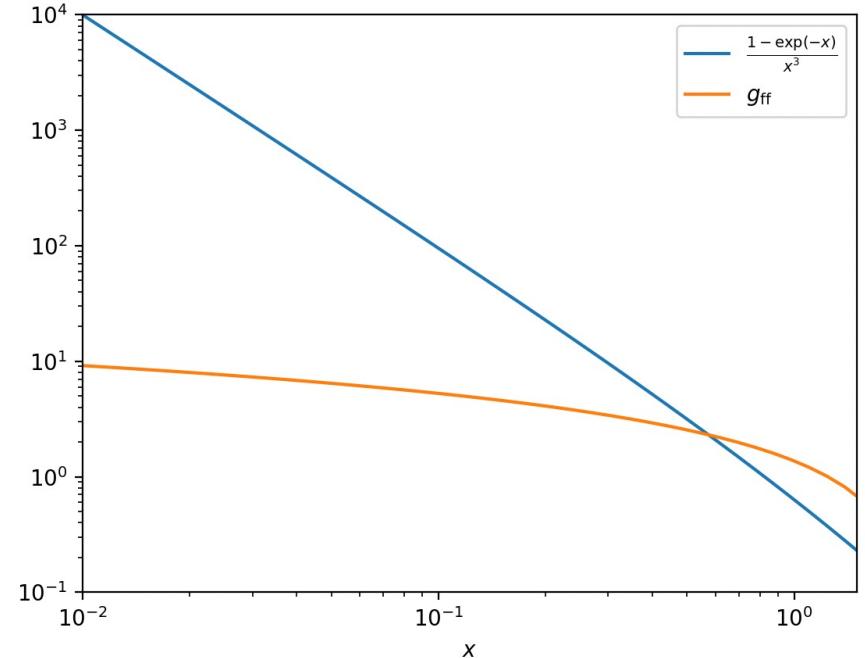
In this case, the thermally- and frequency-averaged Gaunt factor is

$$\langle g_{\text{ff}} \rangle \sim \frac{3}{\sqrt{\pi}} \ln \left(\frac{2.246}{x} \right) \quad \text{Eq. 3 of } \underline{\text{Felten \& Rees (1972)}}$$

Assuming at $x_0 = h\nu_0/kT$, we have $n_e \sigma_T = \alpha_{\nu}^{\text{ff}}(\nu_0)$

- ❖ For $x \ll x_0$ (low frequencies), i.e., $n_e \sigma_T \ll \alpha_{\nu}^{\text{ff}}(\nu_0)$, Thomson scattering can be neglected. The (absorption) mean free path of the photons is $1/\alpha_{\nu}^{\text{ff}}(\nu)$, we get the blackbody spectrum

$$x = \frac{h\nu}{kT}$$



slide 2.2.3

$$\bar{l}_{\nu} = \frac{1}{\alpha_{\nu}}$$

cf. Thomson scattering (high frequency)

prev. sl.

$$\frac{\alpha_{\nu}^{\text{ff}}(\nu)}{n_e \sigma_T} = 61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{\text{K}}\right)^{\frac{5}{2}} \langle g_{\text{ff}}(x) \rangle n_{\text{H}}$$

Here, the thermally- and frequency-averaged Gaunt factor is

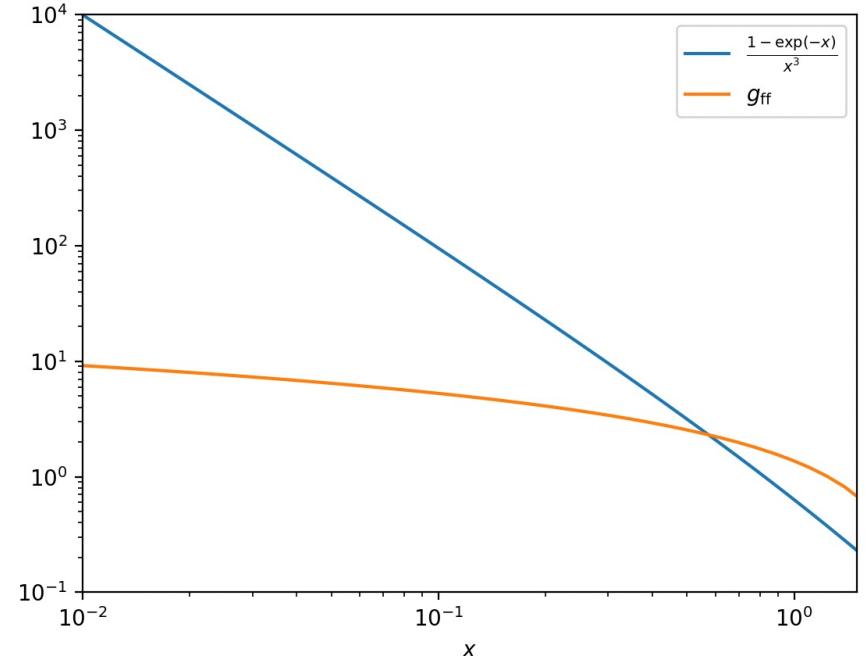
$$\langle g_{\text{ff}}(x) \rangle \sim \frac{3}{\sqrt{\pi}} \ln \left(\frac{2.246}{x} \right) \quad \text{Eq. 3 of Felten & Rees (1972)}$$

Assuming at $x_0 = h\nu_0/kT$, we have $n_e \sigma_T = \alpha_{\nu}^{\text{ff}}(\nu_0)$

- ❖ For $x \gg x_0$ (high frequencies), i.e., $n_e \sigma_T \gg \alpha_{\nu}^{\text{ff}}(\nu_0)$, Thomson scattering will modify the spectrum. Photons will experience random walk, the (scattering and absorption) mean free path is

$1/\sqrt{\alpha_{\nu}^{\text{ff}}(\nu)(\alpha_{\nu}^{\text{ff}}(\nu) + n_e \sigma_T)} \sim 1/\sqrt{\alpha_{\nu}^{\text{ff}}(\nu)n_e \sigma_T}$, we get the **modified** blackbody spectrum

$$x = \frac{h\nu}{kT}$$



slide 2.5.4

$$l_{\nu}^{\text{eff}} = \frac{1}{\sqrt{\alpha_{\nu}(\alpha_{\nu} + \sigma_{\nu}^{\text{sca}})}}$$

Modified blackbody spectrum

Eq. 7 of Felten & Rees (1972)

$$I_\nu^{\text{MBB}} \sim B_\nu \sqrt{\frac{\alpha_\nu^{\text{ff}}(\nu)}{n_e \sigma_T}}$$

prev. sl.

$$\frac{\alpha_\nu^{\text{ff}}(\nu)}{n_e \sigma_T} = 61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{\text{K}}\right)^{\frac{5}{2}} \langle g_{\text{ff}}(x) \rangle n_{\text{H}}$$

$$I_\nu^{\text{MBB}} \sim B_\nu \left(61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{\text{K}}\right)^{\frac{5}{2}} \langle g_{\text{ff}}(x) \rangle n_{\text{H}} \right)^{1/2}$$

$$x = \frac{h\nu}{kT}$$

- ❖ For $x_0 \ll x \ll 1$ (high frequencies and high temperature), the emission intensity is $\propto \nu$ instead of $\propto \nu^2$

$$I_\nu^{\text{MBB}} \sim \frac{2\nu^2}{c^2} kT \sqrt{61.34} \frac{kT}{h\nu} \left(\frac{T}{\text{K}}\right)^{\frac{5}{4}} \left(\frac{n_{\text{H}}}{\text{cm}^{-3}}\right)^{1/2} \langle g_{\text{ff}}(x) \rangle^{1/2}$$

$$= 5.00 \times 10^{-26} \left(\frac{\nu}{\text{Hz}}\right) \left(\frac{T}{\text{K}}\right)^{\frac{13}{4}} \left(\frac{n_{\text{H}}}{\text{cm}^{-3}}\right)^{1/2} \langle g_{\text{ff}}(x) \rangle^{1/2}$$

Rayleigh-Jeans Law
(asymptotic behavior of Planck's law with $x \ll 1$)

$$I_\nu^{\text{RJ}} = \frac{2\nu^2}{c^2} kT$$

$$\frac{1 - \exp(-x)}{x^3} \sim \frac{1}{x^2} + \dots$$

X-ray spectrum of ICM

Clusters of galaxies (or galaxy clusters) are the largest gravitational bounded structures in the Universe.

The hot (X-ray emitting) intracluster medium (ICM) is a key component of ClG.

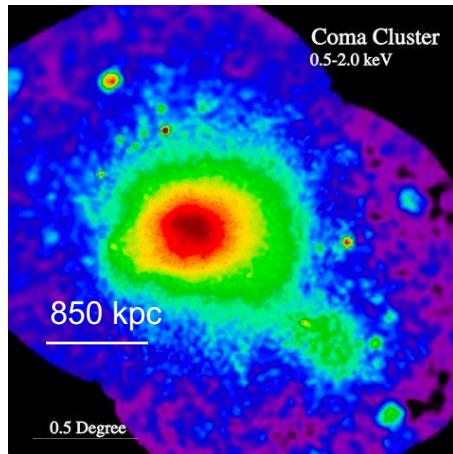
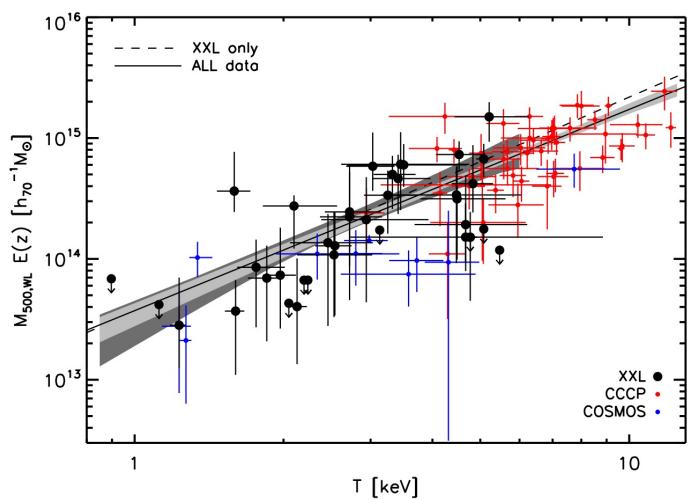
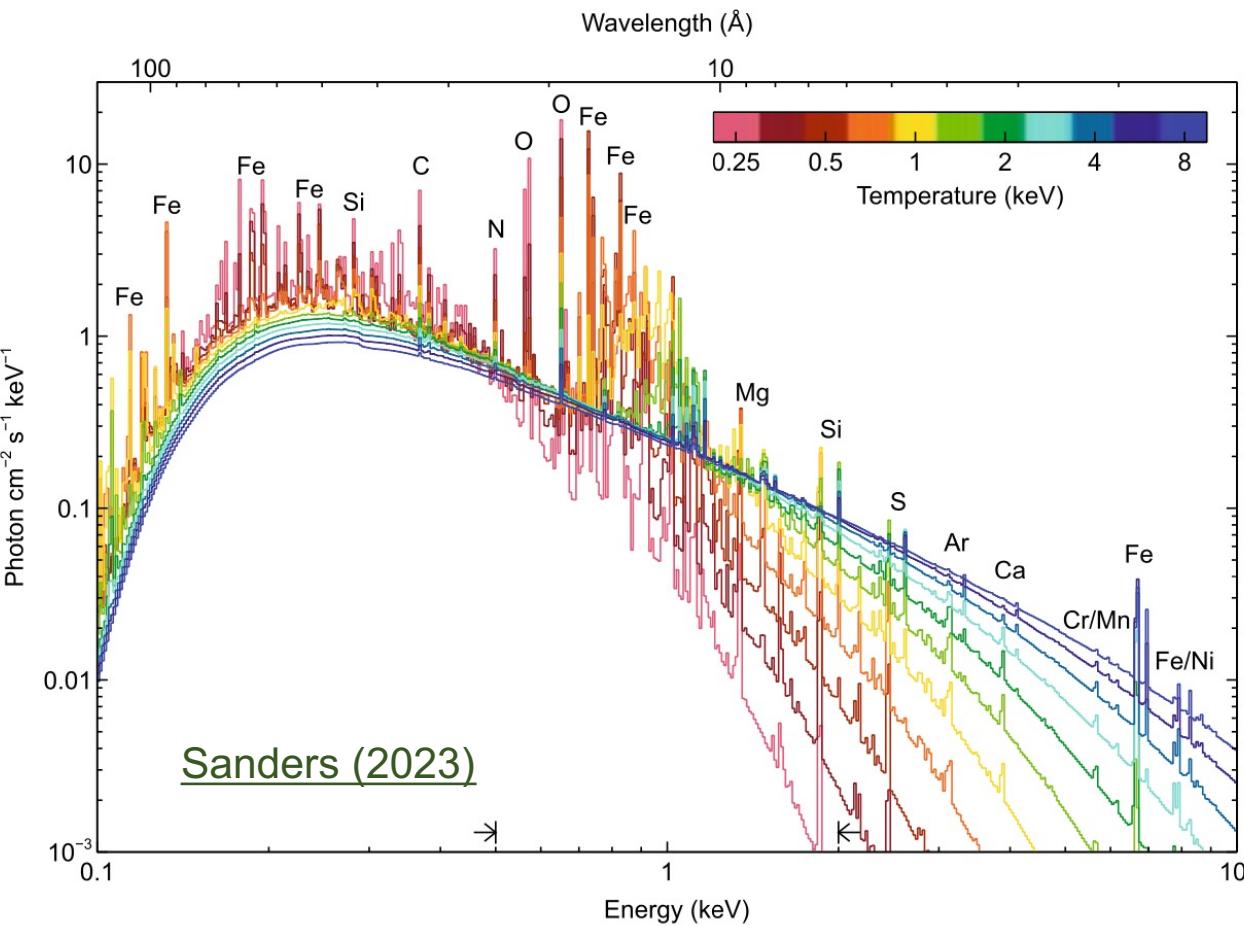


Image credit: ROSAT/MPE/S.
L. Snowden



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prev. sl.

lines and continua

Spectral diagnostics of characteristic **emission** and **absorption features** in the observed atomic spectra can quantify the physical parameters ...

Bremsstrahlung continuum in ICM

Böhringer & Werner (2010)

The continuum of ICM consists of

- Recombination
- Two-photon
- Bremsstrahlung (free-free)

A sense of ICM temperature

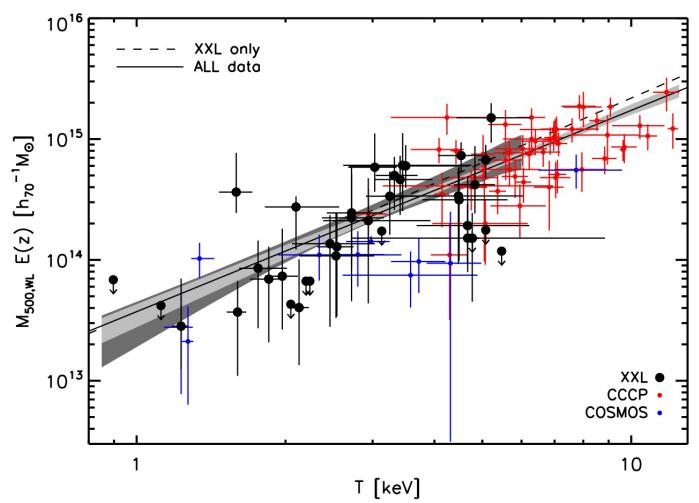
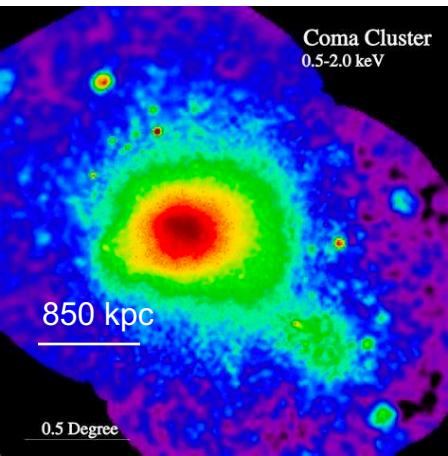


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