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Physics of the Interstellar Medium

Homework 2

Question 1. (20 points) Assume that you have both emission and absorption spectra for the 21 cm line in a particular direction, and can therefore determine τ_ν across the line profile.

1) Show that the measured excitation temperature $T_{\rm exc}$ in any line of sight is actually the harmonic mean of all components with various excitation temperatures in that direction.

2) Typical measured excitation temperatures for lines of sight near the galactic plane are $T_{\rm exc} \sim 150K$. Assume that all emission is optically thin, and all the HI gas is either in cold neutral clouds or in a warm neutral phase. What is the fraction of the HI in each phase?

Question 2. (20 points) Consider a region containing only partially-ionized hydrogen. Let ζ be the ionization rate per H atom, and let α be the recominbation coefficient.

1) Determine the steady-state ionization fraction $x_{\rm ss}$ in terms of $n_{\rm H} \equiv n(H^0) + n(H^+)$, ζ, and α.

2) Suppose that the fractional ionization at time $t = 0$ is given by $x(0) = x_{\rm ss} + \delta$. If $|\delta| \ll x_{\rm ss}$, determine the solution x(t) by assuming $n_{\rm H}$, ζ, and α to be constant.

Question 3. (20 points) In class, we worked out the methodology for the determination of the electron temperature in a photoionized medium using the 4363 Å and 5007 Å lines of [OIII], because the electron density dependence is cancelled out in this ratio.

1) If we add the 4959 Å in the diagnostics, derive the expression for the $j(4959\text{Å} + 5007\text{Å})/j(5363\text{Å})$ by also including the term for collisional de-excitation of the ${}^1D_2$ level. Solve for the relation between the observed emissivity ratio and the temperature. [Hint: in this solution, the temperature-dependent is again through the collisional rate, but there will be an additional term that includes the electron density.]

2) Make a plot of the predicted observed ratio as a function of temperature for electron density of 10^3, 10^4, and 10^5cm^{-3}. [Hint: The appendix E and F in Draine's book contain some useful data.]

3) What does this exercise tell you?

Question 4. (20 points) Derive the Rankine-Hugoniot jump condition for the density, pressure, and temperature ratios across the shock front with Mach number $\mathcal{M}_1$ from the mass, momentum and energy conservation law of hydrodynamics.

Question 5. (20 points) The evolution of supernova remnant in the Sedov-Taylor phase can be derived by simple dimensional analysis, as is shown in class: $R_s = AE^{1/5}\rho_0^{-1/5}t^{2/5}$.

1) Obtain an estimate of the dimensionless factor A by assuming that 50% of the total energy will be in ordered kinetic energy $Mv_s^2/2$, where M is the swept-up mass.

2) Above we considered the case of uniform ambient density ρ and constant total energy E. Suppose that we instead assume that the ambient density decreases as

$$\rho = \rho_0(r/r_0)^\delta,$$

and energy is increasing with time as a power law:

$$E = E_0(t/t_0)^\epsilon,$$

where $\delta > -3$ and $\epsilon \geq 0$. Find γ in the time evolution of the radius of the blastwave $Rs \propto t^\gamma$ as a function of δ and ϵ.

3) If $Rs \propto t^\gamma$, how does the shock temperature T_s vary with time?

4) Suppose that the density profile in the ambient medium is $\rho \propto r^{-2}$, as would applyy to a constant-velocity steady stellar wind pre-process the gas before supernova explosion. Suppose that there is a sudden explosion depositing an energy $E_0 = constant$. What will be γ for this case?