AY2024F

Junjie Mao@THU

Radiative Processes in Astrophysics

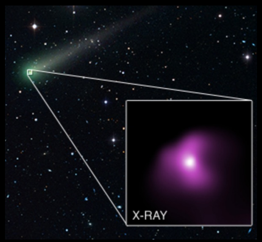
Observation

Up to cosmic size scale

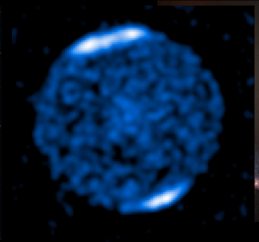
Theory

Down to atomic size scale

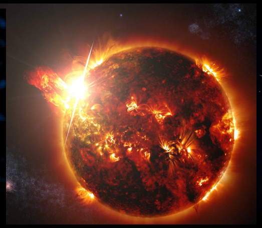
C/2012S1 (comet)



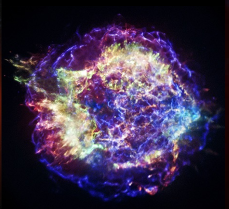
Jupiter (planet)



Sun (star)

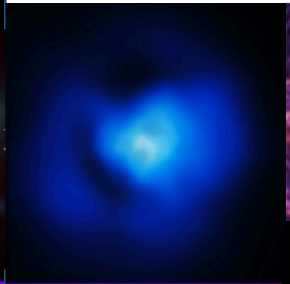


Cas A (SNR)



M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

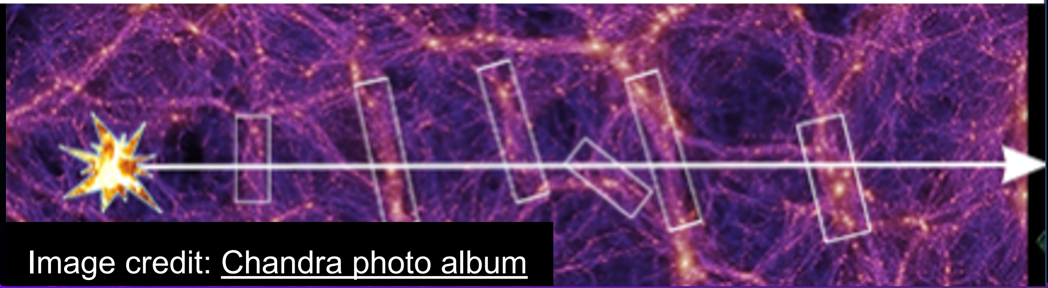


Image credit: Chandra photo album

Chpt.5 Atomic processes

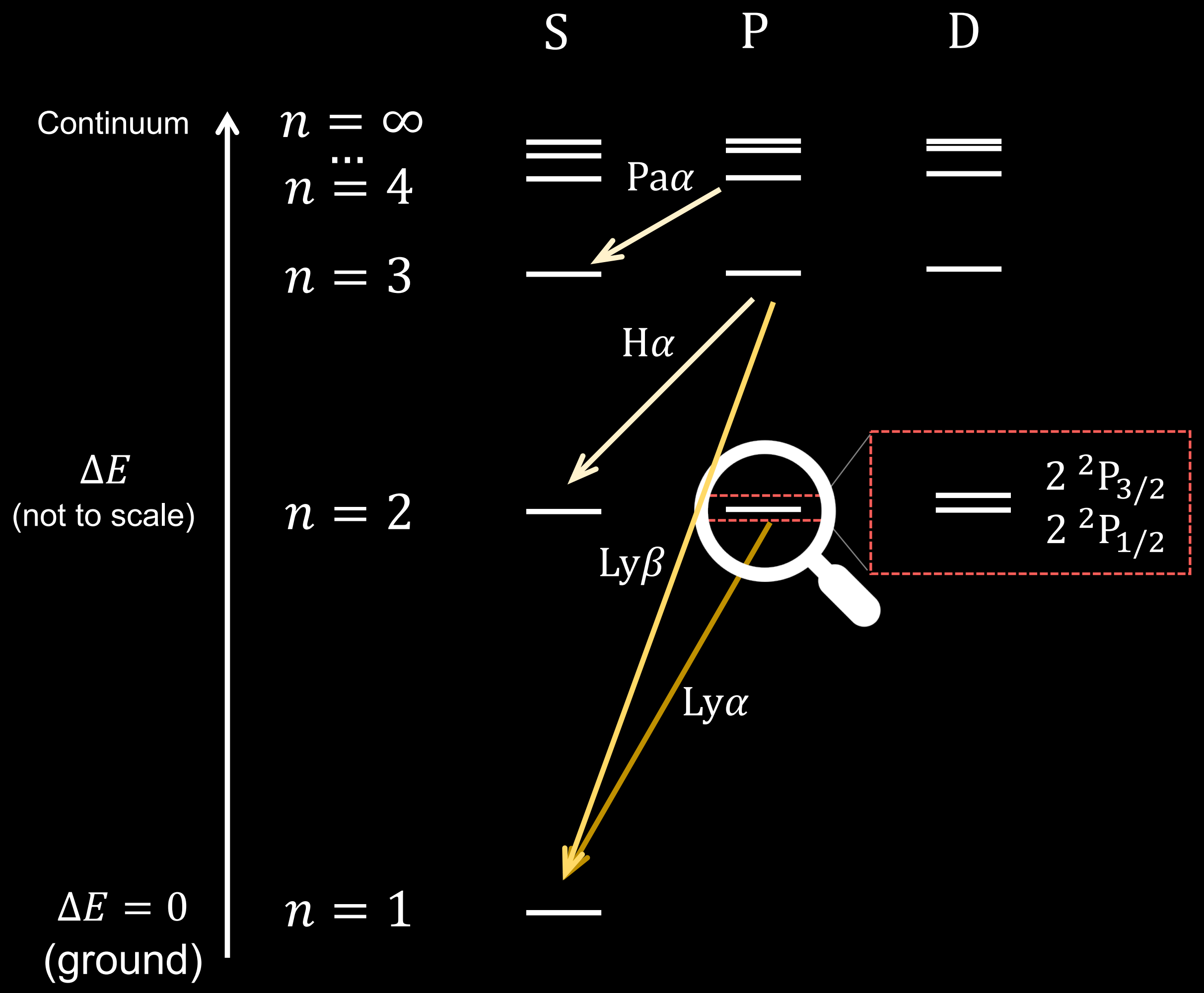
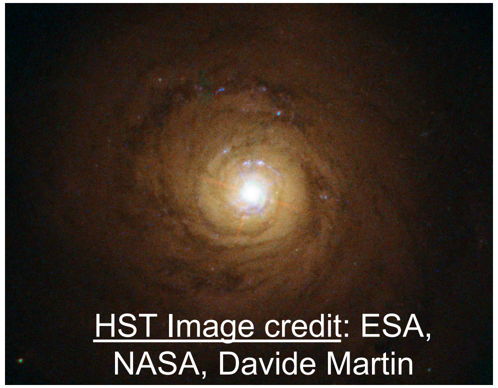


Image credit: Junjie Mao

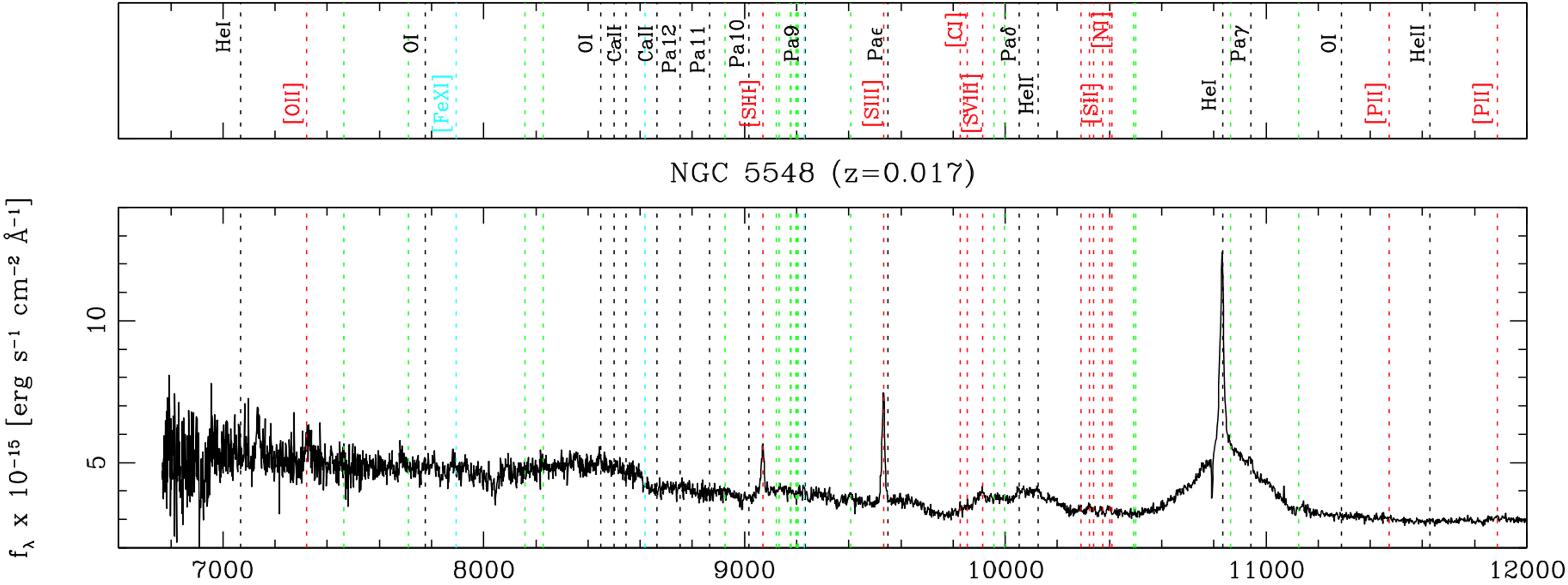
Exemplary NIR spectra



HST Image credit: ESA, NASA, Davide Martin

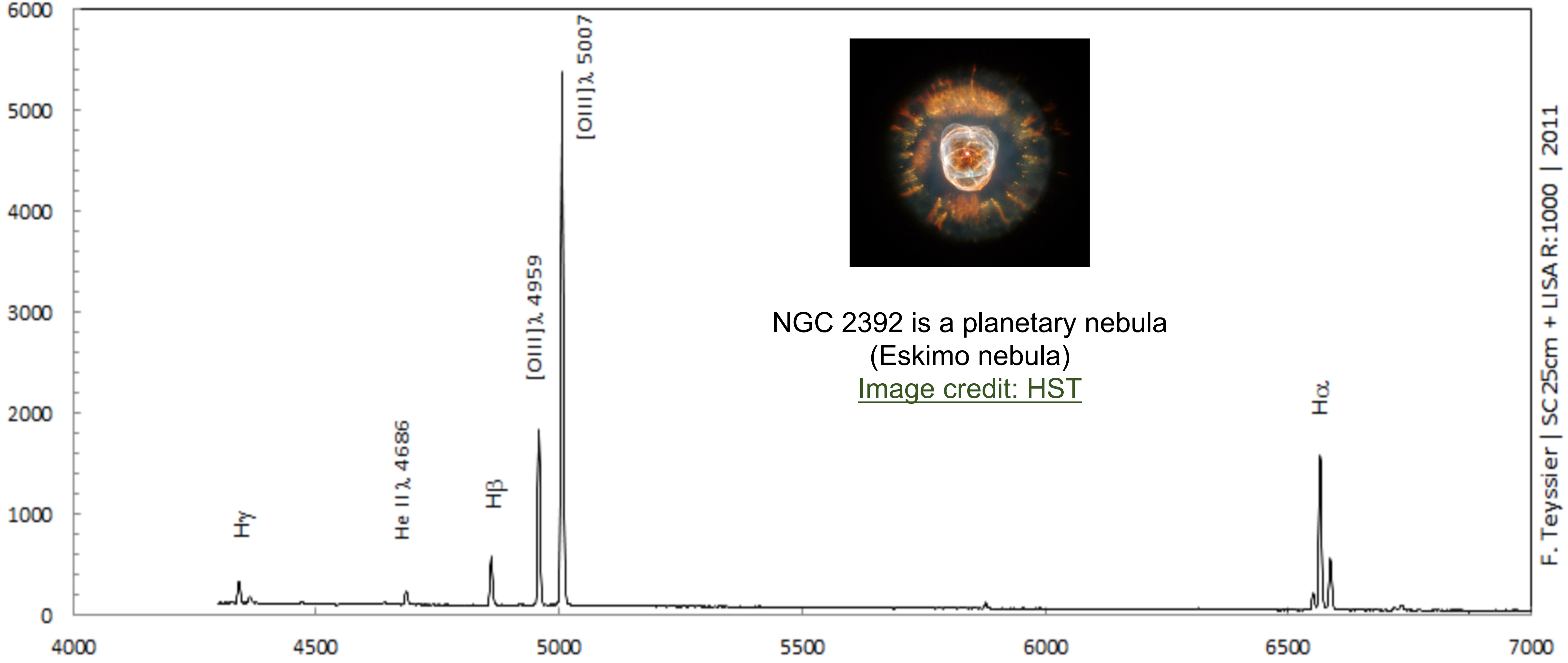
Landt et al. 2019

NGC 5548 is a nearby galaxy host a Seyfert 1 AGN.



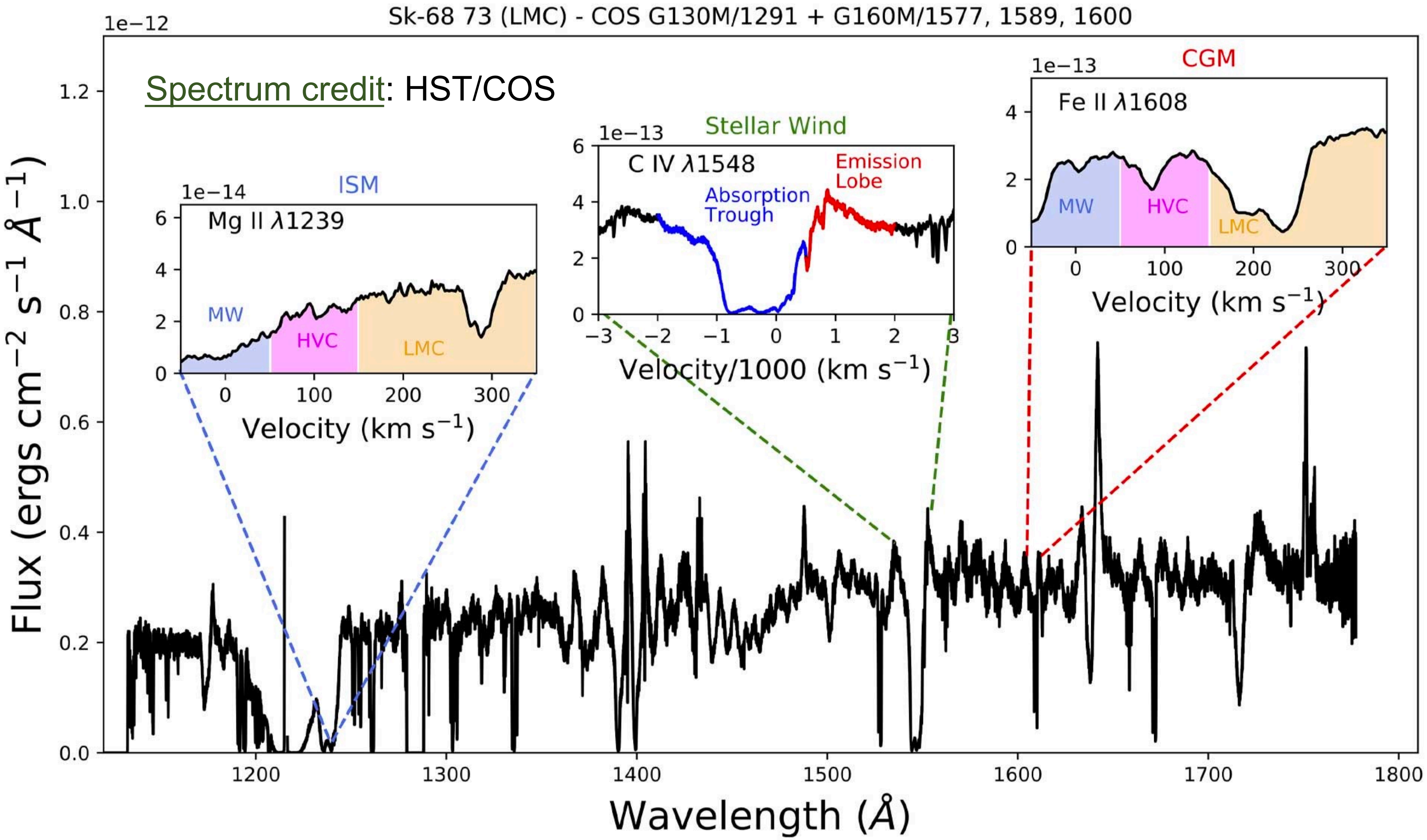
Exemplary optical spectrum

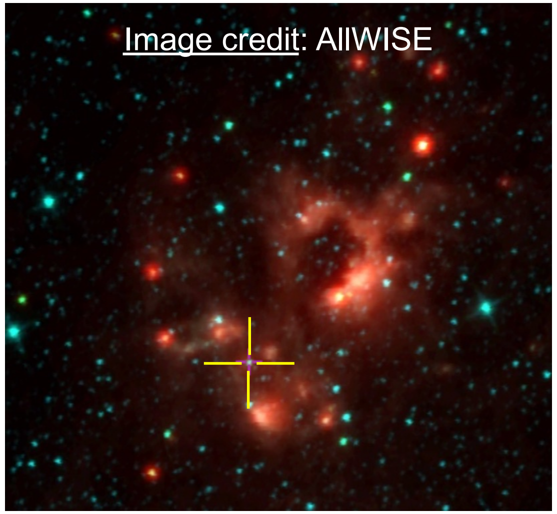
Spectrum credit: F. Teyssier



NGC 2392 is a planetary nebula
(Eskimo nebula)
Image credit: HST

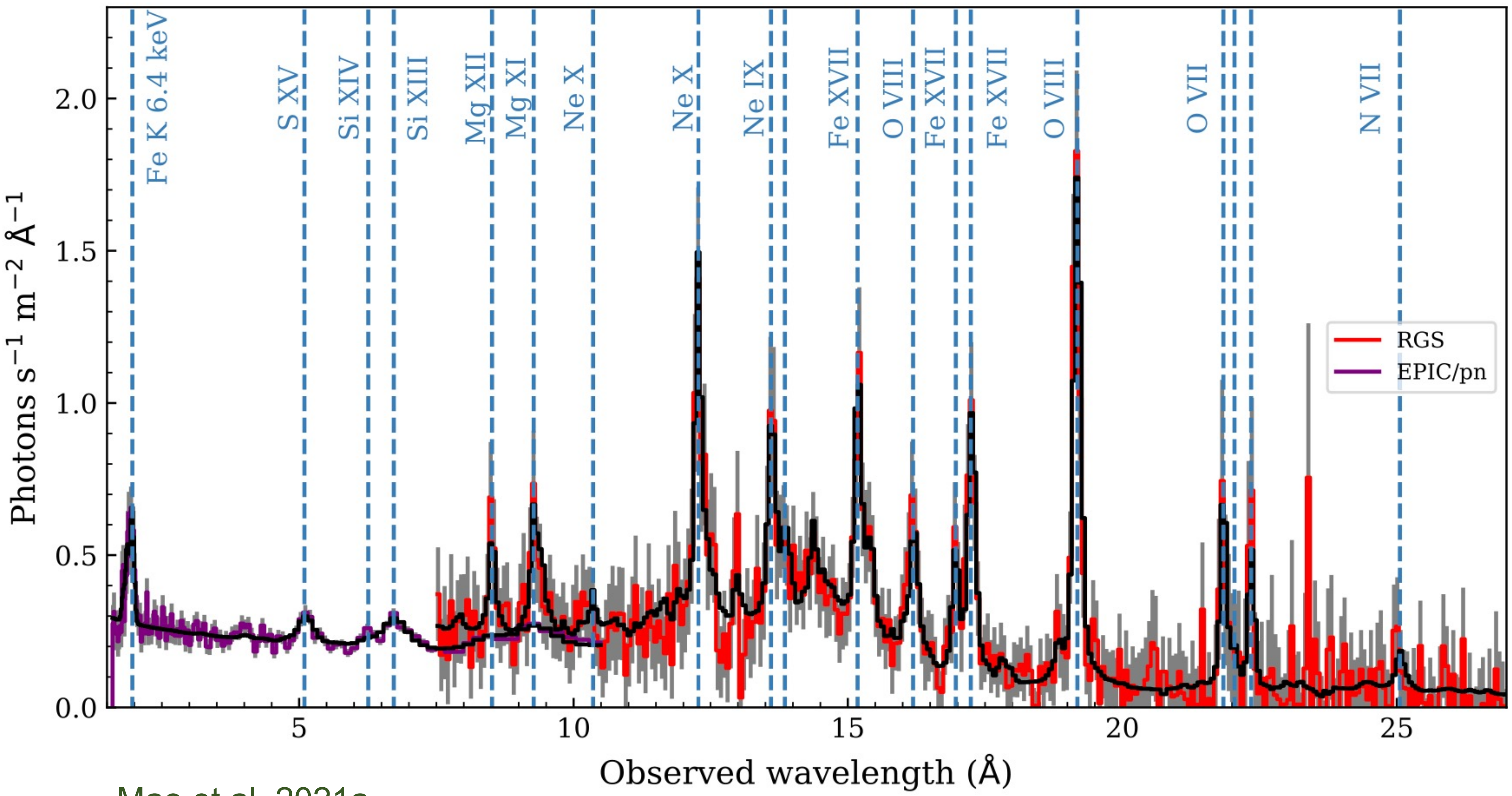
Exemplary UV spectrum





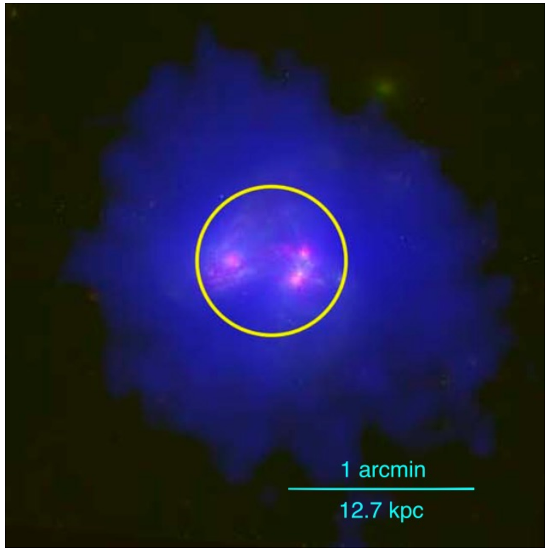
Sk-68 73 is a massive Wolf-Rayet star in the LMC

Exemplary X-ray spectrum



Mao et al. 2021a

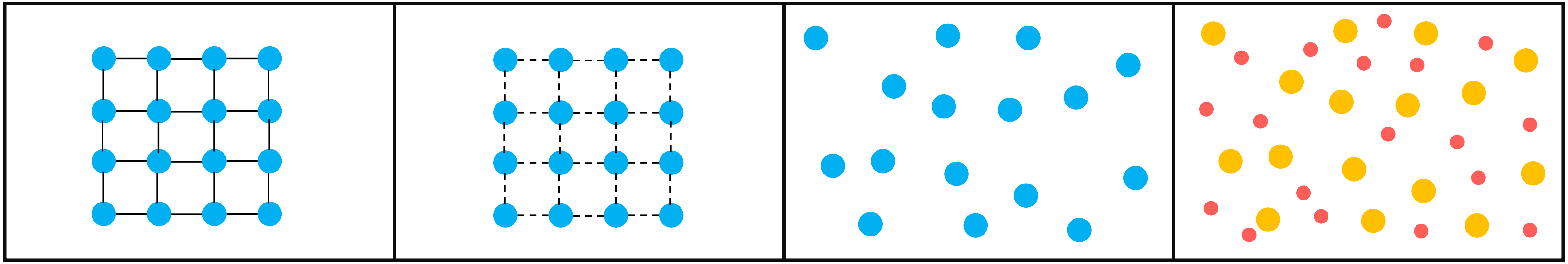
X-ray (XMM-Newton purple)
+ optical (Hubble, green)
+ IR (Spitzer, red)



NGC 3690 is a pair of merging galaxies with intensive star-forming activities.

Four states of matter

add thermal energy



sublimate

solid $\underset{\text{freeze}}{\overset{\text{melt}}{\rightleftharpoons}}$ liquid $\underset{\text{condense}}{\overset{\text{vaporize}}{\rightleftharpoons}}$ gas $\underset{\text{deionize}}{\overset{\text{ionize}}{\rightleftharpoons}}$ plasma

deposit

definite shape and volume
(strong bound)

definite volume
(weak bound)

no definite volume and shape

Astrophysical plasmas (cont.)

Ezoe et al. 2021

Most of the baryons in the Universe are in the form of ionized plasmas with a temperature of $\sim 10^{4-9}$ K (e.g., Bregman et al. 2023)

All natural elements in the periodic table are baryons

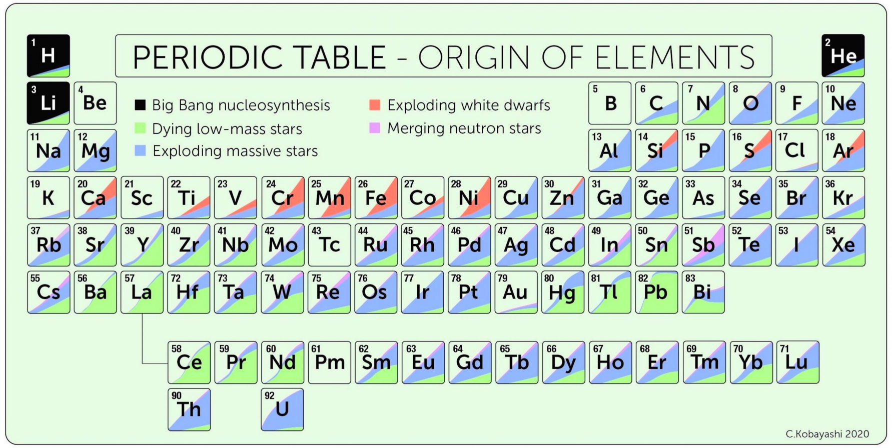
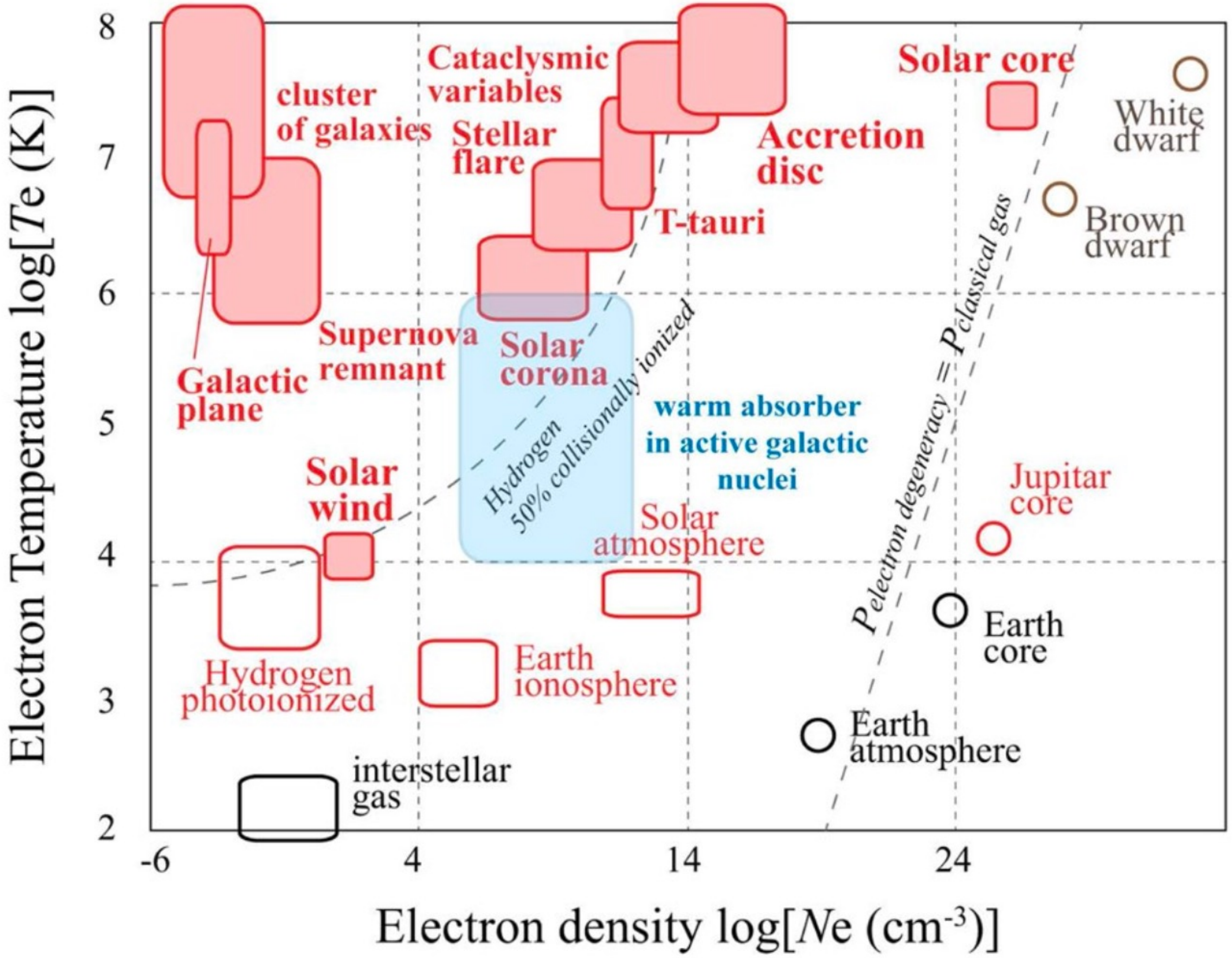


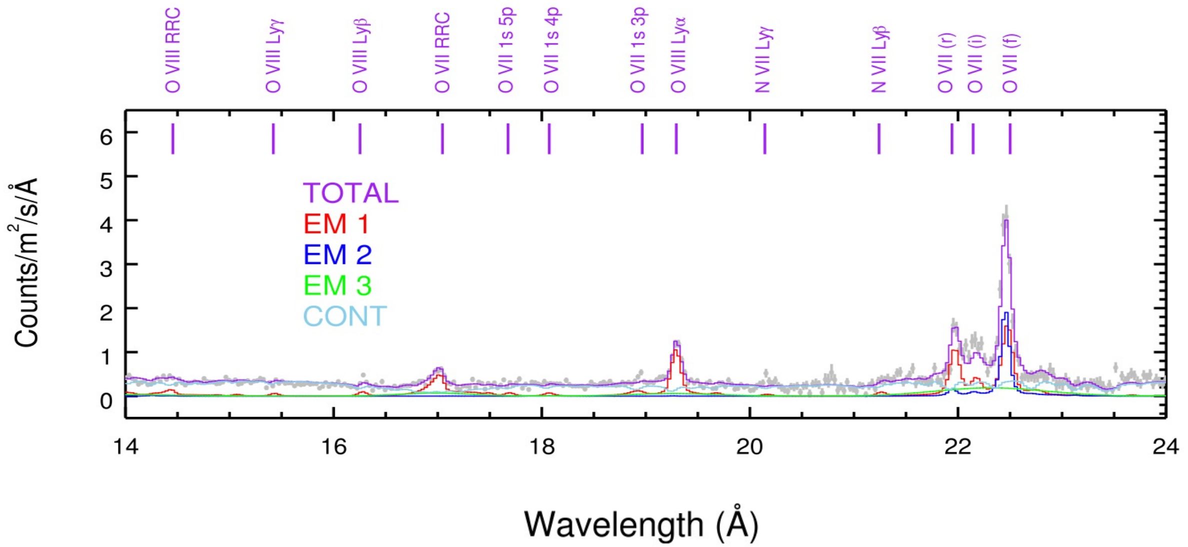
Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily



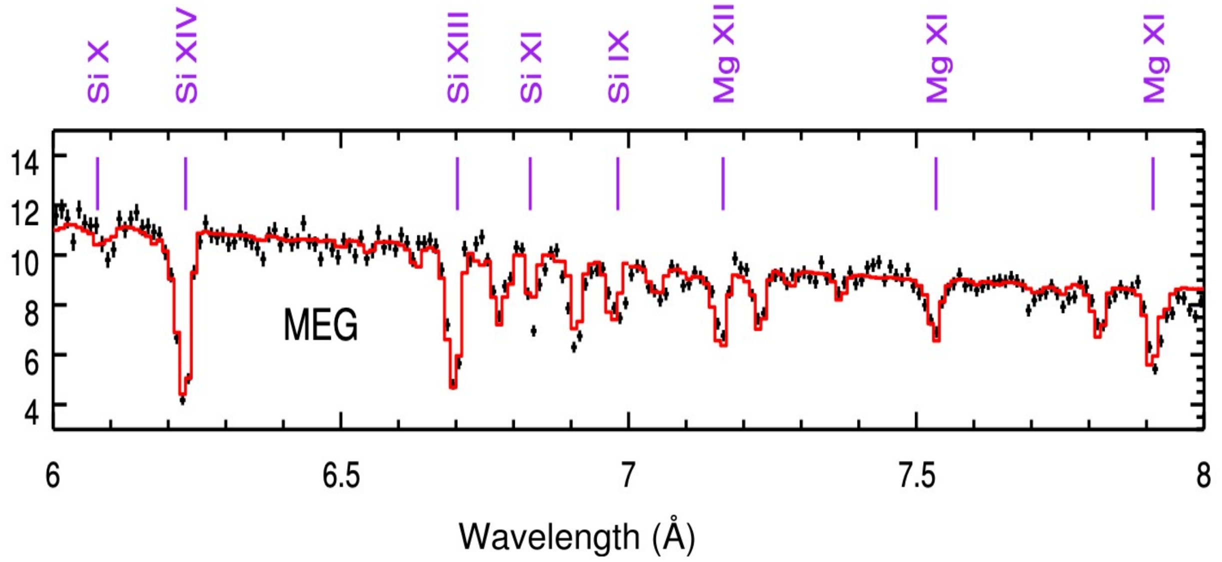
Astro plasma spectra

lines and continua

Spectral diagnostics of characteristic emission and absorption features in the observed atomic spectra can quantify the physical parameters of the observing targets (e.g., elemental abundances, density, temperature, kinematics).

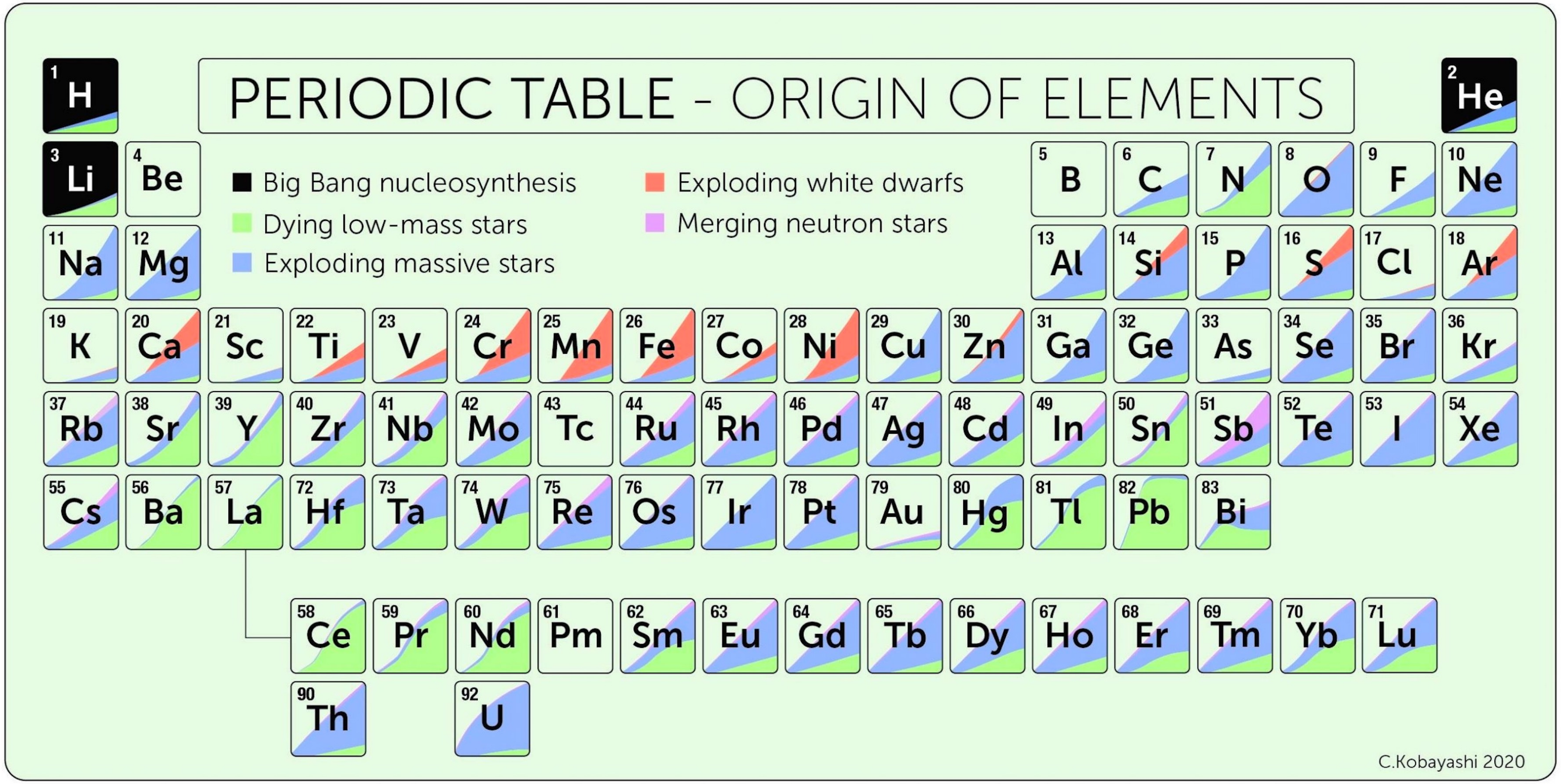


Mao et al. 2018



Mao et al. 2019b

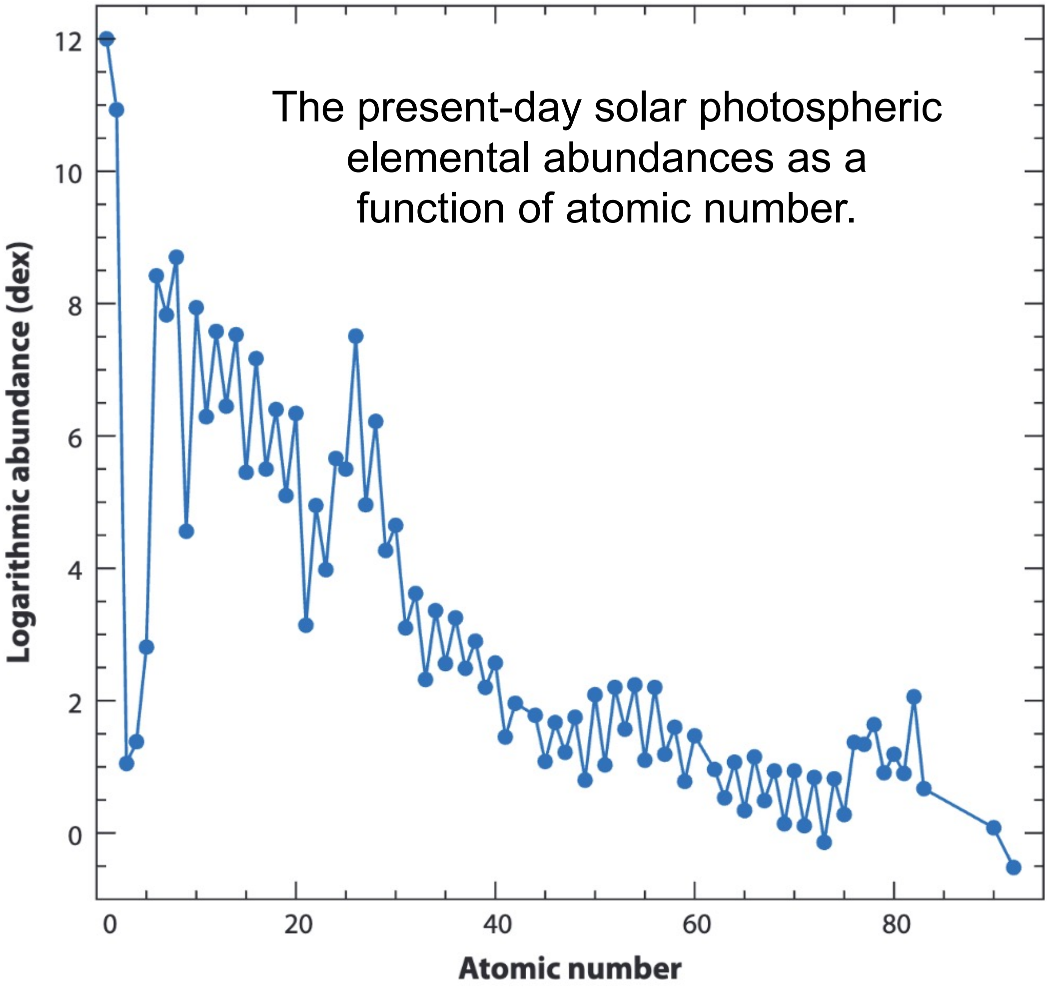
Periodic table – Origin of elements



Ideally, we would like to study as many (naturally occurring) elements as possible.

Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily

Cosmic elemental abundances



Asplund et al. 2009

Solar mass-fractions (Basu & Antia, 2004):

- ❑ $X = 0.7389$ for H
- ❑ $Y = 0.2463$ for He
- ❑ $Z = 0.0148$ for metals (Li to U)
- ❑ $Z/X = 0.020$

Astro abundance scales

$$A(\epsilon_i) = 12 + \log_{10}\frac{n(\epsilon_i)}{n(\mathrm{H})}$$

- ❖ By definition, $A(\mathrm{H}) = 12$ for H.
- ❖ $A(\mathrm{O}) = 8.69$ means that O is a factor of ~ 2000 less than H.

dex refers to the decadic logarithmic units where $1\ \mathrm{dex}$ stands for a factor of 10 and $0.1\ \mathrm{dex}$ stands for a factor of $10^{0.1} \sim 1.259$

Astro periodic table

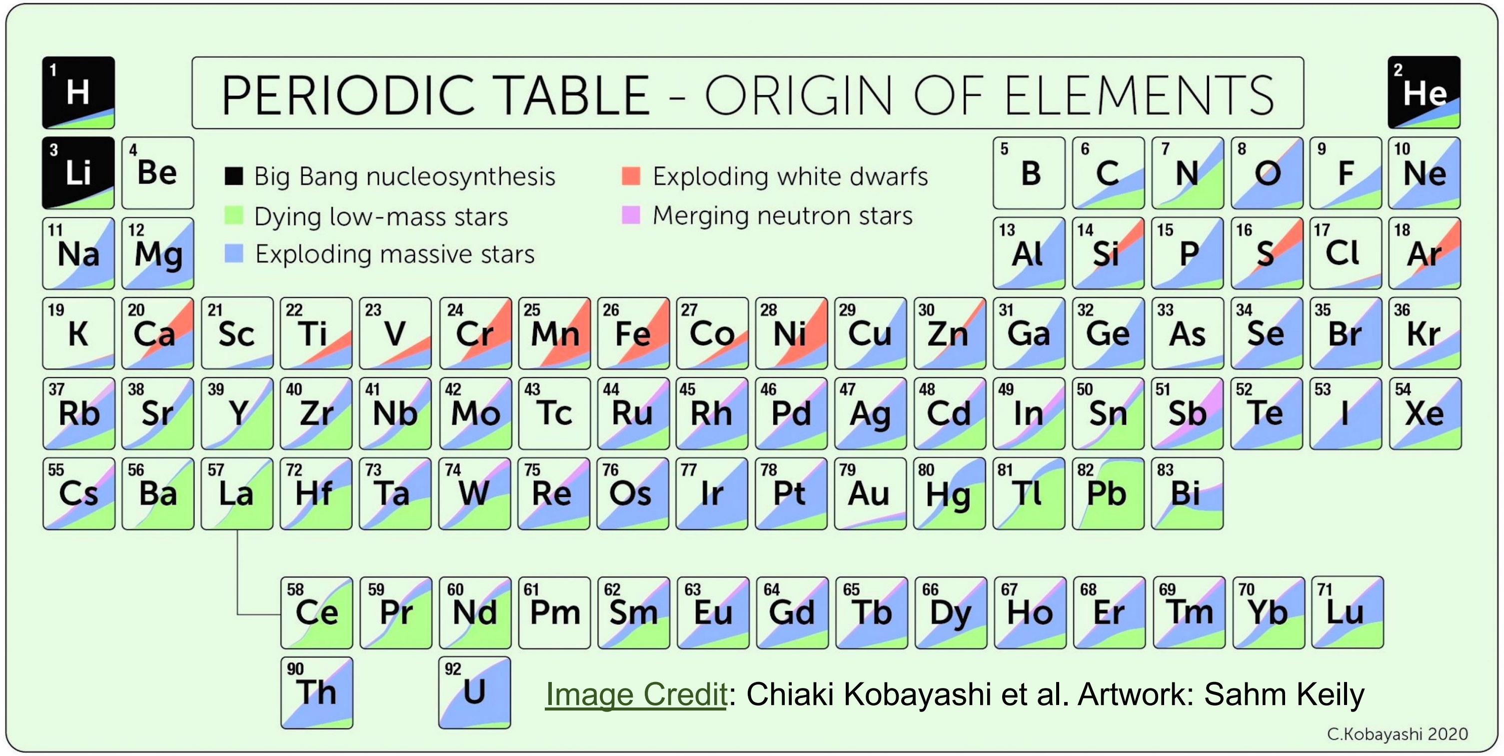
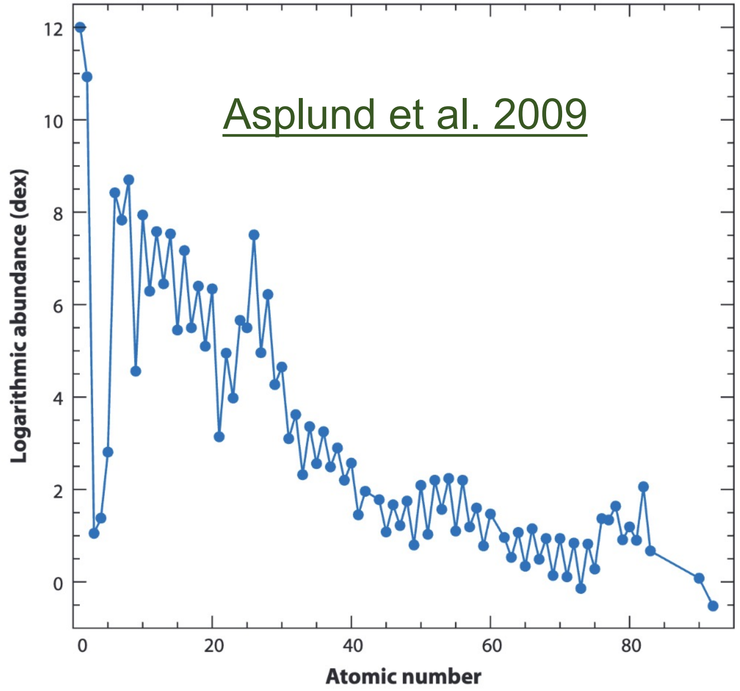


Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily



Asplund et al. 2009

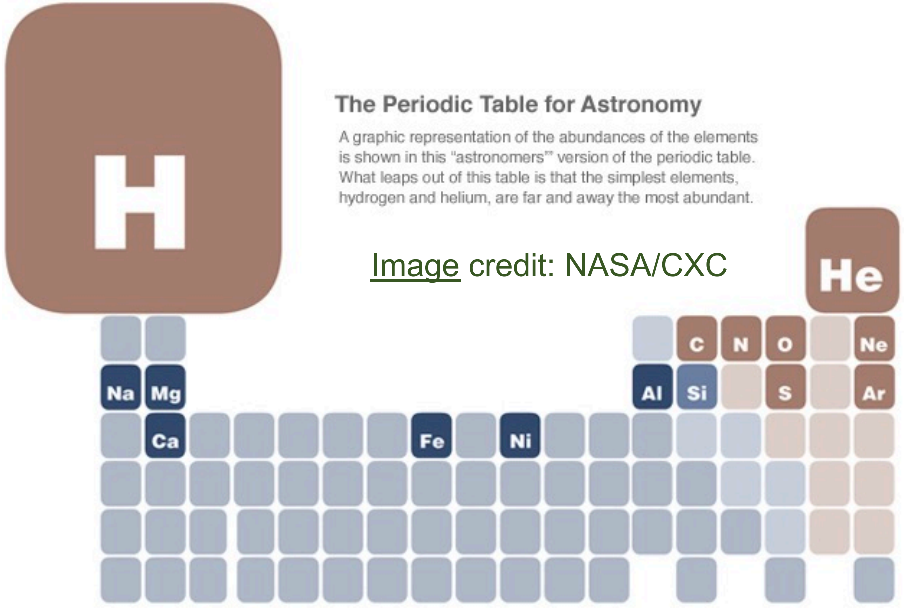


Image credit: NASA/CXC

Ideally, we would like to study as many (naturally occurring) elements as possible. But we might pick and chose in practice.

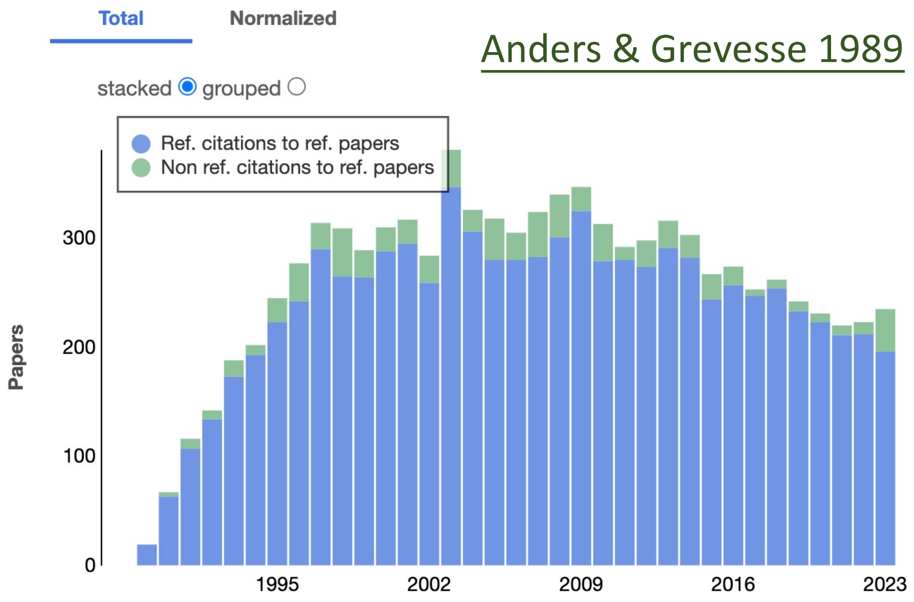
Solar abundance tables

Sometimes, “solar abundance” is used without reference, but technically, various solar abundance tables exist and they do differ.

Source	X	Y	Z	Z/X
Present-day photosphere:				
Anders & Grevesse (1989)[a]	0.7314	0.2485	0.0201	0.0274
Grevesse & Noels (1993)[a]	0.7336	0.2485	0.0179	0.0244
Grevesse & Sauval (1998)	0.7345	0.2485	0.0169	0.0231
Lodders (2003)	0.7491	0.2377	0.0133	0.0177
Asplund, Grevesse & Sauval (2005)	0.7392	0.2485	0.0122	0.0165
Lodders, Palme & Gail (2009)	0.7390	0.2469	0.0141	0.0191
Present work	0.7381	0.2485	0.0134	0.0181
Protosolar:				
Anders & Grevesse (1989)	0.7096	0.2691	0.0213	0.0301
Grevesse & Noels (1993)	0.7112	0.2697	0.0190	0.0268
Grevesse & Sauval (1998)	0.7120	0.2701	0.0180	0.0253
Lodders (2003)	0.7111	0.2741	0.0149	0.0210
Asplund, Grevesse & Sauval (2005)	0.7166	0.2704	0.0130	0.0181
Lodders, Palme & Gail (2009)	0.7112	0.2735	0.0153	0.0215
Present work	0.7154	0.2703	0.0142	0.0199

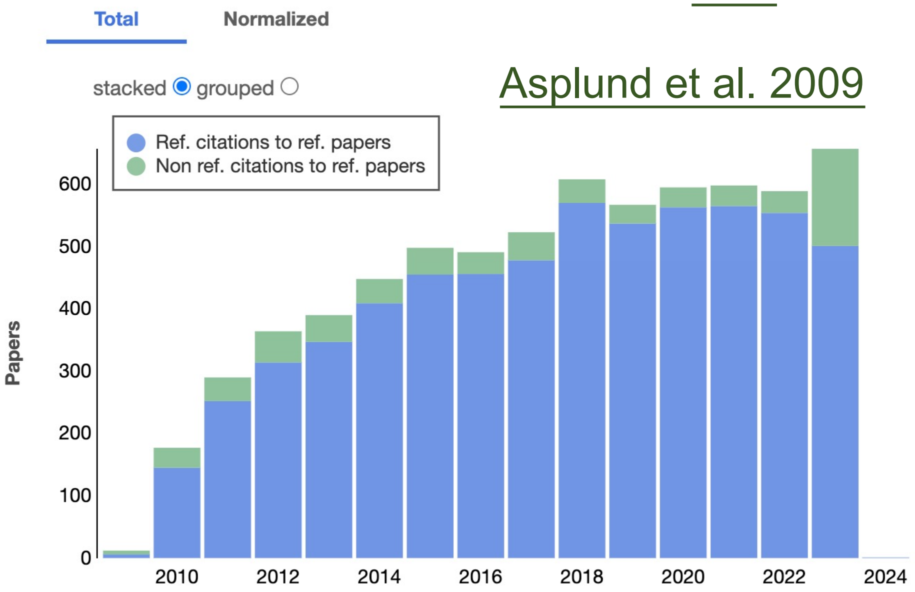
Asplund et al. 2009

Anders & Grevesse 1989



Credit: ADS

Asplund et al. 2009



Photosphere solar abundance

Photosphere abundance: the composition of the present day Sun's outer convection zone (~2% of the Sun's total mass with ongoing diffusion and gravitational settling into the Sun's interior over the solar lifetime).

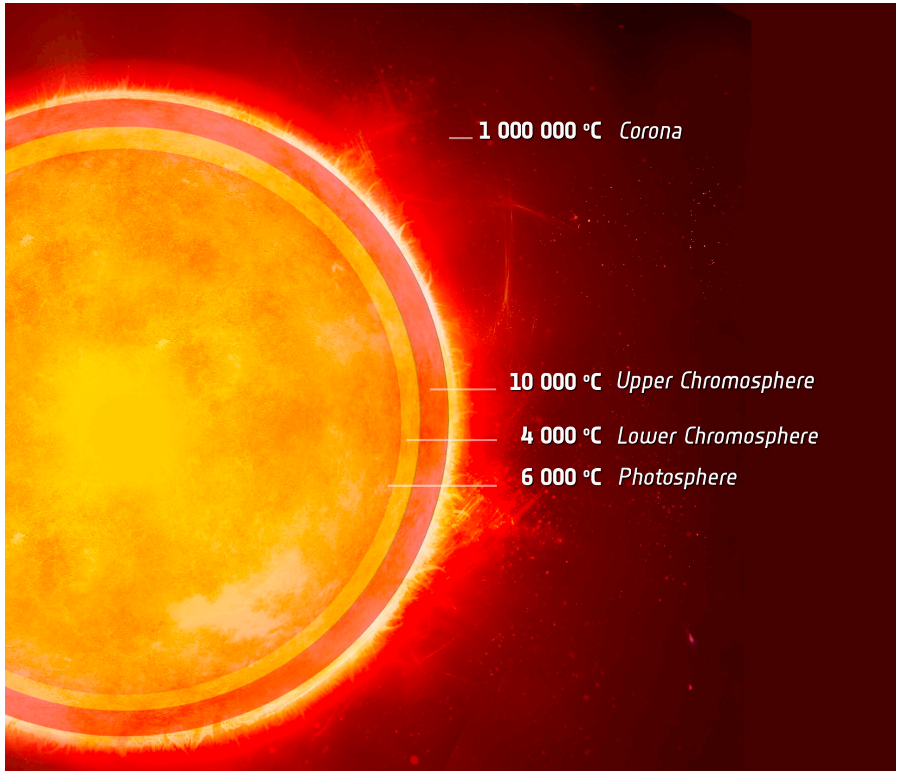
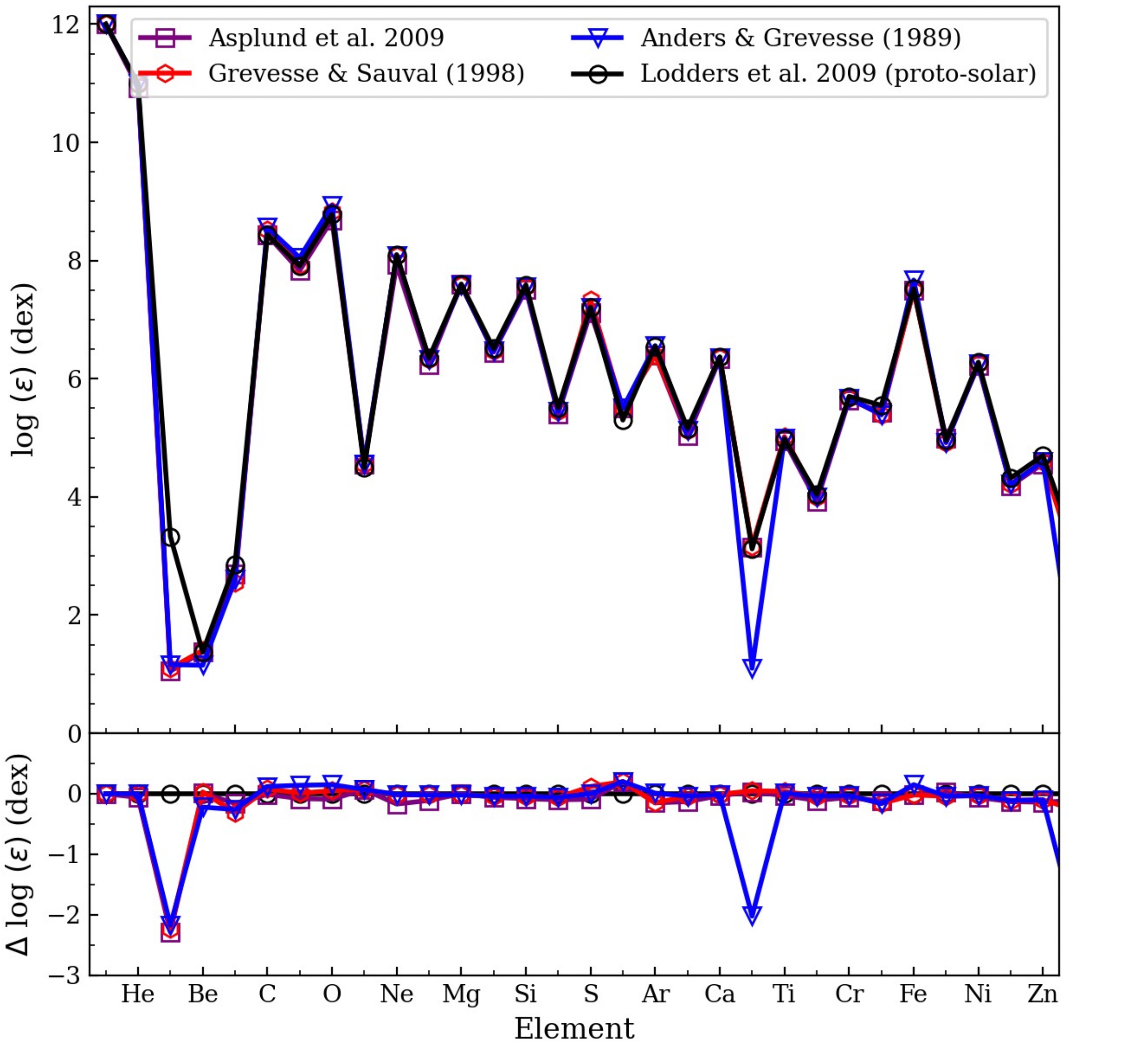
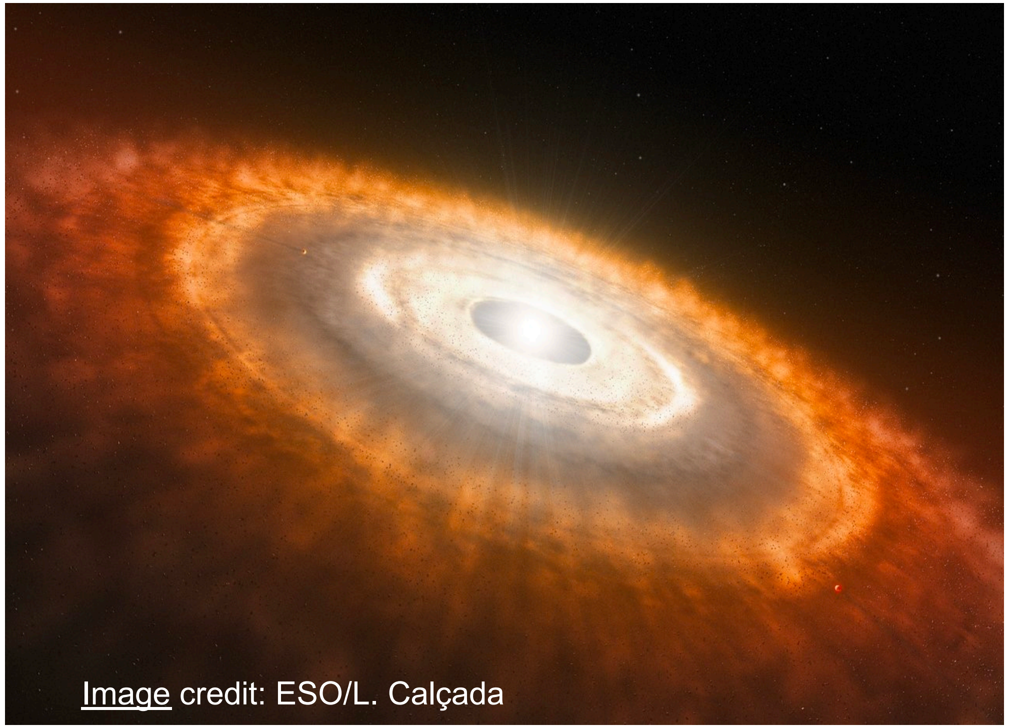


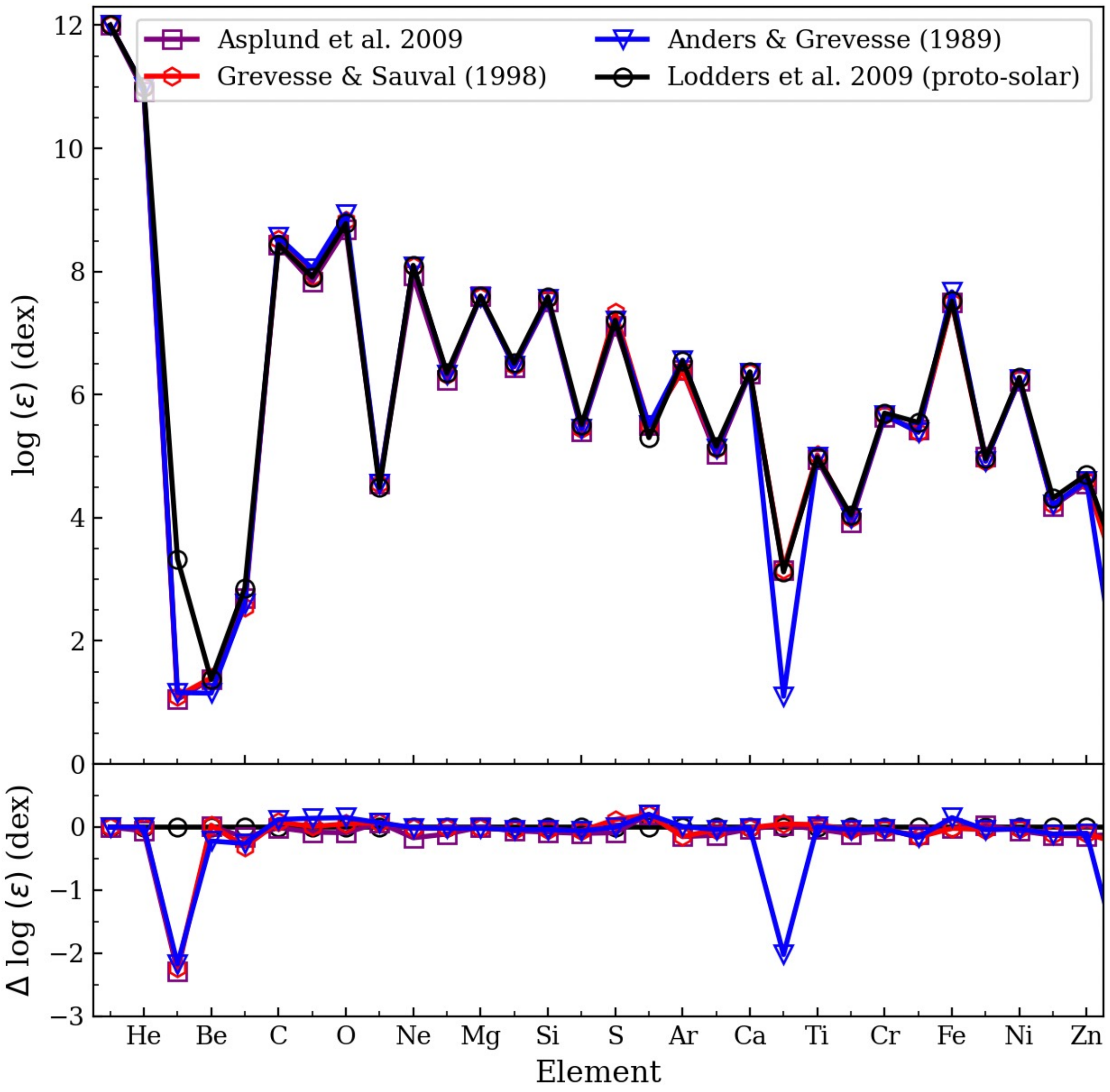
Image credit: ESA



Solar abundance tables

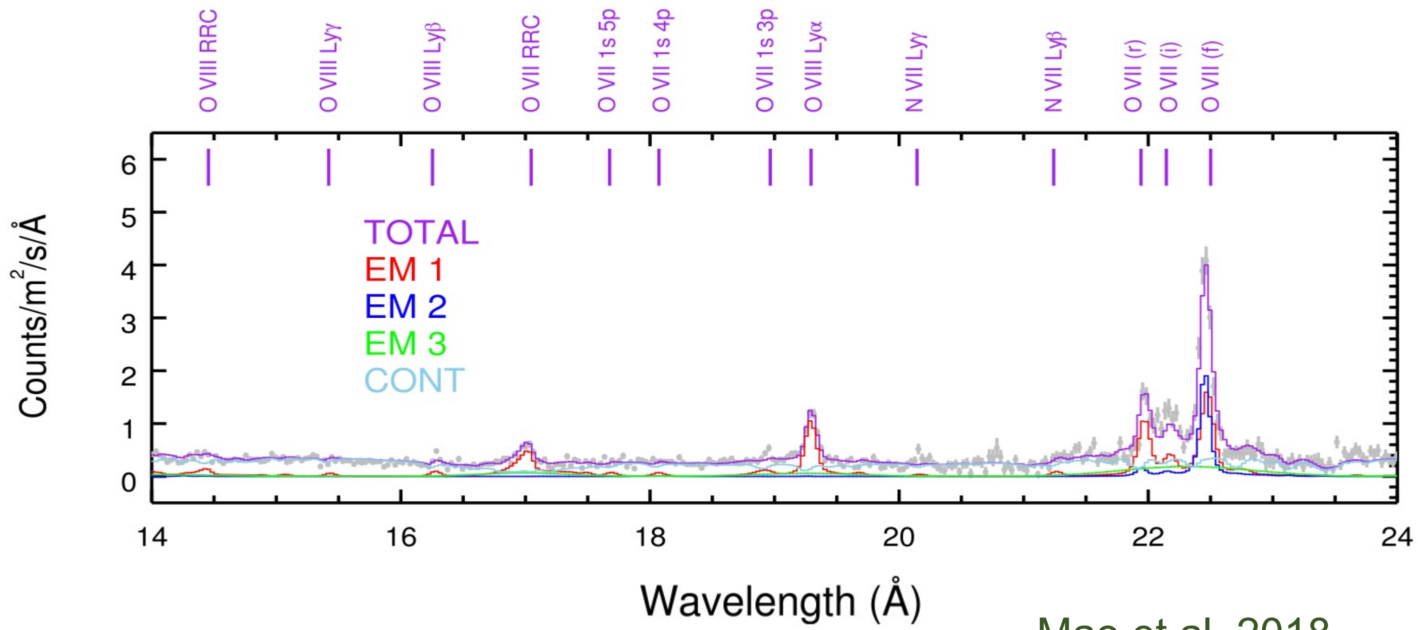
Proto-solar abundance: the overall composition of the molecular cloud within the ISM from which the solar system formed 4.567 Gyr ago.





Ionization stage

The nomenclature for atoms and ions in the astronomy community is slightly different from that in the atomic physics community. We use the combination of an element symbol (e.g., H, C, O) and a roman numeral (e.g., I, IV, XVII) to indicate the charge/ionization stage of the element.



Mao et al. 2018

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

11	12	14	15	16	19	20	30	40	50
XI	XII	XIV	XV	XVI	XIX	XX	XXX	XL	L

Latex trick (case sensitive!)

O {\sc vii}

For A&A only

\ion{Si}{VII}

Ionization stage (cont.)

1	2	3	4	5
I	II	III	IV	V

6	7	8	9	10
VI	VII	VIII	IX	X

11	12	14	15	16
XI	XII	XIV	XV	XVI

19	20	30	40	50
XIX	XX	XXX	XL	L

Atomic number + 1 – Roman numeral = Number of electrons

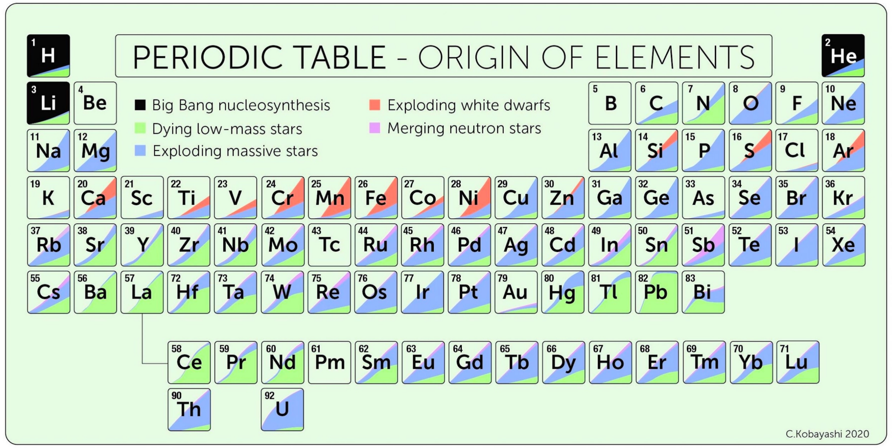


Image Credit: Chiaki Kobayashi et al. Artwork: Sahm Keily

Examples:
Neutral hydrogen ($Z = 1$): H I = H
Isoelectronic sequence: H-like (i.e., one bounded electron)
Ionic charge (used by the atomic physics community): 0

Triply ionized carbon ($Z = 6$): C IV = C^{3+}
Isoelectronic sequence: Li-like (i.e., three bounded electrons)
Ionic charge (used by the atomic physics community): 3

Triply ionized iron ($Z = 26$): Fe XVII = Fe^{16+}
Isoelectronic sequence: Ne-like (i.e., ten bounded electrons)
Ionic charge (used by the atomic physics community): 16

Fully ionized oxygen ($Z = 8$): O IX = O^{8+}
Isoelectronic sequence: N/A (no bounded electrons)
Ionic charge (used by the atomic physics community): 8

Atomic state

Examples:
Neutral hydrogen ($Z = 1$): H I = H
Isoelectronic sequence: H-like (i.e., one bounded electron)
Ionic charge (used by the atomic physics community): 0

Lyman	Balmer	Paschen
Lyα $n = 2 \rightarrow 1$	Hα $n = 3 \rightarrow 2$	Paα $n = 4 \rightarrow 3$
Lyβ $n = 3 \rightarrow 1$	Hβ $n = 4 \rightarrow 2$	Paβ $n = 5 \rightarrow 3$
Lyγ $n = 4 \rightarrow 1$	Hγ $n = 5 \rightarrow 2$	Paγ $n = 6 \rightarrow 3$
…	…	…

Where can this electron stay?

An one-electron atomic state is defined by either one of the following two combinations of quantum numbers

principle quantum number

$$n \; l \; m_l \; m_s$$

$$n \; l \; j \; m_j$$

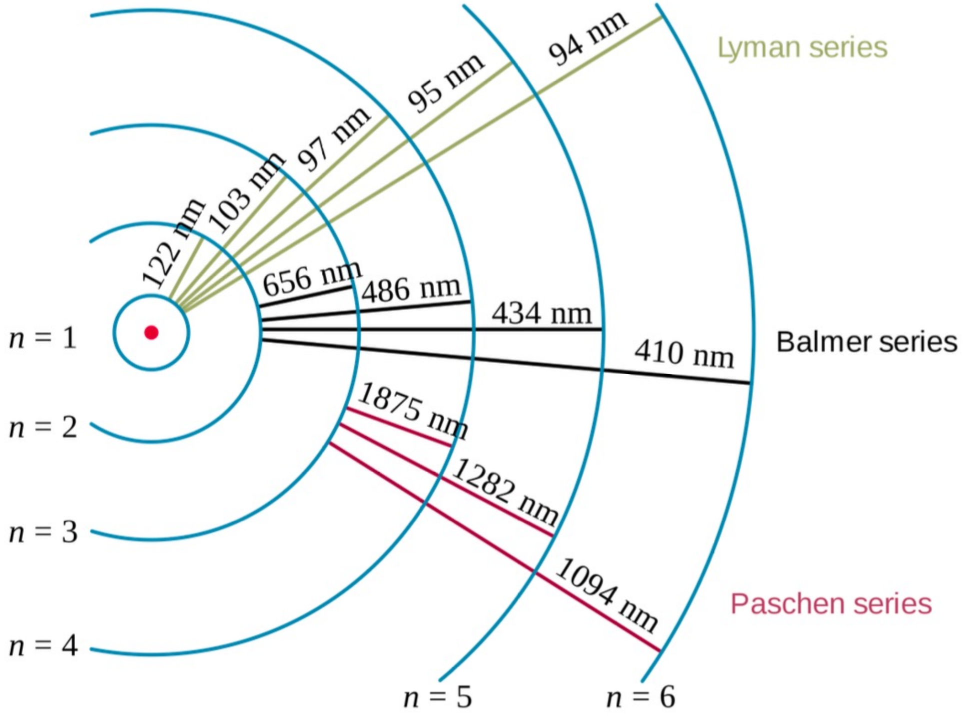


Image credit: wiki

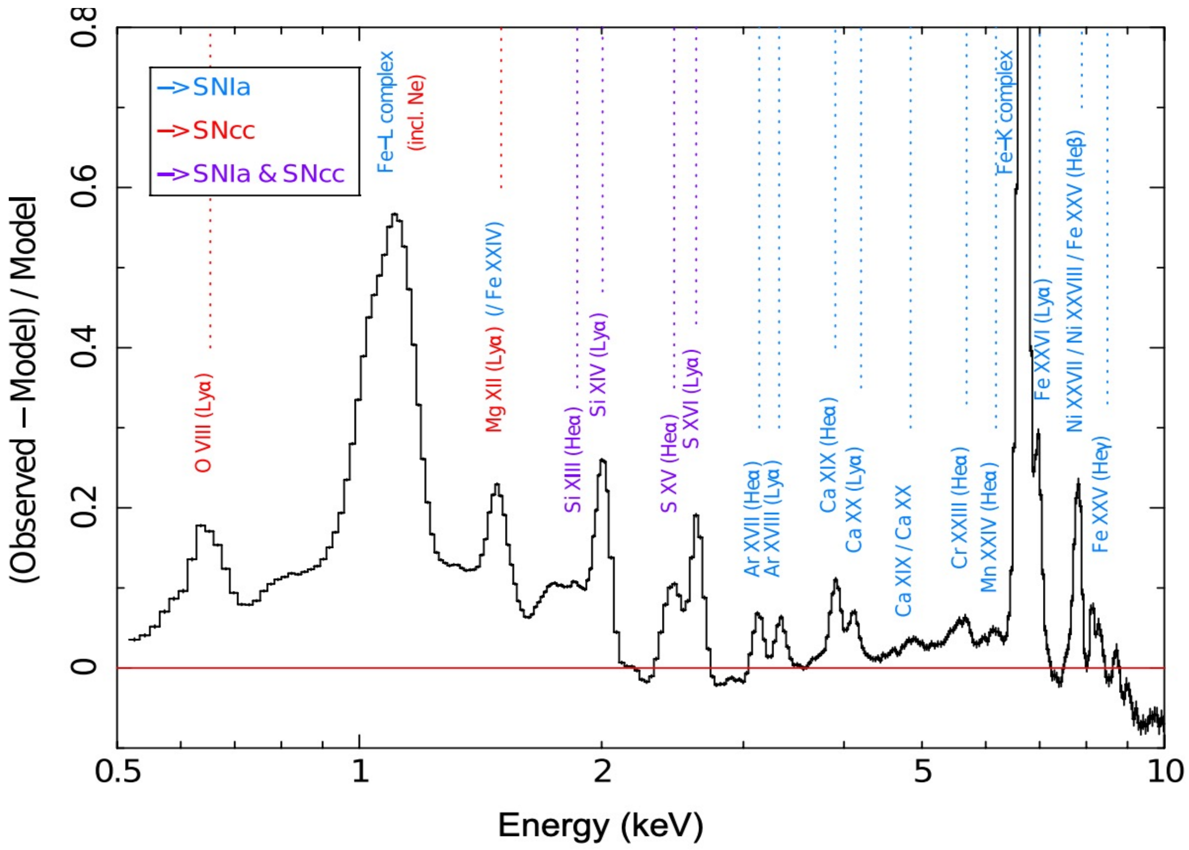
Shell and subshell

principle quantum number $n = 1, \ldots$ $\longrightarrow$ $n\, l$ (subshell) $\leftarrow$ orbital angular momentum quantum number $l = 0, .., n-1$

n	shell
1	K
2	L
3	M
4	N
5	O
6	P
7	Q

l	orbit
0	s
1	p
2	d
3	f
4	g
5	h
6	i
7	k

Mernier et al. 2016a



Caution that Fe K and L lines are not just one line …
(e.g., Kallman et al. 2004, Gu et al. 2019)

Quantum numbers

An one-electron atomic state is defined by either one of the following two combinations of quantum numbers

$$n \; l \; m_l \; m_s$$

$$n \; l \; j \; m_j$$

principle quantum number
$n = 1, \ldots$

orbital angular momentum quantum number
$l = 0, 1, \ldots, n-1$

spin quantum number
$s = 1/2$

orbital magnetic quantum number
$m_l = 0, 1, \ldots, \pm(l-1), \pm l$

spin magnetic quantum number
$m_s = \pm s$

total angular momentum quantum number
$j = l-s, l-s+1, \ldots, l+s-1, l+s$

total angular momentum magnetic quantum number
$m_j = -j, -j+1, \ldots, j-1, j$

Shell and subshell degeneracy

principle quantum number
$n = 1, \ldots$

subshell

$\longrightarrow n\, l$

orbital angular momentum quantum number
$l = 0, \ldots, n-1$

Pauli's exclusion principle

no two electrons in the same atom can have identical values for all of the four quantum numbers $n\ l\ m_l\ m_s$ or
$n\ l\ j\ m_j$.

subshell	max. # of e^-
1s	2
2s	2
2p	6
3s	2
3p	6
3d	10
…	…

The maximum number of electrons in each shell is

$$\sum_{l=0}^{n-1}(2s+1)(2l+1) = \sum_{j=|l-s|}^{|l+s|} 2j+1$$

$$= \sum_{l=0}^{n-1} 2(2l+1) = 2n^2$$

n	shell	max. # of e^-
1	K	2
2	L	8
3	M	18
4	N	32
5	O	50
…	…	…

Electron configurations

principle quantum number $n = 1, \ldots$

conf.

$$nl^x$$

orbital angular momentum quantum number $l = 0, \ldots, n-1$

number of electrons in this subshell $x = 1\ (\text{usually omitted}), 2, \ldots$

closed shell (can be omitted)

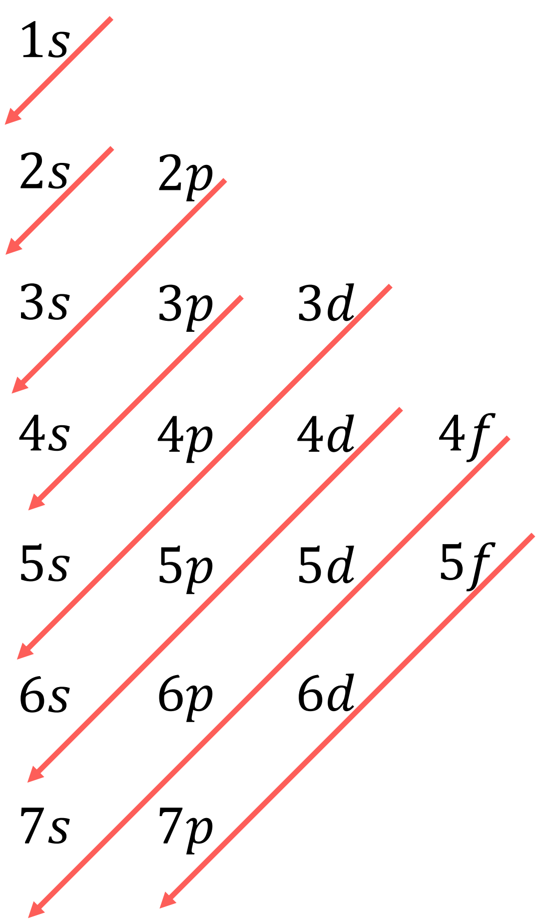
Fe atom: $[1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2]\ 3d^6$

Boron atom: $2p$

open shell

B-like Fe XXII: $2p$

Order of subshell filling



n	shell	max. # of e^-
1	K	2
2	L	8
3	M	18
4	N	32
5	O	50
…	…	…

Coupling scheme

For elements with $Z \leq 30$, the spin-orbit coupling is weak. In this case, the orbital (spin) angular momenta of individual electrons form a total orbital (spin) angular momentum $L = \sum_i l_i$ ($S = \sum_i s_i$). This is the LS (Russell-Saunders) coupling.

$2S+1$	multiplicity
1	singlet
2	doublet
3	triplet
4	quartet
5	quintet
6	sextet
7	septet
8	octet

L	orbit
0	S
1	P
2	D
3	F
4	G
5	H
6	I
7	K

term

$$\boxed{{}^{\boxed{2S+1}}L}$$

multiplicity of the term

Conf.	Term
ns	${}^2S$
ns^2	${}^1S$
np, np^5	${}^2P$
np^2, np^4	${}^1S$, ${}^1D$, ${}^3P$
np^3	${}^2P$, ${}^2D$
np^6	${}^1S$
nd, nd^9	${}^2D$
nd^2, nd^8	${}^1S$, ${}^1D$, ${}^1G$, ${}^3P$, ${}^3F$

For elements with $Z > 30$, jj coupling is needed.

Spectroscopic notation

To describe an energy level, electron configurations, spectroscopic notations (LSJ), and level energies are required.

conf.

$$nl^x$$

term

level

$$^{2S+1}L_J$$

total angular quantum number

$$J = L - S, \dots, L + S$$

Examples of H-like ions:

1s $^2S_{1/2}$: $n = 1, S = 1/2, L = 0, J = 1/2$

2p $^2P_{1/2}$: $n = 2, S = 1/2, L = 1, J = 1/2$

2p $^2P_{3/2}$: $n = 2, S = 1/2, L = 1, J = 3/2$

Examples of He-like ions:

1s^2 $^1S_0$: $n = 1, S = 0, L = 0, J = 0$

1s 2s $^1S_0$: $n = 2, S = 0, L = 0, J = 0$

1s 2s $^3S_1$: $n = 2, S = 1, L = 0, J = 1$

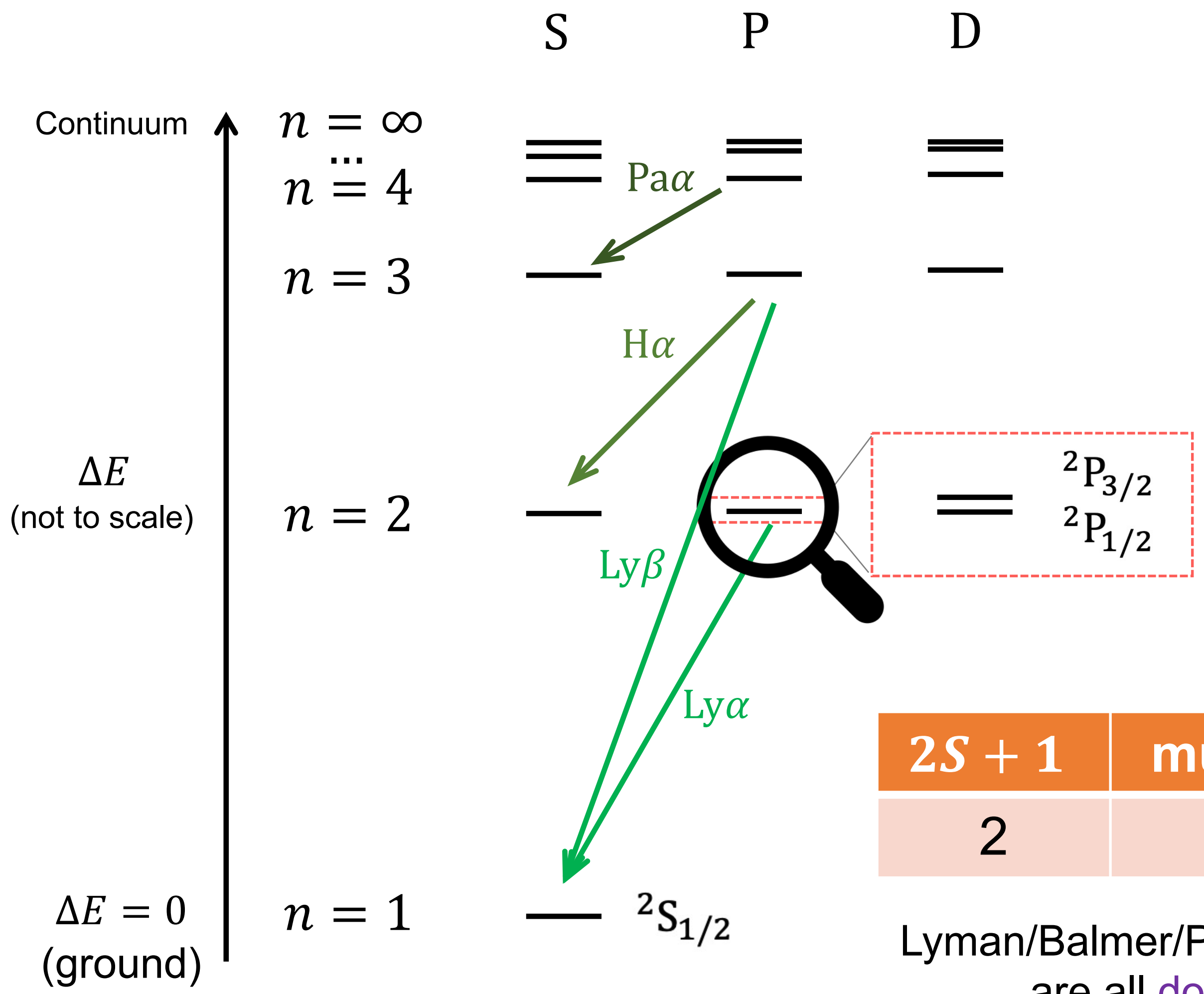
1s 2p $^1P_1$: $n = 2, S = 0, L = 1, J = 1$

1s 2p $^3P_0$: $n = 2, S = 1, L = 1, J = 0$

1s 2p $^3P_1$: $n = 2, S = 1, L = 1, J = 1$

1s 2p $^3P_2$: $n = 2, S = 1, L = 1, J = 2$

Energy levels (H-like)



Grotrian diagram for H-like ions

conf. nl^x

term, level: $^{2S+1}L_J$

multiplicity of the term

$2S+1$	multiplicity
2	doublet

Lyman/Balmer/Paschen lines are all doublets

Examples of H-like ions:
1s $^2S_{1/2}$: $n = 1, S = 1/2, L = 0, J = 1/2$
2p $^2P_{1/2}$: $n = 2, S = 1/2, L = 1, J = 1/2$
2p $^2P_{3/2}$: $n = 2, S = 1/2, L = 1, J = 3/2$

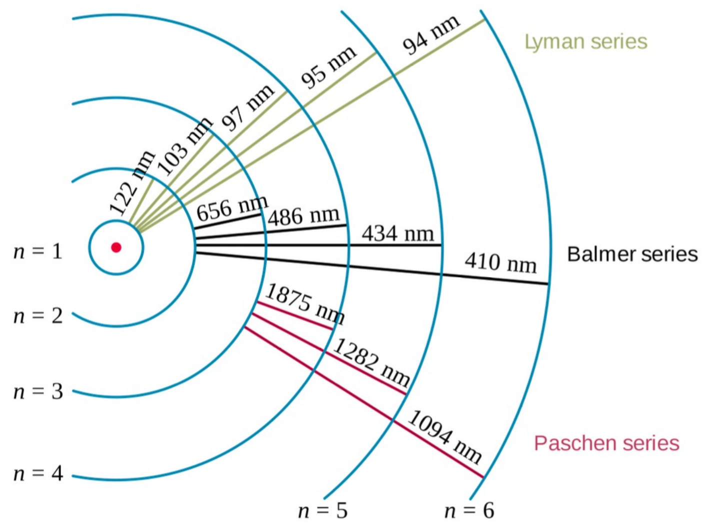
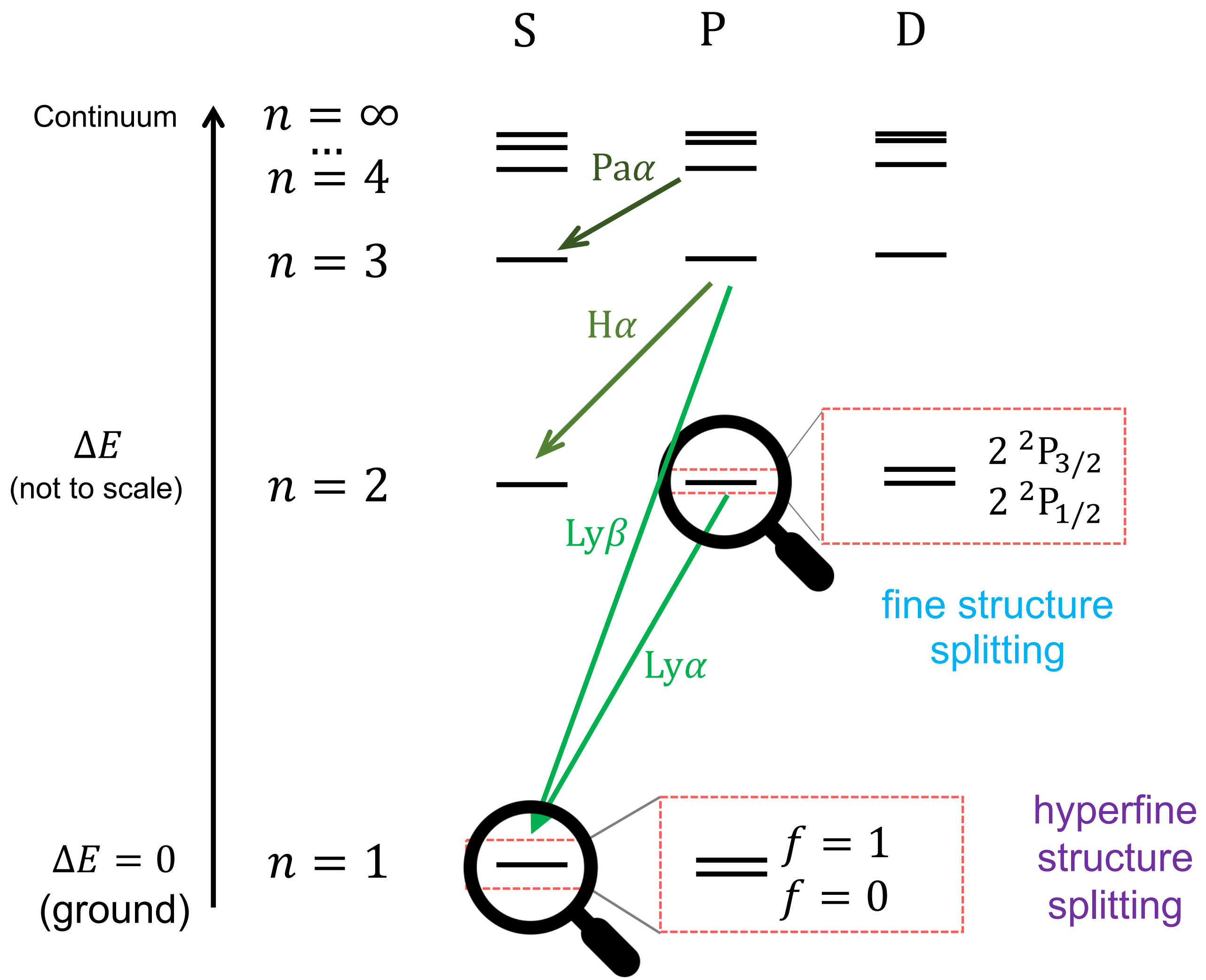


Image credit: wiki

Energy level (splitting)



Grotrian diagram for H-like ions

For H I,

- the $2\mathrm{p}\ ^2\mathrm{P}$ level energy is 10.2 eV
- the $2\mathrm{p}\ ^2\mathrm{P}_{3/2}$ and $2\mathrm{p}\ ^2\mathrm{P}_{1/2}$ level energies differ by $\sim 4.5\times10^{-5}$ eV due to fine structure splitting
- the $1s\ ^2\mathrm{S}_{1/2}$ level can be further split into two levels ($f = 0, 1$) with the level energies differ by $\sim 5.9\times10^{-6}$ eV (i.e., 1420 MHz = 21 cm) due to hyper fine structure splitting

nuclear spin quantum number (for H I)

$i = 1/2$

total spin quantum number

$f = |j - i|, \dots, |j + i|$

Energy levels (He-like)

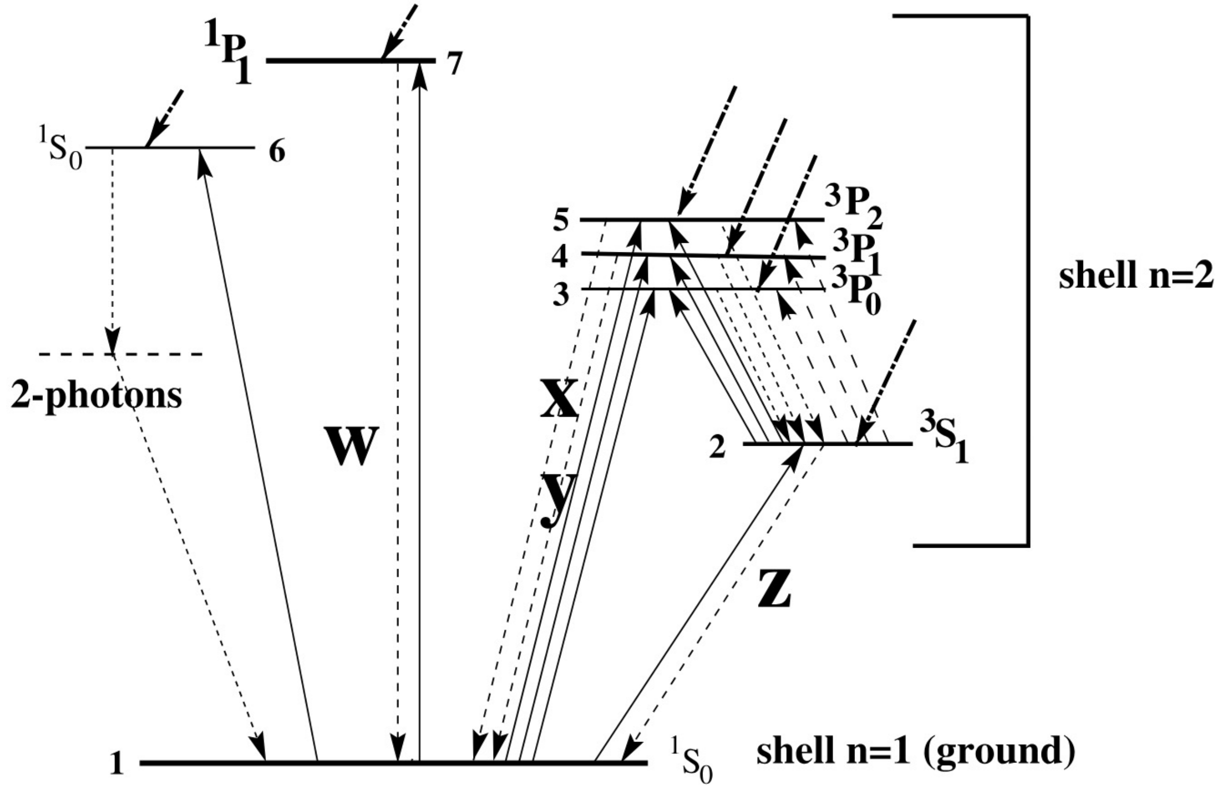
$2S+1$	multiplicity
1	singlet

$2S+1$	multiplicity
3	triplet

term level

conf. nl^x $\quad {}^{2S+1}L_J$

multiplicity of the term



Grotrian diagram for He-like ions (Porquet et al. 2010)

Examples of He-like ions:

$1s^2\ {}^1S_0$: $n=1, S=0, L=0, J=0$

$1s\,2s\ {}^1S_0$: $n=2, S=0, L=0, J=0$

$1s\,2s\ {}^3S_1$: $n=2, S=1, L=0, J=1$

$1s\,2p\ {}^1P_1$: $n=2, S=0, L=1, J=1$

$1s\,2p\ {}^3P_0$: $n=2, S=1, L=1, J=0$

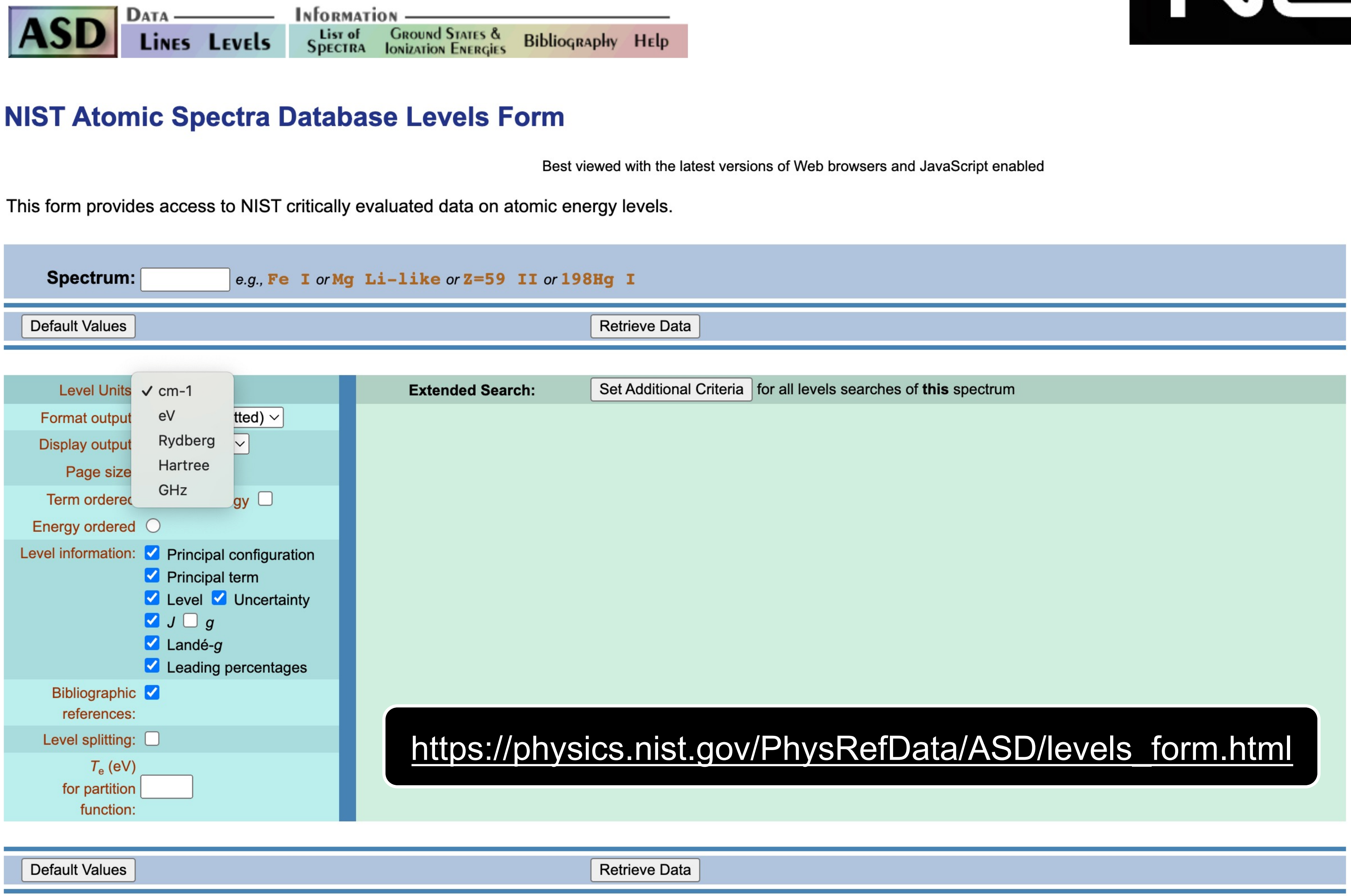
$1s\,2p\ {}^3P_1$: $n=2, S=1, L=1, J=1$

$1s\,2p\ {}^3P_2$: $n=2, S=1, L=1, J=2$

e.g., C V, N VI, O VII, Si XIII, Fe XXV

NIST energy levels

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
U.S. DEPARTMENT OF COMMERCE



ASD | Data: Lines Levels | Information: List of Spectra, Ground States & Ionization Energies, Bibliography, Help

NIST Atomic Spectra Database Levels Form

Best viewed with the latest versions of Web browsers and JavaScript enabled

This form provides access to NIST critically evaluated data on atomic energy levels.

Spectrum: e.g., Fe I or Mg Li-like or Z=59 II or 198Hg I

Default Values | Retrieve Data

Level Units: ✓ cm-1, eV, Rydberg, Hartree, GHz

Format output: ...tted)

Display output

Page size

Term ordered ...gy

Energy ordered

Level information: Principal configuration; Principal term; Level; Uncertainty; *J*; *g*; Landé-*g*; Leading percentages

Bibliographic references:

Level splitting:

T_e (eV) for partition function:

Extended Search: Set Additional Criteria for all levels searches of **this** spectrum

Default Values | Retrieve Data

https://physics.nist.gov/PhysRefData/ASD/levels_form.html

Wavenumber (reciprocal centimeter)

$$E = h\nu = h\frac{c}{\lambda}$$

$$\frac{E}{\mathrm{eV}} = 8065.544\ \mathrm{cm}^{-1}$$

Rydberg

$$1\ \mathrm{Ry} = 13.60569\ \mathrm{eV}$$

$$1\ \mathrm{Ry} = 1.097373\times 10^{5}\ \mathrm{cm}^{-1}$$

Hartree

$$1\ \mathrm{Ha} = 2\ \mathrm{Ry}$$

NIST energy levels for H I

ASD | Data: Lines, Levels | Information: List of Spectra, Ground States & Ionization Energies, Bibliography, Help

NIST Atomic Spectra Database Levels Data

H I   106 Levels Found

***Z* = 1, H isoelectronic sequence**

NIST Energy Levels and Wavelengths Bibliographic Reference # 15291

A Critical Compilation of Experimental Data on Spectral Lines and Energy Levels of Hydrogen, Deuterium, and Tritium,
A. E. Kramida,
At. Data Nucl. Data Tables **96**, 586–644 (2010); Erratum: **126**, 295–298 (2019)
DOI:10.1016/j.adt.2010.05.001

Configuration	Term	*J*	Level (eV)	Uncertainty (eV)	Leading percentages	Reference
1*s*	^2S	$^1/_2$	0.00000000000000	0.00000000000012	*100*	
2*p*	$^2P°$	$^1/_2$	*10.19880615024*	*0.00000000004*	*100*	L15291
		$^3/_2$	*10.19885151459*	*0.00000000005*	*100*	L15291
2*s*	^2S	$^1/_2$	10.19881052514816	0.00000000000012	*100*	L15291

skipping many rows

Configuration	Term	*J*	Level (eV)	Uncertainty (eV)	Leading percentages	Reference
	Limit	---	(13.598434599702)	0.000000000012		HDEL

NIST energy levels for other ions

ASD | Data: Lines, Levels | Information: List of Spectra, Ground States & Ionization Energies, Bibliography, Help

NIST Atomic Spectra Database Levels Data

O VII 87 Levels Found

***Z* = 8, He isoelectronic sequence**

Ion	iso. seq.	# of levels
C IV	Li-like	91
N III	B-like	347
Fe XXII	B-like	61
O III	C-like	188
Ar XIII	C-like	44
O II	N-like	287
Si VII	O-like	65
S VIII	F-like	54

Configuration	Term	*J*	Level (eV)	Uncertainty (eV)	Reference
$1s^2$	^1S	0	0.0000		L11554
$1s2s$	^3S	1	560.98386		
$1s2p$	$^3P^o$	0	568.54442		
		1	568.55186		
		2	568.62005		
$1s2s$	^1S	0	568.88662		
$1s2p$	$^1P^o$	1	573.94778		
$1s3s$	^3S	1	661.92932		
$1s3p$	$^3P^o$	2	664.01845		
		1	664.01845		
		0	664.01845		
$1s3s$	^1S	0	664.11144		
$1s3d$	^3D	1	665.09711		
		2	665.10455		
		3	665.10579		

NIST energy levels for other ions (cont.)

Ion	iso. seq.	# of levels
Fe XVII	Ne-like	67
Ni XIX	Ne-like	73
P V	Na-like	68
Si II	Al-like	151
Fe VII	Ca-like	210
Fe II	Mn-like	1028
La V	I-like	37
Eu VII	La-like	2

ASD | Data: Lines, Levels | Information: List of Spectra, Ground States & Ionization Energies, Bibliography, Help

NIST Atomic Spectra Database Levels Data

Si II 151 Levels Found

***Z* = 14, Al isoelectronic sequence**

Configuration	Term	*J*	Level (eV)	Uncertainty (eV)	Leading percentages				Reference
$3s^2 3p$	$^2P^\circ$	$1/2$	0.000000		94	3	$3p^3$	$^2P^\circ$	L5815
		$3/2$	0.035613		94	3	$3p^3$	$^2P^\circ$	
$3s3p^2$	^4P	$1/2$	5.309535						
		$3/2$	5.322966						
		$5/2$	5.344700						
$3s3p^2$	^2D	$3/2$	6.857485		66	30	$3s^2 3d$	^2D	
		$5/2$	6.859448		66	30	$3s^2 3d$	^2D	

Lanthanide series ($Z = 57 - 71$)

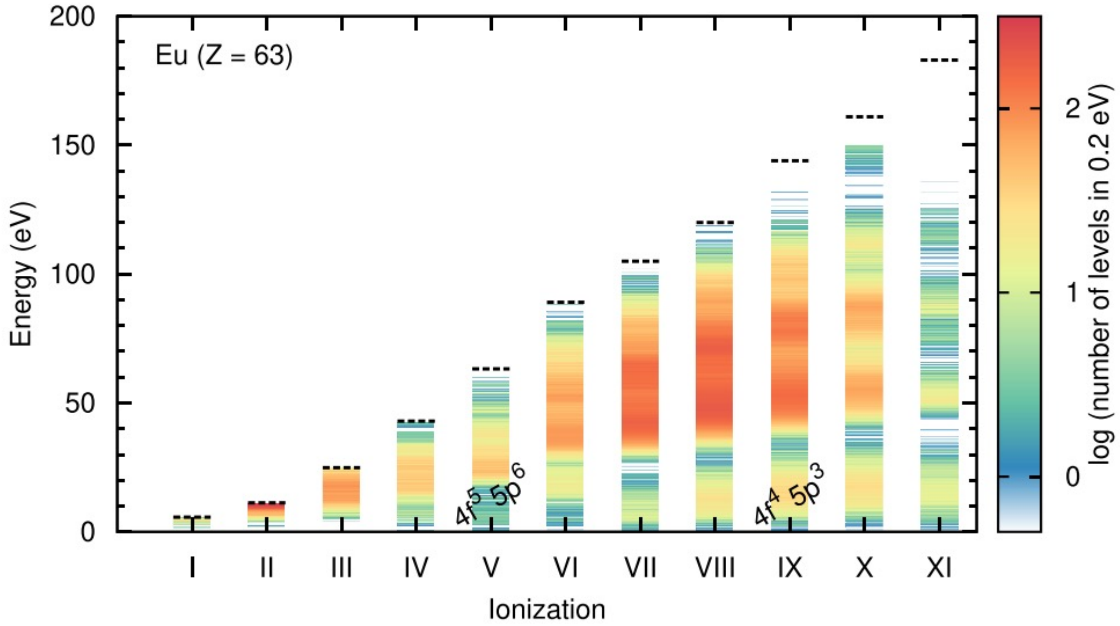
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

NIST not for all

Eu VII 2 Levels Found

***Z* = 63, La isoelectronic sequence**

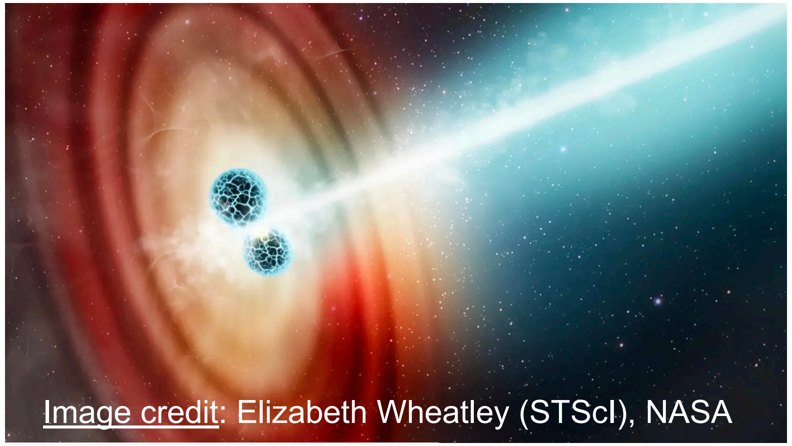
Configuration	Term	*J*	Level (eV)	Uncertainty (eV)	Reference
$4f^5 5s^2 5p^4$	°		0.0000		L582
Eu VIII ($4f^5 5s^2 5p^3$)	Limit	---	(105)	5	L582

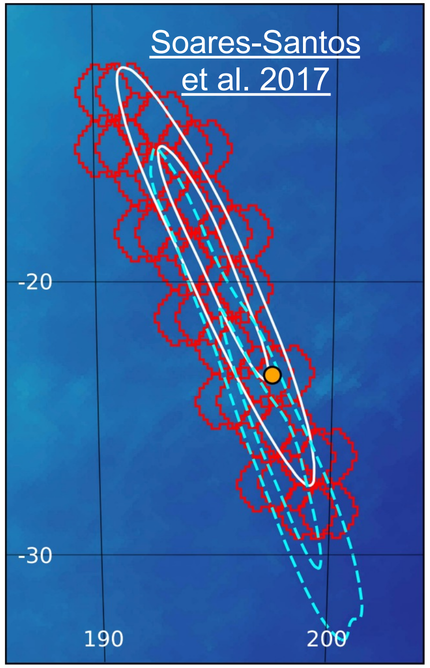


Ion	Configurations	N_{level}	N_{line}
Eu I	$\mathbf{4f}^7 6s^2$, $4f^7 5d6s$, $4f^7 6s6p$, $4f^6 5d6s^2$, $4f^7 5d6p$, $4f^7 6s7s$, $4f^6 5d^2 6s$, $4f^7 5d^2$, $4f^7 6s7p$, $4f^7 6s6d$, $4f^7 6s8s$, $4f^7 6s5f$, $4f^7 6s8p$, $4f^7 6s7d$, $4f^7 6p^2$	103229	741430825
Eu II	$\mathbf{4f}^7 6s$, $4f^7 5d$, $4f^7 6p$, $4f^6 5d6s$, $4f^6 5d^2$	22973	21396542
Eu III	$\mathbf{4f}^7$, $4f^6 5d$, $4f^6 6s$, $4f^6 6p$	5323	2073702
Eu IV	$\mathbf{4f}^6$, $4f^5 5d$, $4f^5 6s$, $4f^5 6p$	3737	1045697
Eu V	$\mathbf{4f}^5 5p^6$, $4f^6 5p^5$, $4f^4 5p^6 6s$, $4f^4 5p^6 6p$, $4f^4 5p^6 5d$, $4f^4 5p^6 7s$	3897	1140035
Eu VI	$\mathbf{4f}^4 5p^6$, $4f^5 5p^5$, $4f^4 5p^5 6s$, $4f^4 5p^5 6p$, $4f^4 5p^5 5d$, $4f^4 5p^5 7s$	13065	12823350
Eu VII	$\mathbf{4f}^4 5p^5$, $4f^3 5p^6$, $4f^4 5p^4 6s$, $4f^4 5p^4 6p$, $4f^4 5p^4 5d$, $4f^4 5p^4 7s$	29465	60643899
Eu VIII	$\mathbf{4f}^4 5p^4$, $4f^3 5p^5$, $4f^4 5p^3 6s$, $4f^4 5p^3 6p$, $4f^4 5p^3 5d$, $4f^4 5p^3 7s$	40241	113753012
Eu IX	$\mathbf{4f}^4 5p^3$, $4f^3 5p^4$, $4f^4 5p^2 6s$, $4f^4 5p^2 6p$, $4f^4 5p^2 5d$, $4f^4 5p^2 7s$	31393	73355941
Eu X	$\mathbf{4f}^4 5p^2$, $4f^3 5p^3$, $4f^3 5p^2 6s$, $4f^3 5p6s^2$, $4f^4 5p6s$, $4f^4 5p6p$, $4f^4 5p5d$, $4f^4 5p7s$	15515	18807502

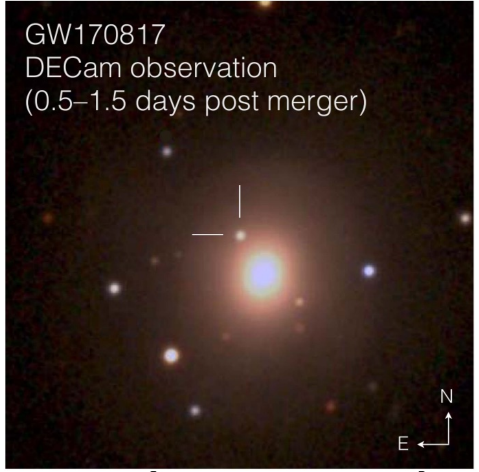
Smaranika et al. 2022

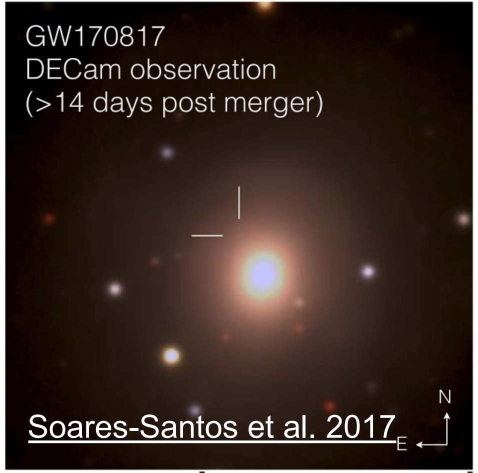
GW170817 kilonova



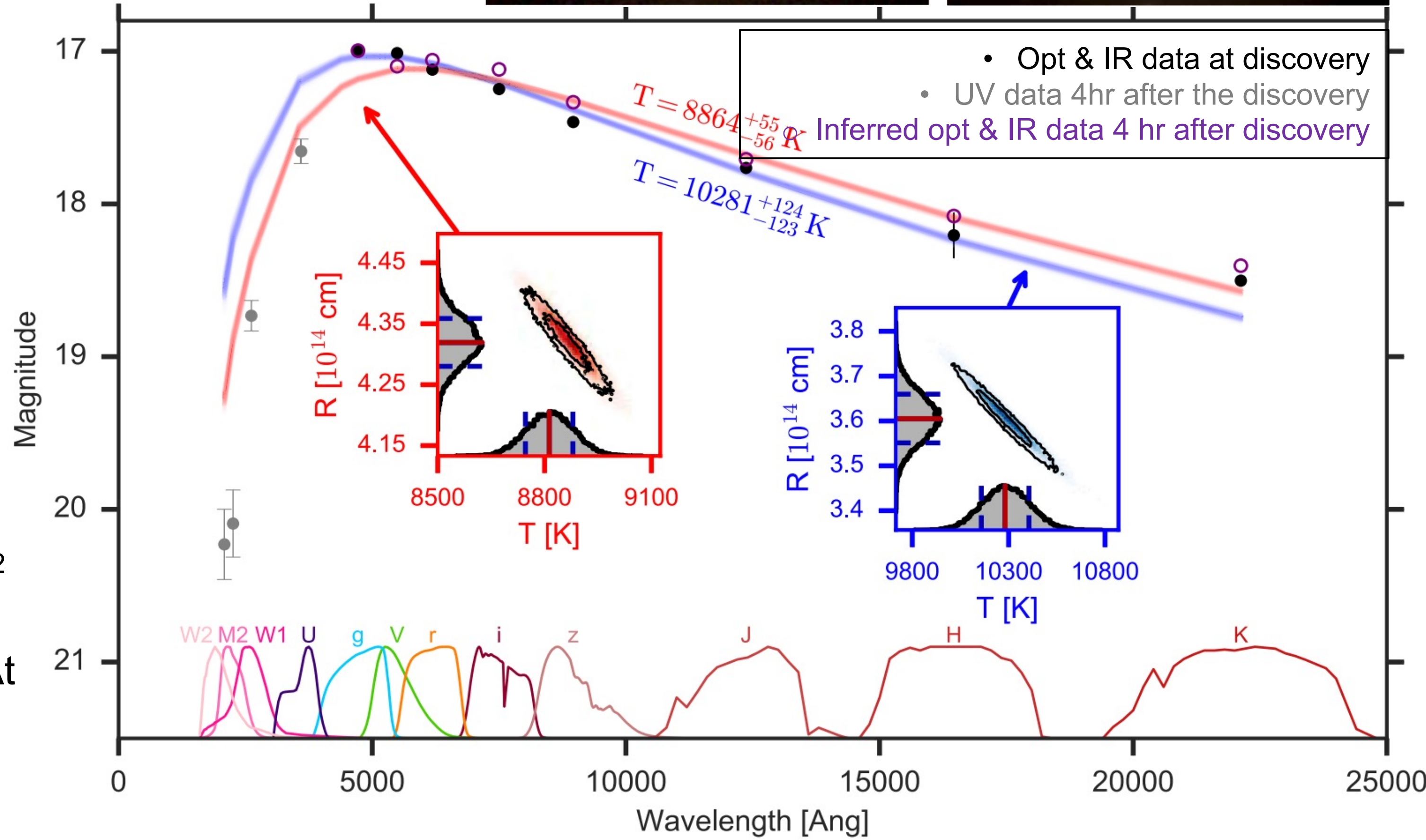


10.5 hr after the localization of GW 170817, Dark Energy Cam era covered 70 deg^2 (i and z bands) to search for optical counterparts. At 11.4 hr post-merger, a kilonova was found.





Arcavi (2018)



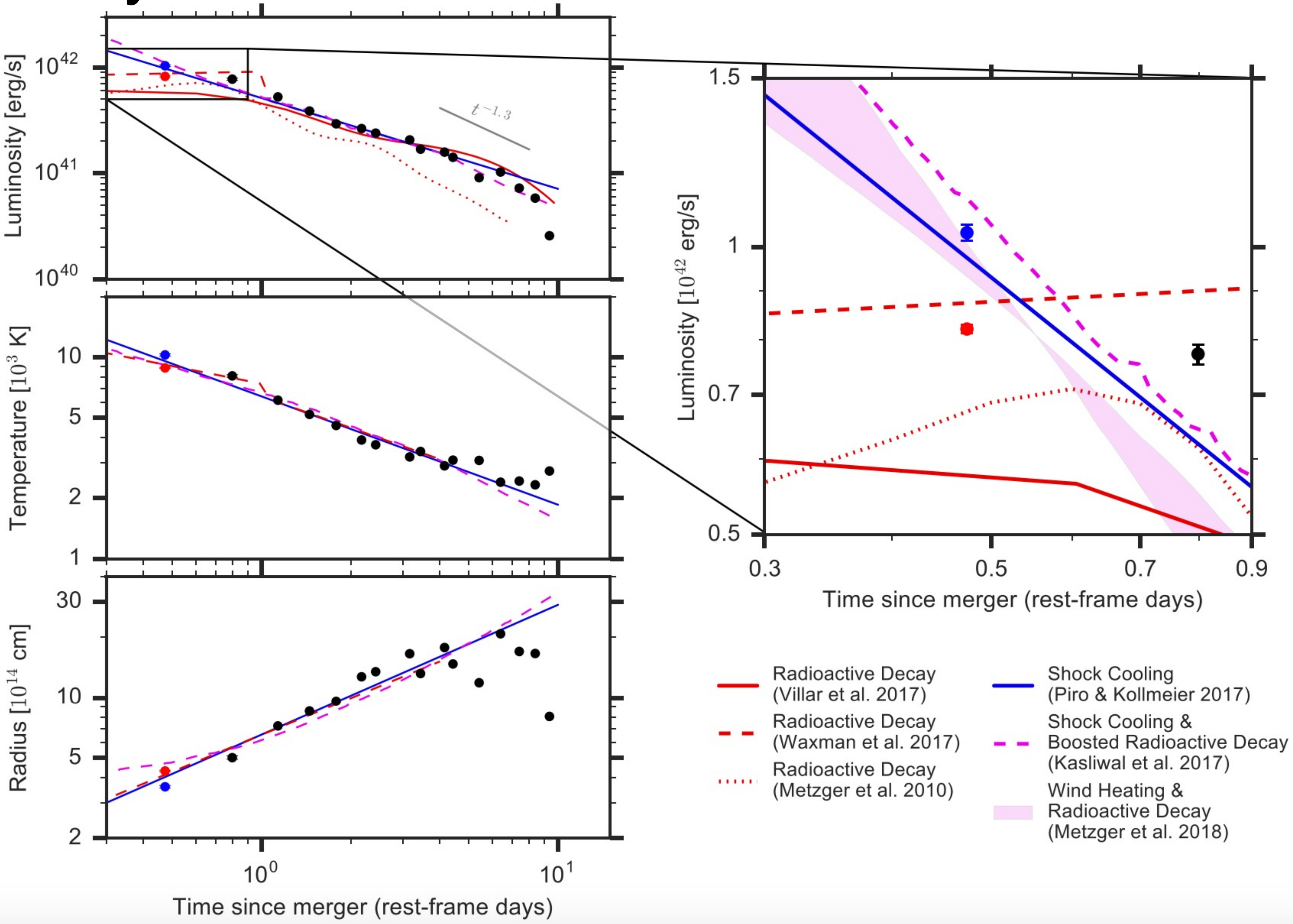
GW071807 kilonova early emission

While the late time ($t > 1$ d) lightcurve can be well explained with the kilonova, the origin and evolution of the early emission has been puzzling.

- ❖ Radiative decay in low-opacity ejecta
- ❖ Relativistic boosting of radiative decay in high-velocity ejecta
- ❖ Shock cooling
- ❖ Wind heating
- ❖ Some or all above

The opacity of the lanthanides is one of the bottlenecks.

Arcavi (2018)



NIST lines

https://physics.nist.gov/PhysRefData/ASD/lines_form.html

ASD | Data: Lines, Levels | Information: List of Spectra, Ground States & Ionization Energies, Bibliography, Help

NIST National Institute of Standards and Technology Physical Meas. Laboratory

NIST Atomic Spectra Database Lines Form

Best viewed with the latest versions of Web browsers and JavaScript enabled

Main Parameters **Spectrum** [ ] e.g., Fe I or Na;Mg; Al or mg i-iii or 198Hg I

Limits for Wavelengths **Lower:** [ ]
Upper: [ ]

Wavelength Units: nm

Reset input | Retrieve Data | Show Graphical Options | Show Advanced Settings

Can you please provide some feedback to improve our database?

Observed Wavelength Air (Å)	Ritz Wavelength Air (Å)	Rel. Int. (?)	A_{ki} (s^{-1})	Acc.	E_i (cm^{-1})		E_k (cm^{-1})	Lower Level Conf., Term, J			Upper Level Conf., Term, J			Type	TP Ref.	Line Ref.
O VI	1 031.912		4.16e+08	A	0.0	-	96 907.5	$1s^22s$	^2S	$^1/_2$	$1s^22p$	$^2P°$	$^3/_2$		T5118LS	
	1 037.613		4.09e+08	A	0.0	-	96 375.0	$1s^22s$	^2S	$^1/_2$	$1s^22p$	$^2P°$	$^1/_2$		T5118LS	
	1 080.880		1.53e+09	A	863 333.8	-	955 851	$1s^24p$	$^2P°$	$^1/_2$	$1s^25d$	^2D	$^3/_2$		T5118LS	
	1 081.522		1.83e+09	A	863 397.7	-	955 860	$1s^24p$	$^2P°$	$^3/_2$	$1s^25d$	^2D	$^5/_2$		T5118LS	
	1 081.627		3.05e+08	A	863 397.7	-	955 851	$1s^24p$	$^2P°$	$^3/_2$	$1s^25d$	^2D	$^3/_2$		T5118LS	
	1 126.351		3.16e+06	B	867 077.7	-	955 860	$1s^24f$	$^2F°$	$^5/_2$	$1s^25d$	^2D	$^5/_2$		T3127LS	
	1 126.465		6.63e+07	B	867 077.7	-	955 851	$1s^24f$	$^2F°$	$^5/_2$	$1s^25d$	^2D	$^3/_2$		T3127LS	
	1 126.469		6.31e+07	B	867 087.0	-	955 860	$1s^24f$	$^2F°$	$^7/_2$	$1s^25d$	^2D	$^5/_2$		T3127LS	
	1 171.561		4.92e+08	A	863 333.8	-	948 690	$1s^24p$	$^2P°$	$^1/_2$	$1s^25s$	^2S	$^1/_2$		T5118LS	
	1 172.439		9.82e+08	A	863 397.7	-	948 690	$1s^24p$	$^2P°$	$^3/_2$	$1s^25s$	^2S	$^1/_2$		T5118LS	

O VI: 126 Lines of Data Found

***Z* = 8, Li isoelectronic sequence**

Ion	iso. seq.	# of lines
C IV	Li-like	228
N III	B-like	1378
Fe XXII	B-like	104
O III	C-like	1005
Ar XIII	C-like	71
O II	N-like	1630
Si VII	O-like	233
S VIII	F-like	254

NIST lines + pulications

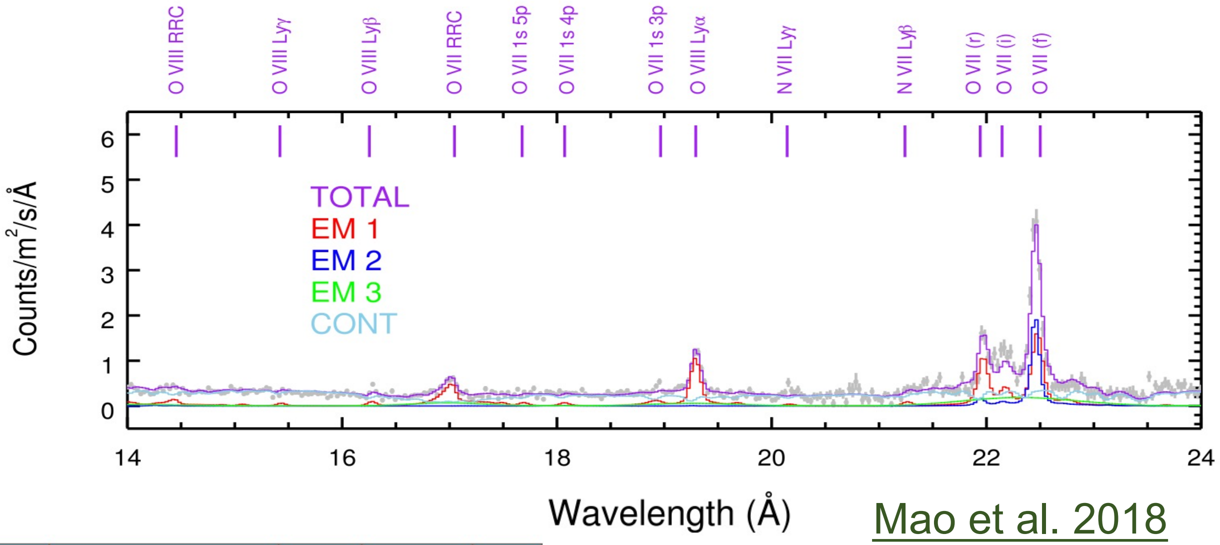
ASD Data: Lines | Levels — Information: List of Spectra | Ground States & Ionization Energies | Bibliography | Help

NIST Atomic Spectra Database Lines Data

O VII: 189 Lines of Data Found

***Z* = 8, He isoelectronic sequence**

Observed Wavelength Vac (Å)	Ritz Wavelength Vac (Å)	Rel. Int. (?)	A_{ki} (s^{-1})	Acc.	E_i (cm^{-1})		E_k (cm^{-1})	Lower Level Conf., Term, J			Upper Level Conf., Term, J			Type	TP Ref.	Line Ref.
	17.8052		1.84e+02	C	0	-	5 616 340.0	$1s^2$	^1S	0	$1s4s$	^3S	1	M1	T4905	
	18.6270		9.365e+11	AA	0	-	5 368 550.0	$1s^2$	^1S	0	$1s3p$	$^1P°$	1		T5997	
	18.6377		1.451e+08	AA	0	-	5 365 470.0	$1s^2$	^1S	0	$1s3d$	^1D	2	E2	T3908	
	18.7307		3.57e+02	C	0	-	5 338 820.0	$1s^2$	^1S	0	$1s3s$	^3S	1	M1	T4905	
	21.6020		3.309e+12	AA	0	-	4 629 201.0	$1s^2$	^1S	0	$1s2p$	$^1P°$	1		T5997	
	21.8044		3.31e+05	A	0	-	4 586 230.0	$1s^2$	^1S	0	$1s2p$	$^3P°$	2	M2	T1494	
	21.8070				0	-	4 585 680.0	$1s^2$	^1S	0	$1s2p$	$^3P°$	1			
	22.1012		1.04e+03	AA	0	-	4 524 640.0	$1s^2$	^1S	0	$1s2s$	^3S	1	M1	T1896	
	78.8619		4.61e+09	A	4 585 620.0	-	5 853 660.0	$1s2p$	$^3P°$	0	$1s7d$	^3D	1		T5231LS	
	78.8656		6.22e+09	A	4 585 680.0	-	5 853 660.0	$1s2p$	$^3P°$	1	$1s7d$	^3D	2		T5231LS	



Mao et al. 2018

H- and He-like C to Zn
(Mao et al. 2022c)

Ion	ID	λ (Å)
O VIII	$Ly\alpha_{1/2}$	18.973
O VIII	$Ly\alpha_{3/2}$	18.967
O VIII	$Ly\beta_{1/2}$	16.007
O VIII	$Ly\beta_{3/2}$	16.006
O VIII	$Ly\gamma_{1/2}$	15.176
O VIII	$Ly\gamma_{3/2}$	15.176
O VIII	$Ly\delta_{1/2}$	14.821
O VIII	$Ly\delta_{3/2}$	14.820

AtomDB/CHIANTI lines

❖ AtomDB (esp. for the X-ray community):
http://www.atomdb.org/Webguide/webguide.php

ATOMDB ATOMIC DATA FOR ASTROPHYSICISTS

❖ CHIANTI (esp. for the solar community)
https://db.chiantidatabase.org/

CHIANTI

An Atomic Database for Spectroscopic Diagnostics of Astrophysical Plasmas.

H I

λ (Å)	gf	A (s^{-1})	Transition
1215.674	2.780e-01	6.260e+08	1s 2S1/2 - 2p 2P1/2
1215.668	5.550e-01	6.260e+08	1s 2S1/2 - 2p 2P3/2
6564.564	2.720e-02	2.100e+06	2p 2P1/2 - 3s 2S1/2
6564.722	5.440e-02	4.210e+06	2p 2P3/2 - 3s 2S1/2

O VIII

Energy level 1	
Electron configuration	1s1
Energy above ground (eV)	0.0
Quantum state	n=1, L=0, S=0.5, degeneracy=2, parity=0
Energy Level Data Source	NIST ASD 5.3
Photoionization Data Source	from XSTAR

Energy level 3	
Electron configuration	2p1
Energy above ground (eV)	653.49365
Quantum state	n=2, L=1, S=0.5, degeneracy=2, parity=1
Energy Level Data Source	NIST ASD 5.3
Photoionization Data Source	from XSTAR

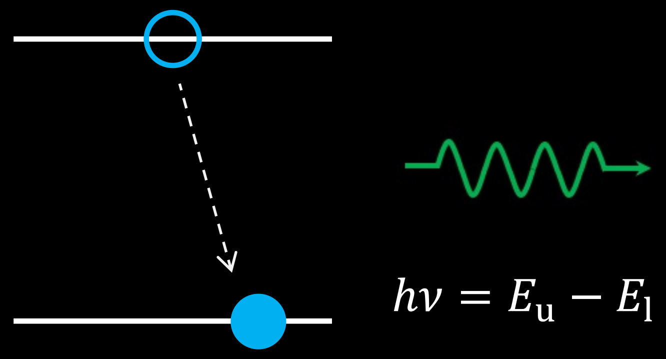
Level 1 → 3 Interactions	
Electron collision rate	Nonzero (for 1 → 3)
Electron collision reference	2003JPhB...36.3707B
Wavelength (theory)*	18.972525Å
Wavelength (lab/observed)†	18.972517Å
Transition rate/Einstein A	2.57e+12 s^{-1}
Transition type*	E1
Oscillator Strength $f_{3 \to 1}$	1.385085e-01
Wavelength (theory) reference	NIST ASD 5.3 RITZ
Wavelength (lab/observed) reference	1977JPCRD...6..831E
Transition rate reference	FAC

Chpt.5 Atomic processes

5.1 Atomic data for astrophysics

5.2 Two-level system
5.2.1 Einstein coefficients
5.2.2 Thermodynamic equilibrium
5.2.3 Einstein relations
5.2.4 Emission and absorption coefficients
5.2.5 Collisional (de-)excitation
5.2.6 Emission line regime
5.2.7 Non-thermal emission

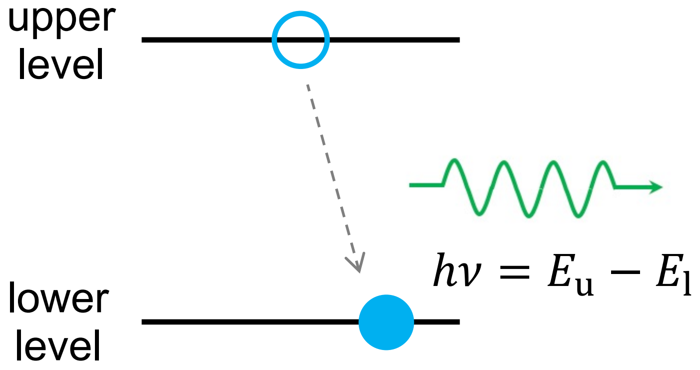
5.3 Bremsstrahlung
5.4 Recombination and photoionization
5.5 Collisional excitation
5.6 Other atomic processes
5.7 Astrophysical plasma models



$h\nu = E_{\mathrm{u}} - E_{\mathrm{l}}$

Einstein A coefficient

Two-level system



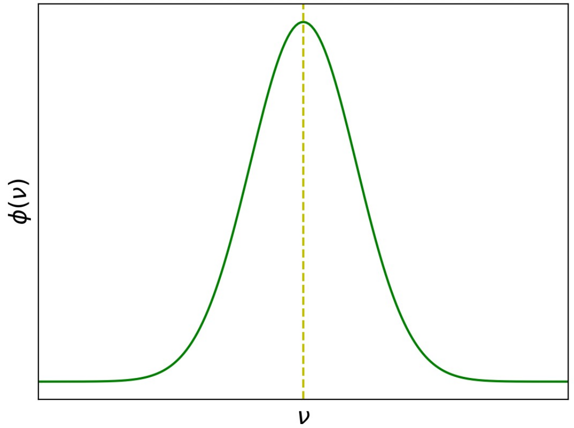
transition from an upper energy level to a lower one → emission line
transition from a lower energy level to an upper one → absorption line

$$\frac{dn_{\mathrm{u}}}{dt} = -n_{\mathrm{u}} A_{\mathrm{ul}}$$

Spontaneous emission probability
(a. k. a. Einstein A coefficient, in s^{-1})

the larger the A-value, the stronger the transition

Einstein B coefficients



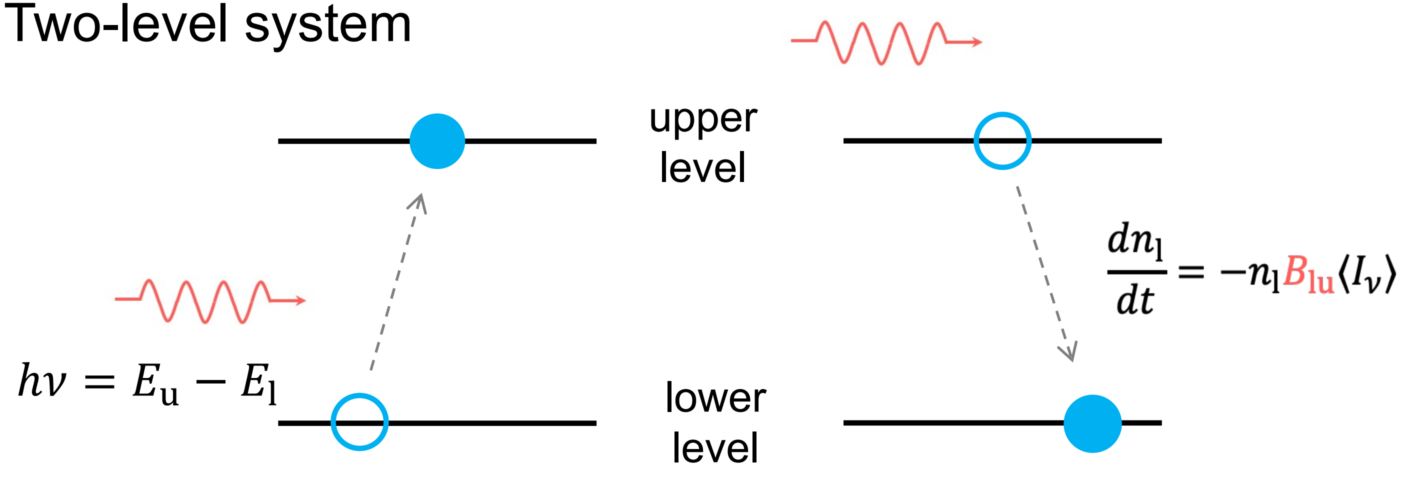
$$\langle I_\nu \rangle = \int \frac{d\Omega}{4\pi} \int I_\nu \phi(\nu) d\nu$$

normalization to the 4π solid angle (isotropic emission)

line profile (e.g., natural broadening due to the Heisenberg uncertainty principle, $\Delta E \Delta t \geq \frac{h}{4\pi}$)

$$1 = \int_0^\infty \phi(\nu) d\nu$$

Two-level system



$$\frac{dn_\mathrm{l}}{dt} = -n_\mathrm{l} B_\mathrm{lu} \langle I_\nu \rangle$$

Stimulated absorption coefficient

$$\frac{dn_\mathrm{l}}{dt} = n_\mathrm{u} B_\mathrm{ul} \langle I_\nu \rangle$$

Stimulated emission coefficient

Einstein B coefficients $(\mathrm{erg}^{-1}\ \mathrm{cm}^2)$

Thermodynamic equilibrium

TE is also known as detailed balance, where all atomic processes are balanced by their thermodynamically inverse processes.

- ✓ radiation field is isotropic
- ✓ free electrons follow the Maxwellian distribution
- ✓ level populations follow Maxwell-Boltzmann equation

Accordingly, the radiation temperature equals the electron temperature.

Photons emitted — Photons absorbed

$$n_\mathrm{u}(A_\mathrm{ul} + B_\mathrm{ul}\langle I_\nu\rangle) = n_\mathrm{l} B_\mathrm{lu}\langle I_\nu\rangle$$

n_u ↑ upper level population

n_l ↑ lower level population

Boltzmann equation for level populations (applies to LTE as well)

$$\frac{n_\mathrm{l}}{n_\mathrm{u}} = \frac{g_\mathrm{l}}{g_\mathrm{u}} \exp\left(\frac{E_\mathrm{u} - E_\mathrm{l}}{kT}\right)$$

- $g_{\mathrm{l/u}}$ are statistical weights (i.e., degeneracy)
- $E_{\mathrm{l/u}}$ are level energies

$$\langle I_\nu\rangle = \frac{n_\mathrm{u} A_\mathrm{ul}}{n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul}} = \frac{A_\mathrm{ul}/B_\mathrm{ul}}{\frac{n_\mathrm{l} B_\mathrm{lu}}{n_\mathrm{u} B_\mathrm{ul}} - 1}$$

$$= \frac{A_\mathrm{ul}/B_\mathrm{ul}}{\frac{g_\mathrm{l} B_\mathrm{lu}}{g_\mathrm{u} B_\mathrm{ul}} \exp\left(\frac{h\nu}{kT}\right) - 1}$$

Einstein relations

prev. sl.

$$\langle I_\nu \rangle = \frac{A_{\mathrm{ul}}/B_{\mathrm{ul}}}{\frac{g_{\mathrm{l}} B_{\mathrm{lu}}}{g_{\mathrm{u}} B_{\mathrm{ul}}} \exp\left(\frac{h\nu}{kT}\right) - 1}$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

$$g_{\mathrm{l}} B_{\mathrm{lu}} = g_{\mathrm{u}} B_{\mathrm{ul}} \qquad A_{\mathrm{ul}} = \frac{2h\nu^3}{c^2} B_{\mathrm{ul}}$$

- ✓ connect properties of the atom (must holds even out of TE)
- ✓ manifestation of the detailed balance relations (TE)

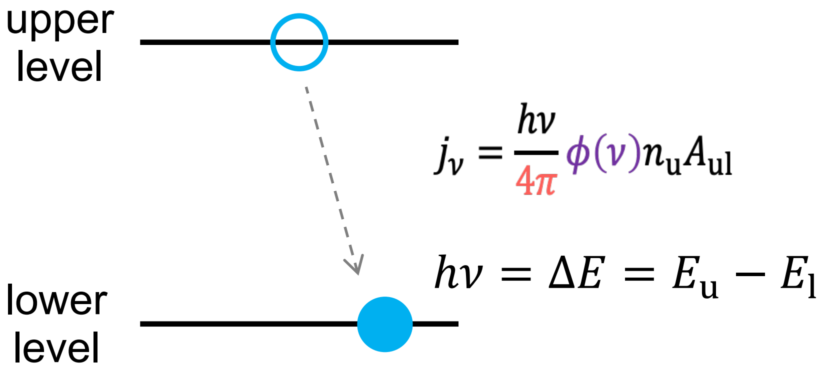
$$\langle I_\nu \rangle = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

Spontaneous emission coefficient

$$dE_\nu = \frac{h\nu}{4\pi}\phi(\nu)\, n_\mathrm{u} A_\mathrm{ul}\, dV\, d\nu\, d\Omega\, dt$$

normalization to the 4π solid angle (isotropic emission)

line profile (e.g., natural broadening due to the Heisenberg uncertainty principle, $\Delta E \Delta t \geq \frac{h}{4\pi}$)

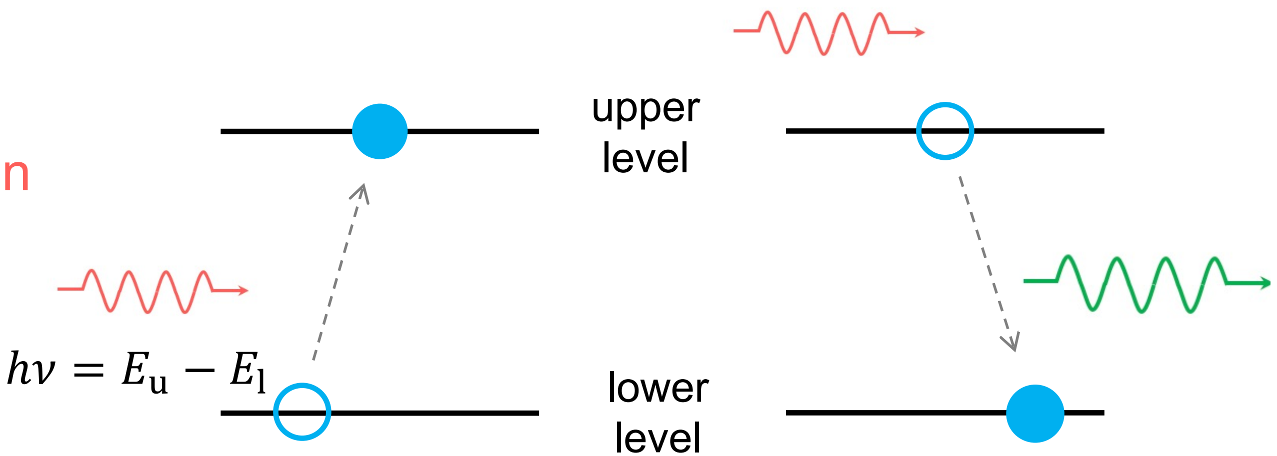


$$j_\nu = \frac{h\nu}{4\pi}\phi(\nu) n_\mathrm{u} A_\mathrm{ul}$$

Sect. 2.2.1

$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

Absorption coefficient



The total energy absorbed via stimulated absorption in unit time and unit volume is

$$\delta E_\mathrm{SA} = -h\nu\, n_\mathrm{l}\, B_\mathrm{lu} \langle I_\nu \rangle\, dV\, dt$$

Treating the stimulated emission as negative absorption, total energy absorbed in unit time and unit volume is

$$\delta E_\mathrm{SE} = h\nu\, n_\mathrm{u}\, B_\mathrm{ul} \langle I_\nu \rangle\, dV\, dt$$

Combining the above two

$$\delta E = -\frac{h\nu}{4\pi} (n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul})\, I_\nu\, \phi(\nu)\, d\nu\, d\Omega\, dV\, dt$$

$$dI_\nu = -\frac{h\nu}{4\pi} (n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul})\, I_\nu\, \phi(\nu)\, ds$$

$$\alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu) (n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul})$$

$dV = dA\, ds$

prev. sl.

$$\langle I_\nu \rangle = \int \frac{d\Omega}{4\pi} \int I_\nu \phi(\nu) d\nu$$

Sect. 2.1.3

$$dE = I_\nu\, dA_\perp\, d\Omega\, dt\, d\nu$$

Sect. 2.2.2

$$dI_\nu = -\alpha_\nu\, I_\nu\, ds$$

Source function

prev. sl.

$$j_\nu = \frac{h\nu}{4\pi}\phi(\nu) n_\mathrm{u} A_\mathrm{ul}$$

prev. sl.

$$\alpha_\nu = \frac{h\nu}{4\pi}\phi(\nu)(n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul})$$

prev. sl.

$$g_\mathrm{l} B_\mathrm{lu} = g_\mathrm{u} B_\mathrm{ul}$$

$$A_\mathrm{ul} = \frac{2h\nu^3}{c^2} B_\mathrm{ul}$$

Source function

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{n_\mathrm{u} A_\mathrm{ul}}{n_\mathrm{l} B_\mathrm{lu} - n_\mathrm{u} B_\mathrm{ul}} = \frac{2h\nu^3}{c^2} \frac{1}{\frac{g_\mathrm{u} n_\mathrm{l}}{g_\mathrm{l} n_\mathrm{u}} - 1}$$

With the Boltzmann equation for level population (caution: not always the case)

prev. sl.

$$\frac{n_\mathrm{l}}{n_\mathrm{u}} = \frac{g_\mathrm{l}}{g_\mathrm{u}} \exp\left(\frac{E_\mathrm{u} - E_\mathrm{l}}{kT}\right)$$

Kirchoff's law

$$j_\nu = \alpha_\nu B_\nu(T)$$

$$S_\nu = B_\nu(T)$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

Collisional (de-)excitation

collisional excitation

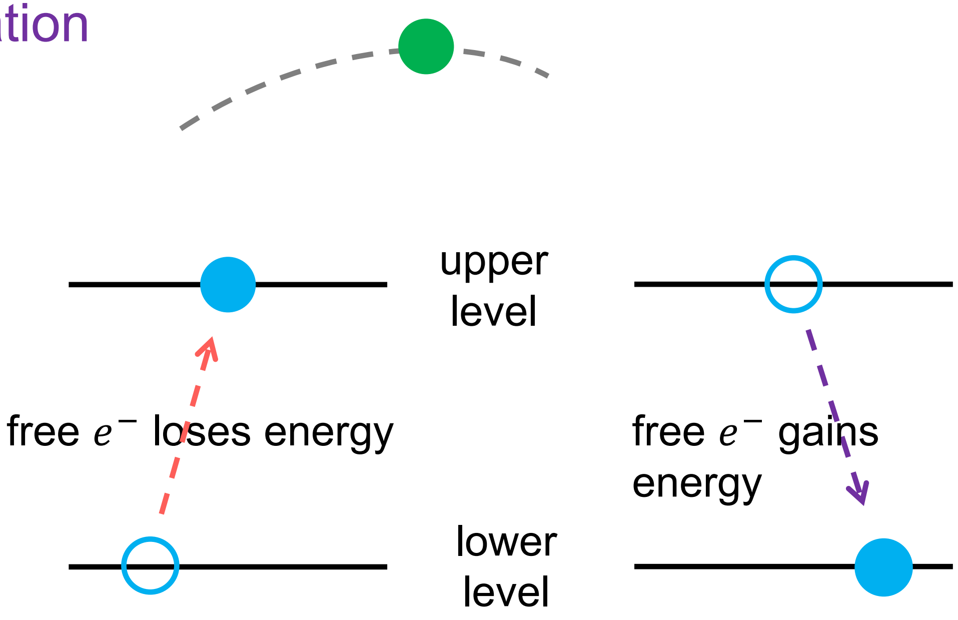
$$n_\mathrm{u}(A_\mathrm{ul} + B_\mathrm{ul}\langle I_\nu\rangle + n_e C_\mathrm{ul}) = n_\mathrm{l}(B_\mathrm{lu}\langle I_\nu\rangle + n_e C_\mathrm{lu})$$

electron number density collisional de-excitation

prev. sl. (under TE)

$$n_\mathrm{u}(A_\mathrm{ul} + B_\mathrm{ul}\langle I_\nu\rangle) = n_\mathrm{l} B_\mathrm{lu}\langle I_\nu\rangle$$

$$n_\mathrm{u} C_\mathrm{ul} = n_\mathrm{l} C_\mathrm{lu}$$



$$\left|\frac{1}{2}m_e v_i^2 - \frac{1}{2}m_e v_f^2\right| = E_\mathrm{u} - E_\mathrm{l}$$

Again, this is the manifestation of the detailed balance, where all atomic processes are balanced by their thermodynamically inverse processes.

Collisional (de-)excitation (cont.)

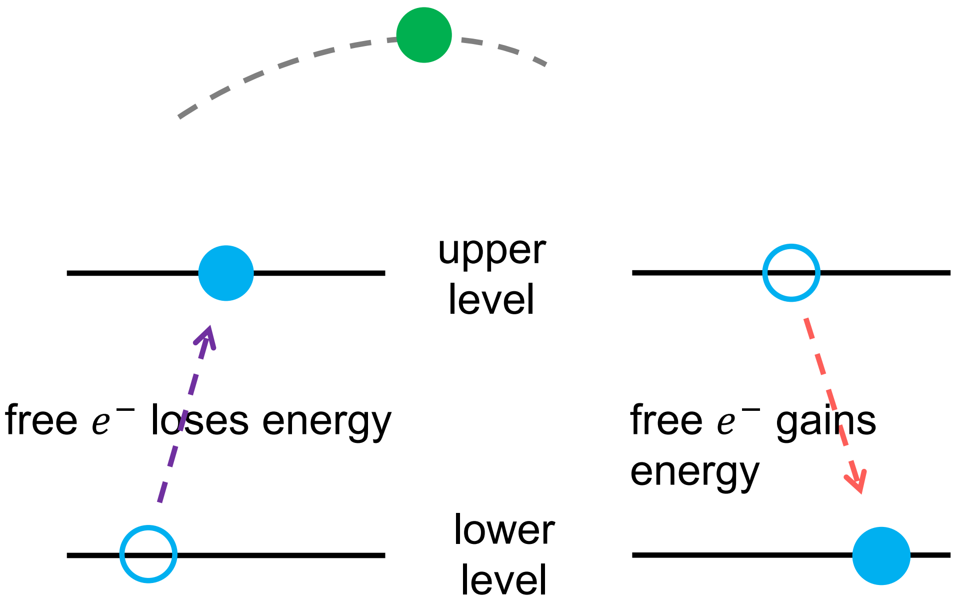
prev. sl. (Boltzmann distribution)

$$\frac{n_\mathrm{l}}{n_\mathrm{u}} = \frac{g_\mathrm{l}}{g_\mathrm{u}} \exp\left(\frac{E_\mathrm{u} - E_\mathrm{l}}{kT}\right)$$

prev. sl. (under TE)

$$n_\mathrm{u} C_\mathrm{ul} = n_\mathrm{l} C_\mathrm{lu}$$

$$\frac{C_\mathrm{ul}}{C_\mathrm{lu}} = \frac{n_\mathrm{l}}{n_\mathrm{u}} = \frac{g_\mathrm{l}}{g_\mathrm{u}} \exp\left(\frac{E_\mathrm{u} - E_\mathrm{l}}{kT}\right)$$



$$\left|\frac{1}{2} m_e v_i^2 - \frac{1}{2} m_e v_f^2\right| = E_\mathrm{u} - E_\mathrm{l}$$

Emission line regime

NGC 2392 is a planetary nebula
(Eskimo nebula)
Image credit: HST

prev. sl.

$$n_\mathrm{u}(A_\mathrm{ul} + B_\mathrm{ul}\langle I_\nu \rangle + n_e C_\mathrm{ul}) = n_\mathrm{l}(B_\mathrm{lu}\langle I_\nu \rangle + n_e C_\mathrm{lu})$$

In the absence of external emission (i.e., $I_\nu = 0$), which is the case for planetary nebulae

$$n_\mathrm{u}(A_\mathrm{ul} + n_e C_\mathrm{ul}) = n_\mathrm{l} n_e C_\mathrm{lu}$$

If $A_\mathrm{ul} \gg n_e C_\mathrm{ul}$, we can ignore the collisional de-excitation term

$$n_\mathrm{u} A_\mathrm{ul} \simeq n_\mathrm{l} n_e C_\mathrm{lu}$$

If $A_\mathrm{ul} \ll n_e C_\mathrm{ul}$, we can ignore the spontaneous emission term

$$n_\mathrm{u} n_e C_\mathrm{ul} \simeq n_\mathrm{l} n_e C_\mathrm{lu}$$

Critical density

$$n_e^\mathrm{crit} = \frac{A_\mathrm{ul}}{C_\mathrm{ul}}$$

High-density emission line regime

prev. sl.

$$n_{\mathrm{u}}(A_{\mathrm{ul}} + n_e C_{\mathrm{ul}}) = n_{\mathrm{l}} n_e C_{\mathrm{lu}}$$

Critical density

$$n_e^{\mathrm{crit}} = \frac{A_{\mathrm{ul}}}{C_{\mathrm{ul}}}$$

Consider a high-density with $n_e \gg n_e^{\mathrm{crit}}$, we can ignore the spontaneous emission term

$$n_{\mathrm{u}} n_e C_{\mathrm{ul}} \simeq n_{\mathrm{l}} n_e C_{\mathrm{lu}}$$

$$\frac{n_{\mathrm{l}}}{n_{\mathrm{u}}} \simeq \frac{C_{\mathrm{ul}}}{C_{\mathrm{lu}}}$$

prev. sl.

$$\frac{C_{\mathrm{ul}}}{C_{\mathrm{lu}}} = \frac{n_{\mathrm{l}}}{n_{\mathrm{u}}} = \frac{g_{\mathrm{l}}}{g_{\mathrm{u}}} \exp\left(\frac{E_{\mathrm{u}} - E_{\mathrm{l}}}{kT}\right)$$

✓ The level population is follows the Boltzmann distribution

Low-density emission line regime

prev. sl.

$$n_{\mathrm{u}}(A_{\mathrm{ul}} + n_e C_{\mathrm{ul}}) = n_{\mathrm{l}} n_e C_{\mathrm{lu}}$$

Critical density

$$n_e^{\mathrm{crit}} = \frac{A_{\mathrm{ul}}}{C_{\mathrm{ul}}}$$

Consider a low-density with $n_e \ll n_e^{\mathrm{crit}}$, we can ignore the collisional de-excitation term

$$n_{\mathrm{u}} A_{\mathrm{ul}} \simeq n_{\mathrm{l}} n_e C_{\mathrm{lu}}$$

$$\frac{n_{\mathrm{u}}}{n_{\mathrm{l}}} \simeq \frac{n_e C_{\mathrm{lu}}}{A_{\mathrm{ul}}}$$

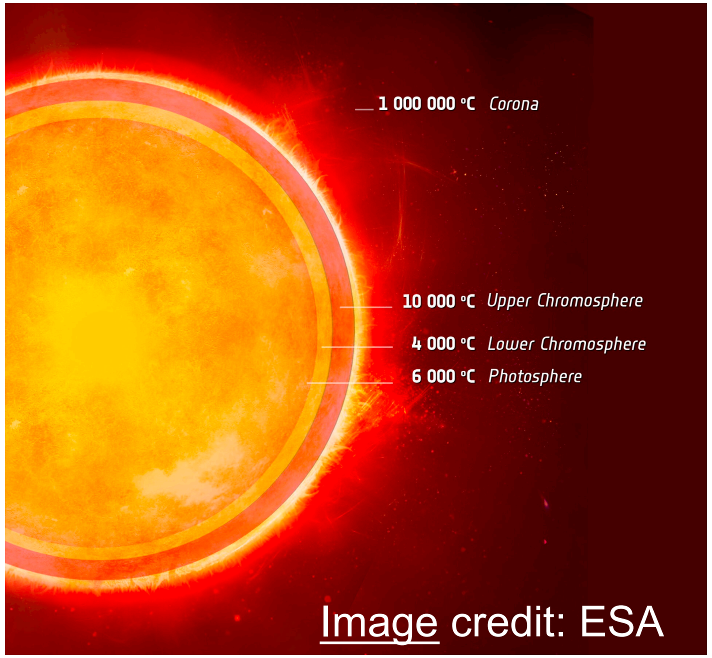


Image credit: ESA

This is also known as the coronal approximation, where the dominant processes are collisional excitation and spontaneous emission.

- ✓ The coronal approximation can be applied to the stellar corona (hence the name)
- ✓ The level population is no longer given by the Boltzmann distribution

Non-thermal emission

Boltzmann equation for level populations (applies to TE & LTE)

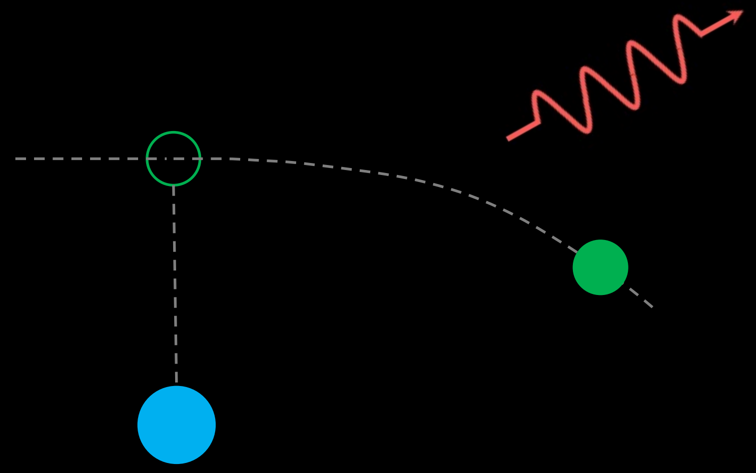
prev. sl.

$$\frac{n_\mathrm{l}}{n_\mathrm{u}} = \frac{g_\mathrm{l}}{g_\mathrm{u}} \exp\left(\frac{E_\mathrm{u} - E_\mathrm{l}}{kT}\right)$$

For non-thermal emission, the level populations do not follow the Boltzmann equation (e.g., astro maser), this might occur if

- ✓ radiating particles do not have a Maxwellian velocity distribution
- ✓ when scattering cannot be neglected

Chpt.5 Atomic processes

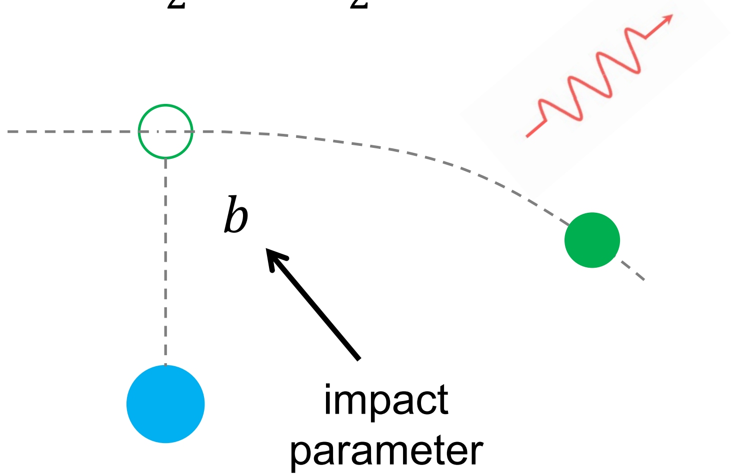


Bremsstrahlung (free-free) processes

$$\Delta E = \frac{1}{2} m_e v^2 - \frac{1}{2} m_e (v - \Delta v)^2 = h\nu$$

Emission due to the deceleration of a charge (e.g., a free electron) in the Coulomb field (no strong external B-field) of another charge with a different charge-to-mass ratio (e.g., an ion)

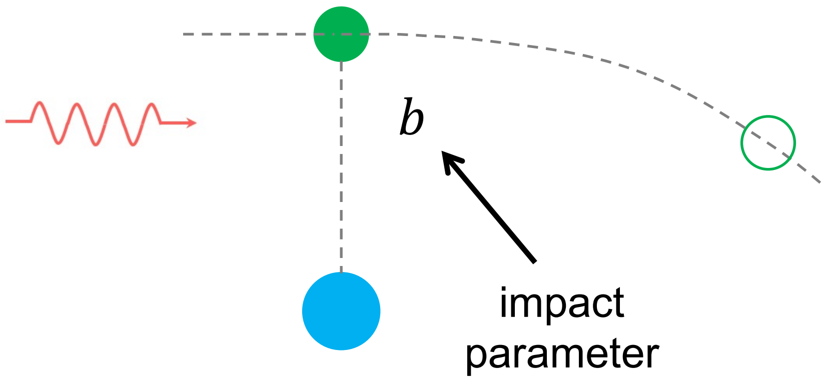
- ✓ a cooling process for the electrons
- ✓ continuum emission spectral features



$$\Delta E = \frac{1}{2} m_e (v + \Delta v)^2 - \frac{1}{2} m_e v^2 = h\nu$$

Absorption due to the acceleration of a charge (e.g., a free electron) in the Coulomb field of another charge with a different charge-to-mass ratio (e.g., an ion)

- ✓ a heating process for the electrons



Free-free kinematics for single-speed electrons

Consider free-free emission from single-speed (non-relativistic) electrons (charge: e) with an ion (charge: Ze) and an impact parameter of b

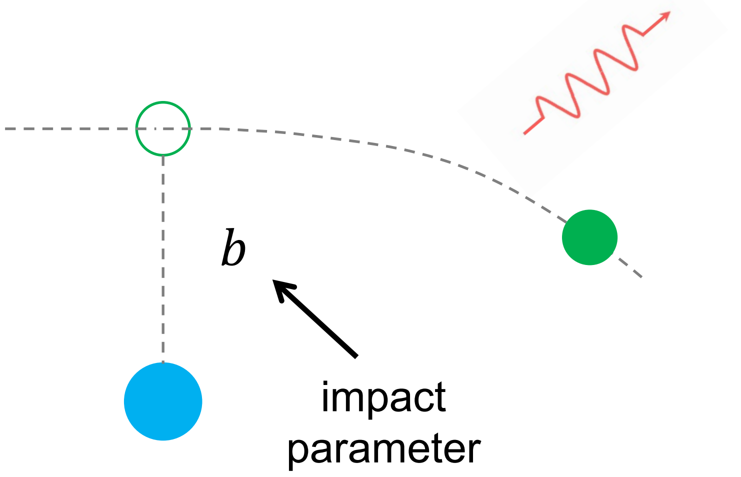
$$\Delta E = \frac{1}{2} m_e v^2 - \frac{1}{2} m_e (v - \Delta v)^2 = h\nu$$

$$\Delta v = \frac{2Ze^2}{b m_e v}$$

$$\tau = \frac{1}{\omega} \simeq \frac{b}{v}$$

collision time

see Sect. 5.1 of the REF book (p156-157) by Rybicki & Lightman



see Sect. 6.1 of 《天体物理中的辐射机制》(p266-270) by 尤峻汉

$$a = \frac{Ze^2}{m_e v^2} \qquad \epsilon = \sqrt{1 + \frac{b^2}{a^2}} \qquad \frac{1}{\tan(\theta_{\rm sca}/2)} = \frac{b}{a}$$

eccentricity

scattering angle of the free electron

For a certain velocity, the smaller the impact parameter, the higher the radiation frequency

Free-free power for single-speed electrons

$$P(v,\omega) = \frac{dE(v,\omega)}{d\omega\, dV\, dt} = \frac{16e^6}{3c^3 m_e^2 v}\, n_e n_i Z^2 \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Coulomb logarithm

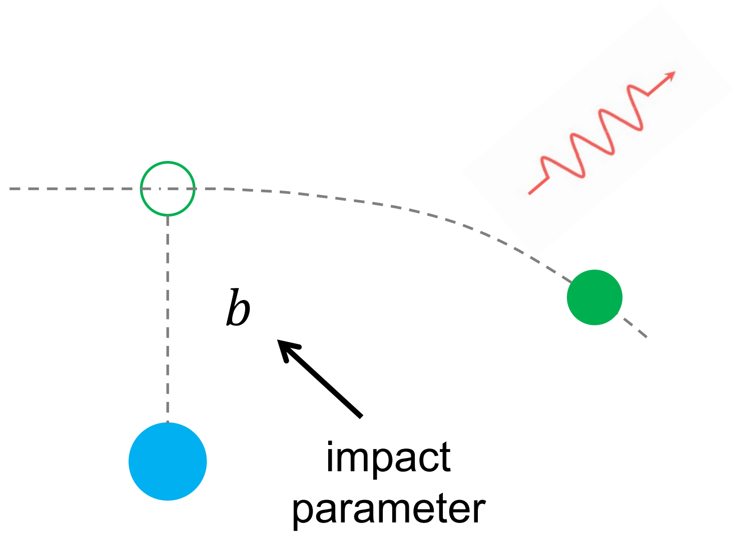
$$b_{\max} = \frac{v}{\omega}$$

$$b_{\min} = \max\left(\frac{h/2\pi}{m_e v}, \frac{2Ze^2}{m_e v^2}\right)$$

see Sect. 5.1 of the REF book (p158-159) by Rybicki & Lightman

Heisenberg uncertainty principle

$$\Delta v = \frac{2Ze^2}{b m_e v} = v$$



Introducing the Gaunt factor

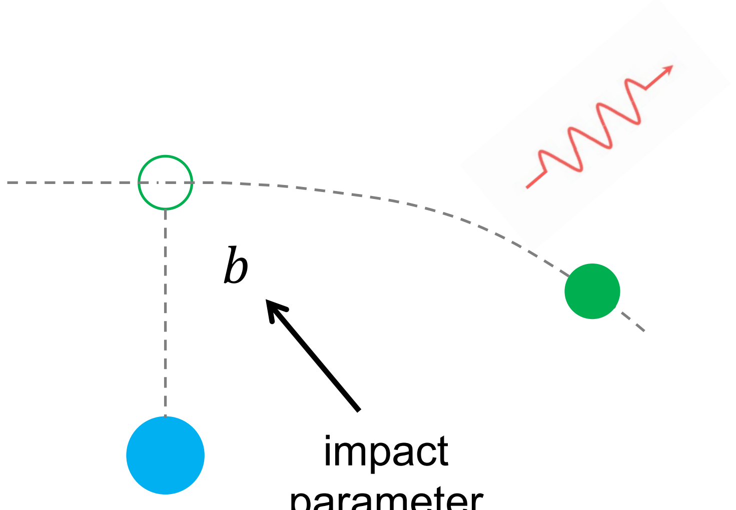
$$P(v,\omega) = \frac{dE}{d\omega\, dV\, dt} = \frac{16\,\pi\, e^6}{3\sqrt{3}\, c^3 m_e^2 v}\, n_e n_i Z^2 g_{\mathrm{ff}}(v,\omega)$$

$$g_{\mathrm{ff}}(v,\omega) = \frac{\sqrt{3}}{\pi} \ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

Thermal bremsstrahlung emission

Averaging the single-speed power over a thermal distribution of electron speeds

$$\Delta E = \frac{1}{2} m_e v^2 - \frac{1}{2} m_e (v - \Delta v)^2 = h\nu$$



Maxwell velocity distribution

$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT_e}\right)$$

prev. sl.

$$P(v,\omega) = \frac{dE(v,\omega)}{d\omega\, dV\, dt} = \frac{16\,\pi\, e^6}{3\sqrt{3}\, c^3 m_e^2 v} n_e n_i Z^2 g_{\mathrm{ff}}(v,\omega)$$

$$\frac{dE(T,\omega)}{dV\, dt\, d\omega} = \frac{\int_{v_{\min}}^{\infty} \frac{dE(v,\omega)}{d\omega\, dV\, dt}\, v^2 \exp(-\frac{m_e v^2}{2kT_e})\, dv}{\int_0^{\infty} v^2 \exp(-\frac{m_e v^2}{2kT_e})\, dv}$$

photon discreteness

$$\frac{1}{2} m_e v_{\min}^2 = h\nu$$

Spectral shape of thermal bremsstrahlung emission

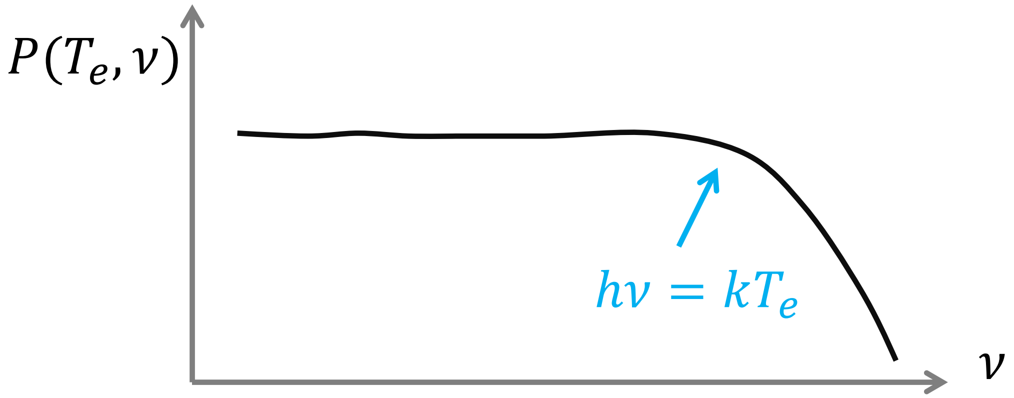
$$P(T_e,\nu) = \frac{dE(T_e,\nu)}{d\nu\, dV\, dt} = \frac{2^5\,\pi e^6}{3m_e c^3}\left(\frac{2\pi}{3km_e}\right)^{1/2} T_e^{-1/2}\, Z^2 \langle g_{\rm ff}(T_e,\nu)\rangle\, n_e n_i \exp\left(-\frac{h\nu}{kT_e}\right)$$

$$\frac{P(T_e,\nu)}{\mathrm{erg\ s^{-1}\ cm^{-3}\ Hz^{-1}}} = 6.84\times10^{-38}\left(\frac{\mathrm{K}}{T_e}\right)^{1/2} Z^2 \langle g_{\rm ff}(T_e,\nu)\rangle \left(\frac{n_e}{\mathrm{cm^{-3}}}\right)\left(\frac{n_i}{\mathrm{cm^{-3}}}\right)\exp\left(-\frac{h\nu}{kT_e}\right)$$

$\langle g_{\rm ff}(T_e,\nu)\rangle$ is the thermally-averaged Gaunt factor

see Sect. 5.2 of the REF book (p159-160) by Rybicki & Lightman

The thermal bremsstrahlung emission spectrum is rather flat at the low-frequency end with an exponential decay towards the high-frequency end. The cutoff frequency is $h\nu{\sim}kT_e$.



Total power of thermal bremsstrahlung emission

$$\int_0^\infty \exp(-a\,x)\,dx = \frac{1}{a}$$

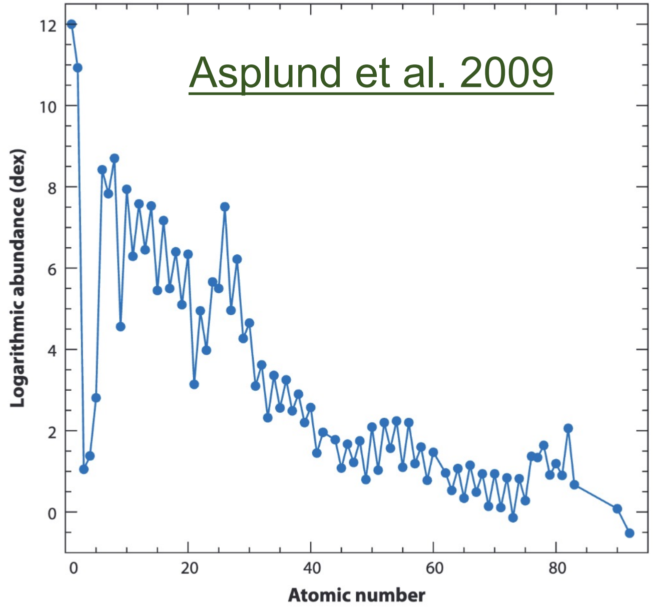
$$P(T_e) = \int P(T_e, \nu)d\nu = \frac{2^5\,\pi e^6}{3hm_e c^3}\left(\frac{2\pi k}{3m_e}\right)^{1/2} T_e^{1/2} Z^2 \langle g_{\rm ff}(T_e)\rangle\, n_e n_i$$

$$\frac{P(T_e)}{\mathrm{erg\,s^{-1}cm^{-3}}} = 1.43\times10^{-27} Z^2 T_e^{1/2} \langle g_{\rm ff}(T_e)\rangle\, n_e n_i$$

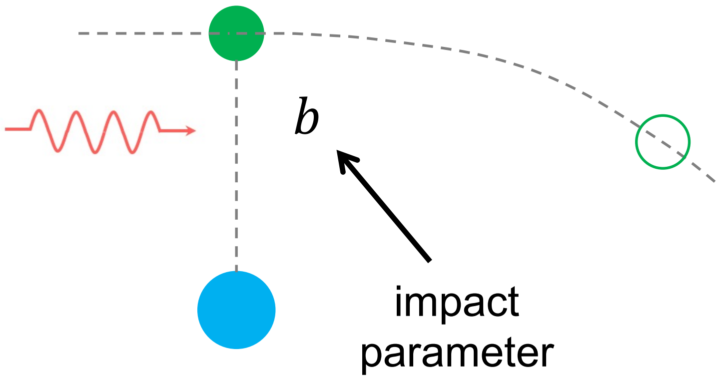
see Sect. 5.2 of the REF book (p161-162) by Rybicki & Lightman

$\langle g_{\rm ff}(T_e)\rangle$ is the thermally- and frequency-averaged Gaunt factor, which is of the order of unity.

Thermal bremsstrahlung emission is dominated by hydrogen and helium, if the plasma has elemental abundances similar to those of the Sun.



Thermal bremsstrahlung absorption



Kirchoff's law

$$j_\nu = \alpha_\nu B_\nu(T)$$

Planck's spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

prev. sl.

$$\frac{dE(T_e,\nu)}{d\nu\, dV\, dt} = \frac{2^5\,\pi e^6}{3m_e c^3}\left(\frac{2\pi}{3km_e}\right)^{1/2} T^{-1/2}\, Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle\, n_e n_i \exp\left(-\frac{h\nu}{kT_e}\right)$$

$$j_\nu^{\mathrm{ff}} = \frac{1}{4\pi}\frac{dE(T_e,\nu)}{d\nu\, dV\, dt}$$

$$\alpha_\nu^{\mathrm{ff}} = \frac{j_\nu^{\mathrm{ff}}}{B_\nu(T)} = \frac{4\,e^6}{3m_e hc}\left(\frac{2\pi}{3km_e}\right)^{1/2} \nu^{-3} T^{-1/2}\, Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \left(1 - \exp\left(-\frac{h\nu}{kT_e}\right)\right)$$

$$\frac{\alpha_\nu^{\mathrm{ff}}}{\mathrm{cm}^{-1}} = 3.69\times 10^8 \nu^{-3} T^{-1/2}\, Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \left(1 - \exp\left(-\frac{h\nu}{kT_e}\right)\right)$$

Asymptotic behavior

prev. sl.

$$\frac{\alpha_\nu^{\mathrm{ff}}}{\mathrm{cm}^{-1}} = 3.69\times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \left(1 - \exp\left(-\frac{h\nu}{kT_e}\right)\right)$$

For $h\nu \gg kT_e$
$$\frac{\alpha_\nu^{\mathrm{ff}}}{\mathrm{cm}^{-1}} = 3.69\times 10^8 \nu^{-3} T^{-1/2} Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \propto \nu^{-3}$$

For $h\nu \ll kT_e$
$$\frac{\alpha_\nu^{\mathrm{ff}}}{\mathrm{cm}^{-1}} = 1.77\times 10^{-2} \nu^{-2} T^{-3/2} Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \propto \nu^{-2}$$

$1 - e^{-x} = x + \cdots$

the lower the frequency, the larger the thermal bremsstrahlung absorption coefficient

cf. Thomson scattering

prev. sl.

$$\frac{\alpha_\nu^{\mathrm{ff}}}{\mathrm{cm}^{-1}} = 3.69\times10^8 \nu^{-3} T^{-1/2}\, Z^2 \langle g_{\mathrm{ff}}(T_e,\nu)\rangle n_e n_i \left(1-\exp\left(-\frac{h\nu}{kT_e}\right)\right)$$

consider hydrogen only

$$\frac{\alpha_\nu^{\mathrm{ff}}(\nu)}{n_e \sigma_{\mathrm{T}}} = \frac{3.69\times10^8 \nu^{-3} T^{-1/2} \langle g_{\mathrm{ff}}(T_e,\nu)\rangle\, n_e n_{\mathrm{H}} \left(1-\exp\left(-\frac{h\nu}{kT_e}\right)\right)}{n_e\, 6.65\times10^{-25}}$$

$$= 5.55\times10^{32} x^{-3}(1-\exp(-x)) \left(\frac{h}{k}\right)^3 \left(\frac{T}{\mathrm{K}}\right)^{\frac{5}{2}} \langle g_{\mathrm{ff}}(T_e,\nu)\rangle\, n_{\mathrm{H}}$$

$$x = \frac{h\nu}{kT}$$

$$= 61.34 \frac{1-\exp(-x)}{x^3} \left(\frac{T}{\mathrm{K}}\right)^{\frac{5}{2}} \langle g_{\mathrm{ff}}\rangle\, n_{\mathrm{H}}$$

Thomson scattering cross section

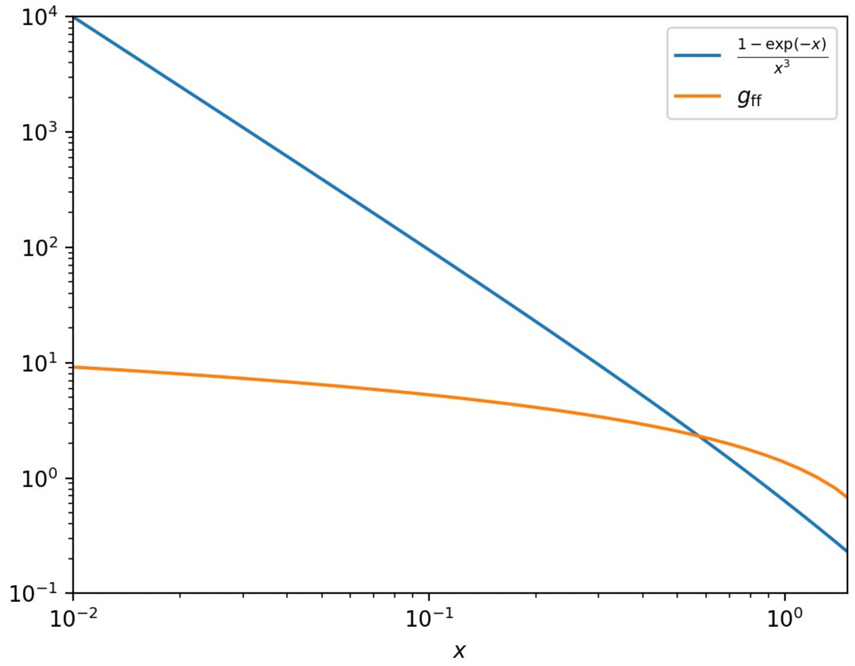
$$\sigma_{\mathrm{T}} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

cf. Thomson scattering (low frequency)

$$x = \frac{h\nu}{kT}$$

prev. sl.

$$\frac{\alpha_\nu^{\rm ff}(\nu)}{n_e \sigma_{\rm T}} = 61.34 \frac{1 - \exp(-x)}{x^3} \left(\frac{T}{\rm K}\right)^{\frac{5}{2}} \langle g_{\rm ff} \rangle \, n_{\rm H}$$



In this case, the thermally- and frequency-averaged Gaunt factor is

$$\langle g_{\rm ff} \rangle \sim \frac{3}{\sqrt{\pi}} \ln\left(\frac{2.246}{x}\right)$$

Eq. 3 of Felten & Rees (1972)

Assuming at $x = x_0 = h\nu_0/kT$, we have $n_e \sigma_{\rm T} = \alpha_\nu^{\rm ff}(\nu_0)$

- For $x \ll x_0$ (low frequencies), i.e., $n_e \sigma_{\rm T} \ll \alpha_\nu^{\rm ff}(\nu_0)$, Thomson scattering can be neglected. The (absorption) mean free path of the photos is $1/\alpha_\nu^{\rm ff}(\nu)$, we get the blackbody spectrum

slide 2.2.3

$$\bar{l}_\nu = \frac{1}{\alpha_\nu}$$

cf. Thomson scattering (high frequency)

$$x = \frac{h\nu}{kT}$$

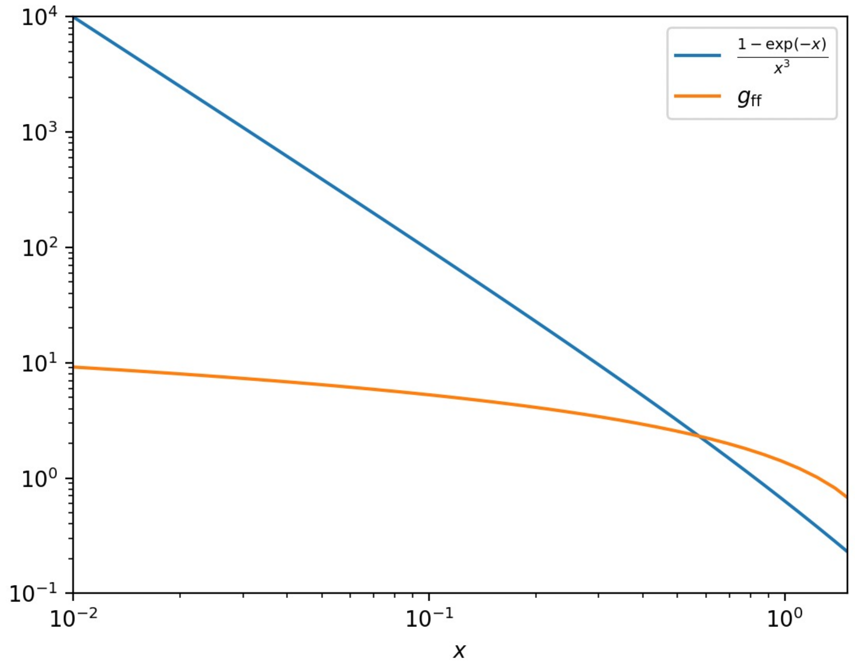
prev. sl.

$$\frac{\alpha_\nu^{\rm ff}(\nu)}{n_e\sigma_{\rm T}} = 61.34\,\frac{1-\exp(-x)}{x^3}\left(\frac{T}{\rm K}\right)^{\frac{5}{2}}\langle g_{\rm ff}(x)\rangle\, n_{\rm H}$$

Here, the thermally- and frequency-averaged Gaunt factor is

$$\langle g_{\rm ff}(x)\rangle \sim \frac{3}{\sqrt{\pi}}\ln\left(\frac{2.246}{x}\right)$$

Eq. 3 of Felten & Rees (1972)



Assuming at $x = x_0 = h\nu_0/kT$, we have $n_e\sigma_{\rm T} = \alpha_\nu^{\rm ff}(\nu_0)$

- For $x \gg x_0$ (high frequencies), i.e., $n_e\sigma_{\rm T} \gg \alpha_\nu^{\rm ff}(\nu_0)$, Thomson scattering will modify the spectrum. Photons will experience random walk, the (scattering and absorption) mean free path is $1/\sqrt{\alpha_\nu^{\rm ff}(\nu)(\alpha_\nu^{\rm ff}(\nu)+n_e\sigma_{\rm T})} \sim 1/\sqrt{\alpha_\nu^{\rm ff}(\nu)n_e\sigma_{\rm T}}$, we get the modified blackbody spectrum

slide 2.5.4

$$l_\nu^{\rm eff} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu+\sigma_\nu^{\rm sca})}}$$

Modified blackbody spectrum

Eq. 7 of Felten & Rees (1972)

$$I_\nu^{\mathrm{MBB}} \sim B_\nu \sqrt{\frac{\alpha_\nu^{\mathrm{ff}}(\nu)}{n_e \sigma_{\mathrm{T}}}}$$

prev. sl.

$$\frac{\alpha_\nu^{\mathrm{ff}}(\nu)}{n_e \sigma_{\mathrm{T}}} = 61.34 \frac{1-\exp(-x)}{x^3} \left(\frac{T}{\mathrm{K}}\right)^{\frac{5}{2}} \langle g_{\mathrm{ff}}(x) \rangle \, n_{\mathrm{H}}$$

$$I_\nu^{\mathrm{MBB}} \sim B_\nu \left(61.34 \frac{1-\exp(-x)}{x^3} \left(\frac{T}{\mathrm{K}}\right)^{\frac{5}{2}} \langle g_{\mathrm{ff}}(x) \rangle \, n_{\mathrm{H}}\right)^{1/2}$$

$$x = \frac{h\nu}{kT}$$

- For $x_0 \ll x \ll 1$ (high frequencies and high temperature), the emission intensity is $\propto \nu$ instead of $\propto \nu^2$

$$I_\nu^{\mathrm{MBB}} \sim \frac{2\nu^2}{c^2} kT \sqrt{61.34} \frac{kT}{h\nu} \left(\frac{T}{\mathrm{K}}\right)^{\frac{5}{4}} \left(\frac{n_{\mathrm{H}}}{\mathrm{cm}^{-3}}\right)^{1/2} \langle g_{\mathrm{ff}}(x) \rangle^{1/2}$$

$$= 5.00 \times 10^{-26} \left(\frac{\nu}{\mathrm{Hz}}\right) \left(\frac{T}{\mathrm{K}}\right)^{\frac{13}{4}} \left(\frac{n_{\mathrm{H}}}{\mathrm{cm}^{-3}}\right)^{1/2} \langle g_{\mathrm{ff}}(x) \rangle^{1/2}$$

Rayleigh-Jeans Law (asymptotic behavior of Planck's law with $x \ll 1$)

$$I_\nu^{\mathrm{RJ}} = \frac{2\nu^2}{c^2} kT$$

$$\frac{1-\exp(-x)}{x^3} \sim \frac{1}{x^2} + \cdots$$

Bremsstrahlung continuum in ICM

Clusters of galaxies (or galaxy clusters) are the largest gravitational bounded structures in the Universe.

The hot (X-ray emitting) intracluster medium (ICM) is a key component of CIG.

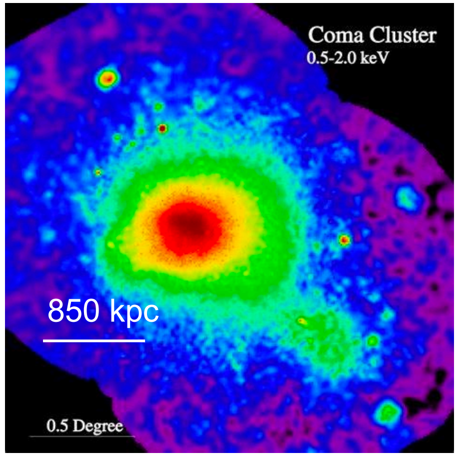
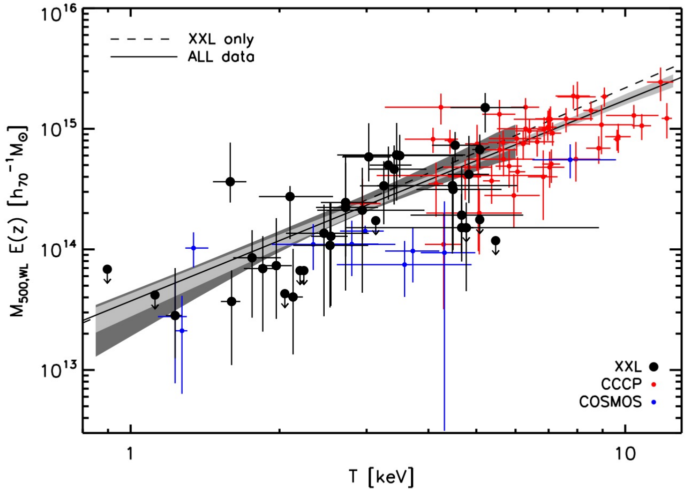


Image credit: ROSAT/MPE/S. L. Snowden



Lieu et al. 2016

Böhringer & Werner (2010)