Final projects

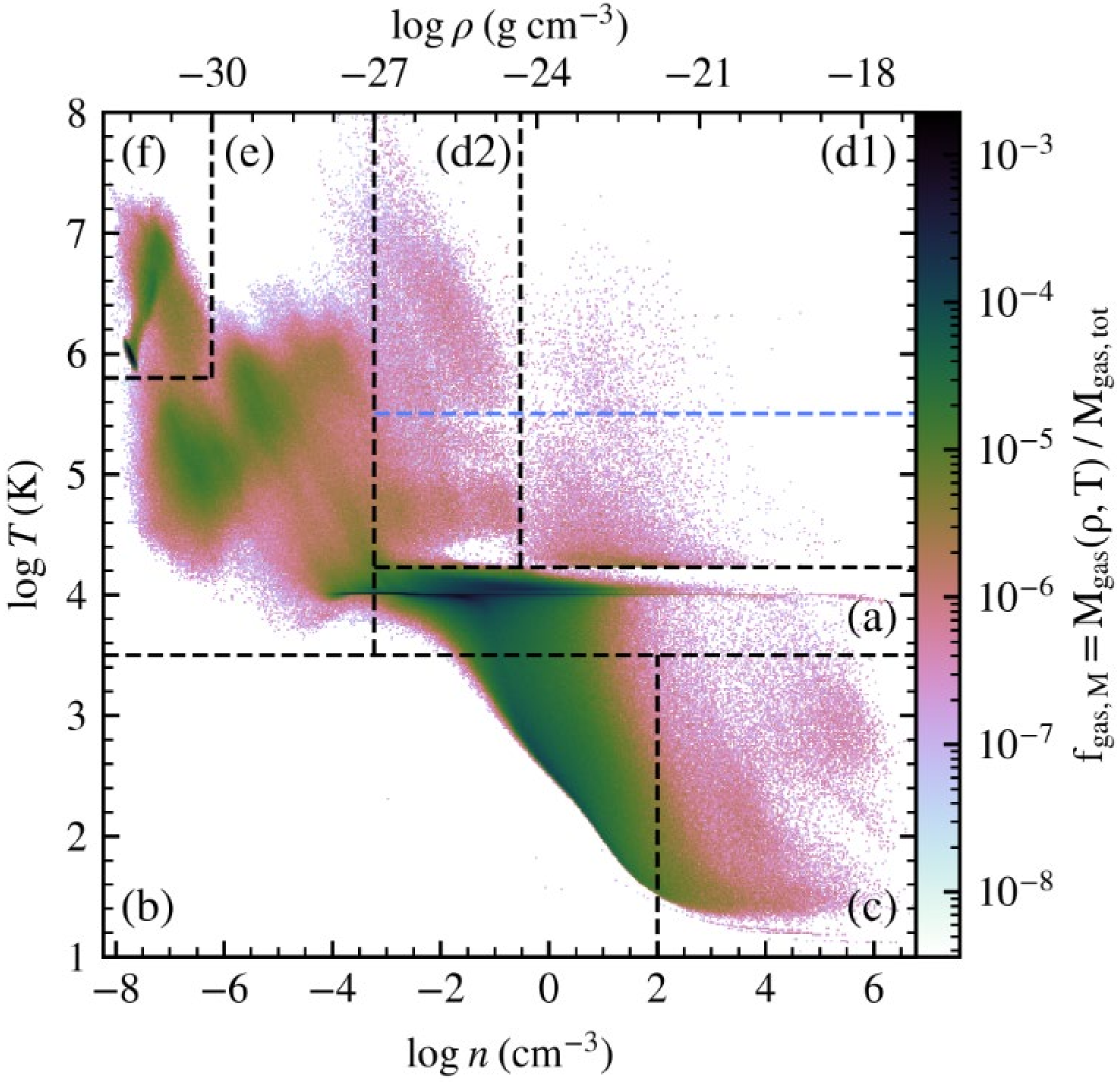
- Write a 3D Eulerian hydrodynamical code and demonstrate e.g. Kelvin-Helmholtz instability.
- Write a SPH hydrodynamical code and simulate e.g. Sedov explosion.
- Use Cloudy to calculate the detailed cooling function of gas in different ISM conditions.
- Use Skirt to convert simulated galaxy samples to mock broadband/narrow band imaging and SED.
- Use Colt to create mock line emission and line shape from galaxy simulations.
- Use RADMC-3D to perform dusty radiative transfer and obtain the mock observation of simulated star-forming regions.
- Use 3D-PDR to explore the ionization structure and chemistry in HII regions.
- Use Chempl to investigate the non-eq molecular chemistry in different ISM conditions.

2. Collisional Processes

Instructor: Hui Li TA: Chengzhe Li
Department of Astronomy
Tsinghua University

The Origin of the Multiphase ISM: Multi-level Equilibrium!

- Simplified 3-phase ISM ->
- pressure eq. ->
- stable and unstable eq. ->
- cooling and heating balance ->
- excitation/ionization balance ->
- kinetic ->
- collisional processes



Collisional Processes

- Everything is about collision.
- Collison depends on cross-sections.
- How to evaluate cross-sections? depends on velocity distribution of different species.
- Maxwellian distribution? Same temperature for all species? Why?
- Collisional timescale between different species.

Collisional Rate Coefficients

- The rate per unit volume of a two-body collision process (A -> B) is

$$n_A n_B \langle \sigma v \rangle_{AB}$$

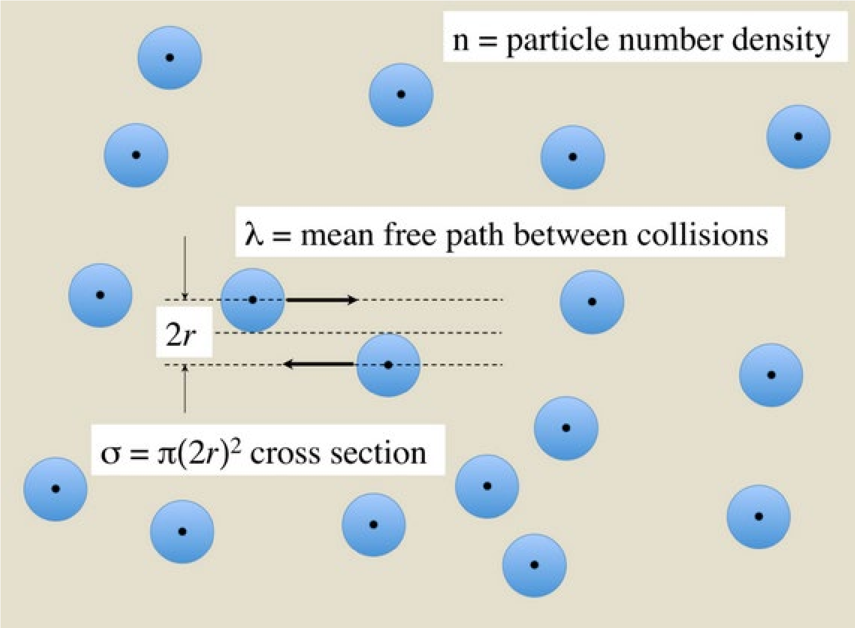
- The two-body collisional rate coefficient is

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv$$

- The mean free path is $\lambda_{\rm mfp} \sim 1/(n\sigma)$

- The collisional timescale is

$$t_{\rm coll} \sim \frac{\lambda_{\rm mfp}}{v_{\rm rms}} \sim \frac{1}{n\sigma}\left(\frac{m}{2\langle E\rangle}\right)^{1/2} .$$



Kinetic Equilibrium

- Maxwellian distribution $f_v\,dv = 4\pi\left(\frac{\mu}{2\pi kT}\right)^{3/2} \mathrm{e}^{-\mu v^2/2kT}\, v^2\, dv$

$$\langle E\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT = 1.293\,\mathrm{eV}\left(\frac{T}{10^4\,\mathrm{K}}\right)$$

$$\langle\sigma v\rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv \quad = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E)\,\frac{E}{kT}\,\mathrm{e}^{-E/kT}\,\frac{dE}{kT} \quad .$$

Close vs. Distance Encounters

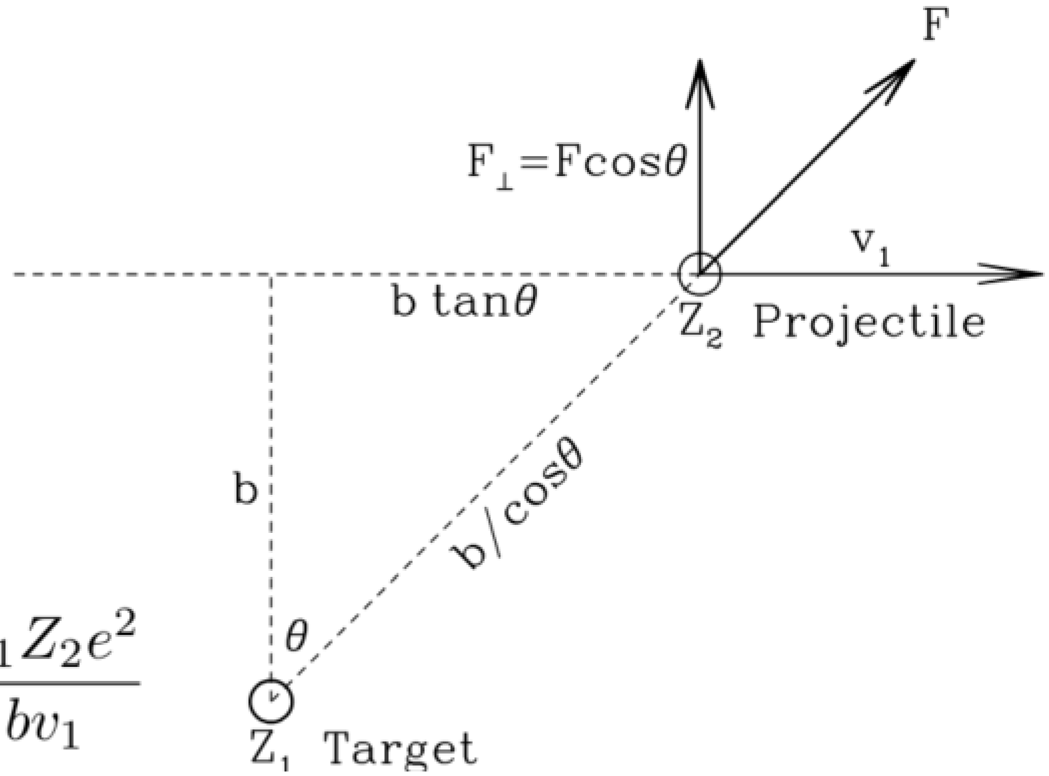
- A close encounter between two charged particles produces relatively large changes in velocity comparable with the initial velocities.
- In a distance encounter these changes are much smaller with negligible deflection angle -> impact approximation.
- For inverse-square forces between particles (gravity, Coulomb), the velocities changes decreases relatively slowly with increasing impact parameters. In consequence, the numerous small velocity changes produced by distance encounters outweigh the effect of rare close encounters.
- This is not true for neutral atom collision!

Inverse-Square Law Forces: Elastic scattering

- The impact approximation

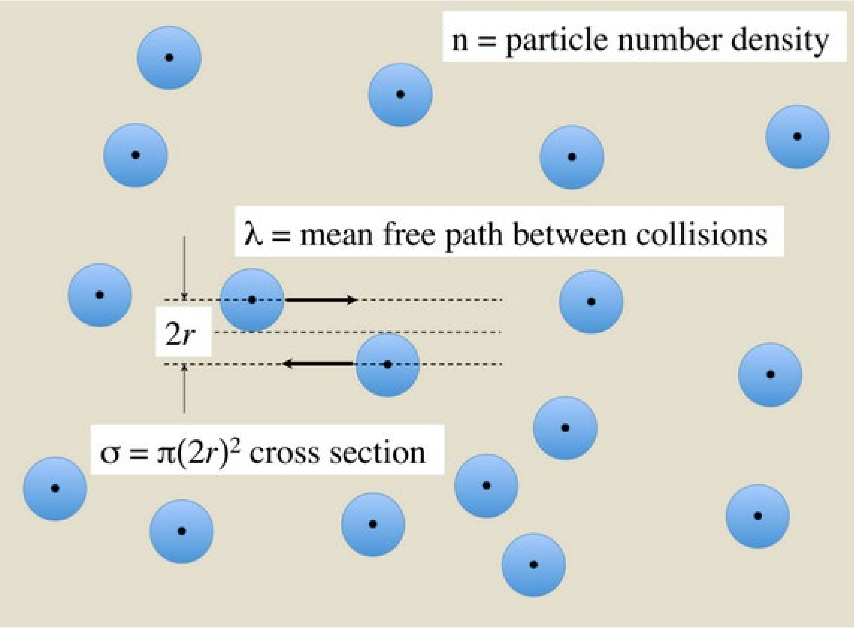
$$F_\perp = \frac{Z_1 Z_2 e^2}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$

$$\Delta p_\perp = \int_{-\infty}^{\infty} F_\perp dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = 2\frac{Z_1 Z_2 e^2}{b v_1}$$



Deflection / Collisional Timescale

- In the impact approximation, each interaction gives an impulse in the direction that is perpendicular to the direction of motion of the projectile.
- The orientation, though, is randomly distributed in this plane. Thus the net vector momentum transferred to the projectile follows a random walk process (dp^2).

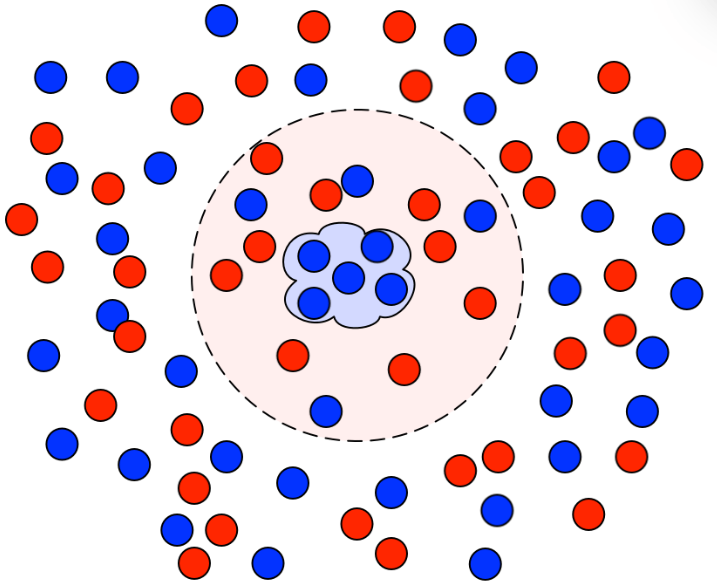


$$\langle \frac{d}{dt}[(\Delta p)_\perp]^2 \rangle = \int_{b_{\rm min}}^{b_{\rm max}} \underbrace{[2\pi b db\, n_2 v_1]}_{d(\text{event rate})} \times \underbrace{\left[\frac{2Z_1 Z_2 e^2}{b v_1}\right]^2}_{(\Delta p_\perp)^2}$$

$$= \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \int_{b_{\rm min}}^{b_{\rm max}} \frac{db}{b} \quad .$$

The min and max distance in the logarithmic.

- The integral is logarithmically divergent at both the min and max impact parameter b, so we have to find some physical limits for the lower and upper cutoffs, b_min and b_max.
- b_min can be estimated at the separation where the Coulomb energy is the same as the kinetic energy of the particle. (The impact approximation fails). b_min~Z1Z2e^2/E
- b_max is the Debye radius, beyond which the plasma maintain electrical neutrality.



$$L_{\mathrm{D}} \equiv \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\,\mathrm{cm}\left(\frac{T}{10^4\,\mathrm{K}}\right)^{1/2}\left(\frac{\mathrm{cm}^{-3}}{n_e}\right)^{1/2}$$

Values of the logarithmic term

- The typical values of Lambda is around 20-40.
- The value much larger than one suggests that weak distant frequent encounters dominate the momentum changes.
- The impact approximation is quite accurate for normal ISM conditions.

$$\Lambda \equiv \frac{b_{\max}}{b_{\min}} = \frac{E}{kT}\frac{(kT)^{3/2}}{(4\pi n_e)^{1/2} Z_1 Z_2 e^3}$$

$$= 4.13 \times 10^9 \left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,\mathrm{K}}\right)^{3/2}\left(\frac{\mathrm{cm}^{-3}}{n_e}\right)^{1/2}$$

$$\ln\Lambda = 22.1 + \ln\left[\left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,\mathrm{K}}\right)^{3/2}\left(\frac{\mathrm{cm}^{-3}}{n_e}\right)\right] .$$

Timescale estimates: deflection and energy loss timescales

$$t_{\rm defl} = \frac{(m_1 v_1)^2}{\langle (d/dt)[(\Delta p)_\perp]^2 \rangle} = \frac{m_1^2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} = 7.6\times 10^3\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

$$\begin{aligned}{\rm mfp} = v_1 t_{\rm defl} &= \frac{m_1^2 v_1^4}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} \\ &= 5\times 10^{17}\,{\rm cm}\left(\frac{T}{10^6\,{\rm K}}\right)^2\left(\frac{0.01\,{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

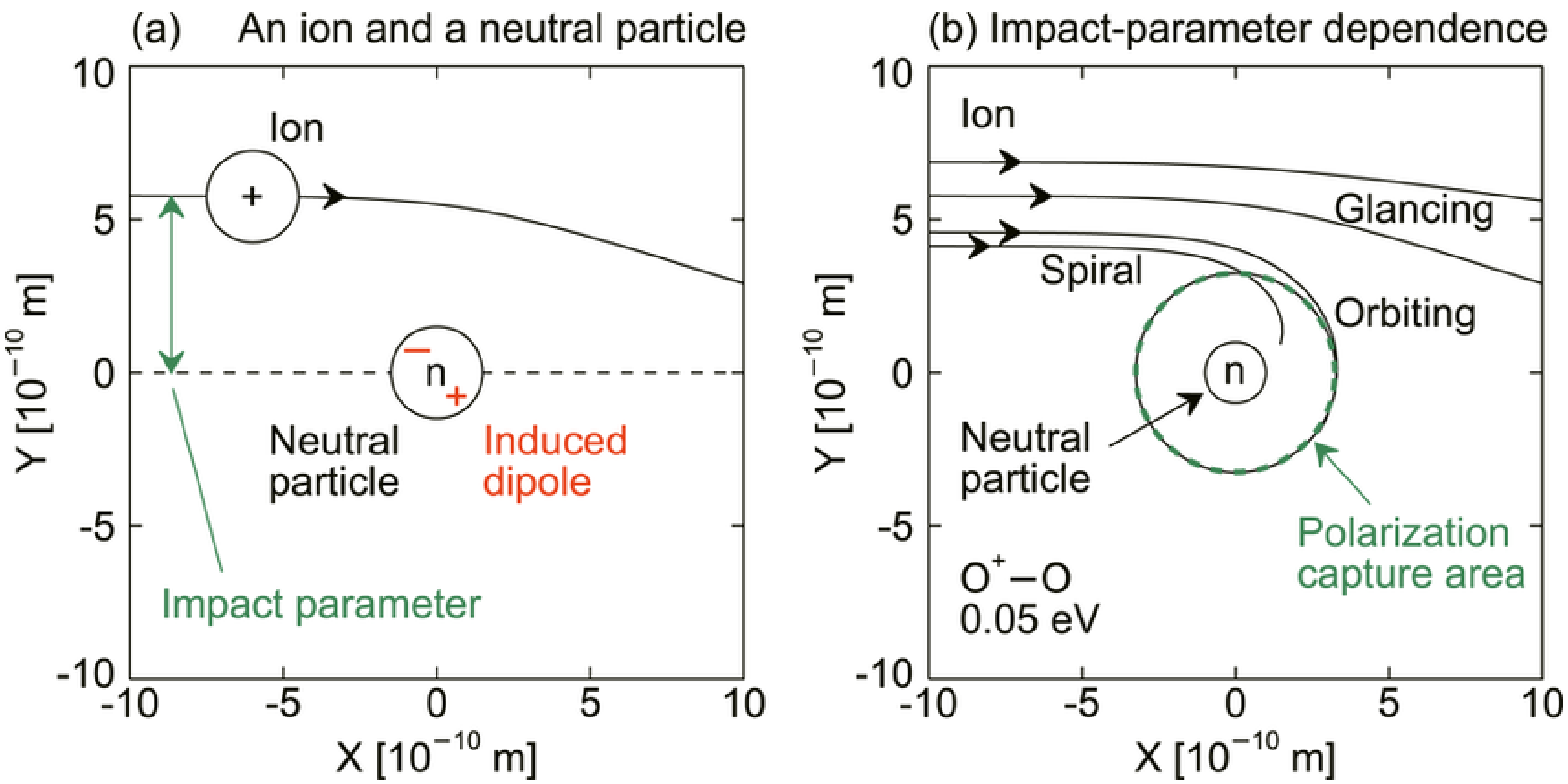
$$\begin{aligned}t_{\rm loss} \equiv \frac{E}{\langle (dE/dt)_{\rm loss}\rangle} &= \frac{m_1 v_1^2}{\langle (d/dt)[(\Delta p)_\perp]^2\rangle/m_2} = \frac{m_1 m_2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} \\ &= 1.4\times 10^7\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

Ion-Neutral Collision: short range interaction!

- The 1/r Coulomb potential means that the potential drop relatively slow for longer distant.
- The physics of Ion-Neutral collision: polarization of the neutral caused by the electric field of the ion.

$$U(r) = -\frac{1}{2}\frac{\alpha_N Z^2 e^2}{r^4}$$

- The cross-section only depends on the short-range impact!



Neutral-Neutral Collision

- If we ignore the very weak van der Waals attraction, we can assume the simplest "billiard balls" model.
- R~ 2*a0, where a0 is the Bohr radius. $a_0 = 5.292 \times 10^{-9}\,\mathrm{cm}$

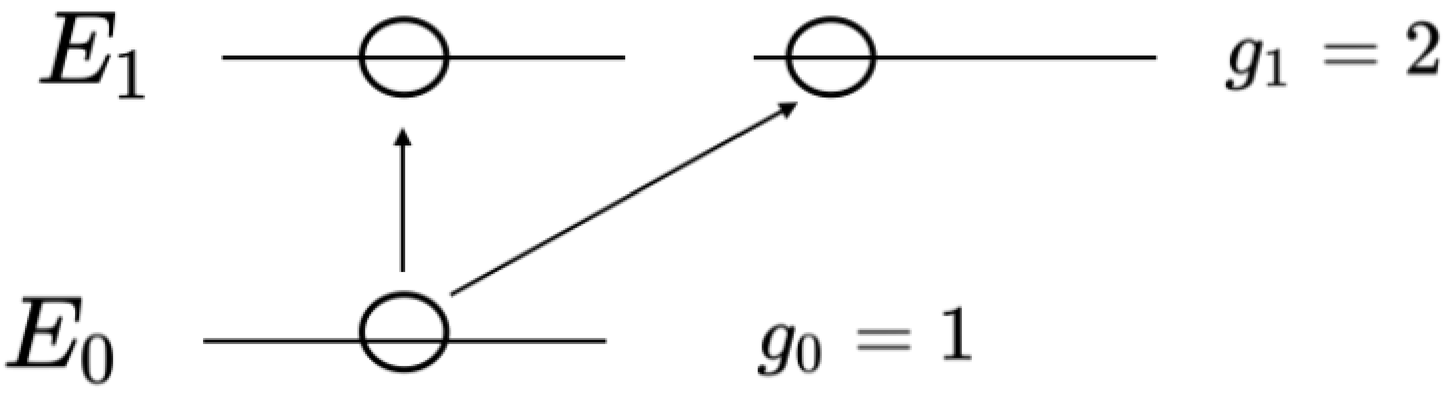
$$\langle \sigma v \rangle = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \pi (R_1 + R_2)^2$$

$$= 1.81 \times 10^{-10} \left(\frac{T}{10^2\,\mathrm{K}}\right)^{1/2} \left(\frac{m_{\mathrm{H}}}{\mu}\right)^{1/2} \left(\frac{R_1 + R_2}{2\,\text{Å}}\right)^2 \mathrm{cm}^3\,\mathrm{s}^{-1}$$

$$t_{\mathrm{coll}} \sim \frac{\lambda_{\mathrm{mfp}}}{v_{\mathrm{rms}}} \sim \frac{1}{n\sigma}\left(\frac{m}{2\langle E\rangle}\right)^{1/2} \sim 2 \times 10^8\,\mathrm{s}\left(\frac{n_{\mathrm{H}}}{1\,\mathrm{cm}^{-3}}\right)^{-1}\left(\frac{\langle E\rangle}{1\,\mathrm{eV}}\right)^{-1/2}$$

Minimalists' Statistical Mechanism: Boltzmann distribution!

- Example of a two-level quantum system:



$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-(E_1 - E_0)/kT}$$

Derivation of Boltzmann Distribution

- Entropy is the natural logarithm of the number of microstates, W.

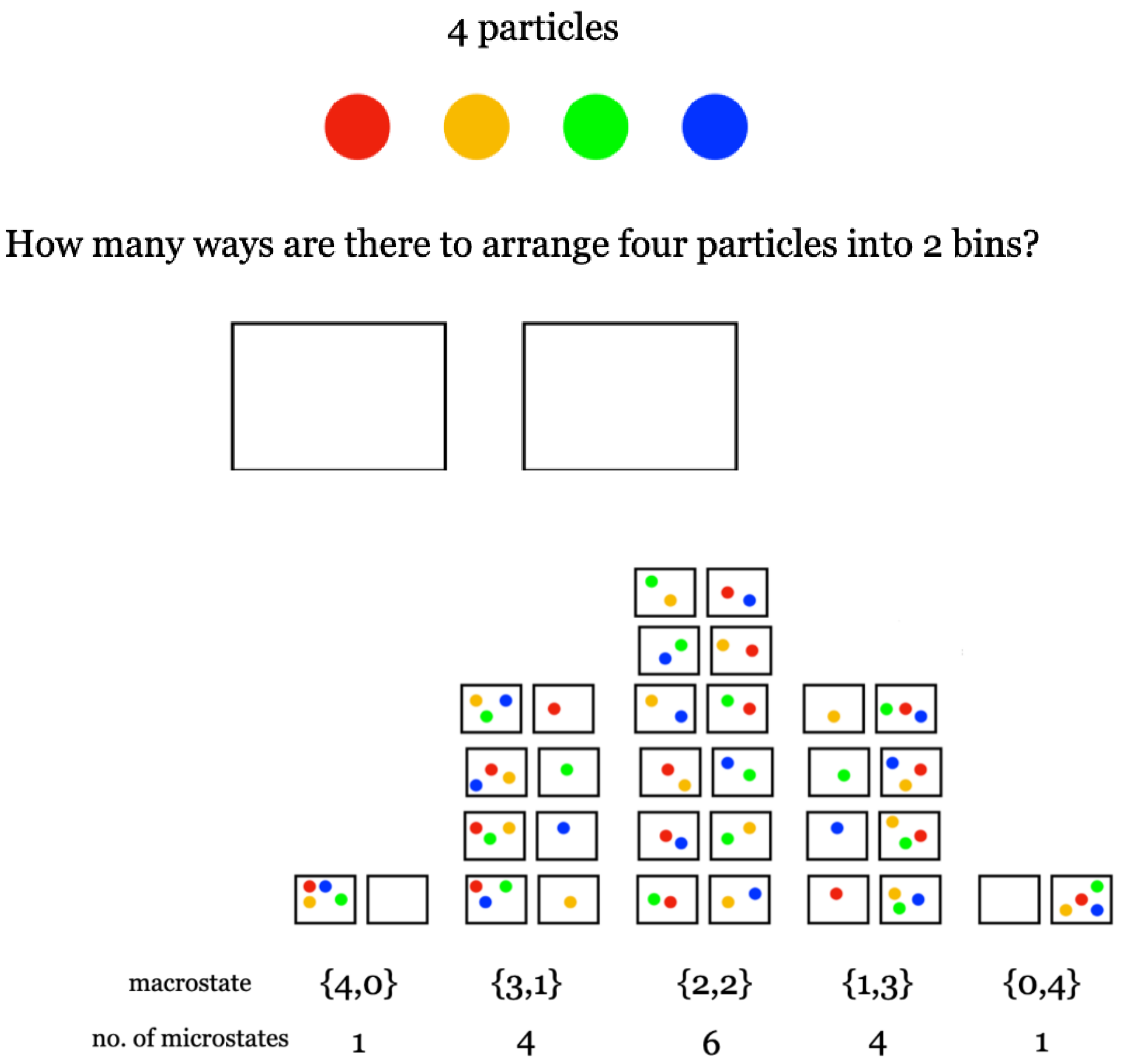
$$S = k_B \ln W$$

- Specifically, for N level systems with probability p_i for level I,

$$S = -k_\mathrm{B} \sum_i p_i \ln p_i$$

with two constraint conditions:

$$\sum_i p_i \epsilon_i = E, \qquad \sum_i p_i = 1.$$



Derivation of Boltzmann Distribution

$$S = -k_{\mathrm{B}} \sum_i p_i \ln p_i$$

- Using Lagrange multiplier method, we can obtain the general solution to maximize entropy.

Boltzmann distribution

$$p_j = Ae^{-\beta \epsilon_j}$$

$$\beta = \frac{1}{k_B}\frac{\partial S}{\partial U} = \frac{1}{k_B T}$$

Excitation Temperature

- Excitation temperature is simply a convenient way to parameterize the excitation state of a two-level system, not a measure of kinetic temperature of the gas.
- In excitation equilibrium, excitation temperature is the same as the kinetic temperature.
- In general, however, Texc is far from Tkin.
- One extreme case is astrophysical maser, where a population is over-excited by shocks/strong radiation. The excitation temperature is negative in this case.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT_{\rm exc}}\right),$$

$$kT_{\rm exc} \equiv \frac{E_{u\ell}}{\ln[(g_u/g_\ell)(n_\ell/n_u)]}.$$

Ionization Equilibrium

- The first ionization energy, I, of important elements is ~ 10 eV, which corresponds to ~1e5 K.

Element	ppm by number	percentage by mass	atomic number	1st ionization energy [eV]
hydrogen (H)	910 630	71.10%	1	13.60
helium (He)	88 250	27.36%	2	24.59
oxygen (O)	550	0.68%	8	13.62
carbon (C)	250	0.24%	6	11.26
neon (Ne)	120	0.18%	10	21.56
nitrogen (N)	75	0.08%	7	14.53
magnesium (Mg)	36	0.07%	12	7.65
silicon (Si)	35	0.08%	14	8.15
iron (Fe)	30	0.13%	26	7.90
sulfur (S)	15	0.04%	16	10.36

[a] *Data from Lodders 2010*

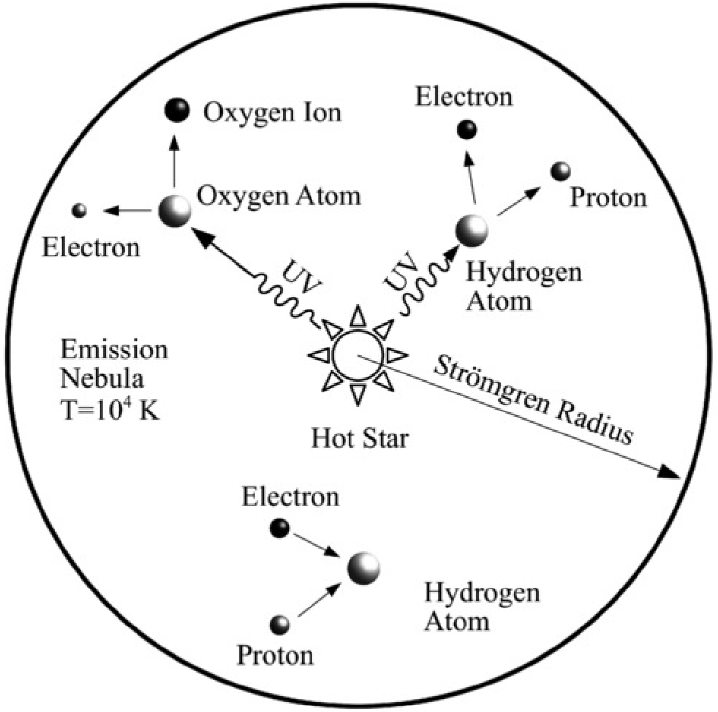
- Only regions with temperature > 1e5K (rare) are effective collisional ionization of neutral atoms.
- In cooler regions, most ionization comes from photo-ionization from UV photons.

Photo-ionization vs. Radiative Recombination

$$X^i + \gamma \rightarrow X^{i+1} + e^-, \qquad X^{i+1} + e^- \rightarrow X^i + \gamma.$$

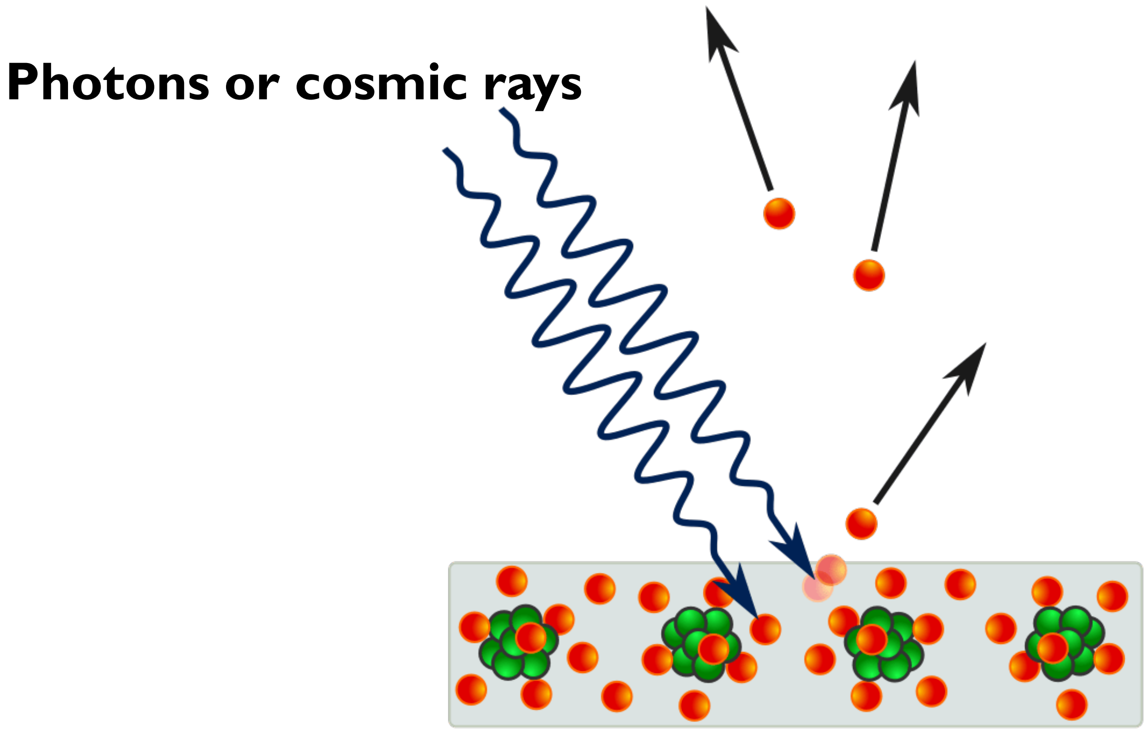
- Ionization equilibrium means the balance between the two processes.

$$n(X^i) n_\gamma \sigma_{\rm pho} c = n(X^{i+1}) n_e \sigma_{\rm rr} v$$



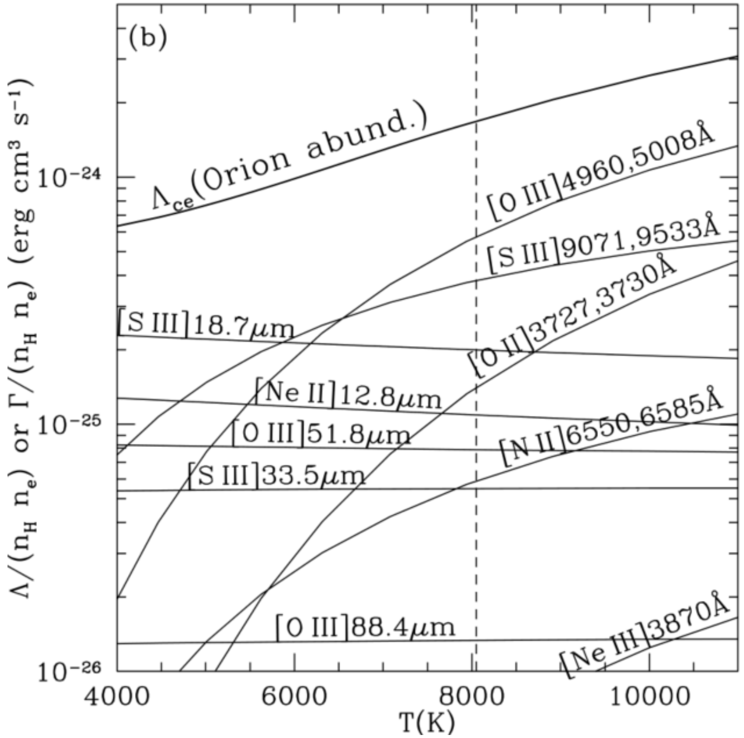
Heating of the ISM

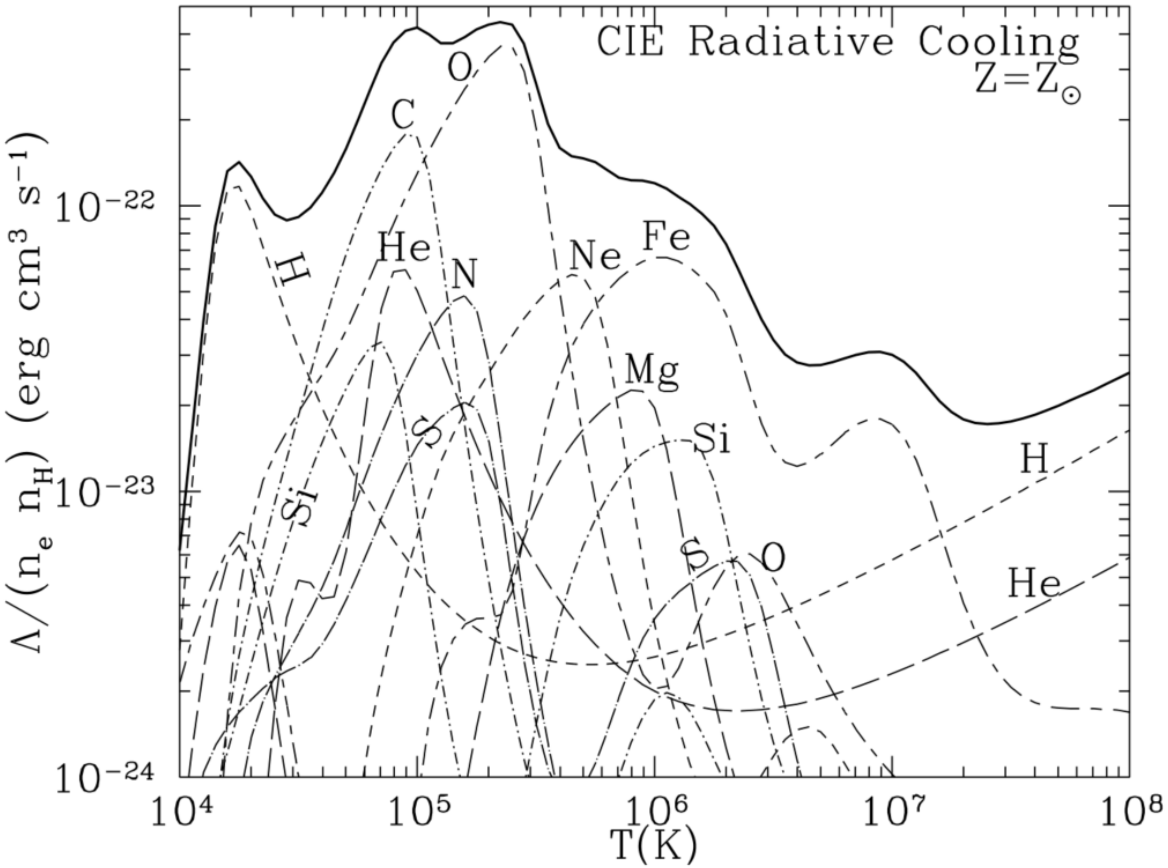
- Heating is the process of converting energy of other forms into thermal energy of the gas.
- Heating is dominated by suprathermal electrons produced via photo-ionization, photoelectric effect on dust grains, and cosmic rays. Hydrodynamical shocks can also convert kinetic energy into thermal energy by the thermalization process at the shock front.



Cooling of the ISM

- Collisional excited line emissions (H, O, C, S for warm medium, Fe for hot gas).
- Recombination radiation (mostly H+ cascade).
- Free-free emission of hot plasma.

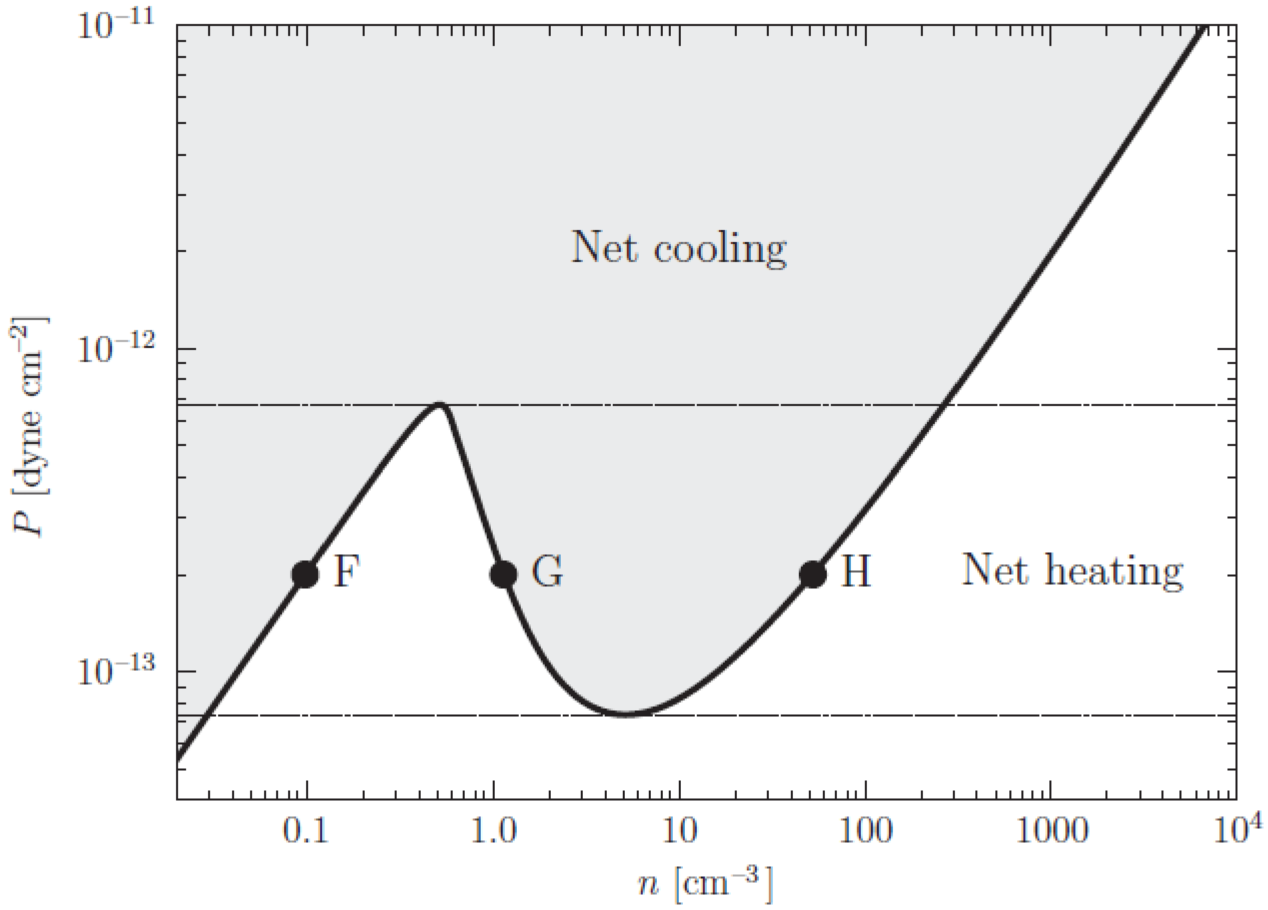




Phases of the ISM (on the order of temperature)

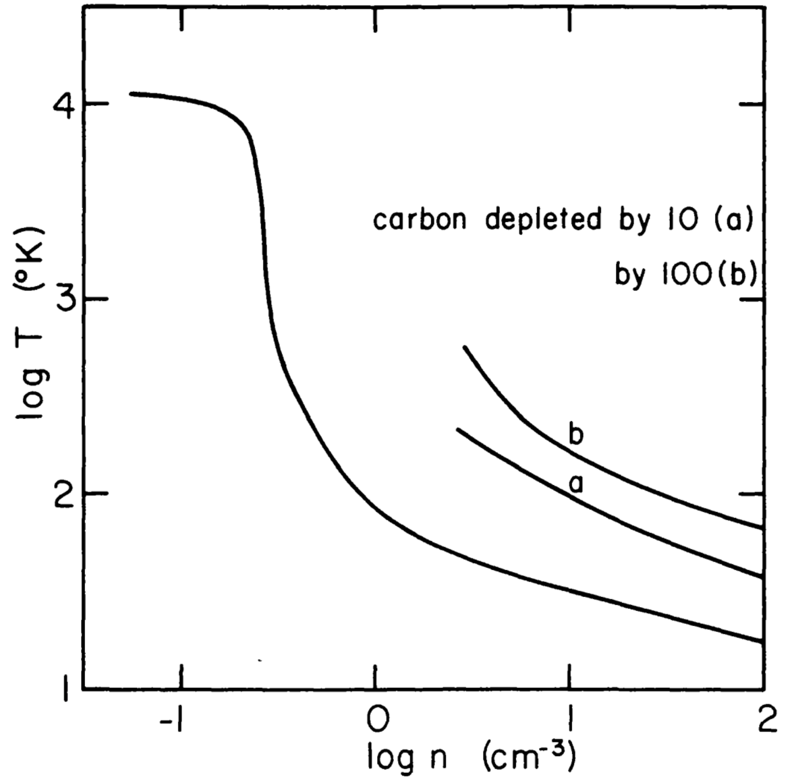
Phase	$T(\mathrm{K})$	$n_{\mathrm{H}}(\mathrm{cm}^{-3})$	Comments
Coronal gas (HIM) $f_V \approx 0.5?$ $\langle n_{\mathrm{H}}\rangle f_V \approx 0.002\,\mathrm{cm}^{-3}$ ($f_V \equiv$ volume filling factor)	$\gtrsim 10^{5.5}$	~ 0.004	Shock-heated Collisionally ionized Either expanding or in pressure equilibrium Cooling by: ◇ Adiabatic expansion ◇ X ray emission Observed by: • UV and x ray emission • Radio synchrotron emission
H II gas $f_V \approx 0.1$ $\langle n_{\mathrm{H}}\rangle f_V \approx 0.02\,\mathrm{cm}^{-3}$	10^4	$0.3 - 10^4$	Heating by photoelectrons from H, He Photoionized Either expanding or in pressure equilibrium Cooling by: ◇ Optical line emission ◇ Free–free emission ◇ Fine-structure line emission Observed by: • Optical line emission • Thermal radio continuum
Warm H I (WNM) $f_V \approx 0.4$ $n_{\mathrm{H}} f_V \approx 0.2\,\mathrm{cm}^{-3}$	$\sim$5000	0.6	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Pressure equilibrium Cooling by: ◇ Optical line emission ◇ Fine structure line emission Observed by: • H I 21 cm emission, absorption • Optical, UV absorption lines
Cool H I (CNM) $f_V \approx 0.01$ $n_{\mathrm{H}} f_V \approx 0.3\,\mathrm{cm}^{-3}$	~ 100	30	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◇ Fine structure line emission Observed by: • H I 21-cm emission, absorption • Optical, UV absorption lines
Diffuse H_2 $f_V \approx 0.001$ $n_{\mathrm{H}} f_V \approx 0.1\,\mathrm{cm}^{-3}$	$\sim 50\,\mathrm{K}$	~ 100	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◇ Fine structure line emission Observed by: • H I 21-cm emission, absorption • CO 2.6-mm emission • optical, UV absorption lines
Dense H_2 $f_V \approx 10^{-4}$ $\langle n_{\mathrm{H}}\rangle f_V \approx 0.2\,\mathrm{cm}^{-3}$	$10 - 50$	$10^3 - 10^6$	Heating by photoelectrons from dust Ionization and heating by cosmic rays Self-gravitating: $p > p(\mathrm{ambient\ ISM})$ Cooling by: ◇ CO line emission ◇ C I fine structure line emission Observed by: • CO 2.6-mm emission • dust FIR emission
Cool stellar outflows	$50 - 10^3$	$1 - 10^6$	Observed by: • Optical, UV absorption lines • Dust IR emission • H I, CO, OH radio emission

Stable and Unstable Equilibrium



Two phase ISM model: a sharp insight from Field 1969

since it is thermally stable. A third stable phase should exist above 10^6 ° K, with bremsstrahlung the chief cooling process.



THE ASTROPHYSICAL JOURNAL, Vol. 155, March 1969

COSMIC-RAY HEATING OF THE INTERSTELLAR GAS

G. B. FIELD, D. W. GOLDSMITH, AND H. J. HABING*
Department of Astronomy, University of California, Berkeley

Received December 16, 1968; revised January 17, 1969

ABSTRACT

We present a model of the interstellar medium based on detailed calculations of heating by low-energy cosmic rays. The model contains two thermally stable gas phases that coexist in pressure equilibrium, one at $T = 10^4$ ° K and one at $T < 300$° K. The hot gas occupies most of interstellar space. Gravitation in the z-direction compresses about 75 per cent of the gas into the cool, dense phase to form clouds. By choosing three parameters (the cosmic-ray ionization rate, the amount by which trace elements are depleted in sticking to dust grains, and the magnetic-field strength), we are able to predict six previously unrelated observational parameters to within a factor of 2.

McKee Ostriker (1977) standard model of three-phase ISM