Hui Li
Physics of the Interstellar Medium

Homework 3

Question 1. (20 points) Suppose that in the optical and near-UV band, the extinction efficiency $Q_{\rm ext}(a,\lambda) \approx 2(\pi a/2\lambda)^\beta$ for $\pi a/\lambda < 2$; and $Q_{\rm ext} \approx 2$ for $\pi a/\lambda > 2$. Assume that the dust density is proportional to $n_{\rm H}$, with a simple power-law size distribution

$$\frac{1}{n_{\rm H}}\frac{dn}{da} = \frac{A_0}{a_0}\left(\frac{a}{a_0}\right)^{-p} \qquad 0 < a \le a_{\rm max},$$

where $a_0 = 0.1\mu$m is a fiducial length, A_0 is dimentionless and $p < 4$. Let $\sigma_{\rm ext}(\lambda)$ be the extinction cross section per H at wavelength λ.

1) Assume that $a_{\rm max} < 0.28\mu$m. Obtain an expression for $\sigma_{\rm ext}(\lambda)/A_0\pi a_0^2$ that would be valid for $\lambda = \lambda_V$ or λ_B. Evaluate this ratio for $\beta = 1.5$, $p = 3.5$, $a_{\rm max} = 0.25\mu$m, and $\lambda = \lambda_V$.

2) For $a_{\rm max} < 0.28\mu$m, using your result from 1) to obtain an expression for the ratio $\sigma_{\rm ext}(\lambda_B)/\sigma_{\rm ext}(\lambda_V)$, and evaluate this ratio for $\beta = 1.5$.

3) If assuming $a_{\rm max} < 0.28\mu$m, obtain an expression for R_V and evaluate this for $\beta = 1.5$.

4) Suppose that $a_{\rm max} > 2\lambda/\pi$. Obtain an expression for $\sigma_{\rm ext}(\lambda)/A_0\pi a_0^2$.

5) If $a_{\rm max} = 0.35\mu$m, $p = 3.5$, and $\beta = 2$, evaluate $\sigma_{\rm ext}(\lambda_V)/A_0\pi a_0^2$, $\sigma_{\rm ext}(\lambda_B)/A_0\pi a_0^2$ and R_V.

6) Plot R_V as a function of p from 3-4 for different $a_{\rm max}$ (e.g. 0.25, 0.35, 0.45, 0.55, 0.65 μm).

Question 2. (20 poihnts) Consider a hot plasma with density $n_{\rm H}$ in an elliptical galaxy. Suppose that planetary nebulae and other stellar outflows are injecting dust into the plasma with a rate per unit graine radius

$$\left(\frac{d\dot{N}_d}{da}\right)_{\rm inj} = \frac{A_0}{a_{\rm max}}\left(\frac{a}{a_{\rm max}}\right)^{-p}.$$

1) Obtain an expression for the total rate $(dM_d/dt)_{\rm inj}$ at which dust mass is being injected into the plasma, in terms of A_0, $a_{\rm max}$, p, and the density ρ of the grain material.

2) Upon injection into the plasma, the grains are subject to sputtering at a rate $da/dt = -\beta n_{\rm H}$, where β is a constant. Find the steady state solution for dN_d/da, where $N_d(a)$ is the number of dust grains present with radii $\le a$.

3) Obtain an expression for the steady-state dust mass $M_{\rm dust}$ and the characteristic survival time $\tau_{\rm surv} \equiv M_{\rm dust}/(dM_{\rm dust}/dt)_{\rm inj}$ in terms of $a_{\rm max}$, p, and da/dt.

4) Consider a passive elliptical galaxy NGC 4564 containing hot plasma $kT \approx 0.5$ keV and a core density $n_{\rm H} \approx 0.01/{\rm cm}^3$. Assuming $\beta = 10^{-6}\mu{\rm m\,cm^3/yr}$, $p = 3.5$, and $a_{\rm max} = 0.3\mu$m, estimate the survival time $\tau_{\rm surv}$. If the dust injection rate from evolved stars in the central kpc is $1.3 \times 10^{-4} M_\odot$/yr, estimate the estimated steady-state dust mass $M_{\rm dust}$. The observed upper limit of the dust mass from Clemens et al. (2010) is $M_{\rm dust} < 8700 M_\odot$. Is your estimate in agreement with observations?

Question 3. (20 points) Consider a diffuse molecular cloud with $n_{\rm H} = 100/{\rm cm}^3$. The hydrogen is predominatly molecular, with $n(H_2) = 50/{\rm cm}^3$. The oxygen is primarily atomic, with $n(O) \approx 4 \times 10^{-4} n_{\rm H}$. Assume that cosmic ray ionization maintains an abundance $n(H_3^+) \approx 5 \times 10^{-8} n_{\rm H}$ and cosmic ray ionization plus starlight photoionization of metals maintains $n_e \approx 10^{-4} n_{\rm H}$. Consider the reaction network for OH formation in Section 33.2.2 in Draine's textbook:

1) What is the steady-state density $n(OH^+)$?

2) What is the steady-state density $n(H_2O+)$?

3) What is the steady-state OH abundance relative to hydrogen, $n(OH)/n_{\rm H}$?

4) There is more than one reaction that can reproduce OH. Which is the most important for the diven conditions?

Question 4. (20 points) Consider the collapse of a uniform density (ρ_0) spherical cloud with no gas pressure to counteract gravity, the so-called free-fall condition. Derive the free-fall time $\tau_{\rm ff}$, which is the time it takes for a given gas shell starting at radius r_0 to collapse to the center of the cloud. How does $\tau_{\rm ff}$ depend on the starting radius r_0? What does this dependence mean?

Question 5. (20 points) A pulsar is observed at 1610 and 1660 MHz. The plane of polarization at these two frequencies differs by 57.5^o..

1) What is the minimum possible magnitude of the rotation measure |RM| toward this source? Why is it a minimum? What would be the next largest possible value of |RM|?

2) If the source has a dispersion measure $DM = 200 \mathrm{pc/cm}^3$. Using the minimum |RM| derived in 1), what is the electron-density-weighted component of magnetic field along the line-of-sight?