AY2024F

Junjie Mao

Radiative Processes in Astrophysics

Observation

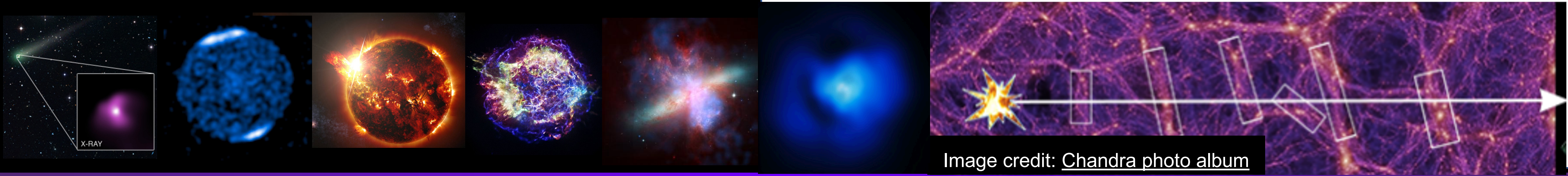
Up to cosmic size scale

Goal

Theory

Down to atomic size scale

C/2012S1 (comet) | Jupiter (planet) | Sun (star) | Cas A (SNR) | M82 (galaxy) | Phoenix (gal. cluster) | Cosmic web filament



Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.1.1 Radiative flux
2.1.2 Radiative intensity
2.1.3 Specific intensity
2.1.4 Specific flux and momentum
2.1.5 Specific energy density

2.2 Radiative transfer
2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

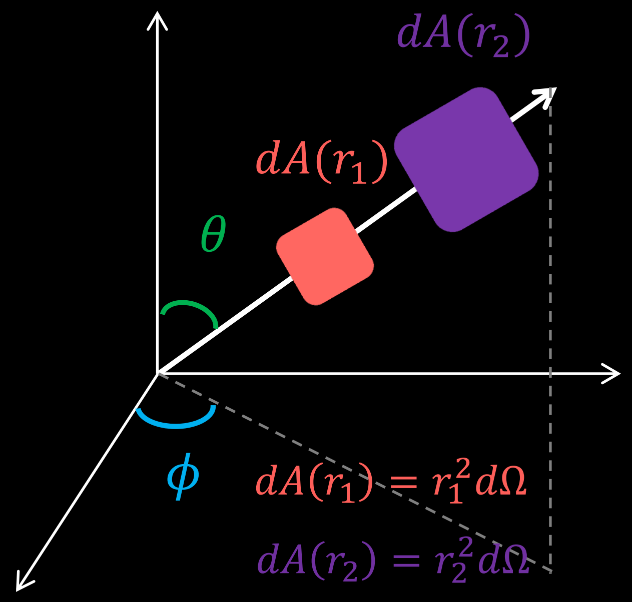


Image credit: Junjie Mao

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Radiative flux

For a detector element (with a photon collecting area dA) exposed to a radiation source for a time interval dt, the number of photons or the amount of energy received is

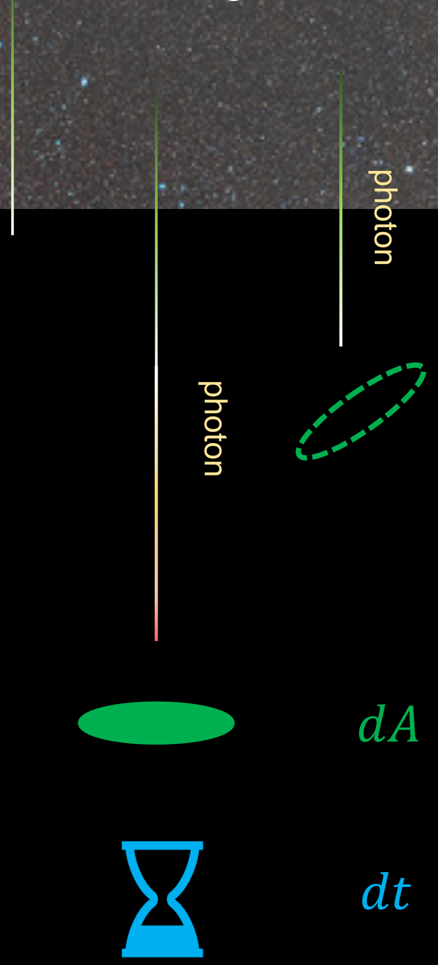
energy or photon flux

$\mathrm{erg\ cm^{-2}\ s^{-1}}$
$\mathrm{ph\ cm^{-2}\ s^{-1}}$

$$dF \equiv \frac{dE}{dA\,dt}$$

photon collecting area (cm^2)

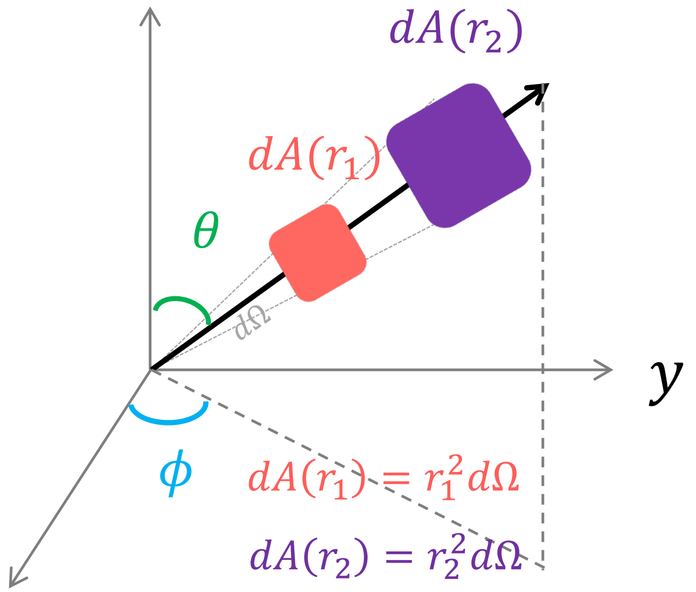
integration time (s)



F can vary for different orientations of the detector element

Inverse square law

Consider an **isotropic** source emitting photons/energy equally in all directions (e.g., a spherically symmetric and isolated star)



energy flux $(\mathrm{erg\,cm^{-2}\,s^{-1}})$

luminosity $(\mathrm{erg\,s^{-1}})$

$$F(r) \equiv \frac{L}{4\pi r^2}$$

distance to the source (cm)

Conservation of energy (no loss or gain)

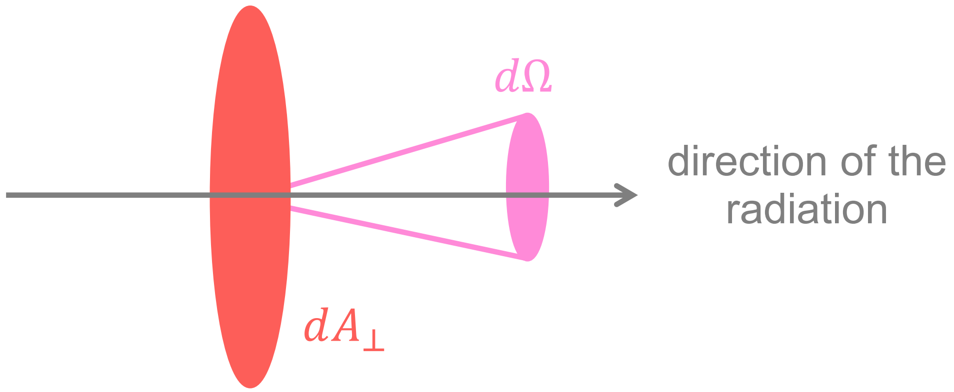
$$F(r) = \frac{\text{constant}}{r^2}$$

$$dF(r) = \frac{dE}{dA(r)dt} = \frac{dE}{r^2 d\Omega dt}$$

$$d\Omega = \sin\theta \, d\theta \, d\phi$$

Radiative intensity

Since flux changes with distance to the source, it is useful to a concept that is **intrinsic** to the properties of the source, e.g., luminosity (L), intensity (I)



Solar luminosity

$$L_\odot = 3.8\times10^{33}\ \mathrm{erg\ s^{-1}}$$

luminosity ($\mathrm{erg\ s^{-1}}$)

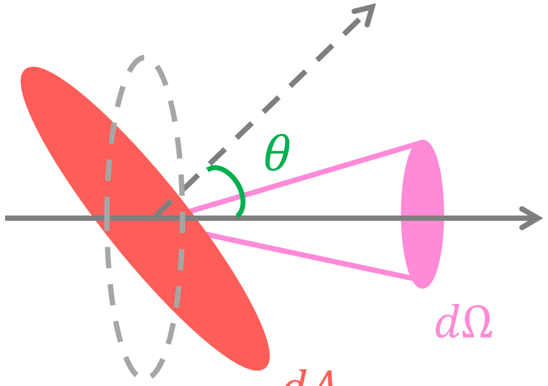
$$L \equiv \frac{dE}{dt}$$

$$I \equiv \frac{dE}{dt\, dA_\perp\, d\Omega}$$

By definition, $dA_\perp$ is normal to the direction of the radiation

Intensity and flux

Consider a radiation field (radiation in all directions), for a detector element (with photon collecting area dA) at some arbitrary orientation, we have



$$dF = I \cos\theta \, d\Omega$$

prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

prev. sl.

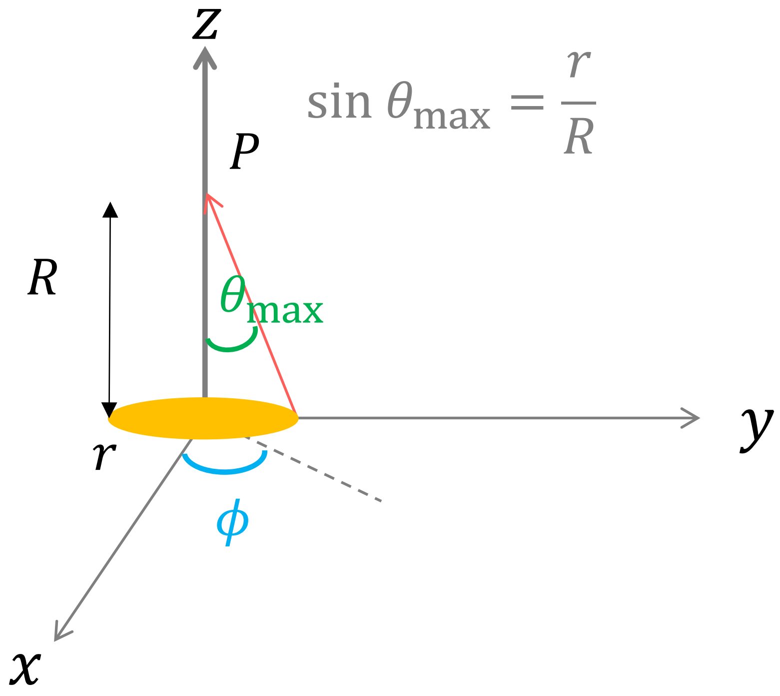
$$dF = \frac{dE}{dt \, dA}$$

effective area: $dA_\perp = dA \cos\theta$

Intensity and flux (example #1)

prev. sl. $dF = I\cos\theta\, d\Omega$

For an **isotropic** source $I(\theta,\phi) = \text{const.}$ with a flat sheet geometry (radiation into the upper hemisphere), the radiative flux of at P is



$d\Omega = \sin\theta\, d\theta\, d\phi$

$$F(R) = \int I\cos\theta\ d\Omega = I\int_0^{2\pi} d\phi \int_0^{\theta_{\max}} \cos\theta\sin\theta\, d\theta$$

$$= \pi I \sin^2\theta_{\max} = \pi I \left(\frac{r}{R}\right)^2$$

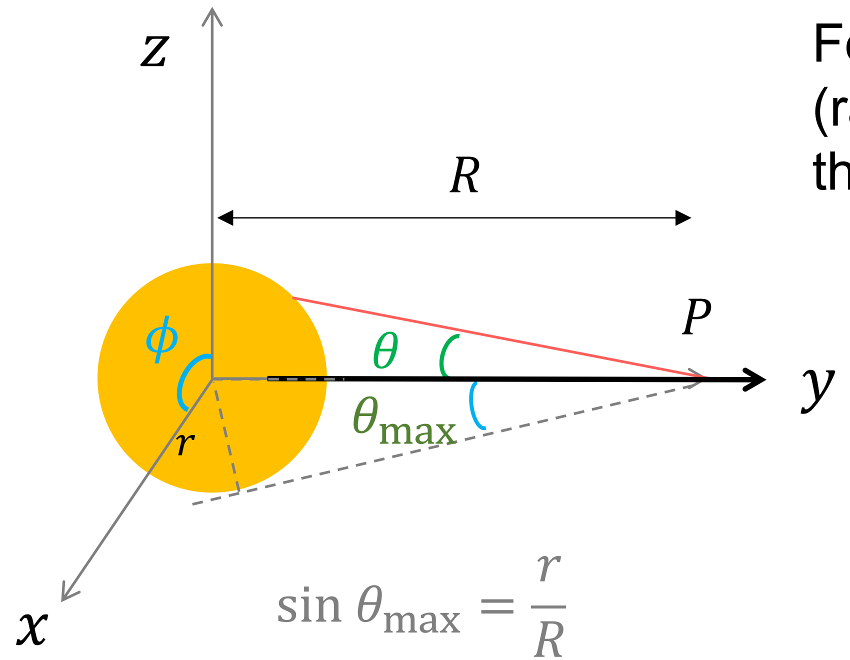
$$F(R=r) = \pi I$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\int_0^{\pi/2} \cos\theta\sin\theta\, d\theta = \frac{1}{2}\sin^2\frac{\pi}{2} = \frac{1}{2}$$

Intensity and flux (example #2)

prev. sl. $dF = I \cos\theta \, d\Omega$



$\sin\theta_{\max} = \frac{r}{R}$

For an **isotropic** source $I(\theta, \phi) = \mathrm{const.}$ with a spherical geometry (radiation from the surface of the sphere and within the sphere but the sphere is not transparent), the radiative flux at P

$$F(R) = \int I \cos\theta \, d\Omega = I \int_0^{2\pi} d\phi \int_0^{\theta_{\max}} \sin\theta \cos\theta \, d\theta$$

$$= \pi I \sin^2\theta_{\max} = \pi I \left(\frac{r}{R}\right)^2$$

$$\int_0^t \cos\theta \sin\theta \, d\theta = \frac{\sin^2 t}{2}$$

At $R = r$ (i.e. the surface of the sphere), we have

$$F = \pi I$$

Specific intensity

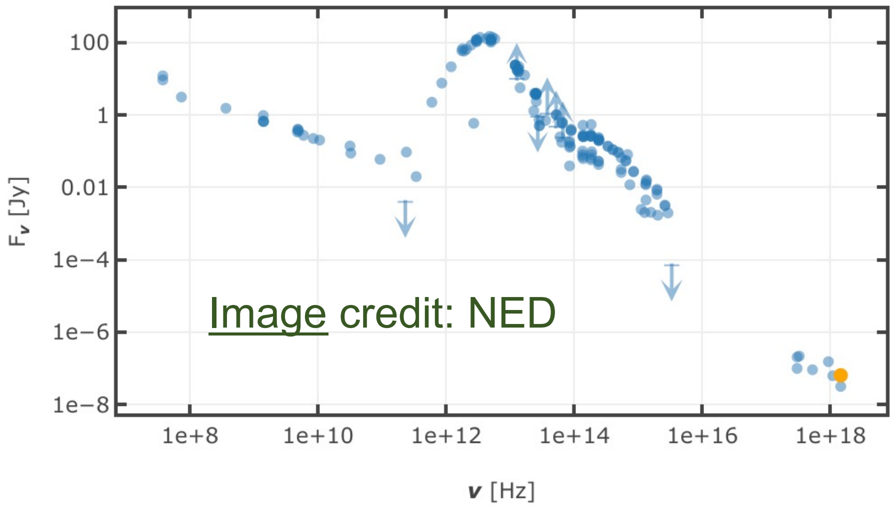
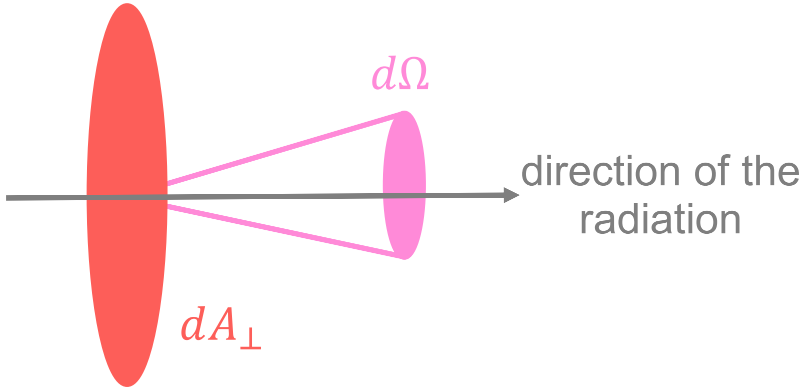


Image credit: NED

Radiative energy in a specific frequency crossing unit area ($dA_\perp$) in unit time in unit solid angle

specific intensity



$$dE_\nu = I_\nu \, dA \, d\nu \, d\Omega \, dt$$

$$I_\nu \equiv \frac{dE_\nu}{dA_\perp \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\,Hz^{-1}cm^{-2}s^{-1}sr^{-1}}$

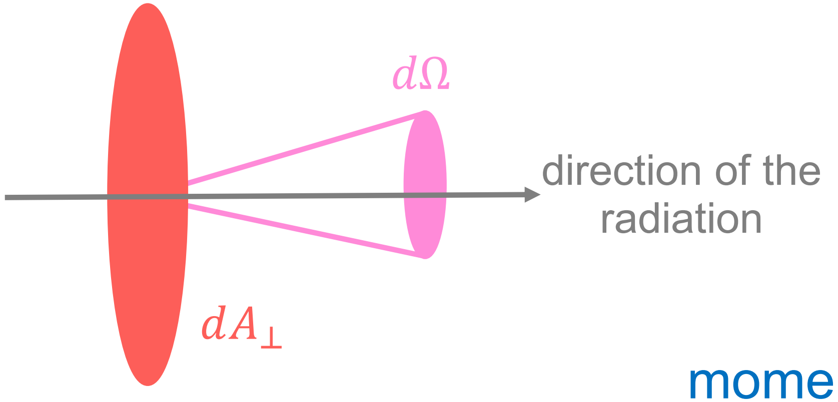
$$I_\nu = \frac{\lambda}{\nu} I_\lambda$$

prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

Specific flux and momentum

prev. sl. $F = \pi I$



$$F_\nu = \pi I_\nu$$

specific flux

specific intensity

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1}}$

$1\ \mathrm{erg} = 1\ \mathrm{g\ cm^2\ s^{-2}}$

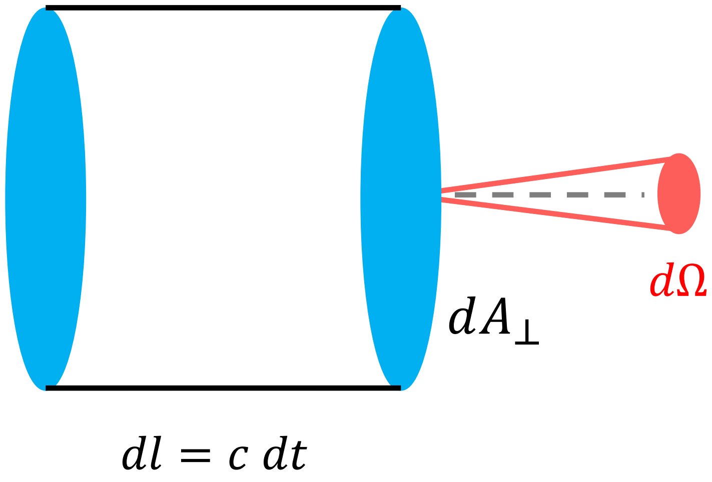
momentum $\longrightarrow$ $p_\nu = \dfrac{F_\nu}{c}$

$\mathrm{dyn\ Hz^{-1} cm^{-2}}$

$1\ \mathrm{dyn} = 1\ \mathrm{g\ cm\ s^{-2}}$

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\ dt$ and its volume $dA_\perp\ c\ dt$



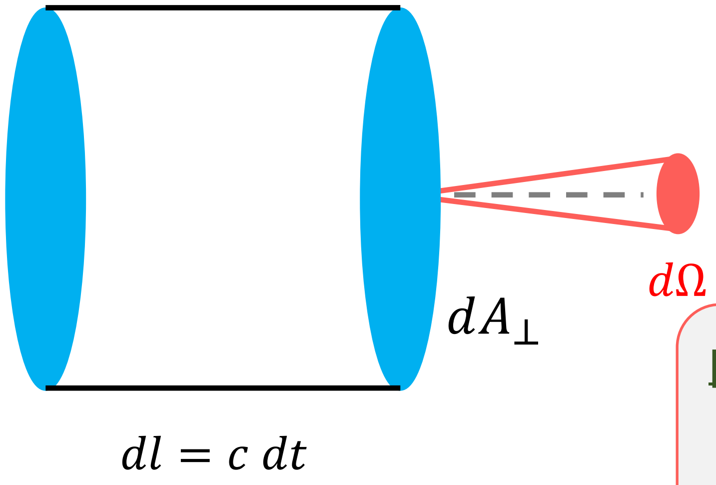
$$dE_\nu = u_\nu(\Omega) d\Omega\, d\nu\, dV$$
$$= u_\nu(\Omega) d\Omega\, d\nu\, dA_\perp\, c\, dt$$

specific energy density per unit solid angle $u_\nu(\Omega)$: Energy in a specific frequency per unit volume per solid angle

In time dt, all the radiation with velocity c in the cylinder will pass through.

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\,dt$ and its volume $dA_\perp\, c\, dt$



$$dE_\nu = u_\nu(\Omega) d\Omega\, d\nu\, dA_\perp c\, dt$$

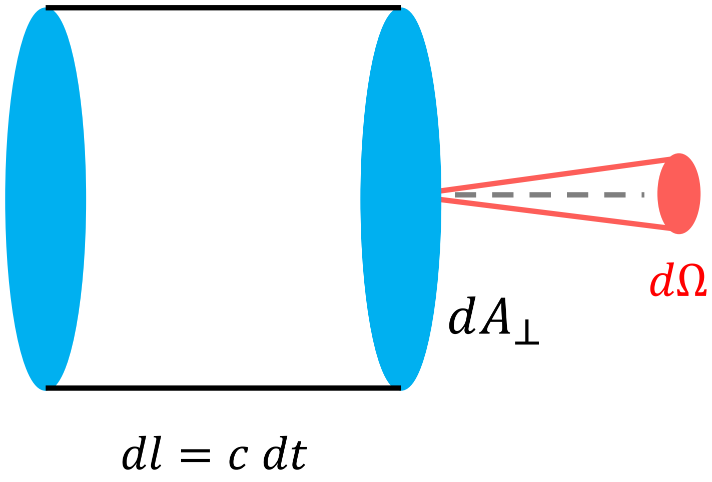
prev. sl.

$$dE_\nu = I_\nu\, dA_\perp d\nu\, d\Omega\, dt$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\,dt$ and its volume $dA_\perp\, c\, dt$



$$dE_\nu = u_\nu(\Omega) d\Omega\, d\nu\, dA_\perp\, c\, dt$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$u_\nu = \frac{1}{c}\int I_\nu\, d\Omega = \begin{cases} \dfrac{4\pi}{c} I_\nu & \text{isotropic} \\ \dfrac{4\pi}{c} \bar{I}_\nu & \text{anisotropic} \end{cases}$$

$\mathrm{erg\ Hz^{-1}\ cm^{-3}}$

specific energy density

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer
2.2.1 Spontaneous emission coefficient
2.2.2 Absorption coefficient
2.2.3 Optical depth and mean free path
2.2.4 Radiative transfer equation

2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

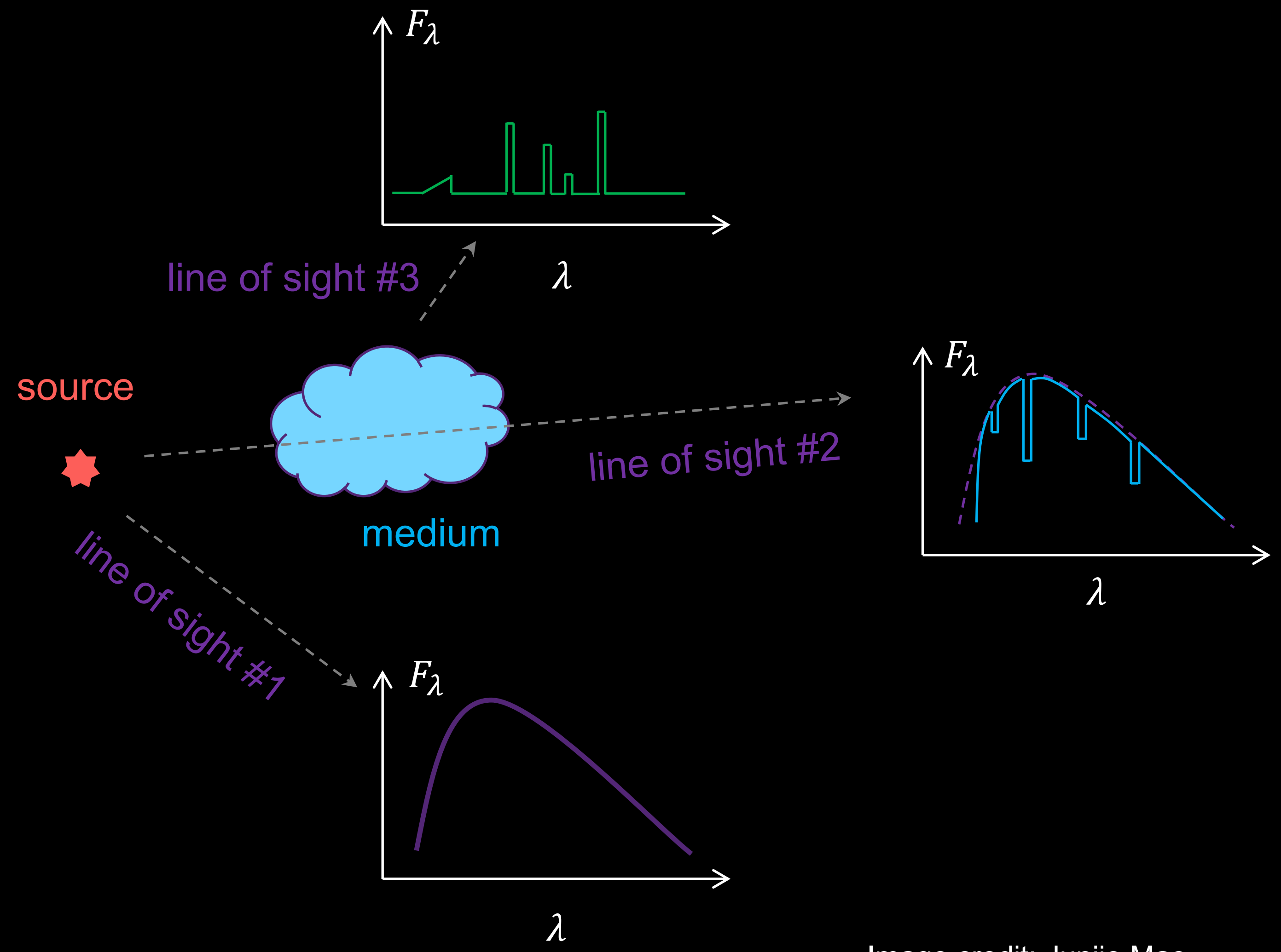


Image credit: Junjie Mao

Radiative transfer

- Radiation is one of the key processes to exchange energy for most celestial bodies
- When a ray passes through matter, the energy exchange process is referred to as radiation transfer

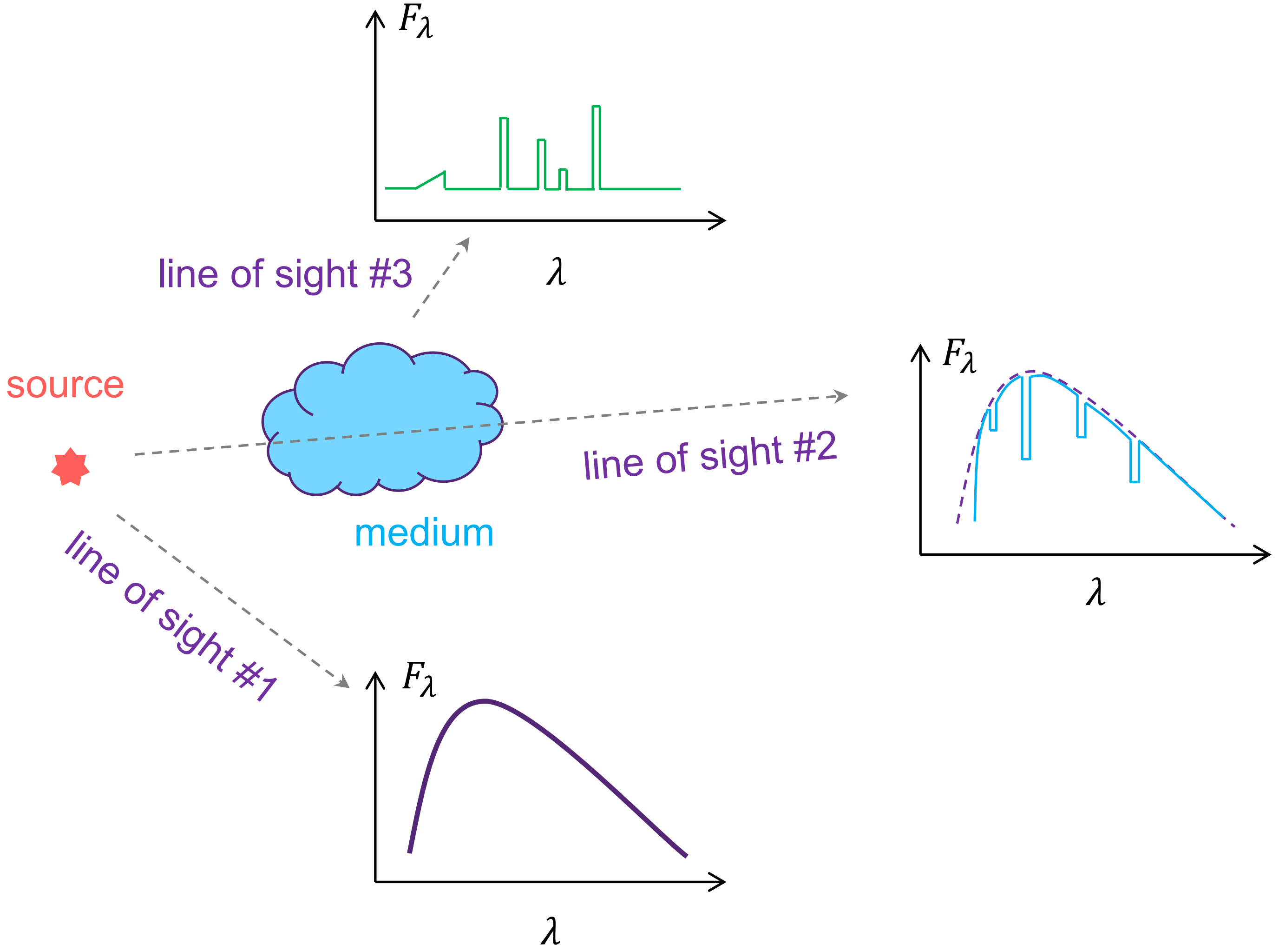
prev. sl.

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

unabsorbed flux

observed flux

$$F_{\mathrm{obs}} \le \frac{L}{4\,\pi\, d^2}$$



Spontaneous emission coefficient

Radiative energy emitted spontaneously in a specific frequency in unit time in unit solid angle in unit volume

spontaneous emission coefficient

$$dE_\nu = j_\nu \, dV \, d\nu \, d\Omega \, dt$$

$$j_\nu \equiv \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-3} s^{-1} sr^{-1}}$

$\mathrm{Energy\ (frequency)^{-1} (volume)^{-1} (time)^{-1} (solid\ angle)^{-1}}$

Spontaneous emission coefficient (cont.)

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$$dV = dA_\perp\, dl$$

prev. sl.

$$I_\nu = \frac{dE_\nu}{dA_\perp\, d\nu\, d\Omega\, dt}$$

$$I_\nu = j_\nu\, dl$$

Emissivity

For an isotropic emitter or a distribution of randomly oriented emitters, the emission can also be characterized via emissivity ϵ_ν, which is the radiative energy (angle integrated) emitted spontaneously in a specific frequency per unit time per unit mass

$$dE_\nu = \epsilon_\nu \rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}$$

normalized fraction of energy radiated into the solid angle $d\Omega$

$$\epsilon_\nu \equiv \frac{dE_\nu}{\rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}}$$

$\mathrm{erg\,Hz^{-1} g^{-1} s^{-1}}$

mass density $\mathrm{g\,cm^{-3}}$

Spontaneous emission coefficient

For an isotropic emitter or a distribution of randomly oriented emitters,

prev. sl.

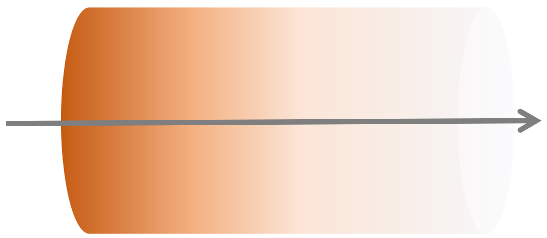
$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$$\epsilon_\nu = \frac{dE_\nu}{\rho\, dV dt\, d\nu \frac{d\Omega}{4\pi}}$$

$$j_\nu = \frac{\epsilon_\nu\, \rho}{4\pi}$$

Absorption coefficient

Loss of intensity in a beam as radiation travels a distance dl



absorption coefficient (positive values for energy taken out of the beam)

$$dI_\nu \equiv -\alpha_\nu \, I_\nu \, dl$$

$$\alpha_\nu = \rho \, \kappa_\nu$$

mass density $\mathrm{g\,cm^{-3}}$

mass absorption coefficient (a. k. a. opacity coefficient)

$$\int_{I_\nu^{\mathrm{in}}}^{I_\nu^{\mathrm{out}}} \frac{dI_\nu}{I_\nu} = -\int \alpha_\nu \, dl$$

$$\ln \frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = -\int \alpha_\nu \, dl$$

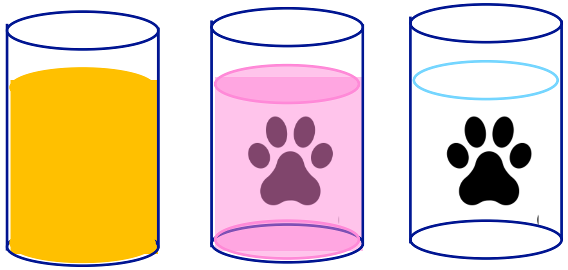
$$\frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = \exp\left(-\int \alpha_\nu \, dl\right)$$

Optical depth

optical depth absorption coefficient

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu) \qquad \tau_\nu \equiv \int \alpha_\nu dl$$

Optical thin	$\tau_\nu \ll 1, I_\nu^{\text{out}} \simeq I_\nu^{\text{in}}$
	$\tau_\nu = 1, I_\nu^{\text{out}} \sim 0.368\, I_\nu^{\text{in}}$
Optical thick	$\tau_\nu \gg 1, I_\nu^{\text{out}} \sim 0$



Mean free path

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu)$$

For homogeneous medium, $\bar{\tau}_\nu = \alpha_\nu \bar{l}_\nu$

$$\bar{\tau}_\nu = \frac{\int_0^\infty \tau_\nu \exp(-\tau_\nu)\, d\tau_\nu}{\int_0^\infty \exp(-\tau_\nu)\, d\tau_\nu} = 1$$

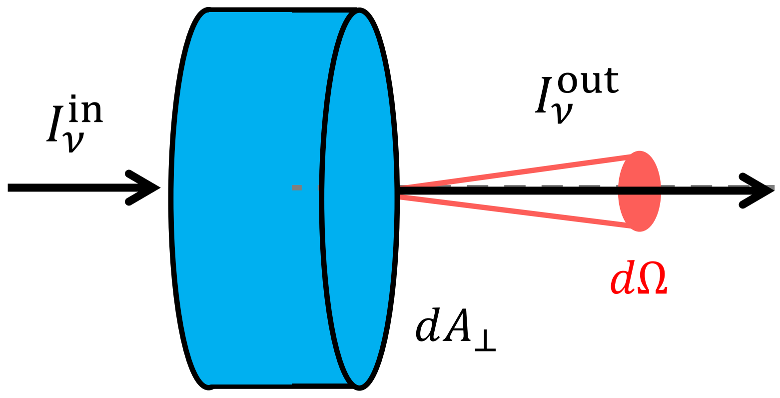
$$\int_0^\infty x \exp(-x)\ dx = 1$$

$$\int_0^\infty \exp(-x)\ dx = 1$$

Mean free path (the average distance a photon can travel within a medium without being absorbed)

$$\bar{l}_\nu = \frac{1}{\alpha_\nu}$$

Radiative transfer equation



$$dE^{\text{in}} = I_\nu \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{out}} = (I_\nu + dI_\nu) \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{em}} = j_\nu \, d\nu \, d\Omega \, dA_\perp \, dl \, dt$$

$$dE^{\text{abs}} = -\alpha_\nu dE^{in} dl$$

Radiative transfer equation (cont.)

Conservation of energy (no loss or gain)

$$dE^{\text{out}} = dE^{\text{in}} + dE^{\text{abs}} + dE^{\text{em}}$$

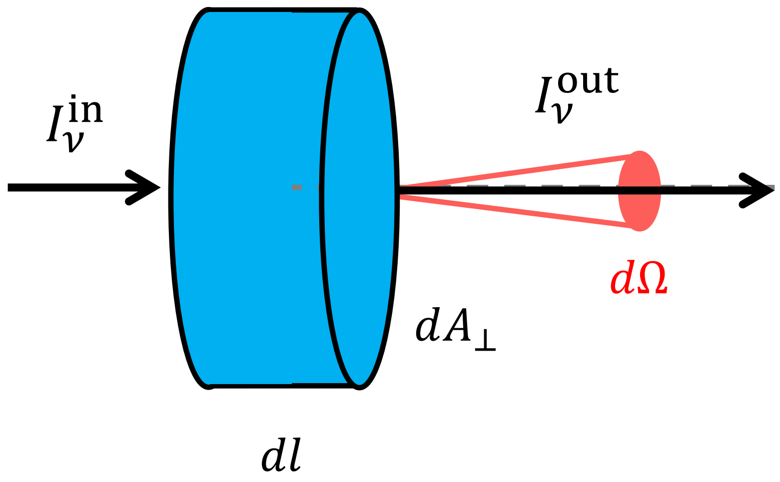
prev. sl.

$$dE^{\text{in}} = I_\nu \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{out}} = (I_\nu + dI_\nu) \, d\nu \, d\Omega \, dA_\perp \, dt$$

$$dE^{\text{em}} = j_\nu \, d\nu \, d\Omega \, dA_\perp \, dl \, dt$$

$$dE^{\text{abs}} = -\alpha_\nu dE^{in} dl$$

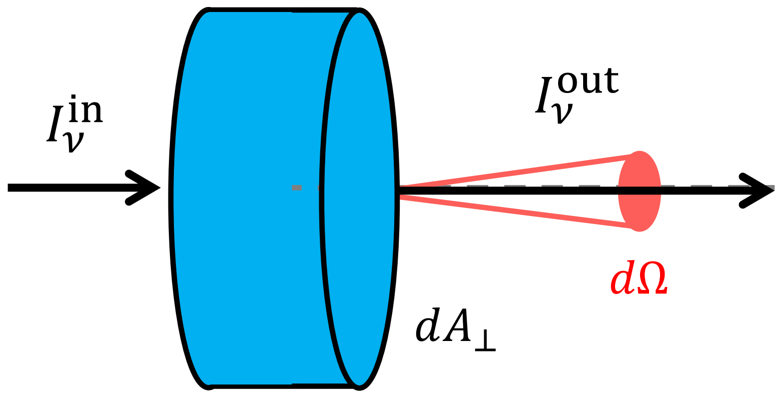


$$\frac{dI_\nu}{dl} = -\alpha_\nu \, I_\nu + j_\nu$$

Radiative transfer equation (cont.)

prev. sl. $$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$d\tau_\nu = \alpha_\nu dl$



$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

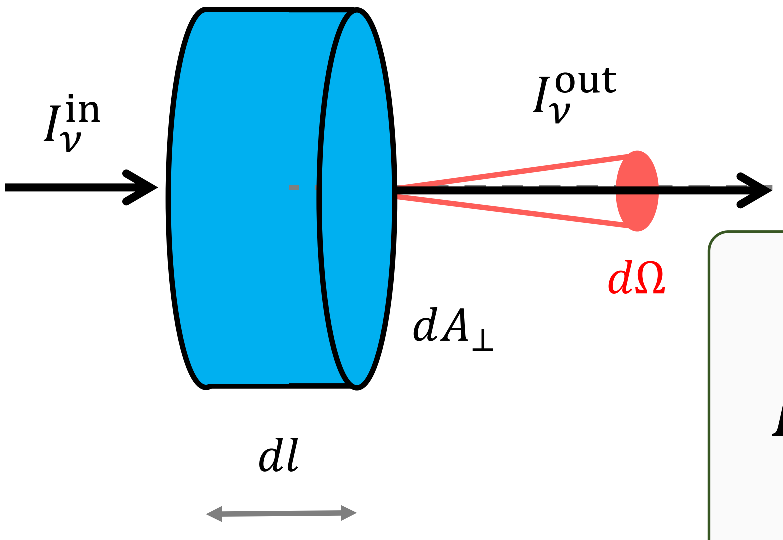
Source function

Solution to the RT equation

prev. sl. $$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} = S_\nu e^{\tau_\nu} - I_\nu e^{\tau_\nu}$$

$$e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = \frac{d}{d\tau_\nu}\left(I_\nu e^{\tau_\nu}\right) = S_\nu e^{\tau_\nu}$$



$$\tau_{\nu,0} = \int \alpha_\nu dl$$

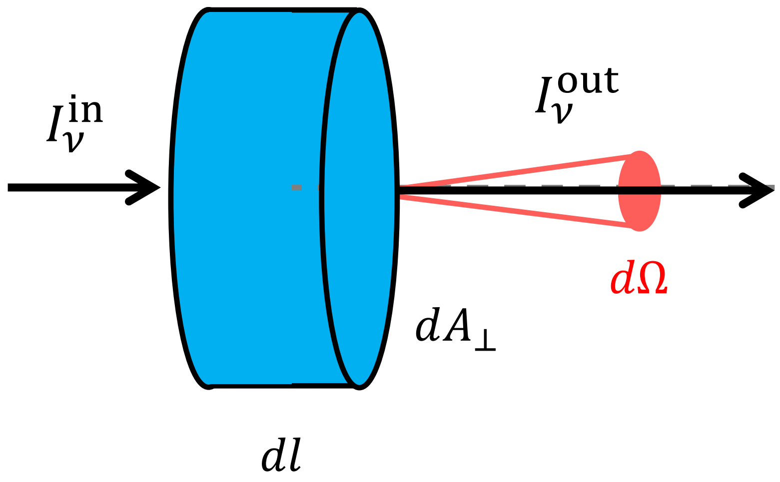
$$I_\nu^{\rm out} = I_\nu^{\rm in} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

see Sect. 3.2 of 《天体物理中的辐射机制》by 尤峻汉 (p81–82)

Radiative transfer equation (no intrinsic emission)

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$



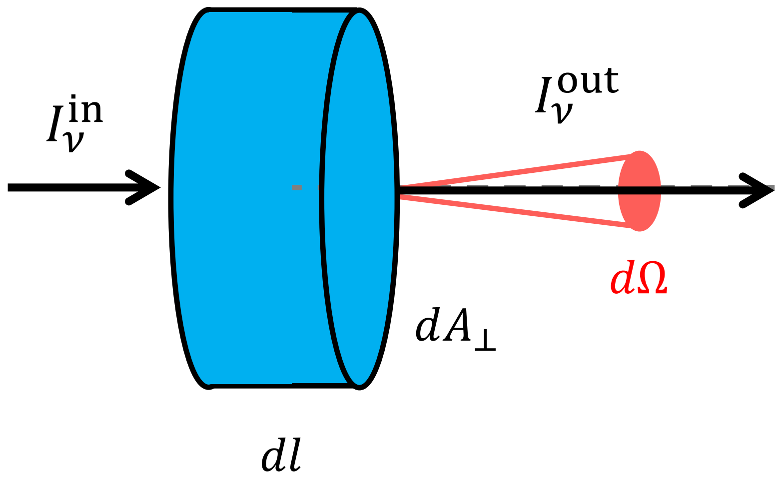
Case #1: no intrinsic emission $(j_\nu = S_\nu = 0)$

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}}$$

Radiative transfer equation (no external emission)

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$



Case #2: no external emission ($I_\nu^{\text{in}} = 0$)

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer

2.3 Thermal radiation
2.3.1 Blackbody radiation spectrum
2.3.2 Blackbody emission intensity
2.3.3 Blackbody emission flux
2.3.4 Thermal equilibrium
2.3.5 Local thermal equilibrium

2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

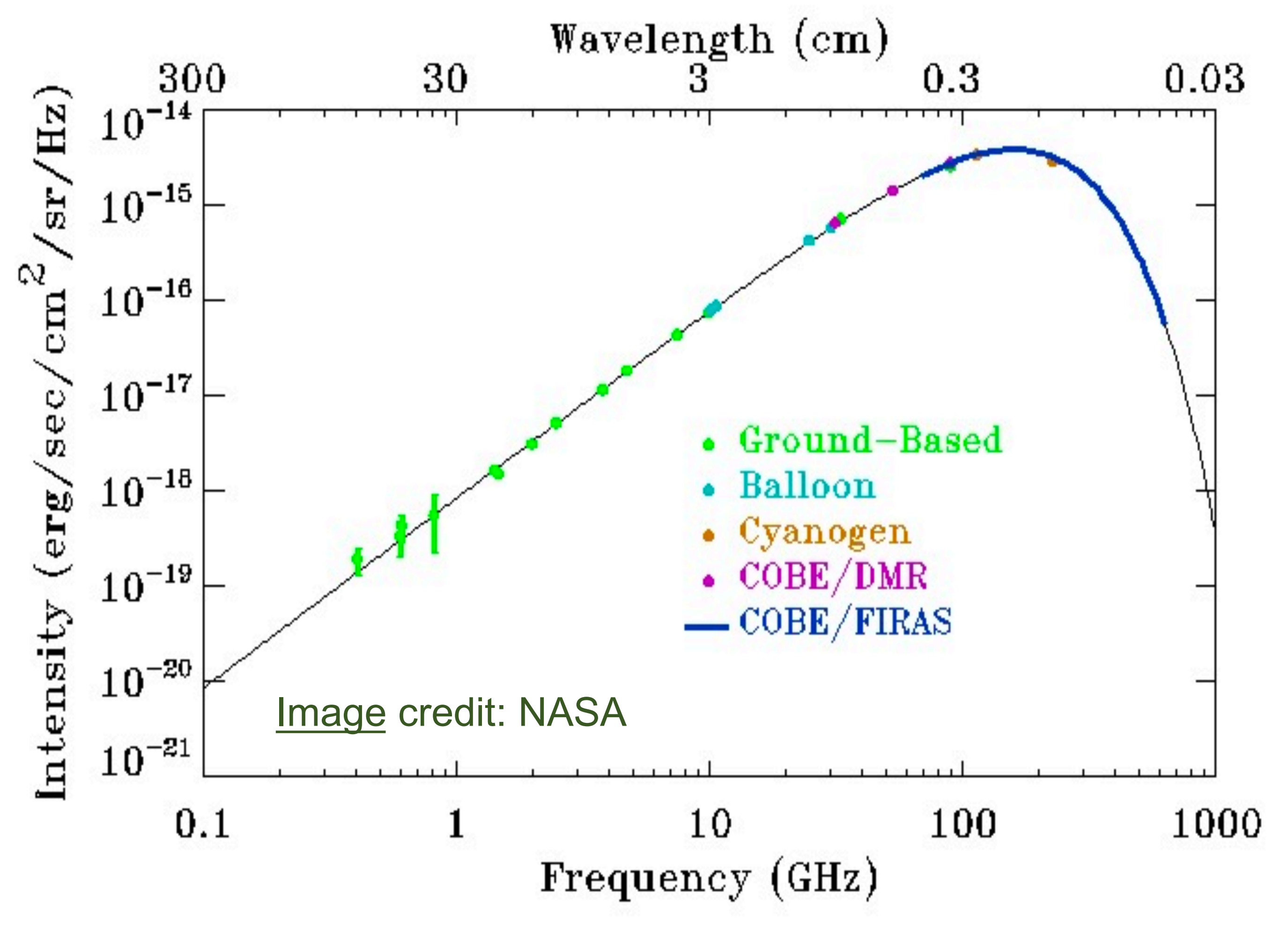


Image credit: NASA

Blackbody radiation spectrum

Thermal radiation is radiation emitted by matter in thermal equilibrium. In this case, the free electrons follows the Maxwellian distribution

$$f(v)dv = 4\pi\left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT_e}\right)$$

Blackbody radiation spectrum is the Planck spectrum

$$h\nu_{\max} = 2.82\ kT$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)^{-1}$$

Caution that even if particles in the matter follow the Maxwellian distribution, radiation emitted by this matter does **not** necessarily have to be thermal radiation

Blackbody emission intensity

$$I_\nu^{\mathrm{BB}} \equiv B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

Blackbody emission intensity

$$I^{\mathrm{BB}} = \int_0^\infty \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} d\nu$$

$$= \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x - 1)^{-1}dx$$

$$x = \frac{h\nu}{kT}$$

Blackbody emission flux

prev. sl.

$$I^{\mathrm{BB}} = \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x-1)^{-1}dx$$

Blackbody emission flux

$$F^{\mathrm{BB}} = \pi I^{\mathrm{BB}} = \sigma T^4$$

$$\int_0^\infty x^3(e^x-1)^{-1}dx = \frac{\pi^4}{15}$$

$$\sigma = \frac{2\pi^5k^4}{15\,h^3c^2}$$

$$= 5.670\times10^{-5}\ \mathrm{erg\,s^{-1}cm^{-2}K^{-4}}$$

Stefan-Boltzmann constant

Blackbody radiation flux depends **only** on the temperature

Asymptotic behavior

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

$$\frac{h\nu}{kT} \ll 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \to \frac{h\nu}{kT} \qquad I_\nu^{\mathrm{RJ}} = \frac{2\nu^2}{c^2} kT$$

Rayleigh-Jeans Law

$$\frac{h\nu}{kT} \gg 1 \qquad \exp\left(\frac{h\nu}{kT}\right) - 1 \to \exp\left(\frac{h\nu}{kT}\right) \qquad I_\nu^{\mathrm{W}} = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$$

Wien Law

Thermodynamic Equilibrium (TE)

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

For medium in thermodynamic equilibrium, Kirchoff's law links the spontaneous emission coefficient and emission source function with Planck's law

$$j_\nu \equiv \alpha_\nu B_\nu(T)$$
$$S_\nu \equiv B_\nu\,(T)$$

TE (cont.)

prev. sl.

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

$$I_\nu^{\text{out}} = B_\nu(T)\big(1 - \exp(-\tau_{\nu,0})\big)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \gg 1 \qquad I_\nu^{\text{out}} = B_\nu(T)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \ll 1 \qquad I_\nu^{\text{out}} = \tau_{\nu,0} B_\nu(T)$$

Only if the medium is in TE and it is optically thick, then its radiation follows the Planck's Law.
For non-thermal processes, the source function can only be derived from j_ν and α_ν.

Local Thermodynamic Equilibrium (LTE)

LTE differs from TE in that the radiation field does not need to follow Planck's law, i.e., $I_\nu \neq B_\nu(T)$.

The assumption of LTE is valid if either of the follow two applies:

- ❑ collisional processes among particles dominate photon-related processes
- ❑ collisional processes among particles are in equilibrium with photo-related processes

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer
2.3 Thermal radiation

2.4 Scattering
2.4.1 Pure scattering
2.4.2 Emission, absorption, and scattering
2.4.3 Absorption and scattering probabilities
2.4.4 Effective mean free path

2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

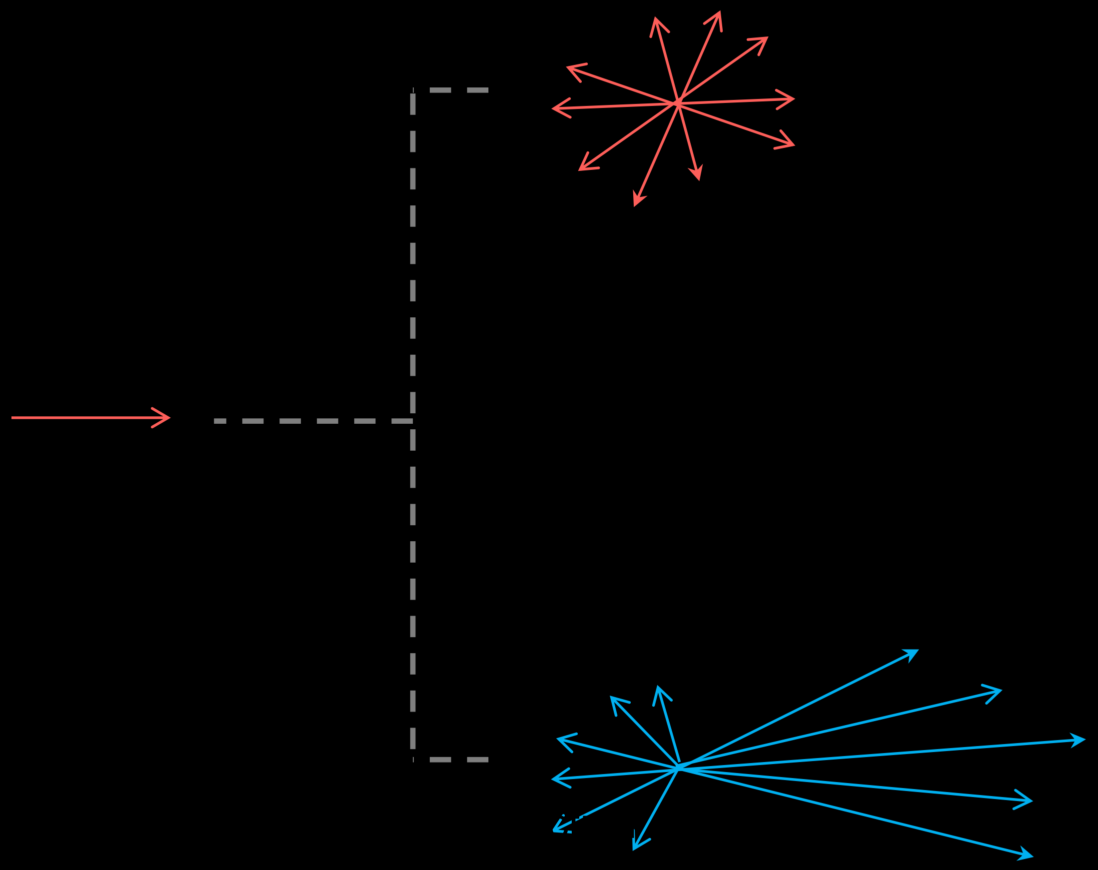
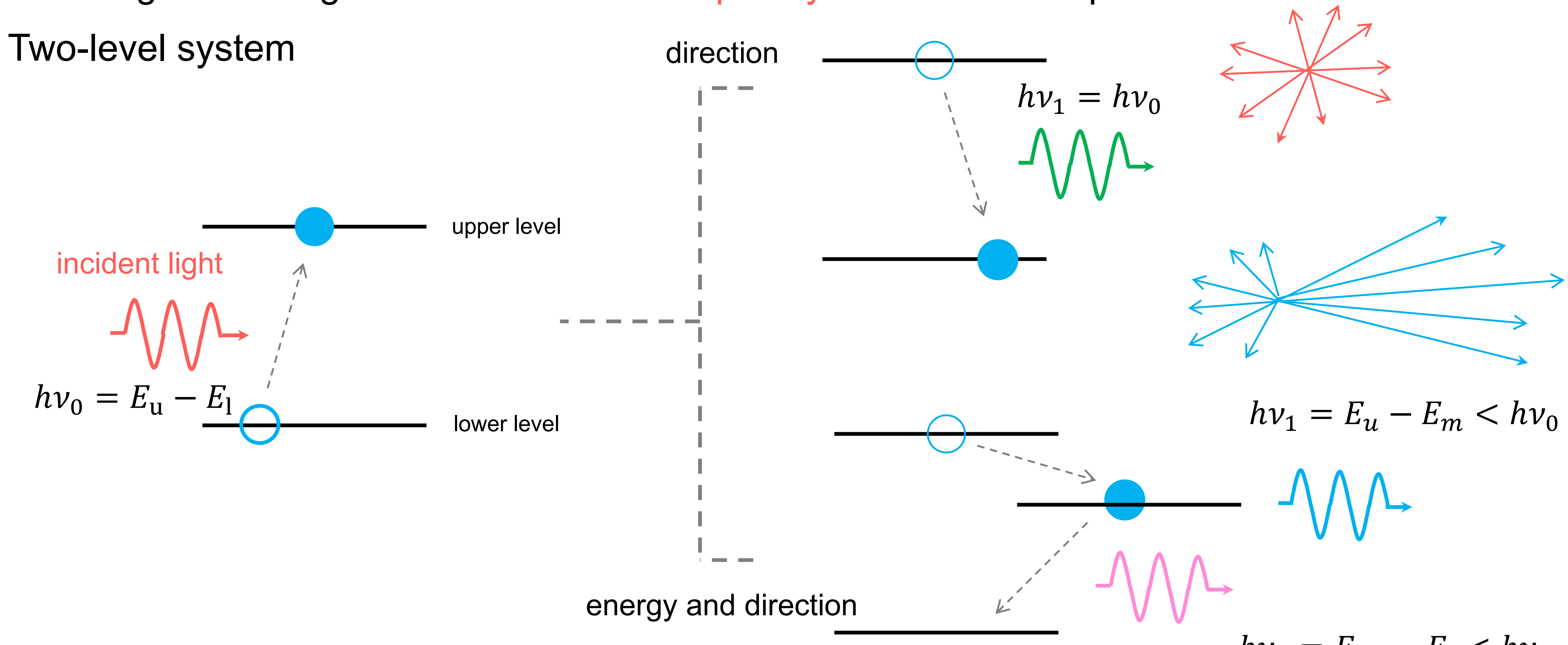


Image credit: Junjie Mao

Scattering

Scattering significantly complicates the radiative transfer.
Scattering can change the direction and frequency of the incident photon.



Pure scattering

Consider a medium has only isotropic scattering (no emission or absorption), which only changes the direction of the incident photon

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}$$

scattered outside the line of sight

scattered back into the line of sight

scattering coefficient

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{sca}}} = -I_\nu + S_\nu^{\mathrm{sca}} \qquad d\tau_\nu^{\mathrm{sca}} \equiv \sigma_\nu^{\mathrm{sca}} dl$$

scattering optical depth

Source function for scattering

Random walk

The mean displacement traveled by photons is (order-of-magnitude estimation)

see Sect. 1.7 of the REF book (p35) by Rybicki & Lightman

$$\bar{l}_\nu^{\mathrm{RM}} \sim \sqrt{N}\,\bar{l}_\nu$$

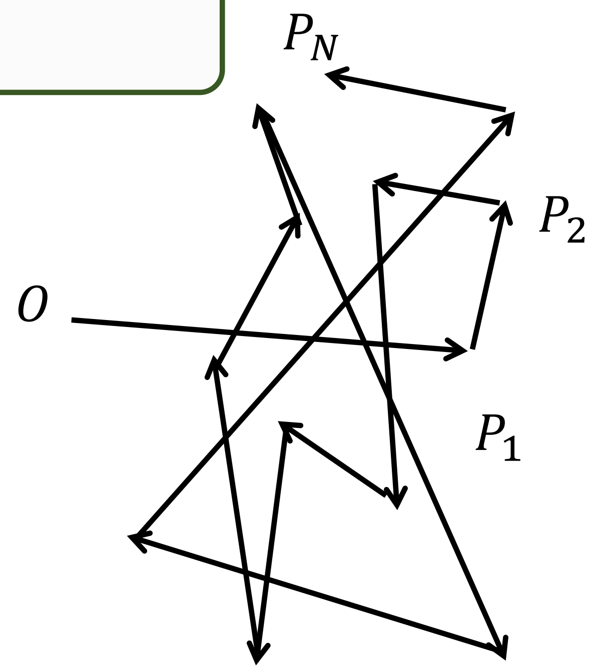
$\bar{l}_\nu = \frac{1}{\sigma_\nu^{\mathrm{sca}}}$: scattering mean free path

Number of scattering

size scale of the medium

for $\tau_\nu^{\mathrm{sca}} \gg 1$ $\quad N \sim \frac{L^2}{\bar{l}_\nu^2} = (\sigma_\nu^{\mathrm{sca}} L)^2 \sim (\tau_\nu^{\mathrm{sca}})^2$

for $\tau_\nu^{\mathrm{sca}} \ll 1$ $\quad N \sim \sigma_\nu^{\mathrm{sca}} L \sim \tau_\nu^{\mathrm{sca}}$



Emission + absorption + scattering

Considering scattering + absorption + emission for a medium in TE

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + j_\nu - \alpha_\nu I_\nu$$

$$j_\nu = \alpha_\nu B_\nu$$

$$\frac{dI_\nu}{dl} = -(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) I_\nu + \ (\sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + \alpha_\nu B_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{AS}}} = -I_\nu + S_\nu^{\mathrm{EAS}}$$

$$S_\nu^{\mathrm{EAS}} \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Source function for emission, absorption, and scattering

$$(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) dl = d\tau_\nu^{\mathrm{AS}}$$

Absorption and scattering probability

An alternative way to define the source function

$$S_\nu^{\mathrm{EAS}} = \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Introducing absorption and scattering probabilities

absorption probability

$$\epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

scattering probability

$$1 - \epsilon_\nu = \frac{\sigma_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

$$S_\nu^{\mathrm{EAS}} = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) S_\nu^{\mathrm{sca}}$$

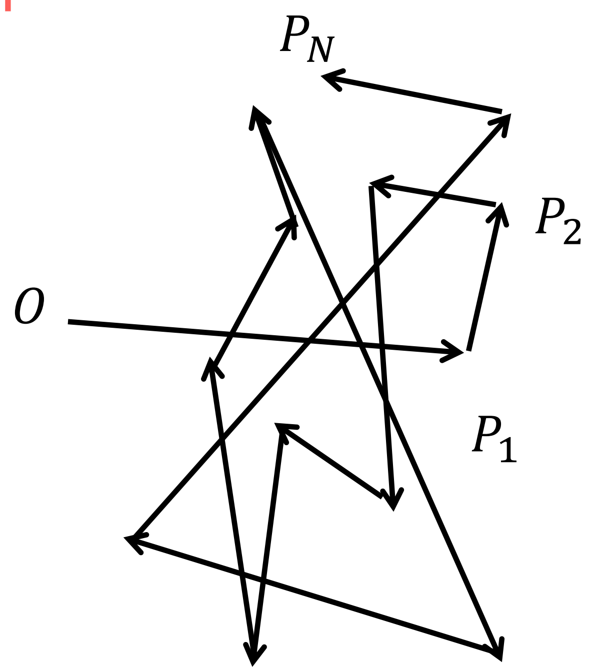
Effective mean free path

prev. sl. $\epsilon_\nu = \dfrac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$

After $N = \epsilon_\nu^{-1}$ steps, a photon is **truly** absorbed

$\bar{l}_\nu^{\mathrm{AS}} = \frac{1}{\alpha_\nu+\sigma_\nu^{\mathrm{sca}}}$: mean free path

$$\left(\bar{l}_\nu^{\mathrm{RM}}\right)^2 = N\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2 = \frac{\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2}{\epsilon_\nu} = \frac{1}{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}$$



$$l_\nu^{\mathrm{eff}} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}}$$

effective mean free path
diffusion length
thermalization length

for $\alpha_\nu \ll \sigma_\nu^{\mathrm{sca}}$ $\quad l_\nu^{\mathrm{eff}} = \dfrac{1}{\sqrt{\alpha_\nu \sigma_\nu^{\mathrm{sca}}}}$

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Escape probability

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

Without external emission ($I_\nu^{\text{in}} = 0$)

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

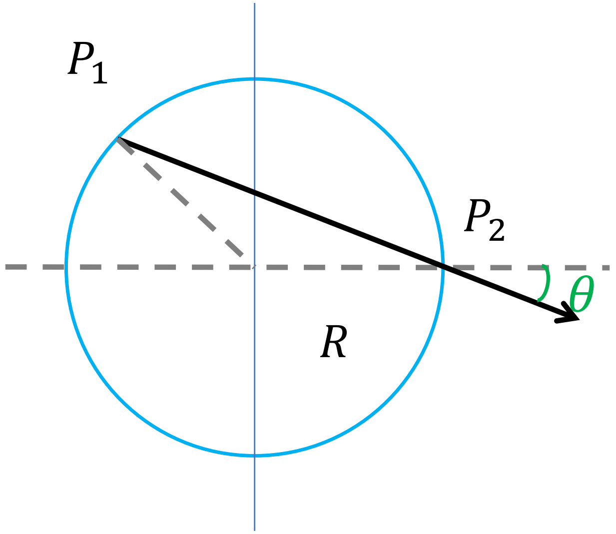
If we know the escape probability P_{esc}, it might be easier to calculate the emergent emission than a full radiative transfer calculation.

$$j_\nu^{\text{eff}} \equiv j_\nu P_{\text{esc}}$$

Spherical geometry + homogenous medium

homogenous→ $\tau_\nu = \alpha_\nu R$

Radius: R, absorption coefficient α_ν, line emission coefficient j_ν



Consider a photon emitted from P_1, when traveling to P_2, the optical depth is $2\tau_\nu \cos\theta$

prev. sl. $I_\nu^{\mathrm{out}} = \int S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$

prev. sl. $\frac{j_\nu}{\alpha_\nu} = S_\nu$

$$I_\nu^{\mathrm{out}} = \int_0^{2R\cos\theta} j_\nu e^{-\alpha_\nu r} dr$$

$$= \frac{j_\nu}{\alpha_\nu}\left(1 - \exp(-2\tau_\nu \cos\theta)\right)$$

$$\int_0^b \exp(-a\, r)\, dr = \frac{1 - \exp(-a\, b)}{a}$$

Flux

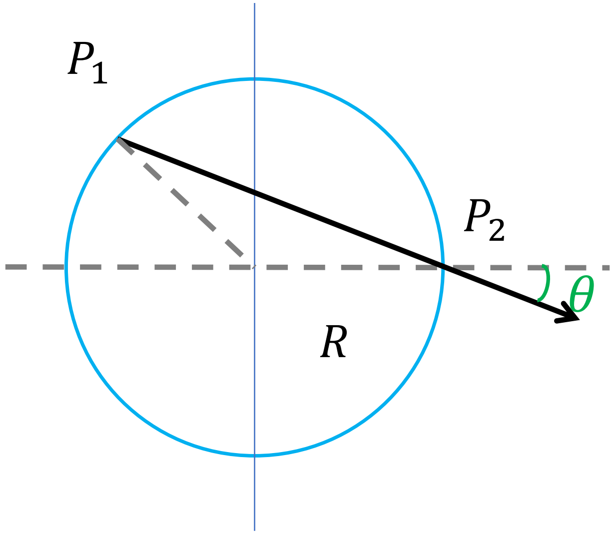
prev. sl.

$$I_\nu^{\rm out} = \frac{j_\nu}{\alpha_\nu}\left(1 - \exp(-2\tau_\nu \cos\theta)\right)$$

prev. sl.

$$dF = I\cos\theta\, d\Omega$$

homogenous→ $\tau_\nu = \alpha_\nu R$



Flux emitted from P_2

$$F_{\nu,\rm esc} = \int I_\nu^{\rm out}\cos\theta\;\, d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_\nu^{\rm out}\cos\theta\sin\theta\, d\theta$$

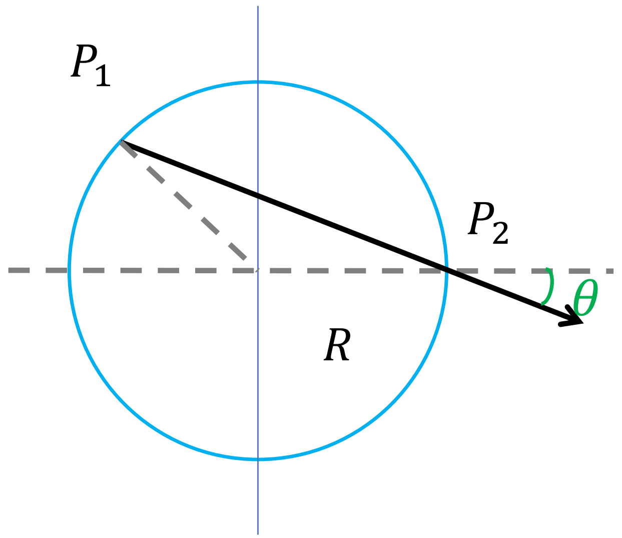
$$= \frac{\pi j_\nu}{2\alpha_\nu\tau_\nu^2}\left(2\tau_\nu^2 - 1 + e^{-2\tau_\nu}(2\tau_\nu + 1)\right)$$

$$\int_0^{\pi/2} \cos\theta\sin\theta\,(1 - \exp(-2\,a\cos\theta))d\theta = \frac{2a^2 - 1 + \exp(-2a)\,(2a+1)}{4\,a^2}$$

Escape probability

prev. sl.

$$F_{\nu,\mathrm{esc}} = \frac{\pi j_\nu}{2\alpha_\nu \tau_\nu^2}\left(2\tau_\nu^2 - 1 + e^{-2\tau_\nu}(2\tau_\nu + 1)\right)$$



c.f. a photon emitted from P_1, when travelling to P_2 without any absorption

$$I_{\nu,0}^{\mathrm{out}} = \int_0^{2R\cos\theta} j_\nu dr = j_\nu 2R\cos\theta$$

prev. sl.

$$I_\nu^{\mathrm{out}} = \int_0^{2R\cos\theta} j_\nu e^{-\alpha_\nu r} dr$$

$$F_{\nu,0} = \int_0^{2\pi} d\phi \int_0^{\pi/2} I_{\nu,0}^{\mathrm{out}} \cos\theta \sin\theta \, d\theta = \frac{4\pi R\, j_\nu}{3}$$

$$\int_0^{\pi/2} \cos^2\theta \sin\theta \, d\theta = \frac{1}{3}$$

prev. sl.

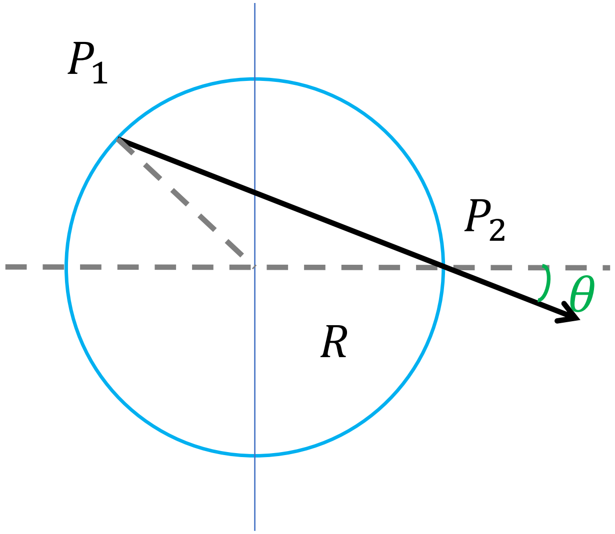
$$dF = I\cos\theta \, d\Omega$$

$$P_{\mathrm{esc}} = \frac{F_{\nu,\mathrm{esc}}}{F_{\nu,0}} = \frac{3}{8\tau_\nu^3}\left(2\tau_\nu^2 - 1 + e^{-2\tau_\nu}(2\tau_\nu + 1)\right)$$

Asymptotic behavior

prev. sl.

$$P_{\text{esc}} = \frac{F_{\nu,\text{esc}}}{F_{\nu,0}} = \frac{3}{8\tau_\nu^3}\left(2\tau_\nu^2 - 1 + e^{-2\tau_\nu}(2\tau_\nu + 1)\right)$$



$$\tau_\nu \gg 1 \;\to P_{\text{esc}} \sim \frac{3}{4\,\tau_\nu} \sim \frac{1}{\tau_\nu} = \frac{1}{\alpha_\nu R}$$

$$\tau_\nu \sim 0 \;\to P_{\text{esc}} \sim 1$$

$$\frac{x^2}{2} - 1 + e^{-x}(x+1), x = 2\tau_\nu$$

$$= \frac{x^2}{2} - 1 + \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6}\right)(x+1)$$

$$\exp(-x) = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$$

$$\frac{3}{x^3}\left(\frac{x^2}{2} - 1 + x + 1 - x^2 - x + \frac{x^3}{2} + \frac{x^2}{2} - \frac{x^4}{6} - \frac{x^3}{6}\right) = \frac{3}{x^3}\left(\frac{x^3}{3} - \frac{x^4}{6}\right) \to 1$$

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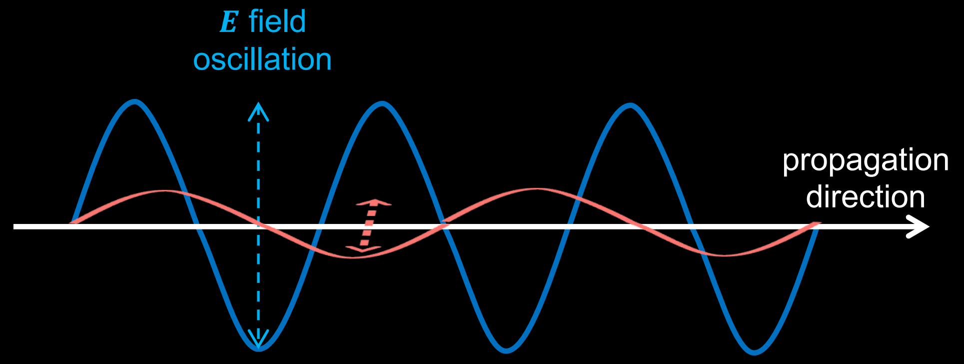
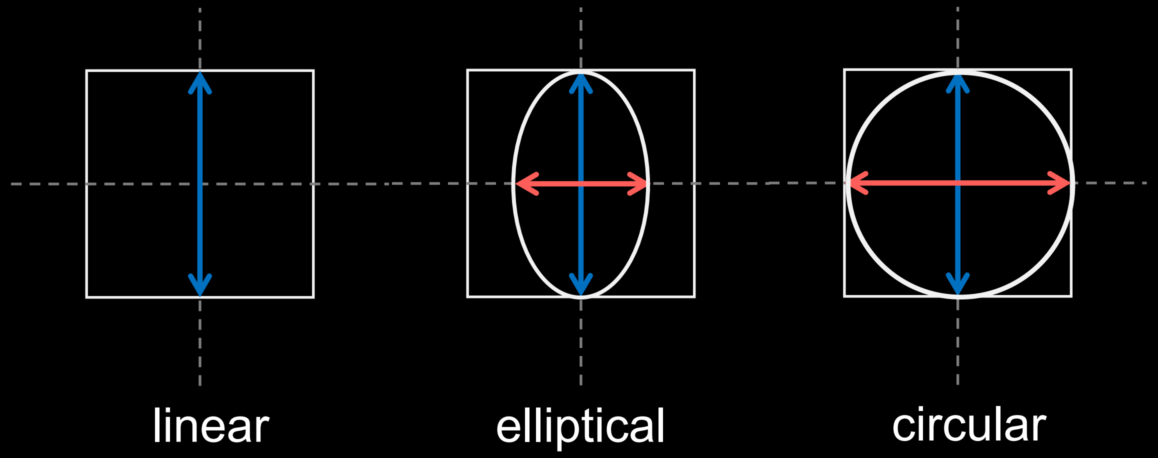
2.5 Scattering

2.6 Polarization
2.6.1 Stokes parameters
2.6.2 Polarization fraction
2.6.3 CMB polarization

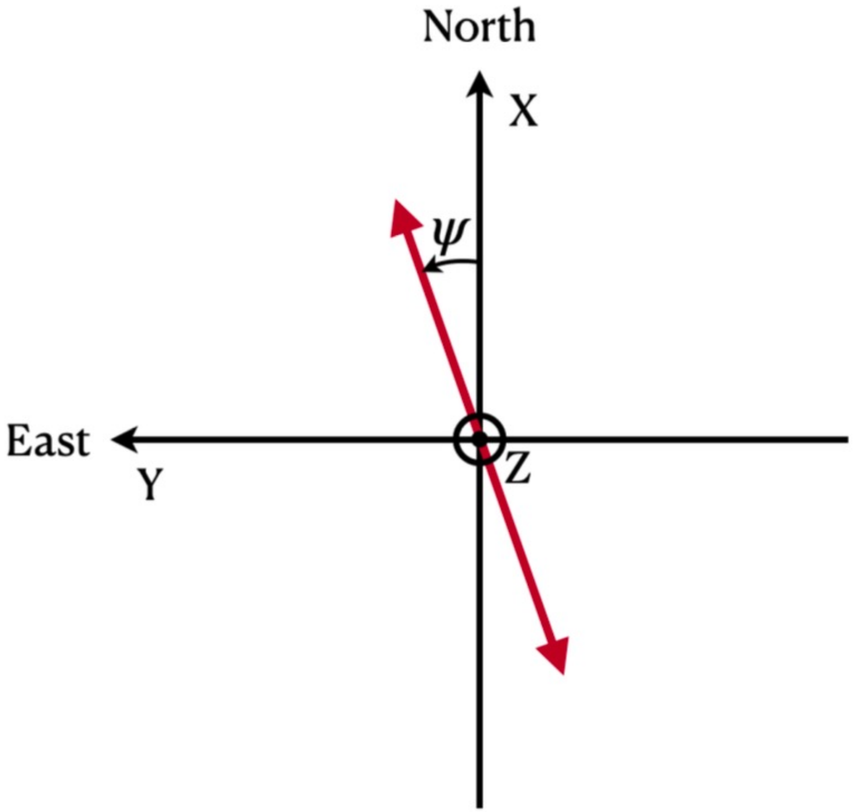
2.7 Dispersion and rotation measures

Image credit: Junjie Mao





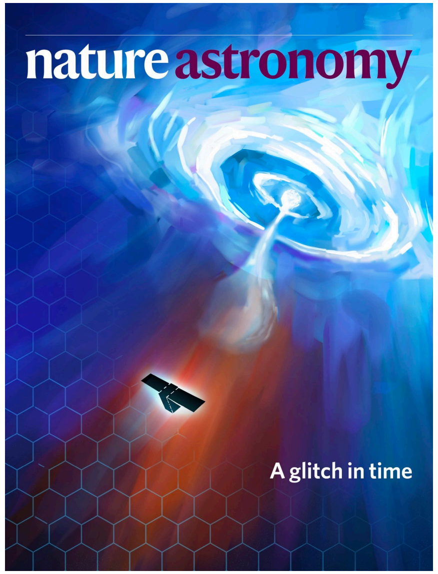
Polarization



IAU reference frame for polarization
(e.g., Ferriere et al. 2021)

IAU definition:

- ✓ X and Y are the Cartesian coordinates of the plane of the sky
- ✓ Z is along the line of sight (+Z towards the observer)
- ✓ ψ: linear polarization angle (measured counterclockwise from the North)



PolarLight is a compact soft X-ray polarimeter onboard a CubeSat, which was launched into a low-Earth orbit on Oct. 29, 2018 (Feng et al. 2020)

Stokes parameters

Stokes parameters, all related to intensity, are **observable**

- ❑ I: Intensity
- ❑ $\sqrt{Q^2 + U^2}$: Intensity of linear polarization
- ❑ V: Intensity of circular polarization

$$S_0 \equiv I$$
$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$
$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2 + U^2}}, -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$$

$$\tan 2\psi = U/Q, 0 \le \psi \le \pi$$

linear polarization angle

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

polarization fraction

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized

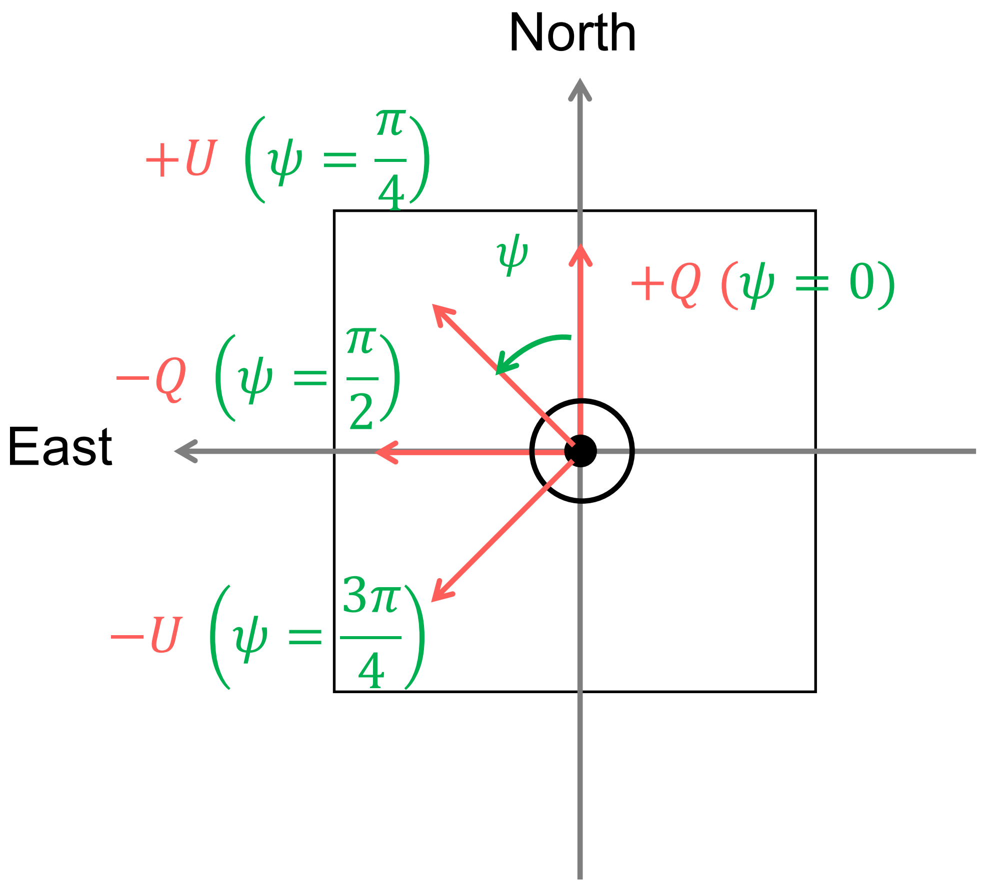
Linear polarization

For $V = 0$ $(\chi = 0)$, we have the linear polarization

$$S_0 = I$$
$$S_1 = Q = Ip\cos 2\psi$$
$$S_2 = U = Ip\sin 2\psi$$
$$S_3 = V = 0$$

$\cos\pi = -1$

$\sin\left(\frac{3\pi}{2}\right) = -1$



prev. sl.

$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\psi = U/Q$$

linear polarization angle

e.g., Hamaker & Bregman (1996)

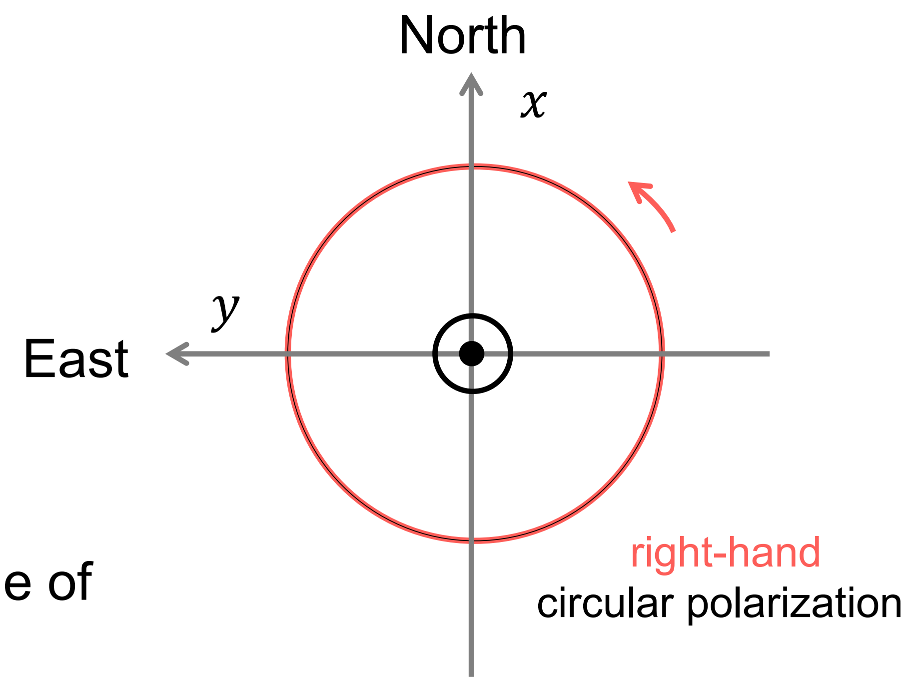
Circular polarization

prev. sl.

$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$

For $Q = U = 0$ ($\chi = \pm\pi/4$), we have the circular polarization

$$S_0 = I$$
$$S_1 = Q = 0$$
$$S_2 = U = 0$$
$$S_3 = V = Ip\sin 2\chi$$



IAU definition:

- ✓ $+V$ for the right-handed circular polarization
- ✓ For the right-handed circular polarization, the position angle of $\vec{E}$ at any point **increases** with time
- ✓ Along the $+z$ direction, $\vec{E}$ traces a left-hand circle at any time

e.g., Hamaker & Bregman (1996)

Linear/circular polarization fraction

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - pI \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} pI \\ Q \\ U \\ V \end{pmatrix}$$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - p_L I - p_C I \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} p_L I \\ Q \\ U \\ 0 \end{pmatrix} + \begin{pmatrix} p_C I \\ 0 \\ 0 \\ V \end{pmatrix}$$

polarization fraction

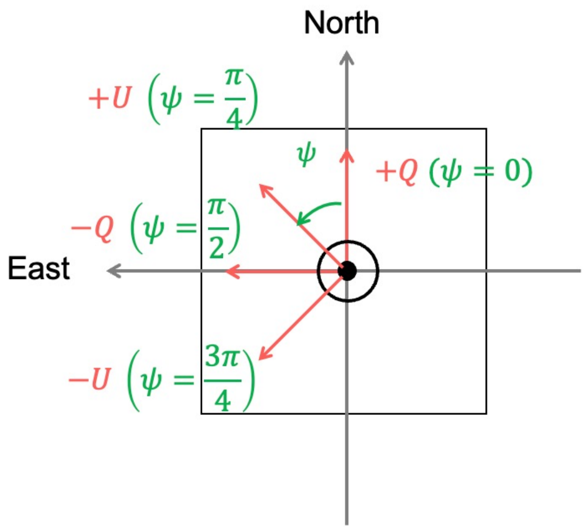
$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

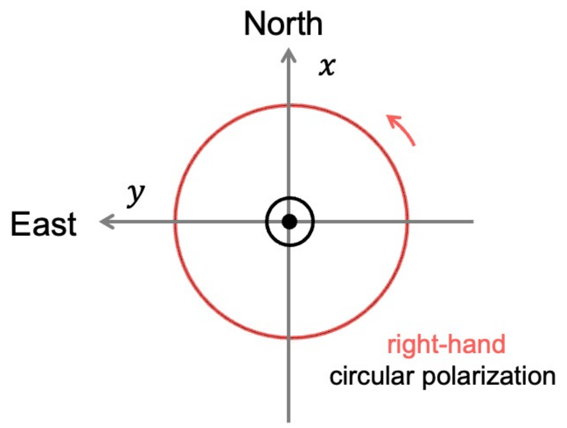
linear or circular polarization fraction

$$p_L = \sqrt{Q^2 + U^2}/I$$
$$p_C = V/I$$

prev. sl.

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized





Cosmic Microwave Background

CMB, the leftover radiation from the Big Bang, was discovered in 1965 by Arno Penzias and Robert Wilson using the 6-meter Holmdel Horn Antenna.

As CMB radiation evolve in the expanding Universe, it carries valuable cosmological information.

Wrong data excluded

Image credit: wikipedia

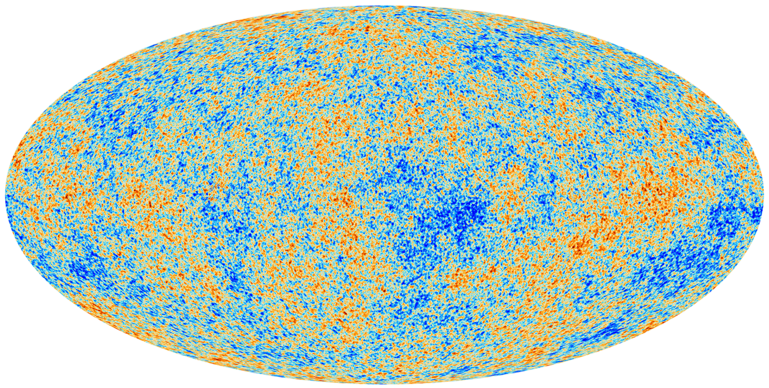


Image credit: ESA

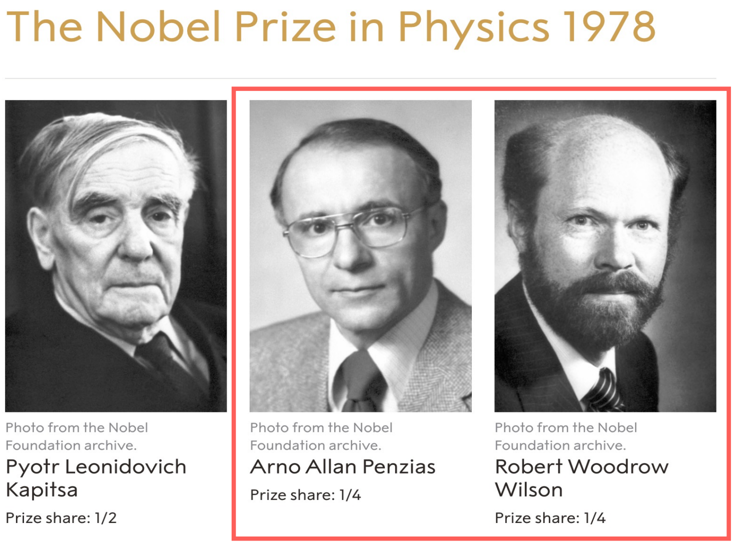


Image credit: nobelprize.org

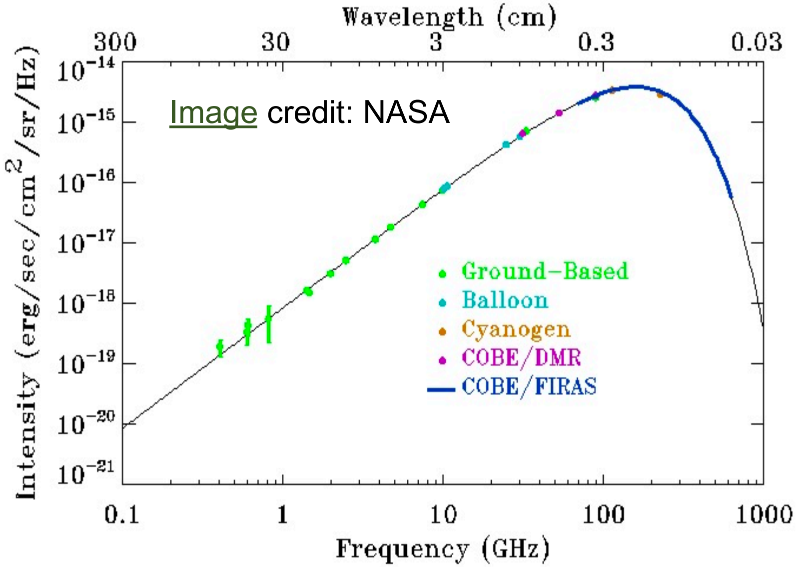
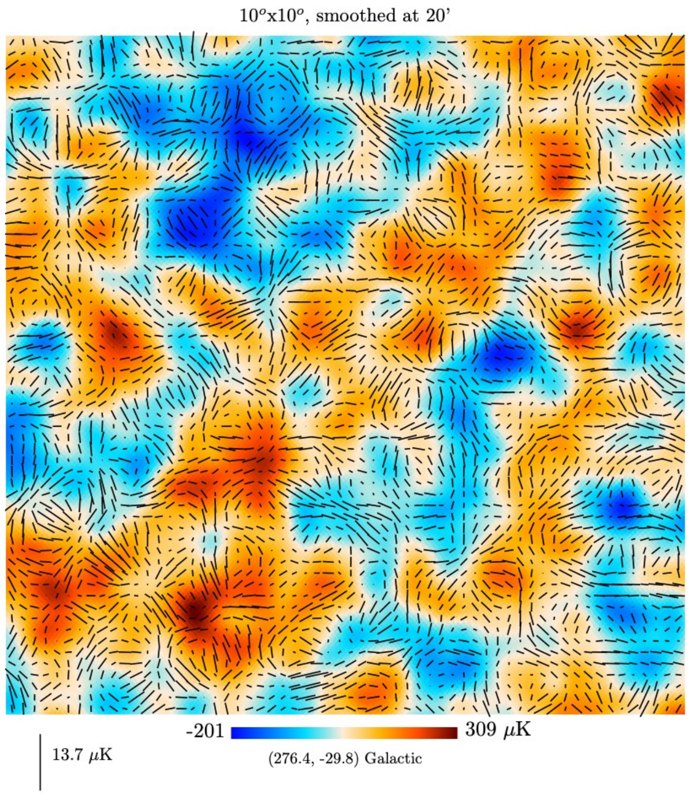
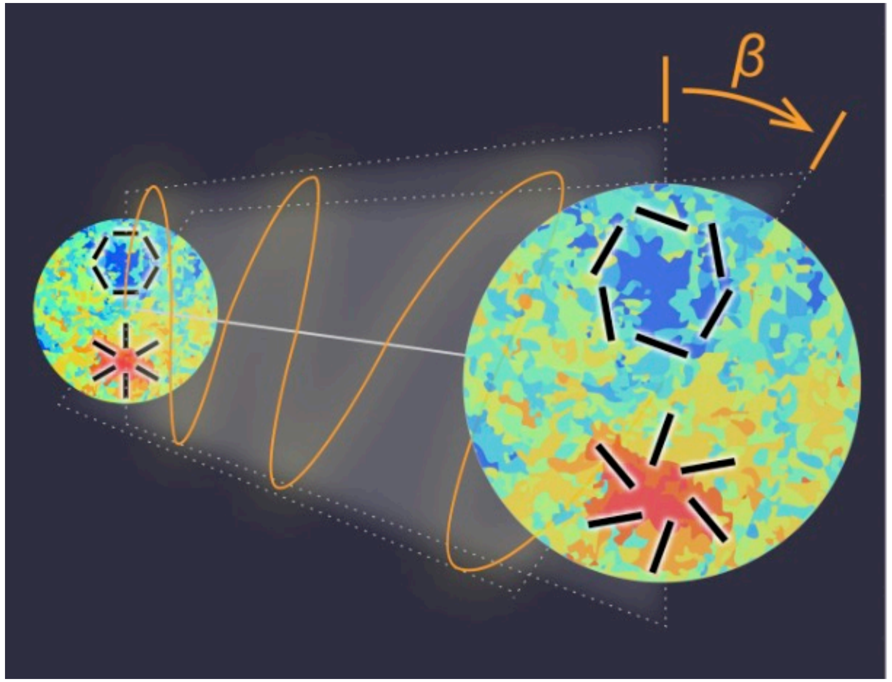


Image credit: NASA

Cosmic Microwave Background polarization





[Komatsu (2022, review)]

B-mode (curl-like)

E-mode (gradient-like)

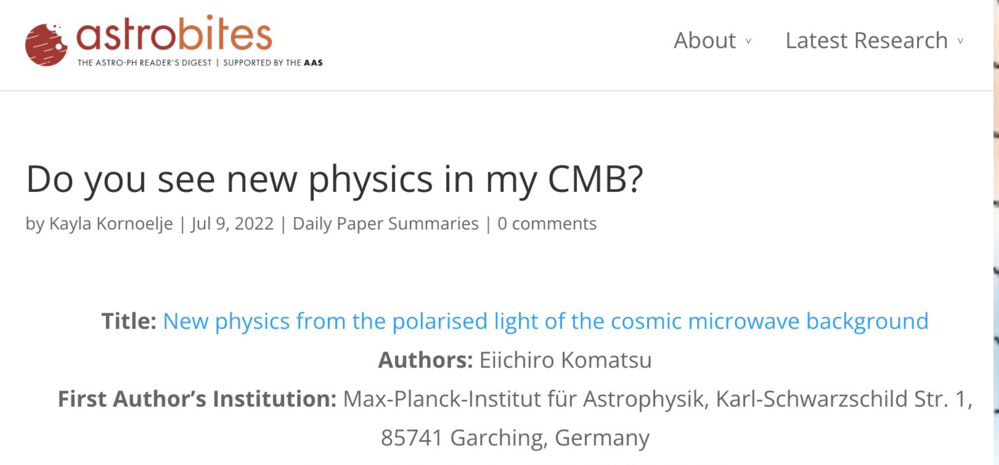
β is the cosmic-birefringence-induced rotation angle

CMB is polarized (E-mode and B-mode). CMB polarization might unravel the nature of Dark Matter and Dark Energy, cosmic inflation, etc.

CMB B-mode

The detection of B-mode polarization will provide strong constraints for cosmic inflationary models.

The B-mode power is $\propto r^2$. The tensor-to-scalar ratio r is no larger than 0.07 (BICEP2 collaboration 2016).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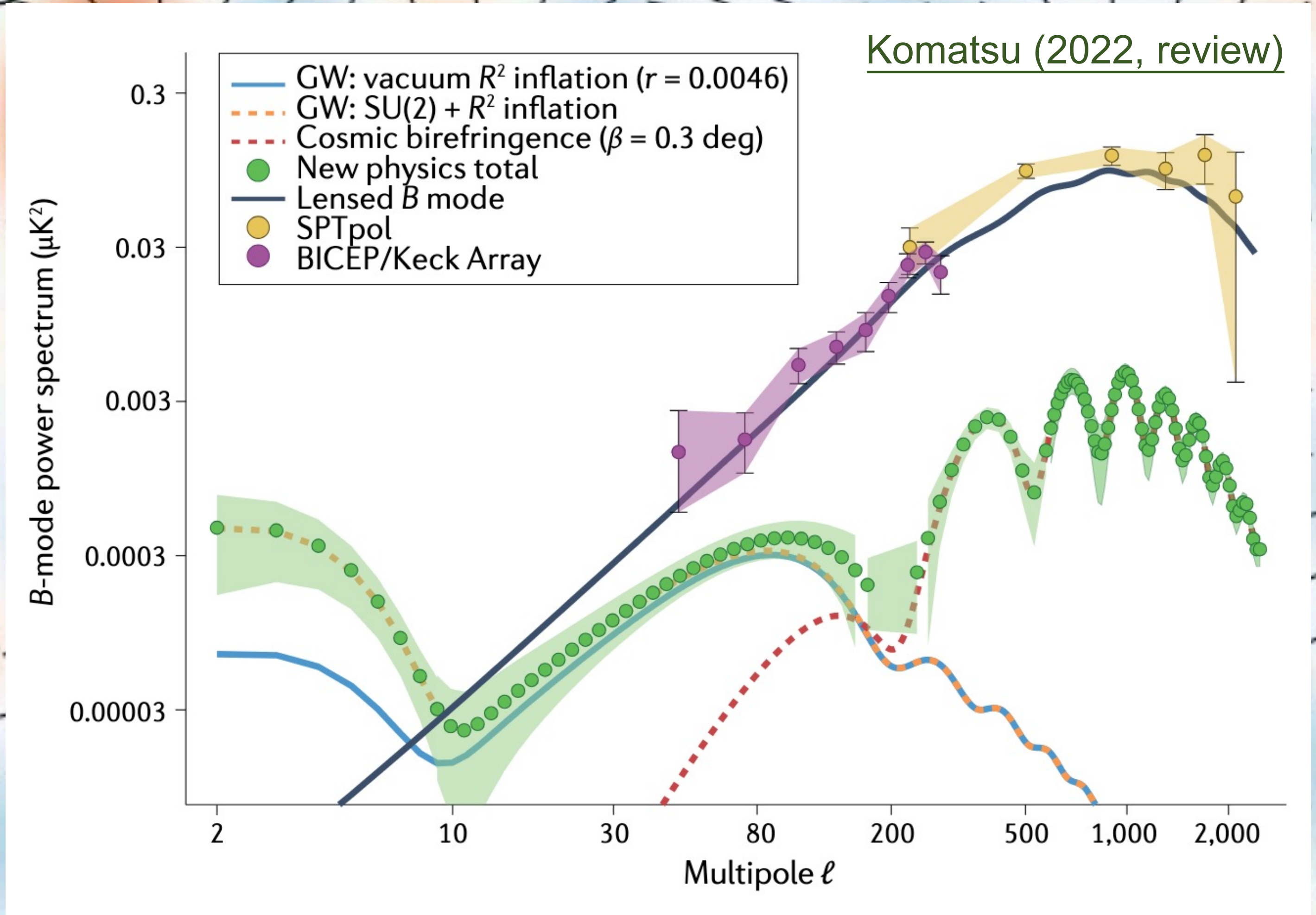
Do you see new physics in my CMB?

by Kayla Kornoelje | Jul 9, 2022 | Daily Paper Summaries | 0 comments

Title: New physics from the polarised light of the cosmic microwave background

Authors: Eiichiro Komatsu

First Author's Institution: Max-Planck-Institut für Astrophysik, Karl-Schwarzschild Str. 1, 85741 Garching, Germany



Komatsu (2022, review)

Image credit: bicepkeck.org

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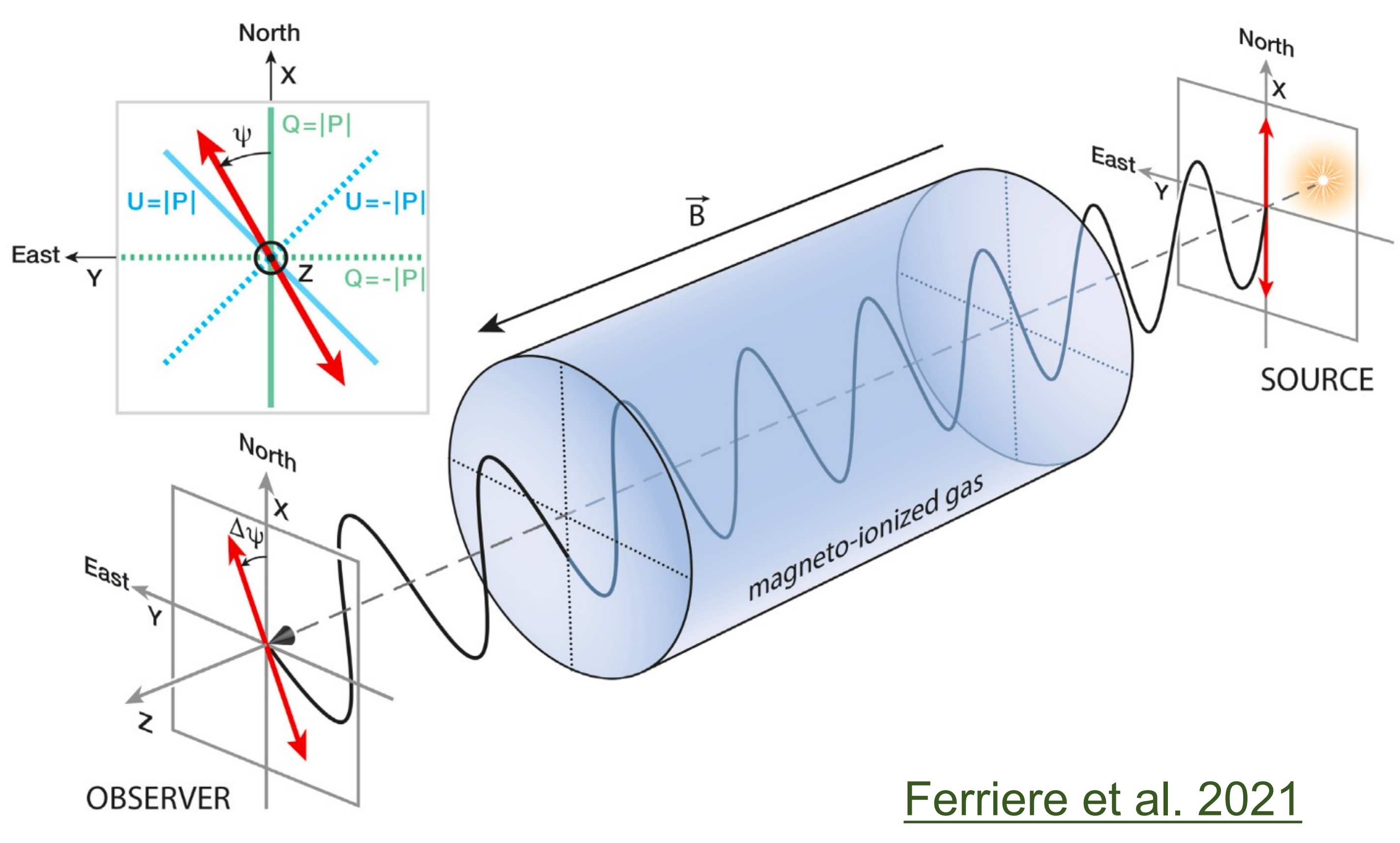
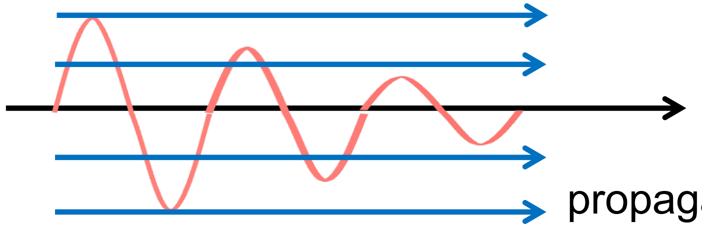
2.7.1 Dispersion relation
2.7.2 Time of arrival difference
2.7.3 Dispersion measure
2.7.4 Fast Radio Bursts
2.7.5 Phase difference
2.7.6 Rotation measure



Ferriere et al. 2021

Dispersion relation

Consider a pair of left- and right-hand circular polarized wave propagates along the B-field ($\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$), the dispersion relation is



Eq. 1 of Ferriere et al. 2021

$$\omega^2 = c^2 k^2 + \frac{\omega_{\mathrm{p}}^2}{1 \mp \frac{\omega_{\mathrm{L}}}{\omega}}$$

angular frequency
$\omega = 2\pi\nu$

wave number
$\lambda = \frac{2\pi}{k}$

plasma frequency

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

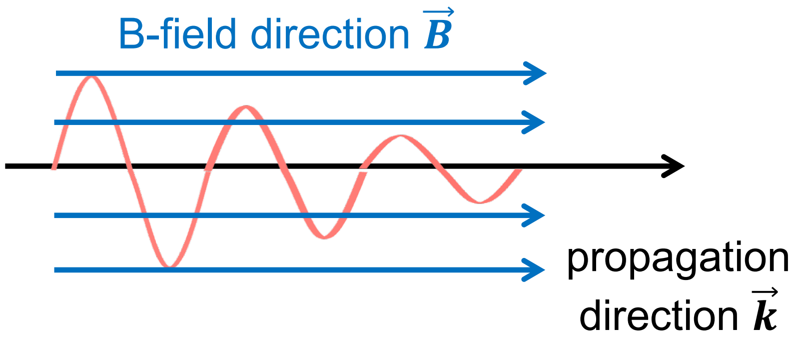
Larmor frequency

$$\omega_L \equiv \frac{eB}{m_e c}$$

《原子物理学》杨福家
3rd edition, p153 - 155

- Right circular polarized (RCP) mode: –
- Light circular polarized (LCP) mode: +

Dispersion relation (cont.)



For astrophysical waves of interest, the following always apply (Assignment #3: Plasma frequency)

$$\omega_{\mathrm{p}} \ll \omega, \qquad \omega_L \ll \omega$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

prev. sl.

$$\omega^2 = c^2k^2 + \frac{\omega_{\mathrm{p}}^2}{1 \mp \frac{\omega_{\mathrm{L}}}{\omega}}$$

$$\frac{1}{1+x} = 1 - x + \ \ldots$$

$$= c^2k^2 + \omega_{\mathrm{p}}^2 \pm \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega} \sim c^2k^2$$

EM wave in free space

electric force

magnetic force

$$\omega = ck\sqrt{1 + \frac{\omega_{\mathrm{p}}^2}{c^2k^2} \pm \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega c^2k^2}}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \ \ldots$$

$$= ck\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega} \pm \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{2\omega^3}\right)$$

Time of arrival difference

Consider a pulse propagates in a non-magnetized medium ($B = 0, \omega_{\mathrm{L}} = 0$), the group velocity is

$$v_{\mathrm{grp}}(\omega) = \frac{d\omega}{dk} = \frac{c^2 k}{\omega} = c^2 k \sqrt{\frac{1}{c^2 k^2 + \omega_{\mathrm{p}}^2}}$$

$$= c\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}}$$

$$\frac{1}{1+x} = 1 - x + \ \dots$$

This leads to a frequency-dependent time of arrival

$$t_{\mathrm{arv}}(\omega) = \int_0^l \frac{1}{v_{\mathrm{grp}}(\omega)} dl$$

prev. sl.

$$\omega^2 = c^2 k^2 + \omega_{\mathrm{p}}^2 \pm \frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{L}}}{\omega} \simeq c^2 k^2$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency — Larmor frequency

Lorimer & Kramer (2012)

Dispersion measure

For $\omega \gg \omega_p$ (normally valid, see Assignment #3), the time of arrival of a specific frequency is

traveling distance in the medium

Lorimer & Kramer (2012)

$$t_{\mathrm{arv}}(\omega) \sim \int_0^l \frac{1}{c}\left(1 + \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2\right) dl$$

$$\frac{1}{\sqrt{1+x}} = 1 - x/2 + \ \ldots$$

prev. sl.

$$v_{\mathrm{grp}}(\omega) = c\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}}$$

$$t_{\mathrm{arv}}(\omega) = \int_0^l \frac{1}{v_{\mathrm{grp}}(\omega)} dl$$

Waves with different frequencies arrive at different times

$$\Delta t = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\mathrm{low}}^2} - \frac{1}{\omega_{\mathrm{high}}^2}\right)\int_0^l n_e dl$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Fast Radio Bursts (FRBs)

FRBs are radio bursts with extremely short duration (a few milliseconds) yet rather energetic ($\gtrsim 10\,L_\odot$). The first FRB was discovered in 2001 (Lorimer et al. 2007)

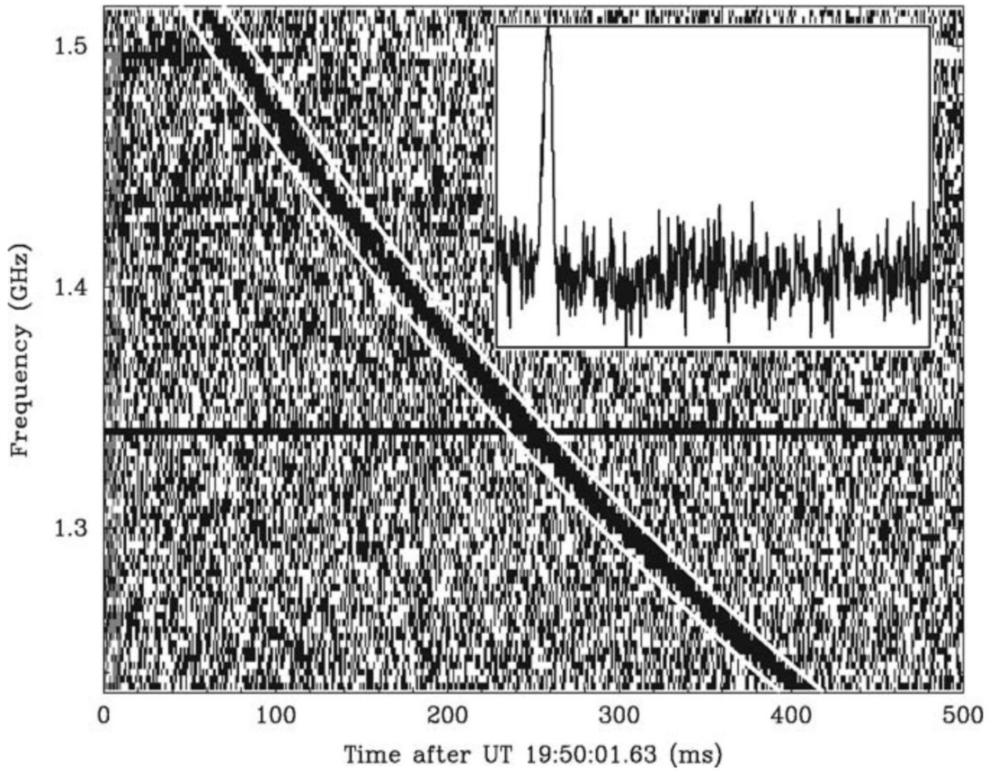
prev. sl.

$$\Delta t = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\text{low}}^2} - \frac{1}{\omega_{\text{high}}^2}\right)\int_0^l n_e dl$$

Dispersion Measure (DM)

$$= \frac{4.149}{\text{ms}}\left(\left(\frac{1\text{ GHz}}{\nu_{\text{low}}}\right)^2 - \left(\frac{1\text{ GHz}}{\nu_{\text{high}}}\right)^2\right)\left(\frac{\text{DM}}{\text{pc cm}^{-3}}\right)$$

Waterfall plot
measured DM= $375 \pm 1\text{pc cm}^{-3}$

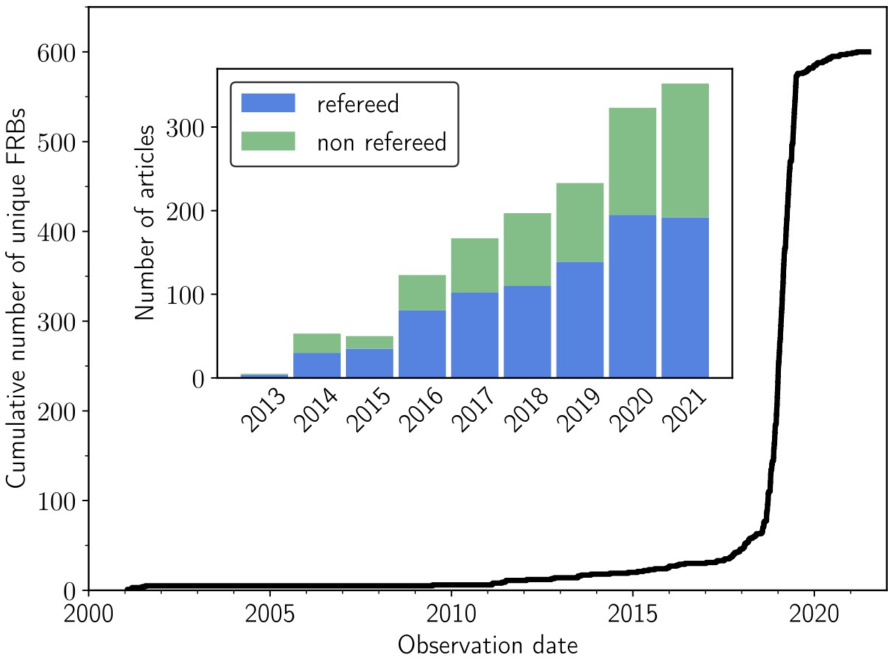


Lorimer et al. 2007

Fast Radio Bursts (FRBs)

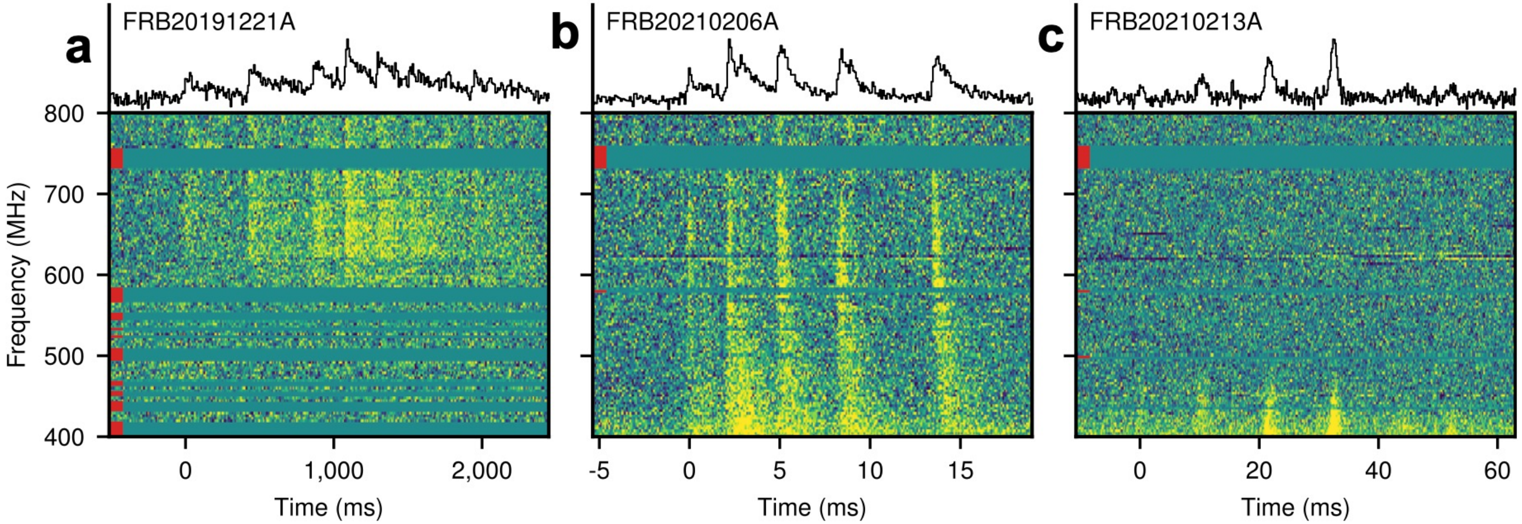
FRBs have been put in the spot light in the past decade:
- Repeating FRBs
- FRBs with known hosts
- FRBs with relatively long durations (slow)

But, we still don't know much about FRBs …



Publications on FRBs from ADS

repeating FRBs (a small yet growing population) Petroff et al. 2022

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Some "not so fast" fast radio bursts

by Alice Curtin | Nov 22, 2022 | Daily Paper Summaries | 1 comment

Title: Four New Fast Radio Bursts Discovered in the Parkes 70-cm Pulsar Survey Archive

Authors: F. Crawford, S. Hisano, M. Golden, T. Kikunaga, A. Laity, D. Zoeller

First Author's Institution: Franklin & Marshall College

Phase difference

In a homogeneous plasma, the phase can be defined as one of the following

Eq. 4 of Ferriere et al. 2021

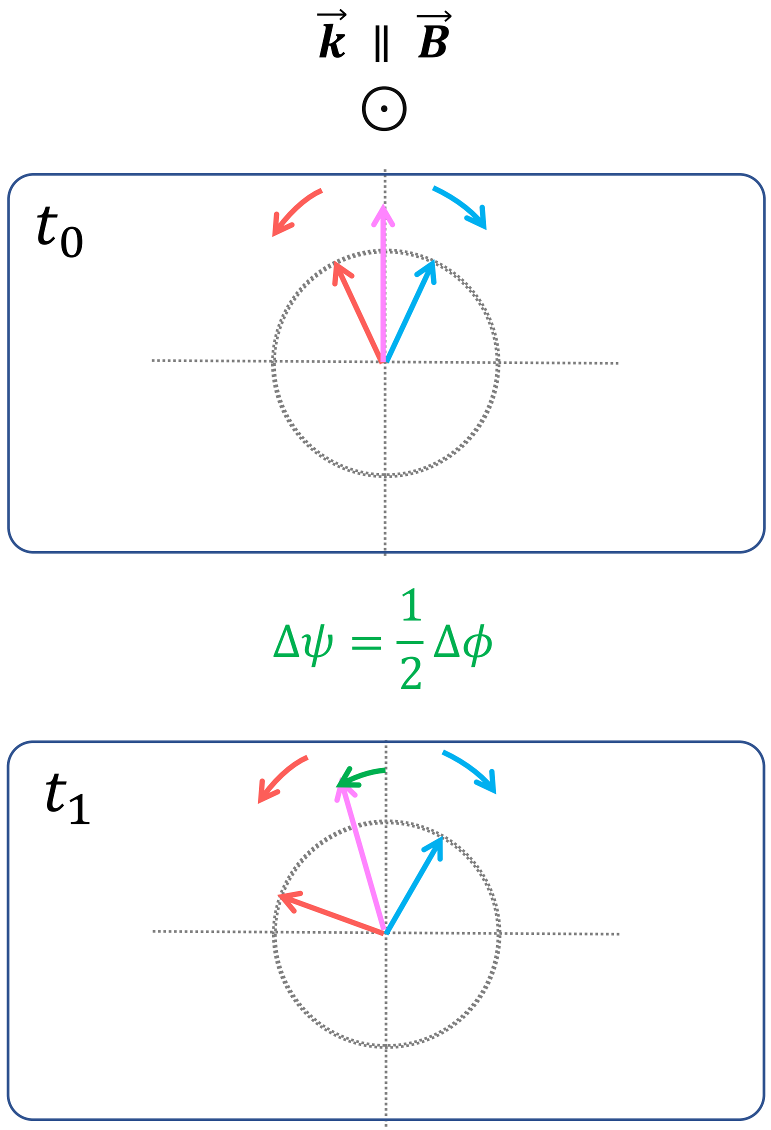
$$\phi \equiv \omega t - \int_0^l k \, dl$$

Consider a pair of left- and right-hand circular polarized waves propagate along the B-field ($\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$), at the source ($l = 0$), the two waves have the same phase, but a phase difference will occur

prev. sl.

$$\omega = ck\left(1 + \frac{\omega_\mathrm{p}^2}{2\omega} \pm \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{2\omega^3}\right)$$

$$\phi_\mathrm{RCP} > \phi_\mathrm{LCP}$$



- Right circular polarized (RCP) mode: +
- Light circular polarized (LCP) mode: –

Phase difference (cont.)

prev. sl.

$$\omega_{\rm p} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

prev. sl.

$$\omega = ck\left(1 + \frac{\omega_{\rm p}^2}{2\omega} \pm \frac{\omega_{\rm p}^2\omega_{\rm L}}{2\omega^3}\right) \simeq ck = \frac{2\pi c}{\lambda}$$

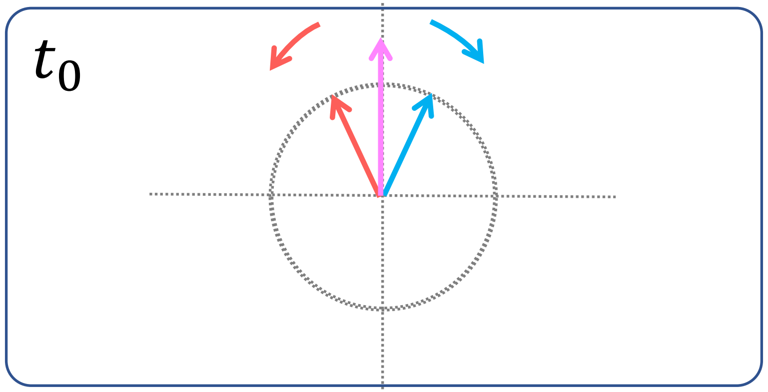
Eq.9 of Ferriere et al. 2021

$$\Delta\psi = \frac{1}{2}\Delta\phi = \frac{1}{2}\int_0^l \frac{\omega_{\rm p}^2\,\omega_{\rm L}}{c\omega^2}dl$$

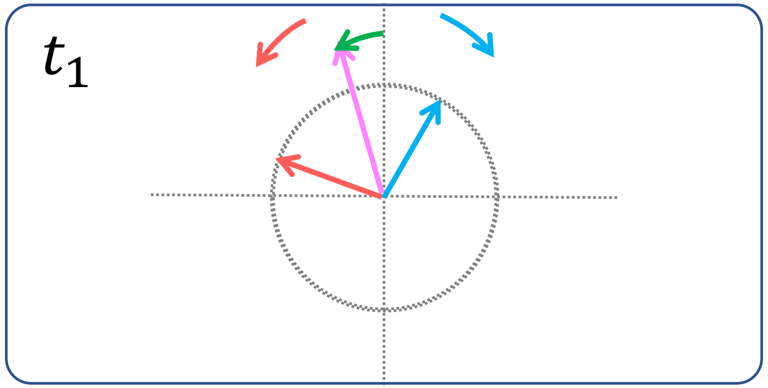
$$= \frac{e^3}{2\pi\, m_e^2\, c^4}\lambda^2(\pm B_\parallel)\int_0^l n_e dl$$

$\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: – $\otimes$



$$\Delta\psi = \frac{1}{2}\Delta\phi$$



- Right circular polarized (RCP) mode
- Light circular polarized (LCP) mode

Rotation measure

prev. sl.

$$\Delta\psi = \frac{e^3}{2\pi\, m_e^2\, c^4}\lambda^2(\pm B)\int_0^l n_e dl$$

- $\vec{B}$ points towards the observer: + ⊙
- $\vec{B}$ points away from the observer: – ⊗

$$\mathrm{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4}\int_0^l n_e\, B_{\parallel} dl$$

Rotation Measure (RM)

$$\psi_{\mathrm{obs}} = \psi_{\mathrm{src}} + \mathrm{RM}\cos\theta\,\lambda^2$$