AY2024F

Junjie Mao

Radiative Processes in Astrophysics

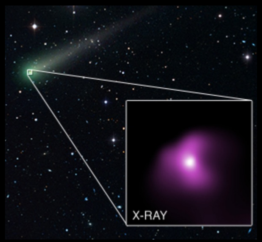
Observation

Up to cosmic size scale

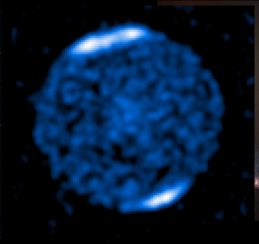
Theory

Down to atomic size scale

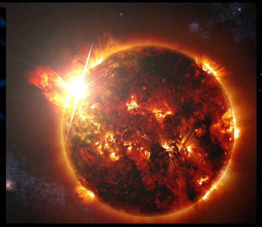
C/2012S1 (comet)



Jupiter (planet)



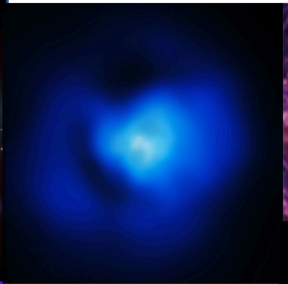
Sun (star)



Cas A (SNR)

M82 (galaxy)

Phoenix (gal. cluster)



Cosmic web filament

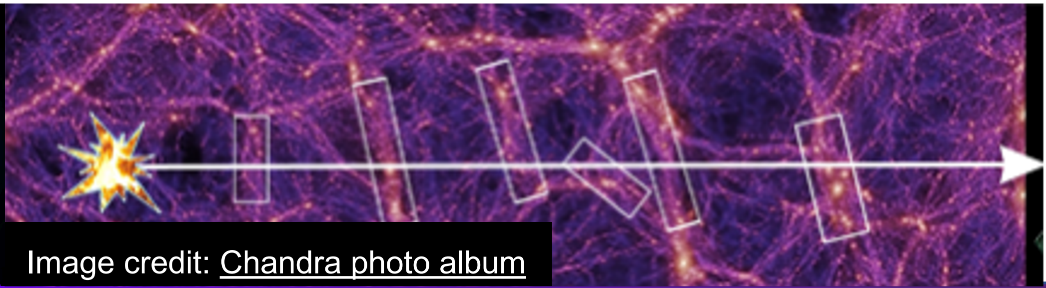


Image credit: Chandra photo album

Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge
3.1.1 Electric and magnetic field
3.1.2 Radiative power angular distribution
3.1.3 Asymptotic behavior (non-relativistic)
3.1.4 Asymptotic behavior (relativistic)
3.1.5 Pulsar spin-down

3.2 Thomson scattering
3.3 Cyclotron radiation
3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

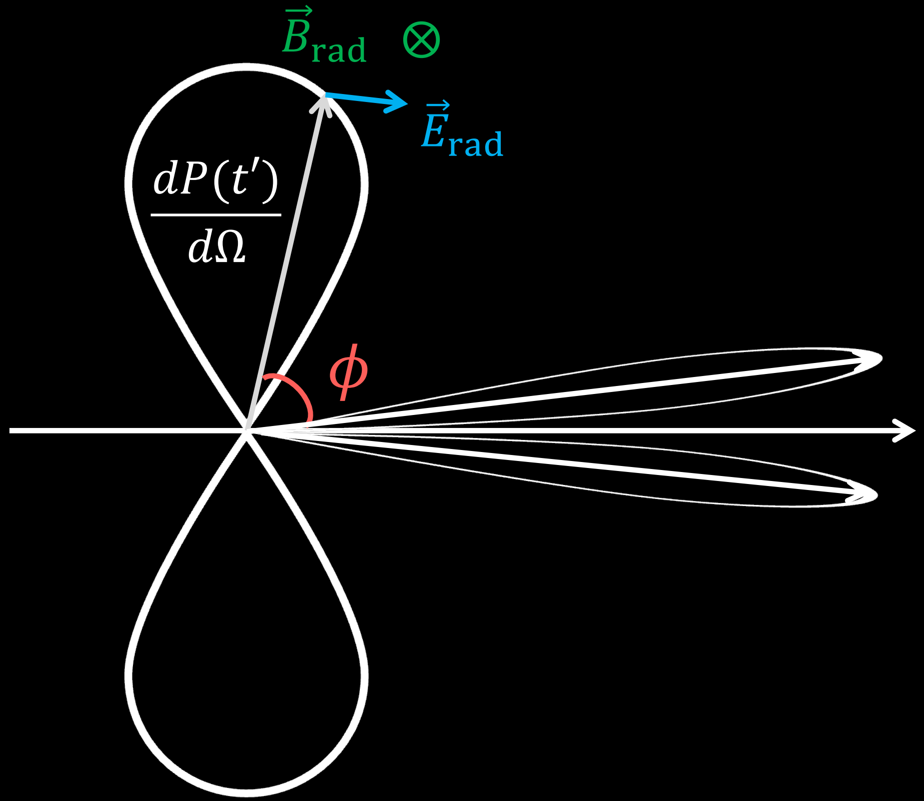


Image credit: Junjie Mao

Electric and magnetic field of an accelerated charge

Consider a moving point charge, its electric and magnetic fields are

charge

$$\vec{E}(r,t) = \frac{q}{\kappa^3 R^2}\left(\vec{e}_\mathrm{R} - \vec{\beta}\right)\left(1-\beta^2\right) + \frac{q}{c\kappa^3 R}\vec{e}_\mathrm{R}\times\left(\left(\vec{e}_\mathrm{R} - \vec{\beta}\right)\times\frac{\partial\vec{\beta}}{\partial t}\right)$$

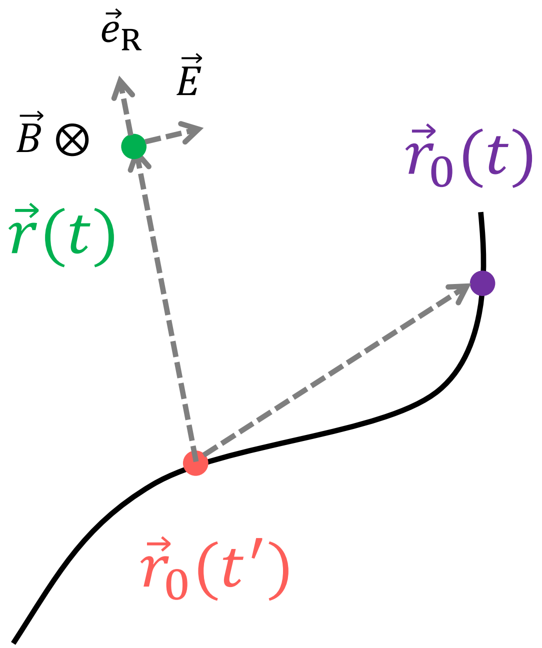
$$\vec{B}(r,t) = \vec{e}_\mathrm{R}\times\vec{E}(r,t)$$

$$R = \left|\vec{r}(t) - \vec{r}_0(t')\right|, \vec{e}_R = \frac{\vec{R}}{R}$$

see Chpt. 3 of the REF book (p77 - 80) by Rybicki & Lightman

$$\vec{\beta} = \frac{\vec{v}(t')}{c}, \beta = \frac{v(t')}{c}, v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\overrightarrow{e_R}\cdot\vec{v}(t')}{c} = 1 - \vec{e}_R\cdot\vec{\beta}$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

Retarded time

prev. sl.

Consider a moving point charge, its electric and magnetic fields are

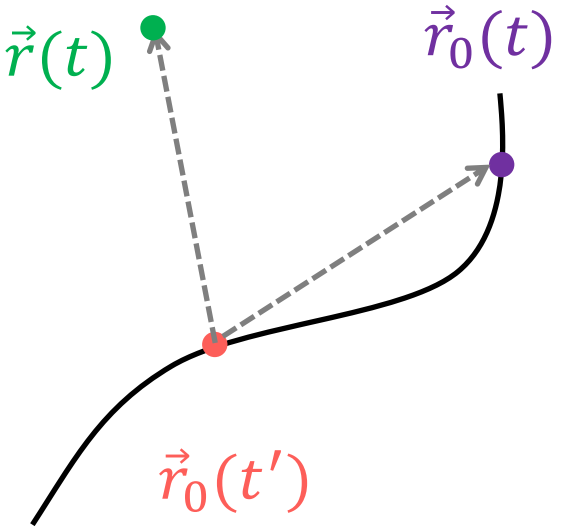
$$\vec{E}(r,t) = \cdots$$

$$\vec{B}(r,t) = \vec{e}_{\mathrm{R}} \times \vec{E}(r,t)$$

$$R = |\vec{r}(t) - \vec{r}_0(t')|, \vec{e}_R = \frac{\vec{R}}{R}$$

$$\vec{\beta} = \frac{\vec{v}(t')}{c}, \beta = \frac{v(t')}{c}, v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\vec{e}_{\mathrm{R}} \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

The propagation of electromagnetic waves has a finite speed. Accordingly, the radiation properties we observed at time t was emitted at an earlier time t'

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} = t - \frac{|\vec{r}_0(t) - \vec{r}_0(t')|}{c}$$

retarded time

Velocity field of an accelerated charge

prev. sl.

$$\vec{E}(r,t) = \frac{q}{\kappa^3\, R^2}(\vec{e}_R - \vec{\beta})(1-\beta^2) + \frac{q}{c\kappa^3 R}\vec{e}_R \times \left((\vec{e}_R - \vec{\beta}) \times \frac{\partial \vec{\beta}}{\partial t} \right)$$

velocity field $\vec{E}_{\text{vel}}$

$$\vec{e}_R = \frac{\vec{R}}{R}$$

$$\kappa = 1 - \frac{\vec{e}_R\, \vec{v}(t')}{c}$$

$$\beta = \frac{v(t')}{c}$$

For a constantly $(\partial\beta/\partial t = 0)$ moving charge, $\vec{E} = \vec{E}_{\text{vel}}$

$$\vec{E}_{\text{vel}} \propto \frac{1}{R^2}$$

little contribution to radiation at large distance (R)

$$\rightarrow \frac{q}{R^2}\vec{e}_R \text{ for } \beta \ll 1$$

Coulomb's law

Radiation field of an accelerated charge

prev. sl.

$$\vec{E}(r,t) = \frac{q}{\kappa^3\, R^2}\left(\vec{e}_R - \vec{\beta}\right)(1-\beta^2) + \frac{q}{c\kappa^3 R}\,\vec{e}_R \times \left(\left(\vec{e}_R - \vec{\beta}\right)\times \frac{\partial \vec{\beta}}{\partial t}\right)$$

$$\vec{e}_R = \frac{\vec{R}}{R}$$

$$\kappa = 1 - \frac{\vec{e}_R\, \vec{v}(t')}{c}$$

$$\beta = \frac{v(t')}{c}$$

acceleration field $\vec{E}_{\mathrm{acc}}$

$\vec{B}_{\mathrm{acc}} = \overrightarrow{e_R} \times \vec{E}_{\mathrm{acc}}$

radiation field

$\vec{E}$ refers to $\vec{E}_{\mathrm{acc}}$ hereafter

Only accelerated $(\partial\beta/\partial t \neq 0)$ particles can give rise to radiation!

Acceleration can be caused by

(1) external B-fields (e.g., cyclotron and synchrotron);

(2a) collision with photons (e.g., Compton);

(2b) collision among electrons (bremsstrahlung caused by Coulomb force)

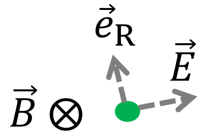
Radiative power

Poynting vector (radiative power vector per unit area)

$$\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{B} = \frac{c}{4\pi}\vec{E}\times\left(\vec{e}_R\times\vec{E}\right) = \frac{c}{4\pi}\vec{e}_R\left|\vec{E}\right|^2$$

Sect. 1.3.1 of 《天体物理中的辐射机制》by 尤峻汉 (p10)

prev. sl. $\vec{B}(r,t) = \vec{e}_R\times\vec{E}(r,t)$



$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = (\vec{a}\cdot\vec{c})\,\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$$

Along the direction of the Poynting vector $\vec{e}_R$, the radiative power received by an observer at the distance R with a collecting area $d\vec{A} = \vec{e}_R\, R^2 d\Omega$

$$dP(t)dt = \vec{S}\cdot d\vec{A} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2\, d\Omega$$

Radiative power received by an observer per unit solid angle

$$\frac{dP(t)}{d\Omega} = \frac{dE(t)}{dt\, d\Omega} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2$$

Radiative power (cont.)

prev. sl.

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} = t - \frac{|\vec{r}_0(t) - \vec{r}_0(t')|}{c}$$

$$\kappa = 1 - \frac{\vec{e}_R \cdot \vec{v}(t')}{c} = 1 - \vec{e}_R \cdot \vec{\beta}$$

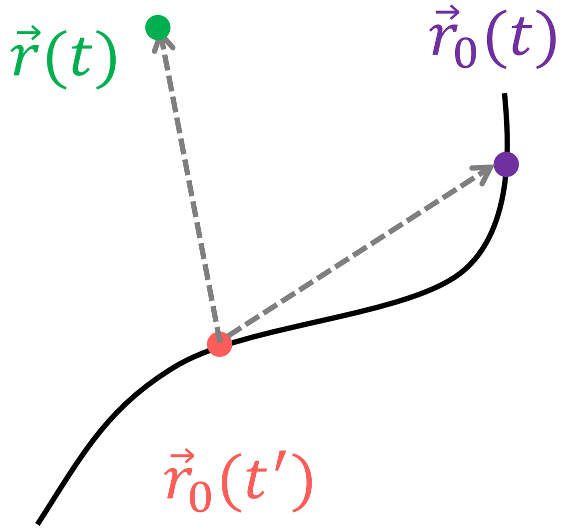
Conservation of energy: Energy received by an observer equals energy emitted by the emitter

$$dP(t)dt = dP(t')dt'$$

$$\vec{R} = |\vec{r}(t) - \vec{r}_0(t')| \approx |\vec{r}(t)| - \vec{e}_R \cdot \vec{r}_0(t') \text{ for } |\vec{r}_0(t')| \ll |\vec{r}|$$

$$t' = t - \frac{|\vec{r}(t) - \vec{r}_0(t')|}{c} \approx t - \frac{1}{c}\left(|\vec{r}(t)| - \vec{e}_R \cdot \vec{r}_0(t')\right)$$

$$dt' = dt + \frac{1}{c}\vec{e}_R \cdot d\vec{r}_0(t')$$



$\vec{r}(t)$: field point
$\vec{r}_0(t)$: current position of the moving charge
$\vec{r}_0(t')$: retarded position of the moving charge
t: observing time
t': retarded time

Radiative power angular distribution

prev. sl.

$$dt = dt' - \frac{1}{c}\vec{e}_R \cdot d\vec{r}_0(t')$$

$$\frac{dt}{dt'} = 1 - \frac{1}{c}\ \vec{e}_R \cdot \frac{d\vec{r}_0(t')}{dt'} = \ \kappa$$

$$\frac{dP(t')}{d\Omega} = \frac{dP(t)}{d\Omega}\frac{dt}{dt'} = \frac{dP(t)}{d\Omega}\kappa$$

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c\kappa^5}\left|\vec{e}_R \times \left((\vec{e}_R - \vec{\beta})\times\dot{\vec{\beta}}\right)\right|^2$$

prev. sl.

$$v(t') = \dot{r}_0(t')$$

$$\kappa = 1 - \frac{\vec{e}_R \cdot \vec{v}(t')}{c} = 1\ -\ \vec{e}_R \cdot \vec{\beta}$$

prev. sl.

$$dP(t)dt = dP(t')dt'$$

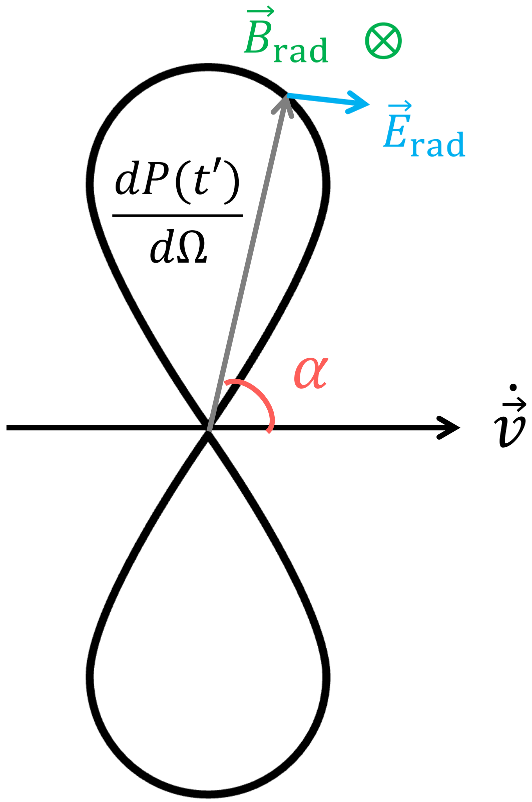
prev. sl.

$$\frac{dP(t)}{d\Omega} = \frac{dE(t)}{dt\ d\Omega} = \frac{c}{4\pi}\left|\vec{E}\right|^2 R^2$$

prev. sl.

$$\vec{E} = \frac{q}{c\kappa^3 R}\vec{e}_R \times \left((\vec{e}_R - \vec{\beta})\times\dot{\vec{\beta}}\right)$$

Asymptotic behavior (non-relativistic)



prev. sl.

$$\frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c\kappa^5}\left|\vec{e}_R\times\left((\vec{e}_R - \vec{\beta})\times\dot{\vec{\beta}}\right)\right|^2$$

For non-relativistic particles with $v \ll c$ $(t \approx t')$, and α the angle between $\vec{e}_R$ and $\dot{\vec{v}}$

$$\kappa = 1 - \frac{\vec{e}_R \cdot \vec{v}}{c} \to 1$$

$$\beta = \frac{v}{c} \to 0$$

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\,\dot{v}^2 \sin^2\alpha$$

$$\vec{e}_R\times(\vec{e}_R\times\dot{\vec{v}}) = (\vec{e}_R\cdot\dot{\vec{v}})\vec{e}_R - (\vec{e}_R\cdot\vec{e}_R)\dot{\vec{v}} = (\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}$$

$$\left|\vec{e}_R\times\vec{e}_R\times\dot{\vec{v}}\right|^2 = \left((\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}\right)\left((\dot{v}\cos\alpha)\vec{e}_R - \dot{\vec{v}}\right)$$

$$= (\dot{v}\cos\alpha)^2(\vec{e}_R\cdot\vec{e}_R) - 2(\dot{v}\cos\alpha)(\dot{\vec{v}}\cdot\vec{e}_R) + \dot{v}^2$$

$$= \dot{v}^2(\cos^2\alpha - 2\cos^2\alpha + 1) = \dot{v}^2\sin^2\alpha$$

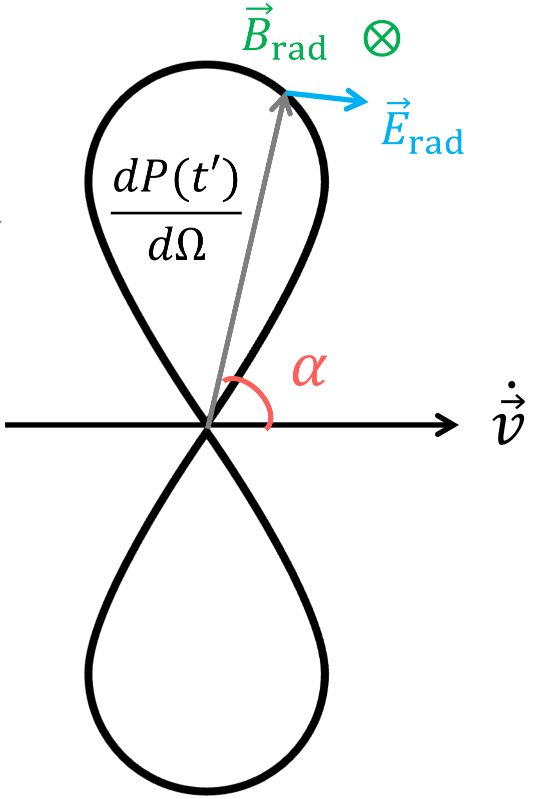
Larmor's formula (non-relativistic)

prev. sl.

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\dot{v}^2 \sin^2\alpha$$

$$d\Omega = \int_0^{2\pi} d\phi \int_{-\pi/2}^{\pi/2} \sin\alpha \, d\alpha = 2\pi \int_{-1}^{1} d\cos\alpha$$

$$\int_{-1}^{1} (1-\cos^2\theta) d\cos\theta = \frac{4}{3}$$



$$P = \frac{q^2}{4\pi c^3}\dot{v}^2 \int \sin^2\alpha \, d\Omega = \frac{q^2}{2c^3}\dot{v}^2 \int_{-1}^{1} (1-\cos^2\alpha) d\cos\alpha$$

$$\boxed{P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}} \longleftarrow \quad \ddot{d} = q\,\ddot{r} = q\,\dot{v}$$

see Chpt. 3 of the REF book (p85 - 88) by Rybicki & Lightman for more discussions on the dipole approximation

- ✓ This dipole angular distribution is symmetrically perpendicular to the acceleration direction $\dot{\vec{v}}$
- ✓ The maximum radiation is perpendicular to $\dot{\vec{v}}$ and there is no radiation along $\dot{\vec{v}}$

Asymptotic behavior (relativistic)

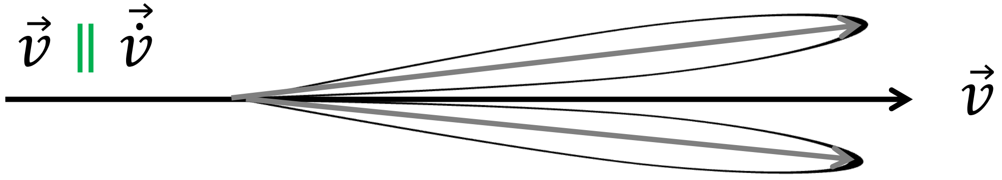
For parallel acceleration,

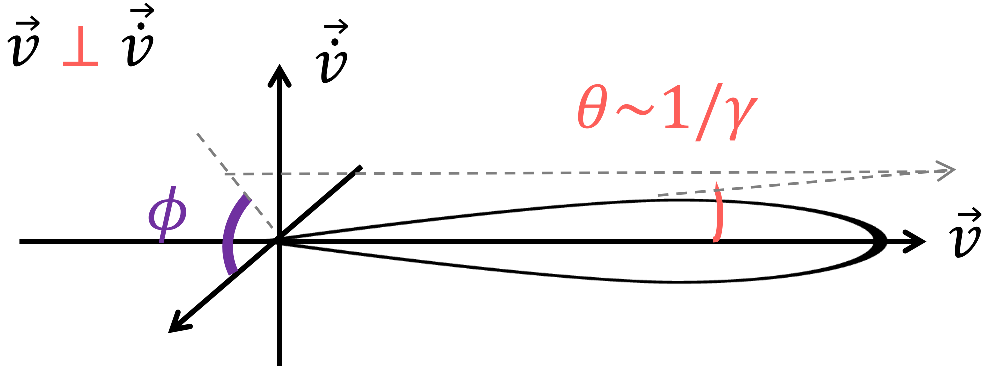
see Chpt. 4 of Ribicki & Lightman (p138-145)

$$\frac{dP_{\parallel}}{d\Omega} \approx \frac{16q^2\,\dot{v}_{\parallel}^2}{\pi c^3}\,\gamma^{10}\,\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6}$$

For perpendicular acceleration,

$$\frac{dP_{\perp}}{d\Omega} \approx \frac{4q^2\,\dot{v}_{\perp}^2}{\pi c^3}\,\gamma^{8}\,\frac{1-2\gamma^2\theta^2\cos 2\phi+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^6}$$





Lorentz factor

$$\gamma = \frac{mc^2}{m_0c^2} = \frac{1}{\sqrt{1-\beta^2}}$$

rest mass of the particle

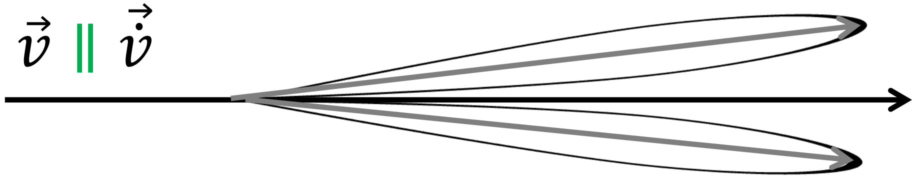
the larger the Lorentz factor,
the larger the radiative power

Parallel acceleration (relativistic)

prev. sl.

$$\frac{dP_\parallel}{d\Omega} \approx \frac{16q^2\,\dot{v}_\parallel^2}{\pi c^3}\,\gamma^{10}\,\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^6}$$

$\vec{v} \parallel \dot{\vec{v}}$

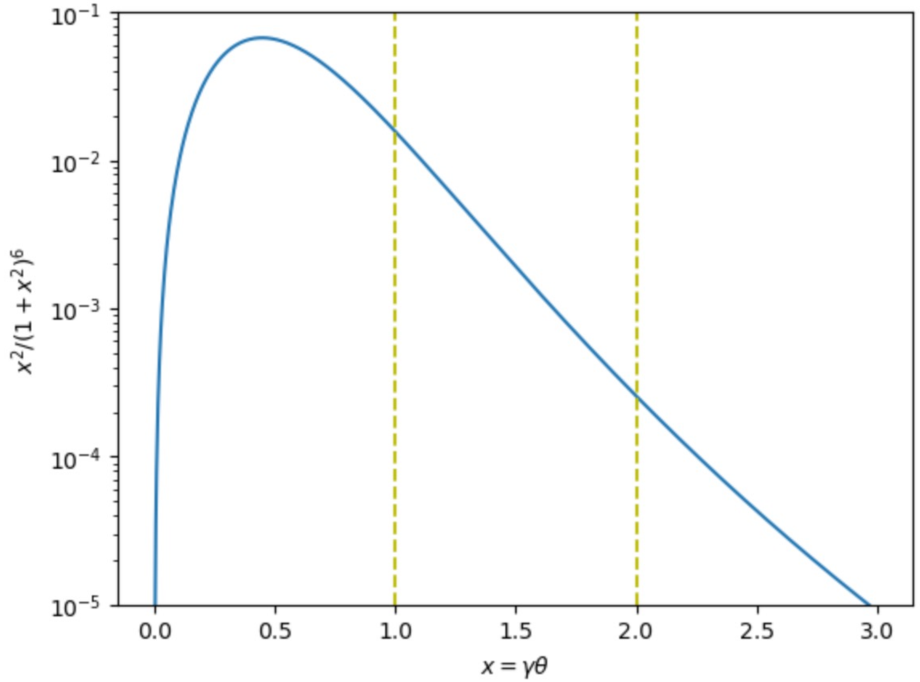


For $\theta = 0$ $$\frac{dP_\parallel}{d\Omega} = 0$$

For $\theta = 1/\gamma$ $$\frac{dP_\parallel}{d\Omega} \approx \frac{16q^2\,\dot{v}_\parallel^2}{\pi c^3}\,\gamma^{10}\,\frac{1}{2^6}$$

$$\frac{1/2^6}{2^2/(1+2^2)^6} \sim 60$$

For $\theta = 2/\gamma$ $$\frac{dP_\parallel}{d\Omega} \approx \frac{16q^2\,\dot{v}_\parallel^2}{\pi c^3}\,\gamma^{10}\,\frac{2^2}{(1+2^2)^6}$$



Perpendicular acceleration (relativistic)

prev. sl.

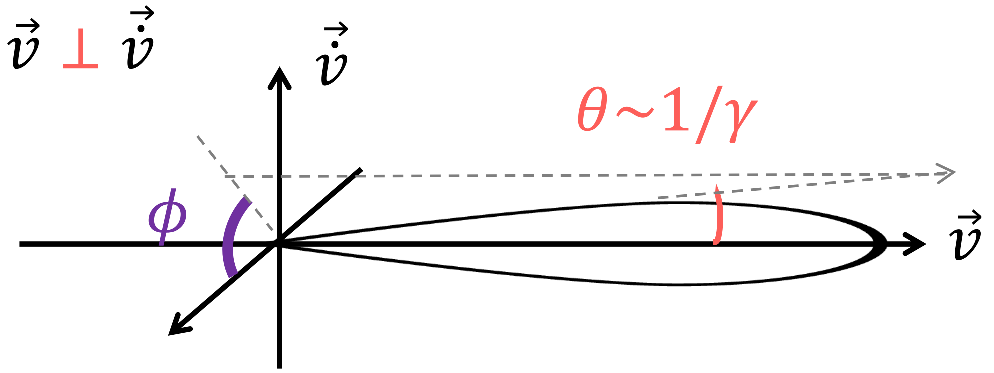
$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\gamma^8\frac{1-2\gamma^2\theta^2\cos 2\phi+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^6}$$

For $\theta = 0$

$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\gamma^8$$

For $\theta = 1/\gamma$

$$\frac{dP_\perp}{d\Omega} \approx \frac{4q^2\,\dot{v}_\perp^2}{\pi c^3}\gamma^8\frac{(1-\cos^2 2\phi)}{2^5}$$



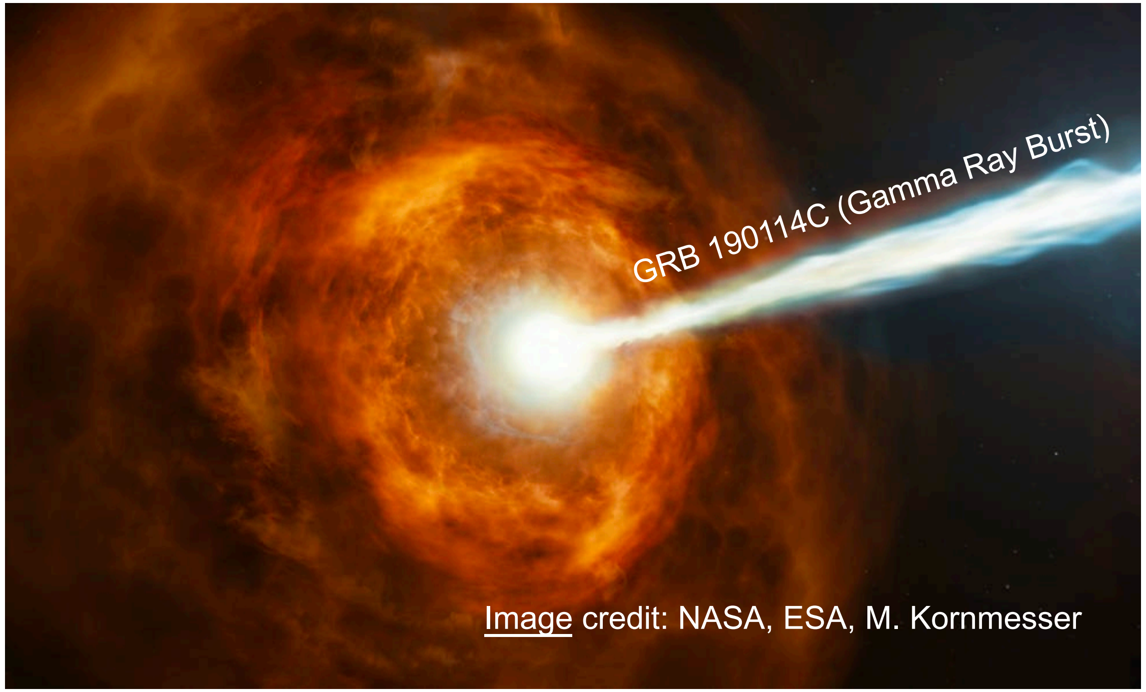
The radiation is smaller by a factor of $\sim 2^5 = 32$ at the half opening angle of $\phi = \gamma^{-1}$
The larger the Lorentz factor, the smaller the opening angle

Gamma-Ray bursts (GRBs)

GRBs are the most luminous explosions in the Universe (Meszaro 2006) with a typical energy of $10^{51}\ \mathrm{erg\,s^{-1}}$ (Frail et al. 2001).

cf. $L(\mathrm{FRB}) \sim 10\ L_{\odot} \sim 10{\times}3.8{\times}10^{33}\ \mathrm{erg\,s^{-1}}$

GRBs are discovered serendipitously in the 1967 by Vela satellites (Klebesadel et al. 1973)



The Lorentz factor of GRBs can be $\gtrsim 10^2$ (Lithwick & Sari 2001, Ghirlanda et al. 2018)

Total radiative power (relativistic)

the radiative power is significantly larger ($\propto \gamma^n$) than the non-relativistic one

Chpt. 4 of the REF book (p137-140) by Ribicki & Lightman

$$P = \frac{2q^2}{3c^3}\gamma^4\left(\dot{v}_\perp^2 + \gamma^2\dot{v}_\parallel^2\right)$$

Larmor's formula

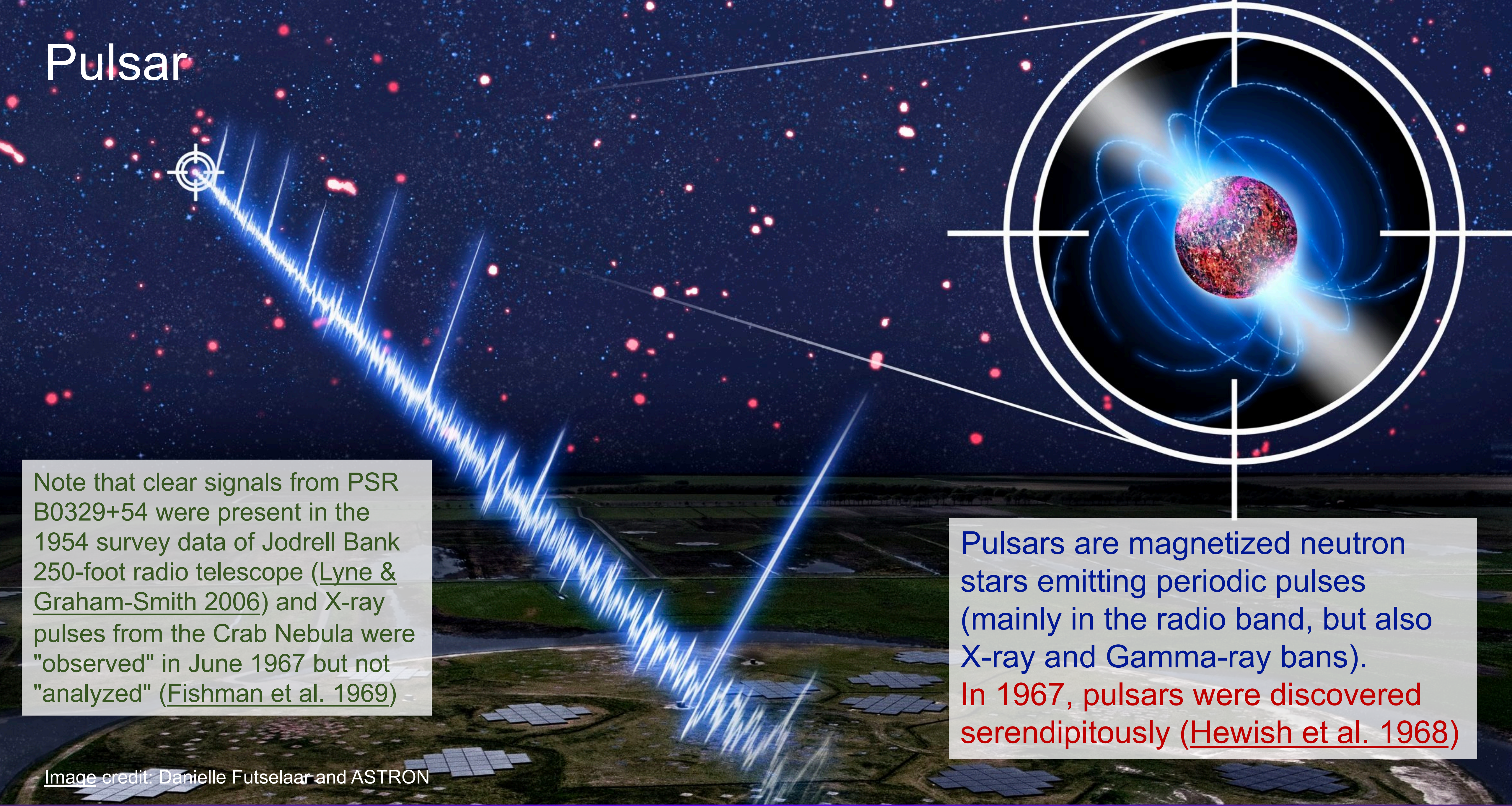
$$P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}$$

When $\beta \to 0$ $(\gamma \to 1)$, the radiative power is identical to that of the Larmor's formula

$$\dot{v} \propto \frac{1}{m_0}$$

The smaller the particle mass, the larger the radiative power
→ Radiation of electrons are much larger than that of protons

Pulsar



Note that clear signals from PSR B0329+54 were present in the 1954 survey data of Jodrell Bank 250-foot radio telescope (Lyne & Graham-Smith 2006) and X-ray pulses from the Crab Nebula were "observed" in June 1967 but not "analyzed" (Fishman et al. 1969)

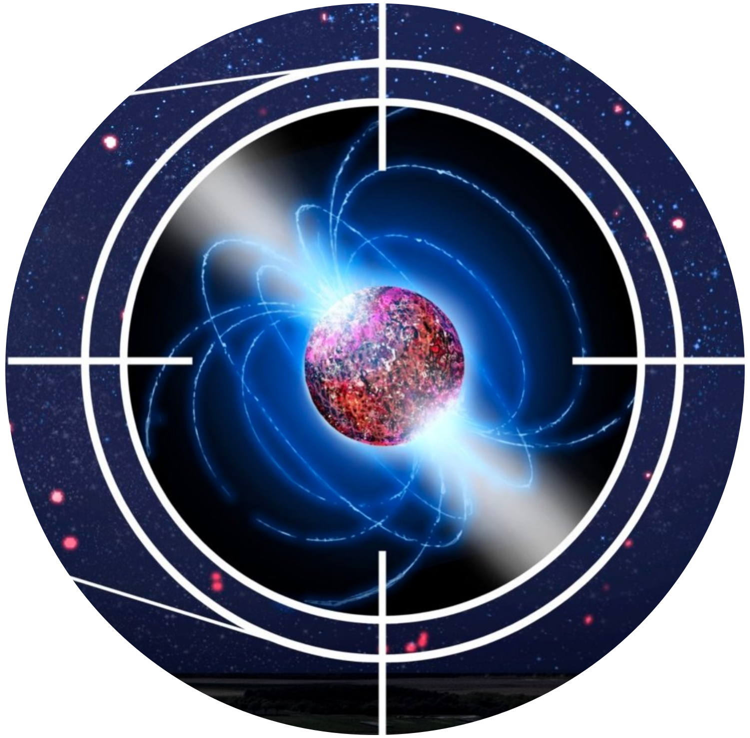
Pulsars are magnetized neutron stars emitting periodic pulses (mainly in the radio band, but also X-ray and Gamma-ray bans).
In 1967, pulsars were discovered serendipitously (Hewish et al. 1968)

Image credit: Danielle Futselaar and ASTRON

Neutron star (NS)

A feeling of fear or anxiety about something that may happen

The concept of neutron stars was envisioned with trepidation by Baade & Zwicky (1934): “With all reserve we advance the view that a supernova represents the transition of an ordinary star into a new form of star, the neutron star, which would be the endpoint of stellar evolution. Such a star may possess a very small radius and an extremely high density.”



PSR J0250+5854

$P_{\mathrm{spin}} = 23.5\ \mathrm{s}$

Tan et al. 2018

Normal star

$B\sim1\ \mathrm{G}$

$R\sim10^6\ \mathrm{km}$

$P_{\mathrm{spin}} \sim 1\ \mathrm{yr}$

Conservation of angular momentum

$L \propto m\, r^2\, \omega$

collapse

Conservation of magnetic flux

$\Phi = \int \vec{B} \cdot \vec{e}_A dA$

Neutron star

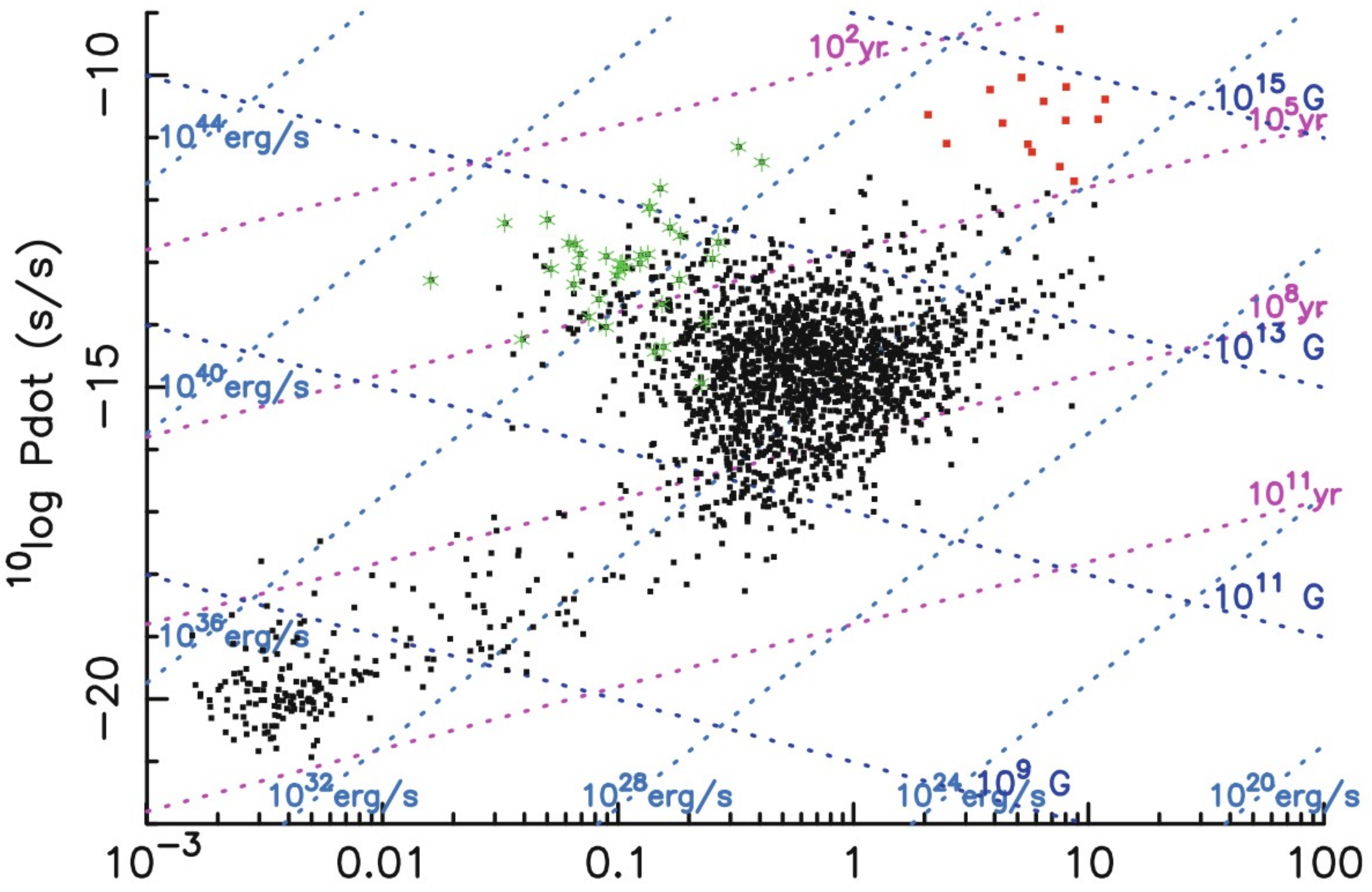
$R\sim10\ \mathrm{km}$

$B\sim10^{10}\ \mathrm{G}$

$P_{\mathrm{spin}} \sim 10^{-3}\ \mathrm{s}$

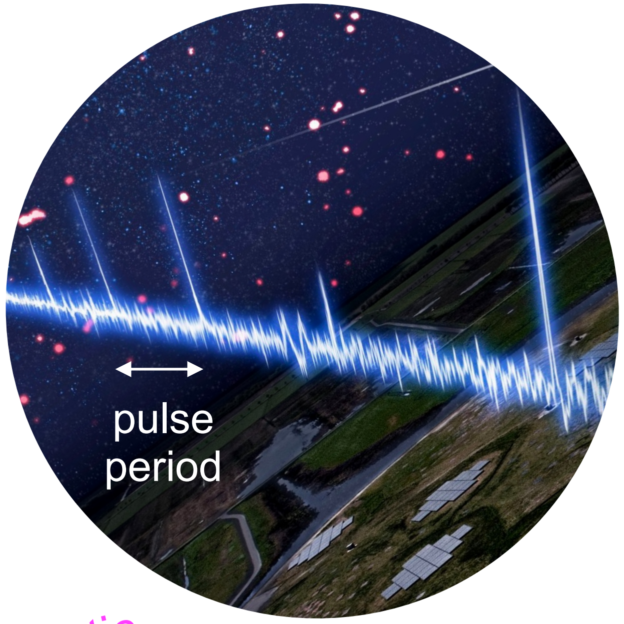
Why is PSR J0250+5854 so slow?

P-Pdot diagram for pulsars



pulse period derivative

pulse period



characteristic pulsar age

surface B-field strength

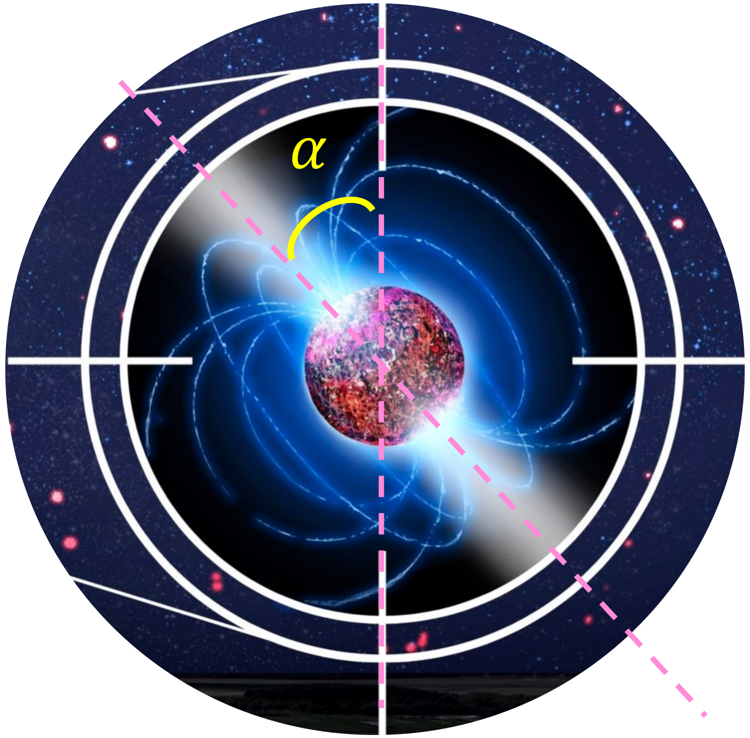
red squares: magnetars
green stars: pulsars in pulsar wind nebulae

Vink (2021)

Magnetic dipole

A young neutron star can be viewed as a rapidly rotating dipole

The power of magnetic dipole rotation is similar to that of electric dipole



Why are some pulsars so slow?

$$P = \frac{2\,(\ddot{\mu}\sin\alpha)^2}{3c^3}$$

$$= \frac{2}{3}\frac{\mu^2\omega^4}{c^3}\sin^2\alpha$$

magnetic dipole momentum $\longrightarrow$ $\mu = \frac{1}{2} B R_{\mathrm{NS}}^3$

$$\frac{d\vec{\mu}}{dt} = \vec{\omega}\times\vec{\mu}$$

$$\ddot{\mu} = \mu\,\omega^2$$

$$\omega = \frac{2\pi}{P}$$

pulse period

Larmor's formula

$$P = \frac{2q^2\dot{v}^2}{3c^3} = \frac{2\,\ddot{d}^2}{3c^3}$$

Pulsar spin down and B-field strength

Lattimer & Schutz, 2005

$$\mu = \frac{1}{2} B R_{\mathrm{NS}}^3$$

$$\omega = \frac{2\pi}{P}$$

$$I \simeq \frac{1}{5} M_{\mathrm{NS}} R_{\mathrm{NS}}^2$$

Assuming the spin-down of pulsars is due to the energy and momentum loss of the magnetic dipole radiation,

$$\frac{d}{dt}\left(-\frac{1}{2} I \omega^2\right) = \frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \alpha$$

$$\frac{d}{dt}\left(-\frac{1}{2} I \omega^2\right) = \frac{4\pi^2 \, M_{\mathrm{NS}} R_{\mathrm{NS}}^2}{2 \times 5} \frac{2\dot{P}}{P^3}$$

$$\frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \alpha = \frac{1}{6} \frac{B^2 R_{\mathrm{NS}}^6}{c^3} \frac{(4\pi^2)^2}{P^4} \sin^2 \alpha$$

$$B = \left(\frac{3c^3}{10\pi^2 \sin^2 \alpha} P\dot{P} \frac{M_{\mathrm{NS}}}{R_{\mathrm{NS}}^4}\right)^{1/2} = \frac{2.4 \times 10^{19}}{\sin \alpha} \left(P\dot{P}\right)^{\frac{1}{2}} \left(\frac{M_{\mathrm{NS}}}{1.4 \, M_{\odot}}\right)^{\frac{1}{2}} \left(\frac{10 \text{ km}}{R_{\mathrm{NS}}}\right)^2 \text{ G}$$

Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge

3.2 Thomson scattering
3.2.1 Scattered power and angular distribution
3.2.2 Differential cross section
3.2.3 Total cross section
3.2.4 Thomson scattering of unpolarized radiation
3.2.5 Eddington limit
3.2.6 Compton-thick AGN

3.3 Cyclotron radiation
3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

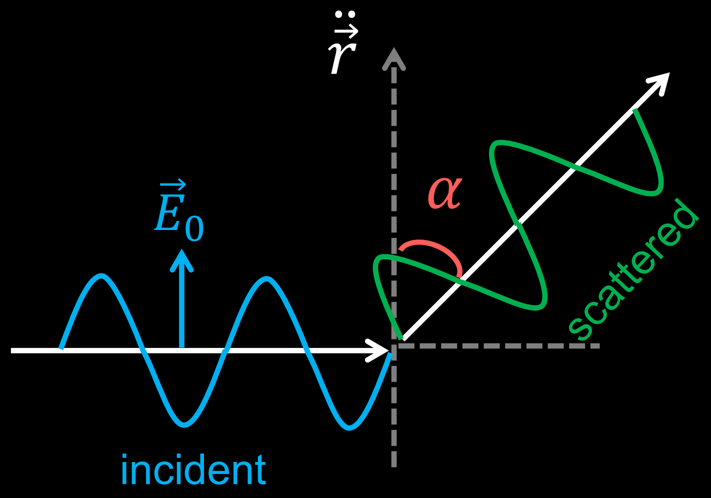
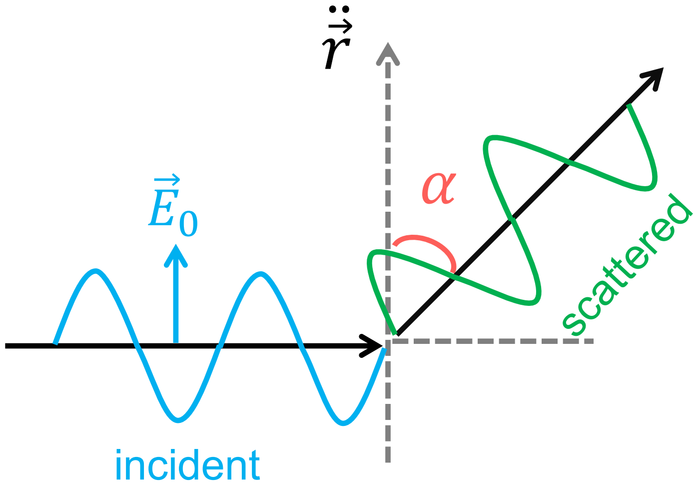


Image credit: Junjie Mao

Scattered power and angular distribution

Consider the response of a free electron to a linearly polarized incident EM wave, the time-averaged acceleration is



$$\langle \ddot{r}^2 \rangle = \frac{1}{2}\left(\frac{eE}{m_e}\right)^2$$

see Chpt. 3 of the REF book (p90-91) by Rybicki & Lightman

prev. sl.

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\dot{v}^2 \sin^2\alpha$$

$$\langle \frac{dP}{d\Omega} \rangle = \frac{e^2}{4\pi c^3}\langle \ddot{r}^2 \rangle \sin^2\alpha = \frac{e^4 E^2}{8\pi m_e^2 c^3}\sin^2\alpha$$

$$\langle P \rangle = \int \langle \frac{dP}{d\Omega} \rangle \, d\Omega = \frac{e^4 E^2}{8\pi m_e^2 c^3}\int_0^{2\pi} d\phi \int_{-1}^{1} (1-\cos^2\alpha)\, d\cos\alpha = \frac{e^4 E^2}{3\, m_e^2\, c^3}$$

$$\int_{-1}^{1}(1-\cos^2\theta) d\cos\theta = \frac{4}{3}$$

Differential cross section

The time-averaged incident flux is

$$\langle S \rangle = \frac{cE^2}{8\pi}$$

see Chpt. 3 of the REF book (p90) by Rybicki & Lightman

prev. sl.

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2}{4\pi c^3} \langle \ddot{r}^2 \rangle \sin^2 \alpha = \frac{e^4 E^2}{8\pi m_e^2 c^3} \sin^2 \alpha$$

The angular distribution of scattered power relates to the time-averaged incident flux and the differential cross section ($d\sigma$) for scattering into solid angle $d\Omega$ as

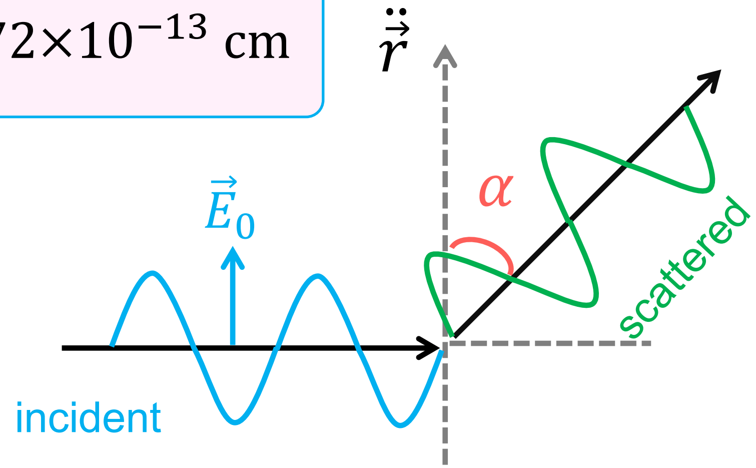
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \langle S \rangle \frac{d\sigma}{d\Omega} = \frac{cE^2}{8\pi} \frac{d\sigma}{d\Omega}$$

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} = 2.72 \times 10^{-13} \text{ cm}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2 \alpha = r_0^2 \sin^2 \alpha$$

The scattered radiation is linearly polarized



Total cross section

prev. sl.

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4} \sin^2\alpha = r_0^2 \sin^2\alpha$$

$$r_0 = \frac{e^2}{m_e c^2} = 2.72\times10^{-13}\ \mathrm{cm}$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = r_0^2 \int_0^{2\pi} d\phi \int_{-1}^{1} (1-\cos^2\alpha)\, d\cos\alpha$$

$$\int_{-1}^{1} (1-\cos^2\theta)\, d\cos\theta = \frac{4}{3}$$

Thomson scattering cross section

$$\sigma_\mathrm{T} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

Alternative approach to get the total cross section

prev. sl.

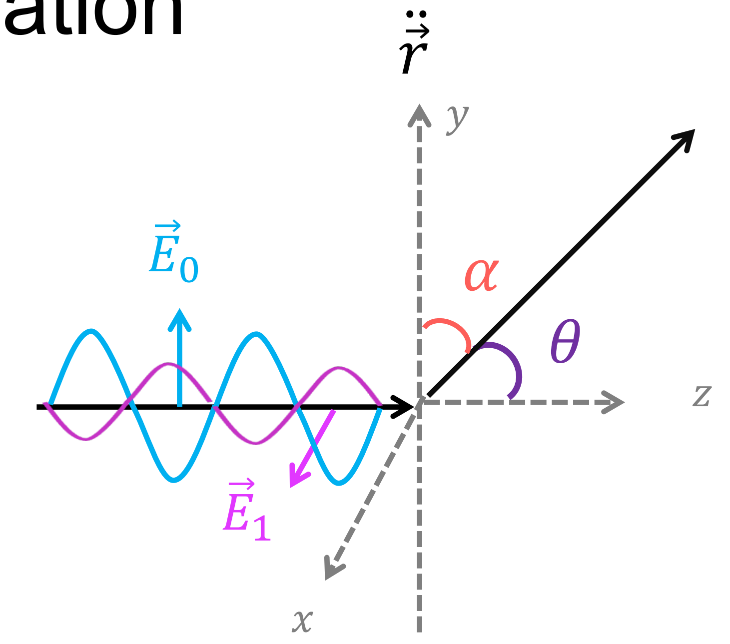
$$\langle P\rangle = \frac{e^4 E^2}{3\, m_e^2\, c^3}$$

prev. sl.

$$\langle S\rangle = \frac{cE^2}{8\pi}$$

$$\sigma = \frac{\langle P\rangle}{\langle S\rangle}$$

Differential cross section of unpolarized radiation



Thomson scattering of unpolarized radiation can be obtained by decomposing the radiation into a pair of two orthogonally linear-polarized radiations ($\vec{E}_0 \perp \vec{E}_1$), the differential cross section of unpolarized radiation

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}(\pi/2)\right)_{\text{pol}} + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}(\alpha)\right)_{\text{pol}}$$

$$= \frac{1}{2} r_0^2 (1 + \sin^2\alpha)$$

$$= \frac{1}{2} r_0^2 (1 + \cos^2\theta)$$

scattering angle $\theta = \pi/2 - \alpha$

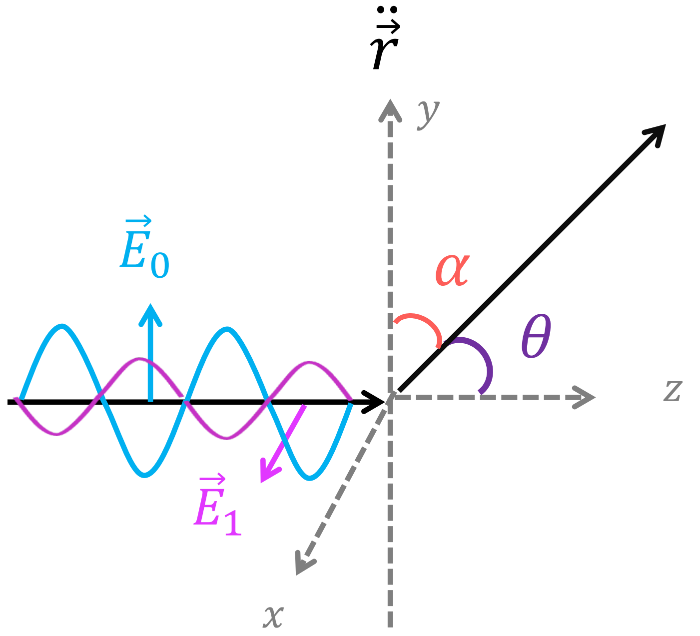
prev. sl.

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4}\sin^2\alpha = r_0^2 \sin^2\alpha$$

The differential cross section is symmetric for $\pm\theta$

Total cross section of unpolarized radiation

prev. sl. $$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2} r_0^2 (1 + \cos^2\theta)$$



$$\sigma_{\text{unpol}} = \int \left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} d\Omega$$

$$\int_{-1}^{1} (1 + \cos^2\theta) d\cos\theta = \frac{8}{3}$$

$$= \frac{r_0^2}{2} \int_0^{2\pi} d\phi \int_{-1}^{1} (1 + \cos^2\theta)\, d\cos\theta$$

$$= \frac{8}{3} \pi r_0^2$$

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652 \times 10^{-25}\ \mathrm{cm}^2$$

While the differential cross sections differ for polarized and unpolarized radiations, the total cross section is identical.

✓ Only valid for $h\nu \ll m_e c^2$ (i.e. $\ll 511$ keV). Below this threshold, the (differential and total) Thomson scattering cross section is **independent** of the incident photon energy.

Gravity vs Radiation

Consider a test particle with mass m at the distance r with respect to a black hole with mass M_{BH} and luminosity L

Gravitational force

$$f_{\mathrm{grav}} = \frac{G M_{\mathrm{BH}} m}{r^2}$$

Radiation pressure

$$p_{\mathrm{rad}} = \frac{1}{c} \frac{L}{4\pi r^2}$$

slide Sect. 2.1.4

$$p = \frac{F}{c}$$

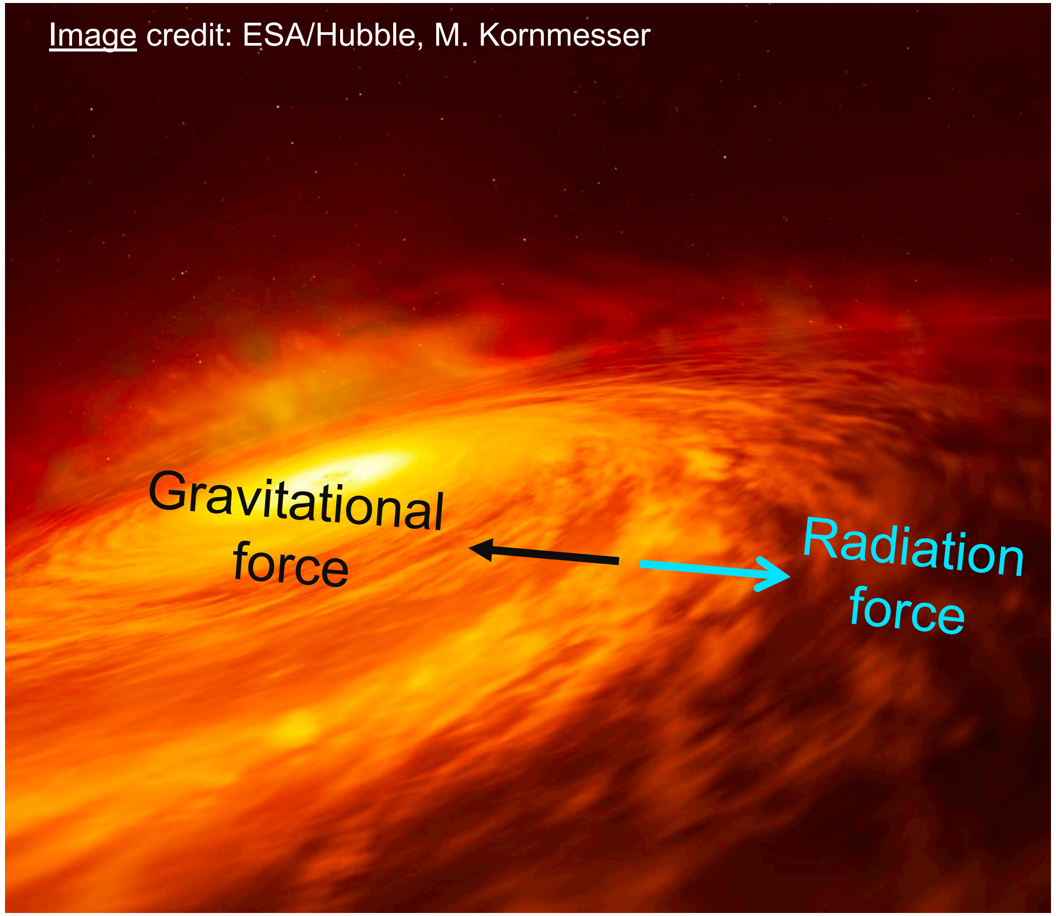
Radiation force

opacity in $\mathrm{g}^{-1}\,\mathrm{cm}^2$

$$f_{\mathrm{rad}} = p_{\mathrm{rad}}\, \kappa\, m = \frac{\kappa\, m}{c} \frac{L}{4\pi r^2}$$

slide Sect. 2.2.2

$$\alpha = \rho\, \kappa$$



Eddington luminosity

prev. sl.

$$f_{\mathrm{grav}} = \frac{G M_{\mathrm{BH}} m}{r^2} \quad ? \quad f_{\mathrm{rad}} = \frac{\kappa\, m}{c} \frac{L}{4\pi r^2}$$

$$L_{\mathrm{balance}} = \frac{4\pi c\, G M_{\mathrm{BH}}}{\kappa}$$

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

For a fully ionized plasma, Thomson scattering (scattering of photons by free electrons) provides an opacity of $\kappa_{\mathrm{T}} = \frac{\sigma_{\mathrm{T}}}{m_{\mathrm{H}}} = 0.398\ \mathrm{g}^{-1}\ \mathrm{cm}^2$

$$L_{\mathrm{Edd}} = \frac{4\pi c\, G M_{\mathrm{BH}}}{\kappa_{\mathrm{T}}} = 1.26\times10^{38} \left(\frac{M_{\mathrm{BH}}}{M_{\odot}}\right) \mathrm{erg\ s}^{-1}$$

Thomson scattering optical depth

HI4PI collaboration (2016)

slides Sect. 2.4.1

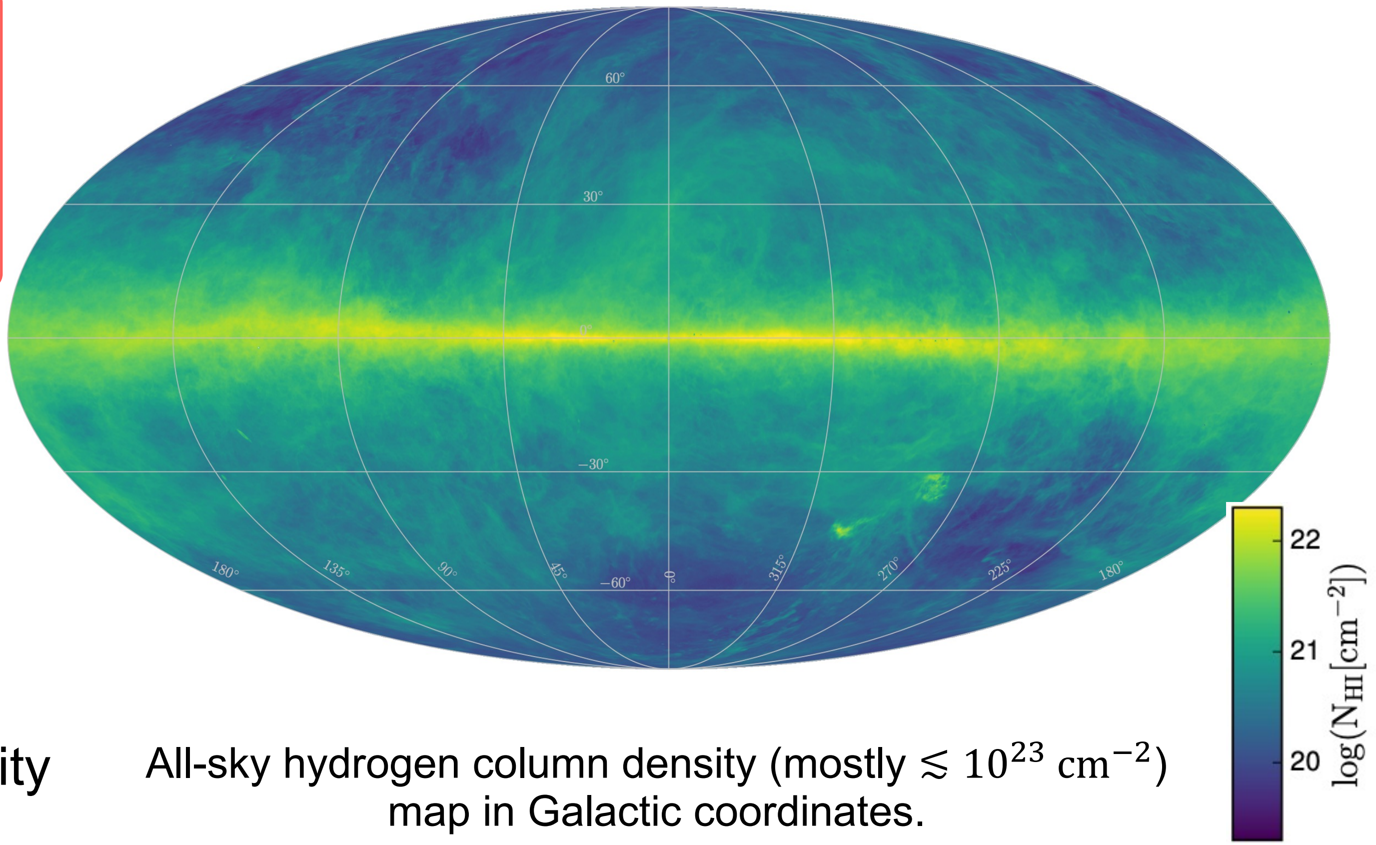
scattering coefficient

$$\tau_\nu^{\mathrm{sca}} \equiv \int \sigma_\nu^{\mathrm{sca}} dl$$

Thomson scattering cross section (ν independent)

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652\times10^{-25}\ \mathrm{cm}^2$$

When the line of sight hydrogen column density (N_{H}) exceeds $1.5\times10^{24}\ \mathrm{cm}^{-2}$, the Thomson scattering optical depth $\tau = N_{\mathrm{H}}\,\sigma_{\mathrm{T}} > 1$

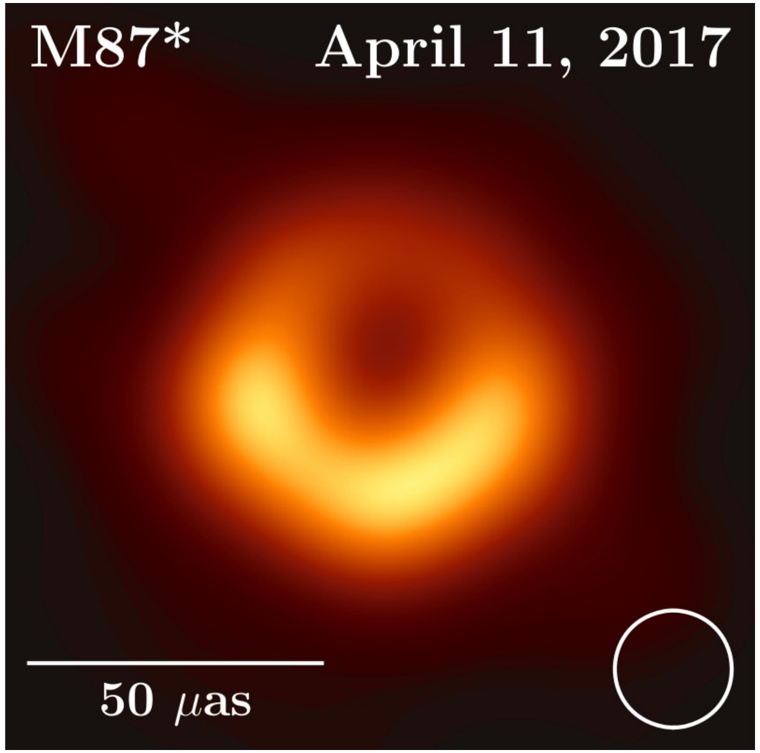


All-sky hydrogen column density (mostly $\lesssim 10^{23}\ \mathrm{cm}^{-2}$) map in Galactic coordinates.

Active Galactic Nuclei

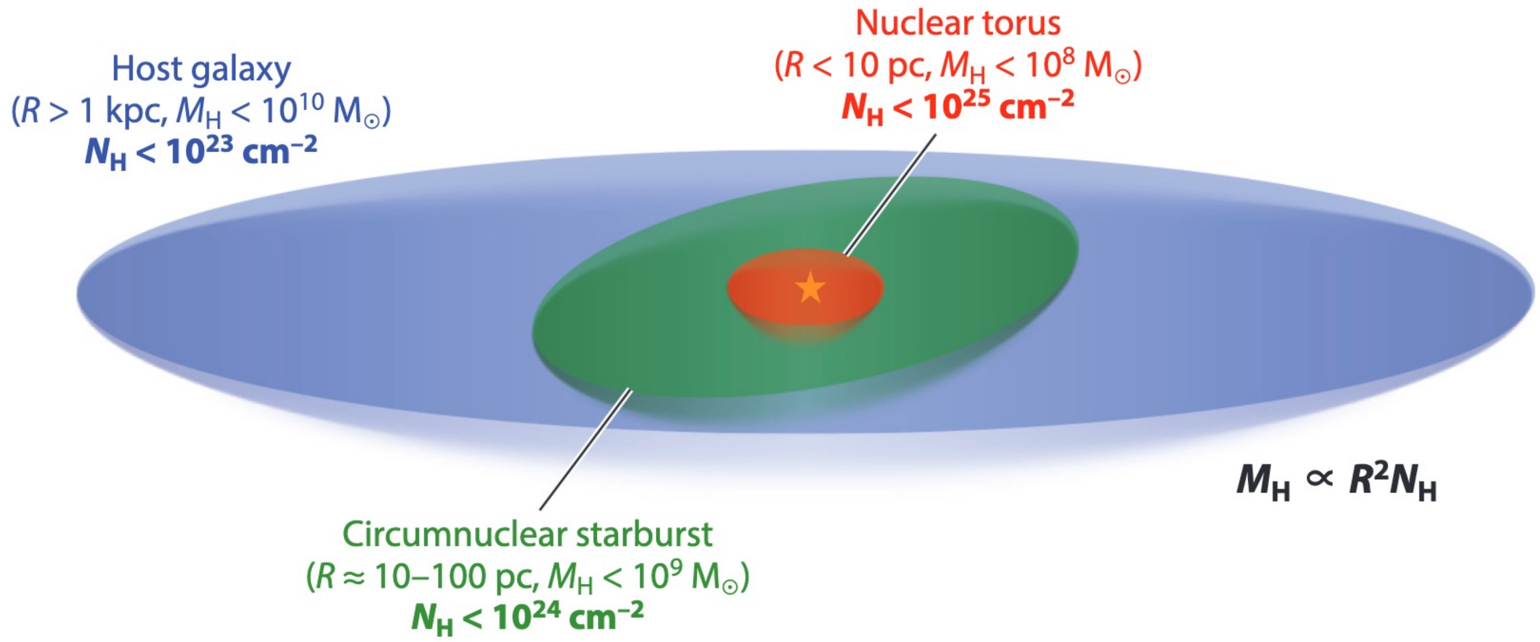
At the centre of almost all massive galaxies sit a supermassive black hole (SMBH)

Active: above 0.005% Eddington limit.



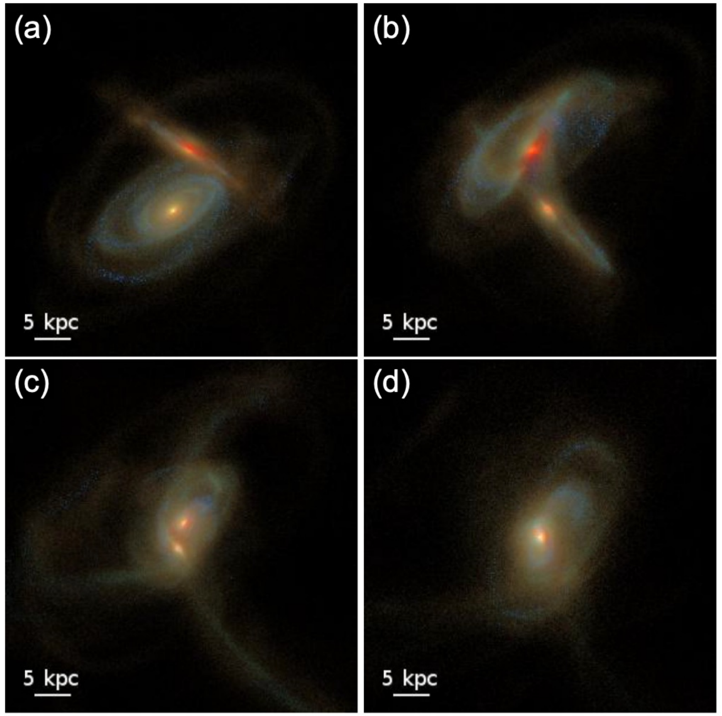
The Event Horizon Telescope network has obtained the first image of a SMBH — at the centre of the galaxy M87 (EHT Collaboration 2019)

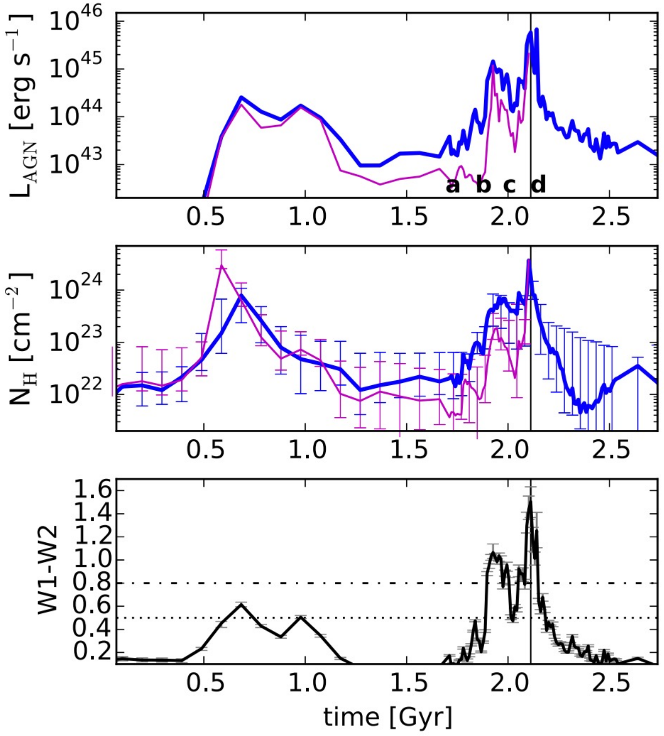
Netzer (2015, ARAA)

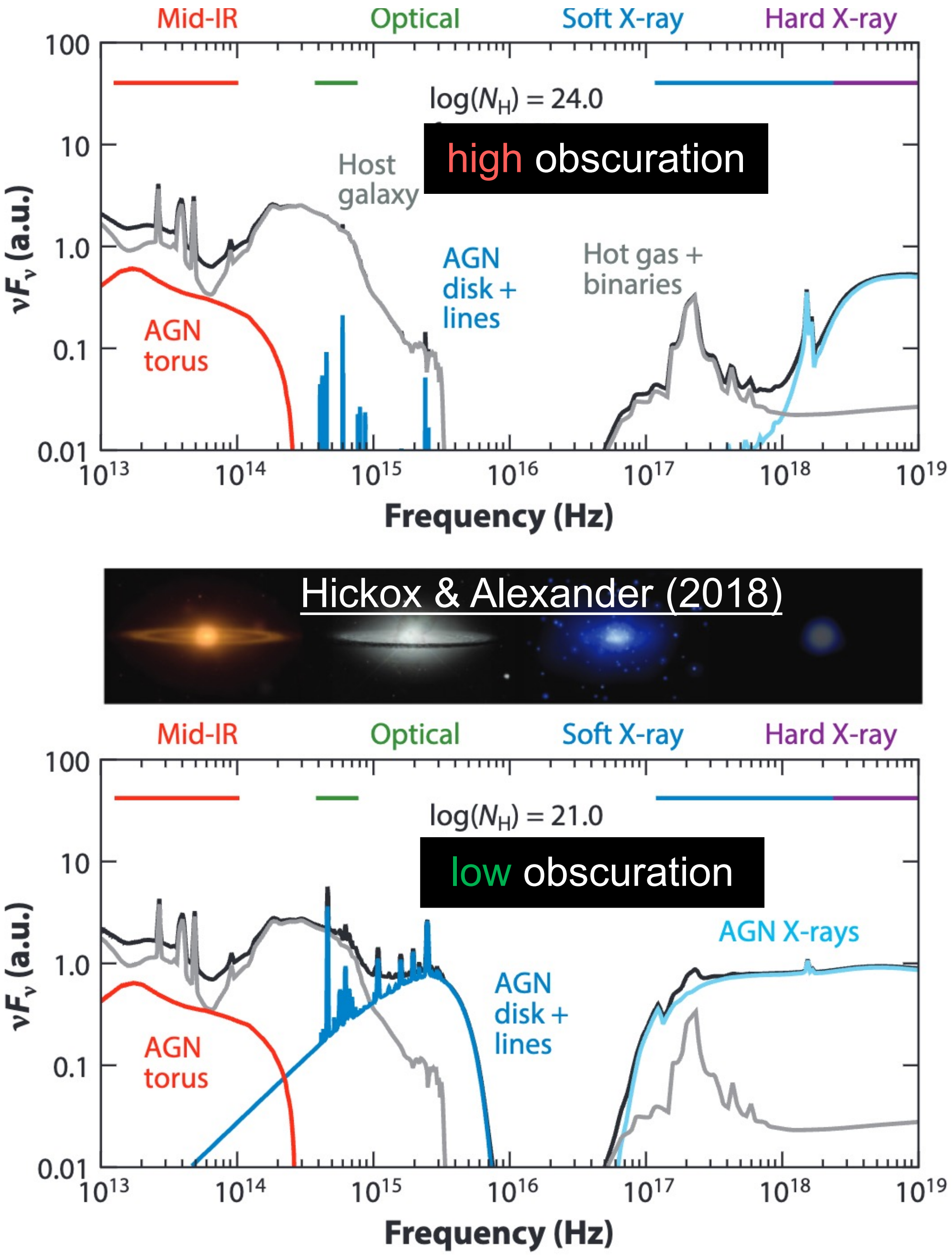


Hickox & Alexander (2018)

Compton-thick AGN







An illustration of supermassive black hole (SMBH) fuelling and obscuration during a galaxy merger (Blecha et al. 2018)

Chpt.3 Radiation from accelerating charges

3.1 Radiation field of an accelerated charge
3.2 Thomson scattering

3.3 Cyclotron radiation
3.3.1 Helical motion of non-relativistic electron
3.3.2 Radiative power of cyclotron radiation
3.3.3 Angular distribution of cyclotron radiation
3.3.4 Polarization of cyclotron radiation

3.4 Cyclotron resonance scattering features
3.5 Synchrotron radiation

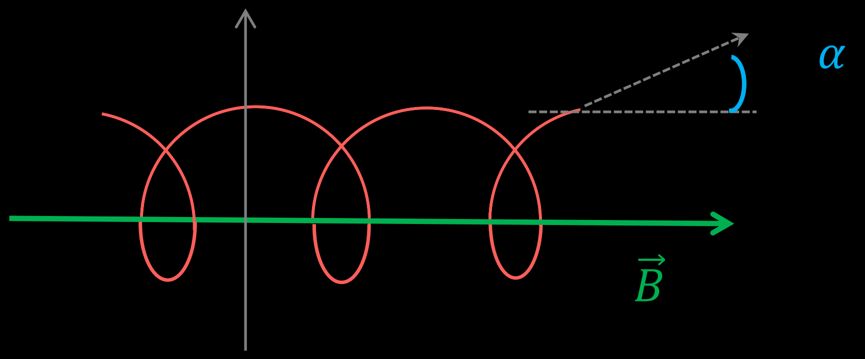
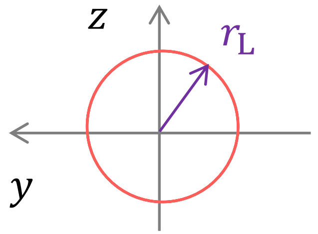
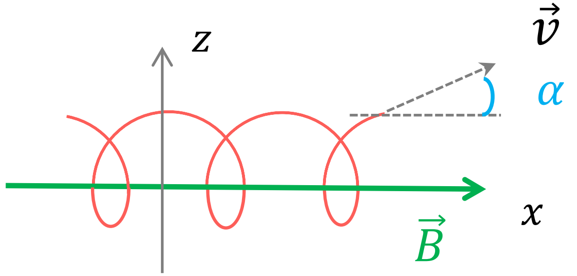


Image credit: Junjie Mao

Helical motion of non-relativistic electrons



For non-relativistic electrons, when moving in a homogenous B-field with α the angle between v and B, they will spiral along the field line.



The projected motion in the (y, z) plane is a circle with

$$r_\mathrm{L} = \frac{v \sin \alpha}{\omega_\mathrm{L}}$$

Larmor frequency

$$\omega_\mathrm{L} = \frac{eB}{m_e c}$$

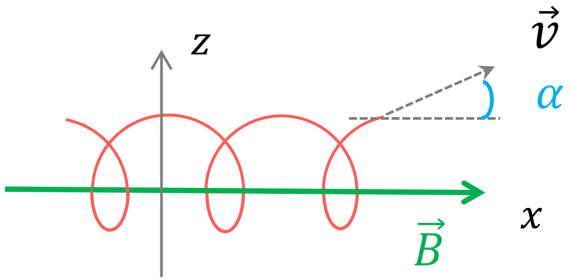
- If non-relativistic electrons do not move along the field line ($v \sin \alpha = 0$), the radiation has only one frequency (ω_L).
- Otherwise, harmonic frequencies $2\omega_\mathrm{L}, 3\omega_\mathrm{L}$, … can also be observed.
- The radiative power of cyclotron radiation declines rapidly for higher-order harmonic frequencies

Sect. 4.1.3 of 《天体物理中的辐射机制》by 尤峻汉 (p161)

$$\frac{P_{n+1}}{P_n} \sim \beta^2, n = 1, 2, 3, \ldots$$

Radiative power of cyclotron radiation

For non-relativistic electrons, the radiative power (consider only the base frequency radiation) simply follows Larmor's formula



Larmor's formula

$$P = \frac{2e^2\dot{v}^2}{3c^3}$$

Sect. 4.1.2 of 《天体物理中的辐射机制》by 尤峻汉 (p156)

$$m_e\dot{v} = -\frac{e}{c}\, v\, B \sin\alpha$$

Often is the case that we want to know the radiative power given the B-field strength and the velocity of electrons

$$P(\alpha) = \frac{2}{3}\frac{e^4}{m_e^2\, c^5}\; v^2 B^2 \sin^2\alpha = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2\alpha$$

$$= 1.59\times10^{-15}\, \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \sin^2\alpha \;\; \mathrm{erg\,s^{-1}}$$

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2} = 2.72\times10^{-13}\ \mathrm{cm}$$

Mean radiative power of cyclotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2\alpha$$

$$\int_0^{\pi} \sin^3\alpha \; d\alpha = \frac{4}{3}$$

If the velocity distribution of the electrons is isotropic, we have

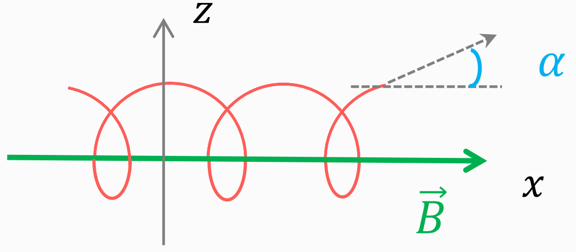
$$\langle \sin^2\alpha \rangle = \frac{\int \sin^2\alpha \; d\Omega}{\int d\Omega} = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\alpha \; d\alpha = \frac{2}{3}$$

$$\langle P \rangle = \frac{4}{9} c\, r_0^2 \beta^2 B^2 = 1.06\times 10^{-15}\, \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \; \mathrm{erg\,s^{-1}}$$

Angular distribution of cyclotron radiation

The angular distribution of cyclotron radiation (again, referring to the base frequency radiation) received by a distant observer is

$$\frac{dP_1}{d\Omega} \sim \frac{\pi e^2 \nu_L^2 \beta^2}{2c} (1 + \cos^2 \alpha)$$



- Along the direction of the B-field ($\alpha = 0$), the radiative power reaches its maximum.
  - cyclotron radiation is circularly polarized
- Perpendicular to the B-field ($\alpha = \pi/2$), the radiative power reaches its minimum, yet only smaller by a factor of 2 compared to the maximum value.
  - cyclotron radiation is linearly polarized
- At other directions, radiative power has intermediate values
  - cyclotron radiation is elliptically polarized

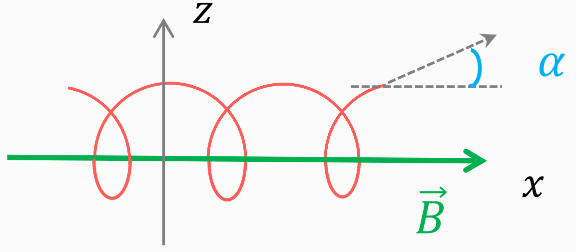
see Sect. 4.1.4 of 《天体物理中的辐射机制》 by 尤峻汉(p164-165)

Polarization of cyclotron radiation

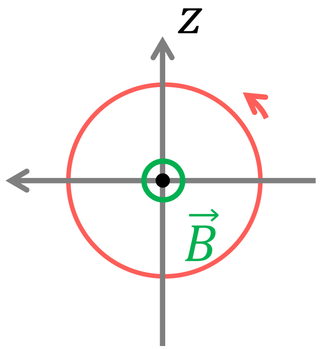
The polarization of cyclotron radiation (again, referring to the base-frequency radiation) is

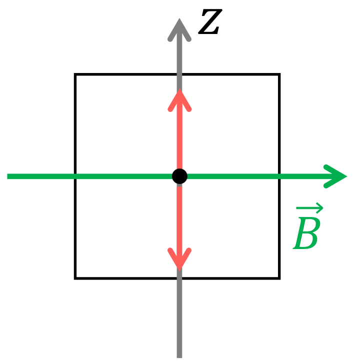
see Sect. 4.1.4 of 《天体物理中的辐射机制》 by 尤峻汉(p164-165)

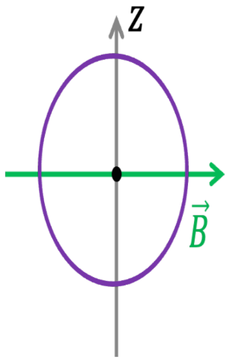
$$\vec{E}_1 = \left(-\frac{1}{2\nu_0}\beta\cos^2\alpha\right)\vec{x} + \left(-\frac{-i\,\beta}{2\nu_0}\right)\vec{y} + \left(\frac{\beta}{2\nu_0}\cos\alpha\sin\alpha\right)\vec{z}$$



- ❑ Along the direction of the B-field ($\alpha = 0$), cyclotron radiation is circularly polarized
- ❑ Perpendicular to the B-field ($\alpha = \pi/2$), cyclotron radiation is linearly polarized
- ❑ At other directions, cyclotron radiation is elliptically polarized





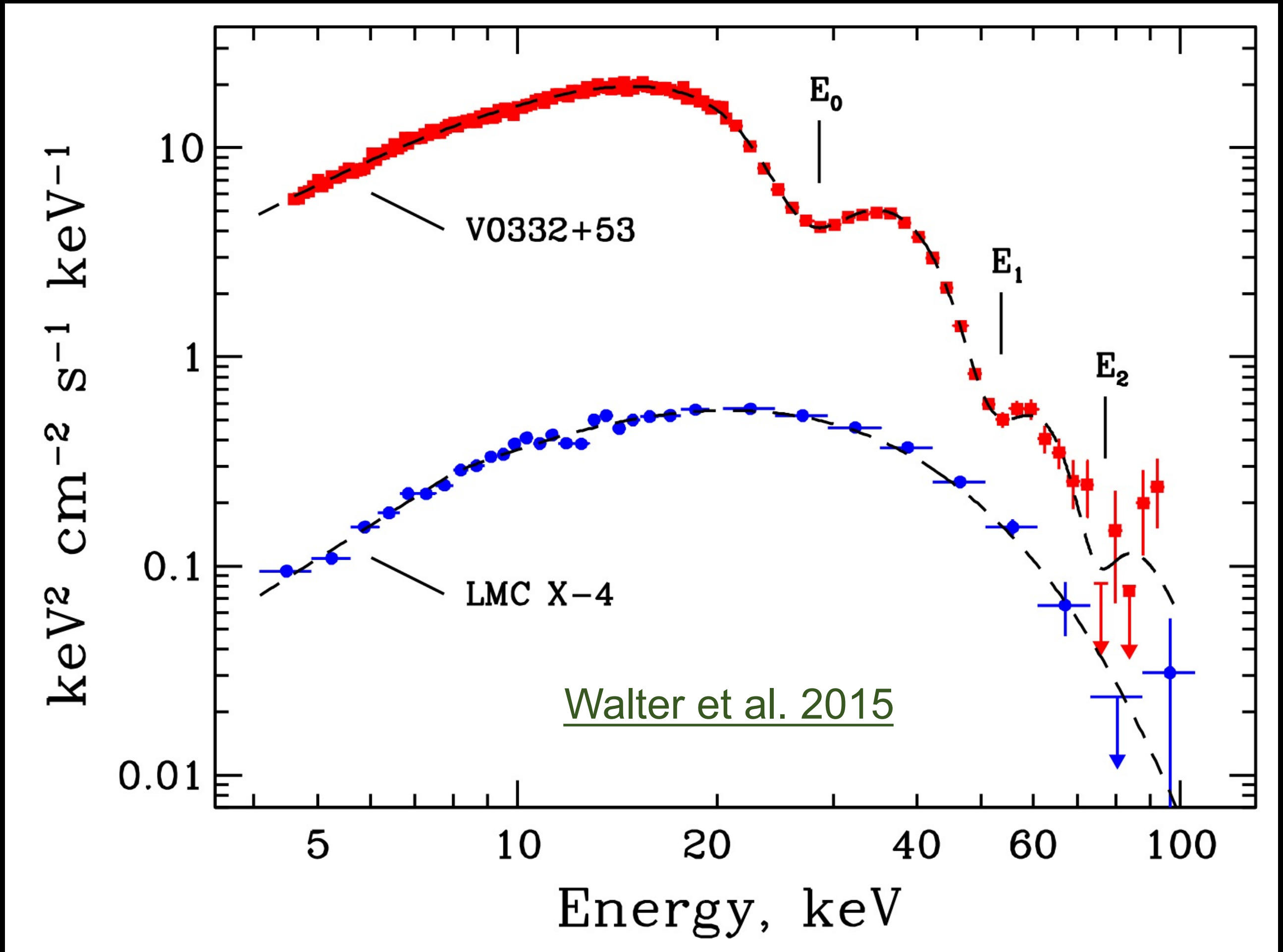


Chpt.3 Radiation from accelerating charges

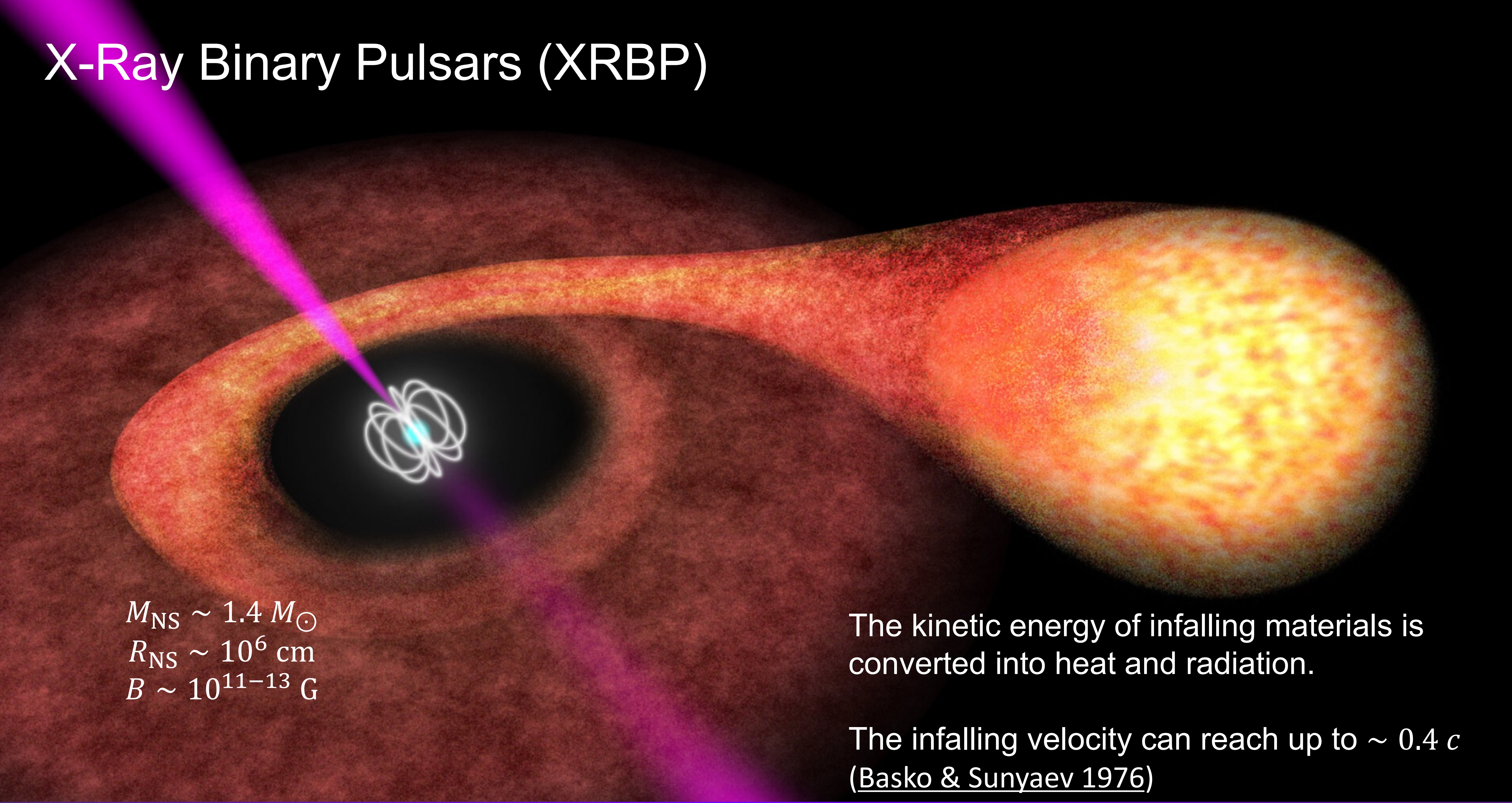
3.1 Radiation field of an accelerated charge
3.2 Thomson scattering
3.3 Cyclotron radiation

3.4 Cyclotron resonance scattering features

3.5 Synchrotron radiation



X-Ray Binary Pulsars (XRBP)

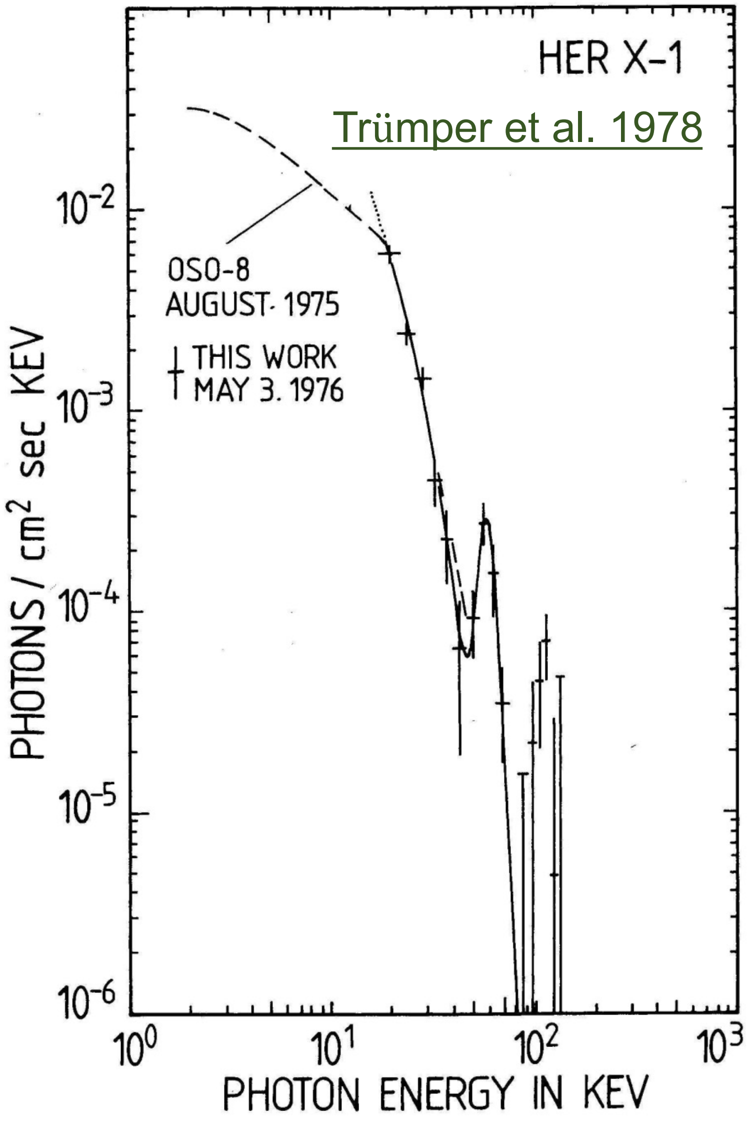


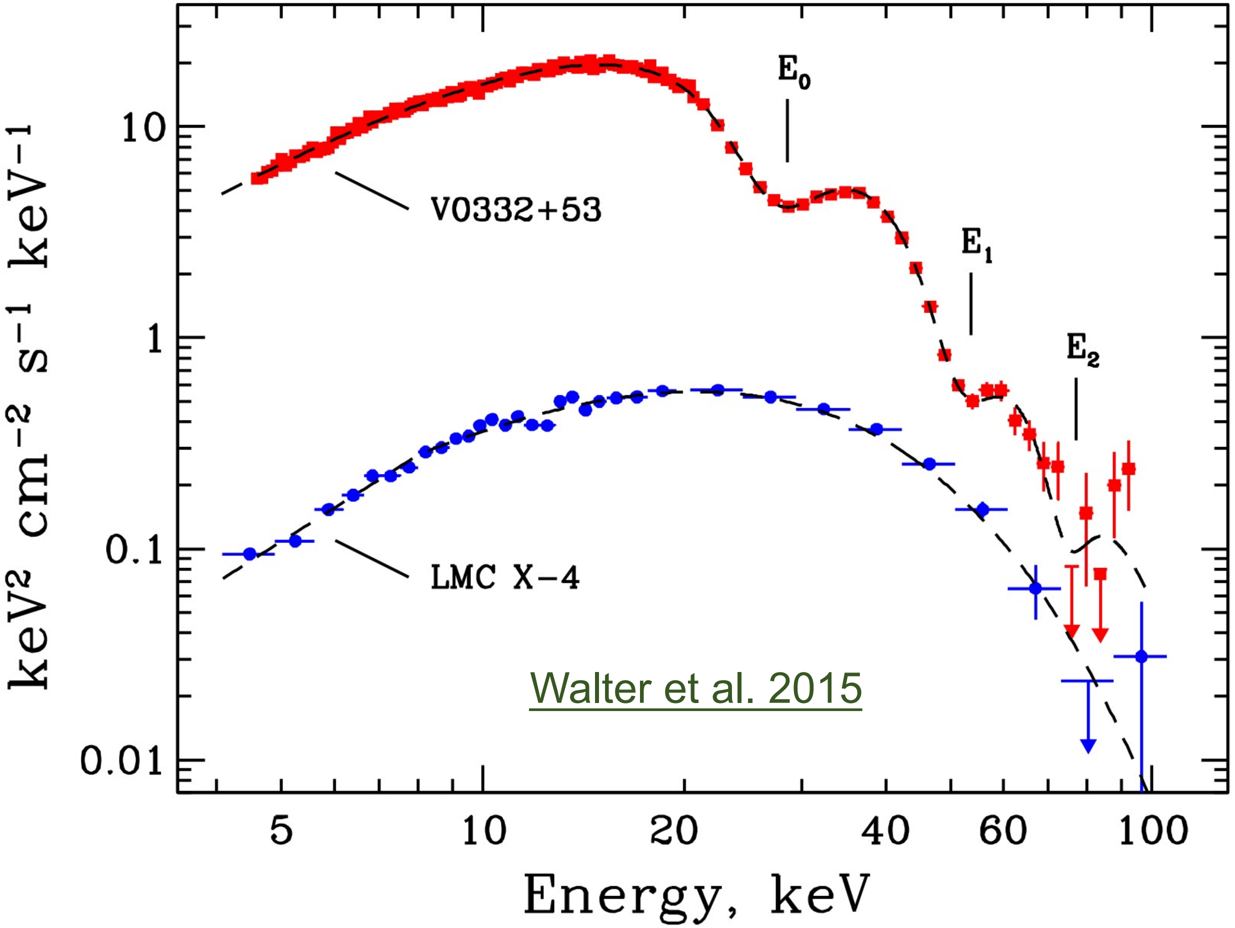
$M_{\mathrm{NS}} \sim 1.4\ M_\odot$
$R_{\mathrm{NS}} \sim 10^6\ \mathrm{cm}$
$B \sim 10^{11-13}\ \mathrm{G}$

The kinetic energy of infalling materials is converted into heat and radiation.

The infalling velocity can reach up to $\sim 0.4\ c$ (Basko & Sunyaev 1976)

X-ray absorption features

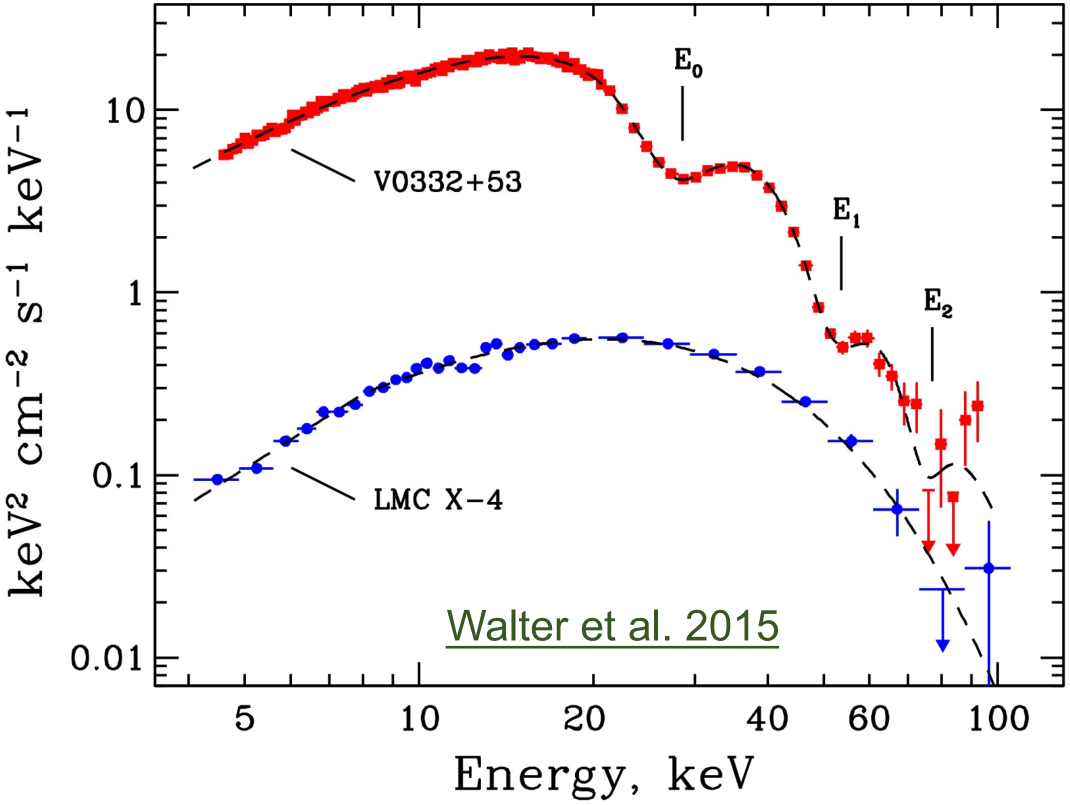




Before leaving the accretion column, these photons undergo scattering with (relativistic) electrons. The cross section is resonant at discrete energies of the so-called Landau levels (Schönherr et al. 2007)

Cyclotron resonance scattering features (CRSFs)

The scattering process gives rise to resonance absorption features in the X-ray spectrum (e.g., Staubert et al. 2019)



Walter et al. 2015

$n = 1, 2, 3, \ldots$ Landau level

12-B-12 rule

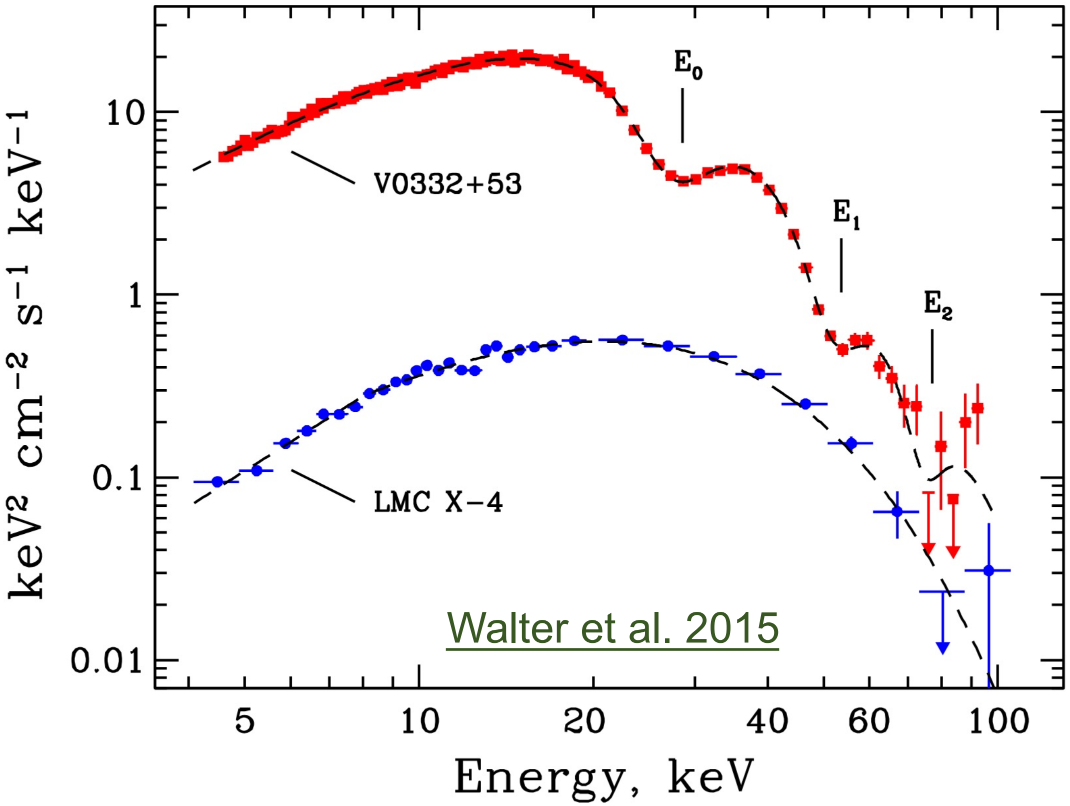
$$E_{\mathrm{CRSF}} = \frac{n}{1+z}\frac{h}{2\pi}\omega_{\mathrm{L}} \sim \frac{n}{1+z}\frac{11.6}{\mathrm{keV}}\frac{B}{10^{12}\ \mathrm{G}}$$

Gravitational redshift

$$z = \left(1 - \frac{2GM_{\mathrm{NS}}}{R_{\mathrm{NS}}\, c^2}\right)^{-1/2} - 1$$

for typical XRBP, $z \sim 0.3$

Broadening by inhomogeneous B-field strength



Walter et al. 2015

Larmor frequency

$$\omega_{\mathrm{L}} = \frac{eB}{m_e c}$$

$$\Delta\nu = \frac{e}{2\pi m_e c}\,\Delta B$$

Note: There are other line broadening mechanisms at play here.

Chpt.3 Radiation from accelerating charges

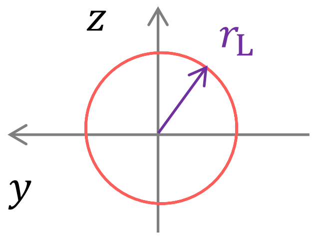
Helical motion of synchrotron radiation

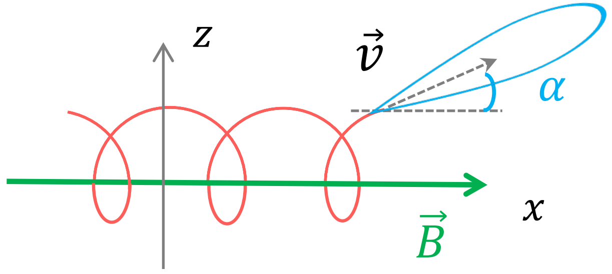
The base frequency of synchrotron radiation is a factor of γ (Lorentz factor) lower than Larmor's frequency

Larmor frequency
$$\omega_\mathrm{L} = \frac{eB}{m_e c}$$

$$\omega_0 = \frac{1}{\gamma}\,\omega_\mathrm{L}$$

Sect. 6.1 of the REF book (p167-168) by Rybicki & Lightman





The reduced base frequency leads to a large radius of the helical motion ($r\sim c/\nu_0$)

- Considering a relativistic electron with $\gamma = 100$ travelling in a normal spiral galaxy with a typical interstellar B-field strength of $10\ \mu\mathrm{G}$ (Condon & Ransom 2016), we have $\nu_\mathrm{L} \sim 30\ \mathrm{Hz}$ and $\nu_0 = \frac{\nu_\mathrm{L}}{\gamma} \sim 0.3\ \mathrm{Hz}$. The radius of the helical motion is of the order of $10^{11}\ \mathrm{cm}$.
- Almost linear at small scales ($r \ll c/\nu_0$).
- Harmonic frequencies $2\omega_0, 3\omega_0, \ldots$ can also be observed.
- The base and harmonic frequencies are much closer to each other in the frequency domain

Time duration of synchrotron radiation

- The radiation is strongly beamed with the half-angle of the radiation cone equals $1/\gamma$.
- We only see a short pulse of radiation in each orbit when the beam is pointing towards us.
- The time duration (Δt) of the radiation emitted between P_1 and P_2 is

$$\gamma m_e \frac{\Delta v}{\Delta t} = \frac{e}{c}\, v\, B \sin\alpha$$

$$\frac{\Delta\theta}{2} = \frac{1}{\gamma}$$

$$\Delta v = v\,\Delta\theta = \frac{2v}{\gamma}$$

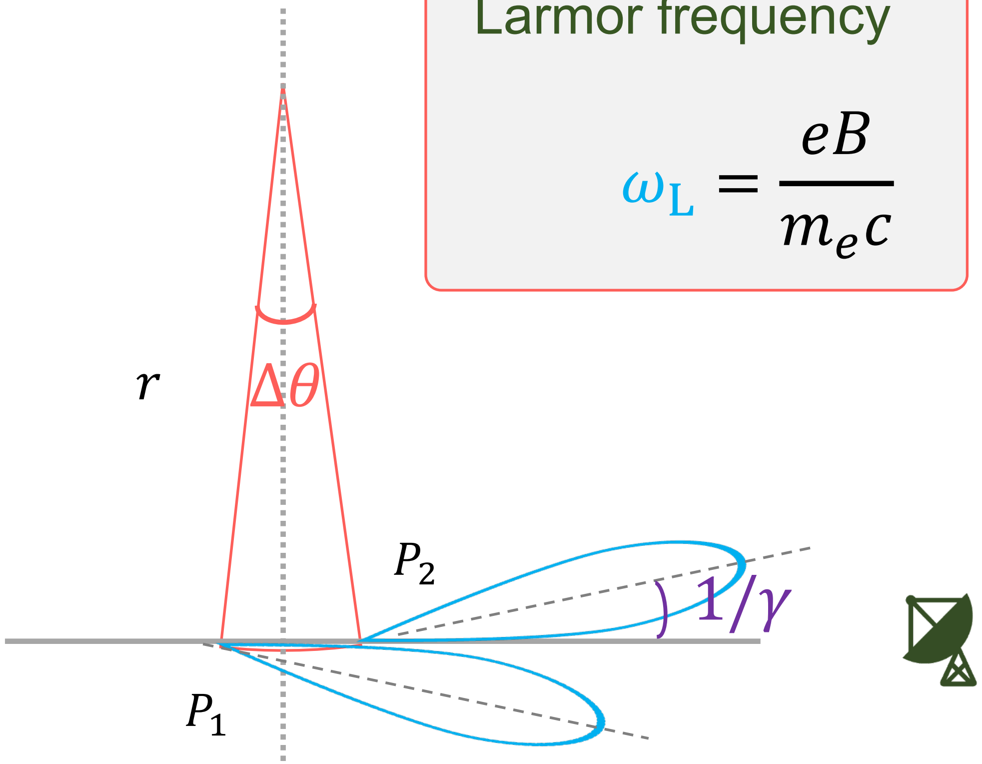
$$\Delta t = \frac{\gamma m_e c\,\Delta\theta}{e\,B\sin\alpha} = \frac{\gamma\,\Delta\theta}{\omega_{\mathrm{L}}\sin\alpha} = \frac{2}{\omega_{\mathrm{L}}\sin\alpha}$$

Sect. 4.1.2 of 《天体物理中的辐射机制》by 尤峻汉 (p156)

$$m_e\dot{v} = -\frac{e}{c}\, v\, B\sin\alpha$$

Larmor frequency

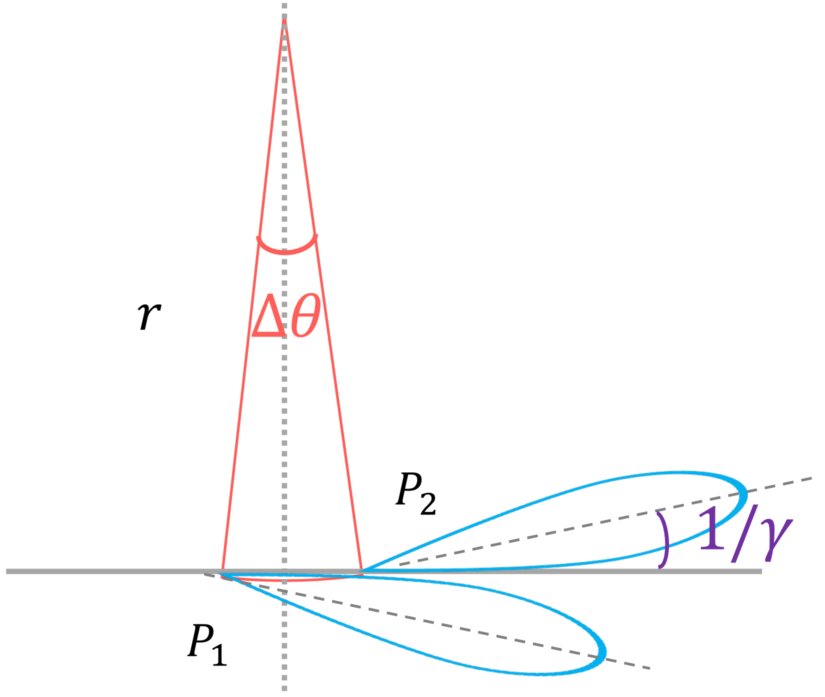
$$\omega_{\mathrm{L}} = \frac{eB}{m_e c}$$



Observed time duration of synchrotron radiation

prev. sl.

$$\Delta t = \frac{\gamma m c\, \Delta\theta}{e\, B \sin\alpha} = \frac{\gamma\, \Delta\theta}{\omega_{\mathrm{L}} \sin\alpha} = \frac{2}{\omega_{\mathrm{L}} \sin\alpha}$$



The observed time duration is a factor of $(1-\beta)$ smaller

$$\gamma^2 = \frac{1}{(1-\beta)(1+\beta)} \sim \frac{1}{2(1-\beta)}, \gamma \gg 1$$

$$\Delta t_{\mathrm{obs}} = \Delta t(1-\beta) \sim \frac{\Delta t}{2\gamma^2} = \frac{1}{\gamma^2\, \omega_{\mathrm{L}} \sin\alpha}$$

Lorentz factor

$$\gamma = \frac{mc^2}{m_0 c^2} = \frac{1}{\sqrt{1-\beta^2}}$$

Radiative power of synchrotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 B^2 \sin^2 \alpha$$

Compared to the radiative power of cyclotron, the radiative power of synchrotron is increased by a factor of γ^2

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \gamma^2 \beta^2 B^2 \sin^2 \alpha$$

$$= 1.587{\times}10^{-15}\, \gamma^2 \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \sin^2 \alpha \;\; \mathrm{erg\, s^{-1}}$$

Sect. 6.1 of the REF book (p167-169) by Rybicki & Lightman

Mean radiative power of synchrotron radiation

prev. sl.

$$P(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 \gamma^2 B^2 \sin^2 \alpha$$

prev. sl.

$$\langle \sin^2 \alpha \rangle = = \frac{2}{3}$$

If the velocity distribution of the electrons is isotropic, we have

$$\langle P \rangle = \frac{4}{9} c\, r_0^2 \gamma^2 \beta^2 B^2 = \frac{4}{3}\, \sigma_{\mathrm{T}}\, c \beta^2 \gamma^2 U_B$$

$$= 1.058{\times}10^{-15}\, \gamma^2 \beta^2 \left(\frac{B}{\mathrm{G}}\right)^2 \ \mathrm{erg\ s^{-1}}$$

Magnetic energy density

$$U_B = \frac{B^2}{8\pi}$$

Sect. 2.1 of the REF book (p51-53) by Rybicki & Lightman

Thomson scattering cross section

$$\sigma_{\mathrm{T}} = \frac{8\pi r_0^2}{3} = 6.652{\times}10^{-25}\ \mathrm{cm^{-2}}$$

Spectral shape of synchrotron radiation

The observed spectrum of synchrotron radiation will spread over a wide (frequency) range

$$\frac{dP(\nu)}{d\nu} = \frac{2\pi\sqrt{3}e^2\nu_{\mathrm{L}}}{c} F\left(\frac{\nu}{\nu_c}\right) \quad \mathrm{erg\,s^{-1}Hz^{-1}}$$

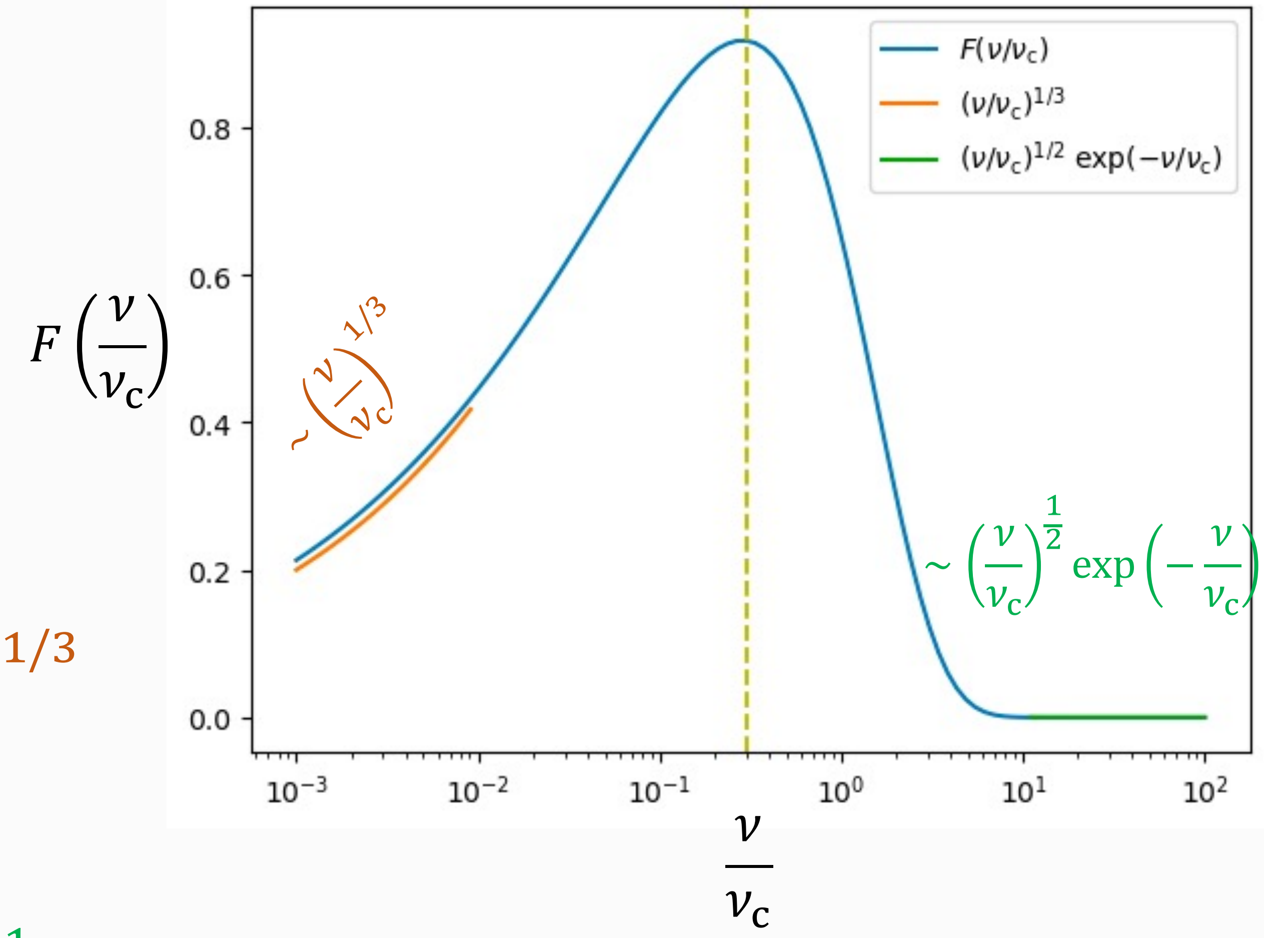
$$\nu_c = \frac{3}{2}\gamma^2\nu_{\mathrm{L}}$$

$$F\left(\frac{\nu}{\nu_c}\right) = \left(\frac{\nu}{\nu_c}\right)\int_{\nu/\nu_c}^{\infty} K_{5/3}(t)dt$$

$$\propto \left(\frac{\nu}{\nu_c}\right)^{1/3}$$

$$\propto \left(\frac{\nu}{\nu_c}\right)^{\frac{1}{2}} \exp\left(-\frac{\nu}{\nu_c}\right)$$

modified Bessel function of the 5/3 order



see Chpt. 4.2.4 of 《天体物理中的辐射机制》 by 尤峻汉(p184-188)

Spectrum of synchrotron radiation (cont.)

The observed spectrum of synchrotron radiation will spread over a wide (frequency) range

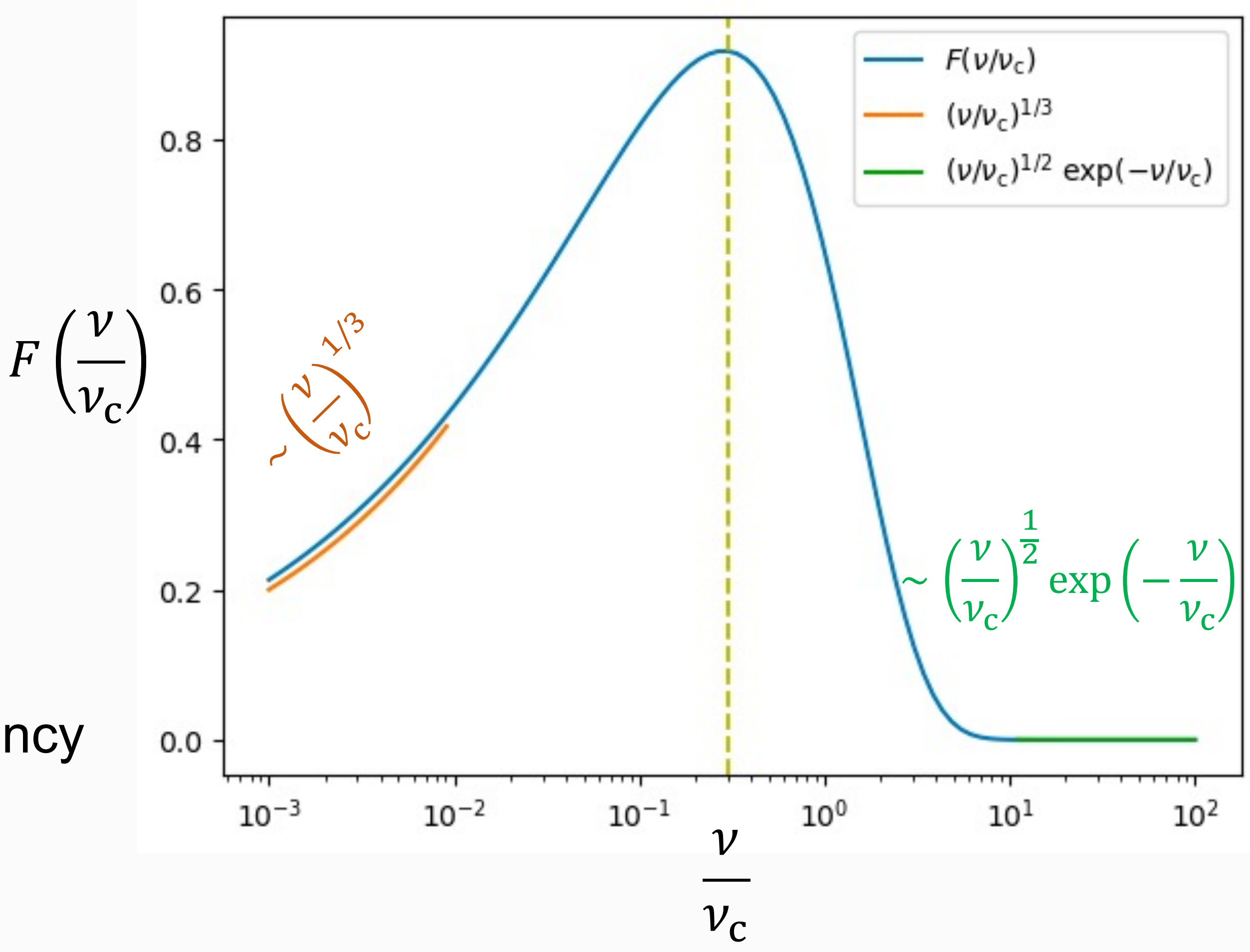
$$\frac{dP(\nu)}{d\nu} = \frac{2\pi\sqrt{3}e^2\nu_{\mathrm{L}}}{c} F\left(\frac{\nu}{\nu_c}\right) \ \ \mathrm{erg\,s^{-1}Hz^{-1}}$$

$$\nu_c = \frac{3}{2}\gamma^2\nu_{\mathrm{L}}$$

Most of the energy is radiated away at the frequency

$$\nu_{\mathrm{peak}} \sim 0.3\,\nu_c \sim 0.45\,\gamma^2\nu_{\mathrm{L}}$$

$$\sim 1.256{\times}10^6\,\gamma^2\left(\frac{B}{\mathrm{G}}\right)\ \mathrm{Hz}$$



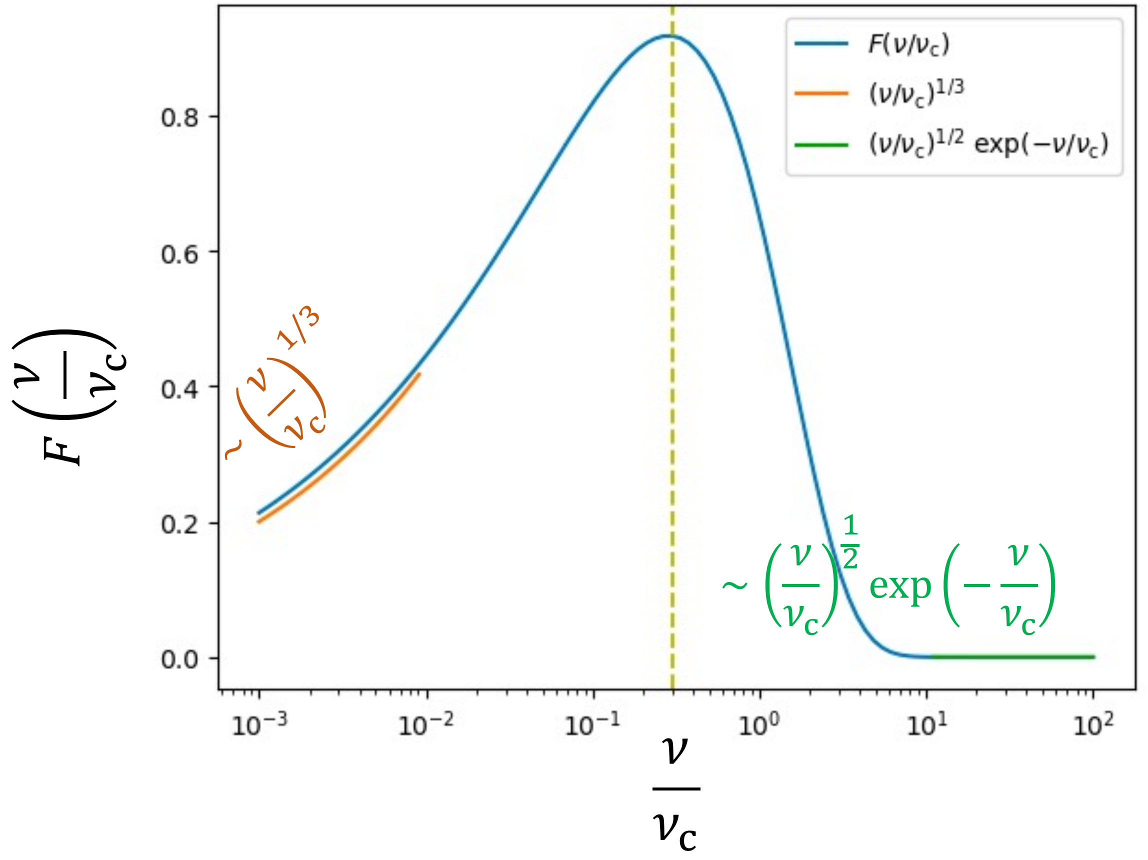
see Chpt. 4.2.4 of 《天体物理中的辐射机制》 by 尤峻汉(p184-188)

Synchrotron spectrum of power law electron distribution

Previous slides are limited to properties of a single relativistic electron. The observed spectral properties come from a large population of relativistic electrons with a power law energy distribution

$$N(\gamma)d\gamma \propto \gamma^{-p}d\gamma, \gamma_{\min} < \gamma < \gamma_{\max}$$

$$P(\nu) \propto P(\nu)\gamma^{-p}d\gamma \propto \int_{\gamma_{\min}}^{\gamma_{\max}} F\left(\frac{\nu}{\nu_c}\right)\gamma^{-p}d\gamma$$



Each single relativistic electron with γ gives rise to a broad emission "bump"

Synchrotron spectrum (cont.)

prev. sl.

$$P(\nu) \propto P(\nu)\gamma^{-p} d\gamma \propto \int_{\gamma_{\min}}^{\gamma_{\max}} F\left(\frac{\nu}{\nu_c}\right) \gamma^{-p} d\gamma$$

prev. sl.

$$\frac{dP(\nu)}{d\nu} \propto F\left(\frac{\nu}{\nu_c}\right)$$

$$\nu_c = \frac{3}{2}\gamma^2 \nu_{\mathrm{L}}$$

$$P(\nu) \propto \nu^{-(p-1)/2} \int_{x_{\min}}^{x_{\max}} F(x) x^{(p-3)/2} dx$$

$$x = \frac{\nu}{\nu_c} \propto \frac{\nu}{\gamma^2}, \gamma \propto \left(\frac{\nu}{x}\right)^{\frac{1}{2}}$$

$$d\gamma \propto -\frac{1}{2}\frac{\nu^{1/2}}{x^{3/2}} dx$$

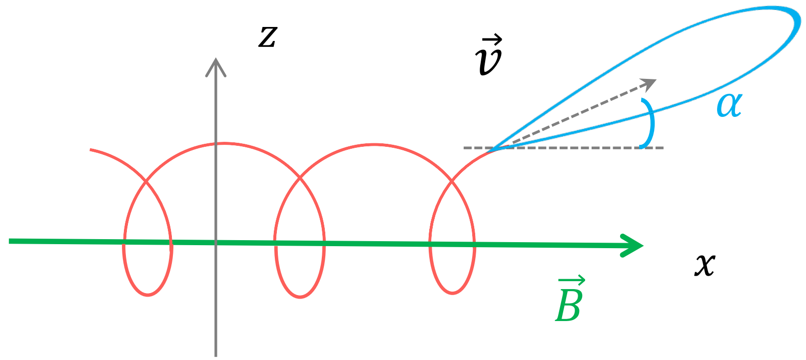
If $x_{\min} \to 0$ and $x_{\max} \to \infty$

$$\int_0^{\infty} F(x) x^{(p-3)/2} dx \sim \text{const.}$$

$$P(\nu) \propto \nu^{-(p-1)/2}$$

Polarization of synchrotron radiation

For a population of relativistic electrons with the same γ factor but different radiation angle $\theta \simeq \alpha \pm 1/\gamma$, the synchrotron radiation is linearly polarized

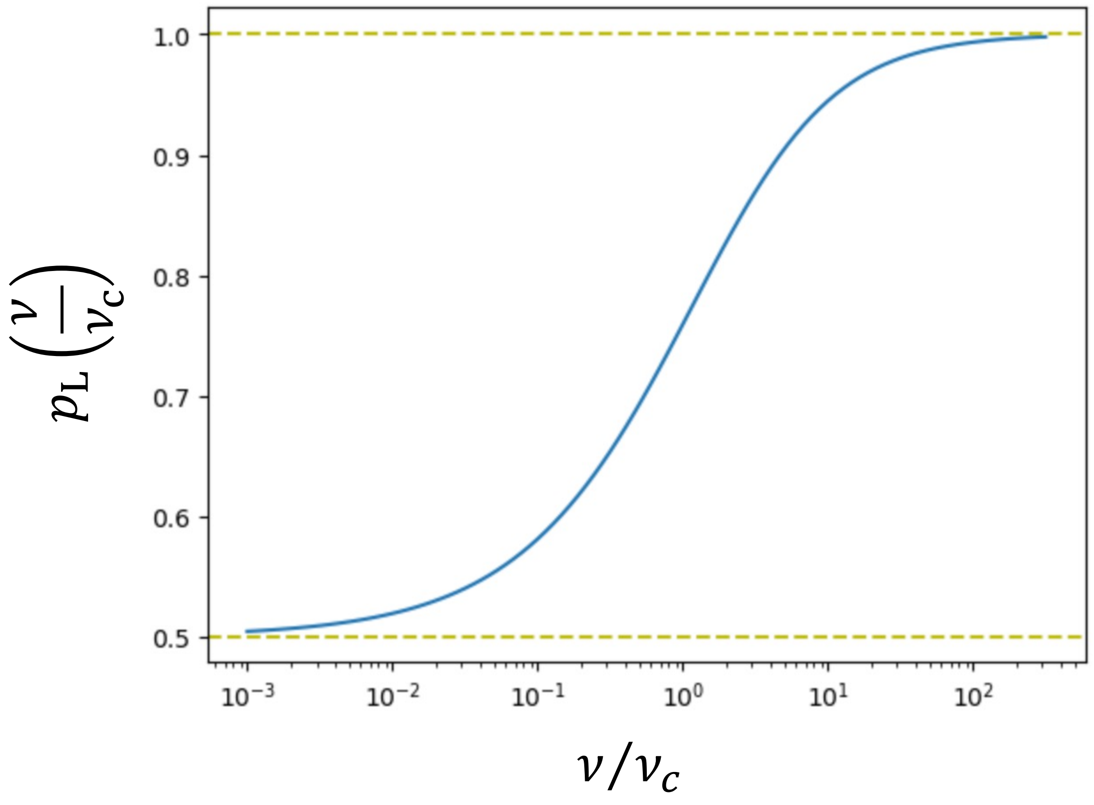


$$p_L = \frac{K_{2/3}(\nu/\nu_c)}{\int_{\nu/\nu_c}^{\infty} K_{5/3}(t)dt} \quad \begin{cases} 0.5 \text{ for } \nu \ll \nu_c \\ 1.0 \text{ for } \nu \gg \nu_c \end{cases}$$

Westfold (1959)

prev. sl.

$$\nu_c = \frac{3}{2}\gamma^2\nu_L$$



Synchrotron polarization of power law electron distribution

The observed average linear polarization fraction come from a large population of relativistic electrons with a power law energy distribution

$$N(\gamma)d\gamma \propto \gamma^{-p}d\gamma, \gamma_{\min} < \gamma < \gamma_{\max}$$

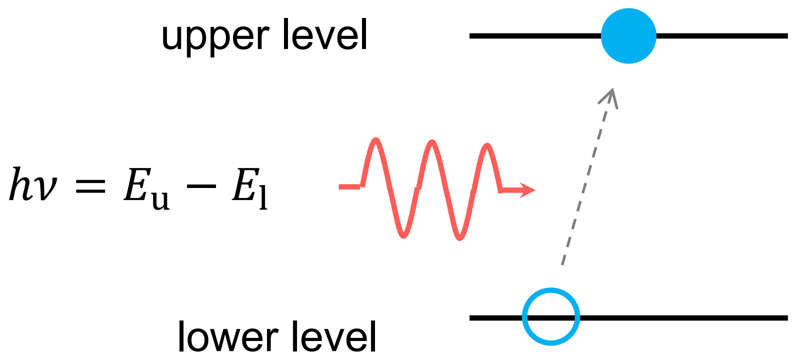
$$\langle p_L \rangle = \frac{p+1}{p+\frac{7}{3}}$$

See Chpt. 6 of the REF book (p180 - 181) by Rybicki & Lightman

- ✓ The average linear polarization fraction depends only on the power-law index of the relativistic electron population.
- ✓ The average linear polarization fraction is **<u>frequency-independent</u>**.

Synchrotron absorption

A photon interacts with a charge in a magnetic field and is absorbed, giving up its energy to the charge



Similar to the absorption of lines (two quantum states), synchrotron self-absorption can happen between two continuum states.

Synchrotron self-absorption

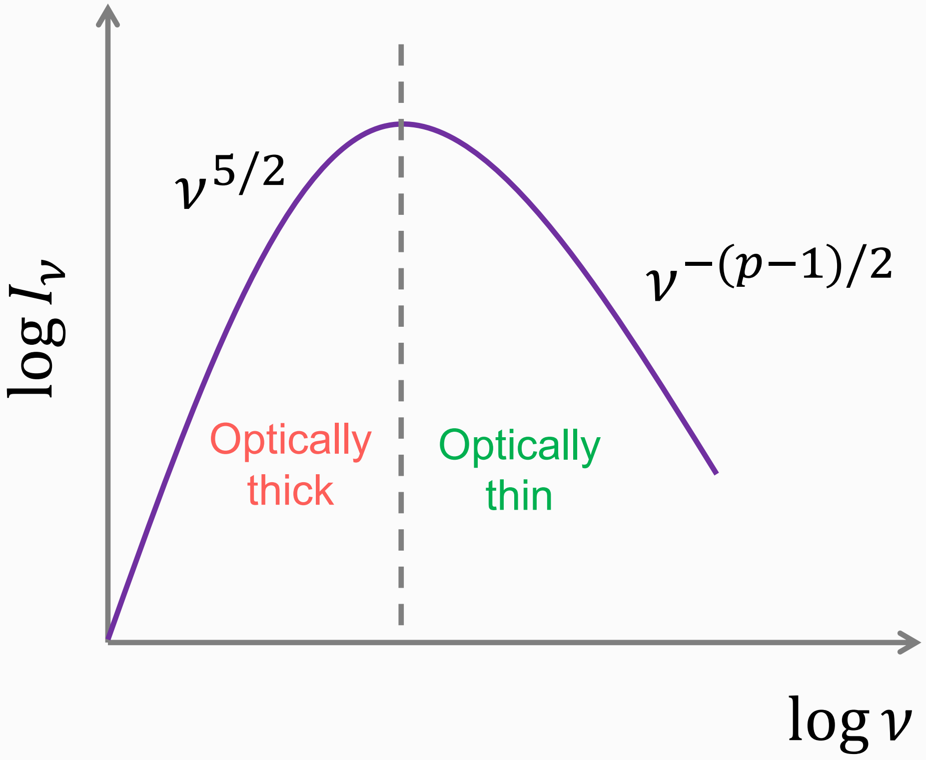
Consider a power-law distribution of electrons with $N(\gamma)d\gamma \propto \gamma^{-p}d\gamma$, for $h\nu \ll \gamma m_0 c^2$, the absorption coefficient and source function of synchrotron radiation

$$j_\nu = P_\nu$$

$$\alpha_\nu \propto \nu^{-(p+4)/2}$$

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = \frac{P_\nu}{4\pi\,\alpha_\nu} \propto \nu^{5/2}$$

See Chpt. 6 of the REF book (p186-190) by Rybicki & Lightman



$$I_\nu \propto \begin{cases} P_\nu\,, \text{optically thin} \\ S_\nu, \text{optically thick} \end{cases}$$

prev. sl.

$$P(\nu) \propto \nu^{-(p-1)/2}$$

Lifetime of synchrotron radiation

prev. sl.

$$P_{\text{syn}}(\alpha) = \frac{2}{3} c\, r_0^2 \beta^2 \gamma^2 B^2 \sin^2 \alpha$$

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$, it can lose energy via synchrotron radiation

classical electron radius

$$r_0 = \frac{e^2}{m_e c^2}$$

$$\frac{d}{dt}(\gamma m_e c^2) = -P_{\text{syn}} = -\frac{2}{3} \frac{e^4}{m_e^2 c^3} \frac{v^2}{c^2} B^2 \gamma^2 \sin^2 \alpha$$

$$= -A\, m_e c^2 (1 - \gamma^{-2}) \gamma^2$$

$$\frac{v^2}{c^2} = 1 - \gamma^{-2}$$

$$\frac{d\gamma}{dt} = -A\,(\gamma^2 - 1) \sim -A\gamma^2 \text{ for } \gamma \gg 1$$

$$A = \frac{2}{3} \frac{e^4}{m_e^3 c^5} B^2 \sin^2 \alpha$$

$$-\gamma^2\, d\gamma \sim A\, dt$$

$$\frac{1}{\gamma(t)} - \frac{1}{\gamma_0} = At$$

$$\gamma(t) = \frac{\gamma_0}{1 + A\,\gamma_0 t}$$

Half-energy lifetime due to synchrotron radiation

For a relativistic electron with initial kinetic energy $E_0 = \gamma_0 m_e c^2$ and emits synchrotron radiation, the half-energy lifetime is

$$\frac{\gamma_0}{2} = \frac{\gamma_0}{1 + A\,\gamma_0 t_{\mathrm{syn},1/2}}$$

$$t_{\mathrm{syn},1/2} = \frac{1}{\gamma_0 A} = \frac{3 m_e^3 c^5}{2e^4} \frac{1}{\gamma_0} B^{-2} (\sin\alpha)^{-2}$$

$$\sim 5.16\times 10^8 \frac{1}{\gamma_0} \left(\frac{B}{\mathrm{G}}\right)^{-2} (\sin\alpha)^{-2}\ \mathrm{s}$$

$$t_{\mathrm{syn},1/2} = 5.78\times 10^{11} \left(\frac{\nu_{\mathrm{peak}}}{\mathrm{Hz}}\right)^{-\frac{1}{2}} \left(\frac{B}{\mathrm{G}}\right)^{-3/2} (\sin\alpha)^{-2}\ \mathrm{s}$$

prev. sl.

$$\gamma(t) = \frac{\gamma_0}{1 + A\,\gamma_0 t}$$

$$A = \frac{2e^4 B^2 \sin^2\alpha}{3\, m_e^3 c^5}$$

prev. sl.

$$\nu_{\mathrm{peak}} \sim 1.256\times 10^6\, \gamma^2 \left(\frac{B}{\mathrm{G}}\right)\ \mathrm{Hz}$$

Requirement of synchrotron radiation

To ensure the energy loss is negligible in a specific cycle, the following requirement needs to be met on $t_{\mathrm{syn},1/2}$ and the period of circular motion $T = 2\pi/\omega_0$

prev. sl.

$$t_{\mathrm{syn},1/2} = \frac{1}{\gamma_0 A} = \frac{3m_e^3c^5}{2e^4}\frac{1}{\gamma_0}B^{-2}(\sin\alpha)^{-2}$$

$$\frac{2\pi}{\omega_0 t_{\mathrm{syn},1/2}} = \frac{2\pi\gamma_0}{\omega_\mathrm{L}}\frac{2e^4\gamma_0 B^2\sin^2\alpha}{3m_e^3c^5} = \frac{4\pi e^3}{3m_e^2c^4}\gamma_0^2 B\sin^2\alpha$$

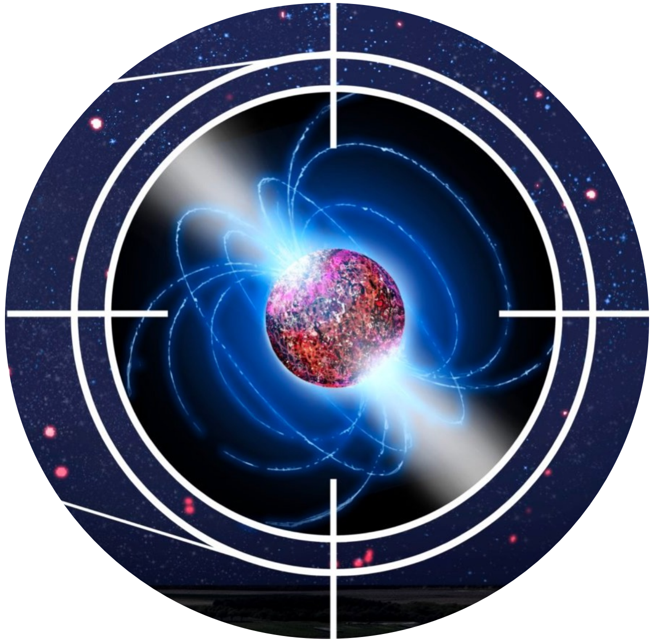
$$\sim 6.93\times 10^{-16}\,\gamma_0^2\left(\frac{B}{\mathrm{G}}\right)\sin^2\alpha \ll 1$$

prev. sl.

$$\nu_\mathrm{peak} \sim 1.256\times 10^6\,\gamma^2\left(\frac{B}{\mathrm{G}}\right)\ \mathrm{Hz}$$

If $\nu_\mathrm{peak} \ll 10^{21}\ \mathrm{Hz}$, the above requirement can be met

Synchrotron radiation in strong B-fields



prev. sl.

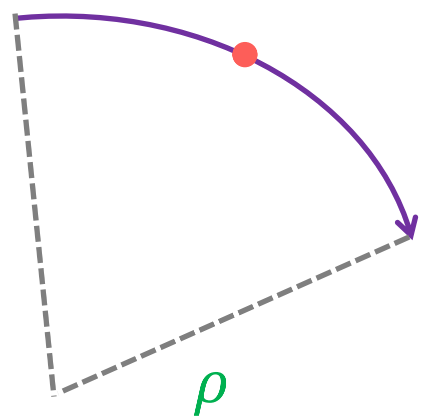
$$t_{\mathrm{syn},1/2} \sim 5.78\times10^{8}\,\gamma_0^{-1}\left(\frac{B}{\mathrm{G}}\right)^{-2}(\sin\alpha)^{-2}\ \mathrm{s}$$

For a neutron star with a B-field strength of 10^{12} G, if it emits synchrotron radiation with $\gamma \sim 100$, $t_{\mathrm{syn},1/2}$ is as short as 6×10^{-18} s!

$$\frac{1}{\nu_0} \sim \frac{\gamma}{\nu_\mathrm{L}} \sim \frac{\gamma}{2.8\times10^{6}\left(\frac{B}{\mathrm{G}}\right)}\ \mathrm{s}$$

For comparison, the gyration time is $\sim 4\times10^{-16}$ s.

Curvature process



If the pitch angle α is not too small, the relativistic electron still has a velocity component along the B-field line.

For curved B-field lines, such a process can give rise to curvature radiation. The instantaneous behavior is similar to synchrotron radiation.

$$\nu_0^{\mathrm{crv}} \simeq \frac{c}{2\pi\rho}$$

$\longleftarrow$ (instantaneous) curvature radius

$$\nu_0^{\mathrm{syn}} = \frac{c}{2\pi r_0}$$

$\longleftarrow$ radius of the helical motion

Decomposing the kinetic energy of relativistic electrons along the B-field line ($K_{E,\parallel}$) and perpendicular to the B-field line ($K_{E,\perp}$) . Curvature radiation consumes $K_{E,\parallel}$ while synchrotron radiation consumes $K_{E,\perp}$.