AY2024F

Junjie Mao

Radiative Processes in Astrophysics

Observation

Up to cosmic size scale

Goal

Theory

Down to atomic size scale

C/2012S1 (comet)

Jupiter (planet)

Sun (star)

Cas A (SNR)

M82 (galaxy)

Phoenix (gal. cluster)

Cosmic web filament

Image credit: Chandra photo album

Chpt.1 Astrophysical messengers

1.1 Electromagnetic waves
1.2 Neutrinos
1.3 Cosmic rays
1.4 Gravitational waves

GW170817 (GW)
+GRB170817A (γ-ray)
+AT2017gfo (NIR to UV)
+ GRB afterglow (X-ray + radio)

Image credit: Branchesi (2023)

Electromagnetic waves

Image credit: ESA

speed of light

wavelength

frequency

$$c = \lambda\nu$$

Planck constant

$$E = h\nu$$

$$T = E/k$$

Boltzmann constant

Atmospheric window

Space missions (almost all UV, X-ray, and γ-ray missions)

Image credit: NASA

Ground facilities (mainly radio, IR, and optical telescopes)

Background image credit: CMS

Not just atmospheric window

Background image credit: NOIRLab/NSF/AURA, P. Marenfeld

7	6	5	4	3	2	1	0
Excellent Dark Sky Site	Dark Sky Site	Rural Sky	Suburban/Rural Transition	Suburban Sky	Bright Suburban Site	City/Suburbia Transition	City/Inner City Sky

EM observing facilities

Image credit: NASA

LOFAR (radio)
Image credit: ASTRON

Planck (microwave)
Image credit: ESA

HST (IR to UV)
Image credit: HST.org

Fermi (γ-ray)
Image credit: NASA

ALMA (microwave)
Image credit: ESO

JWST (IR to optical)
Image credit: NASA

Gemini (IR to optical)
Image credit: NOIRLab

XRISM(X-ray)
Image credit: JAXA

EM observing facilities (cont.)

Image credit: NASA

ASO-S (UV to X-ray)
Image credit: PMO

FAST (radio)
Image credit: Nature

LAMOST (optical)
Image credit: NAOC

insight-HMXT
(X-ray to γ-ray)
Image credit: IHEP

Neutrinos

The neutrino was postulated by Wolfgang Pauli in the early 1930s but could only be detected for the first time in the 1950s

- Symbol is lowercase Greek letter: ν
  - Three flavors: electron ν_e, muon ν_μ, tau ν_τ
  - Oscillates between flavors
- Produced by nuclear processes
- Basic properties
  - Electrically neutral
  - Nearly massless
  - Travels close to the speed of light
- Interaction
  - Reacts to gravity and the weak nuclear force
  - Mostly passes through matter
  - Billions of neutrinos pass through our body every second

IceCube (neutrinos)
Image credit: ESA

Neutrinos (IceCube)

High energy (~290 TeV) neutrino from IceCube-170922A was detected in association with a known gamma-ray blazar TXS 0506+056 ($z \sim 0.3365$). This is the third detections of individual astrophysical sources of neutrinos, after the Sun and SN1987A.

Image credit: IceCube collaboration (2023)

~5800 photoelectrons in ~3000 ns (from left to right)

Cosmic rays

In 1912, Victor Hess discovered cosmic rays via a balloon flight
- Ionization radiation at 5300 m is 3 times that at the see level
- Penetrating radiation was entering the atmosphere from above.

- High-energy particles
  - ~89% protons (nuclei of hydrogen)
  - ~10% nuclei of helium
  - ~1% nuclei of heavier elements
- Origin
  - Mostly produced within the Milky Way (Galactic Cosmic Rays, GCRs)
  - The rest have extragalactic origins
- When passing through the Earth atmosphere, a shower of particles are produced
  - Mostly pion
    - Pion decay → muons (and more)
  - Muon
    - 207 times heavier than electrons
    - do not interact strongly with matter
    - Muon decay → 1 electron + 2 neutrinos

Image credit: A. Chantelauze, S. Staffi, L. Bret, Pierre Auger Collabration

Cosmic rays (LHAASO)

Image credit: Taylor (2016)

Large High Altitude Air Shower Observatory (LHAASO) (cosmic rays)

Image credit: Science

Clues about the origin of cosmic rays come from both their composition and their energy spectra.

Tera (trillion): $\mathrm{TeV} = 10^{12}$ eV
Peta (quadrillion): $\mathrm{PeV} = 10^{15}$ eV
Exa (quintillion): $\mathrm{EeV} = 10^{18}$ eV
Zetta (sextillion): $\mathrm{ZeV} = 10^{21}$ eV

Gravitational wave

Image credit: V. Altounian/Science

astrobites
THE ASTRO-PH READER'S DIGEST | SUPPORTED BY THE AAS

link: Astrobites

Guide to Gravitational Waves

by Astrobites | Nov 8, 2023 | Daily Paper Summaries, Guides | 0 comments

- Gravitational waves
  - "Ripples" in space-time caused
- Origin
  - Mostly produced by mergers of massive objects

Gravitational waves (LIGO)

Image credit: Caltech and R. Hurt (IPAC)

Image credit: Abbott et al. 2016

Image credit: V. Altounian/Science

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Image credit: Chandra photo album

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.1.1 Radiative flux
2.1.2 Radiative intensity
2.1.3 Specific intensity
2.1.4 Specific flux and momentum
2.1.5 Specific energy density

2.2 Radiative transfer
2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

Image credit: Junjie Mao

Radiative flux

Image credit: APOD

For a detector element (with a photon collecting area dA) exposed to a radiation source for a time interval dt, the number of photons or the amount of energy received is

energy or photon flux

$\mathrm{erg\ cm^{-2}\ s^{-1}}$
$\mathrm{ph\ cm^{-2}\ s^{-1}}$

$$dF \equiv \frac{dE}{dA\,dt}$$

photon collecting area ($\mathrm{cm^2}$)

integration time (s)

F can vary for different orientations of the detector element

Inverse square law

Consider an **isotropic** source emitting photons/energy equally in all directions (e.g., a spherically symmetric and isolated star)

energy flux ($\mathrm{erg\ cm^{-2}\ s^{-1}}$)

luminosity ($\mathrm{erg\ s^{-1}}$)

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

distance to the source (cm)

$$dF(r) = \frac{dE}{dA(r)dt} = \frac{dE}{r^2 d\Omega dt}$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

Conservation of energy (no loss or gain)

$$F(r) = \frac{\text{constant}}{r^2}$$

Radiative intensity

Since flux changes with distance to the source, it is useful to a concept that is **intrinsic** to the properties of the source, e.g., luminosity (L), intensity (I)

luminosity ($\mathrm{erg\ s^{-1}}$)

$$L \equiv \frac{dE}{dt}$$

Solar luminosity

$$L_\odot = 3.8\times10^{33}\ \mathrm{erg\ s^{-1}}$$

$$I \equiv \frac{dE}{dt\ dA_\perp\ d\Omega}$$

By definition, $dA_\perp$ is normal to the direction of the radiation

Intensity and flux

Consider a radiation field (radiation in all directions), for a detector element (with photon collecting area dA) at some arbitrary orientation, we have

$$dF = I \cos\theta \, d\Omega$$

prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

prev. sl.

$$dF = \frac{dE}{dt \, dA}$$

effective area: $dA_\perp = dA \cos\theta$

Specific intensity

Radiative energy in a specific frequency crossing unit area ($dA_\perp$) in unit time in unit solid angle

specific intensity

$$dE_\nu = I_\nu \, dA \, d\nu \, d\Omega \, dt$$

$$I_\nu \equiv \frac{dE_\nu}{dA_\perp \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1} sr^{-1}}$

prev. sl.

$$I = \frac{dE}{dt \, dA_\perp \, d\Omega}$$

$$I_\nu = \frac{\lambda}{\nu} I_\lambda$$

Specific flux and momentum

prev. sl. $F = \pi I$

$$F_\nu = \pi I_\nu$$

specific flux

specific intensity

$\mathrm{erg\ Hz^{-1} cm^{-2} s^{-1}}$

$1\ \mathrm{erg} = 1\ \mathrm{g\ cm^2\ s^{-2}}$

momentum $\longrightarrow$ $$p_\nu = \frac{F_\nu}{c}$$

$\mathrm{dyn\ Hz^{-1} cm^{-2}}$

$1\ \mathrm{dyn} = 1\ \mathrm{g\ cm\ s^{-2}}$

Specific energy density

Radiative energy in a specific frequency in unit solid angle in a cylinder with its length $dl = c\,dt$ and its volume $dA_\perp\, c\,dt$

$$dE_\nu = u_\nu(\Omega)\, d\Omega\, d\nu\, dA_\perp\, c\, dt$$

$$u_\nu(\Omega) = \frac{I_\nu}{c}$$

$$u_\nu = \frac{1}{c}\int I_\nu\, d\Omega = \begin{cases} \dfrac{4\pi}{c} I_\nu & \text{isotropic} \\ \dfrac{4\pi}{c} \bar{I}_\nu & \text{anisotropic} \end{cases}$$

$\mathrm{erg\ Hz^{-1}\ cm^{-3}}$

specific energy density

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation

2.2 Radiative transfer
2.2.1 Spontaneous emission coefficient
2.2.2 Absorption coefficient
2.2.3 Optical depth and mean free path
2.2.4 Radiative transfer equation

2.3 Thermal radiation
2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

Image credit: Junjie Mao

Radiative transfer

- Radiation is one of the key processes to exchange energy for most celestial bodies

- When a ray passes through matter, the energy exchange process is referred to as radiation transfer

prev. sl.

$$F(r) \equiv \frac{L}{4\pi\, r^2}$$

unabsorbed flux

observed flux

$$F_{\mathrm{obs}} \leq \frac{L}{4\,\pi\, d^2}$$

Spontaneous emission coefficient

Radiative energy emitted spontaneously in a specific frequency in unit time in unit solid angle in unit volume

spontaneous emission coefficient

$$dE_\nu = j_\nu \, dV \, d\nu \, d\Omega \, dt$$

$$j_\nu \equiv \frac{dE_\nu}{dV \, d\nu \, d\Omega \, dt}$$

$\mathrm{erg\ Hz^{-1} cm^{-3} s^{-1} sr^{-1}}$

$\mathrm{Energy\ (frequency)^{-1} (volume)^{-1} (time)^{-1} (solid\ angle)^{-1}}$

Spontaneous emission coefficient (cont.)

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV\, d\nu\, d\Omega\, dt}$$

$$dV = dA_\perp\, dl$$

prev. sl.

$$I_\nu = \frac{dE_\nu}{dA_\perp\, d\nu\, d\Omega\, dt}$$

$$I_\nu = j_\nu\, dl$$

Emissivity

For an isotropic emitter or a distribution of randomly oriented emitters, the emission can also be characterized via emissivity ϵ_ν, which is the radiative energy (angle integrated) emitted spontaneously in a specific frequency per unit time per unit mass

$$dE_\nu = \epsilon_\nu \rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}$$

normalized fraction of energy radiated into the solid angle $d\Omega$

$$\epsilon_\nu \equiv \frac{dE_\nu}{\rho \, dV dt \, d\nu \frac{d\Omega}{4\pi}}$$

$\mathrm{erg\ Hz^{-1} g^{-1} s^{-1}}$

mass density $\mathrm{g\ cm^{-3}}$

Spontaneous emission coefficient

For an isotropic emitter or a distribution of randomly oriented emitters,

prev. sl.

$$j_\nu = \frac{dE_\nu}{dV\ d\nu\ d\Omega\ dt}$$

$$\epsilon_\nu = \frac{dE_\nu}{\rho\ dV dt\ d\nu \frac{d\Omega}{4\pi}}$$

$$j_\nu = \frac{\epsilon_\nu\ \rho}{4\pi}$$

Absorption coefficient

Loss of intensity in a beam as radiation travels a distance dl

absorption coefficient (positive values for energy taken out of the beam)

$$dI_\nu \equiv -\alpha_\nu \, I_\nu \, dl$$

$$\alpha_\nu = \rho \, \kappa_\nu$$

mass absorption coefficient (a. k. a. opacity coefficient)

mass density $\mathrm{g\ cm^{-3}}$

$$\int_{I_\nu^{\mathrm{in}}}^{I_\nu^{\mathrm{out}}} \frac{dI_\nu}{I_\nu} = -\int \alpha_\nu \, dl$$

$$\ln \frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = -\int \alpha_\nu \, dl$$

$$\frac{I_\nu^{\mathrm{out}}}{I_\nu^{\mathrm{in}}} = \exp\left(-\int \alpha_\nu \, dl\right)$$

Optical depth

optical depth

absorption coefficient

$$I_\nu^{\mathrm{out}} = I_\nu^{\mathrm{in}} \exp(-\tau_\nu) \qquad \tau_\nu \equiv \int \alpha_\nu dl$$

Optical thin $\tau_\nu \ll 1, I_\nu^{\mathrm{out}} \simeq I_\nu^{\mathrm{in}}$

$\tau_\nu = 1, I_\nu^{\mathrm{out}} \sim 0.368\, I_\nu^{\mathrm{in}}$

Optical thick $\tau_\nu \gg 1, I_\nu^{\mathrm{out}} \sim 0$

Mean free path

prev. sl.

$$I_\nu^{\text{out}} = I_\nu^{\text{in}} \exp(-\tau_\nu)$$

For homogeneous medium, $\bar{\tau}_\nu = \alpha_\nu \bar{l}_\nu$

$$\bar{\tau}_\nu = \frac{\int_0^\infty \tau_\nu \exp(-\tau_\nu)\, d\tau_\nu}{\int_0^\infty \exp(-\tau_\nu)\, d\tau_\nu} = 1$$

$$\int_0^\infty x \exp(-x)\ dx = 1$$

$$\int_0^\infty \exp(-x)\ dx = 1$$

Mean free path (the average distance a photon can travel within a medium without being absorbed)

$$\bar{l}_\nu = \frac{1}{\alpha_\nu}$$

Radiative transfer equation (cont.)

Conservation of energy (no loss or gain)

$$dE^{\mathrm{out}} = dE^{\mathrm{in}} + dE^{\mathrm{abs}} + dE^{\mathrm{em}}$$

$$\frac{dI_\nu}{dl} = -\alpha_\nu\, I_\nu + j_\nu$$

prev. sl.

$$dE^{\mathrm{in}} = I_\nu\, d\nu\, d\Omega\, dA_\perp\, dt$$

$$dE^{\mathrm{out}} = (I_\nu + dI_\nu)\, d\nu\, d\Omega\, dA_\perp\, dt$$

$$dE^{\mathrm{em}} = j_\nu\, d\nu\, d\Omega\, dA_\perp\, dl\, dt$$

$$dE^{\mathrm{abs}} = -\alpha_\nu dE^{in} dl$$

Radiative transfer equation (cont.)

prev. sl.

$$\frac{dI_\nu}{dl} = -\alpha_\nu I_\nu + j_\nu$$

$$d\tau_\nu = \alpha_\nu dl$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

Source function

Solution to the RT equation

prev. sl.

$$\frac{dI_\nu}{d\tau_\nu} = \frac{j_\nu}{\alpha_\nu} - I_\nu = S_\nu - I_\nu$$

$$\tau_{\nu,0} = \int \alpha_\nu dl$$

$$e^{\tau_\nu}\frac{dI_\nu}{d\tau_\nu} = S_\nu e^{\tau_\nu} - I_\nu e^{\tau_\nu}$$

$$e^{\tau_\nu}\left(\frac{dI_\nu}{d\tau_\nu} + I_\nu\right) = \frac{d}{d\tau_\nu}(I_\nu e^{\tau_\nu}) = S_\nu e^{\tau_\nu}$$

$$I_\nu^{\rm out} = I_\nu^{\rm in} e^{-\tau_{\nu,0}} + \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

see Sect. 3.2 of 《天体物理中的辐射机制》by 尤峻汉 (p81–82)

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer

2.3 Thermal radiation
2.3.1 Blackbody radiation spectrum
2.3.2 Blackbody emission intensity
2.3.3 Blackbody emission flux
2.3.4 Thermal equilibrium
2.3.5 Local thermal equilibrium

2.4 Scattering
2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

Image credit: NASA

Blackbody radiation spectrum

Thermal radiation is radiation emitted by matter in thermal equilibrium. In this case, the free electrons follows the Maxwellian distribution

$$f(v)dv = 4\pi \left(\frac{m_e}{2\pi k T_e}\right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT_e}\right)$$

Blackbody radiation spectrum is the Planck spectrum

$$h\nu_{\max} = 2.82\ kT$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)^{-1}$$

Caution that even if particles in the matter follow the Maxwellian distribution, radiation emitted by this matter does **not** necessarily have to be thermal radiation

Blackbody emission intensity

$$I_\nu^{\mathrm{BB}} \equiv B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

Blackbody emission intensity

$$I^{\mathrm{BB}} = \int_0^\infty \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1} d\nu$$

$$= \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x - 1)^{-1}dx$$

$$x = \frac{h\nu}{kT}$$

Blackbody emission flux

prev. sl.

$$I^{\mathrm{BB}} = \frac{2k^4T^4}{h^3c^2}\int_0^\infty x^3(e^x-1)^{-1}dx$$

Blackbody emission flux

$$F^{\mathrm{BB}} = \pi I^{\mathrm{BB}} = \sigma T^4$$

$$\int_0^\infty x^3(e^x-1)^{-1}dx = \frac{\pi^4}{15}$$

$$\sigma = \frac{2\pi^5k^4}{15\,h^3c^2}$$

$$= 5.670\times10^{-5}\ \mathrm{erg\ s^{-1}cm^{-2}K^{-4}}$$

Stefan-Boltzmann constant

Blackbody radiation flux depends **only** on the temperature

Asymptotic behavior

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

$\frac{h\nu}{kT} \ll 1$ $\quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \frac{h\nu}{kT}$ $\quad I_\nu^{\mathrm{RJ}} = \frac{2\nu^2}{c^2} kT$ Rayleigh-Jeans Law

$\frac{h\nu}{kT} \gg 1$ $\quad \exp\left(\frac{h\nu}{kT}\right) - 1 \rightarrow \exp\left(\frac{h\nu}{kT}\right)$ $\quad I_\nu^{\mathrm{W}} = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{kT}\right)$ Wien Law

Thermodynamic Equilibrium (TE)

prev. sl.

$$I_\nu^{\mathrm{BB}} = B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp\left(\frac{h\nu}{kT}\right) - 1\right)^{-1}$$

For medium in thermodynamic equilibrium, Kirchoff's law links the spontaneous emission coefficient and emission source function with Planck's law

$$j_\nu \equiv \alpha_\nu B_\nu(T)$$
$$S_\nu \equiv B_\nu\,(T)$$

TE (cont.)

prev. sl.

$$I_\nu^{\text{out}} = \int_0^{\tau_{\nu,0}} S_\nu(\tau_\nu) e^{-\tau_\nu} d\tau_\nu$$

$$I_\nu^{\text{out}} = B_\nu(T)\left(1 - \exp\left(-\tau_{\nu,0}\right)\right)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \gg 1 \qquad I_\nu^{\text{out}} = B_\nu(T)$$

$$\tau_{\nu,0} = \int \alpha_\nu \, dl \ll 1 \qquad I_\nu^{\text{out}} = \tau_{\nu,0} B_\nu(T)$$

Only if the medium is in TE and it is optically thick, then its radiation follows the Planck's Law.
For non-thermal processes, the source function can only be derived from j_ν and α_ν.

Local Thermodynamic Equilibrium (LTE)

LTE differs from TE in that the radiation field does not need to follow Planck's law, i.e., $I_\nu \neq B_\nu(T)$.

The assumption of LTE is valid if either of the follow two applies:

- ❑ collisional processes among particles dominate photon-related processes
- ❑ collisional processes among particles are in equilibrium with photo-related processes

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer
2.3 Thermal radiation

2.4 Scattering
2.4.1 Pure scattering
2.4.2 Emission, absorption, and scattering
2.4.3 Absorption and scattering probabilities
2.4.4 Effective mean free path

2.5 Escape probability
2.6 Polarization
2.7 Dispersion and rotation measures

Image credit: Junjie Mao

Scattering

Scattering significantly complicates the radiative transfer.
Scattering can change the direction and frequency of the incident photon.

Two-level system

Pure scattering

Consider a medium has only isotropic scattering (no emission or absorption), which only changes the direction of the incident photon

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}$$

scattered outside the line of sight

scattered back into the line of sight

scattering coefficient

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{sca}}} = -I_\nu + S_\nu^{\mathrm{sca}}$$

$$d\tau_\nu^{\mathrm{sca}} \equiv \sigma_\nu^{\mathrm{sca}} dl$$

scattering optical depth

Source function for scattering

Random walk

The mean displacement traveled by photons is (order-of-magnitude estimation)

see Sect. 1.7 of the REF book (p35) by Rybicki & Lightman

$$\bar{l}_\nu^{\mathrm{RM}} \sim \sqrt{N}\,\bar{l}_\nu$$

$\bar{l}_\nu = \frac{1}{\sigma_\nu^{\mathrm{sca}}}$: scattering mean free path

Number of scattering

size scale of the medium

for $\tau_\nu^{\mathrm{sca}} \gg 1$ $\quad N \sim \frac{L^2}{\bar{l}_\nu^2} = (\sigma_\nu^{\mathrm{sca}} L)^2 \sim (\tau_\nu^{\mathrm{sca}})^2$

for $\tau_\nu^{\mathrm{sca}} \ll 1$ $\quad N \sim \sigma_\nu^{\mathrm{sca}} L \sim \tau_\nu^{\mathrm{sca}}$

Emission + absorption + scattering

Considering scattering + absorption + emission for a medium in TE

$$\frac{dI_\nu}{dl} = -\sigma_\nu^{\mathrm{sca}} I_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + j_\nu - \alpha_\nu I_\nu$$

$$j_\nu = \alpha_\nu B_\nu$$

$$\frac{dI_\nu}{dl} = -(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) I_\nu + (\sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}} + \alpha_\nu B_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu^{\mathrm{AS}}} = -I_\nu + S_\nu^{\mathrm{EAS}}$$

$$S_\nu^{\mathrm{EAS}} \equiv \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Source function for emission, absorption, and scattering

$$(\sigma_\nu^{\mathrm{sca}} + \alpha_\nu) dl = d\tau_\nu^{\mathrm{AS}}$$

Absorption and scattering probability

An alternative way to define the source function

$$S_\nu^{\mathrm{EAS}} = \frac{\alpha_\nu B_\nu + \sigma_\nu^{\mathrm{sca}} S_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

Introducing absorption and scattering probabilities

absorption probability

$$\epsilon_\nu \equiv \frac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

scattering probability

$$1 - \epsilon_\nu = \frac{\sigma_\nu^{\mathrm{sca}}}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$$

$$S_\nu^{\mathrm{EAS}} = \epsilon_\nu B_\nu + (1 - \epsilon_\nu) S_\nu^{\mathrm{sca}}$$

Effective mean free path

prev. sl. $\epsilon_\nu = \dfrac{\alpha_\nu}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$

After $N = \epsilon_\nu^{-1}$ steps, a photon is **truly** absorbed

$\bar{l}_\nu^{\mathrm{AS}} = \dfrac{1}{\alpha_\nu + \sigma_\nu^{\mathrm{sca}}}$: mean free path

$$\left(\bar{l}_\nu^{\mathrm{RM}}\right)^2 = N\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2 = \frac{\left(\bar{l}_\nu^{\mathrm{AS}}\right)^2}{\epsilon_\nu} = \frac{1}{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}$$

$$l_\nu^{\mathrm{eff}} = \frac{1}{\sqrt{\alpha_\nu(\alpha_\nu + \sigma_\nu^{\mathrm{sca}})}}$$

effective mean free path
diffusion length
thermalization length

for $\alpha_\nu \ll \sigma_\nu^{\mathrm{sca}}$ $\quad l_\nu^{\mathrm{eff}} = \dfrac{1}{\sqrt{\alpha_\nu \sigma_\nu^{\mathrm{sca}}}}$

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2.6 Polarization
2.6.1 Stokes parameters
2.6.2 Polarization fraction

2.7 Dispersion and rotation measures

Image credit: Junjie Mao

IAU standard of polarization

IAU reference frame for polarization
(e.g., Ferriere et al. 2021)

IAU definition:

- ✓ X and Y are the Cartesian coordinates of the plane of the sky
- ✓ Z is along the line of sight (+Z towards the observer)
- ✓ ψ: linear polarization angle (measured counterclockwise from the North)

Stokes parameters

Stokes parameters, all related to intensity, are **observable**

- I: Intensity
- $\sqrt{Q^2+U^2}$: Intensity of linear polarization
- V: Intensity of circular polarization

$$S_0 \equiv I$$
$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$
$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2+U^2}}, -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$$
$$\tan 2\psi = U/Q\,, 0 \le \psi \le \pi$$

linear polarization angle

$$p = \sqrt{Q^2+U^2+V^2}/I$$

polarization fraction

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized

Poincaré sphere

prev. sl.

$$S_0 \equiv I$$
$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$
$$S_3 \equiv V = Ip\sin 2\chi$$

$$\tan 2\chi = \frac{V}{\sqrt{Q^2+U^2}}, -\frac{\pi}{4} \le \chi \le \frac{\pi}{4}$$
$$\tan 2\psi = U/Q\,, 0 \le \psi \le \pi$$

visualization of the Stokes parameters

Image credit: Jiejia Liu

Linear polarization

For $V = 0$ $(\chi = 0)$, we have the linear polarization

$$S_0 = I$$
$$S_1 = Q = Ip\cos 2\psi$$
$$S_2 = U = Ip\sin 2\psi$$
$$S_3 = V = 0$$

$$\cos\pi = -1$$
$$\sin\left(\frac{3\pi}{2}\right) = -1$$

prev. sl.

$$S_3 \equiv V = Ip\sin 2\chi$$
$$\tan 2\psi = U/Q$$

linear polarization angle

e.g., Hamaker & Bregman (1996)

Circular polarization

prev. sl.

$$S_1 \equiv Q = Ip\cos 2\chi \cos 2\psi$$
$$S_2 \equiv U = Ip\cos 2\chi \sin 2\psi$$

For $Q = U = 0$ ($\chi = \pm\pi/4$), we have the circular polarization

$$S_0 = I$$
$$S_1 = Q = 0$$
$$S_2 = U = 0$$
$$S_3 = V = Ip\sin 2\chi$$

IAU definition:

- ✓ $+V$ for the right-handed circular polarization
- ✓ For the right-handed circular polarization, the position angle of $\vec{E}$ at any point **increases** with time
- ✓ Along the $+z$ direction, $\vec{E}$ traces a left-handed circle at any time

e.g., Hamaker & Bregman (1996)

Linear/circular polarization fraction

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - pI \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} pI \\ Q \\ U \\ V \end{pmatrix}$$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I - p_L I - p_C I \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} p_L I \\ Q \\ U \\ 0 \end{pmatrix} + \begin{pmatrix} p_C I \\ 0 \\ 0 \\ V \end{pmatrix}$$

polarization fraction

$$p = \sqrt{Q^2 + U^2 + V^2}/I$$

linear or circular polarization fraction

$$p_L = \sqrt{Q^2 + U^2}/I$$

$$p_C = V/I$$

prev. sl.

$p = 1$	completely polarized
$0 < p < 1$	partially polarized
$p = 0$	unpolarized

Chpt.2 Fundamentals of radiation

2.1 Elementary concepts of radiation
2.2 Radiative transfer
2.3 Thermal radiation
2.4 Escape probability
2.5 Scattering
2.6 Polarization

2.7 Dispersion and rotation measures
2.7.1 Dispersion relation
2.7.2 Refractive index
2.7.3 Time of arrival difference
2.7.4 Dispersion measure
2.7.5 Fast Radio Bursts
2.7.6 Phase difference
2.7.7 Rotation measure
2.7.8 Faraday rotation
2.7.9 CMB polarization

Ferriere et al. 2021

Dispersion relation

Consider a pair of left- and right-handed circularly polarized waves propagate along the B-field with $\vec{k} \parallel \vec{B}$, the dispersion relation is

Eq. 1 of Ferriere et al. 2021

$$\omega^2 = c^2 k^2 + \frac{\omega_p^2}{1 \mp \frac{\omega_L}{\omega}}$$

angular frequency $\omega = 2\pi\nu$

wave number $k = \frac{2\pi}{\lambda}$

plasma frequency

$$\omega_p \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

- Right circular polarized (RCP) mode: −
- Light circular polarized (LCP) mode: +

Larmor (or gyro) frequency

$$\omega_L \equiv \frac{eB}{m_e c}$$

《原子物理学》杨福家
3rd edition, p153 - 155

gyro motion

Dispersion relation (cont.)

For astrophysical waves of interest, the following always apply (Assignment #3: Plasma frequency)

$$\omega_{\mathrm{p}} \ll \omega, \qquad \omega_L \ll \omega$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

prev. sl.

$$\omega^2 = c^2k^2 + \frac{\omega_{\mathrm{p}}^2}{1 \mp \frac{\omega_{\mathrm{L}}}{\omega}}$$

$$\frac{1}{1+x} = 1 - x + \ \ldots$$

$$= c^2k^2 + \omega_{\mathrm{p}}^2 \pm \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega}$$

EM wave in free space electric force magnetic force

$$c^2k^2 = \omega^2 - \omega_{\mathrm{p}}^2 \mp \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega} \simeq \omega^2$$

$$k(\omega) = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} \mp \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega^3}}$$

Refractive index

Consider the propagation of EM waves through an unmagnetized plasma (mainly free electrons), the medium refractive index is

$$n = \sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}}$$

see Sect. 8.1 of the REF book (p227) by Rybicki & Lightman

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Only waves with $\omega > \omega_{\mathrm{p}}$ can propagate, which is not an issue in most cases

$$v_\phi = \frac{\omega}{k} = \frac{c}{n}$$

phase velocity

Time of arrival difference

Consider a pulse propagates in a non-magnetized medium ($B = 0, \omega_{\mathrm{L}} = 0$), the group velocity is

prev. sl.

$$c^2k^2 = \omega^2 - \omega_{\mathrm{p}}^2 \mp \frac{\omega_{\mathrm{p}}^2\omega_{\mathrm{L}}}{\omega} \simeq \omega^2$$

$$v_{\mathrm{grp}}(\omega) = \frac{d\omega}{dk} = \frac{c^2k}{\omega} = c\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}}$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

This leads to a frequency-dependent time of arrival

$$t_{\mathrm{arv}}(\omega) = \int_0^l \frac{1}{v_{\mathrm{grp}}(\omega)} dl = \int_0^l \frac{1}{c}\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\right) dl$$

$$\frac{1}{\sqrt{1-x}} = 1 + \frac{x}{2} + \ \ldots$$

Dispersion measure

For $\omega \gg \omega_p$ (Assignment #3: Plasma frequency), the time of arrival for a specific frequency is

traveling distance in the medium

prev. sl.

$$t_{\mathrm{arv}}(\omega) = \int_0^l \frac{1}{c}\left(1 + \frac{\omega_{\mathrm{p}}^2}{2\omega^2}\right) dl$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}}$$

plasma frequency

Waves with different frequencies arrive at different times

$$\Delta t_{\mathrm{arv}} = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\mathrm{low}}^2} - \frac{1}{\omega_{\mathrm{high}}^2}\right)\int_0^l n_e dl$$

High-frequency waves arrive earlier than the low-frequency ones

Fast Radio Bursts (FRBs)

FRBs are radio bursts with extremely short duration (a few milliseconds) yet rather energetic ($\gtrsim 10\ L_\odot$). The first FRB was discovered in 2001 (Lorimer et al. 2007)

prev. sl.

$$\Delta t_{\mathrm{arv}} = \frac{2\pi e^2}{m_e c}\left(\frac{1}{\omega_{\mathrm{low}}^2} - \frac{1}{\omega_{\mathrm{high}}^2}\right)\int_0^l n_e dl$$

Dispersion Measure (DM)

$$= \frac{4.149}{\mathrm{ms}}\left(\left(\frac{1\ \mathrm{GHz}}{\nu_{\mathrm{low}}}\right)^2 - \left(\frac{1\ \mathrm{GHz}}{\nu_{\mathrm{high}}}\right)^2\right)\left(\frac{\mathrm{DM}}{\mathrm{pc\ cm^{-3}}}\right)$$

Waterfall plot
measured DM= $375 \pm 1\mathrm{pc\ cm^{-3}}$

Lorimer et al. 2007

Fast Radio Bursts (FRBs)

FRBs have been put in the spot light in the past decade:

- Repeating FRBs
- FRBs with known hosts
- FRBs with relatively long durations (slow)

But, we still don't know much about FRBs …

repeating FRBs (a small yet growing population)

Petroff et al. 2022

Publications on FRBs from ADS

Phase difference

In a homogeneous plasma, the phase can be defined as one of the following

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k\, dl$$

Decomposing a linearly polarized wave into a pair of left- and right-hand circular polarized waves. At the source ($l = 0$), the two waves have the same phase, as the waves propagates (with $\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$) in a magnetized medium, phase difference will occur

prev. sl.

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} \mp \frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{L}}}{\omega^3}}$$

$$\phi_{\mathrm{RCP}} > \phi_{\mathrm{LCP}}$$

$\vec{\boldsymbol{k}} \parallel \vec{\boldsymbol{B}}$

$\odot$

- Right circular polarized (RCP) mode: $-$
- Light circular polarized (LCP) mode: $+$

Phase difference introduced by B-field

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k\, dl$$

prev. sl.

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} \mp \frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{L}}}{\omega^3}}$$

- Right circular polarized (RCP) mode: −
- Light circular polarized (LCP) mode: +

When $B = 0$, $\omega_L = \frac{eB}{m_0 c} = 0$, there is no phase difference

$$k_{\mathrm{RCP}} = k_{\mathrm{LCP}}$$
$$\Delta\phi \equiv \phi_{\mathrm{RCP}} - \phi_{\mathrm{LCP}} = 0$$

Phase difference in parallel B-field

$\vec{k} \parallel \vec{B}$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: − $\otimes$

Eq. 4 of Ferriere et al. 2021

$$\phi \equiv \omega t - \int_0^l k\, dl$$

prev. sl.

$$k = \frac{\omega}{c}\sqrt{1 - \frac{\omega_\mathrm{p}^2}{\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{\omega^3}}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \ \ldots$$

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_\mathrm{p}^2}{2\omega^2} \mp \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{2\omega^3}\right)$$

$$\Delta\psi = \frac{1}{2}\Delta\phi = -\frac{1}{2}\int_0^l (k_\mathrm{RCP} - k_\mathrm{LCP})dl$$

$$= \frac{1}{2}\int_0^l \frac{\omega_\mathrm{p}^2 \omega_\mathrm{L}}{c\omega^2}\, dl$$

$$\Delta\psi = \frac{1}{2}\Delta\phi$$

- Right circular polarized (RCP) mode: −
- Light circular polarized (LCP) mode: +

Rotation measure

prev. sl.

$$\Delta\psi = \frac{1}{2}\int_0^l \frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{L}}}{c\omega^2}\, dl$$

prev. sl.

$$\omega_{\mathrm{p}} \equiv \sqrt{\frac{4\pi n_e e^2}{m_e}} \qquad \omega_L \equiv \frac{eB}{m_e c}$$

plasma frequency Larmor frequency

Eq. 9 of Ferriere et al. 2021

$$\Delta\psi = \frac{e^3}{2\pi\, m_e^2\, c^4}\lambda^2 \int_0^l n_e(\pm B_\parallel) dl$$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: – $\otimes$

Rotation Measure (RM) $\longrightarrow$ $\mathrm{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4}\int_0^l n_e(\pm B_\parallel) dl$

$$\mathrm{RM} = 0.812\ \mathrm{rad\ m^{-2}} \int_0^l \left(\frac{n_e}{\mathrm{cm^{-3}}}\right)\left(\pm\frac{B_\parallel}{\mu\mathrm{G}}\right) d\left(\frac{l}{\mathrm{pc}}\right)$$

Faraday rotation

When a linearly polarized wave travels through a magnetized medium, its polarization angle will rotate (Burn 1966)

Eq. 9 of Ferriere et al. 2021

$$\Delta\psi = \frac{e^3}{2\pi\, m_e^2\, c^4} \lambda^2 \int_0^l n_e(\pm B_\parallel) dl$$

prev. sl.

$$\mathrm{RM} \equiv \frac{e^3}{2\pi m_e^2 c^4} \int_0^l n_e(\pm B_\parallel) dl$$

$$\psi_{\mathrm{obs}} = \psi_{\mathrm{src}} + \mathrm{RM} \cos\theta\, \lambda^2$$

- $\vec{B}$ points towards the observer: + $\odot$
- $\vec{B}$ points away from the observer: − $\otimes$

Cosmic Microwave Background

CMB is the leftover radiation from the Big Bang. It was firstly predicted by Gamow (1948).

In1965, CMB was observed by Arno Penzias and Robert Wilson using the 6-meter Holmdel Horn Antenna.

Wrong data excluded

Image credit: wikipedia

Image credit: ESA

The Nobel Prize in Physics 1978

Image credit: nobelprize.org

Image credit: NASA

Cosmic Microwave Background polarization

Komatsu (2022, review): β is the cosmic-birefringence-induced rotation angle

- ❑ B-mode (curl-like)
  - ➢ $\perp \vec{\boldsymbol{k}}$ or $\parallel \vec{\boldsymbol{k}}$
  - ➢ dominated by density perturbation
- ❑ E-mode (gradient-like)
  - ➢ $45°$ w.r.t. $\vec{\boldsymbol{k}}$
  - ➢ fingerprints of GW background (too faint to detect directly nowadays) from cosmic inflation

CMB is linearly polarized (E-mode and B-mode), firstly detected in 2002 (Kovac et al. 2002).

Birefringence: the refractive index depends on the polarization direction. Typically, double refraction (ordinary and extraordinary indices)

CMB B-mode

The detection of B-mode polarization will provide strong constraints for cosmic inflationary models.

The B-mode power is $\propto r^2$, where, $r < 0.07$ is the tensor-to-scalar ratio (BICEP2 collaboration 2016).

Image credit: bicepkeck.org