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6. Warm and Hot Ionized Medium

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Ionization processes

- **Photoelectric absorption:** $X + h\nu \rightarrow X^+ + e^-$.
- **Photoelectric absorption followed by the Auger effect:**
 $X + h\nu \rightarrow (X^+)^* + e^- \rightarrow X^{+n} + ne^-$ ($n \geq 2$).
- **Collisional ionization:** $X + e^- \rightarrow X^+ + 2e^-$.
- **Cosmic ray ionization:** $X + CR \rightarrow X^+ + e^- + CR$.
- **Charge exchange:** $X + Y^+ \rightarrow X^+ + Y$.

Spherical cow model of HII regions

- Assumptions: uniform density; fully ionized; static-state solution; hydrogen only
- Stromgren (1939) sphere!

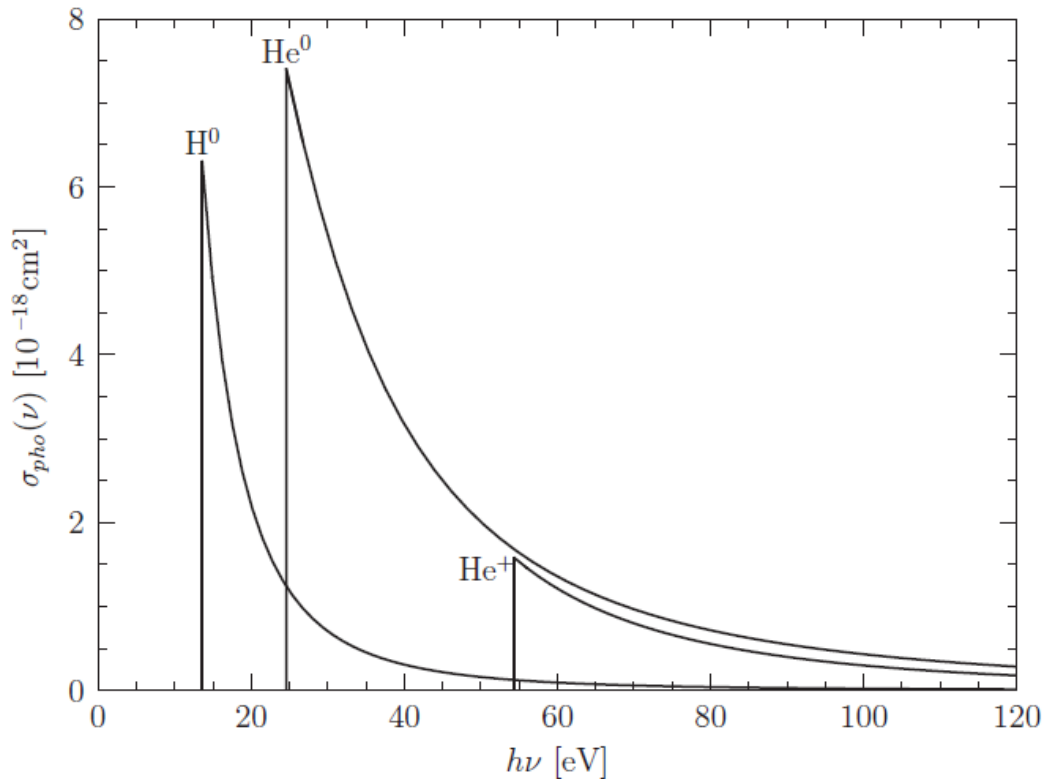
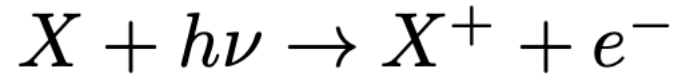
Three processes govern the physics of HII regions:

- **Photoionization Equilibrium**, the balance between photoionization and recombination.
- **Thermal Balance** between heating and cooling.
- **Hydrodynamical response** of the overpressured HII regions surrounded by cold atomic or molecular gas.



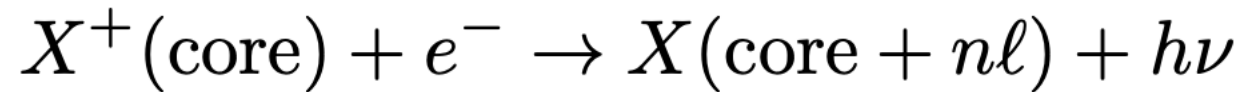
The Rosette Nebula

- Photo-ionization



$$\sigma_{\text{pe}}(\nu) \approx \sigma_0 \left(\frac{h\nu}{Z^2 I_H} \right)^{-3} \quad \sigma_0 = \frac{6.30 \times 10^{-18} \text{ cm}^2}{Z^2} = \frac{0.225 a_0^2}{Z^2}$$

- Radiative Recombination (RR)



Recombination coefficient for RR to an arbitrary nl energy level

$$\alpha_{n\ell}(T) = \left(\frac{8kT}{\pi m_e} \right)^{1/2} \int_0^\infty \sigma_{\text{rr},n\ell}(E) \frac{E}{kT} e^{-E/kT} \frac{dE}{kT}$$

$$\alpha_A(T) \equiv \sum_{n=1}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) \quad ,$$

On The Spot Approximation:

$$\alpha_B(T) \equiv \sum_{n=2}^{\infty} \sum_{\ell=0}^{n-1} \alpha_{n\ell}(T) = \alpha_A(T) - \alpha_{1s}(T)$$

I. Photoionization Equilibrium

rate of emission of
hydrogen-ionizing
photons, i.e.,
 $h\nu > 13.6\text{eV}$

Case B recombination coefficient:
 $\sim 2.6 \times 10^{-13} \text{ cm}^3/\text{s}$ at $T=10^4 \text{ K}$

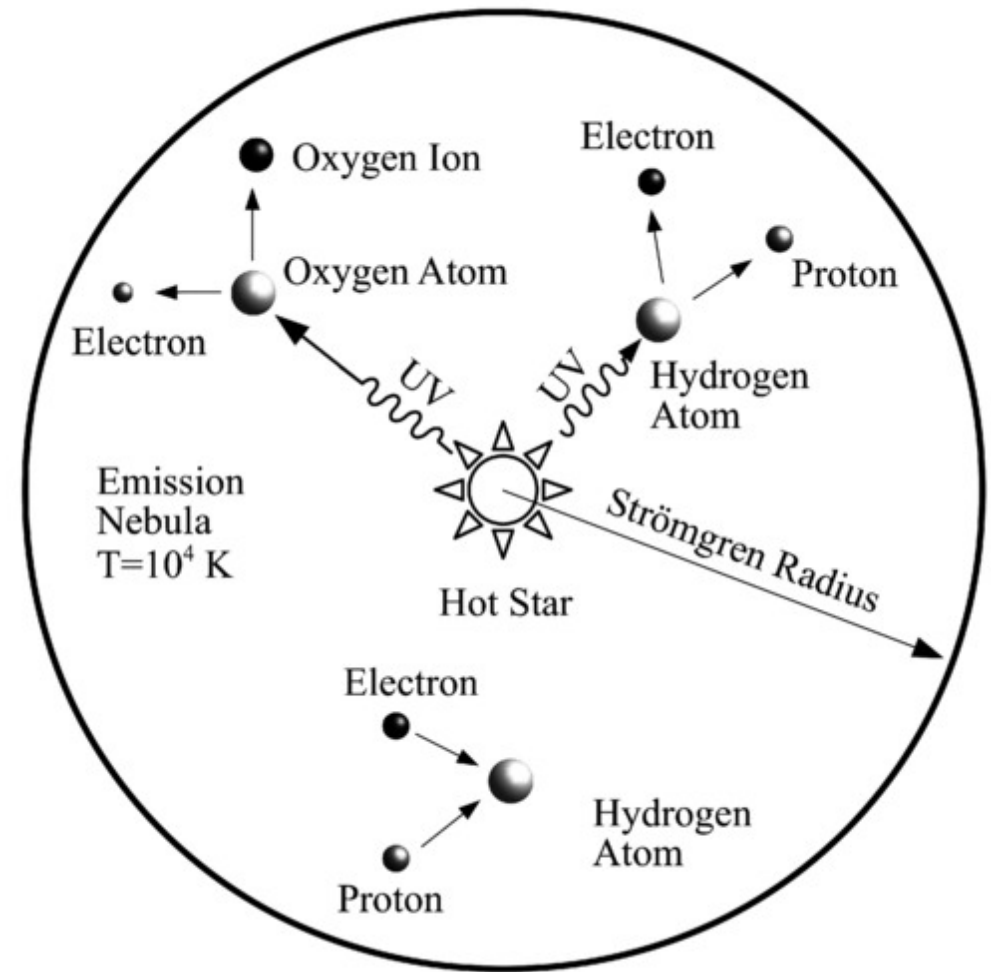
$$Q_0 = \frac{4\pi}{3} R_{S0}^3 \alpha_B n(\text{H}^+) n_e$$

Recombination is a two-body process!

Fully ionized, uniform density: $n(\text{H}^+) = n_e = n_{\text{H}}$

$$R_{S0} \equiv \left(\frac{3 Q_0}{4\pi n_{\text{H}}^2 \alpha_B} \right)^{1/3} = 9.77 \times 10^{18} Q_{0,49}^{1/3} n_2^{-2/3} T_4^{0.28} \text{ cm}$$

assuming the temperature of HII region is around 10^4 K .



Assumption 1: static state?

$$R_{S0} \equiv \left(\frac{3 Q_0}{4\pi n_H^2 \alpha_B} \right)^{1/3}$$

- Ionization timescale for the HII region

$$\tau_{\text{ioniz.}} \equiv \frac{(4/3)\pi R_{S0}^3 n_H}{Q_0} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3 \text{ yr}}{n_2}$$

- Recombination timescale

$$\tau_{\text{rec}} = \frac{1}{\alpha_B n_H} = \frac{1.22 \times 10^3 \text{ yr}}{n_2}$$

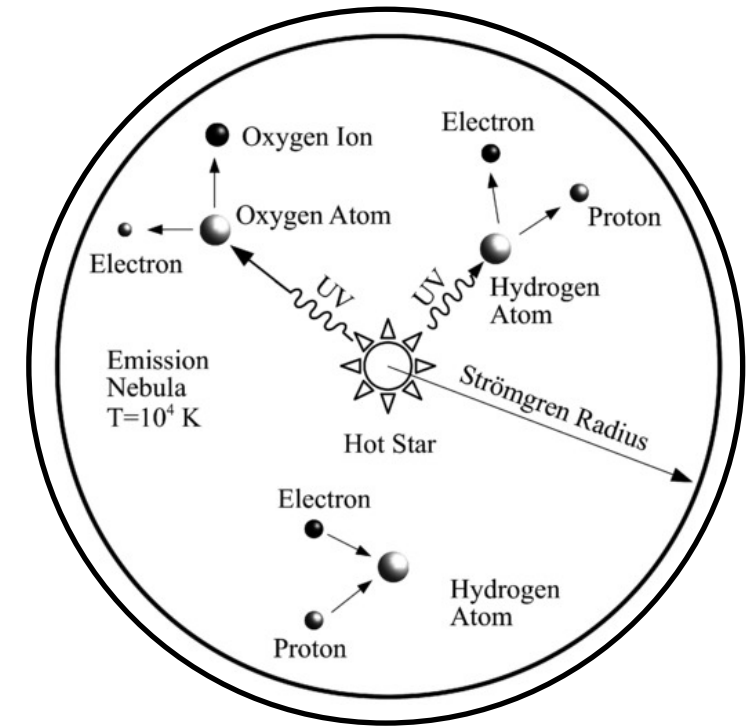
- Recombination timescale is identical to the ionization timescale!
- For density $n > 0.03/\text{cc}$, the timescale is shorter than the stellar lifetime (5Myr) for a massive star.

Assumption 2: fully ionized?

- Whether the transition from ionized gas to neutral gas at the boundary of the HII region is sharp or not?
- Mean-free path for ionizing photons

$$\text{mfp} = \frac{1}{n(\text{H}^0)\sigma_{\text{p.i.}}} = 3.39 \times 10^{17} \left(\frac{\text{cm}^{-3}}{n(\text{H}^0)} \right) \text{cm}$$

photo-ionization
cross section



Better treatment on the ionization structure of HII

Photon number conservation

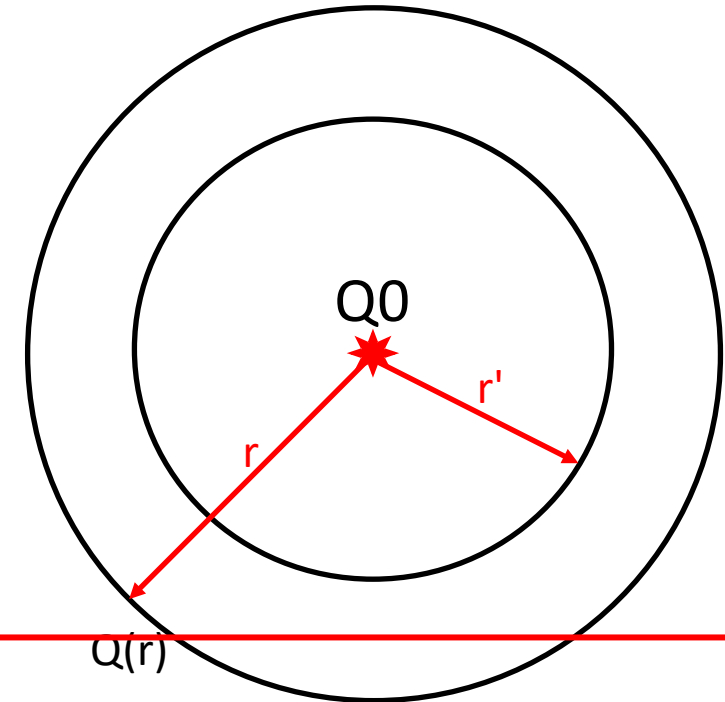
$$Q(r) = Q_0 - \int_0^r n_H^2 \alpha_B x^2 4\pi (r')^2 dr' \\ = Q_0 \left[1 - 3 \int_0^{r/R_{S0}} x^2 y^2 dy \right]$$

Ionization fraction as a function of radius

$$x \equiv n(\text{H}^+)/n_H = n_e/n_H, \quad y \equiv r/R_{S0}$$

Detailed balance between ionization and recombination at each point within the HII region:

$$n_H^2 \alpha_B x^2 = \frac{Q(r)}{4\pi r^2} n_H (1 - x) \sigma_{\text{p.i.}}$$



Homework:

Calculate the ionization structure of HII region numerically
Evaluate the radius where the ionization fraction is 50%.

II. Thermal Balance

Energy equation represents conservation of energy:

$$\frac{D\epsilon}{Dt} = -\frac{P}{\rho} \vec{\nabla} \cdot \vec{u} - \frac{1}{\rho} \vec{\nabla} \cdot \vec{F} + \frac{1}{\rho} \Psi + \boxed{\frac{\Gamma - \Lambda}{\rho}}$$

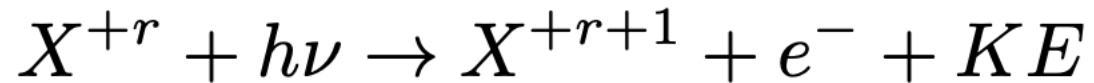
$\Gamma \equiv$ volumetric radiative heating rate $[\text{erg cm}^{-3} \text{s}^{-1}]$

$\Lambda \equiv$ volumetric radiative cooling rate $[\text{erg cm}^{-3} \text{s}^{-1}]$

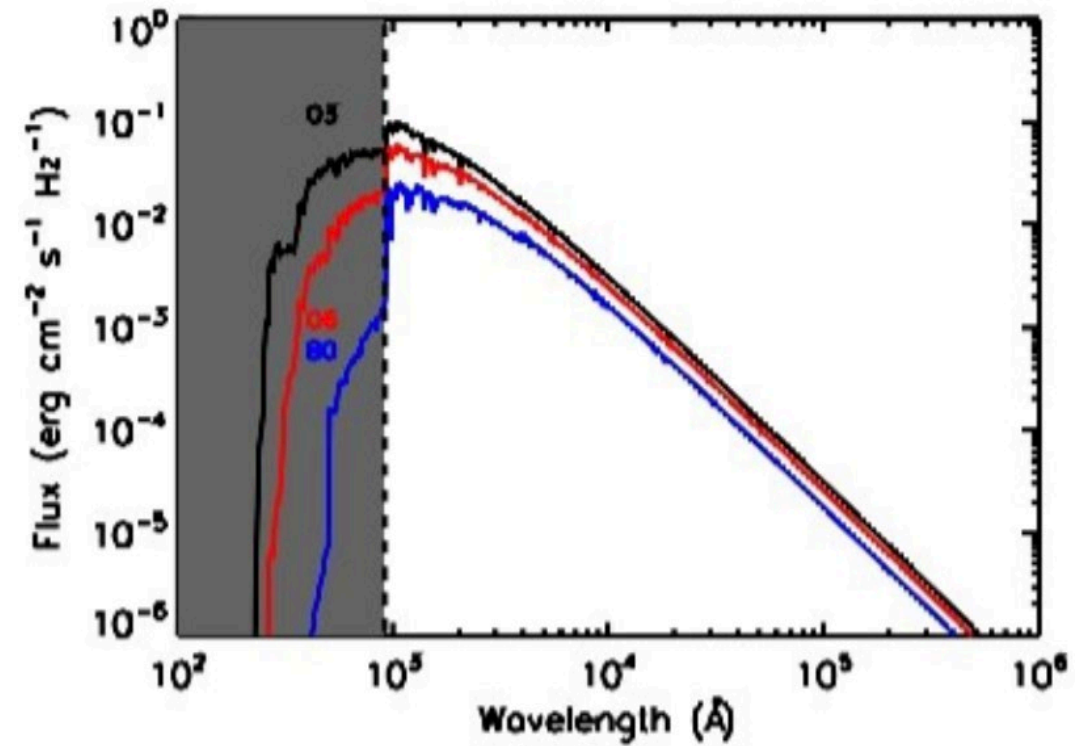
If hydrostatic equilibrium is achieved and external energies such as gravity are ignored, heating and cooling of hot plasma dominate the thermal-dynamical state of the HII region.

Heating

- **Heating by photoionization**



- Photoelectric heating from dust
- Cosmic rays
- X-rays
- Damping of MHD waves



Photon with wavelength shorter than 912 Å can generate photoelectrons that heat up HII region

$$\Gamma_{\text{pe}} = n(X^{+r}) \int_{\nu_0}^{\infty} \sigma_{\text{pe}}(\nu) c \left[\frac{u_{\nu}}{h\nu} \right] (h\nu - h\nu_0) d\nu$$

$$\langle E \rangle = \langle h\nu \rangle - I_{\text{H}}.$$

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Photoionization heating by nearby ionizing sources

- For typical O/B stars, the required 13.6 eV energy lies on the high-energy spectrum, the Wien tail.

$$\varepsilon_\nu \propto \nu^3 \exp\left(-\frac{h\nu}{kT_c}\right)$$

- The average energy for the effects of photoionization heating must be weighted by pi cross section.

$$\langle h\nu \rangle = \frac{\int_{\nu_0}^{\infty} (\varepsilon_\nu/h\nu)(h\nu)\sigma_{\text{pho}}d\nu}{\int_{\nu_0}^{\infty} (\varepsilon_\nu/h\nu)\sigma_{\text{pho}}d\nu} = \frac{h \int_{\nu_0}^{\infty} (\nu^2 e^{-h\nu/kT_c}) \nu \cdot \nu^{-3} d\nu}{\int_{\nu_0}^{\infty} (\nu^2 e^{-h\nu/kT_c}) \nu^{-3} d\nu} = kT_c \frac{\int_{x_0}^{\infty} e^{-x} dx}{\int_{x_0}^{\infty} e^{-x} x^{-1} dx} = \frac{kT_c e^{-x_0}}{\int_{x_0}^{\infty} e^{-x} x^{-1} dx}.$$

$x \equiv h\nu/kT_c$

$$\langle E \rangle = \langle h\nu \rangle - I_{\text{H}}.$$

Photoionization heating by nearby ionizing sources

- The average energy can be derived by the approximation of the first exponential integral:

$$E_1(x_0) \approx \frac{e^{-x_0}}{x_0} \left[1 - \frac{1}{x_0} + O(x_0^{-2}) \right]$$

- The average energy is, to the zeroth order, just the ionization threshold plus the photon temperature:

$$\langle h\nu \rangle \approx h\nu_0 + kT_c \approx I_{\text{H}} + kT_c.$$

- Surprisingly simple, the electrons liberated by photoionization will go off with the mean kinetic energy $\langle E \rangle = \langle h\nu \rangle - I_{\text{H}} \approx kT_c$.

$$\langle E \rangle = \langle h\nu \rangle - I_{\text{H}} \approx kT_{\text{c}}.$$

Photoionization heating by nearby ionizing sources

- In ionization equilibrium, the ionization is balanced by recombination:

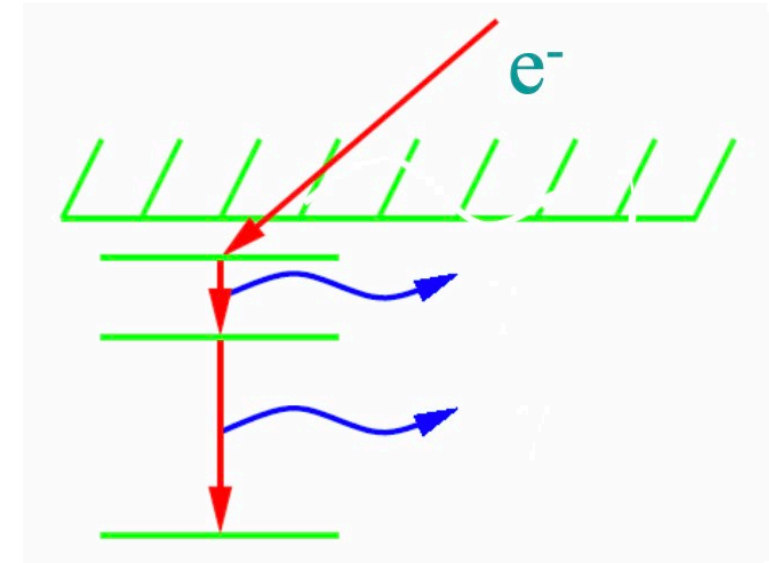
$$n_{\text{HI}}\zeta_{\text{pho}} = n_{\text{e}}n_{\text{HII}}\alpha_{\text{B,H}}.$$

- The volumetric heating rate is the number rate times the average electron energy:

$$g_{\text{pho}} = n_{\text{HI}}\zeta_{\text{pho}}\langle E \rangle \approx n_{\text{e}}n_{\text{HII}}\alpha_{\text{B,H}}kT_{\text{eff}}.$$

Cooling

- Recombination radiation
- Free-free emission
- **Collisionally excited line radiation from heavy elements (metals)**



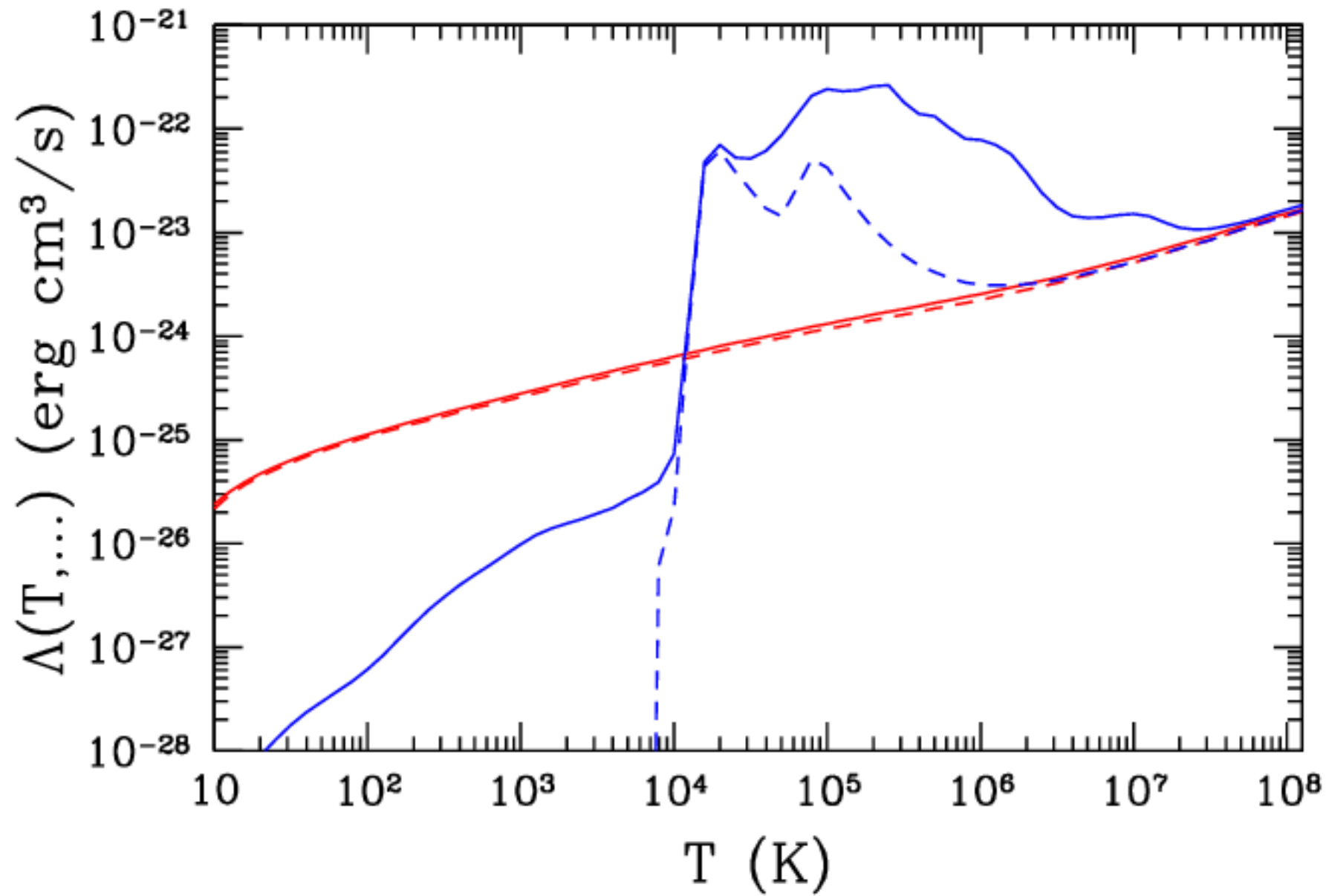


Table 1
Physical Conditions and Oxygen Abundances for the Sample Objects

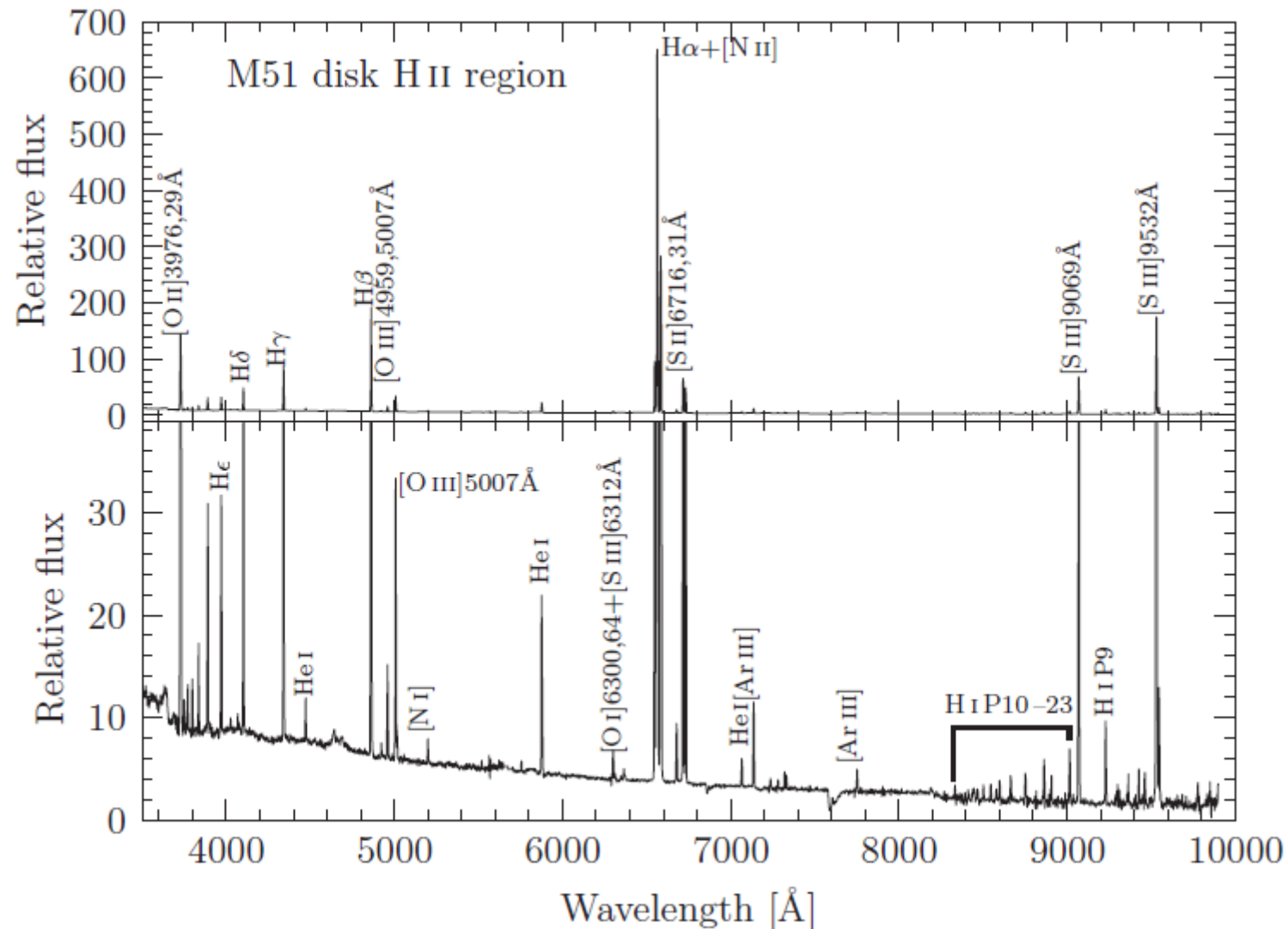
Object	n_e (cm^{-3})	T_e ([N II]) (K)	T_e ([O III]) (K)	$12 + \log (\text{O}/\text{H})$	$\log (\text{O}^+/\text{O}^{++})$	ADF (O^{++})
M8	1550 ± 150	8500 ± 120	8020 ± 100	8.44 ± 0.03	$+0.40 \pm 0.05$	2.2 ± 0.2
M16	980 ± 120	8500 ± 150	7580 ± 150	8.52 ± 0.03	$+0.45 \pm 0.06$	2.4 ± 0.6
M17	420 ± 80	9150 ± 250	7950 ± 100	8.53 ± 0.02	-0.76 ± 0.06	1.8 ± 0.2
M20	220 ± 40	8500 ± 150	7750 ± 250	8.48 ± 0.03	$+0.64 \pm 0.07$	1.9 ± 1.2
M42	6800 ± 600	10100 ± 250	8250 ± 60	8.53 ± 0.02	-0.68 ± 0.05	1.50 ± 0.05
NGC 2467	260 ± 50	9600 ± 200	8900 ± 100	8.35 ± 0.03	$+0.35 \pm 0.05$	1.8 ± 0.2
NGC 3576	1400 ± 200	8950 ± 200	8400 ± 80	8.53 ± 0.02	-0.40 ± 0.06	2.0 ± 0.3
NGC 3603	2350 ± 400	11650 ± 570	9000 ± 150	8.47 ± 0.03	-1.21 ± 0.09	1.9 ± 0.3
30 Doradus	380 ± 50	10800 ± 300	9850 ± 100	8.36 ± 0.02	-0.77 ± 0.06	1.5 ± 0.1

e.g. Rodriguez & Garcia-Rojas 2010

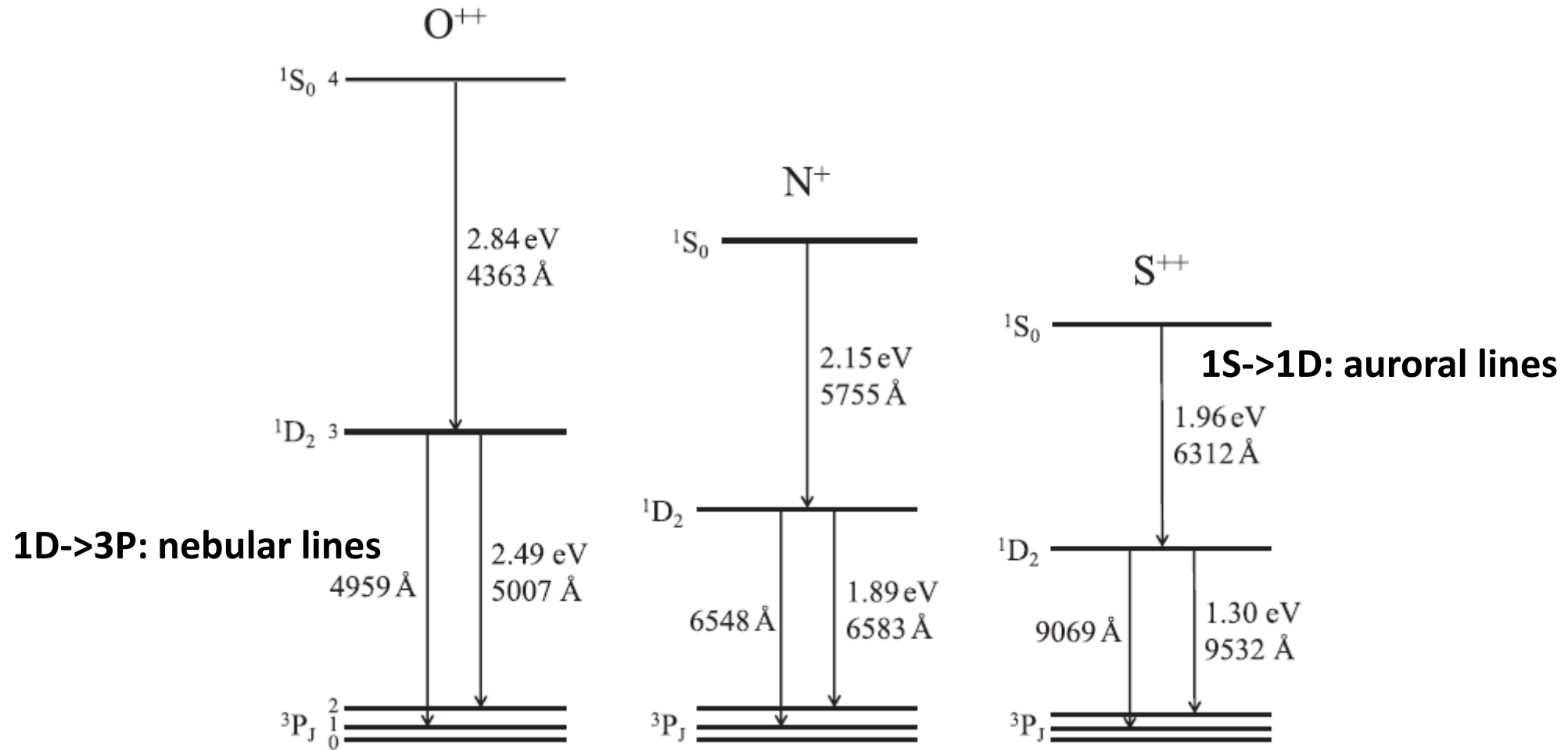
Summary of the physical processes in HII regions

- **Photoionization Equilibrium**, the balance between photoionization and recombination. This determines the structure of the nebula and the rough spatial distribution of ionic states of the elements in the ionized zone.
- **Thermal Balance** between heating and cooling. Heating is dominated by photoelectrons ejected from Hydrogen and Helium with thermal energies of a few eV. Cooling is dominated in most HII regions by electron-ion impact excitation of metal ions followed by emission of “forbidden” lines from low-lying fine structure levels. It is these cooling lines that give HII regions their characteristic spectra.
- **Hydrodynamical response**, including hydro shocks, ionization and photodissociation fronts, and outflows and winds from the embedded stars.

Line diagnostics: how to determine the temperature and density of the HII regions or planetary nebulae?



Line diagnostics in structured energy level system



Temperature diagnostics in structured energy level system

Emissivity from 4→3 level: $j(4 \rightarrow 3) = n_4 \frac{A_{43}}{4\pi} h\nu_{43}.$

Excitation eq for level 4: $n_0 n_e k_{04} = n_4 (A_{43} + A_{41}).$ (4→2 and 4→0 are strongly forbidden.)

$$j(4 \rightarrow 3) = \frac{n_0 n_e}{4\pi} k_{04} \frac{A_{43}}{A_{43} + A_{41}} h\nu_{43}.$$

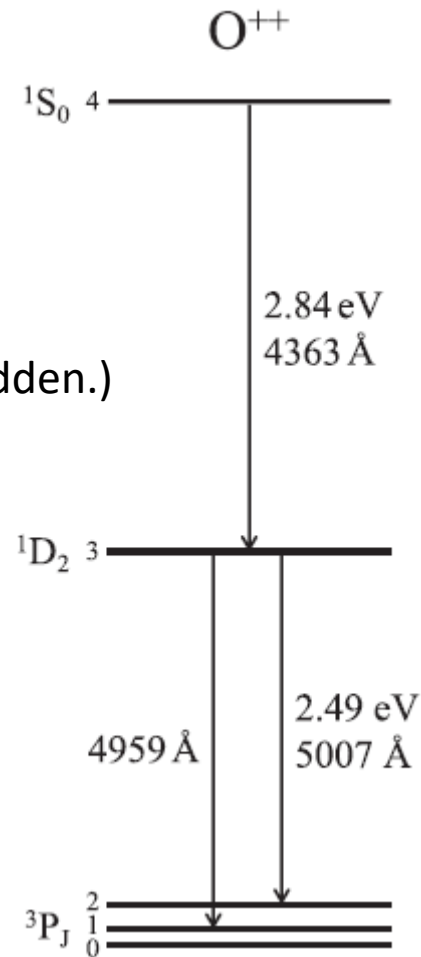
Emissivity from 3→2 level:

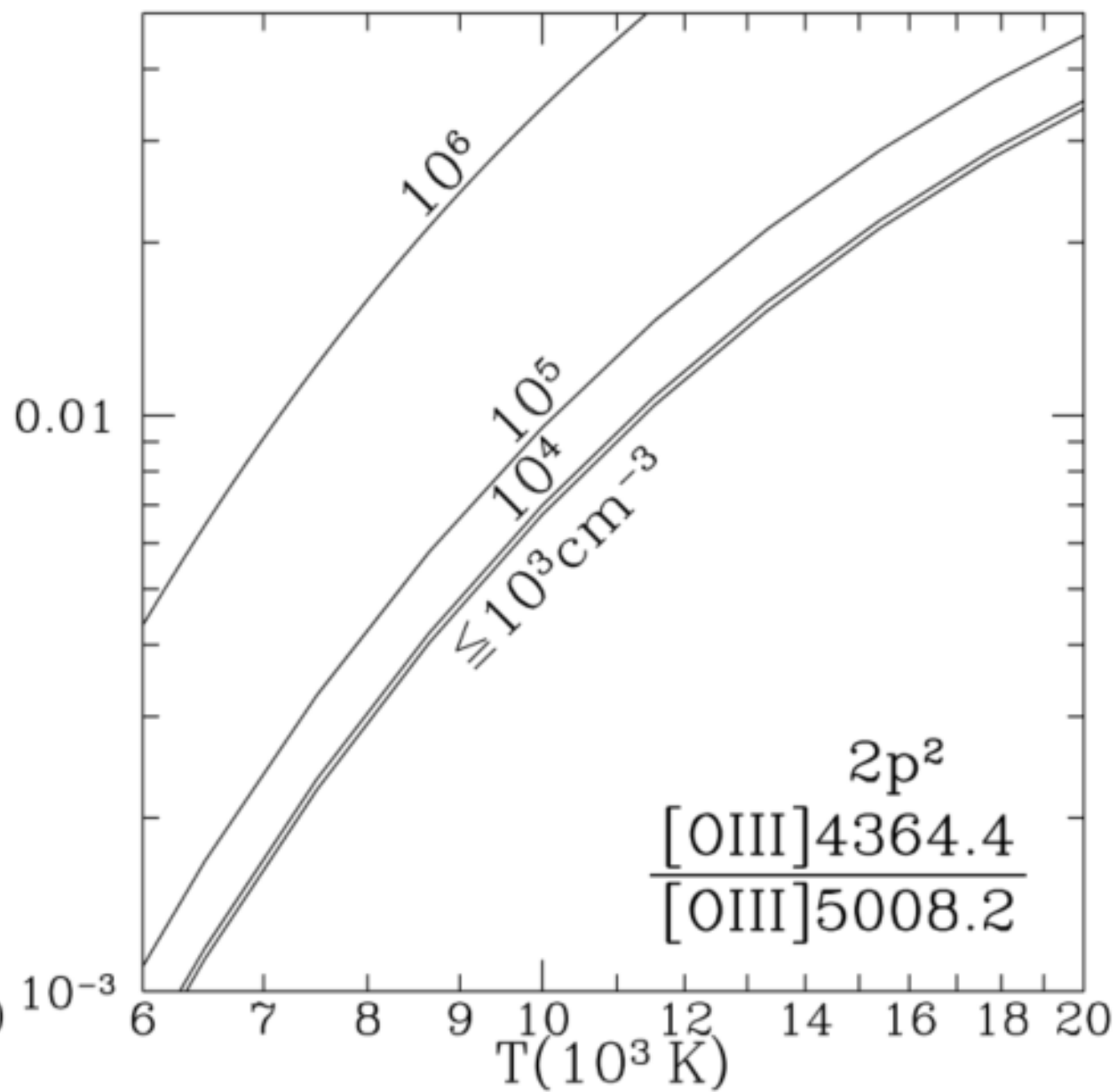
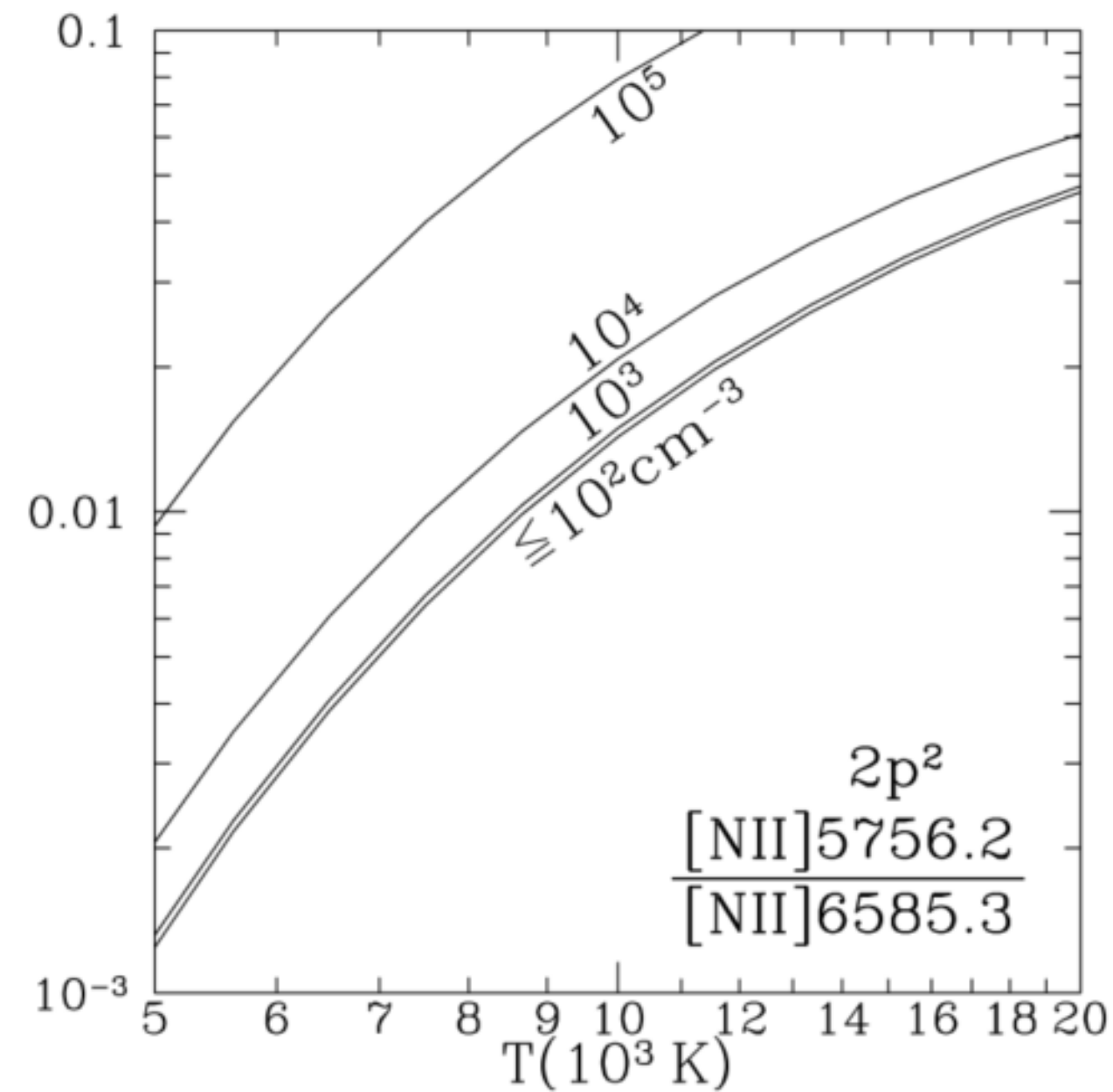
$$j(3 \rightarrow 2) = \frac{n_0 n_e}{4\pi} \left[k_{03} + k_{04} \frac{A_{43}}{A_{43} + A_{41}} \right] \frac{A_{32}}{A_{32} + A_{31}} h\nu_{32}.$$

Line ratio between 4→3 auroral line and 3→2 nebula line:

$$\frac{j(4 \rightarrow 3)}{j(3 \rightarrow 2)} = \frac{A_{43} \nu_{43}}{A_{32} \nu_{32}} \frac{(A_{32} + A_{31}) k_{04}}{(A_{43} + A_{41}) k_{03} + A_{43} k_{04}} = \frac{A_{43} \nu_{43}}{A_{32} \nu_{32}} \frac{(A_{32} + A_{31}) \Omega_{40} \exp(-h\nu_{43}/kT)}{(A_{43} + A_{41}) \Omega_{30} + A_{43} \Omega_{40} \exp(-h\nu_{43}/kT)}.$$

$$k_{0u} = k_{u0} \frac{g_u}{g_0} \exp\left(-\frac{h\nu_{u0}}{kT}\right). \quad k_{u0} = \frac{\beta}{T^{1/2}} \frac{\Omega_{u0}}{g_u}$$





Density diagnostics in structured energy level system

Emissivity from 1-→0 level: $j(1 \rightarrow 0) = n_1 \frac{A_{10}}{4\pi} h\nu_{10}$.

Excitation eq (both collisional and radiative de-excitation is considered) for level 1:

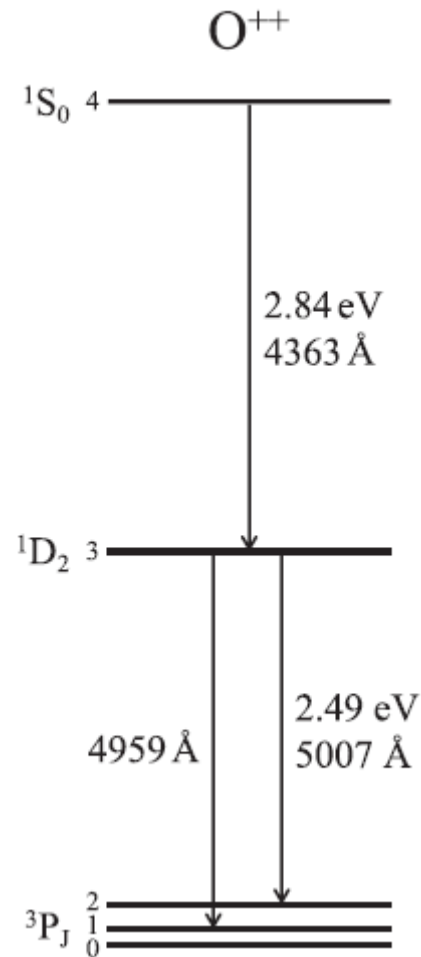
$$n_0 n_e k_{01} = n_1 (A_{10} + n_e k_{10})$$

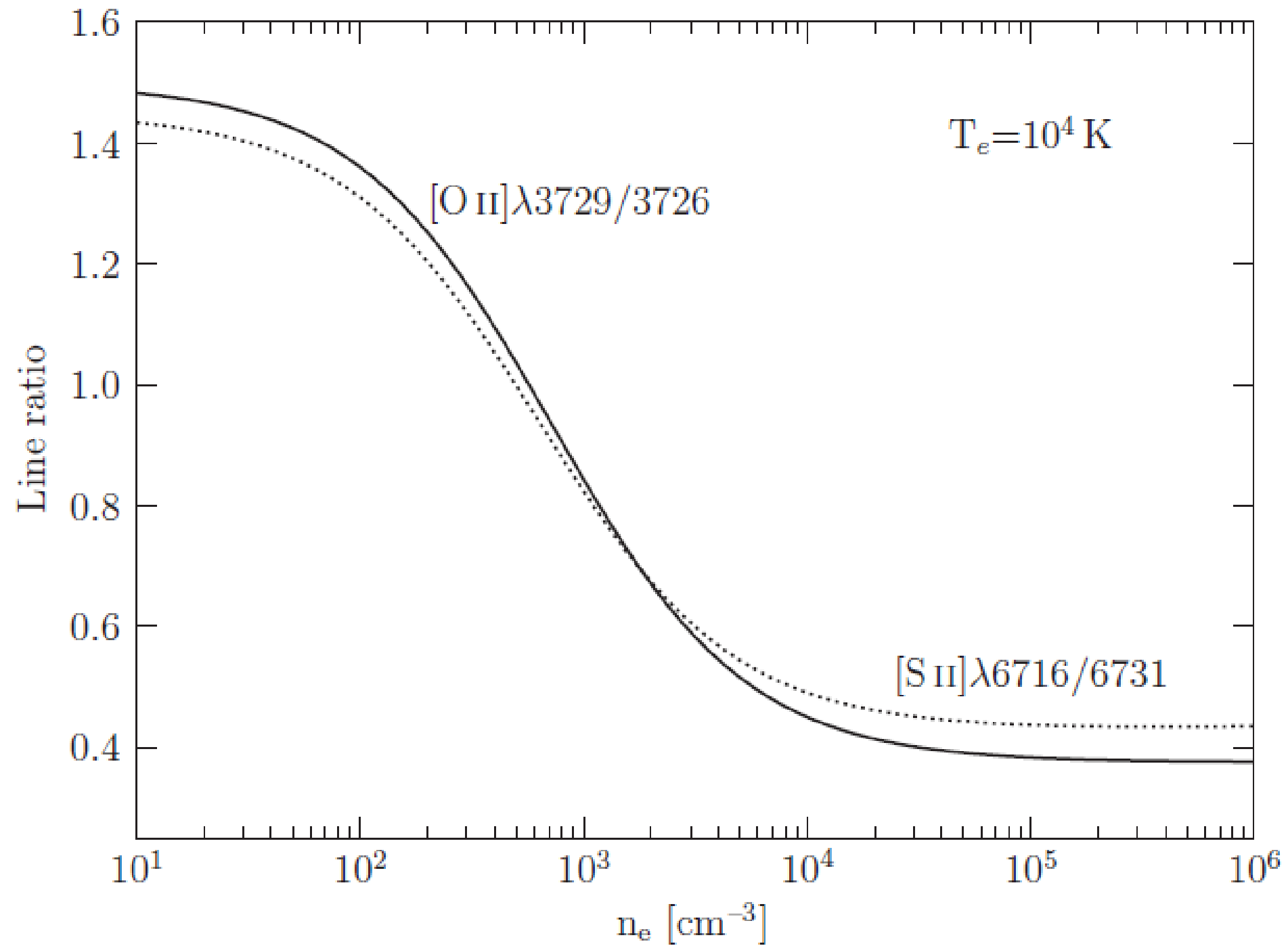
$$\begin{aligned} j(1 \rightarrow 0) &= n_0 n_e \frac{k_{01}}{A_{10} + n_e k_{10}} \frac{A_{10}}{4\pi} h\nu_{10} \\ &= n_0 n_e \frac{k_{01}}{1 + n_e/n_{\text{crit},1}} \frac{h\nu_{10}}{4\pi} \end{aligned}$$

Emissivity from 2-→0 level: $j(2 \rightarrow 0) = n_0 n_e \frac{k_{02}}{1 + n_e/n_{\text{crit},2}} \frac{h\nu_{20}}{4\pi}$.

$$\frac{j(2 \rightarrow 0)}{j(1 \rightarrow 0)} = \frac{\nu_{20}}{\nu_{10}} \frac{k_{02}}{k_{01}} \frac{1 + n_e/n_{\text{crit},1}}{1 + n_e/n_{\text{crit},2}} \approx \frac{\Omega_{20}}{\Omega_{10}} \frac{1 + n_e/n_{\text{crit},1}}{1 + n_e/n_{\text{crit},2}}$$

$$\frac{k_{02}}{k_{01}} = \frac{\Omega_{20}}{\Omega_{10}} \exp(-h\nu_{21}/kT) \approx \frac{\Omega_{20}}{\Omega_{10}}$$





Hot ionize medium (HIM): SNR, CGM and IGM



Volumetric ci rate:

$$\frac{dn(X^i)}{dt} = -n_e n(X^i) k_{ci}$$

$$k_{ci} = 2.32 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1} \left(\frac{kT}{I_H} \right)^{1/2} e^{-I_H/kT} \text{ for hydrogen}$$

$$t_{ci} = \frac{1}{n_e k_{ci}} \quad I_H/k = 1.58 \times 10^5 \text{ K}$$

$$\approx 160 \text{ yr} \left(\frac{T}{10^6 \text{ K}} \right)^{-0.5} \exp \left[0.158 \left(\frac{10^6 \text{ K}}{T} \right) \right] \left(\frac{n_e}{0.004 \text{ cm}^{-3}} \right)^{-1}$$

Collisional ionization equilibrium (CIE): collisional ionization is balanced by radiative recombination.

$$\frac{dn_{\text{HI}}}{dt} = -n_e n_{\text{HI}} k_{ci} + n_e n_{\text{HII}} \alpha_{A,H}$$

Why Case A?

$$n_e n_{\text{HI}} k_{ci} = n_e n_{\text{HII}} \alpha_{A,H} \quad \longrightarrow \quad \frac{n_{\text{HII}}}{n_{\text{HI}}} = \frac{k_{ci}(T)}{\alpha_{A,H}(T)}$$

$$\frac{n_{\text{HII}}}{n_{\text{HI}}} \approx 4.0 \times 10^5 \left(\frac{kT}{I_H} \right)^{1.21} e^{-I_H/kT}$$

CIE of different ions and cooling at high temperature plasma

