4. Cold Neutral Medium

Instructor: Hui Li TA: Chengzhe Li

Department of Astronomy

Tsinghua University

Relative importance of the stimulated emission

- N_gamma, the dimensionless photon occupation number, determines the relative importance of stimulated and spontaneous emission:

$$\left(\frac{dn_\ell}{dt}\right)_{u\to\ell} = -\left(\frac{dn_u}{dt}\right)_{u\to\ell} = n_u\left(A_{u\ell} + B_{u\ell}u_\nu\right)$$

$$\bar{n}_\gamma \equiv \frac{c^2}{2h\nu^3}\bar{I}_\nu = \frac{c^3}{8\pi h\nu^3}u_\nu$$

- Stimulated emission is unimportant when n_gamma << 1, but should be included when n_gamma>1 when analyzing level excitation.

Frequency-dependent absorption cross section

- The three Einstein coefficients summarize the total rate of the transition.
- However, when electrons interacting with photons, they see photons of different energy differently.

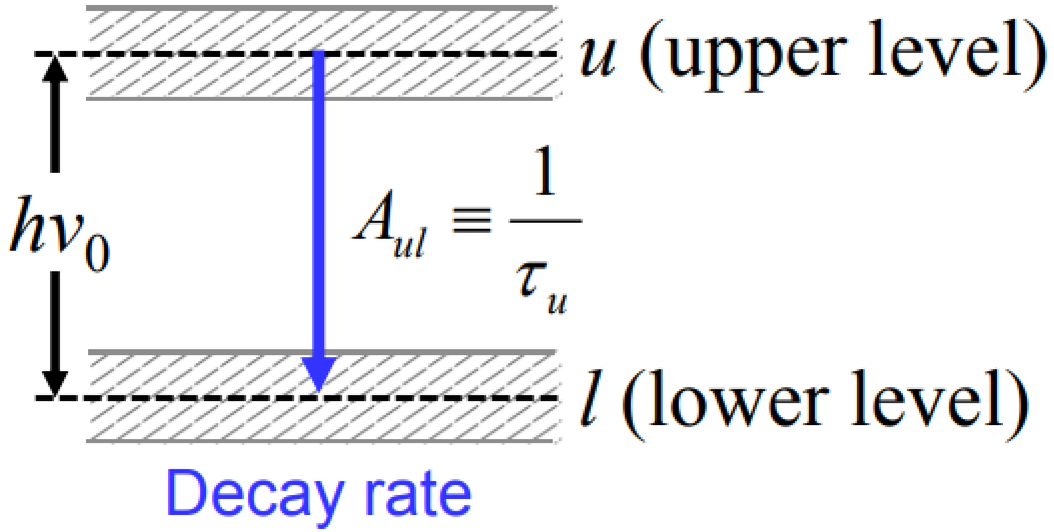
$$\left(\frac{dn_u}{dt}\right)_{\ell\to u} = n_\ell \int d\nu\, \sigma_{\ell u}(\nu) c \frac{u_\nu}{h\nu} \approx n_\ell u_\nu \frac{c}{h\nu} \int d\nu\, \sigma_{\ell u}(\nu)$$

$$B_{\ell u} = \frac{c}{h\nu} \int d\nu\, \sigma_{\ell u}(\nu)$$

$$\sigma_{\ell u}(\nu) = \frac{g_u}{g_\ell} \frac{c^2}{8\pi\nu_{\ell u}^2} A_{u\ell}\phi_\nu \quad \text{with} \quad \int \phi_\nu d\nu = 1$$

Intrinsic Line Profile

- Determined by natural line broadening.
- The physics behind the natural line broadening is the Heisenberg uncertainty principle.



$$\Delta E_u \Delta t_u \geq h / 2\pi$$

Intrinsic Line Profile

- The intrinsic broadening produces a Lorentz profile (damped harmonical oscillator model)

$$\Phi_\nu = \frac{4\gamma_{u\ell}}{16\pi^2(\nu - \nu_{u\ell})^2 + \gamma_{u\ell}^2} \qquad \gamma_{u\ell} = \sum_{E_j<E_u} A_{uj} + \sum_{E_j<E_\ell} A_{\ell j}$$

$$(\Delta\nu)^{\mathrm{intr.}}_{\mathrm{FWHM}} = \frac{\gamma_{u\ell}}{2\pi}$$

$$(\Delta v)^{\mathrm{intr.}}_{\mathrm{FWHM}} = c\frac{(\Delta\nu)^{\mathrm{intr.}}_{\mathrm{FWHM}}}{\nu_{u\ell}} = \frac{\lambda_{u\ell}\gamma_{u\ell}}{2\pi} = 0.0121\frac{\mathrm{km}}{s}\left(\frac{\lambda_{u\ell}\gamma_{u\ell}}{7618\,\mathrm{cm\,s^{-1}}}\right)$$

Doppler Broadening

- Atoms and ions are generally in kinematic equilibrium, where the velocity distribution is Maxwell.
- In one specific line-of-sight, the velocity distribution is simply a Gaussian.

$$p_v = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_v} \mathrm{e}^{-(v-v_0)^2/2\sigma_v^2} = \frac{1}{\sqrt{\pi}} \frac{1}{b} \mathrm{e}^{-(v-v_0)^2/b^2}$$

- b is the thermal broadening parameter: $b = \left(\frac{2kT}{m}\right)^{1/2} = 1.28\,\mathrm{km\,s^{-1}} \left(\frac{T}{100\,\mathrm{K}}\right)^{1/2} \left(\frac{m}{m_{\mathrm{H}}}\right)^{-1/2}$

The Voigt profile

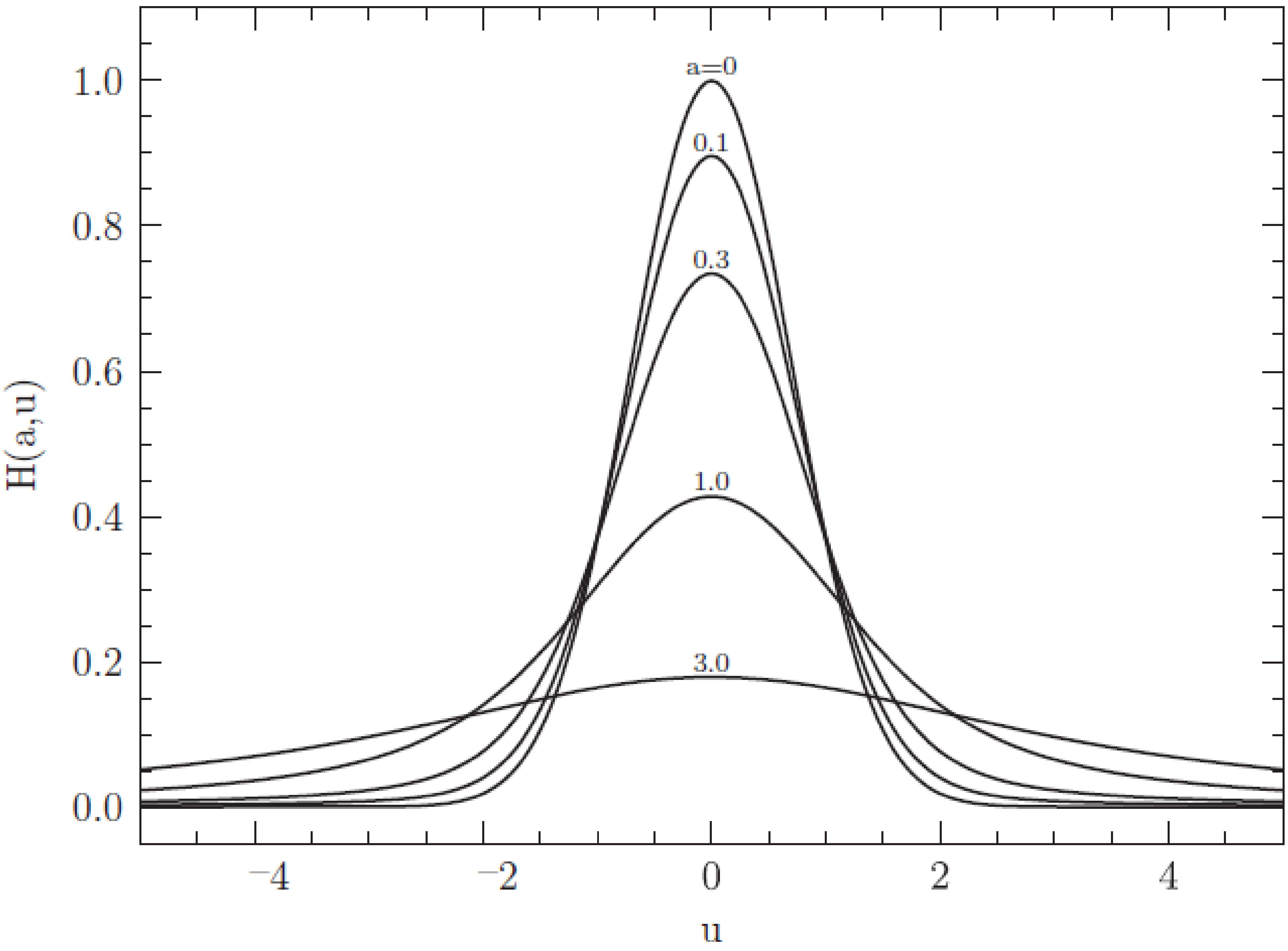
- The final profile is the convolution of the intrinsic profile with the thermal velocity distribution:

$$\phi_\nu = \int dv\, p_v(v)\, \frac{4\gamma_{u\ell}}{16\pi^2 \left[\nu - (1 - v/c)\nu_{u\ell}\right]^2 + \gamma_{u\ell}^2}$$

$$\Phi_\nu^{\mathrm{Voigt}} = \frac{1}{\sqrt{\pi}} \frac{1}{\nu_{u\ell}} \frac{c}{b} H(a,u), \qquad H(a,u) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{(u-y)^2 + a^2},$$

$$a \equiv \frac{\gamma_{u\ell} c}{4\pi\, \nu_{u\ell} b}$$

$$u \equiv \frac{c}{b}\left(1 - \frac{\nu}{\nu_{u\ell}}\right) = \frac{v}{b}.$$



a=0
0.1
0.3
1.0
3.0
1.0
0.8
0.6
0.4
0.2
0.0
H(a,u)
−4
−2
0
2
4
u

Absorption lines from radiative transfer

- Radiative transfer equation along the line-of-sight s: $\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$
- Change variable from s to optical depth: $d\tau_\nu \equiv \kappa_\nu ds.$ $\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$
- The cold neutral medium is the simplest case of the radiative transfer, because nearly all atoms are in their ground state in such low temperature and spontaneous emission can be ignored.

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu}$$

Building absorption lines

- Optical depth can be expressed in the form of profile

$$\tau_\nu = \int \kappa_\nu ds = \int n_\ell \sigma_{\ell u}(\nu) ds = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} \int n_\ell \Phi_\nu ds.$$

- If we assume the absorbers along the line-of-sight have the same temperature and profile:

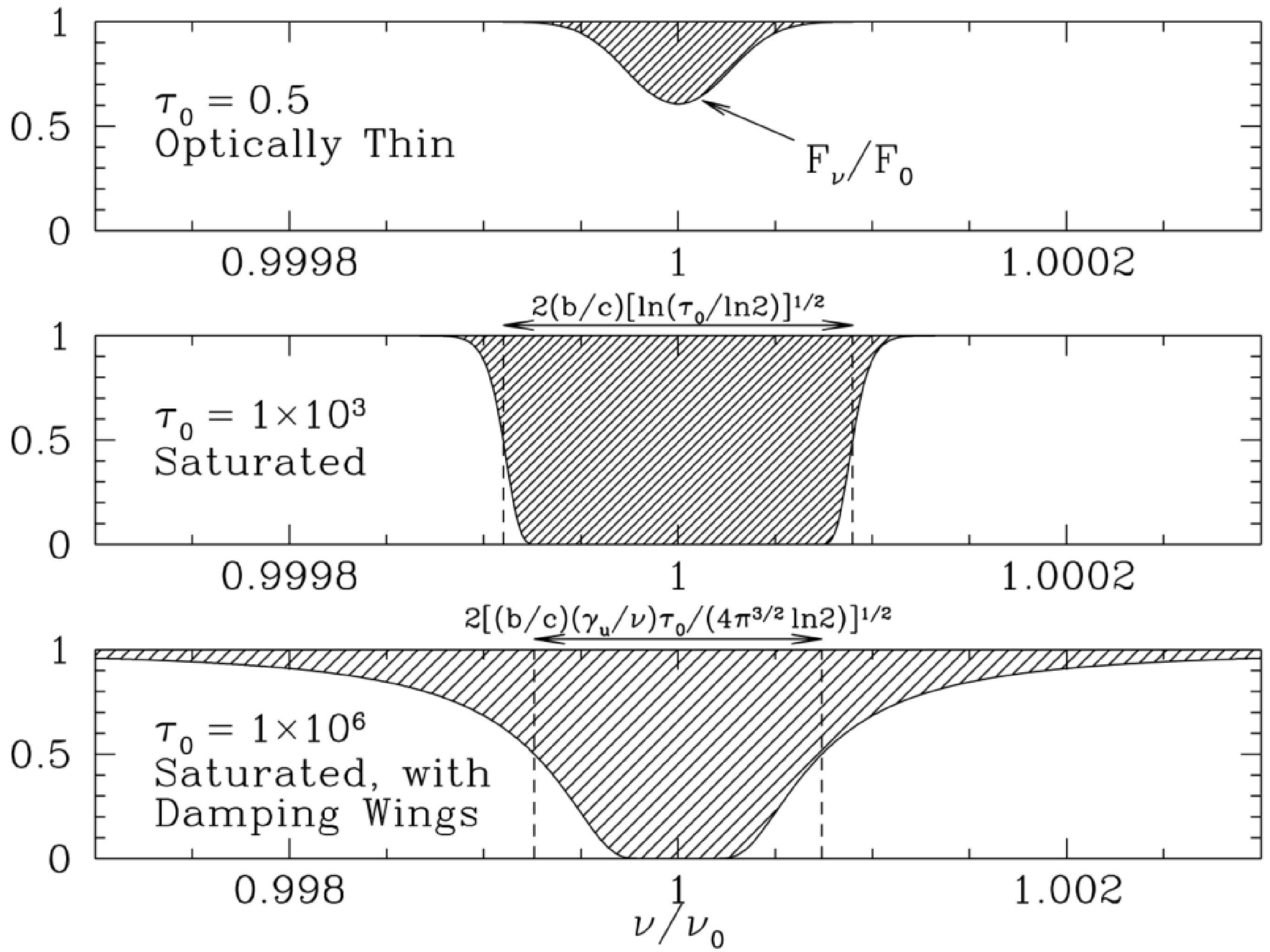
$$\tau_\nu = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} N_\ell \Phi_\nu = \frac{1}{(4\pi)^{3/2}} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^3} A_{u\ell} \right] \frac{c}{b} N_\ell H(a, u).$$

EW in low-res spectral

- Quantify the strength of absorption: the concept of equivalent width.

$$W_\lambda = \int d\lambda [1 - e^{-\tau_\lambda}]$$

- Graphically speaking, the equivalent width is the width of a straight-sided, perfectly black absorption line that has the same integrated flux deficit as the actual absorption line.



Curve of Growth: 1. optically thin regime

- When tau<<1, $W_\lambda = \int d\lambda [1 - \mathrm{e}^{-\tau_\lambda}] \approx \int d\lambda\, \tau_\lambda \approx \tau_0 \int d\lambda\, H(a,u)$
- Also, when tau<<1, the main contribution to the absorption comes from the Gaussian core because it is always that a<<1.

$$W_\lambda \approx \tau_0 \int d\lambda\, \mathrm{e}^{-u^2} \qquad W_\lambda = \frac{b}{c}\lambda_0\tau_0 \int \mathrm{e}^{-u^2} du = \frac{\sqrt{\pi}b}{c}\lambda_0\tau_0$$

- EW grows linearly with tau in the optically thin regime.

$$W_\lambda = \frac{1}{8\pi}\left[\frac{g_u}{g_\ell}\frac{c^2}{\nu_{u\ell}^3}A_{u\ell}\right]\lambda_0 N_\ell$$

Curve of Growth: 2. intermediate regime

$$W_\lambda = \int d\lambda \left[1 - \exp(-\tau_0 e^{-u^2})\right]$$

$$= \frac{b}{c}\lambda_0 \int du \left[1 - \exp(-\tau_0 e^{-u^2})\right]$$

- Consider the above equation as a step function of 1 and 0 with the boundary at $\tau_0 e^{-u^2} > 1$

$$W_\lambda \approx \frac{2b}{c}\lambda_0\sqrt{\ln \tau_0}$$

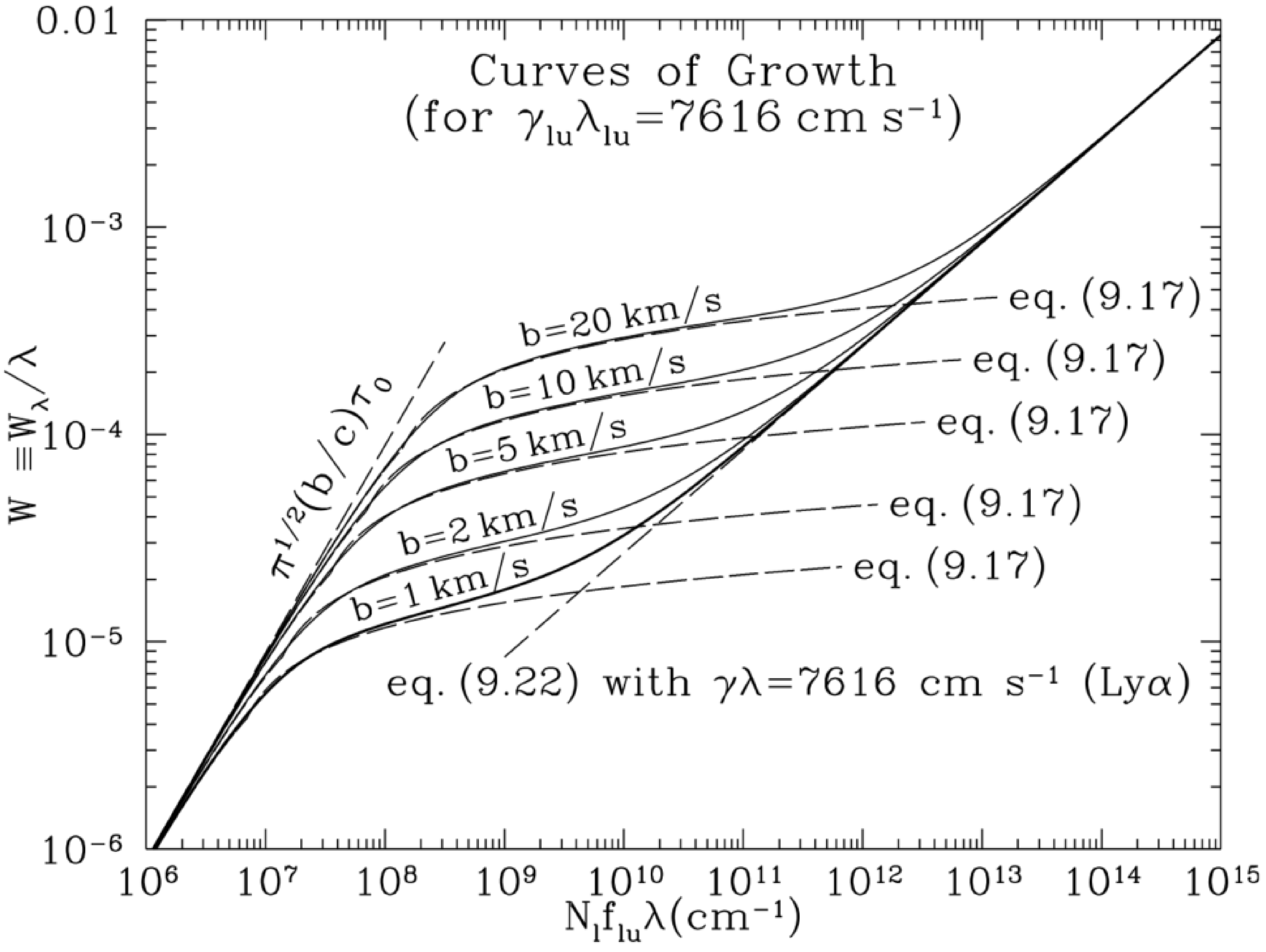
- EW grows very slowly with tau.

Curve of Growth: 3. damped regime

- When damping wing absorption contributes significantly, we are entering the new regime.

$$\tau_\lambda \approx \tau_0 \frac{a}{\sqrt{\pi} u^2}$$

$$\begin{aligned} W_\lambda &\approx \int d\lambda \left[1 - \exp(-\tau_\lambda)\right] \\ &\approx \frac{b}{c}\lambda_0 \int du \left[1 - \exp\left(-\tau_0 \frac{a}{\sqrt{\pi} u^2}\right)\right] \\ &\sim \frac{2b}{c}\lambda_0 \left(\frac{a\tau_0}{\sqrt{\pi}}\right)^{1/2} \end{aligned}$$



Curves of Growth
(for $\gamma_{lu}\lambda_{lu}$=7616 cm s^{-1})
$W \equiv W_\lambda/\lambda$
$N_l f_{lu} \lambda (cm^{-1})$
$\pi^{1/2}(b/c)\tau_0$
b=20 km/s
b=10 km/s
b=5 km/s
b=2 km/s
b=1 km/s
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.22) with $\gamma\lambda$=7616 cm s^{-1} (Lyα)
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{6}
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}