清華大學天文系
Department of Astronomy, Tsinghua University

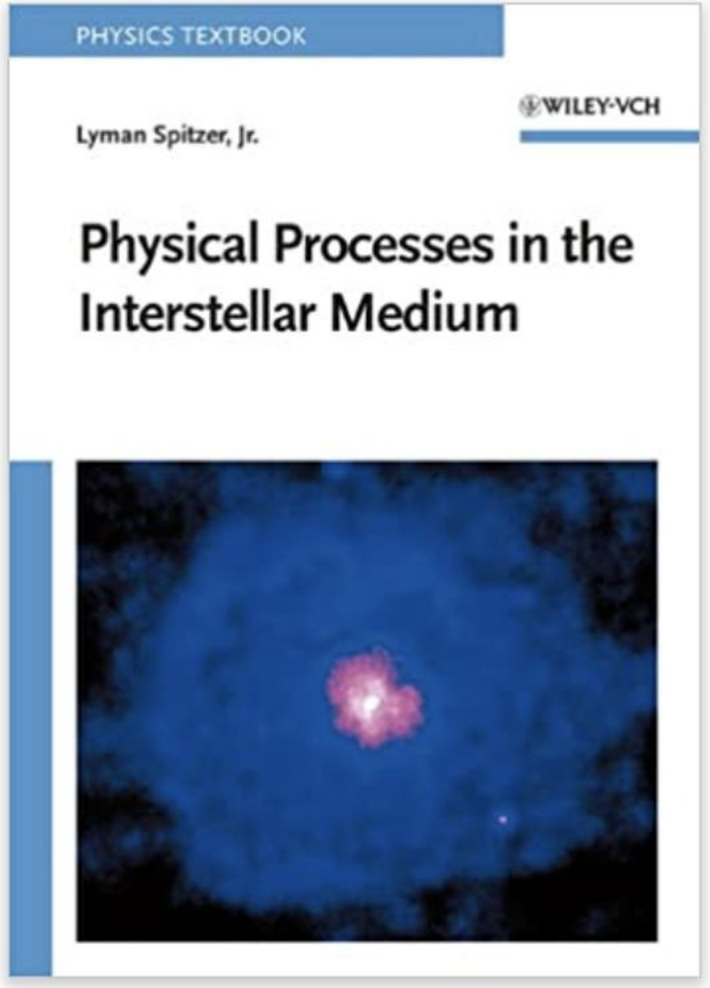
Physics of the Interstellar Medium

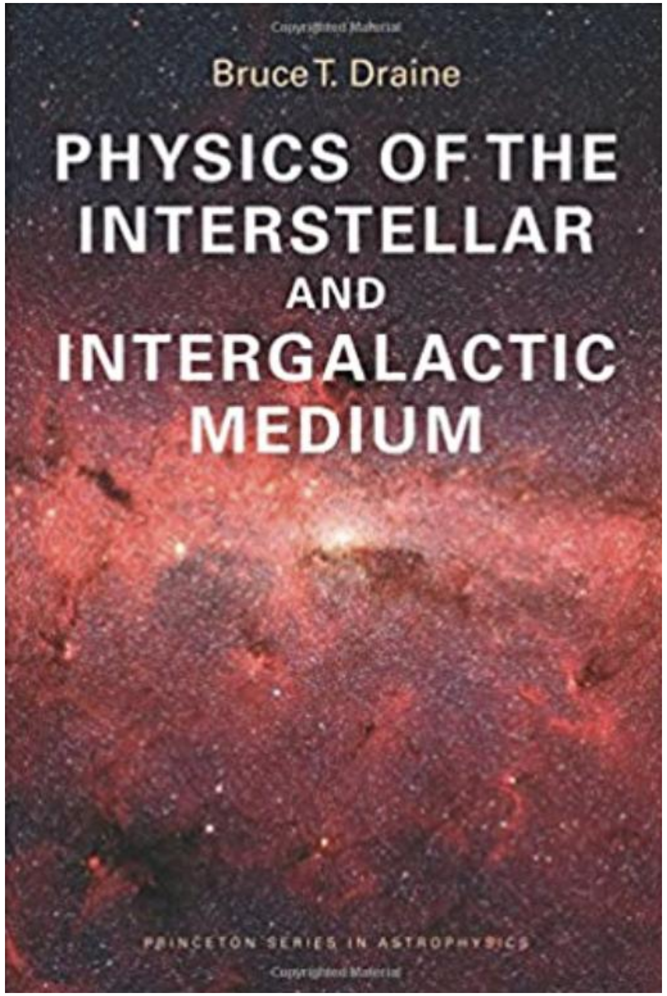
Instructor: Hui Li TA: Chengzhe Li

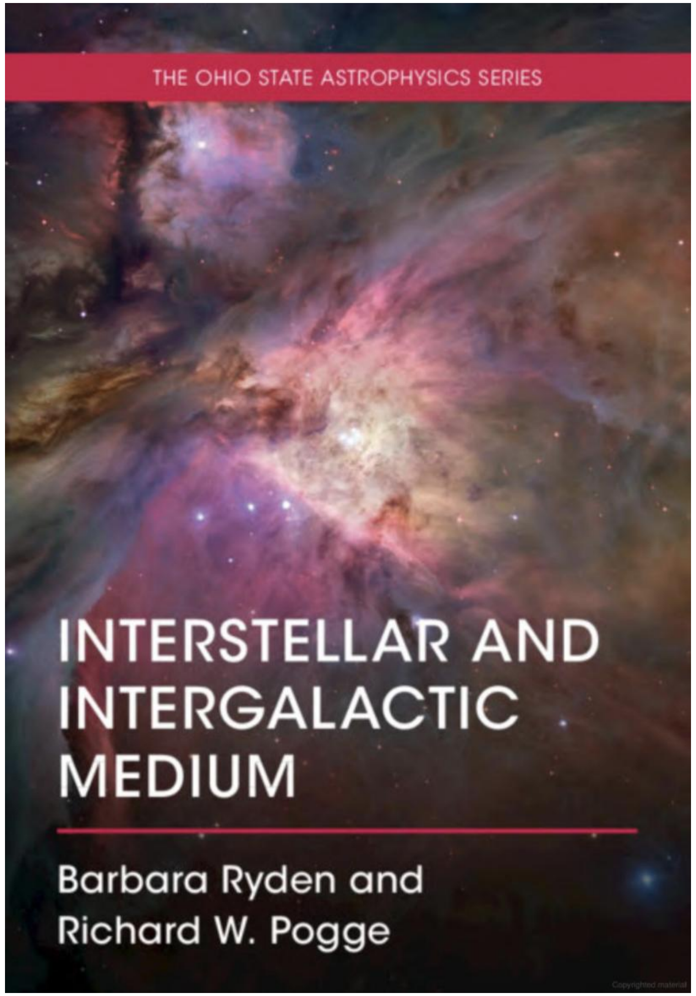
Department of Astronomy

Tsinghua University

Reference books







Course evaluation

- Homework: 5 (?) assignments, in total 40%.
- Mid-term exam: close book, written, 30%.
- Final project: group projects with written report and presentation, 30%.
  - Make your group
  - Select your project
  - Divide your project and distribute efforts to group members
  - Keep on making progress from the very beginning

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2. Collisional Processes

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Collisional Rate Coefficients

- The rate per unit volume of a two-body collision process (A -> B) is

$$n_A n_B \langle \sigma v \rangle_{AB}$$

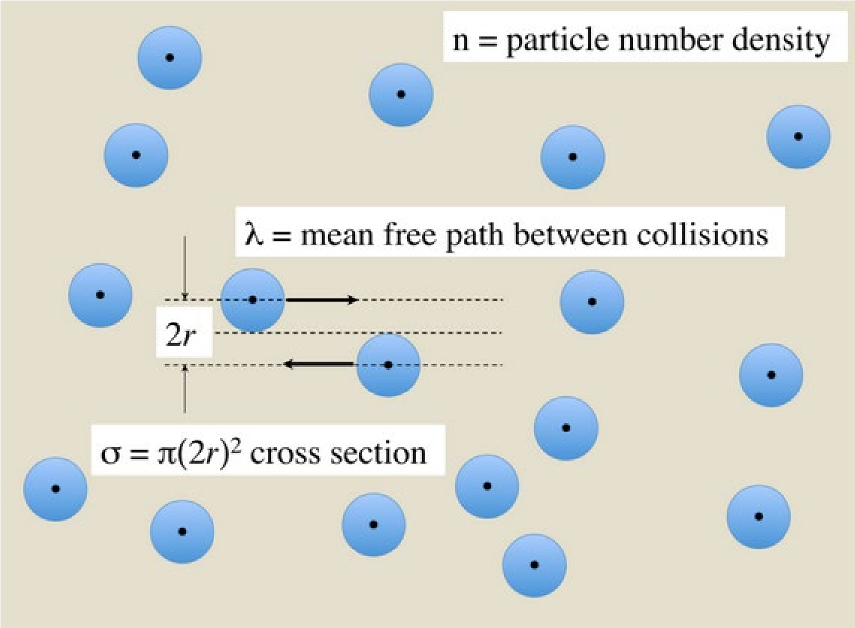
- The two-body collisional rate coefficient is

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv$$

- The mean free path is $\lambda_{\rm mfp} \sim 1/(n\sigma)$

- The collisional timescale is

$$t_{\rm coll} \sim \frac{\lambda_{\rm mfp}}{v_{\rm rms}} \sim \frac{1}{n\sigma}\left(\frac{m}{2\langle E \rangle}\right)^{1/2} .$$



Kinetic Equilibrium

- Maxwellian distribution $f_v\, dv = 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} \mathrm{e}^{-\mu v^2/2kT}\, v^2\, dv$

$$\langle E \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT = 1.293\,\mathrm{eV} \left(\frac{T}{10^4\,\mathrm{K}} \right)$$

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f_v\, dv \quad = \left(\frac{8kT}{\pi \mu} \right)^{1/2} \int_0^\infty \sigma_{AB}(E)\, \frac{E}{kT}\, \mathrm{e}^{-E/kT}\, \frac{dE}{kT} \quad .$$

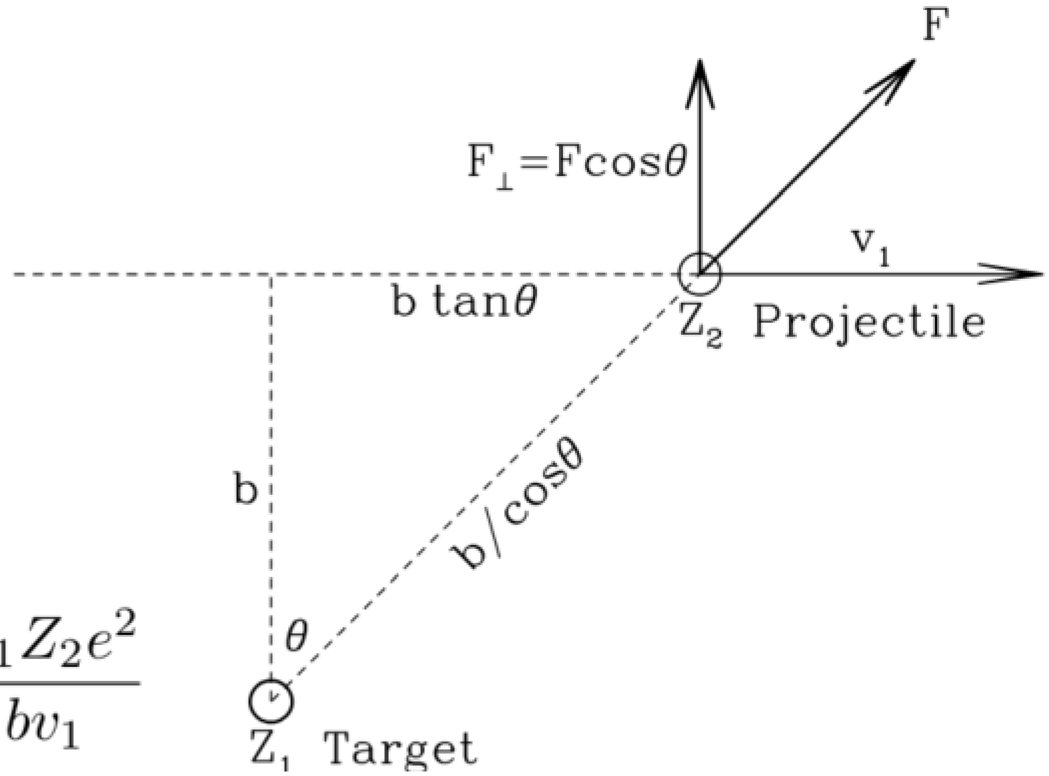
Close vs. Distance Encounters

- A close encounter between two charged particles produces relatively large changes in velocity comparable with the initial velocities.
- In a distance encounter these changes are much smaller with negligible deflection angle -> impact approximation.
- For inverse-square forces between particles (gravity, Coulomb), the velocities changes decreases relatively slowly with increasing impact parameters. In consequence, the numerous small velocity changes produced by distance encounters outweigh the effect of rare close encounters.
- This is not true for neutral atom collision!

Inverse-Square Law Forces: Elastic scattering

- The impact approximation

$$F_\perp = \frac{Z_1 Z_2 e^2}{(b/\cos\theta)^2}\cos\theta = \frac{Z_1 Z_2 e^2}{b^2}\cos^3\theta$$



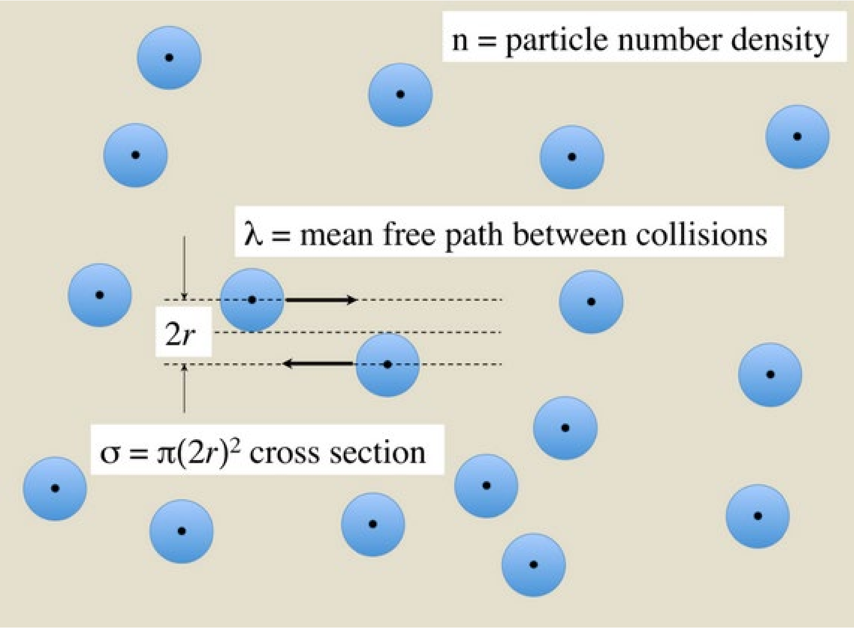
$$\Delta p_\perp = \int_{-\infty}^{\infty} F_\perp dt = \frac{Z_1 Z_2 e^2}{b v_1}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = 2\frac{Z_1 Z_2 e^2}{b v_1}$$

Deflection / Collisional Timescale

- In the impact approximation, each interaction gives an impulse in the direction that is perpendicular to the direction of motion of the projectile.
- The orientation, though, is randomly distributed in this plane. Thus the net vector momentum transferred to the projectile follows a random walk process (dp^2).

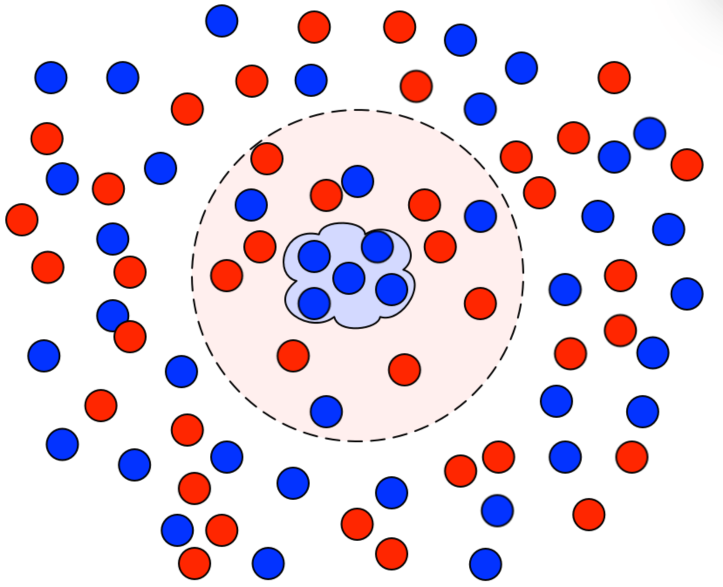
$$\langle \frac{d}{dt}[(\Delta p)_\perp]^2 \rangle = \int_{b_{\min}}^{b_{\max}} \underbrace{[2\pi b db\, n_2 v_1]}_{d(\text{event rate})} \times \underbrace{\left[\frac{2Z_1 Z_2 e^2}{b v_1}\right]^2}_{(\Delta p_\perp)^2}$$

$$= \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} \quad .$$



The min and max distance in the logarithmic.

- The integral is logarithmically divergent at both the min and max impact parameter b, so we have to find some physical limits for the lower and upper cutoffs, b_min and b_max.
- b_min can be estimated at the separation where the Coulomb energy is the same as the kinetic energy of the particle. (The impact approximation fails). b_min~Z1Z2e^2/E
- b_max is the Debye radius, beyond which the plasma maintain electrical neutrality.



$$L_{\rm D} \equiv \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\,{\rm cm}\left(\frac{T}{10^4\,{\rm K}}\right)^{1/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)^{1/2}$$

Values of the logarithmic term

- The typical values of Lambda is around 20-40.
- The value much larger than one suggests that weak distant frequent encounters dominate the momentum changes.
- The impact approximation is quite accurate for normal ISM conditions.

$$\Lambda \equiv \frac{b_{\rm max}}{b_{\rm min}} = \frac{E}{kT}\frac{(kT)^{3/2}}{(4\pi n_e)^{1/2} Z_1 Z_2 e^3}$$

$$= 4.13 \times 10^9 \left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)^{1/2}$$

$$\ln\Lambda = 22.1 + \ln\left[\left(\frac{E}{kT}\right)\left(\frac{T}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\right] \quad .$$

Timescale estimates: deflection and energy loss timescales

$$t_{\rm defl} = \frac{(m_1 v_1)^2}{\langle (d/dt)[(\Delta p)_\perp]^2\rangle} = \frac{m_1^2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda} = 7.6\times 10^3\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

$$\begin{aligned}{\rm mfp} = v_1 t_{\rm defl} &= \frac{m_1^2 v_1^4}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}\\ &= 5\times 10^{17}\,{\rm cm}\left(\frac{T}{10^6\,{\rm K}}\right)^2\left(\frac{0.01\,{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

$$\begin{aligned}t_{\rm loss} \equiv \frac{E}{\langle (dE/dt)_{\rm loss}\rangle} &= \frac{m_1 v_1^2}{\langle (d/dt)[(\Delta p)_\perp]^2\rangle/m_2} = \frac{m_1 m_2 v_1^3}{8\pi n_2 Z_1^2 Z_2^2 e^4 \ln\Lambda}\\ &= 1.4\times 10^7\,{\rm s}\left(\frac{T_e}{10^4\,{\rm K}}\right)^{3/2}\left(\frac{{\rm cm}^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)\end{aligned}$$

Ion-Neutral Collision: short range interaction!

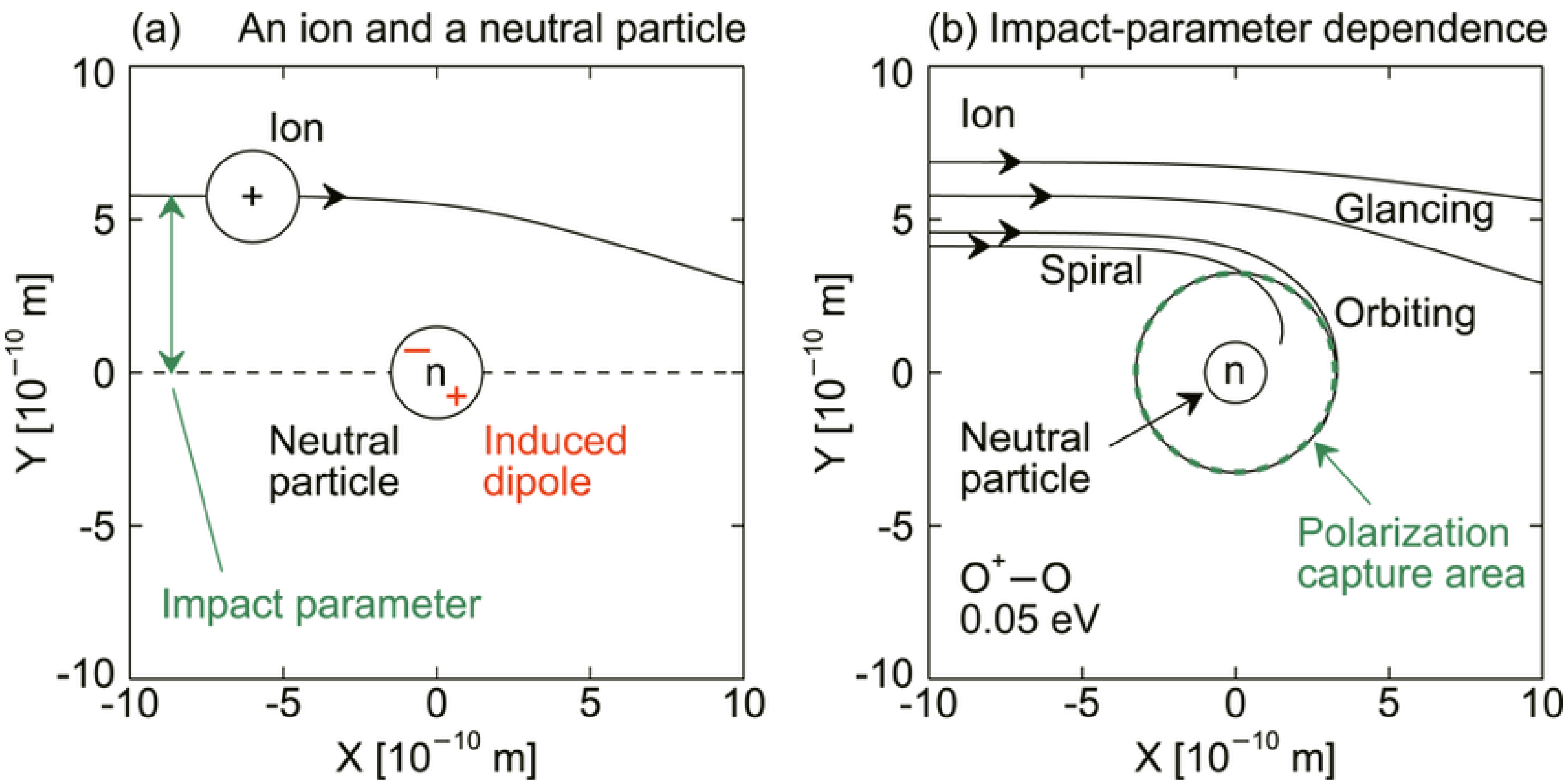
- The 1/r Coulomb potential means that the potential drop relatively slow for longer distant.
- The physics of Ion-Neutral collision: polarization of the neutral caused by the electric field of the ion.

$$U(r) = -\frac{1}{2}\frac{\alpha_N Z^2 e^2}{r^4}$$

- The cross-section only

depends on the short-range

impact!



Neutral-Neutral Collision

- If we ignore the very weak van der Waals attraction, we can assume the simplest "billiard balls" model.
- R~ 2*a0, where a0 is the Bohr radius. $a_0 = 5.292 \times 10^{-9}\,\mathrm{cm}$

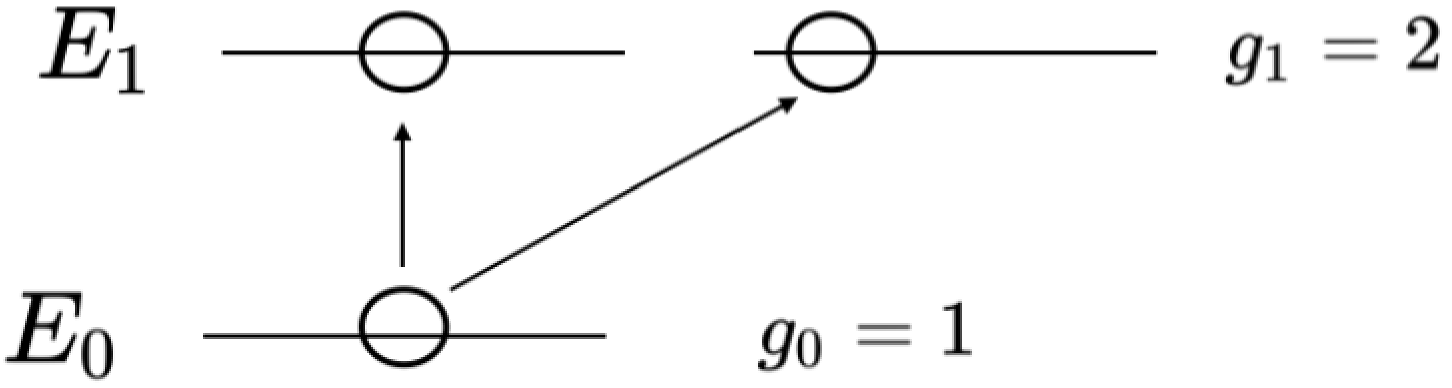
$$\langle \sigma v \rangle = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \pi (R_1 + R_2)^2$$

$$= 1.81 \times 10^{-10} \left(\frac{T}{10^2\,\mathrm{K}}\right)^{1/2} \left(\frac{m_\mathrm{H}}{\mu}\right)^{1/2} \left(\frac{R_1 + R_2}{2\,\text{Å}}\right)^2 \mathrm{cm}^3\,\mathrm{s}^{-1}$$

$$t_\mathrm{coll} \sim \frac{\lambda_\mathrm{mfp}}{v_\mathrm{rms}} \sim \frac{1}{n\sigma}\left(\frac{m}{2\langle E \rangle}\right)^{1/2} \sim 2 \times 10^8\,\mathrm{s} \left(\frac{n_\mathrm{H}}{1\,\mathrm{cm}^{-3}}\right)^{-1} \left(\frac{\langle E \rangle}{1\,\mathrm{eV}}\right)^{-1/2}$$

Minimalists' Statistical Mechanism: Boltzmann distribution!

- Example of a two-level quantum system:



$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-(E_1 - E_0)/kT}$$

Excitation Temperature

- Excitation temperature is simply a convenient way to parameterize the excitation state of a two-level system, not a measure of kinetic temperature of the gas.
- In excitation equilibrium, excitation temperature is the same as the kinetic temperature.
- In general, however, Texc is far from Tkin.
- One extreme case is astrophysical maser, where a population is over-excited by shocks/strong radiation. The excitation temperature is negative in this case.

$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(-\frac{E_{u\ell}}{kT_{\mathrm{exc}}}\right),$$

$$kT_{\mathrm{exc}} \equiv \frac{E_{u\ell}}{\ln[(g_u/g_\ell)(n_\ell/n_u)]}.$$

Ionization Equilibrium

- The first ionization energy, I, of important elements is ~ 10 eV, which corresponds to ~1e5 K.

Element	ppm by number	percentage by mass	atomic number	1st ionization energy [eV]
hydrogen (H)	910 630	71.10%	1	13.60
helium (He)	88 250	27.36%	2	24.59
oxygen (O)	550	0.68%	8	13.62
carbon (C)	250	0.24%	6	11.26
neon (Ne)	120	0.18%	10	21.56
nitrogen (N)	75	0.08%	7	14.53
magnesium (Mg)	36	0.07%	12	7.65
silicon (Si)	35	0.08%	14	8.15
iron (Fe)	30	0.13%	26	7.90
sulfur (S)	15	0.04%	16	10.36

[a] *Data from Lodders 2010*

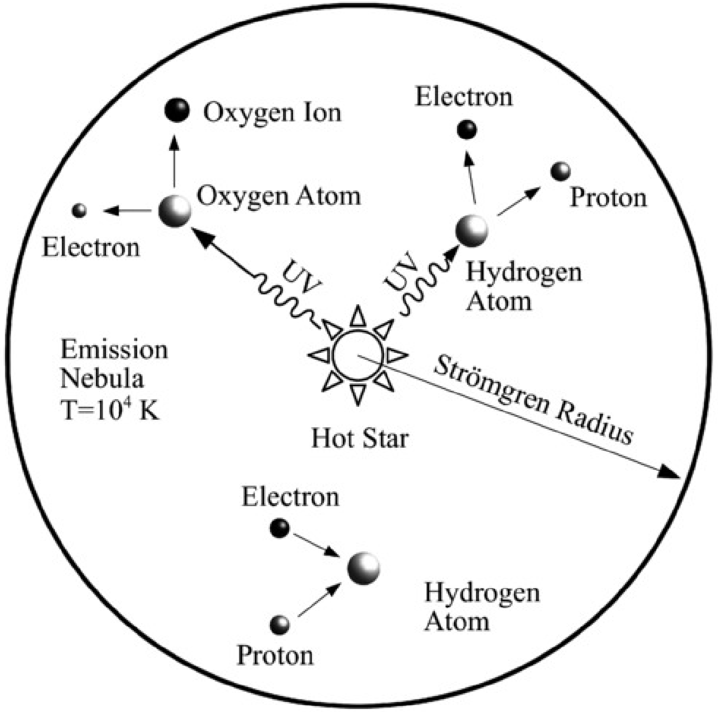
- Only regions with temperature > 1e5K (rare) are effective collisional ionization of neutral atoms.
- In cooler regions, most ionization comes from photo-ionization from UV photons.

Photo-ionization vs. Radiative Recombination

$$X^i + \gamma \rightarrow X^{i+1} + e^-, \qquad X^{i+1} + e^- \rightarrow X^i + \gamma.$$

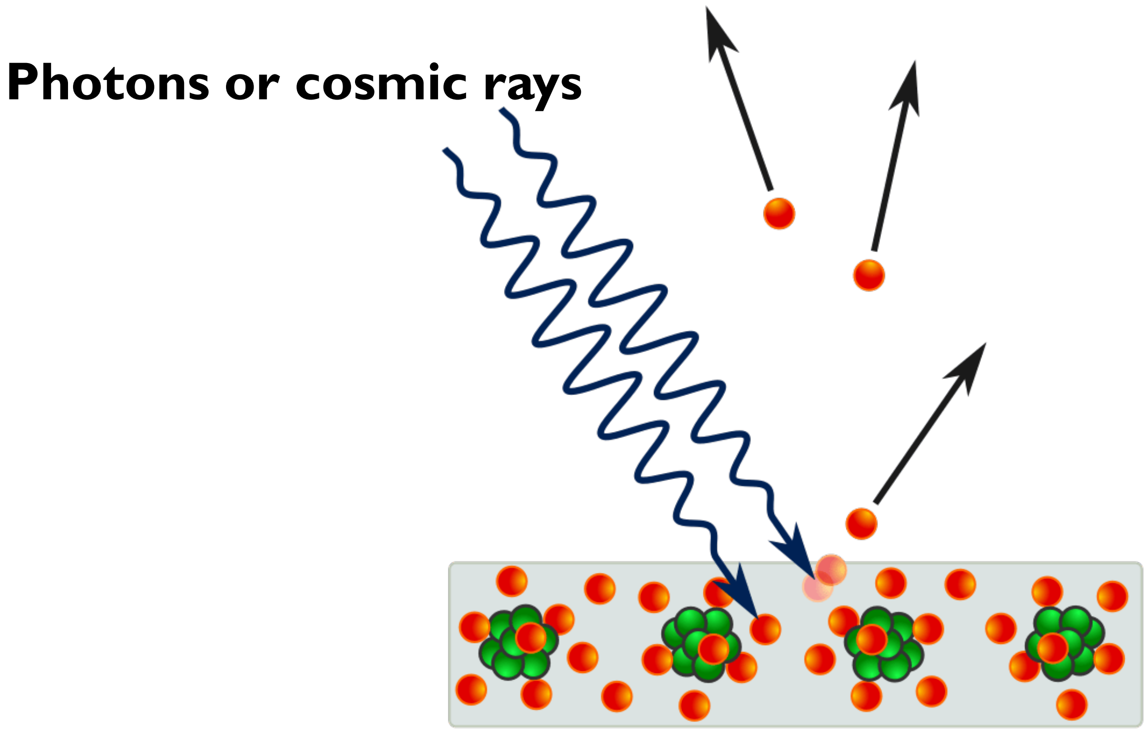
- Ionization equilibrium means the balance between the two processes.

$$n(X^i) n_\gamma \sigma_{\rm pho} c = n(X^{i+1}) n_e \sigma_{\rm rr} v$$



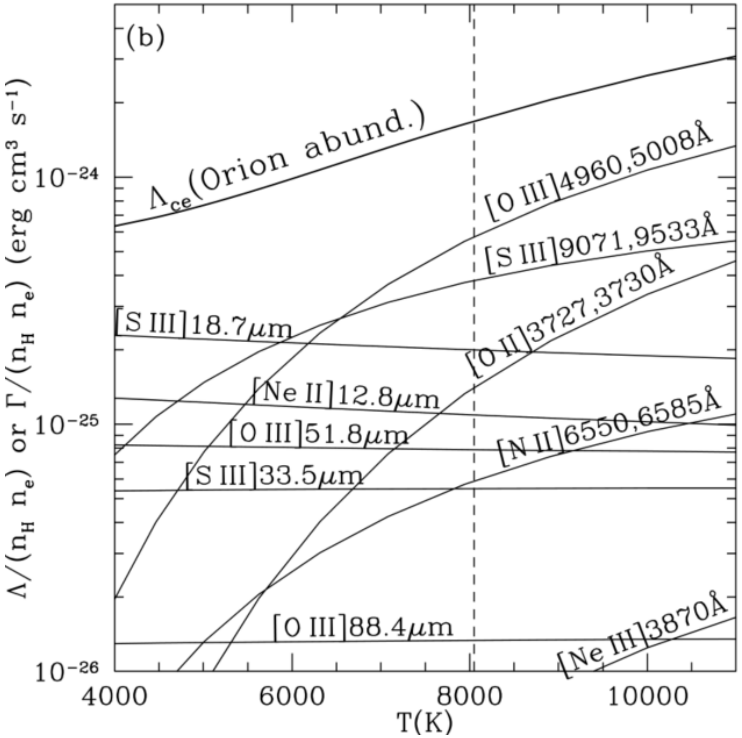
Heating of the ISM

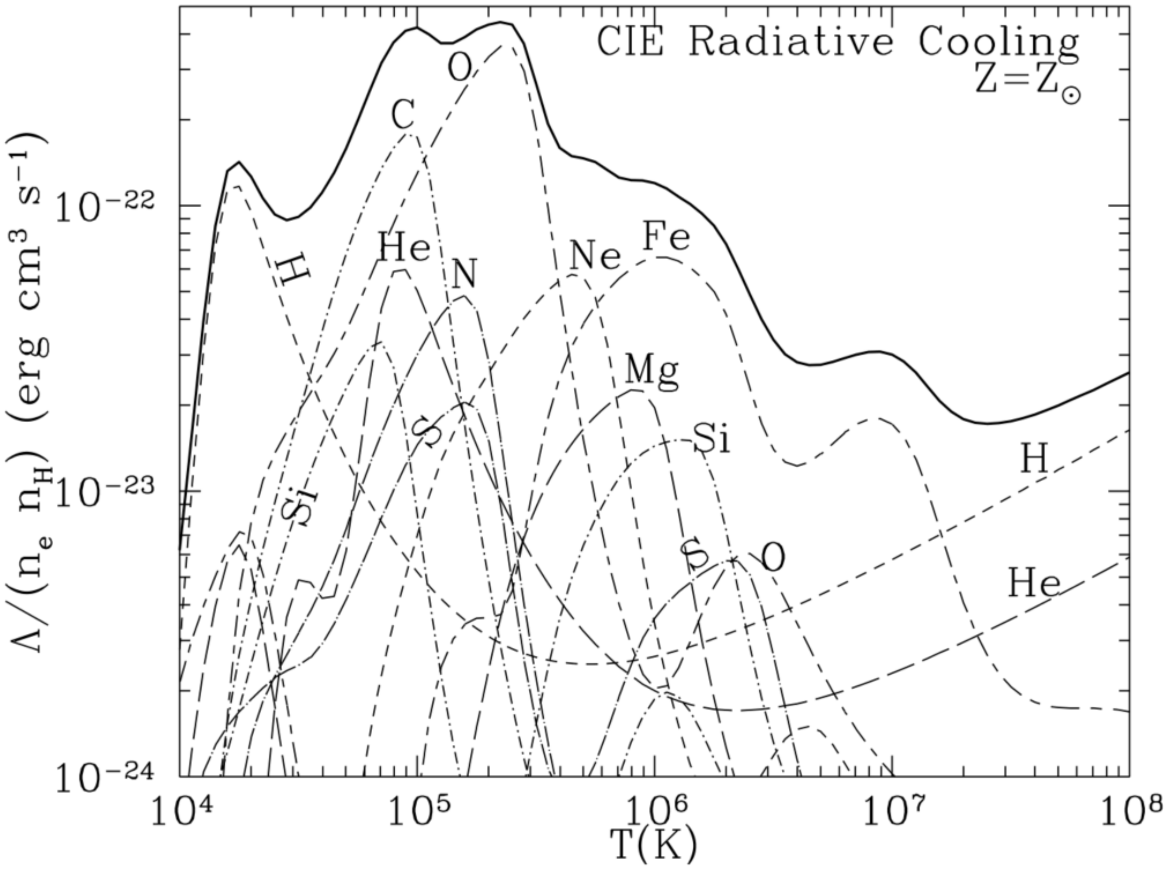
- Heating is the process of converting energy of other forms into thermal energy of the gas.
- Heating is dominated by suprathermal electrons produced via photo-ionization, photoelectric effect on dust grains, and cosmic rays. Hydrodynamical shocks can also convert kinetic energy into thermal energy by the thermalization process at the shock front.



Cooling of the ISM

- Collisional excited line emissions (H, O, C, S for warm medium, Fe for hot gas).
- Recombination radiation (mostly H+ cascade).
- Free-free emission of hot plasma.

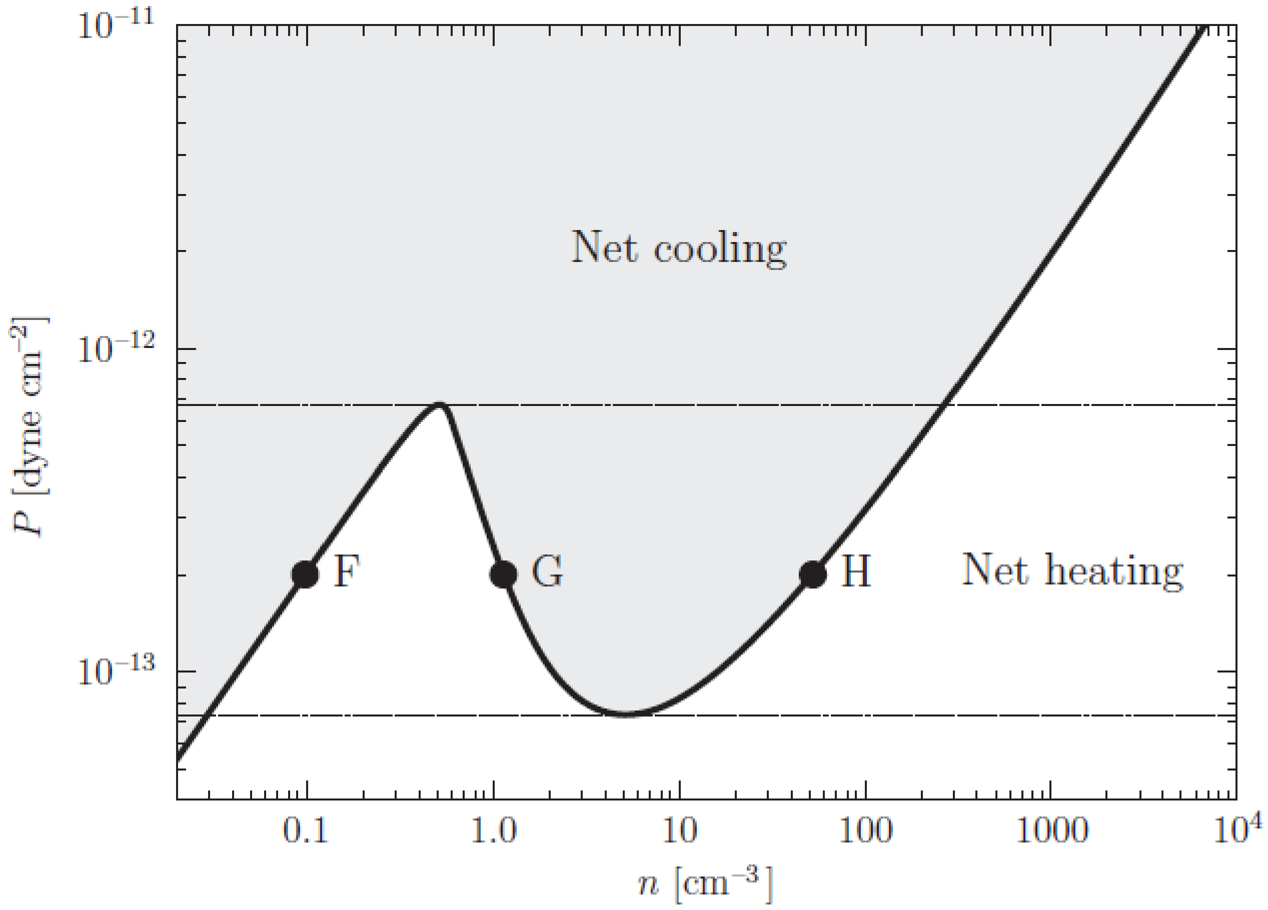




Phases of the ISM (on the order of temperature)

Phase	$T(\mathrm{K})$	$n_H(\mathrm{cm}^{-3})$	Comments
Coronal gas (HIM) $f_V \approx 0.5?$ $\langle n_H \rangle f_V \approx 0.002\,\mathrm{cm}^{-3}$ ($f_V \equiv$ volume filling factor)	$\gtrsim 10^{5.5}$	~ 0.004	Shock-heated Collisionally ionized Either expanding or in pressure equilibrium Cooling by: ◇ Adiabatic expansion ◇ X ray emission Observed by: • UV and x ray emission • Radio synchrotron emission
H II gas $f_V \approx 0.1$ $\langle n_H \rangle f_V \approx 0.02\,\mathrm{cm}^{-3}$	10^4	$0.3 - 10^4$	Heating by photoelectrons from H, He Photoionized Either expanding or in pressure equilibrium Cooling by: ◇ Optical line emission ◇ Free–free emission ◇ Fine-structure line emission Observed by: • Optical line emission • Thermal radio continuum
Warm H I (WNM) $f_V \approx 0.4$ $n_H f_V \approx 0.2\,\mathrm{cm}^{-3}$	~ 5000	0.6	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Pressure equilibrium Cooling by: ◇ Optical line emission ◇ Fine structure line emission Observed by: • H I 21 cm emission, absorption • Optical, UV absorption lines
Cool H I (CNM) $f_V \approx 0.01$ $n_H f_V \approx 0.3\,\mathrm{cm}^{-3}$	~ 100	30	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◇ Fine structure line emission Observed by: • H I 21-cm emission, absorption • Optical, UV absorption lines
Diffuse H_2 $f_V \approx 0.001$ $n_H f_V \approx 0.1\,\mathrm{cm}^{-3}$	$\sim 50\,\mathrm{K}$	~ 100	Heating by photoelectrons from dust Ionization by starlight, cosmic rays Cooling by: ◇ Fine structure line emission Observed by: • H I 21-cm emission, absorption • CO 2.6-mm emission • optical, UV absorption lines
Dense H_2 $f_V \approx 10^{-4}$ $\langle n_H \rangle f_V \approx 0.2\,\mathrm{cm}^{-3}$	$10 - 50$	$10^3 - 10^6$	Heating by photoelectrons from dust Ionization and heating by cosmic rays Self-gravitating: $p > p(\text{ambient ISM})$ Cooling by: ◇ CO line emission ◇ C I fine structure line emission Observed by: • CO 2.6-mm emission • dust FIR emission
Cool stellar outflows	$50 - 10^3$	$1 - 10^6$	Observed by: • Optical, UV absorption lines • Dust IR emission • H I, CO, OH radio emission

Stable and Unstable Equilibrium



Two phase ISM model: a sharp insight from Field 1969

since it is thermally stable. A third stable phase should exist above 10^6 ° K, with brems-strahlung the chief cooling process.

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COSMIC-RAY HEATING OF THE INTERSTELLAR GAS

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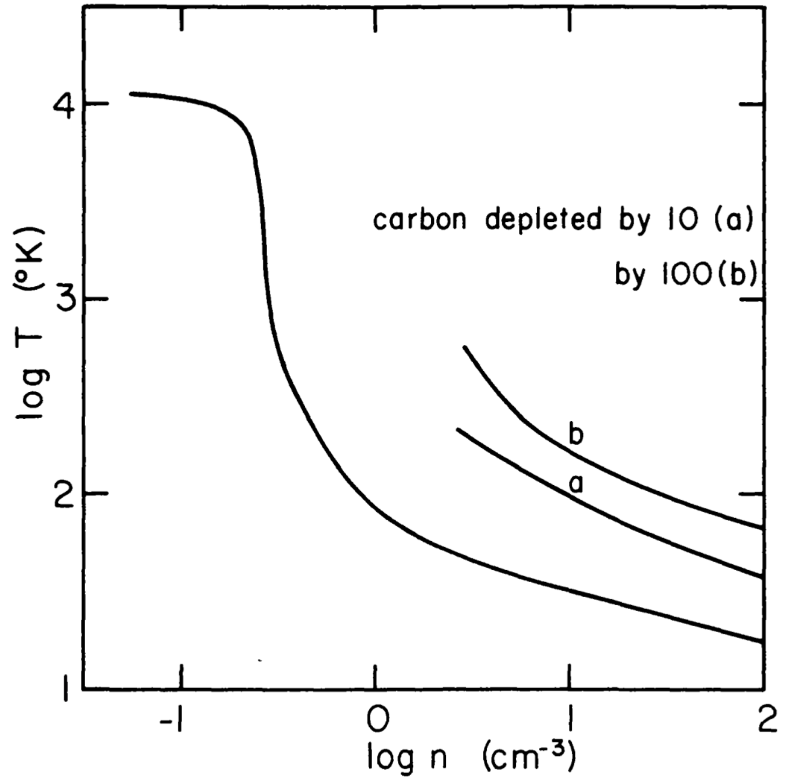
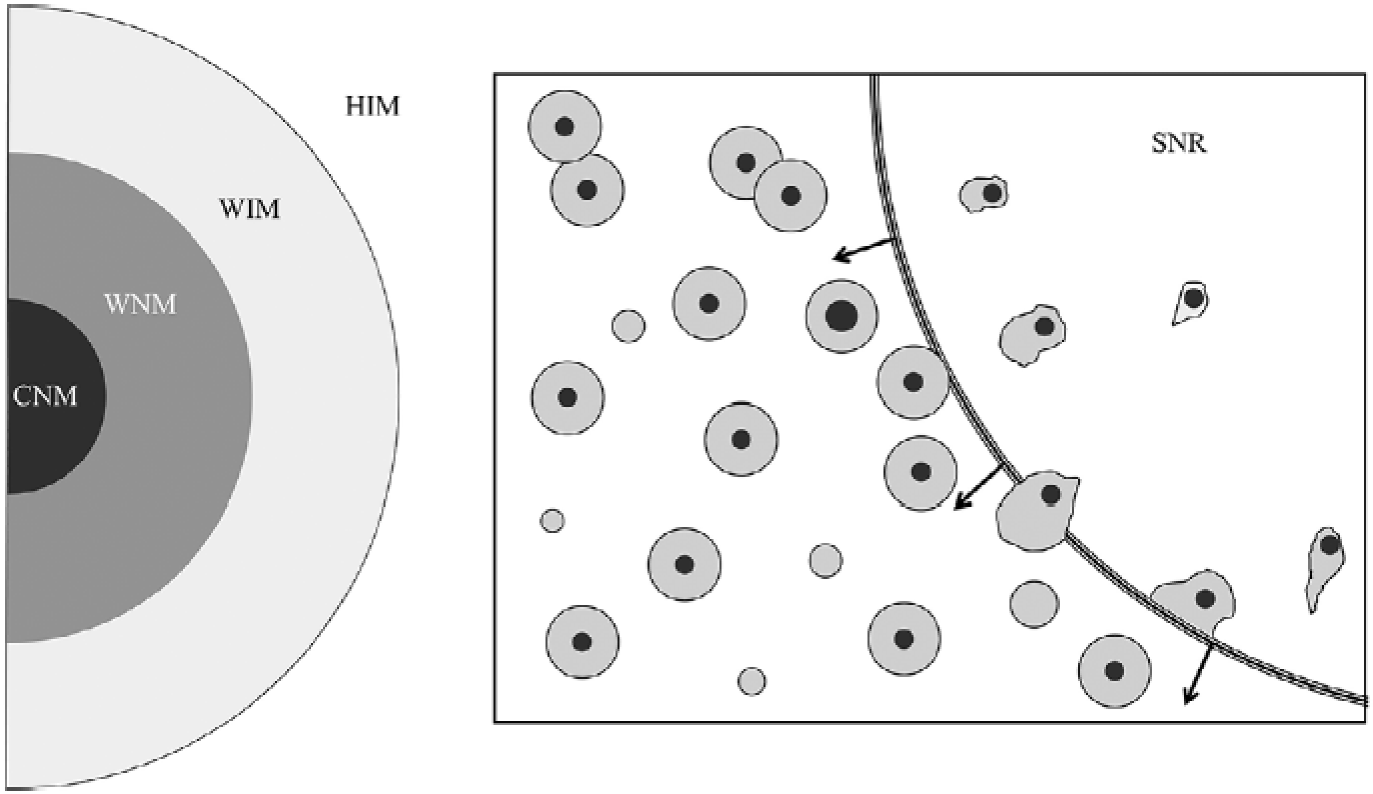
Received December 16, 1968; revised January 17, 1969

ABSTRACT

We present a model of the interstellar medium based on detailed calculations of heating by low-energy cosmic rays. The model contains two thermally stable gas phases that coexist in pressure equilibrium, one at $T = 10^4$ ° K and one at $T < 300$° K. The hot gas occupies most of interstellar space. Gravitation in the z-direction compresses about 75 per cent of the gas into the cool, dense phase to form clouds. By choosing three parameters (the cosmic-ray ionization rate, the amount by which trace elements are depleted in sticking to dust grains, and the magnetic-field strength), we are able to predict six previously unrelated observational parameters to within a factor of 2.



McKee Ostriker (1977) standard model of three-phase ISM



4. Cold Neutral Medium

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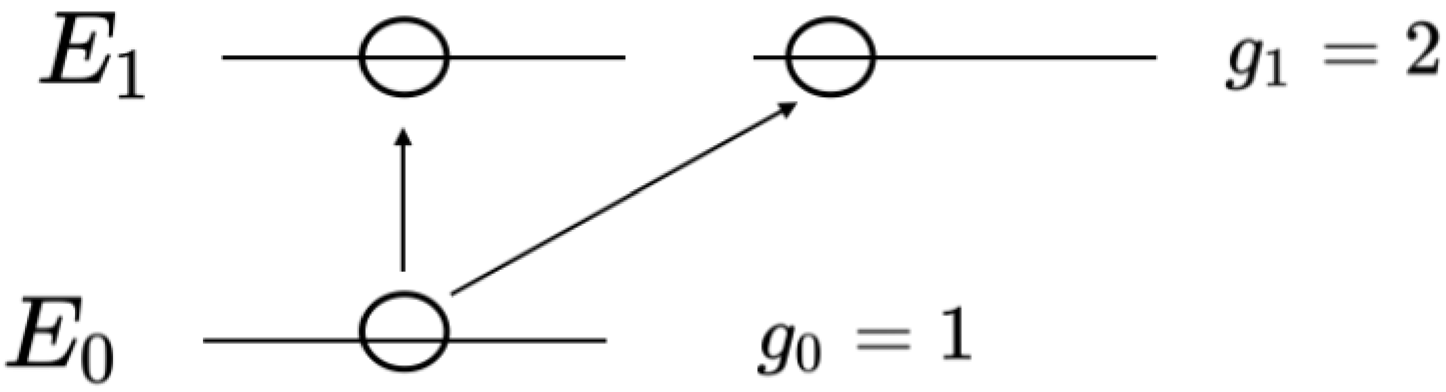
Transitions between quantized energy levels



$$\textbf{absorption}: \quad X_\ell + h\nu \rightarrow X_u\,, \quad h\nu = E_u - E_\ell\,.$$

$$\textbf{spontaneous emission}: \quad X_u \rightarrow X_\ell + h\nu \quad \nu = (E_u - E_\ell)/h$$

$$\textbf{stimulated emission}: \quad X_u + h\nu \rightarrow X_\ell + 2h\nu \quad \nu = (E_u - E_\ell)/h$$

Rates as functions of Einstein coefficient.

$$\left(\frac{dn_u}{dt}\right)_{\ell\to u} = -\left(\frac{dn_\ell}{dt}\right)_{\ell\to u} = n_\ell B_{\ell u} u_\nu \ , \qquad \nu = \frac{E_u - E_\ell}{h}$$

$$\left(\frac{dn_\ell}{dt}\right)_{u\to\ell} = -\left(\frac{dn_u}{dt}\right)_{u\to\ell} = n_u \left(A_{u\ell} + B_{u\ell} u_\nu\right)$$

Thermal equilibrium condition

- In thermal equilibrium, the emitter/absorber system much come to equilibrium with the radiation field.
- Two constraints:
  - 1) l and u levels must be populated based on Boltzmann distribution. Detailed balance of different levels.
  - 2) Radiation field is a blackbody.

$$(u_\nu)_{\mathrm{LTE}} = \frac{4\pi}{c} B_\nu(T) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

$$\begin{aligned} \frac{dn_u}{dt} &= \left(\frac{dn_u}{dt}\right)_{\ell\to u} + \left(\frac{dn_u}{dt}\right)_{u\to\ell} \\ &= n_\ell B_{\ell u} \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} - n_u \left(A_{u\ell} + B_{u\ell} \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}\right) \end{aligned}$$

Relations between three Einstein coefficient

- With the above constraints, we realized that the three coefficients are not independent with each other. The relationships can be derived as:

$$B_{u\ell} = \frac{c^3}{8\pi h\nu^3} A_{u\ell} \quad ,$$

$$B_{\ell u} = \frac{g_u}{g_\ell} B_{u\ell} = \frac{g_u}{g_\ell} \frac{c^3}{8\pi h\nu^3} A_{u\ell}$$

Relative importance of the stimulated emission

- N_gamma, the dimensionless photon occupation number, determines the relative importance of stimulated and spontaneous emission:

$$\left(\frac{dn_\ell}{dt}\right)_{u\rightarrow\ell} = -\left(\frac{dn_u}{dt}\right)_{u\rightarrow\ell} = n_u\left(A_{u\ell} + B_{u\ell}u_\nu\right)$$

$$\bar{n}_\gamma \equiv \frac{c^2}{2h\nu^3}\bar{I}_\nu = \frac{c^3}{8\pi h\nu^3}u_\nu$$

- Stimulated emission is unimportant when n_gamma << 1, but should be included when n_gamma>1 when analyzing level excitation.

Frequency-dependent absorption cross section

- The three Einstein coefficients summarize the total rate of the transition.
- However, when electrons interacting with photons, they see photons of different energy differently.

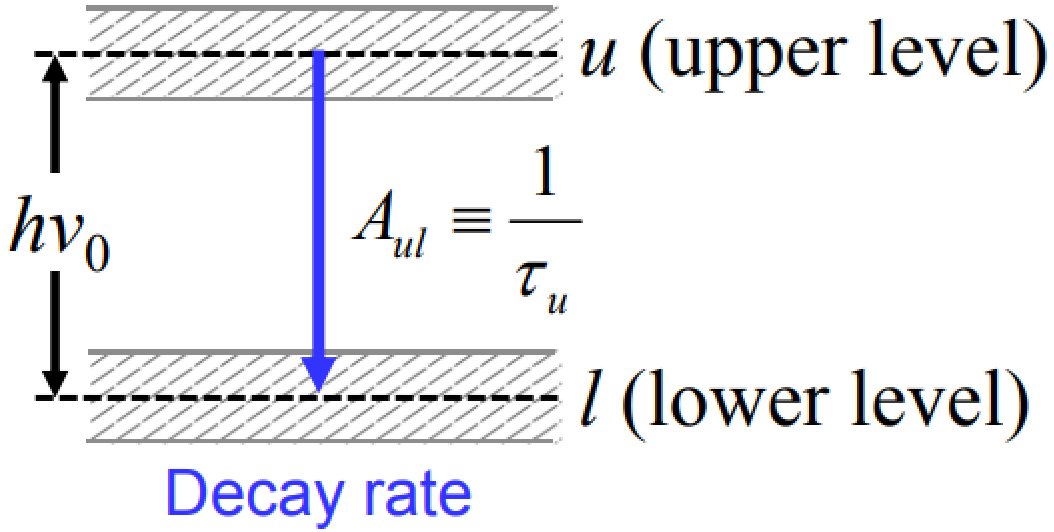
$$\left(\frac{dn_u}{dt}\right)_{\ell\to u} = n_\ell \int d\nu\, \sigma_{\ell u}(\nu) c \frac{u_\nu}{h\nu} \approx n_\ell u_\nu \frac{c}{h\nu} \int d\nu\, \sigma_{\ell u}(\nu)$$

$$B_{\ell u} = \frac{c}{h\nu} \int d\nu\, \sigma_{\ell u}(\nu)$$

$$\sigma_{\ell u}(\nu) = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{\ell u}^2} A_{u\ell} \phi_\nu \quad \text{with} \quad \int \phi_\nu d\nu = 1$$

Intrinsic Line Profile

- Determined by natural line broadening.
- The physics behind the natural line broadening is the Heisenberg uncertainty principle.



$$\Delta E_u \Delta t_u \geq h / 2\pi$$

Intrinsic Line Profile

- The intrinsic broadening produces a Lorentz profile (damped harmonical oscillator model)

$$\Phi_\nu = \frac{4\gamma_{u\ell}}{16\pi^2(\nu-\nu_{u\ell})^2+\gamma_{u\ell}^2}$$

$$\gamma_{u\ell} = \sum_{E_j<E_u} A_{uj} + \sum_{E_j<E_\ell} A_{\ell j}$$

$$(\Delta\nu)^{\mathrm{intr.}}_{\mathrm{FWHM}} = \frac{\gamma_{u\ell}}{2\pi}$$

$$(\Delta v)^{\mathrm{intr.}}_{\mathrm{FWHM}} = c\frac{(\Delta\nu)^{\mathrm{intr.}}_{\mathrm{FWHM}}}{\nu_{u\ell}} = \frac{\lambda_{u\ell}\gamma_{u\ell}}{2\pi} = 0.0121\frac{\mathrm{km}}{s}\left(\frac{\lambda_{u\ell}\gamma_{u\ell}}{7618\,\mathrm{cm\,s^{-1}}}\right)$$

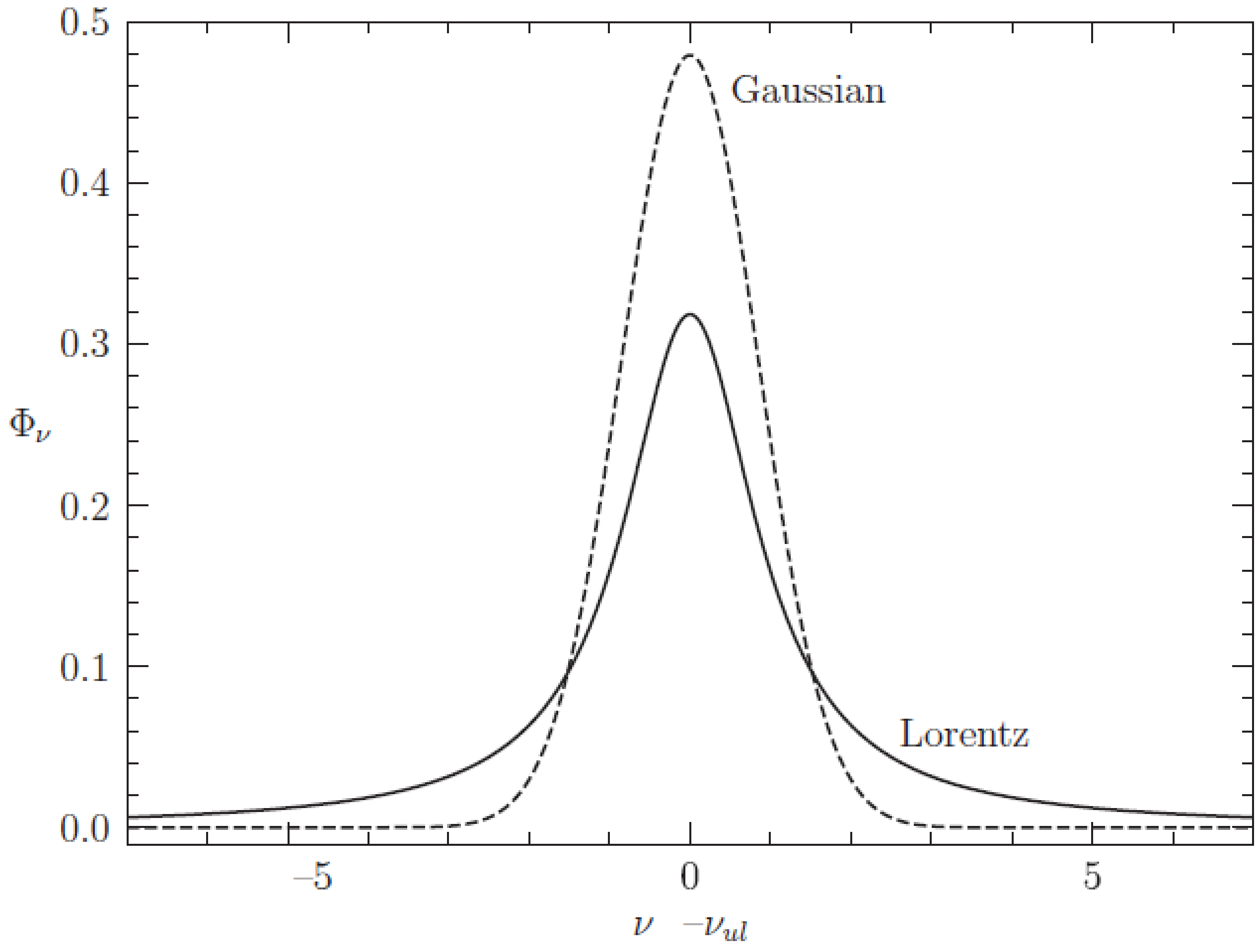
Doppler Broadening

- Atoms and ions are generally in kinematic equilibrium, where the velocity distribution is Maxwell.
- In one specific line-of-sight, the velocity distribution is simply a Gaussian.

$$p_v = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma_v}\,\mathrm{e}^{-(v-v_0)^2/2\sigma_v^2} = \frac{1}{\sqrt{\pi}}\frac{1}{b}\,\mathrm{e}^{-(v-v_0)^2/b^2}$$

- b is the thermal broadening parameter: $b = \left(\frac{2kT}{m}\right)^{1/2} = 1.28\,\mathrm{km\,s^{-1}}\left(\frac{T}{100\,\mathrm{K}}\right)^{1/2}\left(\frac{m}{m_\mathrm{H}}\right)^{-1/2}$

Lorentz vs. Gaussian profile



The Voigt profile

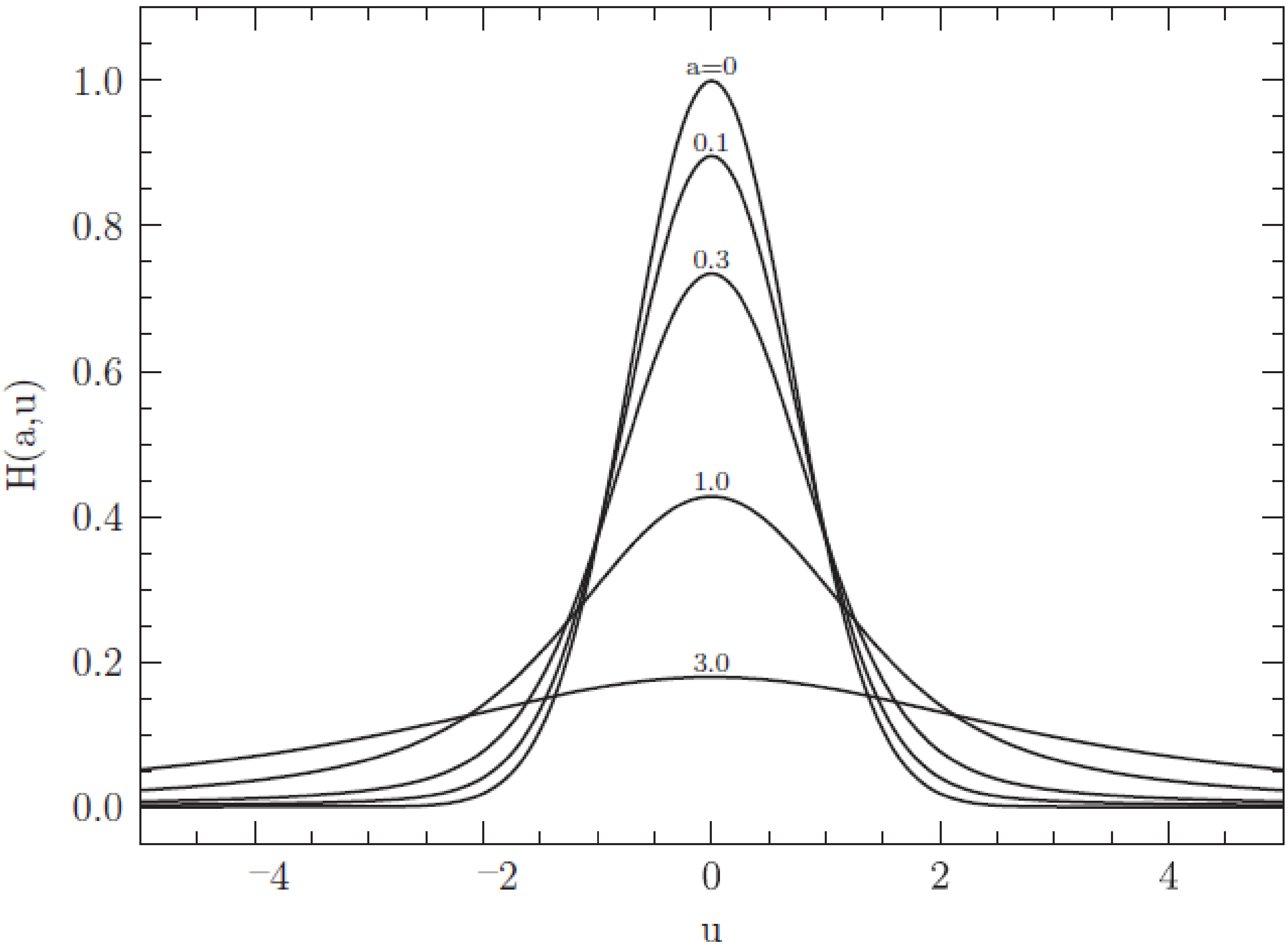
- The final profile is the convolution of the intrinsic profile with the thermal velocity distribution:

$$\phi_\nu = \int dv\, p_v(v) \frac{4\gamma_{u\ell}}{16\pi^2 \left[\nu - (1 - v/c)\nu_{u\ell}\right]^2 + \gamma_{u\ell}^2}$$

$$\Phi_\nu^{\mathrm{Voigt}} = \frac{1}{\sqrt{\pi}} \frac{1}{\nu_{u\ell}} \frac{c}{b} H(a,u), \qquad H(a,u) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-y^2} dy}{(u-y)^2 + a^2},$$

$$a \equiv \frac{\gamma_{u\ell} c}{4\pi\, \nu_{u\ell} b}$$

$$u \equiv \frac{c}{b}\left(1 - \frac{\nu}{\nu_{u\ell}}\right) = \frac{v}{b}.$$



a=0
0.1
0.3
1.0
3.0
H(a,u)
1.0
0.8
0.6
0.4
0.2
0.0
−4
−2
0
2
4
u

Absorption lines from radiative transfer

- Radiative transfer equation along the line-of-sight s: $$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$$
- Change variable from s to optical depth: $$d\tau_\nu \equiv \kappa_\nu ds. \qquad \frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$
- The cold neutral medium is the simplest case of the radiative transfer, because nearly all atoms are in their ground state in such low temperature and spontaneous emission can be ignored.

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu}$$

Building absorption lines

- Optical depth can be expressed in the form of profile

$$\tau_\nu = \int \kappa_\nu ds = \int n_\ell \sigma_{\ell u}(\nu) ds = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} \int n_\ell \Phi_\nu ds.$$

- If we assume the absorbers along the line-of-sight have the same temperature and profile:

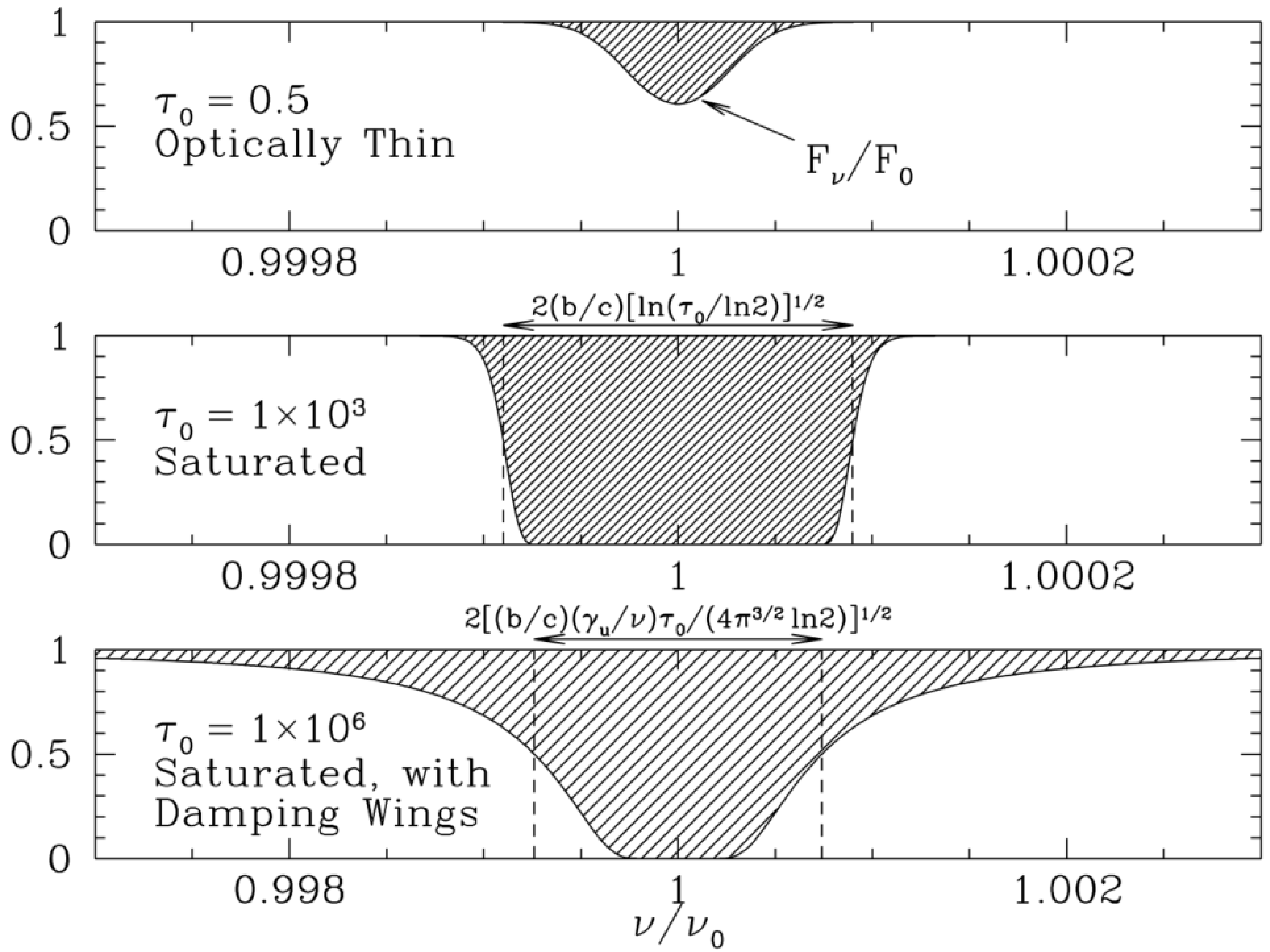
$$\tau_\nu = \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} N_\ell \Phi_\nu = \frac{1}{(4\pi)^{3/2}} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^3} A_{u\ell} \right] \frac{c}{b} N_\ell H(a,u).$$

EW in low-res spectral

- Quantify the strength of absorption: the concept of equivalent width.

$$W_\lambda = \int d\lambda [1 - e^{-\tau_\lambda}]$$

- Graphically speaking, the equivalent width is the width of a straight-sided, perfectly black absorption line that has the same integrated flux deficit as the actual absorption line.



Curve of Growth: 1. optically thin regime

- When tau<<1, $W_\lambda = \int d\lambda [1 - e^{-\tau_\lambda}] \approx \int d\lambda\, \tau_\lambda \approx \tau_0 \int d\lambda\, H(a,u)$

- Also, when tau<<1, the main contribution to the absorption comes from the Gaussian core because it is always that a<<1.

$$W_\lambda \approx \tau_0 \int d\lambda\, e^{-u^2} \qquad W_\lambda = \frac{b}{c}\lambda_0\tau_0 \int e^{-u^2} du = \frac{\sqrt{\pi}\, b}{c}\lambda_0\tau_0$$

- EW grows linearly with tau in the optically thin regime.

$$W_\lambda = \frac{1}{8\pi}\left[\frac{g_u}{g_\ell}\frac{c^2}{\nu_{u\ell}^3}A_{u\ell}\right]\lambda_0 N_\ell$$

Curve of Growth: 2. intermediate regime

$$W_\lambda = \int d\lambda \left[1 - \exp(-\tau_0 e^{-u^2})\right]$$

$$= \frac{b}{c}\lambda_0 \int du \left[1 - \exp(-\tau_0 e^{-u^2})\right]$$

- Consider the above equation as a step function of 1 and 0 with the boundary at $\tau_0 e^{-u^2} > 1$

$$W_\lambda \approx \frac{2b}{c}\lambda_0\sqrt{\ln \tau_0}$$

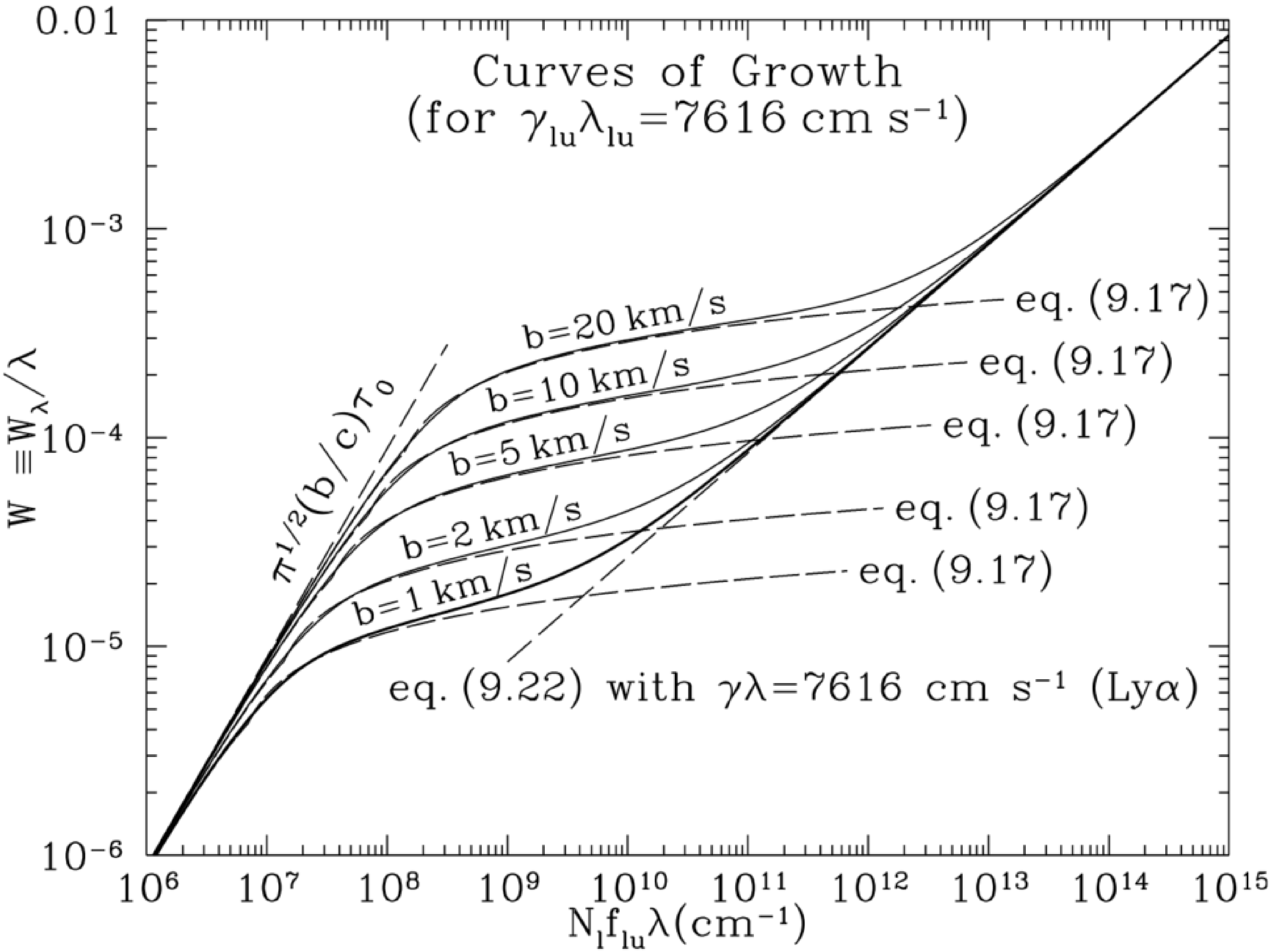
- EW grows very slowly with tau.

Curve of Growth: 3. damped regime

- When damping wing absorption contributes significantly, we are entering the new regime.

$$\tau_\lambda \approx \tau_0 \frac{a}{\sqrt{\pi} u^2}$$

$$\begin{aligned} W_\lambda &\approx \int d\lambda \left[1 - \exp(-\tau_\lambda)\right] \\ &\approx \frac{b}{c}\lambda_0 \int du \left[1 - \exp\left(-\tau_0 \frac{a}{\sqrt{\pi} u^2}\right)\right] \\ &\sim \frac{2b}{c}\lambda_0 \left(\frac{a\tau_0}{\sqrt{\pi}}\right)^{1/2} \end{aligned}$$



Curves of Growth
(for $\gamma_{lu}\lambda_{lu}$=7616 cm s^{-1})
$\pi^{1/2}(b/c)\tau_0$
b=20 km/s
b=10 km/s
b=5 km/s
b=2 km/s
b=1 km/s
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.17)
eq. (9.22) with $\gamma\lambda$=7616 cm s^{-1} (Lyα)
W $\equiv$ W$_\lambda$/λ
N$_l$f$_{lu}\lambda$(cm^{-1})
0.01
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^6
10^7
10^8
10^9
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}

5. Warm Neutral Medium

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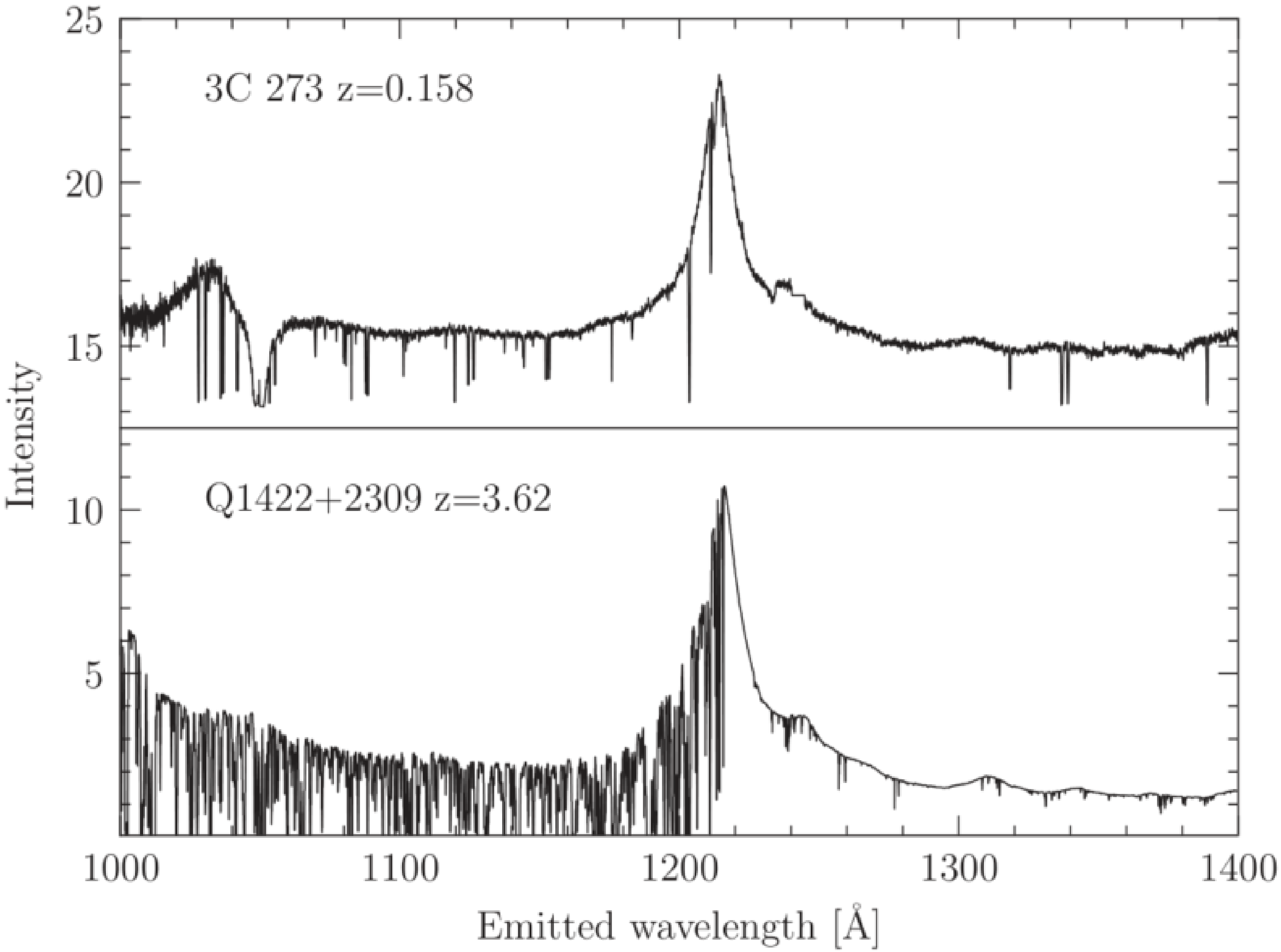
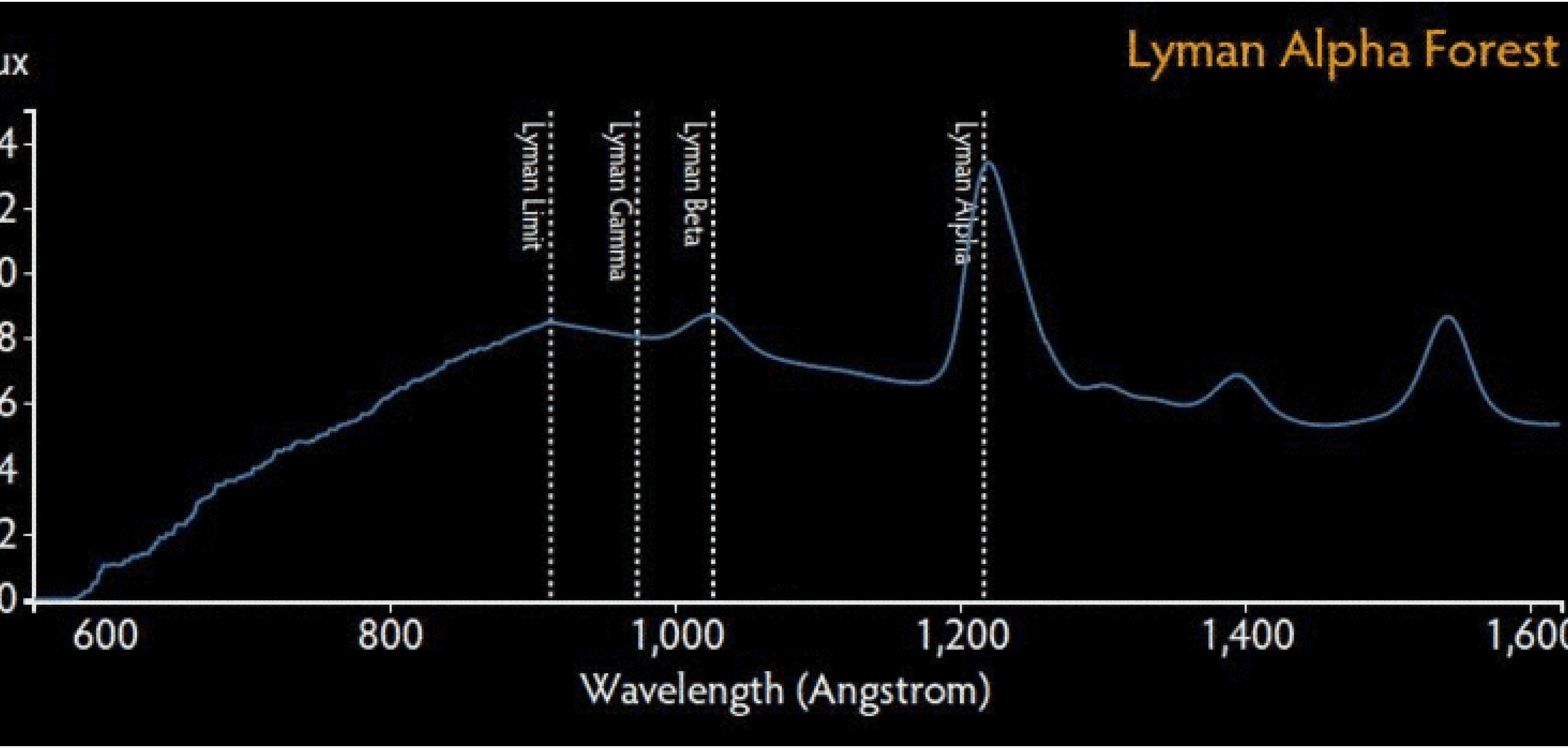
Lyman alpha absorption as a powerful tool to study the IGM



$$\tau_\nu = \int_0^s \kappa_\nu ds = \int_0^s n_\ell \sigma_{\ell u}(\nu) ds$$

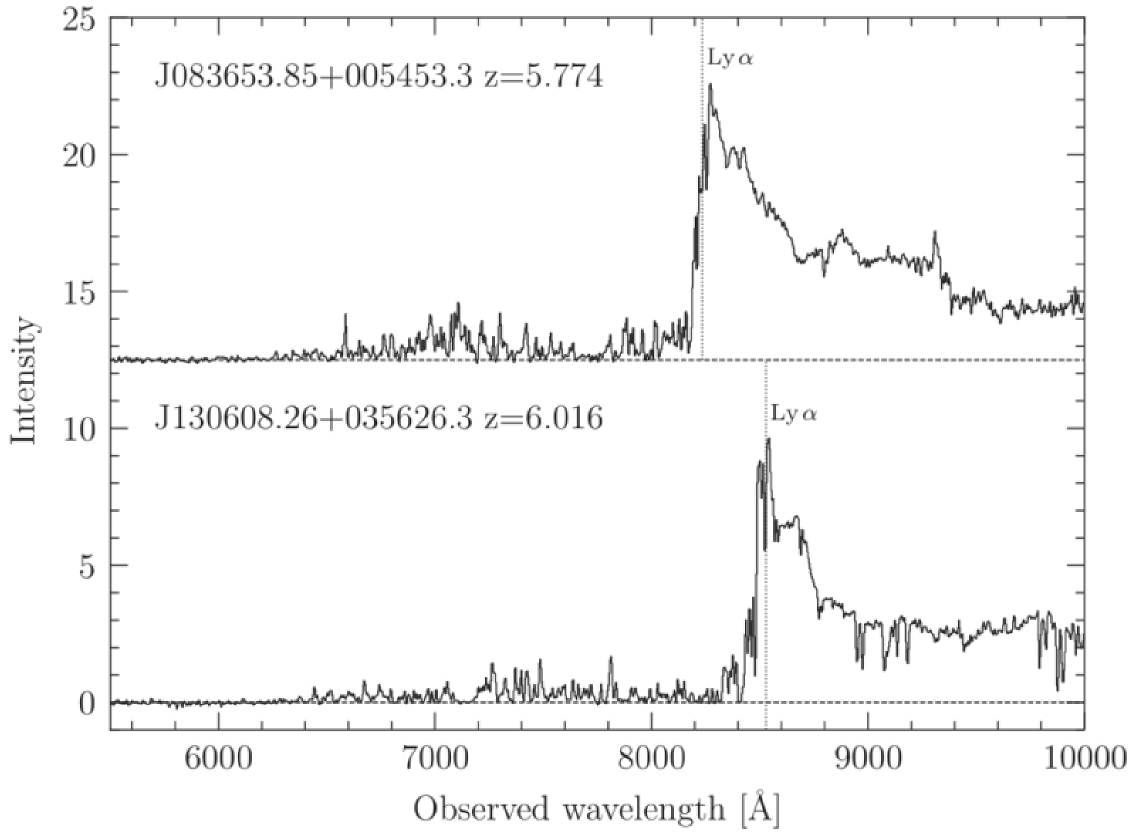
$$= \frac{c}{H_0}\sigma_\alpha n_{\ell,0} \approx 1.5 \times 10^4 \left(\frac{n_{\ell,0}}{\overline{n}_{\text{bary},0}} \right)$$

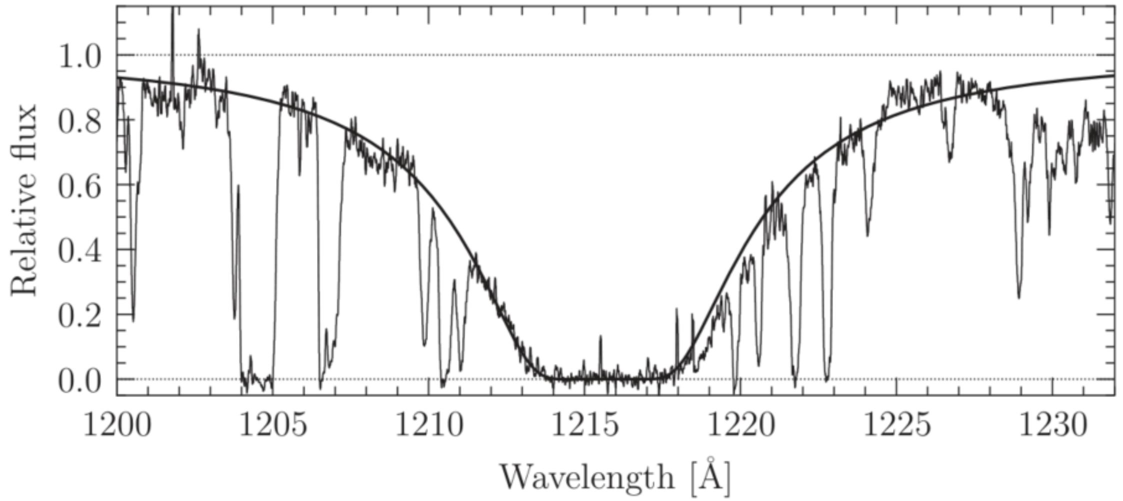
Gunn-Peterson effect: the absence of strong absorption against low-z quasars show that either the IGM has much lower density than the mean baryon density of the universe or the IGM is highly ionized!



Lyman Alpha Forest
Lyman Limit
Lyman Gamma
Lyman Beta
Lyman Alpha
600
800
1,000
1,200
1,400
Wavelength (Angstrom)

Lyman alpha absorption toward high-z quasars





- Gunn-Peterson trough is apparent.
- Damped Ly alpha systems are common.

Detecting WNM (neutral hydrogen)

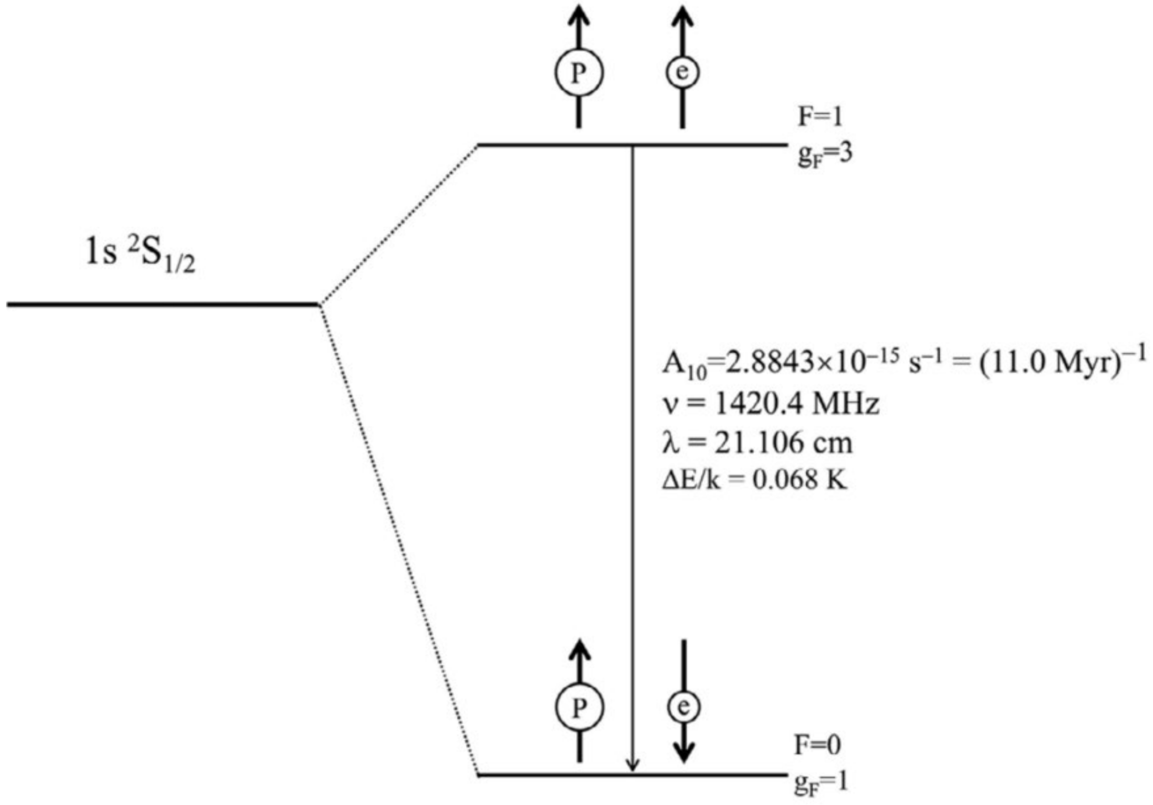
Lyman alpha absorption line

- Abundant.
- Very large Einstein A coefficient. High tau.
- The line has a wavelength of 121.6 nm, at which the Earth's atmosphere is highly opaque.
- The excitation temperature is 1.18e5 K, much higher than the kinetic temperature of the WNM. Absorption only.
- Need strong background UV sources to detect absorption along the line-of-sight. Cannot do a global survey of atomic gas.

HI 21 cm emission line

- Hyperfine structure line and therefore very tiny Einstein A coefficient. A mean lifetime of the upper-level atom is 11 Myr!!
- The line has a very long wavelength of 21 cm, can be observed in the radio band on Earth.
- Correspond to a very low excitation temperature 0.068 K, much lower than the temperature of any gas in the ISM.
- Emission lines from all line-of-sights!

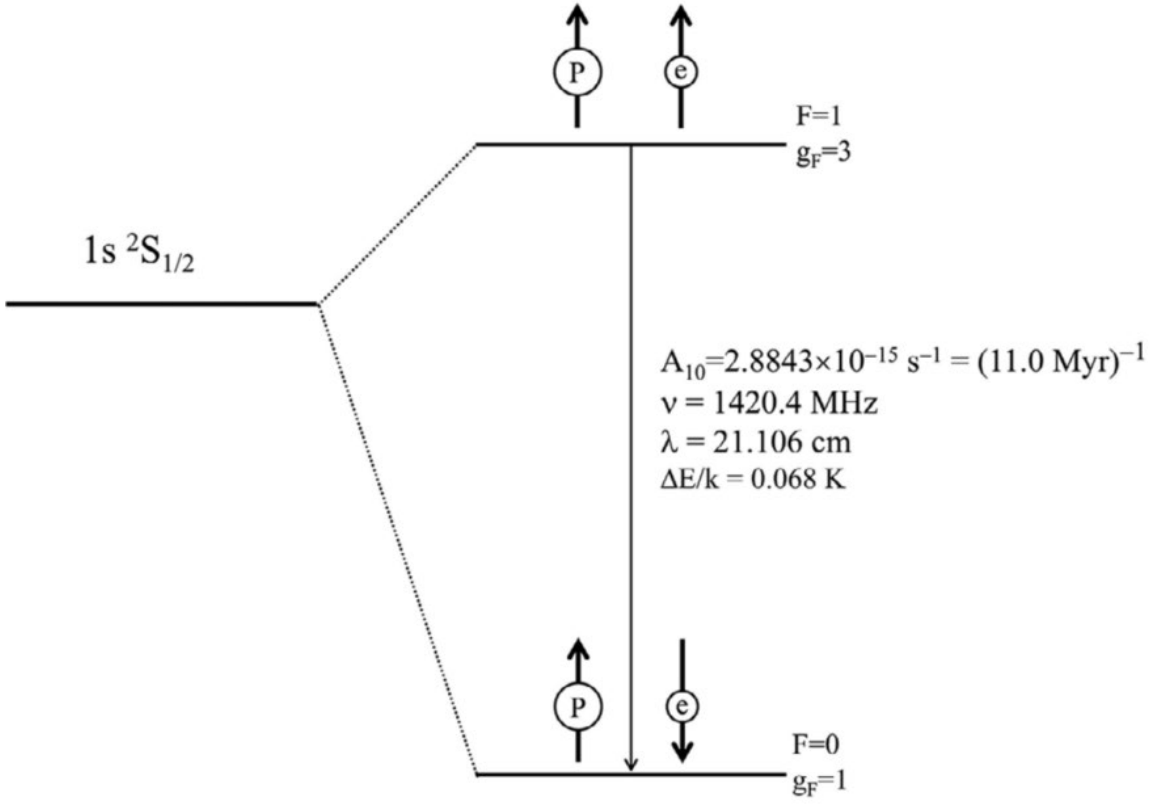
Energy populations of 21 cm hyperfine levels



$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT} = 3e^{-0.0682\,\mathrm{K}/T} \approx 3$$

- Excitation temperature (0.068 K) is much lower than the kinetic temperature of the gas.
- In excitation eq., the upper to lower level ratio is always ~3.

Transitions between quantized energy levels



$$\textbf{absorption}: \quad X_\ell + h\nu \to X_u\,, \quad h\nu = E_u - E_\ell\,.$$

$$\textbf{spontaneous emission}: \quad X_u \to X_\ell + h\nu \quad \nu = (E_u - E_\ell)/h$$

$$\textbf{stimulated emission}: \quad X_u + h\nu \to X_\ell + 2h\nu \quad \nu = (E_u - E_\ell)/h$$

Optical depth of 21cm emission.

$$\left(\frac{dn_u}{dt}\right)_{\mathrm{abs}} = n_\ell B_{\ell u} \varepsilon_\nu$$

$$\left(\frac{dn_u}{dt}\right)_{\mathrm{se}} = -n_u B_{u\ell} \varepsilon_\nu = -n_u \left(\frac{g_\ell}{g_u} B_{\ell u}\right) \varepsilon_\nu$$

$$= -\left(\frac{n_u}{n_\ell}\frac{g_\ell}{g_u}\right)\left(\frac{dn_u}{dt}\right)_{\mathrm{abs}} .$$

- For a system with both absorption and stimulated emission against incoming radiation, absorption coefficient in radiative transfer is:

$$\kappa_\nu = n_\ell \sigma_{\ell u}\left[1 - \frac{g_\ell}{g_u}\frac{n_u}{n_\ell}\right]$$

$$= n_\ell \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\mathrm{exc}}}\right)\right]$$

$$\tau_\nu = \sigma_{\ell u}\left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\mathrm{exc}}}\right)\right] N_\ell$$

Astrophysical Maser (Microwave Amplified by Stimulated Emission of Radiation)

$$\kappa_\nu = n_\ell \sigma_{\ell u} \left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\mathrm{exc}}} \right) \right]$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$$

$$I_\nu(\tau_\nu) = I_\nu(0)\mathrm{e}^{-\tau_\nu}.$$

- Population Inversion: more atoms or molecules are in an upper state than in the lower state due to external energy sources (radiation or shock) exciting the gas molecules.
- Negative excitation temperature!

$$d\tau_\nu \equiv \kappa_\nu ds$$

- Negative absorption coefficient and optical depth!
- Exponential growth of radiation intensity.

Optical depth of 21cm emission.

- In typical ISM condition, $kT_{\rm exc} \gg h\nu_{u\ell}$

$$\kappa_\nu = n_\ell \sigma_{\ell u} \left[1 - \exp\left(-\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right)\right] \approx n_\ell \sigma_{\ell u} \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right) \ll n_\ell \sigma_{\ell u}.$$

- For 21 cm lines, the attenuation coefficient and thus the optical depth is smaller by a factor of $h\nu_{u\ell}/kT_{\rm exc}$ than it would be in the absence of stimulated emission.
- The optical depth in the limit of doppler broadening (why?) is:

$$\tau_0 = \frac{1}{(4\pi)^{3/2}} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^2} A_{u\ell}\right] \frac{c}{b} N_\ell \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right) \approx 0.31 \left(\frac{N_{\rm HI}}{10^{21}\,{\rm cm}^{-2}}\right) \left(\frac{10\,{\rm km\,s}^{-1}}{b}\right) \left(\frac{100\,{\rm K}}{T_{\rm exc}}\right)$$

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu,$$

Radiative Transfer of Line Emission

- The emissivity and attenuation coefficients are related to the Einstein A coefficient.

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell} \Phi_\nu \qquad \kappa_\nu \approx n_\ell \sigma_{\ell u} \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right) \approx n_\ell \frac{g_u}{g_\ell} \frac{c^2}{8\pi \nu_{u\ell}^2} A_{u\ell} \Phi_\nu \left(\frac{h\nu_{u\ell}}{kT_{\rm exc}}\right)$$

- The source function can be simplified because $n_u/n_\ell \approx g_u/g_\ell$:

$$S_\nu \approx \frac{n_u}{n_\ell} \frac{g_\ell}{g_u} \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc} \approx \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc}.$$

- Radiative transfer equation:

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \frac{2\nu_{u\ell}^2}{c^2} kT_{\rm exc}.$$

Antenna temperature as a convention in radio astronomy

- In the Rayleigh-Jeans tail of the Planck function B(Tb), the intensity is $I_\nu = \frac{2\nu^2}{c^2} k T_b.$

- There exists a linear relationship between the radiation intensity and bright temperature of the blackbody radiation. The antenna temperature is thus defined here to simplify radiative transfer calculation:

$$T_\mathrm{A} \equiv \frac{c^2}{2k} \frac{I_\nu}{\nu^2}.$$

- The radiative transfer equation is reduced to temperature-related equation:

$$\frac{dT_\mathrm{A}}{d\tau_\nu} = -T_\mathrm{A} + T_\mathrm{exc}. \quad \Longrightarrow \quad T_\mathrm{A} = T_\mathrm{A}(0)\mathrm{e}^{-\tau_\nu} + T_\mathrm{exc}[1 - \mathrm{e}^{-\tau_\nu}].$$

21 cm HI line emissions toward a dark background ($T_A(0)=0$)

- In optically thin regime with tau<<1,

$$T_A = T_A(0)e^{-\tau_\nu} + T_{exc}[1 - e^{-\tau_\nu}] \approx \tau_\nu T_{exc}.$$

- Tau is inverse propotional to excitation temperature $\tau_0 = \frac{1}{(4\pi)^{3/2}} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^2} A_{u\ell} \right] \frac{c}{b} N_\ell \left(\frac{h\nu_{u\ell}}{kT_{exc}} \right)$

- Therefore, the antenna temperature is independent of the excitation temperature and is only the column density of HI along the line-of-sight!!!

$$\int T_A dv = T_{exc} \int \tau_\nu dv = T_{exc} W_v = \sqrt{\pi} T_{exc} b \tau_0$$

$$= \frac{c}{8\pi} \left[\frac{g_u}{g_\ell} \frac{c^2}{\nu_{u\ell}^2} A_{u\ell} \right] \frac{h\nu_{u\ell}}{k} N_\ell$$

Optically-thin 21cm line as direct tracer of HI column density

$$\int T_A dv = 550\,\mathrm{K\,km\,s^{-1}} \left(\frac{N_{\mathrm{HI}}}{10^{21}\,\mathrm{cm^{-2}}}\right)$$

- The integrated line intensity of the 21 cm line is linearly proportional to the HI column density. The normalization depends solely on quantum mechanical constants without any assumptions about the temperature/excitation states of the HI gas.
- This conclusion is also true for other radio lines, like molecular lines.
- The only assumption here is that the excitation temperature Texc >> 0.068 K!

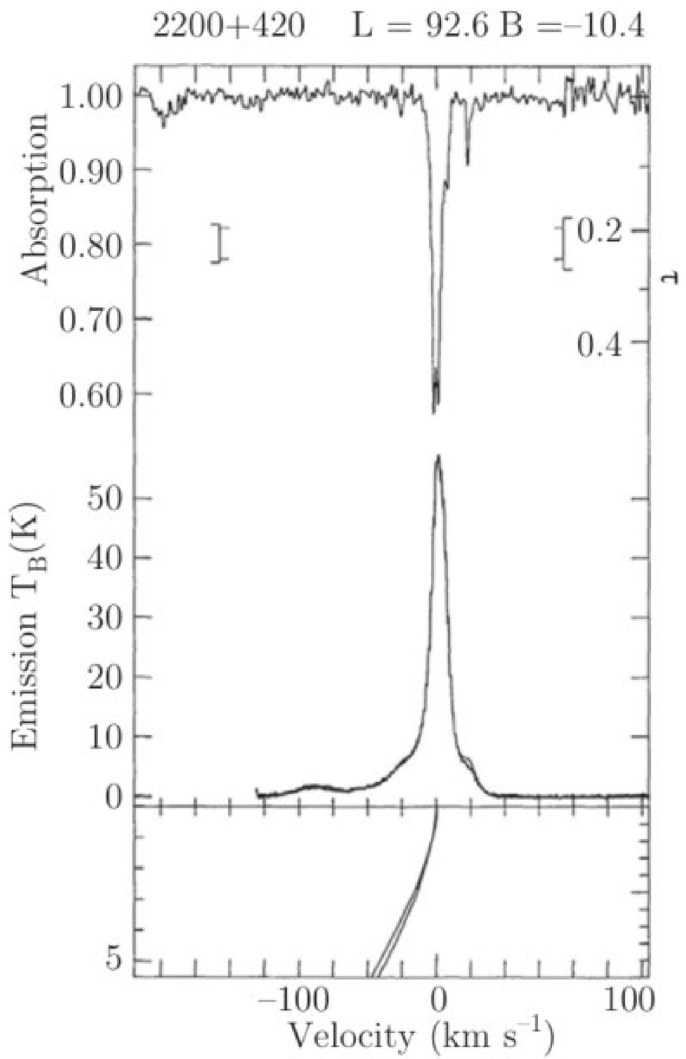
Is excitation temperature really much higher than 0.068K?

- HI absorption toward strong radio source BL Lac.

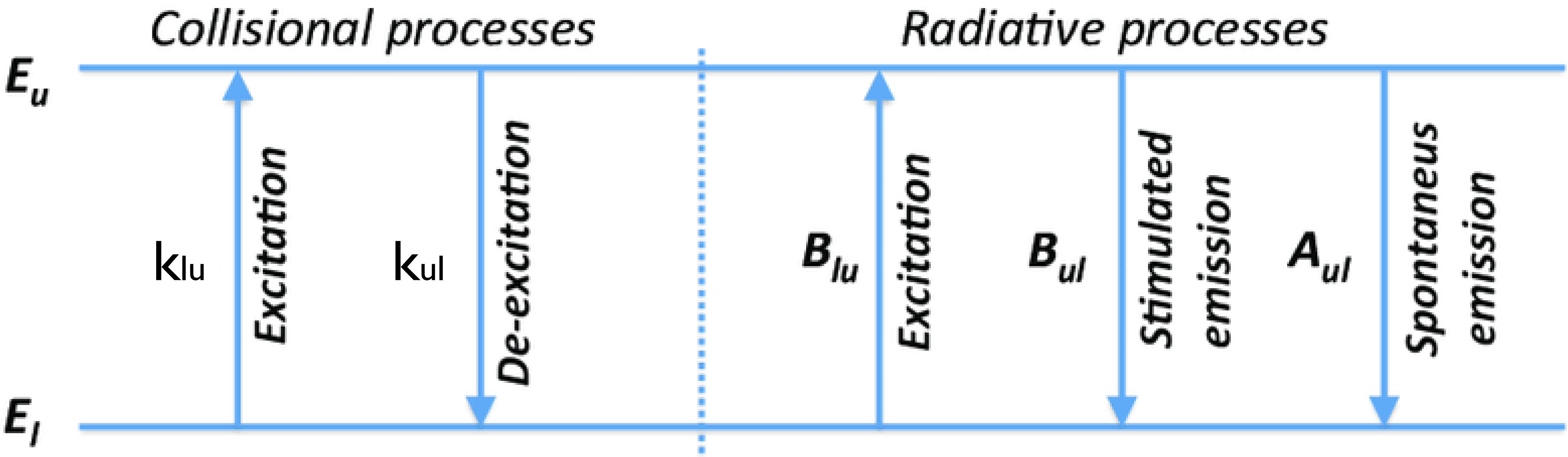
$$T_A(\nu) = T_C e^{-\tau_\nu}$$

- The EW of the absorption line is ~ 7 km/s.
- The HI emission line toward the line-of-sight that is very close to BL Lac provides the integrated line intensity of the emission.

$$T_{\rm exc} = \frac{\int T_A dv}{W_v} = \frac{930\,{\rm K\,km\,s^{-1}}}{7\,{\rm km\,s^{-1}}} \approx 130\,{\rm K}.$$



Radiative vs. Collisional excitation: is 21cm line radiatively excited?



k_{lu} and k_{ul} Rate coefficients

B_{lu}, B_{ul} et A_{ul} : Einstein coefficients

Radiative excitation recap

- The net change of upper level is:

$$\frac{dn_u}{dt} = n_\ell B_{\ell u}\varepsilon_\nu - n_u(A_{u\ell} + B_{u\ell}\varepsilon_\nu).$$

- With the relations among Einstein coefficients and the photon occupation number: $\bar{n}_\gamma \equiv \frac{c^3}{8\pi h\nu^3}\varepsilon_\nu$

$$\frac{dn_u}{dt} = n_\ell A_{u\ell}\frac{g_u}{g_\ell}\frac{c^3}{8\pi h\nu_{u\ell}^3}\varepsilon_\nu - n_u A_{u\ell}\left(1 + \frac{c^3}{8\pi h\nu_{u\ell}^3}\varepsilon_\nu\right) = n_\ell A_{u\ell}\frac{g_u}{g_\ell}\bar{n}_\gamma - n_u A_{u\ell}(1 + \bar{n}_\gamma).$$

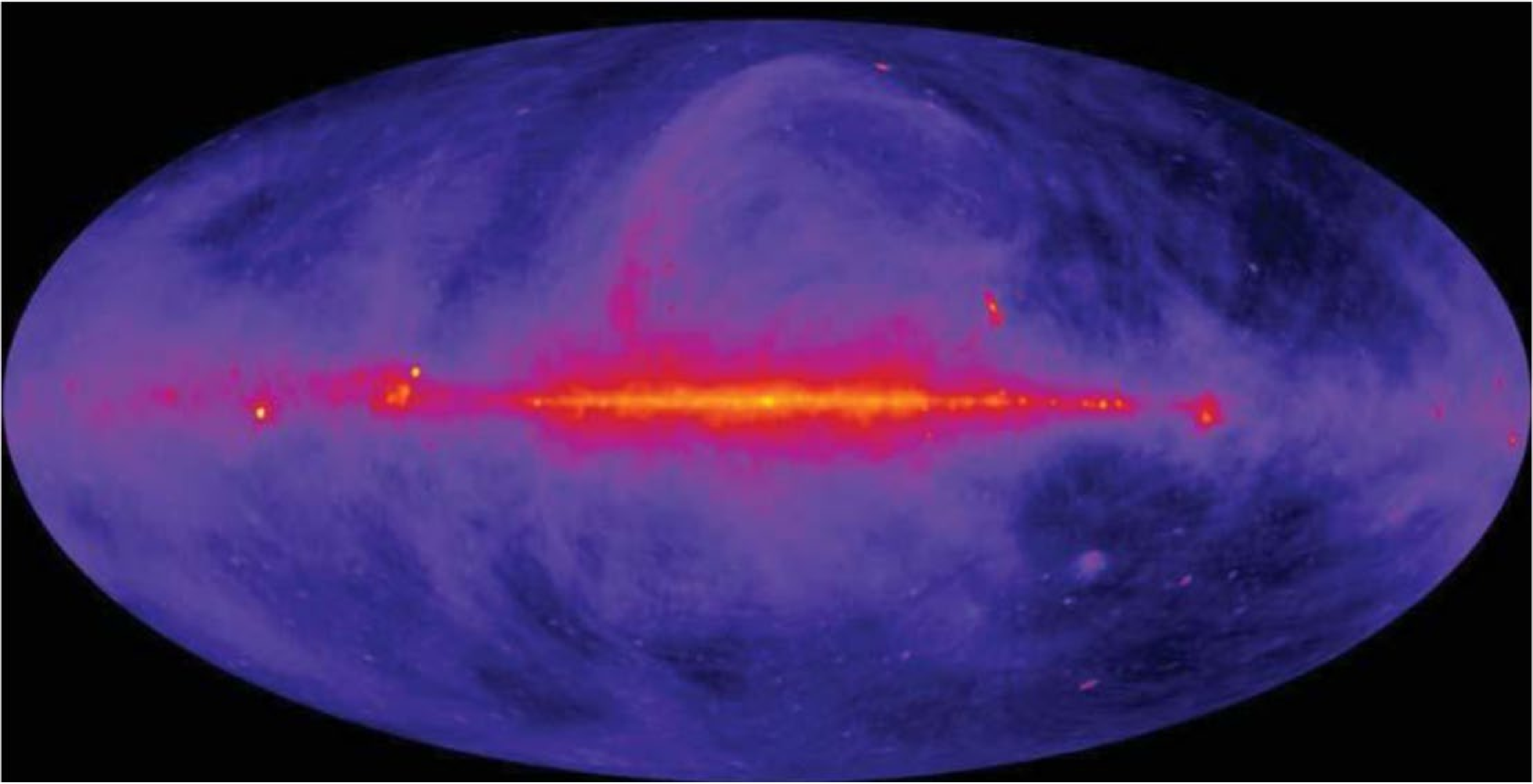
- In excitation eq, the relative population under blackbody radiation is:

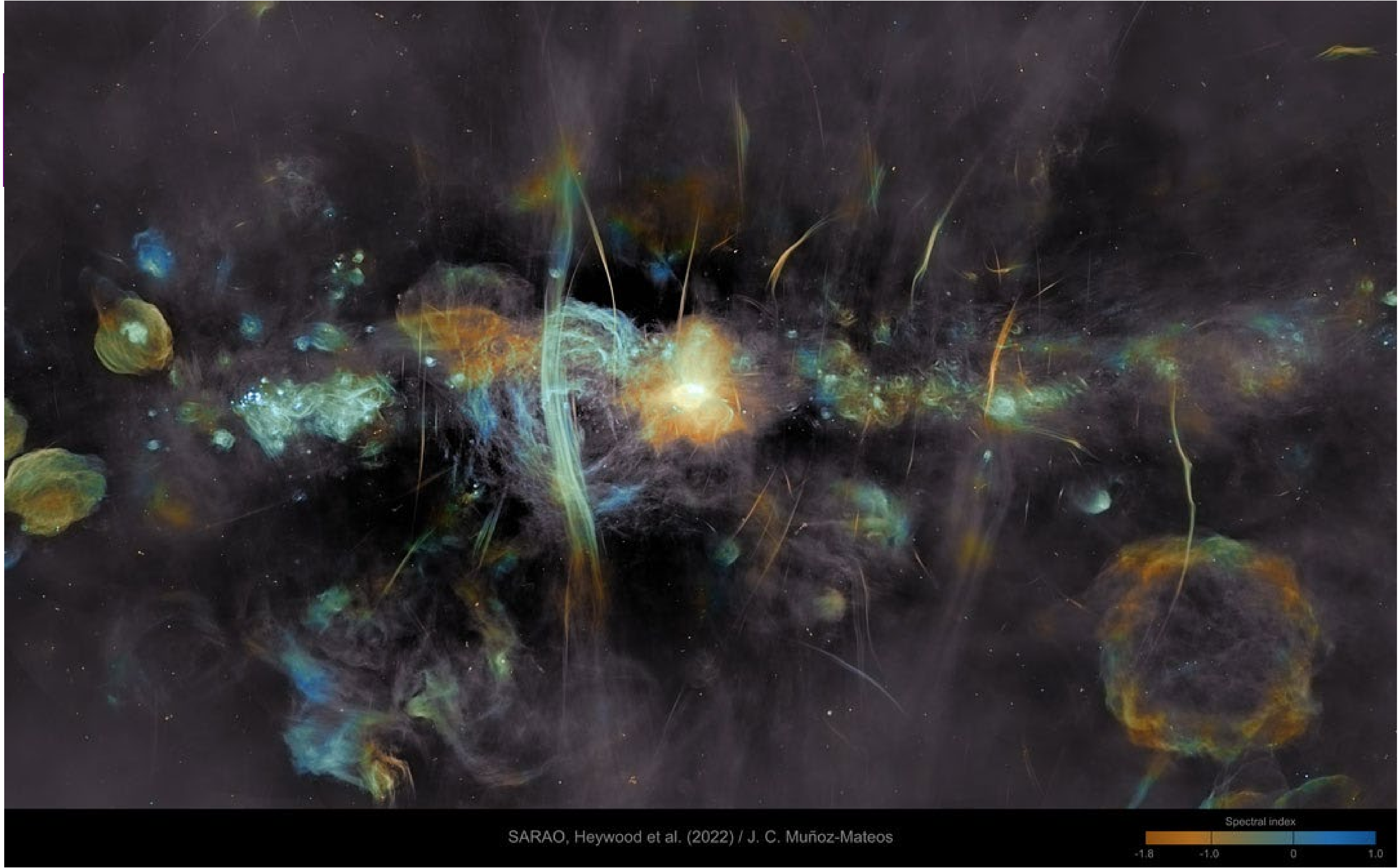
$$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell}\frac{\bar{n}_\gamma}{1 + \bar{n}_\gamma} = \frac{g_u}{g_\ell}\exp\left(-\frac{h\nu_{u\ell}}{kT_{\rm rad}}\right).$$

Boltzmann distribution with radiation temperature!

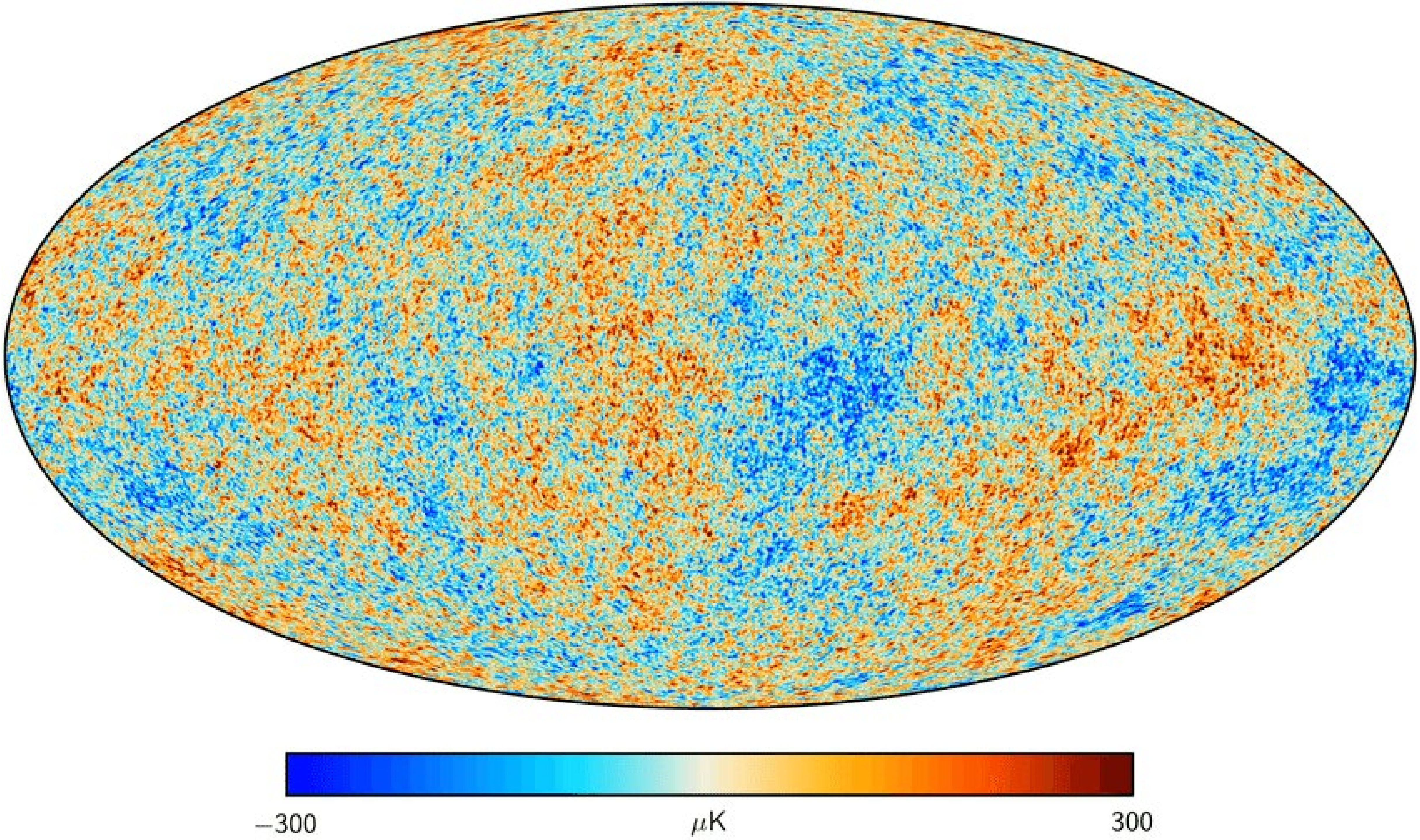
Astrophysical radiation fields in the solar neighborhood

- Galactic synchrotron radiation
- The CMB
- IR emission from thermal emission by dust grains heated by starlight
- Optical and UV emission from starlight
- X-ray emission from hot plasma and unresolved point sources
- Gamma-rays from cosmic rays





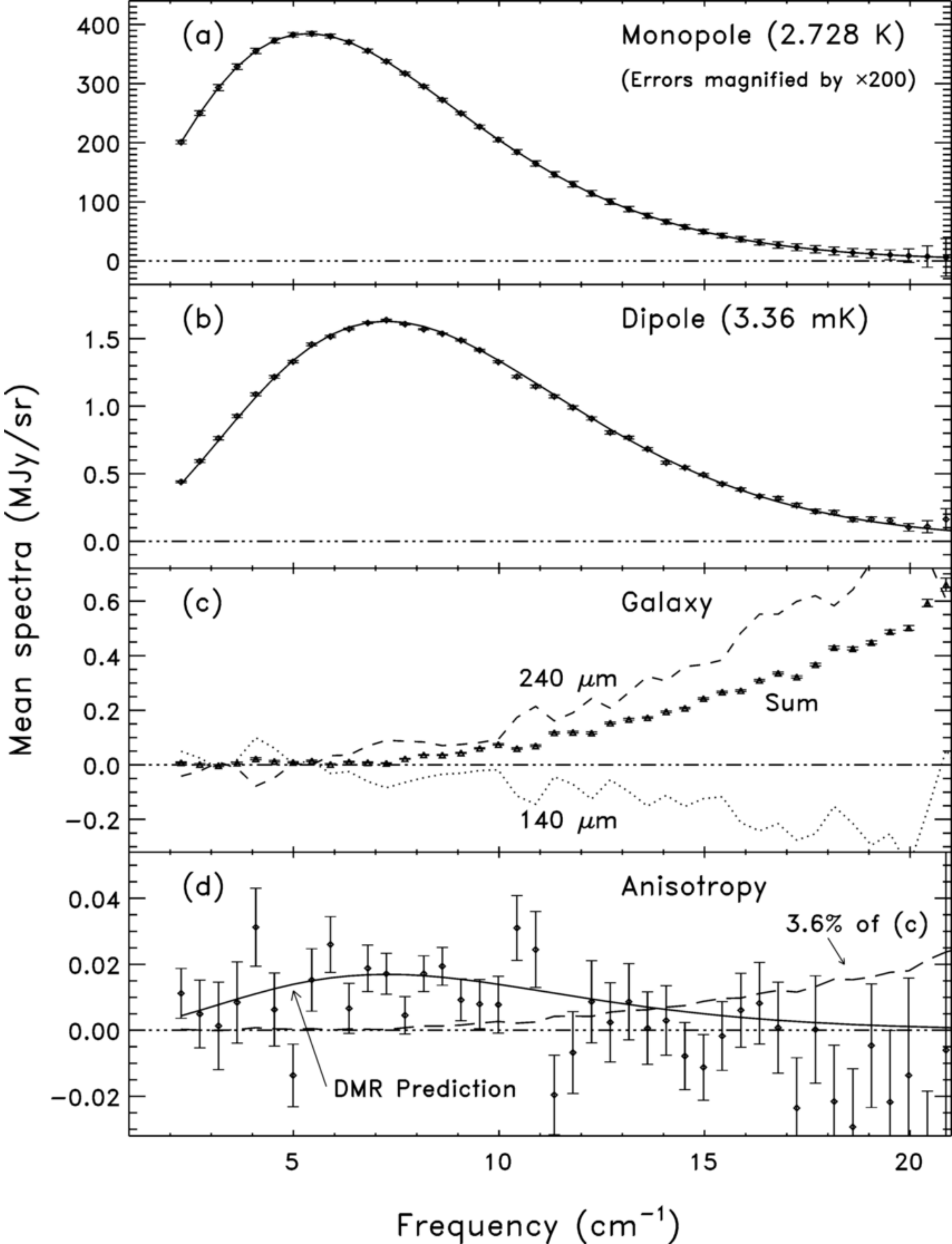
SARAO, Heywood et al. (2022) / J. C. Muñoz-Mateos

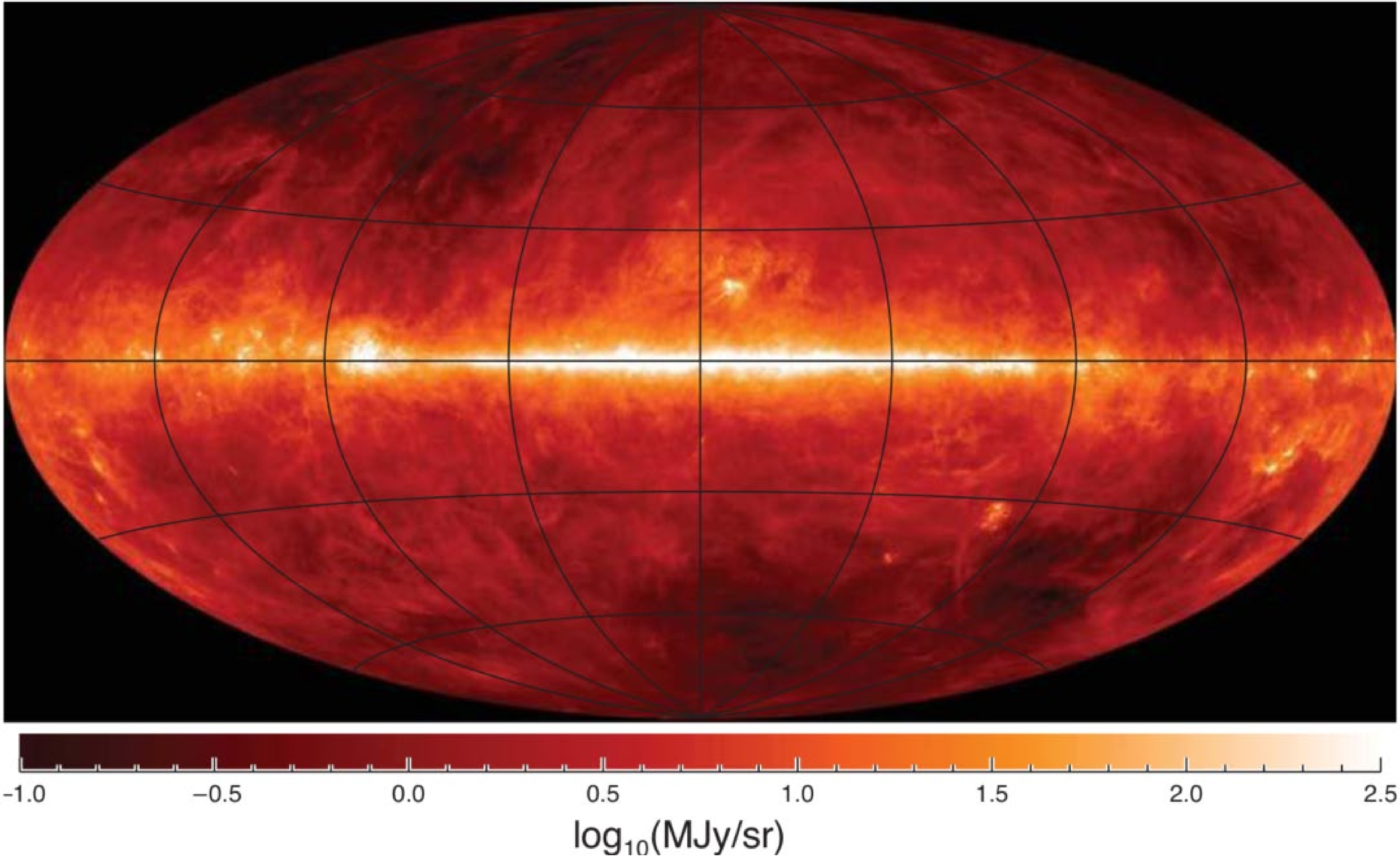


−300
μK
300

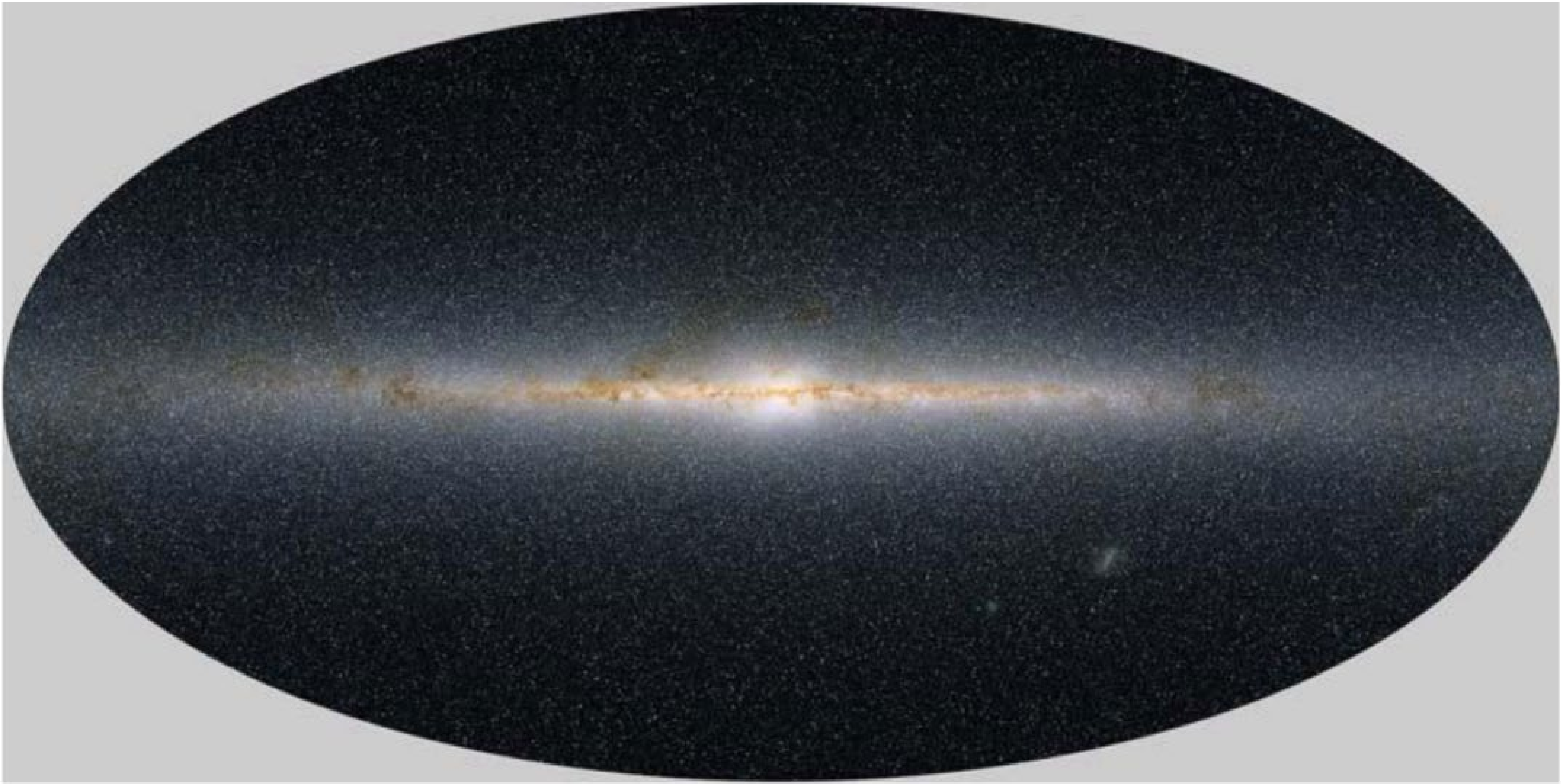
Cosmic Microwave Background

$$u_{CMB} = aT_{CMB}^4 \approx 4.2 \times 10^{-13} erg/cm^3$$





−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
log10(MJy/sr)



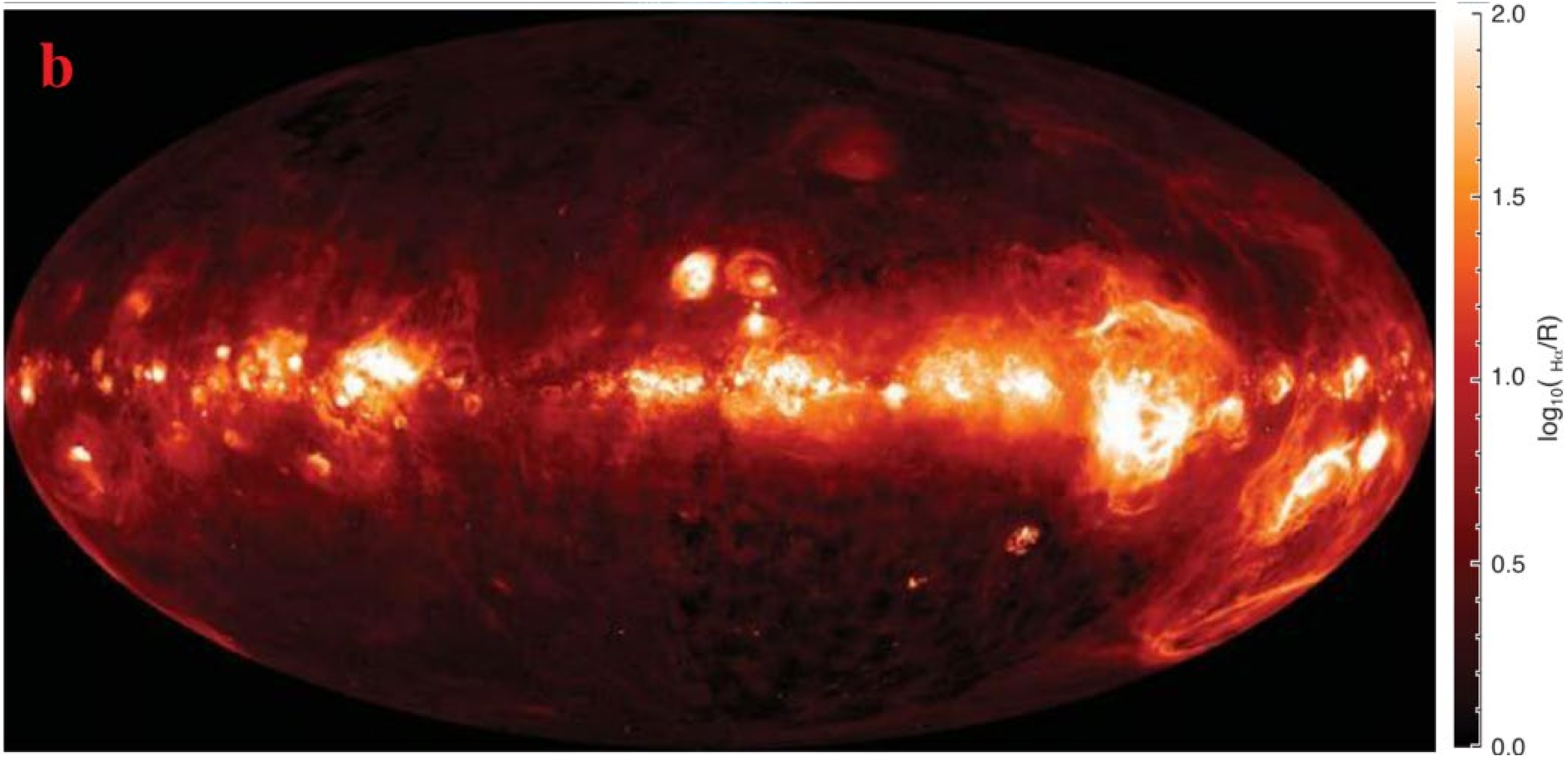
Starlight in 1-13.6 eV

- Can be approximated by a sum of three dilute blackbodies:

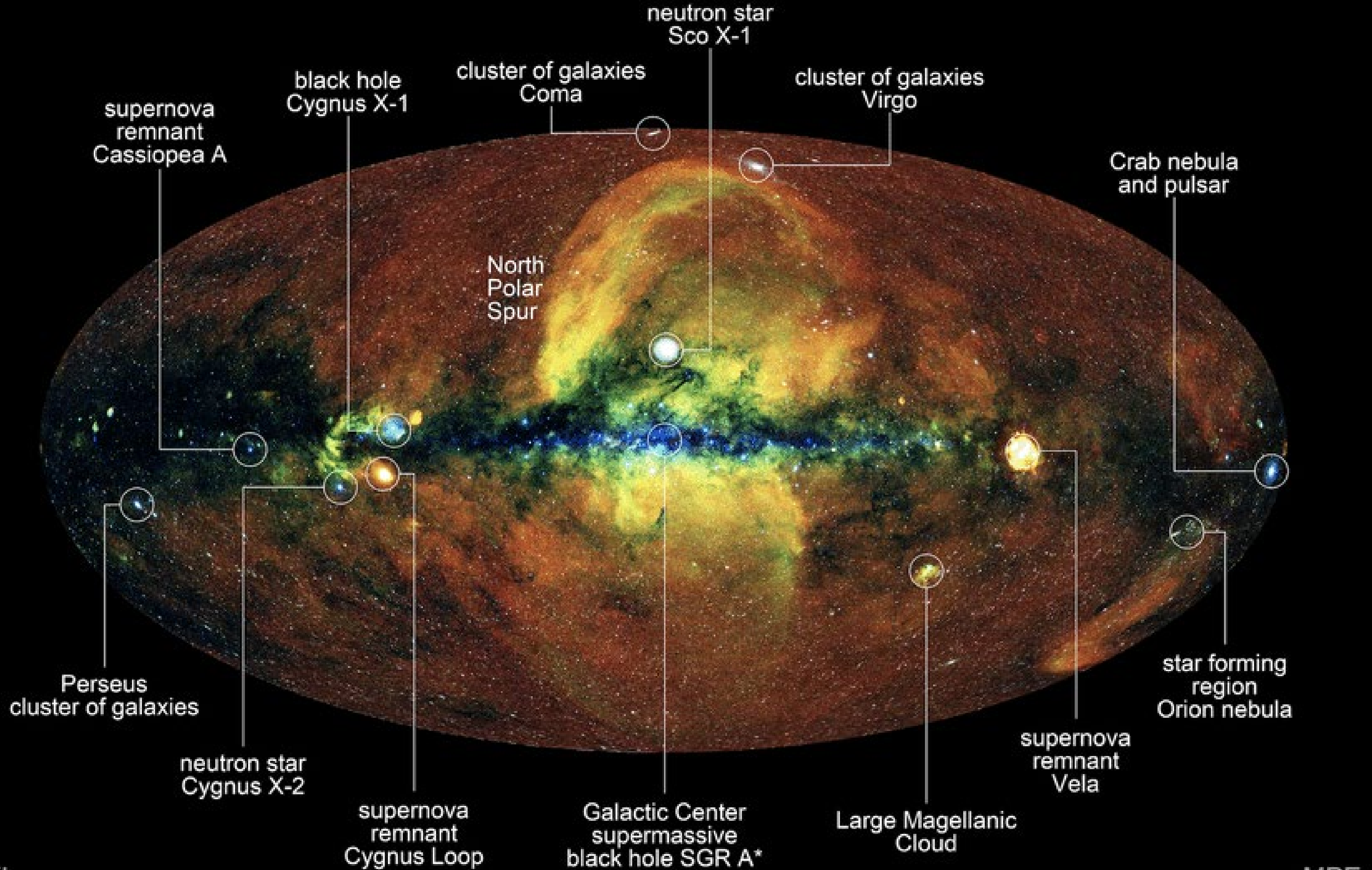
$$\nu u_\nu = \sum_{j=1}^{3} \frac{8\pi h \nu^4}{c^3} \frac{W_j}{e^{h\nu/kT_j} - 1} \qquad \text{for } \lambda > 2450\,\text{Å}$$

- Habing (1968) convention:

$$\chi \equiv \frac{(\nu u_\nu)_{1000\,\text{Å}}}{4 \times 10^{-14}\,\text{erg}\,\text{cm}^{-3}} \qquad\qquad G_0 \equiv \frac{u(6 - 13.6\,\text{eV})}{5.29 \times 10^{-14}\,\text{erg}\,\text{cm}^{-3}}$$

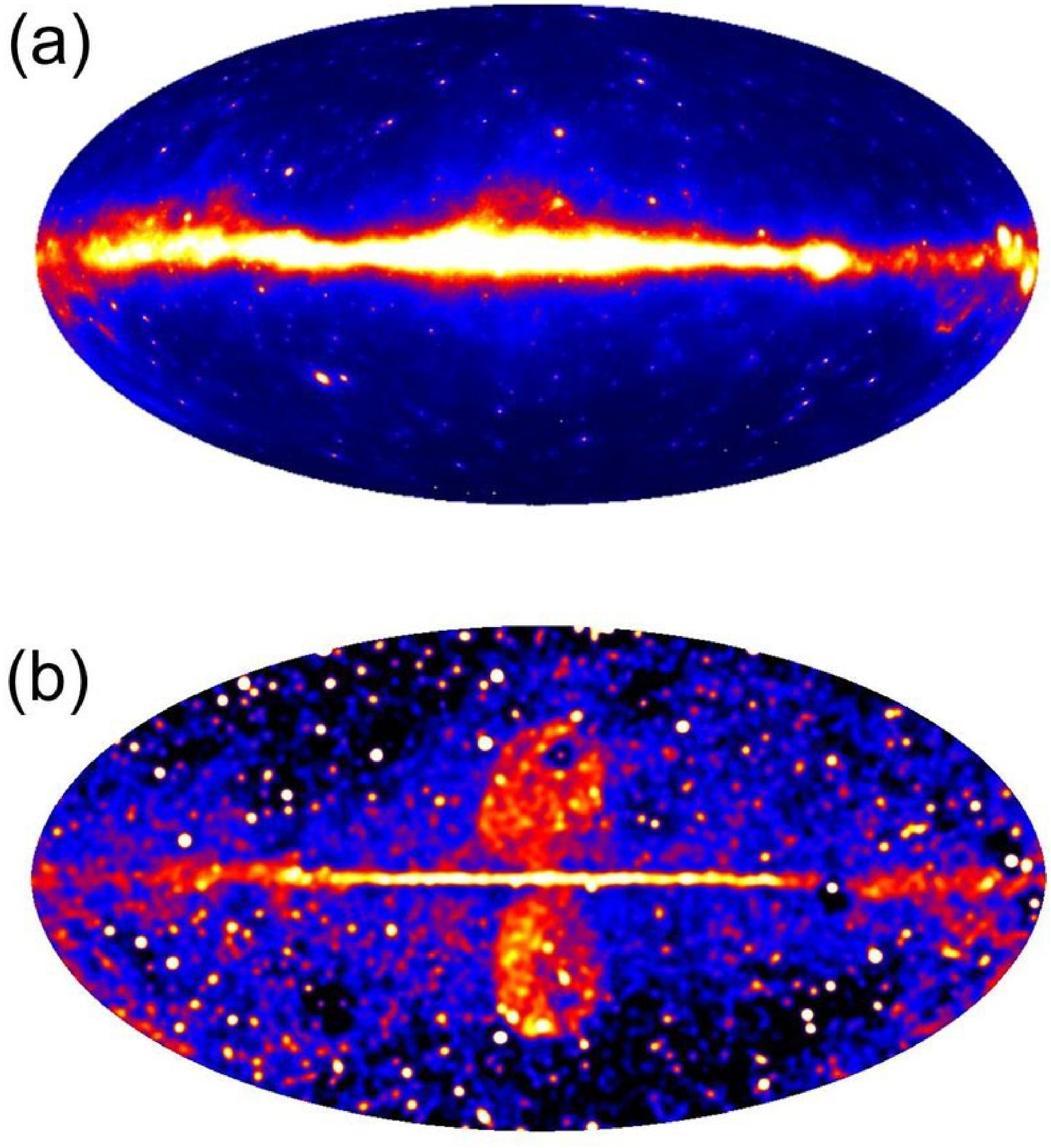

b
2.0
1.5
1.0
0.5
0.0
log10(I_Hα/R)

First SRG/eROSITA all-sky survey: A million of X-ray sources and the Milky Way.

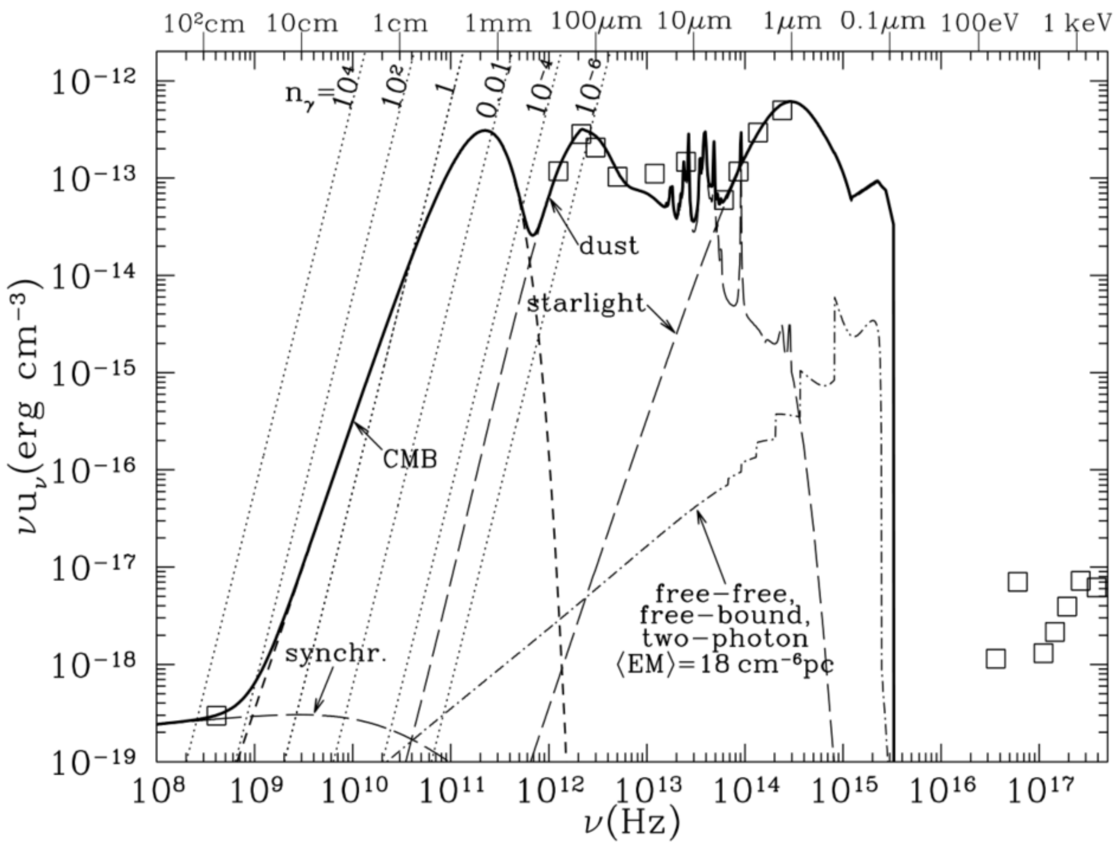


IKI

MPE


(a)
(b)

Interstellar continuum radiation field



Component		$u_{\rm rad}$ $(\mathrm{erg\,cm^{-3}})$
Radio synchrotron [Eq. (12.2)]		2.7×10^{-18}
CMB, $T = 2.725\,\mathrm{K}$		4.19×10^{-13}
Dust emission		5.0×10^{-13}
Free–free,free–bound,two-photon		4.5×10^{-15}
Starlight: $T_1 = 3000\,\mathrm{K}$, $W_1 = 7 \times 10^{-13}$	4.29×10^{-13}	
$T_2 = 4000\,\mathrm{K}$, $W_2 = 1.65 \times 10^{-13}$	3.19×10^{-13}	
$T_3 = 7500\,\mathrm{K}$, $W_3 = 1 \times 10^{-14}$	2.29×10^{-13}	
$\lambda < 2460$ Å UV (Eq. 12.7)	7.11×10^{-14}	
Starlight total		1.05×10^{-12}
Hα		8×10^{-16}
Other $\lambda \geq 3648$ Å H lines = $1.1 \times$ Hα:		9×10^{-16}
$0.1 - 2\,\mathrm{keV}$ x rays		1×10^{-17}
ISRF total		2.19×10^{-12}

Collisional (de-)excitation

- Inelastic collision between the target atoms and colliders!
- The rate equation is similar to the radiative excitation ones, except for the coefficients.

$$\frac{dn_u}{dt} = n_\ell n_c k_{\ell u} - n_u (n_c k_{u\ell} + A_{u\ell})$$

- "K"s are called collisional rate coefficients.

$$k_{u\ell}(T) = \langle \sigma_{u\ell} v \rangle = \left(\frac{8kT}{\pi m_r} \right)^{1/2} \int_0^\infty \sigma_{u\ell} \frac{E}{kT} \exp\left(-\frac{E}{kT} \right) \frac{dE}{kT}$$

- $\sigma_{u\ell}$ is independent of temperature for elastic collision between "billiard balls". Therefore, the rate coefficient has a temperature dependence of $k_{u\ell} \propto T^{1/2}$.
- However, when $\sigma_{u\ell} \propto E^n$, the rate coefficient $k_{u\ell} \propto T^{n+1/2}$.

Relationship between the two collisional rate coefficients

- The same as Einstein coefficients, the two collisional rate coefficients depends on each other.

$$\frac{n_u}{n_\ell} \approx \frac{g_u}{g_\ell} \exp\left(-\frac{h\nu_{u\ell}}{kT}\right) \qquad \frac{n_u}{n_\ell} = \frac{n_c k_{\ell u}}{n_c k_{u\ell} + A_{u\ell}} = \frac{g_u}{g_\ell} \frac{n_c k_{u\ell} e^{-h\nu_{u\ell}/kT}}{n_c k_{u\ell} + A_{u\ell}}$$

$$k_{\ell u} = \frac{g_u}{g_\ell} k_{u\ell} \exp\left(-\frac{h\nu_{u\ell}}{kT}\right)$$

- This can be understood by the fact that excitation from lower to upper levels requires a energy threshold of ~ hv. Gas temperature below this energy has a collisional excitation rate that is much smaller than the collisional de-excitation rate exponentially.

Combining radiative and collisional processes

$$\frac{dn_u}{dt} = n_\ell \left[A_{u\ell} \frac{g_u}{g_\ell} \bar{n}_\gamma + n_c k_{\ell u} \right] - n_u \left[A_{u\ell}(1 + \bar{n}_\gamma) + n_c k_{u\ell} \right].$$

- In the full eq state, the population ratio is determined by both coefficients:

$$\frac{n_u}{n_\ell} = \frac{A_{u\ell}(g_u/g_\ell)\bar{n}_\gamma + n_c k_{\ell u}}{A_{u\ell}(1 + \bar{n}_\gamma) + n_c k_{u\ell}}.$$

- 3 limiting cases:
  - 1) photon occupation number -> 0: pure collisional eq.
  - 2) collider density -> 0: pure radiation eq.
  - 3) blackbody radiation with temperature Trad= Tgas: independent of gas density! Photons alone are sufficient to bring the two-level system into LTE.

Critical density

- The density at which collisional deexcitation equals radiative deexcitation.

$$n_{\rm crit} = \frac{(1+\bar{n}_\gamma)A_{u\ell}}{k_{u\ell}}$$

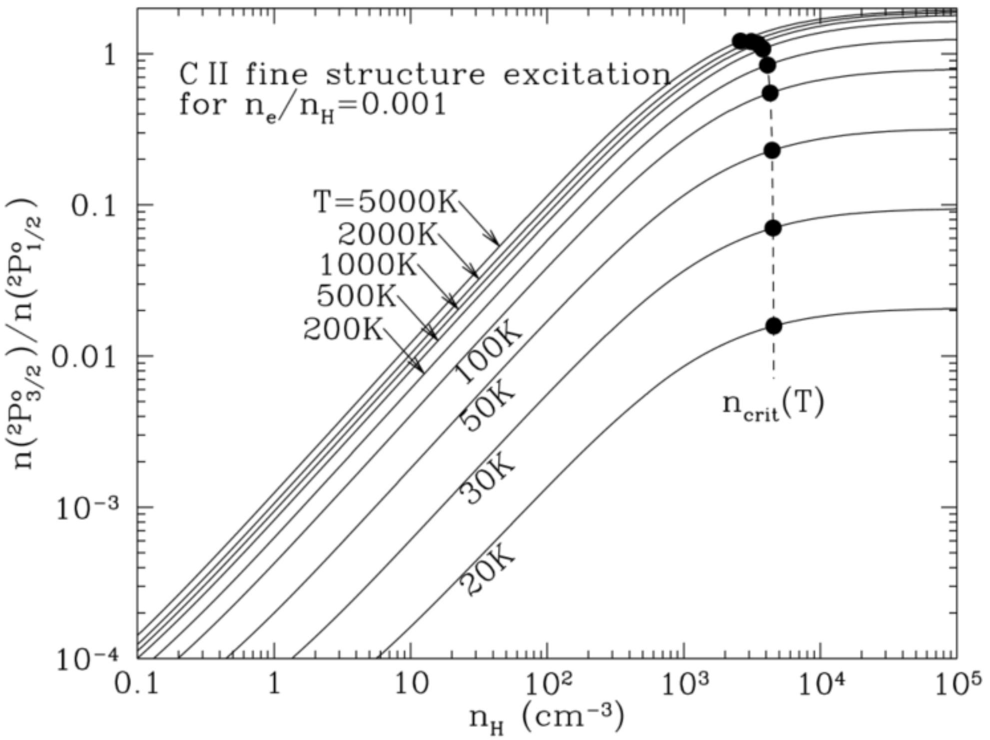
- For 21 cm levels, $k_{u\ell}(\mathrm{HH}) \approx 1.0 \times 10^{-10}\,\mathrm{cm^3\,s^{-1}} \left(\frac{T}{80\,\mathrm{K}}\right)^{0.78}$

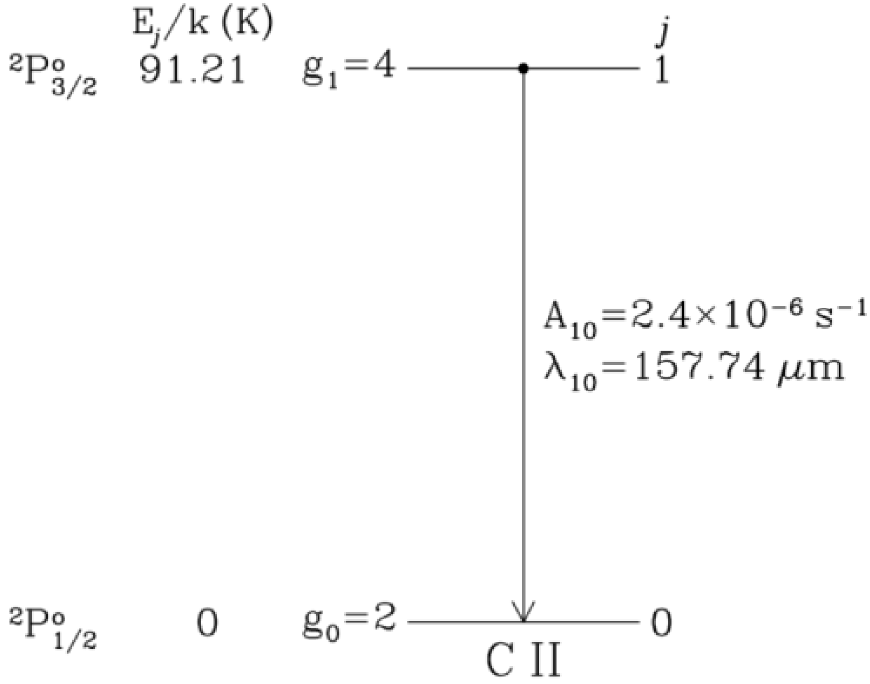
$$n_{\rm crit} = \frac{52.1(2.88 \times 10^{-15}\,\mathrm{s^{-1}})}{1.0 \times 10^{-10}\,\mathrm{cm^3\,s^{-1}}} \approx 0.0015\,\mathrm{cm^{-3}}$$

[CII] 158 micro line as the main coolant of the ISM

- At 158 micro, the photon occupation number << 1 and therefore the stimulated emission can be safely ignored.

$$n_{\rm crit}({\rm H}) \approx 3.2 \times 10^3 T_2^{-0.1281-0.0087\ln T_2}\ {\rm cm}^{-3}$$





$$n_{\rm crit} = \frac{(1+\bar{n}_\gamma)A_{u\ell}}{k_{u\ell}}$$

Critical density of in HI regions and molecular gas

Ion	ℓ	u	E_ℓ/k (K)	E_u/k (K)	$\lambda_{u\ell}$ (μm)	$n_{\mathrm{H,crit}}(u)$ $T = 100\,\mathrm{K}$ (cm^{-3})	$n_{\mathrm{H,crit}}(u)$ $T = 5000\,\mathrm{K}$ (cm^{-3})
C II	$^2\mathrm{P}^\mathrm{o}_{1/2}$	$^2\mathrm{P}^\mathrm{o}_{3/2}$	0	91.21	157.74	2.0×10^3	1.5×10^3
C I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	23.60	609.7	620	160
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	23.60	62.44	370.37	720	150
O I	$^3\mathrm{P}_2$	$^3\mathrm{P}_1$	0	227.71	63.185	2.5×10^5	4.9×10^4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_0$	227.71	326.57	145.53	2.3×10^4	8.4×10^3
Si II	$^2\mathrm{P}^\mathrm{o}_{1/2}$	$^2\mathrm{P}^\mathrm{o}_{3/2}$	0	413.28	34.814	1.0×10^5	1.1×10^4
Si I	$^3\mathrm{P}_0$	$^3\mathrm{P}_1$	0	110.95	129.68	4.8×10^4	2.7×10^4
	$^3\mathrm{P}_1$	$^3\mathrm{P}_2$	110.95	321.07	68.473	9.9×10^4	3.5×10^4

Molecule	Transition	E_u [K]	$n_{\mathrm{crit.}}$ [cm^{-3}]
CO	$(1 \rightarrow 0)$	5.5	$3 \cdot 10^3$
	$(2 \rightarrow 1)$	16.6	$1 \cdot 10^4$
CS	$(1 \rightarrow 0)$	2.4	$1 \cdot 10^5$
	$(2 \rightarrow 1)$	7.1	$7 \cdot 10^5$
HCO^+	$(1 \rightarrow 0)$	4.3	$1.5 \cdot 10^5$
	$(3 \rightarrow 2)$	25.7	$3 \cdot 10^6$
HCN	$(1 \rightarrow 0)$	4.3	$4 \cdot 10^6$
	$(3 \rightarrow 2)$	25.7	$1 \cdot 10^7$
HNC	$(1 \rightarrow 0)$	4.3	$4 \cdot 10^6$
	$(3 \rightarrow 2)$	26.1	$1 \cdot 10^7$