Magnetohydrodynamics Homework 5

(1) 由题意得, $\begin{cases} p_1 = p_1(z)e^{i(ky-\omega t)} \\ \rho_1 = \rho_1(z)e^{i(ky-\omega t)} \\ \vec{B}_1 = \begin{bmatrix} B_{1x}(z) \\ B_{1y}(z) \\ B_{1z}(z) \end{bmatrix} e^{i(ky-\omega t)} = \begin{bmatrix} B_{1x} \\ B_{1y} \\ B_{1z} \end{bmatrix} \end{cases}$ 和 $\vec{B}_0 = B_0(z)\hat{x}$, $\vec{v}_1 = \begin{bmatrix} v_{1x}(z) \\ v_{1y}(z) \\ v_{1z}(z) \end{bmatrix} e^{i(ky-\omega t)} = \begin{bmatrix} v_{1x} \\ v_{1y} \\ v_{1z} \end{bmatrix}$

$$\Rightarrow \nabla\times\vec{B}_1 = \left[\frac{\partial}{\partial y}\left(B_{1z}(z)e^{i(ky-\omega t)}\right) - \frac{\partial}{\partial z}\left(B_{1y}(z)e^{i(ky-\omega t)}\right)\right]\hat{x} + \frac{dB_{1x}(z)}{dz}e^{i(ky-\omega t)}\hat{y} - \frac{\partial}{\partial y}\left(B_{1x}(z)e^{i(ky-\omega t)}\right)\hat{z}$$

$$(\nabla\times\vec{B}_1)\times\vec{B}_0 = -\frac{dB_{1x}(z)}{dz}e^{i(ky-\omega t)}B_0(z)\hat{z} - ikB_{1x}(z)e^{i(ky-\omega t)}B_0(z)\hat{y}$$

$$\nabla\times\vec{B}_0 = \frac{dB_0(z)}{dz}\hat{y}$$

$$(\nabla\times\vec{B}_0)\times\vec{B}_1 = -\frac{dB_0(z)}{dz}B_{1x}(z)e^{i(ky-\omega t)}\hat{z} + \frac{dB_0(z)}{dz}B_{1z}(z)e^{i(ky-\omega t)}\hat{x}$$

$$\nabla p_1 = ikp_1(z)e^{i(ky-\omega t)}\hat{y} + \frac{dp_1(z)}{dz}e^{i(ky-\omega t)}\hat{z} = ikp_1\hat{y} + \frac{dp_1}{dz}\hat{z}$$

对 $\hat{x}$ 方向:

$$\rho_0\frac{\partial}{\partial t}\left[v_{1x}(z)e^{i(ky-\omega t)}\right] = \frac{1}{\mu_0}\frac{dB_0(z)}{dz}B_{1z}$$

$$\Rightarrow -i\omega\rho_0 v_{1x} = \frac{B_{1z}}{\mu_0}\frac{dB_0(z)}{dz}$$

对 $\hat{y}$ 方向:

$$-i\omega\rho_0 v_{1y} = -ikp_1 - \frac{ik}{\mu_0}B_{1x}B_0(z) = -ik\left(p_1 + \frac{B_{1x}B_0(z)}{\mu_0}\right)$$

对 $\hat{z}$ 方向:

$$-i\omega\rho_0 v_{1z} = -\frac{dp_1}{dz} - \frac{1}{\mu_0}\frac{dB_{1x}}{dz}B_0(z) - \frac{1}{\mu_0}\frac{dB_0(z)}{dz}B_{1x} - \rho_1 g$$

$$= -\frac{d}{dz}\left(p_1 + \frac{B_0(z)B_{1x}}{\mu_0}\right) - \rho_1 g$$

(2) $k\to+\infty$时，注意到y方向的速度扰动方程化为：

$$-i\omega\rho_0 v_{1y}(z) = -ik\left[p_1(z)+\frac{B_0(z)B_{1x}(z)}{\mu_0}\right]$$

$ikv_{1y}\sim\frac{dB}{dz}$

因左侧有限，故必有：

$$p_1(z)+\frac{B_0(z)B_{1x}(z)}{\mu_0}=0 \Rightarrow p_1+\frac{B_0(z)B_{1x}}{\mu_0}=0$$

代入z方向速度扰动方程得：

$$-i\omega\rho_0 v_{1z} = -\rho_1 g$$