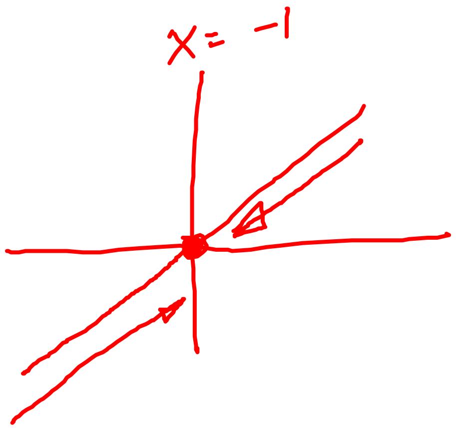
$Octobe r$ 8, 2021

CAL01: CALCULUS 1

SYNCHRONOUS

Continuity of a Function

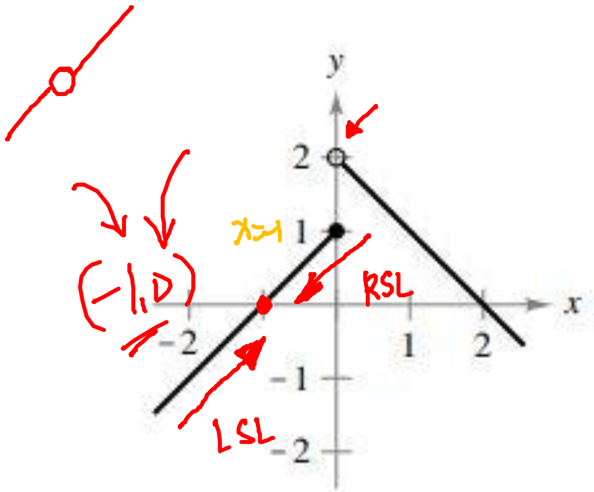


Definition A function f is **continuous at a number** c iff all conditions are satisfied:

$c \in D$

1. $f(c)$ is defined (that is, c is in the domain of f)
2. $\lim\limits_{x \to c} f(x)$ exists   RSL = LSL
3. ✓ $\lim\limits_{x \to c} f(x) = f(c)$

Example No. 1. Determine whether or not f(x) is continuous at x = -1?



✓ i $f(c)$ = defined: ✓

✓ ii $\lim\limits_{x \to c} f(x)$ = exist ✓

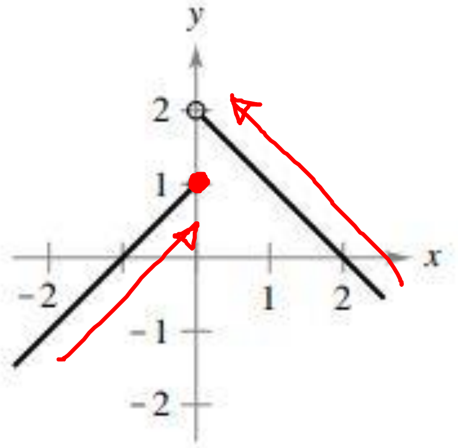
✓ iii $\lim\limits_{x \to c} f(x) = 0 \overset{\checkmark}{=} f(-1)$

3/3

∴ f(x) is continuous @ x = -1

Continuity of a Function

Example No. 2. Determine whether or not f(x) is continuous at x = 0?



Definition A function f is **continuous at a number** c iff all conditions are satisfied:

1. $f(c)$ is defined (that is, c is in the domain of f)
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

✓ (i) $f(c)$ = defined: $f(0) = 1$ ✓

✗ (ii) $\lim_{x \to 0} f(x)$ exists? RSL = LSL

does not Exist $2 \neq 1$

✓ iii

f(x) is discontinuous @ $x = 0$

Types of Discontinuity

Let f be a discontinuous function at $x = a$.

1. If $\lim_{x\to a} f(x)$ exists but, either $f(a)$ does not exist or $f(a) \neq \lim_{x\to a} f(x)$, then f is said to have a **removable discontinuity at $x = a$**.

2. If $\lim_{x\to a} f(x)$ does not exist, then f is said to have an **essential discontinuity at $x = a$**.

   i. If $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ exist but are not equal, then f is said to have a **jump essential discontinuity at $x = a$**.

   ii. If either $\lim_{x\to a^-} f(x) = \pm\infty$ or $\lim_{x\to a^+} f(x) = \pm\infty$, then f is said to have an **infinite essential discontinuity at $x = a$**.

$$f(3) \neq \lim_{x\to 3} f(x) = \frac{2}{3}$$

Example 1. Is f(x) below discontinuous at x = 3? If so, determine its type.

$$f(x) = \frac{x^2 - 2x - 3}{x^2 - 9}$$

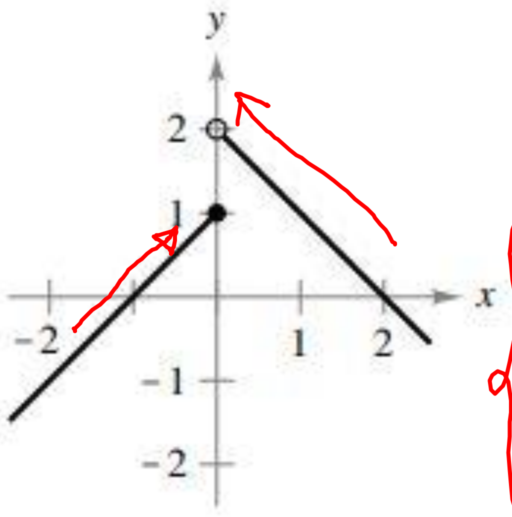
(i) $f(3) = \dfrac{3^2 - 2(3) - 3}{3^2 - 9}$

$f(3) = \dfrac{0}{0}$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2 - 2x - 3}{x^2 - 9}$$

$$\lim_{x\to 3} f(x) \; \frac{(x+1)\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{3+1}{3+3} = \frac{4}{6} = \frac{2}{3}$$

Example No.2. The graph of f(x) is shown below, is f(x) continuous at x = 0? Determine its type.



ii RSL $\neq$ LSL

2 $\neq$ 1

jump essential discontinuity

Continuous from the left

$f(0) = 1$

$\lim_{x \to 0^-} f(x) = LSL = 1$

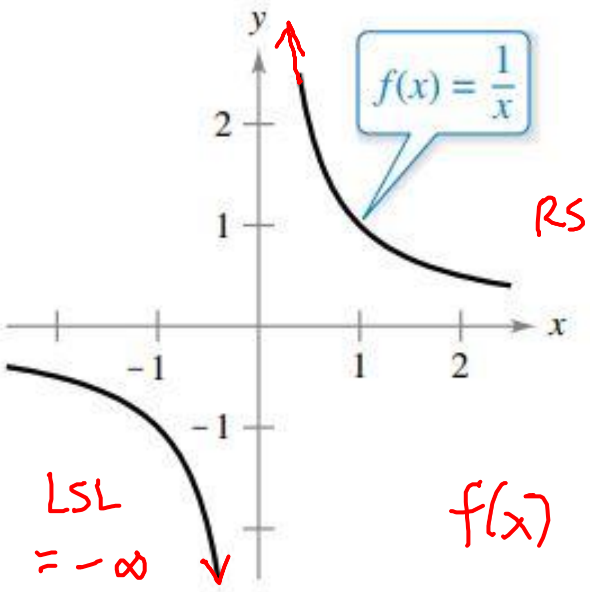
$f(0) = LSL$

$f(0) = 1$

$\lim_{x \to 0^+} f(x) = 2$

$f(0) \neq \lim_{x \to 0^+} f(x)$

Example No.3. Is f(x) continuous at x = 0?



RSL $= +\infty$

LSL $= -\infty$

f(x) is discontinuous

infinite essential discontinuity!

Let f be a real-valued function and $a \in \mathbb{R}$. We say that f is

1. $\boxed{\textbf{continuous from the left}}$ **at** $x = a$ if $f(a) = \lim_{x \to a^-} f(x)$.
2. **continuous from the right at** $x = a$ if $f(a) = \lim_{x \to a^+} f(x)$.

$\boxed{f(a) = LSL}$

$f(a) = RSL$

$$(-2)^2 - (-2)$$

$$G(x) = \begin{cases} \dfrac{x^2 - x - 2}{x + 1} & ; \quad x \le 0 \\ \sqrt{4 - x} & ; \quad 0 \le x \le 4 \\ x^2 - x - 2 & ; \quad x \ge 4 \end{cases}$$

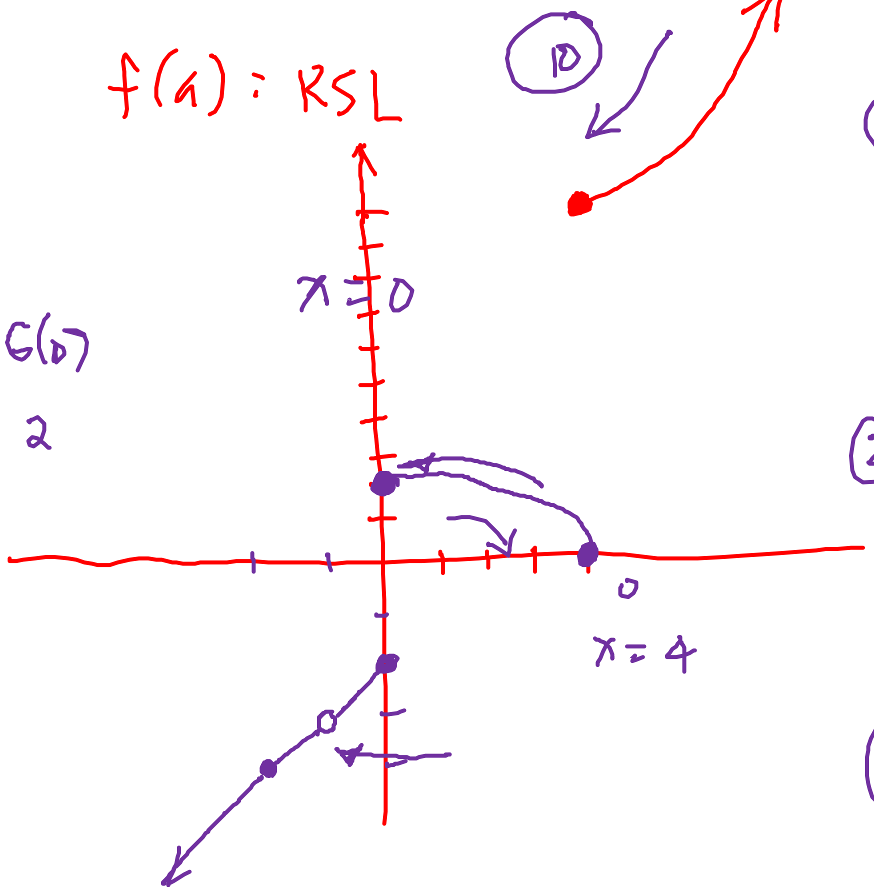
$$\boxed{G(4) = 4^2 - 4 - 2 = 10}$$

2nd piece
$G(0) = 2 \quad (0,2)$
$\boxed{G(4) = 0} \quad (4,0)$

1st pc. $G(0) = \dfrac{0^2 - 0 - 2}{0 + 1}$

$G(-2) = \dfrac{4 + \cancel{2} - \cancel{2}}{-2 + 1} = \dfrac{4}{-1} = -4$

RSL = G(0)
2 = 2



1. f(x) continuous @ x = 4

RSL ≠ LSL

2. RSL = G(4)
10 = 10

LSL = G(4)
0 = 0

$$\frac{(x-2)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$x^2 - x - 2$$

$$(-2)^2 - (-2) - 2 = 4 + \cancel{2} - \cancel{2}$$

Find all the numbers at which the function is discontinuous.

1. $f(x) = x^2 + 4x + 3$

$D: \; x \in \mathbb{R}$

Discontinuous @ $x = \emptyset$

$f(x) = \sin(2x - 3)$

$D: \; x \in \mathbb{R}$

$f(x) = a^{3x-5}$

$D: \; x \in \mathbb{R}$

Find all the numbers at which the function is discontinuous.

2. $f(x) = \dfrac{x-2}{x^2+4x+3}$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

$\boxed{x \neq -3 \quad x \neq -1}$

D: $(-\infty,-3)\cup(-3,-1)\cup(-1,\infty)$

The $f(x)$ is discontinuous @
$x=-3$ and $x=-1$

Find all the numbers at which the function is discontinuous.

3. $f(x) = \sqrt{x^2 - x - 6}$

$x^2 - x - 6 \geq 0 \rightarrow$ DOMAIN

$\boxed{x^2 - x - 6 < 0}$

$(x-3)(x+2) < 0$

C.V: $x = 3$ and $x = -2$

	$x < -2$	$-2 < x < 3$	$x > 3$
$x-3$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
P	$+$	$-$	$+$

$\boxed{(-2, 3)}$

Find all the numbers at which the function is discontinuous.

$A \ln B = \ln B^{A}$

4. $f(x) = \ln\sqrt{\dfrac{x-2}{x+3}}$

$f(x) = \ln\left(\dfrac{x-2}{x+3}\right)^{1/2}$

$f(x) = \dfrac{1}{2}\ln\left(\dfrac{x-2}{x+3}\right)$

Domain: $\dfrac{x-2}{x+3} > 0$

Discontinuity: $\dfrac{x-2}{x+3} \leq 0$

$x = 2$

$x = -3$

	$x<-3$	$-3 \leq x \leq 2$	$x \geq 2$
$x-2$	$-$	$-$	$+$
$x+3$	$-$	$+$	$+$
Quo	$+$	$-$	$+$

@ $x = [-3, 2]$

$$\boxed{\lim_{x \to \frac{1}{2}} (\arcsin 2x)}$$