$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

$$m_{TL} = \left. \frac{dy}{dx} \right|_{x=c}$$

Calculus 1
Synchronous
October 15, 2021

Lesson: BASIC DIFFERENTIATION FORMULAS

Basic Differentiation Formulas

$$D_x k = \boxed{\frac{d}{dx}(k) = 0}, \text{ where k is any constant}$$

Example No. 1. Given the function, find $f'(x)$

1) $f(x) = 1$

$f'(x) = 0$

2) $y = 800\,000$

$y' = 0$

3) $y = 0.000000000000008$

$y' = 0$

4) $y = e$

$y' = 0$

5) $y = e^5$

$y' = 0$

$$\frac{dy}{dx} = y' = \lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

$f(x) = 1$

$f(x+\Delta x) = 1$

$$y' = \lim_{\Delta x \to 0} \left[\frac{\cancel{1 - 1}}{\Delta x} \right] \quad 0$$

$y' = 0$

Power Formula

$$\boxed{d(x^n)} = nx^{n-1} \quad ; \; n \in \mathbb{R}$$

1) $y = x^8$; $n = 8$

$$\boxed{y' = 8x^7}$$

2) $y = t^{3/2}$; $n = 3/2 \in \mathbb{R}$

$$\frac{dy}{dt} = \frac{3}{2} t^{\frac{3}{2}-1}$$

$$\frac{3}{2} - 1 = \frac{3-2}{2} = \frac{1}{2}$$

$$= \frac{3}{2} t^{1/2}$$

$$\frac{dy}{dt} = \frac{3}{2}\sqrt{t}$$

3) $y = \dfrac{1}{\sqrt[7]{x^9}}$

$m = 9$

$n = 7$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$a^{-m} = \frac{1}{a^m}$$

$$y = \frac{1}{x^{9/7}}$$

$y = x^{-9/7}$; $n = -\frac{9}{7}$

$y' = -\frac{9}{7} x^{-\frac{9}{7}-1}$; $\boxed{y' = -\frac{9}{7} x^{-\frac{16}{7}}}$

$y' = -\frac{9}{7}(1)^{-16/7}$

$m_{TL} = -\frac{9}{7}$

$$d[kf(x)] = kd[f(x)]$$

$$\lim_{x \to a} [kf(x)] = k\left(\lim_{x \to a} f(x)\right)$$

① $y = 7x^8$

$y' = 7\frac{d}{dx}(x^8)$

$y' = 7(8x^7)$

$y' = 56x^7$

② $y = 8x^{-6}$

$y' = -48x^{-7}$ ✓

$y' = -\frac{48}{x^7}$

③ $y = \frac{7}{2}x^6$

$y' = \frac{7}{2}(6x^5)$

$y' = 21x^5$

Derivative of Sum

$3 - 2 = 3 + (-2)$

$$d[f(x) \pm g(x) \pm h(x) \pm \cdots] = d[f(x)] \pm d[g(x)] \pm d[h(x)] \pm \cdots$$

Examples: Find the 1st derivative of the ff:

$f(x) = 2x + 5$

$f'(x) = d(2x) + d(5)$

$= 2\frac{dx}{dx}^{1} + \cancel{\frac{d(5)}{dx}}^{0}$

$= 2(1) + 0$

$= 2$

$f(x) = 4x^2 - 5x - 14$

$\frac{dx}{dx} = 1$

$d(x^1) = 1x^{1-1} = x^0 = 1$

$f'(x) = d(4x^2) - d(5x) - \cancel{d(14)}^{0}$

$f'(x) = 4d(x^2) - 5\cancel{d(x)}^{1}$

$= 4(2x^1) - 5$

$= 8x - 5$

$$\boxed{f(x) = (3x+5)^3}$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$a = 3x \quad b = 5$

$3(9)(5)$

$$f(x) = (3x)^3 + \underbrace{3(3x)^2(5)} + 3(3x)(5)^2 + (5)^3$$

$$\rightarrow f(x) = 27x^3 + 135x^2 + 225x + 125$$

$$\boxed{f'(x) = 81x^2 + 270x + 225}$$

The Chain Rule

If y is a differentiable function of u given by $y = f(u)$ and if u is a differentiable function of x given by $u = g(x)$ then y is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$y = f(u) \quad u = g(x)$

$y = f(g(x))$

$y = (f \circ g)(x)$

Example Find $\frac{dy}{dx}$ if $y = 2u^3$ and $\boxed{u = x^2 + 7x}$

$\frac{dy}{du} = 6u^2 \quad \Big| \quad \frac{du}{dx} = 2x+7$

$\frac{dy}{dx} = 6u^2(2x+7)$

$\frac{dy}{dx} = 6(x^2+7x)(2x+7)$

$y = 2(x^2+7x)^3$

$y = (3x+7)^8$

$\boxed{y = u^8 \,;\, u = 3x+7}$

$\frac{dy}{du} = 8u^7 \,;\, \frac{du}{dx} = 3$

$= 8u^7(3)$

$= 24(3x+7)^7$

The General Power Formula

true variable

Using the formula, $\frac{d}{dx}x^n = nx^{n-1}$ enables us to differentiate any power of x.

Using the chain rule, we can differentiate any power of the function of x.

Letting

$$\boxed{y = u^n},\qquad \text{where } u = f(x)$$

then,

$$\frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

$$d(u^n) = nu^{n-1}du$$

$$f(x) = (x^3 - 3x - 2)^5$$

$u = x^3 - 3x - 2$; $n = 5$

$$\frac{du}{dx} = 3x^2 - 3$$

$$f(x) = u^5$$

$$f'(x) = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 5u^4 (3x^2 - 3)$$

$$= 5u^4 (3)(x^2 - 1)$$

$$\boxed{f'(x) = 15(x^3 - 3x - 2)^4 (x^2 - 1)}$$

$$f(x) = (\underbrace{x^3 - 3x - 2}_{u})^5 \qquad n = 5$$

$$d(u^n) = n u^{n-1} \, d\underline{\underline{u}}$$

$$f'(x) = 5(x^3 - 3x - 2)^{5-1} \, \underbrace{d(x^3 - 3x - 2)}$$

$$= 5(x^3 - 3x - 2)^4 (3x^2 - 3)$$

$$= 15(x^3 - 3x - 2)^4 (x^2 - 1)$$

$f(x) = \sqrt{2x-5}$

$u = 2x-5$

$f(x) = \sqrt{u} = u^{1/2}$

$d(u^n) = nu^{n-1}\,du$

$f'(x) = \frac{1}{2} u^{\frac{1}{2}-1}\,du$

$= \frac{1}{2} u^{-1/2}\,du$

$= \frac{du}{2u^{1/2}}$

$\therefore$ $\boxed{d(\sqrt{u}) = \frac{du}{2\sqrt{u}}}$

$f(x) = \sqrt{2x-5}$

$u = 2x-5$

$f'(x) = \frac{d(2x-5)}{2\sqrt{2x-5}}$

$= \frac{2}{2\sqrt{2x-5}}$

$f'(x) = \frac{1}{\sqrt{2x-5}}$

$y = \sqrt{x^2+6x-2}$

$y' = \frac{d(x^2+6x-2)}{2\sqrt{x^2+6x-2}}$

$y' = \frac{\overset{\cancel{2}(x+3)}{2x+6}}{\cancel{2}\sqrt{x^2+6x-2}}$

$\therefore$ $y' = \frac{x+3}{\sqrt{x^2+6x-2}}$

Product Rule

Let $u = f(x)$ and $v = g(x)$ be two functions, then

$$D_x[f(x) \bullet g(x)] = f(x)D_x g(x) + g(x)D_x f(x) \quad \text{or} \quad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$d(uv) = udv + vdu$

$$f(x) = (\overset{u}{2x+7})(\overset{v}{3x+1})$$

$u = 2x+7 \qquad v = 3x+1$

$du = 2 \qquad dv = 3$

$f'(x) = (2x+7)(3) + (3x+1)(2)$

$= 6x + 21 + 6x + 2$

$\boxed{f'(x) = 12x + 23}$

$y = (2x+7)(3x+1)$

$y = 6x^2 + 2x + 21x + 7$

$\boxed{y = 6x^2 + 23x + 7}$

$y' = 12x + 23$

$$f(x) = (3x-2)^3(2x+1)^4$$

u $\quad$ v

$d(uv) = u\,dv + v\,du$

$d(u^n) = n u^{n-1} du$

$\boxed{u = (3x-2)^3}$ $\qquad$ $v = (2x+1)^4$

$\frac{du}{dx} = 3(3x-2)^2 d(3x-2)$ $\quad (3)$ $\qquad$ $\frac{dv}{dx} = 4(2x+1)^3 d(2x+1)$ $\quad (2)$

$\boxed{du = 9(3x-2)^2}$ $\qquad$ $\boxed{dv = 8(2x+1)^3}$

$$d(uv) = f'(x) = (3x-2)^3\left[8(2x+1)^3\right] + \boxed{(2x+1)^4\left[9(3x-2)^2\right]}$$

Factoring: [common factor]:

$$f'(x) = 1(3x-2)^2(2x+1)^3\left[8(3x-2)(1) + 9(1)(2x+1)\right]$$

$24x - 16 + 18x + 9$

$$f'(x) = (3x-2)^2(2x+1)^3(42x-7)$$

$7(6x-1)$

$$\therefore \boxed{f'(x) = 7(3x-2)^2(2x+1)^3(6x-1)}$$

Quotient Rule

Let $u = f(x)$ and $v = g(x)$ be two functions, then

$$D_x\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad \text{or} \quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

Examples. Find the 1st derivative of the ff:

$$f(x) = \frac{2-3x}{4x+5} \quad \begin{array}{l} \longrightarrow u \\ \longrightarrow v \end{array}$$

$u = 2-3x$	$v = 4x+5$
$du = -3$	$dv = 4$

$-12x - 15 \qquad -8 + 12x \qquad -4(2-3x)$

$$f'(x) = \frac{(4x+5)(-3) - (2-3x)(4)}{(4x+5)^2}$$

$$\boxed{f'(x) = \frac{-23}{(4x+5)^2}}$$

$$y = \frac{k \to u}{f(x) \to v}$$; $k =$ constant !

$$y' = d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$y' = \frac{f(x)\,\cancel{d(k)} - k\,dv}{(f(x))^2}$$

0

$$y' = \frac{\cancel{0} - k\,d(f(x))}{[f(x)]^2}$$

$$\boxed{y' = d\left(\frac{k}{v}\right) = \frac{-k\,dv}{v^2}}$$

$$y = \frac{6}{4x - 3}$$

$k = 6$ $\quad v = 4x - 3$

$dv = 4$

$$y' = \frac{-k\,dv}{v^2} = \frac{-(6)(4)}{(4x-3)^2}$$

$$\frac{dy}{dx} = \frac{-24}{(4x-3)^2}$$

$$f(x) = \frac{(2x-3)^5}{(5x-2)^3}$$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$(a^m)^n = a^{mn}$$

$u = (2x-3)^5$

$du = 5(2x-3)^4 \, \underbrace{d(2x-3)}_{(2)}$

$\boxed{du = 10(2x-3)^4}$

$v = (5x-2)^3$

$dv = 3(5x-2)^2 \, d(5x-2)$ 5

$dv = 15(5x-2)^2$

$$f'(x) = \frac{\boxed{(5x-2)^3\left[10(2x-3)^4\right]} - (2x-3)^5\left[15(5x-2)^2\right]}{\left[(5x-2)^3\right]^2}$$

$10x - 4 - 6x + 9$

$$f'(x) = \frac{5\cancel{(5x-2)}^2(2x-3)^4\left[2(5x-2)(1) - 3(2x-3)(1)\right]}{(5x-2)^{\cancel{6}\,4}}$$

$$\boxed{f'(x) = \frac{5(2x-3)^4(4x+5)}{(5x-2)^4}}$$

//

$$f(x)=\frac{(2x-3)^5}{(5x-2)^3}$$

$$\frac{1}{a^m}=a^{-m}$$

$$f(x)=\underset{u}{(2x-3)^5}\,\underset{v}{(5x-2)^{-3}}$$

$$d(uv)=u\,dv+v\,du$$

$$u=(2x-3)^5$$

$$du=5(2x-3)^4\,d(\overbrace{2x-3}^{2})$$

$$du=10(2x-3)^4$$

$$v=(5x-2)^{-3}$$

$$dv=-3(5x-2)^{-4}\,d(\overset{5}{5x-2})$$

$$dv=-15(5x-2)^{-4}$$

$$\frac{(5x-2)^{-3}}{(5x-2)^{-4}}\qquad -3-(-4)=1$$

$$f'(x)=(2x-3)^5(-15)(5x-2)^{-4}+(5x-2)^{-3}(10)(2x-3)^4$$

$$f'(x)=5(2x-3)^4(5x-2)^{-4}\left[\begin{matrix}-6x+9+10x-4\\-3(2x-3)+2(5x-2)(1)\end{matrix}\right]$$

$$f'(x)=5(2x-3)^4(5x-2)^{-4}(4x+5)=\frac{5(2x-3)^4(4x+5)}{(5x-2)^4}$$