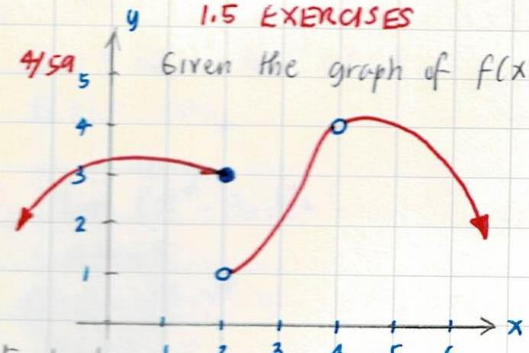
SOLVED PROBLEMS (LIMITS)

Source: CALCULUS by JAMES STEWART (7^{th} Edtn)

1.5 EXERCISES

4/59 Given the graph of $f(x)$



Evaluate:

a) $\lim_{x \to 2^-} f(x) = 3$

b) $\lim_{x \to 2^+} f(x) = 1$

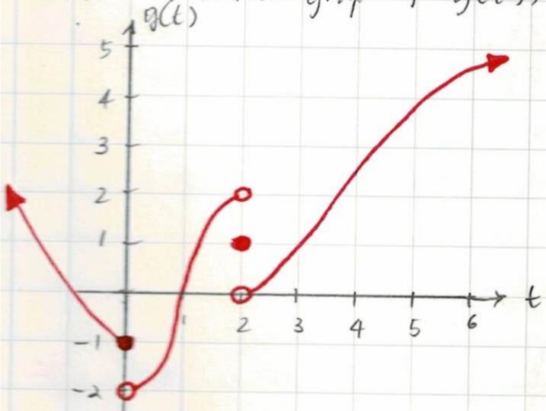
c) $\lim_{x \to 2} f(x) = \text{DNE}$

d) $f(2) = 3$

e) $\lim_{x \to 4} f(x) = 4$

f) $f(4) = \text{undefined}$

7/59. Given the graph of $g(t)$,



Evaluate:

a) $\lim_{t \to 0^-} g(t) = -1$

b) $\lim_{t \to 0^+} g(t) = -2$

c) $\lim_{t \to 0} g(t) = \text{DNE}$

d) $\lim_{t \to 2^-} g(t) = 2$

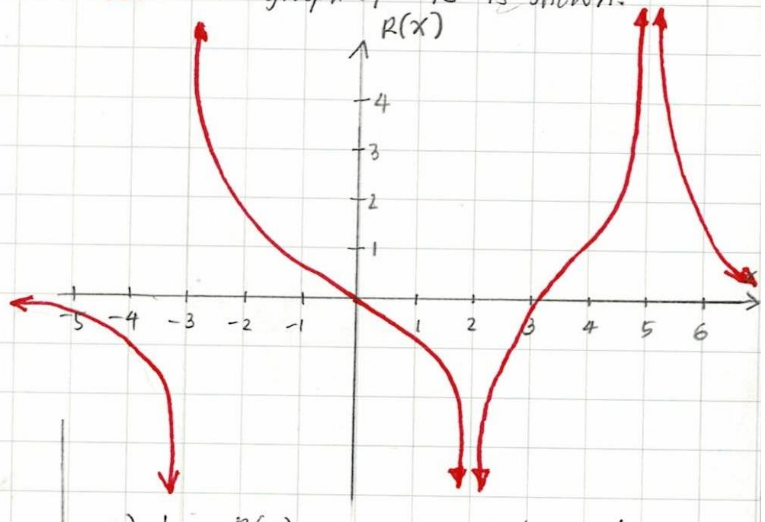
e) $\lim_{t \to 2^+} g(t) = 0$

f) $\lim_{t \to 2} g(t) = \text{DNE}$

g) $g(2) = 1$

h) $\lim_{t \to 4} g(t) = \frac{5}{2}$

8/59 the graph of R is shown.



a) $\lim_{x \to 2} R(x) = -\infty$

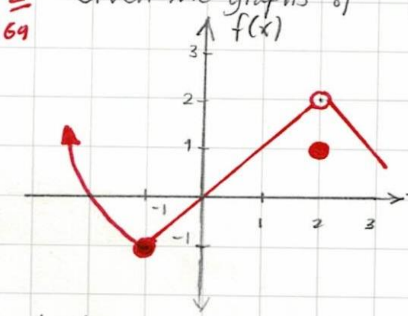
b) $\lim_{x \to 5} R(x) = +\infty$

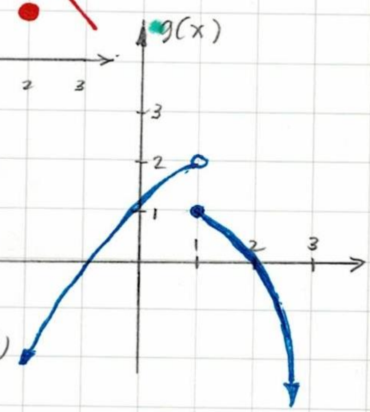
c) $\lim_{x \to -3^-} R(x) = -\infty$

d) $\lim_{x \to -3^+} R(x) = +\infty$

1.6 EXERCISES

2/69 Given the graphs of f and g.





Evaluate:

a) $\lim_{x \to 2} (f(x) + g(x))$

$= \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x)$

$= 2 + 0$

$= 2$

b) $\lim_{x \to 1} [f(x) + g(x)] = \text{DNE}$

c) $\lim_{x \to 0} f(x) g(x) = 0$ $(0)(1)$

d) $\lim_{x \to -1} \frac{f(x)}{g(x)} = \text{undefined}$

e) $\lim_{x \to 2} x^3 f(x)$

$= 2^3 (2)$

$= 16$

f. $\lim_{x \to 1} \sqrt{3 + f(x)}$

$= \sqrt{3 + 1}$

$= 2$

3/70 $\lim_{x\to 3} (5x^3-3x^2+x-6) = f(3)$

$f(3) = 5(3)^3-3(3)^2+3-6$

$f(3) = 105$

4/70 $\lim_{x\to -1}(x^4-3x)(x^2+5x+3) = f(-1)$

$f(-1) = ((-1)^4-3(-1))((-1)^2+5(-1)+3)$

$f(-1) = (1+3)(1-5+3)$

$= (4)(-1)$

$f(-1) = -4$

9/70 $\lim_{x\to 2}\sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\lim_{x\to 2}\frac{2x^2+1}{3x-2}}$

$= \sqrt{f(2)} = \sqrt{\frac{2(2)^2+1}{3(2)-2}}$

$= \sqrt{\frac{9}{4}} = \frac{3}{2}$

11/70 $\lim_{x\to 5}\frac{x^2-6x+5}{x-5} = \lim_{x\to 5}\frac{\cancel{(x-5)}(x-1)}{\cancel{x-5}}$

$= \lim_{x\to 5}(x-1) = 4$

21/70 $\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}\cdot\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}$

$= \lim_{h\to 0}\frac{(9+h)-9}{h(\sqrt{9+h}+3} = \lim_{h\to 0}\left(\frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3}\right)$

$= \frac{1}{\sqrt{9+0}+3} = \frac{1}{6}$

22/70 $\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}\cdot\frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}$

$\lim_{u\to 2}\left(\frac{4u+1-9}{(u-2)\sqrt{4u+1}+3}\right) = \lim_{u\to 2}\left(\frac{4u-8}{(u-2)\sqrt{4u+1}+3}\right)$

$\lim_{u\to 2}\left(\frac{4\cancel{(u-2)}}{\cancel{(u-2)}\sqrt{4u+1}+3}\right) = \lim_{u\to 2}\left(\frac{4}{\sqrt{4u+1}+3}\right)$

$= \frac{4}{\sqrt{4(2)+1}+3} = \frac{4}{6}$

$= \frac{2}{3}$

27/70 $\lim_{x\to 16}\left(\frac{4-\sqrt{x}}{16x-x^2}\right) = \lim_{x\to 16}\frac{4-\sqrt{x}}{x(16-x)}$

$= \lim_{x\to 16}\frac{\cancel{4-\sqrt{x}}}{x(\cancel{4-\sqrt{x}})(4+\sqrt{x})}$

$= \lim_{x\to 16}\frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+4)}$

$= \frac{1}{128}$

28/70 $\lim_{h\to 0}\frac{(3+h)^{-1}-3^{-1}}{h}$

$\lim_{h\to 0}\frac{\frac{1}{3+h}-\frac{1}{3}}{h} = \lim_{h\to 0}\frac{\frac{\overbrace{3-(3+h)}^{-3-h}}{3(3+h)}}{h}$

$= \lim_{h\to 0}\frac{\frac{-h}{3(3+h)}}{h} = \lim_{h\to 0}\frac{-1}{3(3+h)}$

$= \frac{-1}{3(3+0)} = -\frac{1}{9}$

31/70 $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$

$= \lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$

$= \lim_{h\to 0} 3x^2+3xh+h^2$

$= 3x^2+3x(0)+0^2$

$= 3x^2$

DETERMINE WHETHER OR NOT THE FXN IS DISCONTINOUS @ THE INDICATED VALUE of x. IDENTIFY THE TYPE of DISCONTINUITY

17/91 $f(x) = \frac{1}{x+2}$ $a = -2$

$\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = +\infty$

thus, the function is discontinuous at $x = -2$. INFINITE ESSENTIAL DISCONTINUITY

18/91 $f(x) = \begin{cases} \frac{1}{x+2} & x \neq -2 \\ 1 & x = -2 \end{cases}$ $a = -2$

1. $f(-2) = 1$ DEFINED! ✓
2) $\lim_{x \to -2^-} f(x) = -\infty$ $\lim_{x \to -2^+} f(x) = +\infty$
3.) $\lim_{x \to -2} f(x) = \text{DNE}$

THE FUNCTION IS DISCONTINUOUS AT $x = -2$. Since $f(-2)$ is defined, JUMP ESSENTIAL DISCONTINUITY

19/91 $f(x) = \begin{cases} 1 - x^2 & x < 1 \\ 1/x & x \geq 1 \end{cases}$ $a = 1$

1) $f(1) = 1$ defined ✓
2) $\lim_{x \to 1^-} f(x) = 0$ $\lim_{x \to 1^+} f(x) = 1$
3) $\lim_{x \to 1} f(x) = \text{DNE}$

THE fxn is discontinuous @ $x = 1$. Since $f(1)$ is defined, JUMP ESSENTIAL DISCONTINUITY.

23/91 $f(x) = \frac{x^2 - x - 2}{x - 2}$ @ $a = 2$

$f(x) = \frac{\cancel{(x-2)}(x+1)}{\cancel{x-2}}$

$f(x) = x + 1$; $x \in \mathbb{R} \setminus x = 2$

REMOVABLE DISCONTINUITY!

FIND THE INTERVAL TO W/C THE GIVEN FXN IS DISCONTINUOUS.

25/91 $f(x) = \frac{2x^2 - x - 1}{x^2 + 1}$; $x^2 + 1 \neq 0$
$x \neq \pm i$

$F(x)$ is continuous at every point.

26/91 $f(x) = \frac{x^2 + 1}{2x^2 - x - 1}$; $2x^2 - x - 1 \neq 0$
$(2x+1)(x-1) \neq 0$
$x \neq -\frac{1}{2}$ $x \neq 1$

the fxn $f(x)$ is discontinuous @ $x = \frac{1}{2}$ and $x = 1$.

27/91 $Q(x) = \frac{\sqrt[3]{x-2}}{x^3 - 2}$

$x^3 - 2 \neq 0$
$(x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{4}) \neq 0$
$x \neq \sqrt[3]{2}$

the fxn $Q(x)$ is discontinuous @ $x = \sqrt[3]{2}$.

29/91 $h(x) = \cos(1 - x^2)$

DOMAIN: $1 - x^2 \in \mathbb{R}$
$x \in \mathbb{R}$

the fxn $h(x)$ is continuous everywhere.