Numerical Methods Implementation Report

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Introduction

This report covers the implementation of three numerical methods:

1. Bisection Method for root-finding,
2. Golden Section Method for optimizing a unimodal function,
3. Gradient Ascent for finding a maximum of a differentiable function.

Each method's logic, observations about convergence, and implementation challenges are discussed.

1 Task 1: Bisection Method

1.1 Logic

The Bisection Method narrows down a root by repeatedly halving an interval that contains a root. We ensure $f(a) \cdot f(b) < 0$, guaranteeing at least one root. On each iteration, we evaluate $f(m)$ at the midpoint $m = \frac{a+b}{2}$. If $f(m) = 0$, we stop; otherwise, we pick the subinterval where the sign change persists.

1.2 Observations

For the function $f(x)$, we tested the method with intervals $[a_1, b_1]$ and $[a_2, b_2]$. The root at $x = c$ was found very quickly. The choice of the interval affects how fast we converge. A smaller initial bracket or one closer to the actual root reduces the number of steps.

2 Task 2: Golden Section Method

2.1 Logic

The Golden Section Method finds the minimum of a unimodal function by evaluating two interior points determined by the golden ratio. Based on comparing $f(x_1)$ and $f(x_2)$, we discard a portion of the interval and repeat. This process continues until the interval length is below a tolerance ϵ.

2.2 Observations

For the function $f(x)$, we started with points x_1 and x_2. The method quickly converged to a value near x^*. Since the function is strictly unimodal, the method reliably found the global minimum.

3 Task 3: Gradient Ascent Method

3.1 Logic

Gradient Ascent updates the solution by moving in the direction of the gradient. For $f(x)$, we have $\nabla f(x)$. Starting from x_0 and using a learning rate α for 100 iterations, the estimate converged near $x_{\max}$, where the true maximum occurs.

3.2 Observations

The choice of α (learning rate) influences convergence: too large and the method overshoots; too small and it converges slowly.

4 Challenges

- Ensuring that the stopping criteria were met accurately (handling floating-point precision issues).
- Choosing intervals and parameters appropriately to ensure smooth convergence.
- Avoiding infinite loops if a sign change or convergence criterion is not met.

5 Conclusion

All methods worked as expected:

- Bisection Method correctly identified the root at $x = c$.
- Golden Section Method approximated the minimum near x^*.
- Gradient Ascent converged to the maximum near $x_{\max}$.

Parameter choice and interval selection are crucial for efficient convergence and accuracy.