Physics Homework 1

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1 Problem

The figure shows the time dependence of velocity. Do the following:

1. Plot the acceleration and displacement with respect to time. Assume the initial coordinate is $x(0) = 0$ m.
2. Determine the displacement and the average velocity over the time interval $[t_1, t_3]$.

Given: $t_1 = 4$ s, $t_2 = 10$ s, $t_3 = 18$ s.

Solution

1. Acceleration and Displacement Plots

Acceleration

Acceleration is the change of the velocity over the time interval. We have 3 time intervals:

$$t_0 = 0s, \quad t_1 = 4s, \quad t_2 = 10s, \quad t_3 = 18s.$$

Velocity in these intervals:

$$v_0 = 0m/s, \quad v_1 = 4m/s, \quad v_2 = 4m/s, \quad v_3 = 0m/s.$$

Knowing these variables, we can find the acceleration:

$$a_1 = \frac{v_1 - v_0}{t_1 - t_0} = \frac{4-0}{4-0} m/s^2 = 1m/s^2$$

$$a_2 = \frac{v_2 - v_1}{t_2 - t_1} = \frac{4-4}{10-4} m/s^2 = 0m/s^2$$

$$a_3 = \frac{v_3 - v_2}{t_3 - t_2} = \frac{0-4}{18-10} m/s^2 = -0.5m/s^2$$

So the acceleration is $a(t) = \begin{cases} 1 & 0 \le t \le 4 \\ 0 & 4 < t \le 10 \\ -0.5 & 10 < t \le 18 \end{cases}$.

Displacement

We know, that the displacement is the change of the coordinate over the time interval. Coordinate formula:

$$x(t) = x(0) + v(0) \cdot t + a(t) \cdot \frac{t^2}{2} \tag{1.1}$$

As the point goes straight-line motion, the displacement could be found using the formula above. Let us define a displacement over a time interval a:b as $S_{[a:b]}$ Time calculation was omitted due to its simplicity.

$$S_{[0:4]} = 0 + 0 \cdot t + 1 \cdot \frac{t^2}{2} = \frac{4^2}{2}m = 8m$$

$$S_{[4:10]} = 8 + 4 \cdot t + 0 \cdot \frac{t^2}{2} = (8 + 4 \cdot 6)m = 32m$$

$$S_{[10:18]} = 32 + 4 \cdot t - 0.5 \cdot \frac{t^2}{2} = (32 + 4 \cdot 8 - 0.5 \cdot \frac{8^2}{2})m = 48m$$

So the displacement is $S(t) = \begin{cases} 0 & t = 0 \\ 8 & t = 4 \\ 32 & t = 10 \\ 48 & t = 18 \end{cases}$.

Displacement graphs are a little bit trickier, that the acceleration ones. In order to plot the displacement graph, let us clearly state coordinate function next using equation 1.1. We already got the coordinate functions over the intervals in the displacement section above, so let's use them. Let us define the coordinate function on the interval a:b as $x(t)_{[a:b]}$.

$$x(t)_{[0:4]} = \frac{t^2}{2}$$

$$x(t)_{[4:10]} = 4 \cdot t$$

$$x(t)_{[10:18]} = 4 \cdot t - 0.5 \cdot \frac{t^2}{2}$$

But still we can not plot the graph using this functions as we have to transfer time in the future for some of them. For example, $x(t)_{[4:10]}$ should be $x(t-4)_{[4:10]}$ as the body started to move this way only from 4th second. Let us correct the functions:

$$x(t)_{[0:4]} = \frac{(t-0)^2}{2}$$

$$x(t)_{[4:10]} = 4 \cdot (t-4)$$

$$x(t)_{[10:18]} = 4 \cdot (t-10) - 0.5 \cdot \frac{(t-10)^2}{2}$$

Now we are ready for the graph plotting.

Graph Plotting

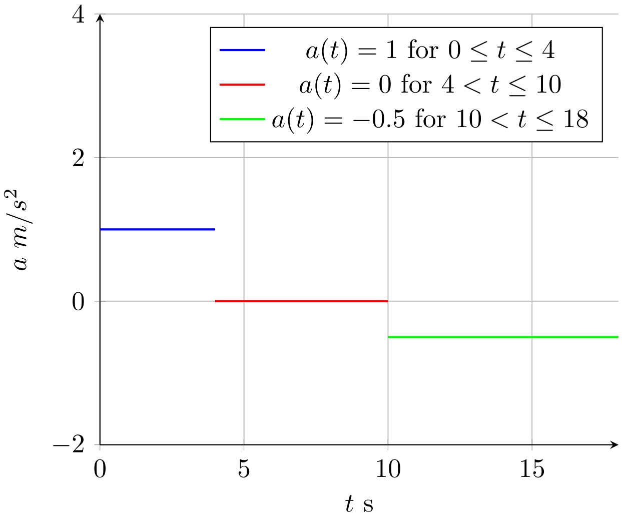

Figure 1.1: Acceleration Plot

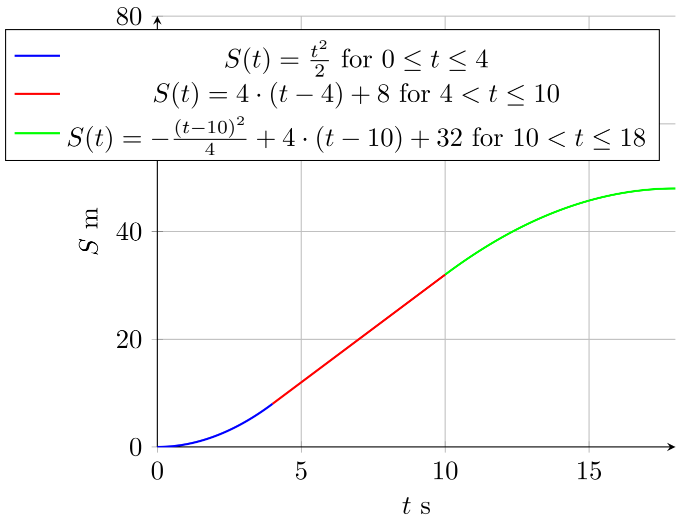

Figure 1.2: Displacement Plot

2. Displacement and Average Velocity

Displacement

To find the displacement over the interval $[t_1, t_3]$ we take displacement over $[t_0, t_3]$ and subtract displacement over $[t_0, t_1]$. This method will work as the movement is straight-line.

$$S_{[t_1:t_3]} = S_{[t_0:t_3]} - S_{[t_0:t_1]} = S_{[0:18]} - S_{[0:4]} = (48 - 8)m = \boxed{40m}$$

Average Velocity

To find the average velocity over the interval $[t_1, t_3]$ we take displacement over $[t_1, t_3]$ and divide it by the time duration.

$$v_{av[t_1:t_3]} = \frac{S_{[t_1:t_3]}}{t_3 - t_1} = \frac{40m}{(18-4)s} \approx \boxed{2.86m/s}$$

ANSWER

1. Graphs: 1.1, 1.2
2. (a) 40m

   (b) 2.86m/s

2 Problem

A ball is thrown horizontally from a height of 20 m and hits the ground with a speed that is three times its initial speed. What is the initial speed? Assume that there is no air drag and therefore acceleration in the horizontal direction is zero

Given:

- $\vec{v_0} = \frac{\vec{v}_{final}}{3}$
- $y_0 = 20\text{m}$
- $|v_{0y}| = 0, |v_{ox}| \neq 0$

Find:

- $|\vec{v}_{init}|$

Solution

Vertical speed

A ball was thrown from the height $h = 20m$. Initial vertical speed was zero. We can find final vertical speed as we know vertical acceleration $g = 9.81m/s^2$

$$S_y = y_0 + v_{y0} \cdot t + a_y \cdot \frac{t^2}{2}$$

Now substitute data we have.

$$0 = 20m + 0m/s - 9.81m/s^2 \cdot \frac{t^2}{2}$$

Solving for t:

$$t = \sqrt{\frac{20m * 2}{9.81m/s^2}} \approx 2s$$

Now we can find v_{yfinal} as we know time the ball travelled:

$$g = \frac{v_{yfinal} - 0}{t}$$

Solving for v_{yfinal}:

$$v_{yfinal} = g * t = 9.81m/s^2 * 2s = 19,62m/s$$

Initial speed

Let's proceed knowing $\vec{v_0} = \frac{\vec{v}_{final}}{3}$:

$$\sqrt{v_{0x}^2 + v_{0y}^2} = \frac{\sqrt{v_{xfinal}^2 + v_{yfinal}^2}}{3}$$

$$\sqrt{v_{0x}^2 + 0} = \frac{\sqrt{v_{xfinal}^2 + (19.62m/s)^2}}{3}$$

There was no horizontal acceleration. This means $v_{0x} = v_{xfinal}$.
Substitute and find initial speed.

$$v_{0x} = \sqrt{\frac{v_{0x}^2 + (19.62m/s)^2}{3}}$$

$$9 \cdot v_{0x}^2 = v_{0x}^2 + 384.94(m/s)^2$$

$$v_{0x} = \sqrt{\frac{384.94(m/s)^2}{8}}$$

$$v_{0x} \approx 6.93m/s$$

v_{0x} is actually the initial speed as no vertical speed existed.

$$|\vec{v}_{init}| \approx \boxed{6.93m/s}$$

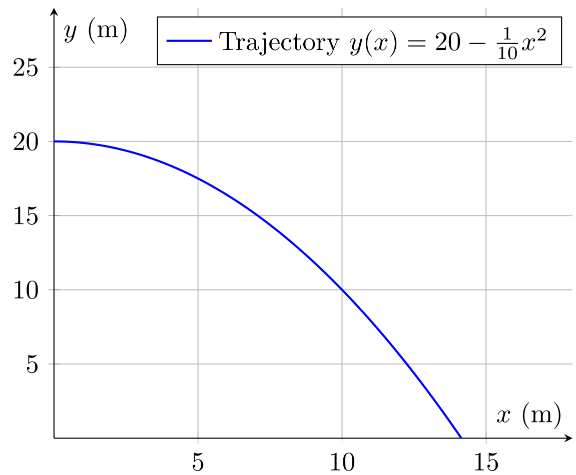

Figure 2.1: Plotted a graph for fun

ANSWER

- Initial Speed: 6.93 m/s

3 Problem

The fisherman on the boat moves up the river. Under the bridge, he drops a bottle into the water. Half an hour later, the fisherman turns back, moves down the river and finds the bottle 5 km down the flow from the bridge. What is the speed of the river if the speed of the boat is constant?

Given:

- $t_{\text{going from bottle}} = 0.5h$
- $x_{\text{bottle destination}} = 5km$

Find:

- v_{river}

Solution

A fisherman guy travelled 0.5 hours with the speed

$$\vec{v}_0 = \vec{v}_{\text{boat}} - \vec{v}_{\text{river}}$$

Then he turns back (immediately) and started to go back with the speed

$$\vec{v}_1 = \vec{v}_{\text{boat}} + \vec{v}_{\text{river}}$$

The distance that he travelled when he was going from the bottle is equal to

$$S_{\text{going from bottle}} = t_{\text{going from bottle}} \cdot (\vec{v}_{\text{boat}} - \vec{v}_{\text{river}})$$

The distance that the bottle has travelled when the guy was going from it:

$$S_{\text{bottle travelling}} = t_{\text{going from bottle}} \cdot \vec{v}_{\text{river}}$$

To reach the bottle again he had to go the same distance he went up the river and also the distance that the bottle travelled.

$$S = S_{\text{going from bottle}} + S_{\text{bottle travelling}}$$

$$S = t_{\text{going from bottle}} \cdot (\vec{v}_{\text{boat}} - \vec{v}_{\text{river}}) + t_{\text{going from bottle}} \cdot \vec{v}_{\text{river}} \tag{3.1}$$

With closing speed of

$$\vec{v}_{\text{closing}} = \vec{v}_{\text{boat}} + \vec{v}_{\text{river}} - \vec{v}_{\text{river}} \tag{3.2}$$

Let's find the time for which he finnaly caught up with the bottle:

$$t_{\text{going for bottle}} = \frac{S}{\vec{v}_{\text{closing}}} \tag{3.3}$$

Substitute equations (3.1) and (3.2):

$$t_{\text{going for bottle}} = \frac{t_{\text{going from bottle}} \cdot (\vec{v}_{\text{boat}} - \vec{v}_{\text{river}}) + t_{\text{going from bottle}} \cdot \vec{v}_{\text{river}}}{\vec{v}_{\text{boat}}}$$

$$t_{\text{going for bottle}} = t_{\text{going from the bottle}}$$

Conclusion

The time of going from the bottle is happened to be the same to the time of going for the bottle back. So, the whole time spent in this situation happened to be $t = 2 \cdot t_{\text{going from the bottle}}$. Also we know, that the bottle itself travelled 5km for this time. Now we can find out it's speed!

$$v_{\text{bottle}} = v_{\text{river}} = \frac{5km}{2 \cdot 0.5h} = \boxed{5km/h}$$

ANSWER

- $5km/h$

4 Problem

The maximum speed of an athlete is 14 m/s. After start, he runs with constant acceleration and then keeps maximum speed for the rest of the race. As a result, it takes him 11 s to cover 100 m distance. What is the acceleration of the athlete?

Given:

- $t = 11s$
- $v_{max} = 14$
- $S = 100m$

Find:

- a

Solution

The whole athele's displacement could be splitted into two parts:

- $S_{\text{with acceleration}} = S_1 = v_0 \cdot t + a \cdot \frac{t^2}{2} = a \cdot \frac{t^2_{\text{max reaching}}}{2}$
- $S_{\text{without acceleration}} = S_2 = v_0 \cdot t + a \cdot \frac{t^2}{2} = v_{max} \cdot (t - t_{\text{max reaching}})$

The whole athele's displacement is equal to:

$$S = S_1 + S_2 = a \cdot \frac{t^2_{\text{max reaching}}}{2} + v_{max} \cdot (t - t_{\text{max reaching}}) = 100m \tag{4.1}$$

We can express acceleration with time:

$$a = \frac{v_1 - v_0}{\Delta t} = \frac{v_{max}}{t_{\text{max reaching}}} \tag{4.2}$$

Substitute equation 4.2 into equation 4.1, we will get:

$$\frac{v_{max} \cdot t^2_{\text{max reaching}}}{t_{\text{max reaching}} \cdot 2} + v_{max} \cdot (t - t_{\text{max reaching}}) = 100m$$

Solve for $t_{\text{max reaching}}$:

$$t_{\text{max reaching}} = 2 \cdot (t - \frac{100m}{v_{max}})$$

$$t_{\text{max reaching}} = 2 \cdot (11 - \frac{100m}{14m/s})$$

$$t_{\text{max reaching}} \approx 7.72s$$

Substite in equation 4.2:

$$a = \frac{14m/s}{7.72s} = \boxed{1.81/s^2}$$

ANSWER

- $1.81/s^2$