Physics Homework 4

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1 Problem

Two boats were slowly moving by inertia (drag forces exerted by water are negligibly small) along parallel courses towards each other. When the boats reached each other, the load of 25 kg was carefully reloaded from first boat to second one. After that, the second loaded boat stopped, first boat continued moving with velocity of 8 m/s. **What were the initial velocities of the boats (m/s, round to 1 decimal place), if the mass of second boat before reloading was 1 ton?**

Given:

- $m = 25\text{kg}$
- $M_2 = 1\text{ton} = 1000\text{kg}$
- $M_2' = M_2 + \text{m} = 1025\text{kg}$ ($'$ means after loading)
- $v_1' = 8\text{m/s}$
- $v_2' = 0\text{m/s}$

Find:

- v_0, v_1

Solution

We will abstract the event as relationship between the load and the second boat. The load was moving on the first boat, hence, had its speed. Then the impulse was created due to reloading. **Proceed with the Conservation of the Momentum:**

$$mv_1 + M_2 v_2 = (m + M_2)v_2'$$

Since the second boat stopped:

$$mv_1 + M_2 v_2 = 0$$

Since boats are moving parallel courses towards each other, we should point they have opposite signs. **Let's then consider v_2 negative.**

$$mv_1 - M_2 v_2 = 0$$

Rearrange:

$$mv_1 = M_2 v_2$$

Little Conversation

At this point we should consider what truly happened during the event. The load from the first boat was carefully reloaded. It implies that the losses in kinetic are approaching zero. From this consideration it is obvious that after the impulse the first boat did not change its speed.

$$\textit{Hence, } v_1 = v_1'$$

Substitute $v_1 = 8$:

$$v_2 = \frac{mv_1}{M_2} = \frac{25 \cdot 8}{1000} = \boxed{0.2\,m/s}$$

ANSWER

- $v_1 = 8\,m/s$
- $v_2 = 0.2\,m/s$

2 Problem

A projectile is shot from a gun at angle $\theta_0 = 60°$ to the flat ground with an initial velocity of $v_0 = 20\,\mathrm{m/s}$. Once the projectile reaches its maximum height, it explodes into 2 pieces of equal mass, with the rear piece having zero horizontal velocity after the explosion (and thus vertically dropping to the ground, as shown in the figure below). **What will be the total travel distance d (m, round to 1 decimal place) of the other piece of projectile from the gun, assuming zero air drag?**

Given:

- $\theta_0 = 60°$
- $v_0 = 20\,\mathrm{m/s}$
- $M = 2 \cdot m$

Find:

- d_{second}

Solution

Let's start by calculating projections of the initial velocity.

$$v_{0x} = v_0 \cos(\theta_0) = 20\,\mathrm{m/s} \times \cos(60°) = 10\,\mathrm{m/s}$$

$$v_{0y} = v_0 \sin(\theta_0) = 20\,\mathrm{m/s} \times \sin(60°) = 17.32\,\mathrm{m/s}$$

Calculate time to reach maximum height. At maximum height, the vertical velocity is zero:

$$t_{\mathrm{up}} = \frac{v_{0y}}{g} = \frac{17.32\,\mathrm{m/s}}{9.8\,\mathrm{m/s}^2} \approx 1.767\,\mathrm{s}$$

Since we know time, let's calculate horizontal distance to this point.

$$x_{\max} = v_{0x} \times t_{\mathrm{up}} = 10\,\mathrm{m/s} \times 1.767\,\mathrm{s} \approx 17.67\,\mathrm{m}$$

At this point the second part of horizontal distance left - travelling after the explosion. *Let's apply conservation of horizontal momentum then.*

$$Mv_{0x} = mv_{1x} + mv_{2x}$$

Since $v_{1x} = 0$ (rear piece drops straight down), we have:

$$Mv_{0x} = 0 + mv_{2x} \implies v_{2x} = \frac{Mv_{0x}}{\frac{M}{2}} = 2v_{0x} = 20\,\mathrm{m/s}$$

The only thing left - to examine how long this piece is going to travel with this speed. We can extract time from vertical distance information.

$$h_{\max} = \frac{v_{0y}^2}{2g} = \frac{(17.32\,\mathrm{m/s})^2}{2 \times 9.8\,\mathrm{m/s}^2} \approx 15.31\,\mathrm{m}$$

$$t_{\text{fall}} = \sqrt{\frac{2h_{\max}}{g}} \implies t_{\text{fall}} = \sqrt{\frac{2 \times 15.31\,\mathrm{m}}{9.8\,\mathrm{m/s}^2}} \approx 1.767\,\mathrm{s}$$

Hence:

$$d_{\text{forward}} = v_{2x} \times t_{\text{fall}} = 20\,\mathrm{m/s} \times 1.767\,\mathrm{s} \approx 35.34\,\mathrm{m}$$

The full distance:

$$d = x_{\max} + d_{\text{forward}} = 17.67\,\mathrm{m} + 35.34\,\mathrm{m} \approx \boxed{53.0\,\mathrm{m}}$$

ANSWER

- $53\,m$

3 Problem

Three blocks are placed on a frictionless surface, with blocks of masses m_2 and m_3 being initially at rest. Block 1 is moving with a velocity of $v_{1i} = 10\,\mathrm{m/s}$. It then collides with block 2 (left figure), which then collides with block 3 (right figure). The third block has mass $m_3 = 6\,\mathrm{kg}$. After the second collision, block 2 becomes stationary again, while block 3 has the velocity of $v_{3f} = 5\,\mathrm{m/s}$. **Assuming all collisions to be elastic, find the final velocity v_{1f} of block 1 (m/s, round to 1 decimal place).**

Given:

- $m_3 = 6\,\mathrm{kg}$
- $v_{3f} = 5\,\mathrm{m/s}$
- $v_{2f} = 0\,\mathrm{m/s}$

Find:

- v_{1f}

Solution

Second collision

Let's examine the event of the collision of second and third blocks a bit closer.
Conservation of Momentum:

$$m_2 v_{2i} + m_3 v_{3i} = m_2 v_{2f} + m_3 v_{3f}$$

Since third block was resting and afterwards second block rest:

$$m_2 v_{2i} + 0 = 0 + m_3 v_{3f}$$

$$m_2 v_{2i} = 6\,\mathrm{kg} \times 5\,\mathrm{m/s} = 30\,\mathrm{kg \cdot m/s} \tag{3.1}$$

Also known, that kinetic energy was also conserved.

$$\frac{1}{2} m_2 v_{2i}^2 + \frac{1}{2} m_3 v_{3i}^2 = \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} m_3 v_{3f}^2$$

$$\frac{1}{2} m_2 v_{2i}^2 + 0 = 0 + \frac{1}{2} m_3 v_{3f}^2$$

$$\frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} \times 6\,\mathrm{kg} \times (5\,\mathrm{m/s})^2 = 75\,\mathrm{J} \tag{3.2}$$

Combine:

$$v_{2i} = \frac{30\,\mathrm{kg \cdot m/s}}{m_2} \quad \text{from 3.1. Putting in 3.2 produces:}$$

$$\frac{1}{2}m_2\left(\frac{30}{m_2}\right)^2 = 75$$

$$\implies m_2 = 6\,\mathrm{kg}$$

Therefore, $v_{2i} = \frac{30}{6} = 5\,\mathrm{m/s}$

First collision

The procedure is the same. **Conservation of Momentum:**

$$m_1v_{1i} = m_1v_{1f} + m_2v_{2i}$$

Plug in known values:

$$m_1 \times 10 = m_1v_{1f} + 6\,\mathrm{kg} \times 5\,\mathrm{m/s}$$

Simplify:

$$m_1(10 - v_{1f}) = 30\,\mathrm{kg}\cdot\mathrm{m/s} \tag{3.3}$$

Conservation of Kinetic Energy:

$$\frac{1}{2}m_1v_{1i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2i}^2$$

Plug in known values:

$$\frac{1}{2}m_1(10)^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2} \times 6\,\mathrm{kg} \times (5\,\mathrm{m/s})^2$$

Simplify:

$$m_1(100 - v_{1f}^2) = 150\,\mathrm{J} \quad (2) \tag{3.4}$$

Proceed:

$$m_1 = \frac{30}{10 - v_{1f}} \quad \text{from 3.3. Substitute in 3.4}$$

$$\left(\frac{30}{10 - v_{1f}}\right)(100 - v_{1f}^2) = 150$$

$$v_{1f}^2 - 5v_{1f} - 50 = 0$$

$$v_{1f} = \frac{5 \pm \sqrt{225}}{2} = \boxed{\frac{5 \pm 15}{2}}$$

Positive value does not make physical sense. $v_{1f} = -5\,\mathrm{m/s}$

ANSWER

- $-5\,\mathrm{m/s}$

4 Problem

In February 1955, a paratrooper fell 370 m from an airplane without being able to open his chute but happened to land in snow, suffering only minor injuries. Assume that his speed at impact was $v_f = 56\,\mathrm{m/s}$ (terminal speed), that his mass (including gear) was 85 kg, and that the magnitude of the constant force on him from the snow was at the survivable limit of $1.2 \times 10^5\,\mathrm{N}$.

Given:

- Event happened February 1995
- $h = 370\,m$
- $v_f = 56\,m/s$
- $m = 85\,kg$
- $F_{snow} = 1.2 \times 10^5\,N$

Find:

- the minimum depth of snow (cm, round to the nearest integer) that would have stopped him safely (15 pts);
- time (s, round to 2 decimal places) required for the trooper to pass through this layer of snow before coming to a full stop (5 pts);
- the magnitude of the impulse p (N·s, round to 2 decimal places) on him from the snow (5 pts).

Solution

In the task's context *safely* means with the given force magnitude, since its survivable limit. Let's calculate how fast snow was changing paratrooper's velocity

$$F_{\text{net}} = mg - F_{\text{snow}} = (85 \times 9.8) - (1.2 \times 10^5) = -119,167\,\mathrm{N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{-119,167}{85} \approx -1,402\,\mathrm{m/s}^2$$

Pretty fast. Now find snow's depth:

$$v_f^2 = v_0^2 + 2ad$$

$$0 = (56)^2 - 2 \times 1,402 \times d$$

$$d = \frac{(56)^2}{2 \times 1,402} = \frac{3,136}{2,804} \approx 1.118\,\mathrm{m} = \boxed{112\,\mathrm{cm}}$$

Proceed using kinematic equation to find time.

$$v_f = v_0 + at$$

$$0 = 56 - 1,402 \times t$$

$$t = \frac{56}{1,402} \approx \boxed{0.04\,\mathrm{s}}$$

And calculate magnitude of Impulse

$$|J| = |F \cdot t| = |120000 \times \frac{56}{1402}| = \boxed{4793.15\,\mathrm{N\,s}}$$

ANSWER

- $112\,cm$
- $0.04\,s$
- $4793.15\,\mathrm{N\,s}$