Physics Homework 3

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November 4, 2024

1 Problem

One particular glitch in the Mafia 3 video game resulted in the player's character getting launched into the air after being intermittently stuck inside a motor boat.

Given:

- $m = 80kg$
- $H = 70m$
- $x = 0.5m$

Find:

- $k = ?$ (If the guy launch at H)
- $v_{max} = ?$

Solution

Finding the stiffness

To solve the task we must understand the processes behind it. Potential energy from the compressed spring is converted into kinetic energy, then into gravitational potential energy. Energy is conserved.

Energy from the spring:

$$E_{spring} = \frac{1}{2}kx^2$$

Gravitational potential energy at height H:

$$E_{gravity} = mgH$$

Energy from the spring converts into gravitational potential energy:

$$\frac{1}{2}kx^2 = mgH$$

Solve for k:

$$k = \frac{2mgH}{x^2} = \frac{2 * 80 * 9.81 * 70}{0.5^2} \approx \boxed{439,000N/m}$$

Calculating the maximum speed

The maximum speed is going to be reached at the maximum of the kinetic energy. It is going to be achieved when potential energy from the spring is fully converted into kinetic energy.

$$E_{spring} = E_{kinetic}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$$

Solve for v_{max}:

$$v_{max} = \sqrt{\frac{kx^2}{m}} = \sqrt{\frac{439,000 * 0.5^2}{80}} \approx \boxed{37m/s}$$

ANSWER

- $439,000N/m$
- $37m/s$

2 Problem

An object slides down an inclined plane set. It successfully slides down, then covers the distance of 50 cm on the horizontal plane, and stops.

Given:

- $m = 0.05kg$
- $\alpha = \angle 30$
- $k = 0.15$
- $v_{init} = v_{final} = 0m/s$
- $s_1 = 0.5m$

Find:

- $W_{friction}$

Solution

The loss in gravitational potential energy should equal the total work done by friction since the object starts and ends at rest. Potential energy at the top:

$$E_{potential} = mgh = -W_{friction} \tag{2.1}$$

Basically, we have to find h. Proceed to find accelerations.

The net force on the incline is given by:

$$ma_1 = mg\sin\alpha - kmg\cos\alpha$$

$$a_1 = g(\sin\alpha - k\cos\alpha) = 9.81\,(0.5 - 0.15 * 0.8660) \approx 3.63m/s^2$$

On the horizontal plane:

$$ma_2 = -kmg$$

$$a_2 = -km = -0.15 * 9.81 = -1.47m/s^2$$

Proceed further.

We will use the formula: $v^2 = v_0^2 + 2as$

1. **Velocity at the start of the horizontal.**

$$v^2 = v_1^2 + 2a_2s_2$$

Since the final velocity $v = 0$:

$$0 = v_1^2 + 2(-1.47)(0.5)$$

$$v_1^2 = 2 * 1.47 * 0.5 = 1.47\,\mathrm{m}^2/\mathrm{s}^2$$

2. **Length of the Incline.**

$$v_1^2 = v_0^2 + 2a_1 s_1$$

Since $v_0 = 0$:

$$1.47 = 2 * 3.63 * s_1$$

$$s_1 = \frac{1.47}{7.26} \approx 0.2027\,\mathrm{m}$$

Now we can find height of the incline.

$$h = s_1 \sin\alpha = 0.2027 * 0.5 \approx 0.1013\,\mathrm{m}$$

Substitue into 2.1:

$$E_{\text{potential}} = mgh = 0.05 * 9.81 * 0.1013 \approx 0.05\,\mathrm{J}$$

$$W_{\text{friction total}} = \boxed{-0.05\,\mathrm{J}}$$

ANSWER

- $0.05\,\mathrm{J}$

3 Problem

A small mass slides down with zero initial velocity from the top of a smooth hill of height H. The foot of the hill has a portion of horizontal surface before the vertical cliff of height h.

Given:

- Mass starts from rest at the top of a smooth hill of height H.
- At the bottom of the hill, there's a horizontal surface leading to a vertical cliff of height h.
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Find:

- What must be the height of the horizontal portion ($h/H =?$) to ensure the maximum distance s covered by the flying mass?
- What the maximum distance s

Solution

Horizontal portion

Since the hill is smooth (no friction), mechanical energy is conserved during the descent.

Potential Energy at the Top:

$$E_{\text{top}} = mgH$$

Potential Energy at Height h:

$$E_{\text{bottom}} = mgh$$

Kinetic Energy at the Bottom:

$$K_{\text{bottom}} = \frac{1}{2}mv_0^2$$

Apply Energy Conservation:

$$E_{\text{top}} = E_{\text{bottom}} + K_{\text{bottom}}$$

$$mgH = mgh + \frac{1}{2}mv_0^2$$

Solving for v_0:

$$\frac{1}{2}mv_0^2 = mg(H - h)$$

$$v_0 = \sqrt{2g(H - h)}$$

Now let's calculate time of flight off the cliff.

Vertical Motion:

$$y = v_{y0}t + \frac{1}{2}at^2$$

Since the mass moves horizontally off the cliff:

$$v_{y0} = 0, \quad a = -g, \quad y = -h$$

So:

$$-h = 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

Proceed with Horizontal Distance s

$$s = v_0 t = \sqrt{2g(H-h)} * \sqrt{\frac{2h}{g}} = 2\sqrt{h(H-h)} \tag{3.1}$$

We need to maximize:

$$s(h) = 2\sqrt{h(H-h)}$$

Let's define:

$$f(h) = h(H-h) = hH - h^2$$

To find the maximum of $f(h)$, take the derivative with respect to h and set it to zero:

$$f'(h) = H - 2h = 0$$

$$2h = H$$

$$\boxed{h = \frac{H}{2}}$$

Maximum distance

Just simply substitute $h = \frac{H}{2}$ into 3.1.

$$s_{\max} = 2\sqrt{\left(\frac{H}{2}\right)\left(H - \frac{H}{2}\right)} = 2\sqrt{\left(\frac{H}{2}\right)\left(\frac{H}{2}\right)} = 2\left(\frac{H}{2}\right) = \boxed{H}$$

ANSWER

- $\frac{H}{2}$
- H

4 Problem

The tank's electric motor has the maximum power of 5 MW. What is the mass of the tank?

Given:

- $P = 5\,\mathrm{MW} = 5,000,000\,\mathrm{W}$
- $\theta = 29°$
- $v_{max} = 37\,\mathrm{km/h} \approx 10.28\,\mathrm{m/s}$

Find:

- m

Solution

Since there is no acceleration, the net force acting is equal to 0. It means, that force, generated by the motor balances the gravitational force.

Gravitational force along the incline:

$$F_{\text{gravity}} = mg\sin\theta$$

Power needed to overcome this force at constant speed:

$$P = F_{\text{gravity}} * v = mg\sin\theta * v$$

Rearrange the Formula:

$$m_{tones} = \frac{P}{g\sin\theta * v * 1000} \approx \boxed{102.39t}$$

ANSWER

- $102.39t$