Physics Homework 2

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1 Problem

In some movie cars were dropped from the sky. What would be the terminal velocities of each car, if they are performing a free-fall headfirst without tumbling in the air?

Given:

- $\rho = 1.007 \frac{\mathrm{kg}}{\mathrm{m}^3}$
- $g = 9.81 \frac{\mathrm{m}}{\mathrm{s}^2}$

Find:

- $v_{\text{terminal of each car}}$

Solution

Formula Deriving

$$\Sigma F = m \cdot a \tag{1.1}$$

The cars are going to reach their terminal velocity after the acceleration become 0, meaning that gravitational force and drag force are balancing each other. Thus:

$$F_g = F_d$$

Formula for drag force:

$$F_d = 0.5 \cdot C_d \cdot \rho \cdot v^2 \cdot A \tag{1.2}$$

Connecting 1.1 and 1.2:

$$m \cdot g = 0.5 \cdot C_d \cdot \rho \cdot v^2 \cdot A$$

Solving for v:

$$v = \sqrt{\frac{2 \cdot m \cdot g}{C_d \cdot \rho \cdot A}} \tag{1.3}$$

Substitute

$$v_{\text{terminal of Jeep}} = \sqrt{\frac{2 \cdot 2000\,kg \cdot 9.8\,\frac{\mathrm{kg}}{s^2}}{0.5 \cdot 1.007 \frac{kg}{m^3} \cdot 2.58\,m^2}} \approx \boxed{173.7 \frac{m}{s}}$$

$$v_{\text{terminal of Dodge}} = \sqrt{\frac{2 \cdot 2450\,kg \cdot 9.8\,\frac{\mathrm{kg}}{s^2}}{0.38 \cdot 1.007 \frac{kg}{m^3} \cdot 2.41\,m^2}} \approx \boxed{228.2 \frac{m}{s}}$$

$$v_{\text{terminal of Subaru}} = \sqrt{\frac{2 \cdot 1550\, kg \cdot 9.8\, \frac{\text{kg}}{s^2}}{0.33 \cdot 1.007 \frac{kg}{m^3} \cdot 2.225\, m^2}} \approx \boxed{202.7 \frac{m}{s}}$$

ANSWER

- $173.7 \frac{m}{s}$
- $228.2 \frac{m}{s}$
- $202.7 \frac{m}{s}$

2 Problem

Object is rotating. Subjects to gravitational force.

Given:

- $m = 0.25kg$
- $R = 0.7m$
- $T_{max} = 30N$
- $g = 9.8m/s^2$

Questions:

- Find the position of the mass on the circular trajectory (exactly) where the string has maximum tension at a given constant speed of the mass
- What is the maximum speed (m/s, round to 1 decimal place), when the string does not break for any position of the object on the whole circle trajectory

Solution

Maximum tension

Tension in the string appears because of the 2 forces: F_{gravit} and F_{centr}
The forces are constant. The maximum tension is going to appear at the **lowest point** of the circle. *Because the forces will point at the same direction.*

Maximum speed

$$T = T_{gravit} + T_{centr}$$

$$T = \frac{mv^2}{R} + mg$$

Now set the tension to the maximum allowed value. Solve for v:

$$v_{max} = \sqrt{\frac{R(T_{max} - mg)}{m}}$$

$$v_{max} = \sqrt{\frac{0.7m(30N - 0.25kg * 9.8m/s^2)}{0.25kg}}$$

$$v_{max} = \sqrt{\frac{0.7m(30N - 0.25kg * 9.8m/s^2)}{0.25kg}}$$

$$v_{max} \approx \boxed{8.78m/s}$$

ANSWER:

- Lowest point
- $8.78 m/s$

3 Problem

Motorboat is moving. Drag force given as $-kv$.

Given:

- $m = 3000\,kg$
- $\vec{F_d} = -k\vec{v}$
- $k = 100\,kg/s$
- $v(33) = 2\,m/s$

Questions:

- Find the maximum speed the boat can reach

Solution

Derivations

The net force could be expressed as:

$$F_{net} = F_{engine} - F_{drag}$$
$$F_{net} = F_{engine} - kv$$

Newton's law:

$$F_{net} = m\frac{dv}{dt}$$

Therefore:

$$F_{engine} - kv = m\frac{dv}{dt}$$

Let us divide the equation by m:

$$\frac{F_{engine}}{m} - \frac{kv}{m} = \frac{dv}{dt}$$
$$\frac{dv}{dt} + \frac{k}{m}v = \frac{F_{engine}}{m} \tag{3.1}$$

Above we just got a first-order liner differential equation. Let us find the integrating factor.

$$\mu(t) = e^{\int \frac{k}{m}dt} = e^{\frac{k}{m}t}$$

Now as we found the integrating factor, we can mul 3.1 by it.

$$e^{\frac{k}{m}t}\frac{dv}{dt} + e^{\frac{k}{m}t}\frac{k}{m}v = e^{\frac{k}{m}t}\frac{F_{engine}}{m}$$
$$\frac{d}{dt}(e^{\frac{k}{m}t} \cdot v) = \frac{F_{engine}}{m}e^{\frac{k}{m}t}$$

Now we simply integrate both sides (with respect to t):

$$\int \frac{d}{dt}(e^{\frac{k}{m}t} \cdot v)dt = \int \frac{F_{engine}}{m}e^{\frac{k}{m}t}dt$$
$$e^{\frac{k}{m}t} \cdot v = \frac{F_{engine}}{k}e^{\frac{k}{m}t} + C$$
$$v(t) = \frac{F_{engine}}{k} + \frac{C}{e^{\frac{k}{m}t}} \tag{3.2}$$

Substitution

Now let us apply initial conditions at 3.2:

At $t = 0$, $v(0) = 0$:

$$0 = \frac{F_{engine}}{k} + C$$
$$C = -\frac{F_{engine}}{k}$$

Reformat 3.2:

$$v(t) = \frac{F_{engine}}{k} + \frac{-\frac{F_{engine}}{k}}{e^{\frac{k}{m}t}}$$
$$v(t) = \frac{F_{engine}}{k}(1 - \frac{1}{e^{\frac{k}{m}t}})$$

At $t = 33$, $v(33) = 2m/s$:

$$2m/s = \frac{F_{engine}}{k}(1 - \frac{1}{e^{\frac{k}{m}33}})$$
$$2m/s = \frac{F_{engine}}{100kg/s}(1 - \frac{1}{e^{\frac{100kg/s}{3000kg}33}})$$

Solve for F_{engine}:

$$F_{engine} = \frac{2m/s \cdot 100kg/s}{1 - \frac{1}{e^{\frac{100kg/s}{3000kg}33}}} \approx 299.9N$$

3.2 Maximal velocity reaches when $t \to \infty$
Then $C \to 0$

Hence:

$$v_{max} = \frac{F_{engine}}{k} = \frac{299.9N}{100kg/s} = \boxed{2.9m/s}$$

ANSWER

- $3m/s$

4 Problem

Blocks A and B are hanging stationary. Find the maximum mass of block A

Given:

- $m_B = 50\,kg$
- $\mu = 0.25$
- $\angle\theta = 30°$
- $g = 9.8\,m/s^2$

Questions:

- Find the maximum mass of block A

Solution

Bodies Diagrams

- *Block B*
  - Weight: $W_B = m_B \cdot g$
  - Normal Force: $N_B = W_B$
  - Friction Force: $f_B = N_B \cdot \mu$
  - Cord's tension: T_B
- *Block A*
  - Weight: $W_A = m_A \cdot g$
  - Cord's tension: T_A
- *Knot*
  - Cord's tension with $\angle\theta$: T
  - Cord's vertical tension: T_A
  - Cord's horizontal tension: T_B

Position of Balance

For the balance to be, the following equations must hold:

$$T_B = f_B \text{ (because the block's on the edge of moving)} \tag{4.1}$$
$$T_A = W_A \tag{4.2}$$

$$T_B = T \cdot cos(\theta) \tag{4.3}$$
$$T_A = T \cdot sin(\theta) \tag{4.4}$$

Formula Deriving

From 4.3

$$T = \frac{T_B}{cos(\theta)}$$

Substitute into 4.4

$$\frac{T_B}{cos(\theta)} = \frac{T_A}{sin(\theta)}$$

$$\frac{T_B \cdot sin(\theta)}{cos(\theta)} = T_A$$

Throw 4.2 into this:

$$\frac{T_B \cdot sin(\theta)}{cos(\theta)} = W_A$$

$$\frac{T_B \cdot sin(\theta)}{cos(\theta)} = m_A \cdot g$$

$$T_B \cdot tg(\theta) = m_A \cdot g$$

And do not forget to add 4.1

$$f_B \cdot tg(\theta) = m_A \cdot g$$

$$N_B \cdot \mu \cdot tg(\theta) = m_A \cdot g$$

$$m_B \cdot g \cdot \mu \cdot tg(\theta) = m_A \cdot g$$

$$m_A = m_B \cdot \mu \cdot tg(\theta)$$

Substitute lol

$$m_A = 50kg \cdot 0.25 \cdot tg(30) \approx \boxed{7.2kg}$$

ANSWER

- $7.2kg$