Esercizio Statistica: 11/03/2019

Sia $\Omega = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$ uno spazio campionario. L'esperimento aleatorio consiste nell'estrazione di un numero da Ω e consideriamo ogni numero equiprobabile. Siano:

$A = \{3,2,7,5\}$

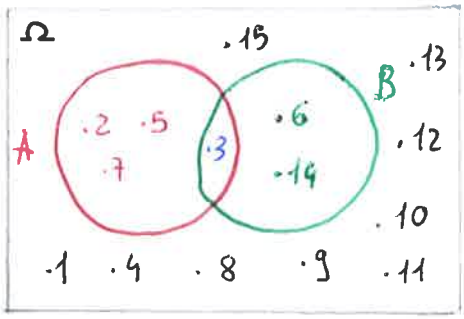
$B = \{6,14,3\}$

due suoi sottoinsiemi dello spazio campionario ($A, B \subset \Omega$)

1. Qual'é la probabilità di $Pr(A|B)$?

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{\frac{1}{15}}{\frac{3}{15}} = \frac{1}{15} \cdot \frac{15}{3} = \frac{1}{3} \simeq 0,33$$

$$Pr(B) = \frac{\#\text{casi favorevoli}}{\#\text{casi totali}} = \frac{\#B}{\#\Omega} = \frac{3}{15}$$



$$A \cap B = \{3,2,7,5\} \cap \{6,14,3\} = \{3\}$$

$$Pr(A \cap B) = \frac{\#\text{casi favorevoli}}{\#\text{casi totali}} = \frac{1}{15}$$

2. Qual'é la probabilità di $P(A \cap B | A \cup B)$?

$$Pr(A \cap B | A \cup B) = \frac{Pr((A \cap B) \cap (A \cup B))}{Pr(A \cup B)} = \frac{\frac{1}{15}}{\frac{6}{15}} = \frac{1}{15} \cdot \frac{15}{6} = \frac{1}{6} \simeq 0,166$$

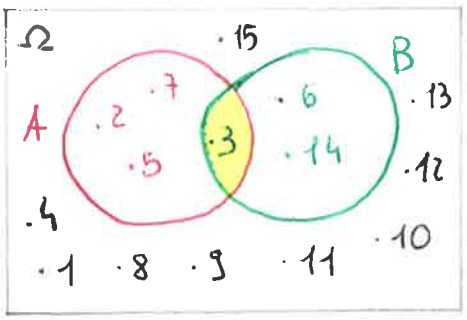
$$A \cup B = \{3,2,7,5\} \cup \{6,14,3\} = \{3,2,7,5,6,14\}$$

$$Pr(A \cup B) = \frac{\#\text{casi favorevoli}}{\#\text{casi totali}} = \frac{6}{15}$$

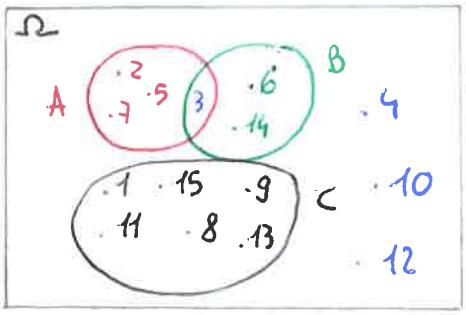
$$(A \cap B) \cap (A \cup B) = \{3\} \cap \{2,3,5,6,7,14\} = \{3\}$$

$$= A \cap B$$

$$Pr((A \cap B) \cap (A \cup B)) = \frac{1}{15}$$



$C = \{1, 11, 9, 13, 15, 8\}$ un altro sottoinsieme dello spazio campionario Ω. Si calcoli la probabilità di $Pr(C \mid A \cup B)$.



$$Pr(C \mid A \cup B) = \frac{Pr(C \cap (A \cup B))}{Pr(A \cup B)} = \frac{\emptyset}{\frac{6}{15}} = \emptyset$$

$A \cup B = \{2, 3, 5, 6, 7, 14\}$

$Pr(A \cup B) = \frac{6}{15}$

$C \cap (A \cup B) = \{1, 11, 9, 13, 15, 8\} \cap \{2, 3, 5, 6, 7, 14\} = \phi$

$Pr(C \cap (A \cup B)) = Pr(\phi) = \emptyset$

4. Qual'è la probabilità che nessuno dei 3 si verifichi?

$$Pr((A \cup B \cup C)^c) = 1 - Pr(A \cup B \cup C) = 1 - \frac{12}{15} = \frac{15-12}{15} = \frac{3}{15} = \frac{1}{5} \simeq \underline{0,2}$$

$A \cup B \cup C = \{3, 2, 5, 7\} \cup \{6, 14, 3\} \cup \{1, 11, 9, 13, 15, 8\}$

$= \{1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 14, 15\}$

$Pr(A \cup B \cup C) = \frac{12}{15}$