## Esercizio Statistica 15/03/2019

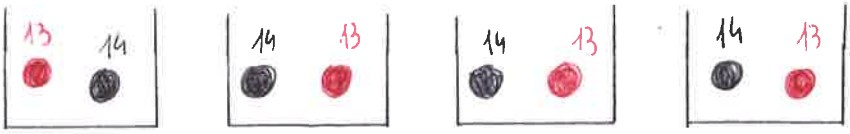
In questo esperimento casuale ci sono 4 sacchetti di biglie colorate e un dado equilibrato a 4 facce.
Si lancia il dado, in base al numero n ottenuto si pesca una biglia da n sacchetti.
In tutti e quattro i sacchetti vi sono 13 biglie rosse "R" e 14 biglie nere "N".

$\Omega_1 = \{R, N\}$ $\quad \mathcal{A}_1 = \mathcal{P}(\Omega_1)$

$\Omega_2 = \{R, N\}$ $\quad \mathcal{A}_2 = \mathcal{P}(\Omega_2)$

$\Omega_3 = \{R, N\}$ $\quad \mathcal{A}_3 = \mathcal{P}(\Omega_3)$

$\Omega_4 = \{R, N\}$ $\quad \mathcal{A}_4 = \mathcal{P}(\Omega_4)$




$A = \{\text{"lancio il dado ed esce un numero"}\} = \{1,2,3,4\}$

$Pr(A) = Pr(\{1\}) = Pr(\{2\}) = Pr(3) = Pr(\{4\}) = \frac{1}{4}$

1- Qual'é la probabilitá che alla fine si peschi esattamente (N,N)?

$\Omega_A = \Omega_1 \times \Omega_2$

$\mathcal{A}_A = \mathcal{A}_1 \otimes \mathcal{A}_2$

$Pr(\{(N,N)\}) = Pr(A) \cdot Pr(\{N,N\})$

$\overset{!}{=} Pr(A) \cdot Pr(N) \cdot Pr(N) = \frac{1}{4} \cdot \frac{14}{27} \cdot \frac{14}{27} = \frac{196}{2916} = \underline{\frac{49}{729}}$

2- Qual'é la probabilitá che alla fine si peschi esattamente (R,N,N)?

$\Omega_B = \Omega_1 \times \Omega_2 \times \Omega_3$

$\mathcal{A}_B = \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_3$

$Pr(\{R,N,N\}) = Pr(A) \cdot Pr(\{R,N,N\}) = Pr(A) \cdot Pr(R) \cdot Pr(\{(N,N)\})$

$\overset{!}{=} Pr(A) \cdot Pr(R) \cdot Pr(N) \cdot Pr(N) = \frac{1}{4} \cdot \frac{13}{27} \cdot \frac{14}{27} \cdot \frac{14}{27} = \frac{2548}{78732} = \underline{\frac{637}{19683}}$

3 - Qual'é la probabilitá che si estraggano almeno 3 N?

$\Omega_C = \Omega_1 \times \Omega_2 \times \Omega_3 \times \Omega_4$

$\mathcal{A}_C = \mathcal{A}_1 \otimes \mathcal{A}_2 \otimes \mathcal{A}_3 \otimes \mathcal{A}_4$

$E = \{\text{"si estraggono almeno 3 N"}\}$

$B = \{\text{"almeno 3 N"}\}$

$\overset{!}{=} \{(N,N,N,N), (N,N,N,R), (N,N,R,N), (N,R,N,N), (R,N,N,N)\}$

$C = \{\text{"3 N"}\}$

$\overset{!}{=} \{(N,N,N)\}$

$$Pr(C) = Pr(A) \cdot Pr(\{N, N, N\})$$

$$= \frac{1}{4} \cdot \left(\frac{14}{27}\right)^3 = \frac{2744}{78732} = \underline{\frac{686}{19683}}$$

$$Pr(B) = Pr(A) \cdot Pr(\{\ , N, N, \cdot\})$$

$$= \frac{1}{4} \cdot \left\{\left(\frac{14}{27}\right)^4 + \left[\left(\frac{14}{27}\right)^3 \cdot \frac{13}{27}\right] + \left[\left(\frac{14}{27}\right)^3 \cdot \frac{13}{27}\right] + \left[\left(\frac{14}{27}\right)^3 \cdot \frac{13}{27}\right] + \left[\left(\frac{14}{27}\right)^3 \cdot \frac{13}{27}\right]\right\}$$

$$= \frac{1}{4}\left\{\frac{38416}{531441} + \frac{35672}{531441} + \frac{35672}{531441} + \frac{35672}{531441} + \frac{35672}{531441}\right\}$$

$$= \frac{1}{4} \cdot \left\{\frac{38416 + 35672 + 35672 + 35672 + 35672}{531441}\right\}$$

$$= \frac{1}{4} \cdot \frac{181104}{531441} = \frac{181104}{2125764} = \underline{\frac{15092}{177147}}$$

$$Pr(E) = Pr(C) + Pr(B)$$

$$= \frac{686}{19683} + \frac{15092}{177147} = \frac{(9 \cdot 686) + 15092}{177147} = \frac{6174 + 15092}{177147} = \underline{\frac{21266}{177147}}$$