

## Esercizio Statistica: 09/05/2019

Sia data la seguente variabile aleatoria bivariata discreta:

	-3.35	-0.51	0.37	1.32	
-2.68	9K	12K	5K	3K	29K
3.56	2K	10K	7K	8K	27K
3.88	11K	6K	4K	1K	22K
	22K	28K	16K	12K	78K

Dove sulle righe abbiamo la variabile X e sulle colonne la variabile Y

1- Calcolare la costante K che deve essere utilizzata per rendere la tabella una funzione di probabilità congiunta.

La somma per righe e colonne della tabella deve essere 1

Dunque:  $9K + 12K + 5K + 3K + 2K + 10K + 7K + 8K + 11K + 6K + 4K + 1K = 1$

$$78K = 1$$

$$K = \frac{1}{78}$$

↙ Equivalentemente la somma delle probabilità marginali deve fare 1.

$$22K + 28K + 16K + 12K = 1 = 29K + 27K + 22K$$

$$78K = 1 = 78K$$

	-3.35	-0.51	0.37	1.32	
-2.68	9/78	12/78	5/78	3/78	29/78
3.56	2/78	10/78	7/78	8/78	27/78
3.88	11/78	6/78	4/78	1/78	22/78
	22/78	28/78	16/78	12/78	1

2- Calcolare la distribuzione di probabilità marginale di X. Determinare  $Pr(X \leq -2.68)$

$$\begin{aligned} Pr(X \leq -2.68) &= Pr(X = -2.68) = 9K + 12K + 5K + 3K \\ &= 29K = \frac{29}{78} \approx 0.3717948718 \end{aligned}$$

Calcolare la distribuzione di probabilità marginale di Y. Determinare  $Pr(Y > -3.35)$ .

$$Pr(Y > -3.35) = Pr(Y = -0.51) + Pr(Y = 0.37) + Pr(Y = 1.32)$$

$$= \frac{28}{78} + \frac{16}{78} + \frac{12}{78} = \frac{56}{78} = \frac{28}{39} \approx 0.7179487179$$

$$= 1 - Pr(Y = -3.35) = 1 - \frac{22}{78} = \frac{56}{78}$$

4 - Calcolare il valore atteso condizionato  $E(X|Y = -0.51)$

	-3.35	-0.51	0.37	1.32
-2.68	9/22	12/28	5/16	3/12
3.56	2/22	10/28	7/16	8/12
3.88	11/22	6/28	4/16	1/12
	1	1	1	1

$P_{X|Y}(x|y)$

non sono stocasticamente indipendenti

$$\begin{aligned} E(X|Y = -0.51) &= \sum_{x \in R_X} x \cdot P_{X|Y}(x|y) = \sum_{x \in R_X} x \cdot \frac{P_{X,Y}(x,y)}{P_Y(y)} \\ &= \left(-2.68 \cdot \frac{12}{28}\right) + \left(3.56 \cdot \frac{10}{28}\right) + \left(3.88 \cdot \frac{6}{28}\right) \\ &= \left(-\frac{268}{100} \cdot \frac{12}{28}\right) + \left(\frac{356}{100} \cdot \frac{10}{28}\right) + \left(\frac{388}{100} \cdot \frac{6}{28}\right) \\ &= -\frac{3216}{2800} + \frac{3560}{2800} + \frac{2328}{2800} = \frac{2672}{2800} = \frac{167}{175} \approx 0.954285714 \end{aligned}$$

5 - Determinare la varianza condizionata  $Var(Y|X = -2.68)$

	-3.35	-0.51	0.37	1.32	
-2.68	9/29	12/29	5/29	3/29	1
3.56	2/27	10/27	7/27	8/27	1
3.88	11/22	6/22	4/22	1/22	1

$P_{Y|X}(y|x)$

$$Var(Y|X = -2.68) = E(Y^2|X = -2.68) - E[(Y|X = -2.68)^2]$$

$$\begin{aligned}
 E(Y|X = -2.68) &= \sum_{y \in R_Y} y \cdot p_{Y|X}(y|x) \rightarrow \sum_{y \in R_Y} y \cdot \frac{p_{X,Y}(x,y)}{P_X(x)} \\
 &= \left[ (-3.35) \cdot \frac{9}{29} \right] + \left[ (-0.51) \cdot \frac{12}{29} \right] + \left[ (0.37) \cdot \frac{5}{29} \right] + \left[ (1.32) \cdot \frac{3}{29} \right] \\
 &= \left( -\frac{335}{100} \cdot \frac{9}{29} \right) + \left( -\frac{51}{100} \cdot \frac{12}{29} \right) + \left( \frac{37}{100} \cdot \frac{5}{29} \right) + \left( \frac{132}{100} \cdot \frac{3}{29} \right) \\
 &= \frac{3015}{2900} - \frac{612}{2900} + \frac{185}{2900} + \frac{396}{2900} \\
 &= -\frac{3046}{2900} = -\frac{1523}{1450} \approx -1.050344827
 \end{aligned}$$

$$\begin{aligned}
 E(Y^2|X = -2.68) &= \sum_{y \in R_Y} y^2 \cdot p_{Y|X}(y|x) \\
 &= \left[ \left( -\frac{335}{100} \right)^2 \cdot \frac{9}{29} \right] + \left[ \left( -\frac{51}{100} \right)^2 \cdot \frac{12}{29} \right] + \left[ \left( \frac{37}{100} \right)^2 \cdot \frac{5}{29} \right] + \left[ \left( \frac{132}{100} \right)^2 \cdot \frac{3}{29} \right] \\
 &= \left( \frac{112225}{10000} \cdot \frac{9}{29} \right) + \left( \frac{2601}{10000} \cdot \frac{12}{29} \right) + \left( \frac{1369}{10000} \cdot \frac{5}{29} \right) + \left( \frac{17424}{10000} \cdot \frac{3}{29} \right) \\
 &= \frac{1010025}{290000} + \frac{31212}{290000} + \frac{6845}{290000} + \frac{52272}{290000} \\
 &= \frac{1100354}{290000} = \frac{550177}{145000} \approx 3.7943241379
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(Y|X = -2.68) &= E(Y^2|X = -2.68) - E[(Y|X = -2.68)^2] \\
 &= \frac{1100354}{290000} - \left( \frac{-3046}{2900} \right)^2 \\
 &= \frac{1100354}{290000} - \frac{9278116}{8410000} \\
 &= \frac{29 \cdot 1100354 - 9278116}{8410000} \\
 &= \frac{31910266 - 9278116}{8410000} = \frac{22632150}{8410000} = \frac{452643}{168200} \approx 2.691099
 \end{aligned}$$