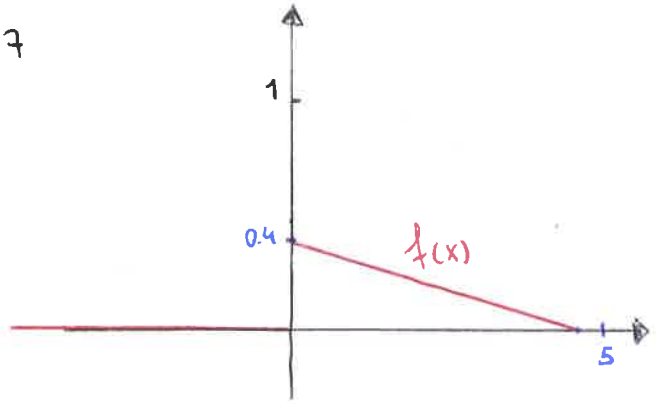
Esercizio Statistica: 18/04/2019

Si consideri una variabile casuale X con funzione di densità:

$$f(x) = \begin{cases} k(9.87 - 2x) & 0 < x < 4.7 \\ 0 & \text{altrimenti} \end{cases}$$

$$f(x) = \begin{cases} \dfrac{(9.87 - 2x)}{24.299} & 0 < x < 4.7 \\ 0 & \text{altrimenti} \end{cases}$$



1- Determinare la costante k affinché $f(x)$ sia funzione di densità

$$\int_{Rx} f(x)\,dx = 1$$

$$\int_{Rx} k(9.87 - 2x)\,dx = 1 \Rightarrow \int_{Rx} 9.87k - 2xk\,dx = 1$$

$$\Rightarrow 9.87xk - kx^2 \Big]_0^{4.7} = 1$$

$$\Rightarrow 9.87 \cdot 4.7 \cdot k - k \cdot (4.7)^2 = 1$$

$$\Rightarrow 46.389k - 22.09k = 1$$

$$\Rightarrow k(46.389 - 22.09) = 1$$

$$\Rightarrow k(24.299) = 1$$

$$\Rightarrow k = \frac{1}{24.299} \simeq \underline{0.0411153957}$$

2- Calcolare il valore atteso di X.

$$\mathbb{E}(X) = \int_{Rx} x \cdot f(x)\,dx = \int_0^{4.7} x \cdot \frac{1}{24.299}(9.87 - 2x)\,dx$$

$$= \frac{1}{24.299} \int_0^{4.7} 9.87x - 2x^2\,dx$$

$$= \frac{1}{24.299} \left[\frac{9.87}{2} x^2 - \frac{2}{3} x^3 \right]_0^{4.7}$$

$$= \frac{1}{24.299} \left[\frac{9.87}{2} \cdot (4.7)^2 - \frac{2}{3}(4.7)^3 \right] = \frac{39.7988167}{24.299} \simeq \underline{1.63787878}$$

- Calcolare il valore atteso di $8.6 + 1.9\,X$:

$$\mathbb{E}(a+bX) = a + b\,\mathbb{E}(X)$$

$$\mathbb{E}(8.6 + 1.9\,X) = 8.6 + 1.9\,\mathbb{E}(X) = 8.6 + 1.9 \cdot (1.63787878)$$

$$= 8.6 + 3.111969682$$

$$= 11.711969682$$

4 - Dato il momento non centrato di ordine 2, calcolare la varianza di X

$$\mathbb{E}(X^2) = 4.016\,64$$

$$\mathbb{V}ar(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$= 4.016364 - (1.63787878)^2$$

$$= 4.016364 - 2.682646898$$

$$= 1.333717$$

$$\mathbb{E}(X^2) = \int_{R_X} x^2 \cdot f(x)\,dx = \int_{R_X} x^2 \cdot \frac{1}{24.299}(9.87 - 2x)\,dx$$

$$= \frac{1}{24.299}\left(\int_0^{4.7} 9.87x^2 - 2x^3\,dx\right)$$

$$= \frac{1}{24.299}\left[\frac{9.87}{3} \cdot x^3 - \frac{2}{4}x^4\right]_0^{4.7}$$

$$= \frac{1}{24.299}\left(\frac{9.87 \cdot (4.7)^3}{3} - \frac{(4.7)^4}{2}\right)$$

$$= \frac{1}{24.299}\left(\frac{1024.733}{3} - \frac{487.9681}{2}\right)$$

$$= \frac{1}{24.299}\left(341.5777 - 243.9841\right)$$

$$= \frac{97.5936}{24.299} = 4.016364$$