

Stochastic block model for multilevel networks unravels structural interdependence between the social and economic networks in a TV program trade fair

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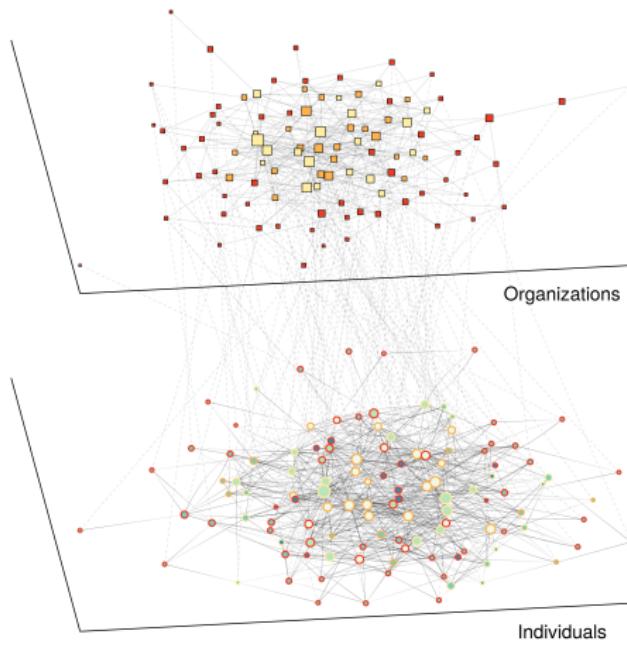
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Outline

- 1 Modeling
- 2 Simulation Studies
- 3 Application to Television Program Trade Fair

Motivation Dataset

Economic and social networks in a television trade fair ¹.



- Economic network: 109 organizations signing deals (undirected interactions)
- Represented on the trade fair by individuals
- Social network: 128 individuals sharing advice (directed interactions)

¹Brailly, 2016

Objective of this work

			n_I		n_O	
Individual 1	0	1	0	1	-	0
⋮	$X_{ii'}^I$		A_{ij}			
Individual n_I	1	1	0	-	1	
Organization 1			1	1		
⋮					$X_{jj'}^O$	
Organization n_O			0	1		
	⋮	⋮	⋮	⋮	⋮	⋮
	Individual 1	Individual n_I	Organization 1	Organization n_O		

Data :

X^I Interactions between individuals

X^O Interactions between organizations

A Affiliations of the individuals to the organizations

$A_{ij} = 1$ if i is affiliated to j

Only one affiliation per individual

Objectives

- Joint probabilistic model on $\mathbf{X} = \{X^I, X^O\}$ given A
- Evaluate the influence of the inter-organizational level on the inter-individual level

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Modeling of a Multilevel SBM



Stochastic Block Model (SBM)^a

^aSnijders and Nowicki, 1997

- Mixture model for graphs
- Latent variables on nodes
- Model heterogeneity of connection

Modeling of a Multilevel SBM

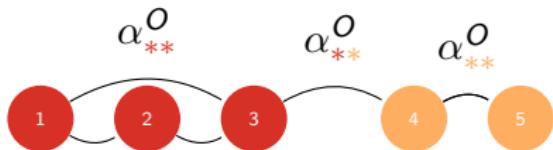


Inter-organizational Level

- n_O organizations into Q_O blocks
- Latent variables are independent
- $Z_j^O = l \Leftrightarrow j \in I, \quad l \in \{1, \dots, Q_O\}$

$$\mathbb{P}(Z_j^O = l) = \pi_l^O$$

Modeling of a Multilevel SBM



Inter-organizational Level

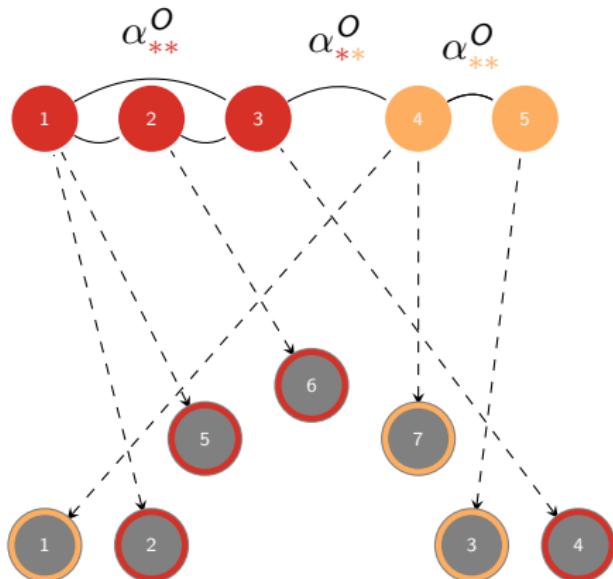
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$$\mathbb{P}(Z_j^O = l) = \pi_l^O$$

- Connections are independent given the latent variables

$$\mathbb{P}(X_{jj'}^O = 1 | Z_j^O = l, Z_{j'}^O = l') = \alpha_{ll'}^O$$

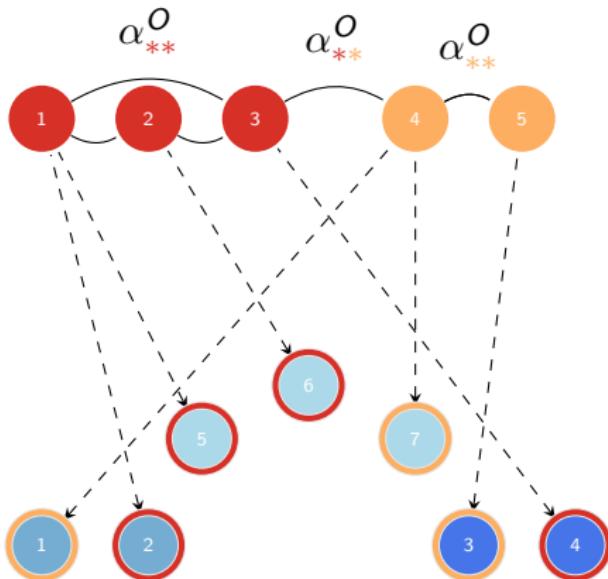
Modeling of a Multilevel SBM



Inter-individual Level

- n_I individuals into Q_I blocks
- The block of an individual depends on the block of her/his organization

Modeling of a Multilevel SBM

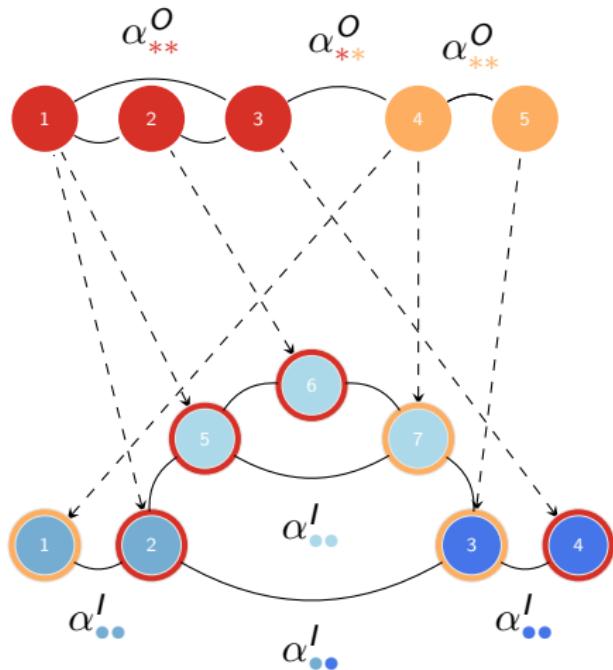


Inter-individual Level

- n_I individuals into Q_I blocks
- The block of an individual depends on the block of her/his organization
- $Z_i^I = k \Leftrightarrow i \in k, k \in \{1, \dots, Q_I\}$

$$\mathbb{P}(Z_i^I = k | A_i = j, Z_j^O = l) = \gamma_{kl}$$

Modeling of a Multilevel SBM



Inter-individual Level

- n_I individuals into Q_I blocks
- The block of an individual depends on the block of her/his organization

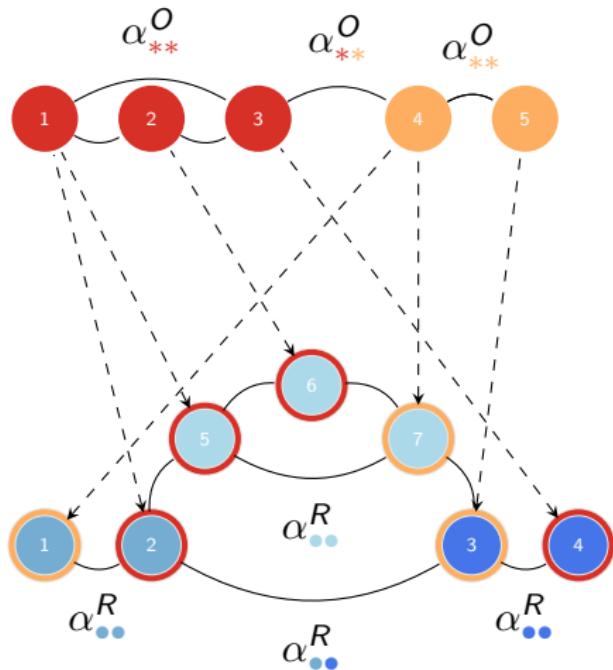
$$Z_i^I = k \Leftrightarrow i \in k, k \in \{1, \dots, Q_I\}$$

$$\mathbb{P}(Z_i^I = k | A_i = j, Z_j^O = l) = \gamma_{kl}$$

- Connections are independent given the latent variables

$$\mathbb{P}(X_{ij'}^I = 1 | Z_i^I = k, Z_j^O = k) = \alpha_{kk'}^I$$

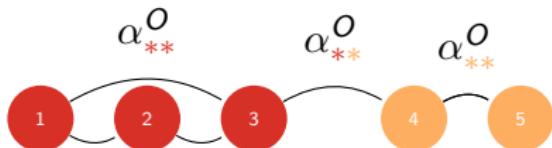
Independence Between Levels



- π^O is a probability vector
- Each column of γ as well
- If $\gamma_{kl} = \gamma_{kl'} \quad \forall l, l'$

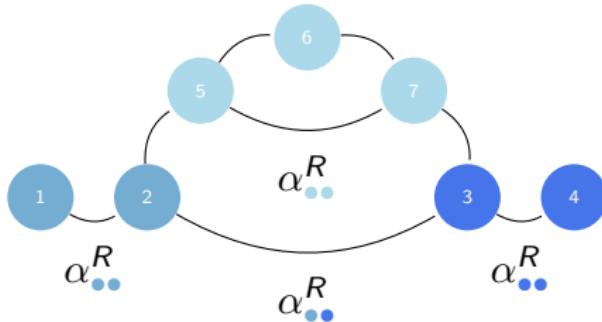
$$\mathcal{L}(X^I, X^O | A) = \mathcal{L}(X^I) \mathcal{L}(X^O)$$

Independence Between Levels



- π^O is a probability vector
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$$\mathcal{L}(X^I, X^O | A) = \mathcal{L}(X^I) \mathcal{L}(X^O)$$



- Each level of the multilevel network is a SBM with $\pi^I = \gamma_1$
- Organizational structure has no influence on the connections of individuals

Inference of the Multilevel SBM

- Inference by maximum likelihood estimation
- Variational EM algorithm for a given number of blocks
- Step-wise procedure to navigate between models
- Model selection by a model based penalized criterion (ICL)
- ICL also used to state on the independence between the inter-individual and the inter-organizational levels

Outline

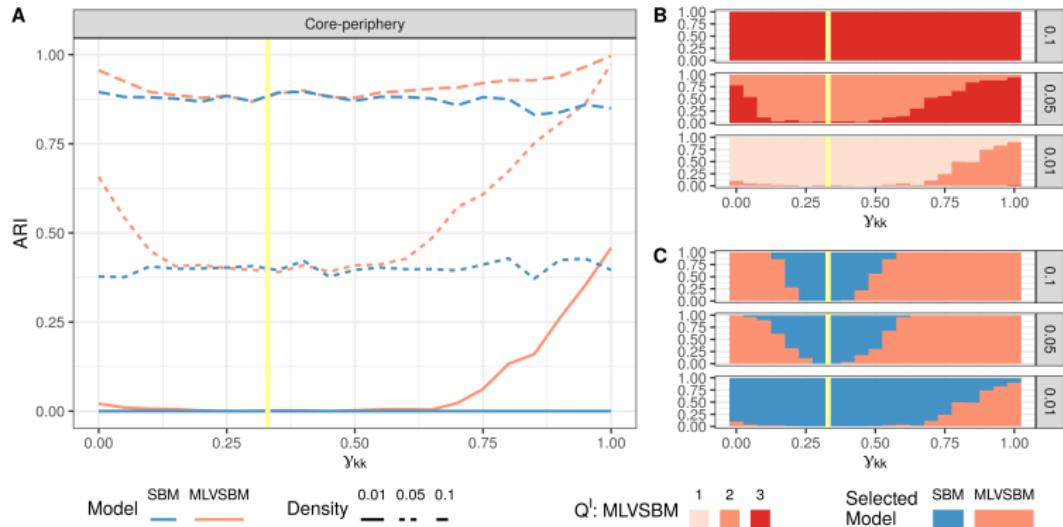
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Simulation Studies

- Strong dependence between levels (γ_{kk} far from 1/3) helps recover the structure of the inter-individual level with the information of the inter-organizational level.
- ICL tends to select model of small size \Rightarrow Good for testing the interdependence.



Outline

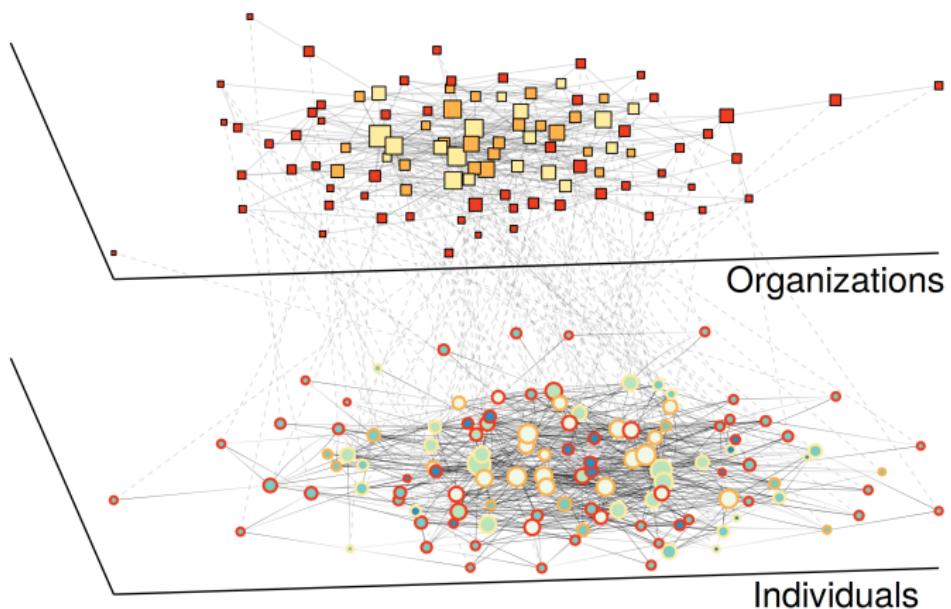
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Application to a Television Program Trade Fair Dataset²

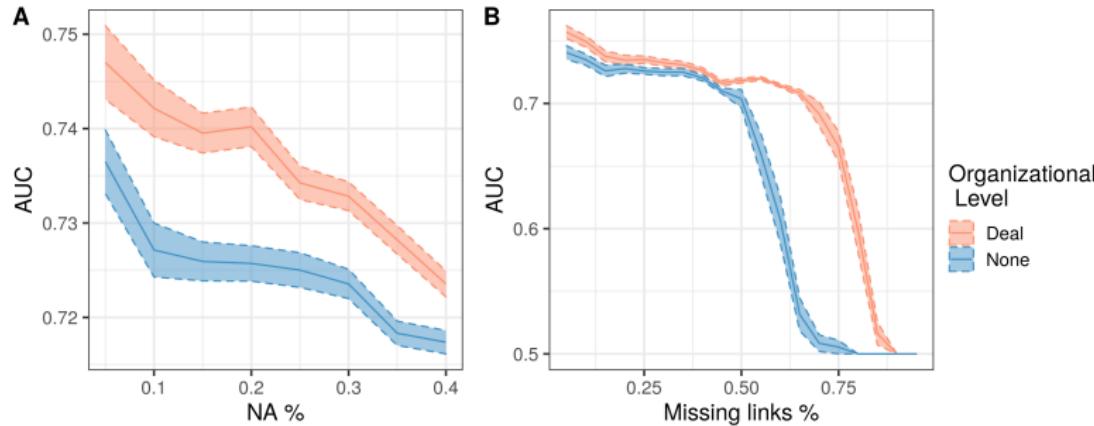
128 individuals (buyers and sellers) with directed interactions (advice) and 109 organizations with undirected interactions (deal).



²Brailly, 2016

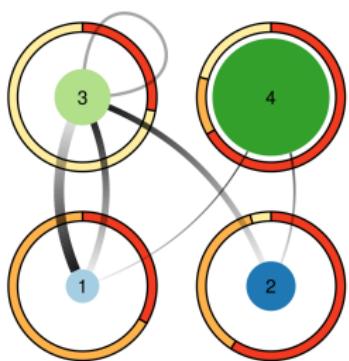
Link Prediction

- The social network and the economic network are interdependent.
- Inter-organizational level helps predicting links on the inter-individual level.

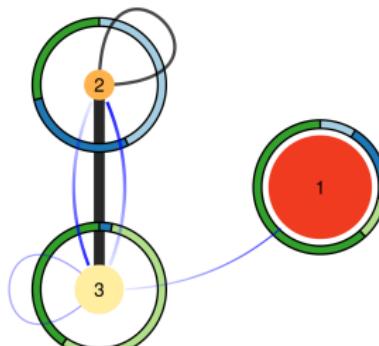


Dataset analysis

Individuals



Organizations



- 4 blocks of individuals and 3 blocks of organizations
- Core-periphery structure for the inter-organizational level
- Mainly inter-block connections for individuals (except block 3, sub-group of sellers)
- Intra-block connection between individuals do not replicate the intra-block connections of their organizations (block 2 and 3)

- Preprint available on arXiv: <https://arxiv.org/abs/1910.10512>
- R package available at
<https://chabert-liddell.github.io/MLVSBM/>
 - Simulation and inference of multilevel networks
 - Handling of missing data on X^I and X^O
 - Prediction on missing dyads, missing links and spurious links
 - Extend to multi-affiliation datasets

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Thank you for your attention!

References

- Brailly, Julien (2016). "Dynamics of networks in trade fairs—A multilevel relational approach to the cooperation among competitors". In: *Journal of Economic Geography* 16.6, pp. 1279–1301.
- Snijders, Tom A.B. and Krzysztof Nowicki (Jan. 1997). "Estimation and Prediction for Stochastic Blockmodels for Graphs with Latent Block Structure". In: *Journal of Classification* 14.1, pp. 75–100. ISSN: 0176-4268. DOI: 10.1007/s003579900004. URL: <http://link.springer.com/10.1007/s003579900004>.