Math 8515 Proj 2

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We use FitzHugh-Nagumo and Hindmarsh-Rose model to simulate neuron network dynamics through phase analysis.

Introduction

A key part of Hodgkin and Huxley's model assumptions was that the memebrane contain channels for potassium and sodium ion flow and the rate of those flows follow first order kinetics. FitzHugh sought to reduce the Hodgkin-Huxley model to a two variable model for which phase plane analysis applies which led to the so called fast-slow phase plane model, of the form

$$\begin{aligned} C_m \frac{dV}{dt} =& - g_k n^4 (V - V_K) - g_{Na} m_\infty^3 (0.8 - n)(V - V_{Na}) - g_L (V - V_L) + I \\ n_\omega(V) \frac{dV}{dt} =& n_\infty(V) - n. \end{aligned}$$

Further observation due to FitzHugh was that the $V-$nullcline had the shape of a cubic function and the $n-$nullcline could be approximated by a straight line, both within the physiological range of the variables. The polynomial reducted and dimensionless form is

$$\begin{aligned} \dot{v} =& - v(v - \alpha)(1 - v) - \omega + I_0 \\ \dot{\omega} =& \epsilon(v - \gamma\omega) \end{aligned}$$

Where v represents the fast variable(potential). ω represents the slow variable(gating variable for sodium). α, γ and ϵ are constants with $0 < \alpha < 1$ and $\epsilon << 1$.
The Hindmarsh and Rose model is the other phenomenological model which may be seen either as a generalization of the FitzHugh model or as a simplification of the physiologically realistic model proposed by Hodgkin and Huxley in the sense that it is simple and capable of mimicking almost all the behavoirs exhibited by real biological neurons. The Hindmarsh-Rose model is described by:

$$\begin{aligned} \dot{x} =& y - x^3 + bx^2 + I - z \\ \dot{y} =& 1 - 5x^2 - y \\ \dot{z} =& \mu(s(x - x_{rest}) - z) \end{aligned}$$

I mimics the membrane input current for biological neurons; b allows to switch between spiking and bursting and control the spiking frequency, μ is a variable similar to ϵ in FitzHugh-Nagumo model which controls the speed of variation of the slow variable z which is the efficiency of the slow channels in exchanging ions for example gating variable for sodium. s govens the adaption while x_{rest} is the resting potential of the neuron in problem.
The fundamental building block of every nervous system is the neuron,. We use the FitzHugh-Nagumo model and Hindmarsh-Rose model to model and simulate neurons and study the behavior of a neuron network built with three cells and fast threshold method coupling through the phase analysis defined in project one. One assumption of the systems is that neurons are udential.

FitzHugh-Nagumo Model

We model the neuronal network with three cells coupled through fast threshold method and then apply the external current for 20 mili-seconds which is a typical time scale for stimulation given to neuron cells. The voltage is scaled to $[-1,\ \ 1]$

$$\begin{aligned}\dot{v_i} =& -v_i(v_i-\alpha)(1-v_i)-\omega_i+\frac{I_0}{1+\frac{e^{20-t}}{0.2}}-g(v_i-V_S)\sum_{j\neq i}S_j\\ \dot{\omega_i} =& \epsilon(v_i-\gamma\omega_i)\\ S_j =& \frac{1}{1+E^{\lambda(v_j-\theta_s)}}\end{aligned}$$

S_j describes the coupling. In this system, the network is symmatric and the coupling is chemical. We fix the parameters: $\alpha = 0.1$, $\epsilon = 0.005$, $I_0 = 0.1$ and $g_i = 0.0051$, $\lambda = -11$, $\theta_s = -0.0$, $V_S = -0.65$ and vary the parameter d with initial values $v_0 = [-0.5; -0.5; -0.5]$, $w_0 = [0.3; 0.302; 0.305]$.

Post Inhibitory Rebound

When $\gamma = 1.0$, the system converge to anti-phase. Since $V_S = -0.65$, which is the reverse potential, always smaller than v_i, $i = 1,\ \ 2,\ \ ,3$, so $v_i - V_S$ is always positive, the coupling of the threee network is mutually inhibitory. From the initial state, we apply external current to all the three cells for 20 ms and the they start

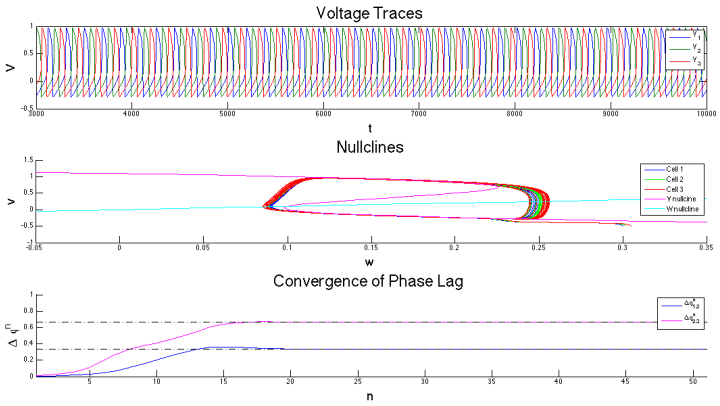


Figure 1: $\gamma = 1.0$

to move to the left. Because of the choice of γ and the mutually inhibition, the ω nullcline is very close to the lower nee point. The three cells are moving very slowly and once of the cells cross the gap between the fast and slow nullclines, the inhibition become much stronger untill the leading cell reach the higher nee point, then the inhibiton from the leading cell(cell #1) is until it reaches the lower part of the fast nullcline. Since the cell #2 and #3 are on the upper side of the fast nucline, the inhibition between them keeps the cell #3 until the cell #2 reaches the lower part of the When the external current equals to zero, the network converges to anti-phase.

Release

By increasing γ, we are elevating the slow nullcline. once γ is large enough, the slow nuclline is close to the upper nee point. The nutual inhibiton is going to move the trace of the cells to the right and narrow down the gap between the slow and fast nullclines to keep the inhibition for longer time. Once the leading cell cross the gap, it'll be released. By setting $\gamma = 3.0$, and compare the simultaion results in figure I and II, we can see, first

of all, in both cases, the system converged to anti-phase, secondly, the system in figure II kept the ϕ_{ij} at 0 for longer and thirdly, they switched position telling that the cell #1 and cell #2 were kept together for longer.

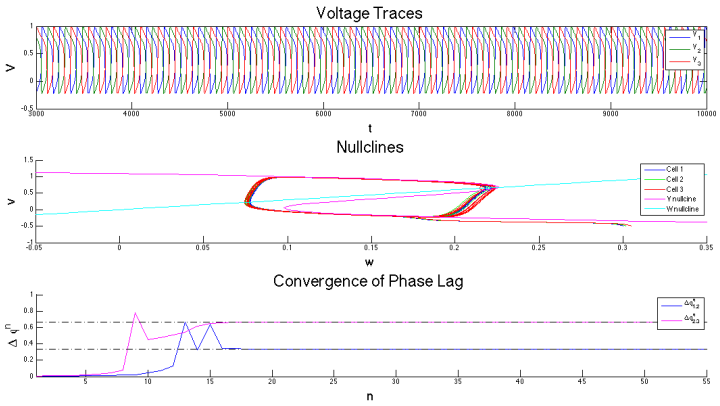


Figure 2: $\gamma = 3.0$

We keep on increasing γ to 3.1, the cell #1 became the pace maker. The phase lag between cell 1 and cell 2 is switching between 1 and 0 for every cycle while ϕ_{23} is kept very close to 0.

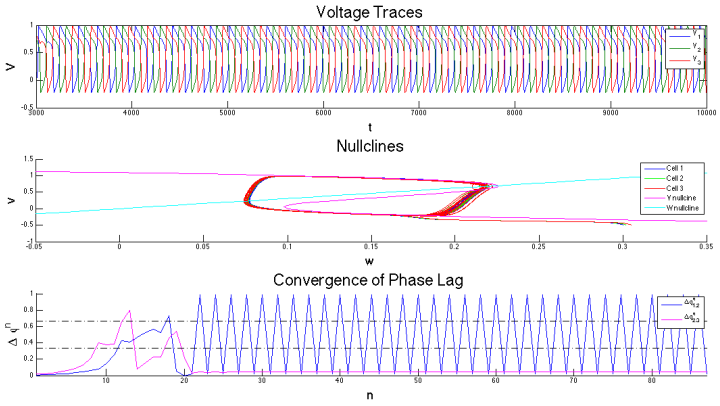
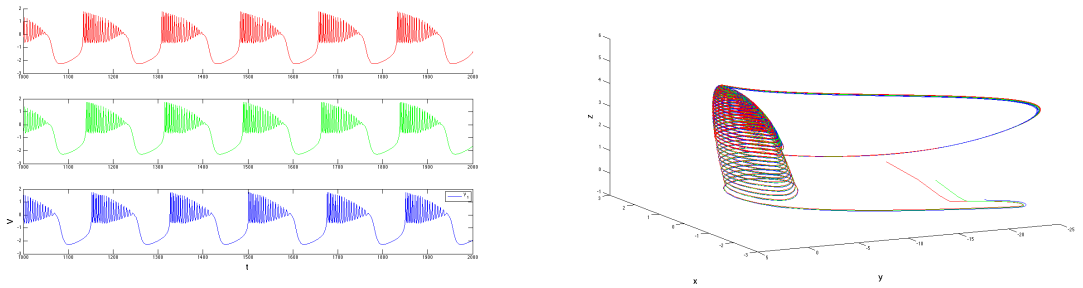


Figure 3: $\gamma = 3.1$

Hindmarsh-Rose

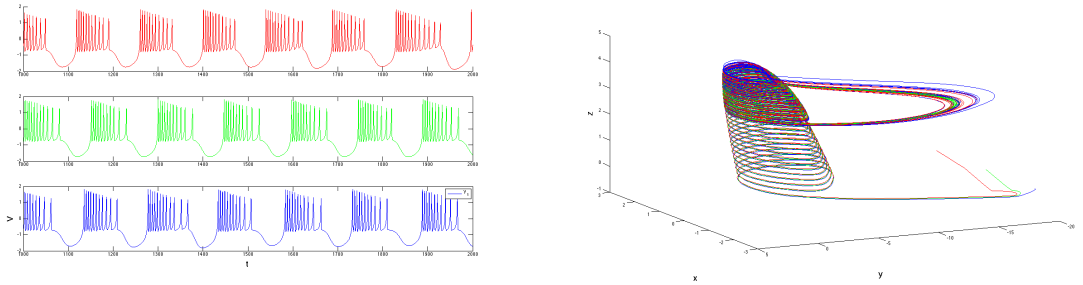
$$
\begin{aligned}
\dot{x_i} =& y_i - x_i^3 + bx_i^2 - z_i + g(x - V_S)\sum_{j \neq i} S_j \\
\dot{y_i} =& 1 - 5x_i^2 - y_i \\
\dot{z_i} =& \mu(s(x_i - x_{rest}) - z_i) \\
S_j =& \frac{1}{1 + E^{\lambda(v_j - \theta_s)}}
\end{aligned}
$$

The parameters are set as: $\lambda = -11$, $\theta_s = 0$, $V_S = -0.68$, $\mu = 0.01$, $s = 4$. The initial condition is:$x_0 = [-2; 0; 2]$;$y_0 = [-20; -20; -20]$; $z_0 = [-0.1; -0.1; -0.1]$;We vary b and $x_{rest} = -1.8$, to study the behavior of the netowork.



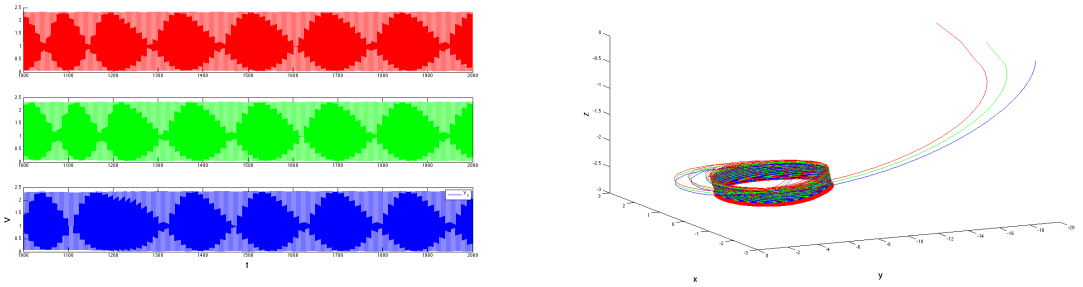
(a) (b)

Figure 4: $b = 2.5$,$x_{rest} = -1.6$, plateau-like bursting.



(a) (b)

Figure 5: $b = 2.7$, $x_{rest} = -1.6$, suqare-wave like bursting.



(a) (b)

Figure 6: $b = 2.5$, $x_{rest} = 1.6$.

In which the $\dot{z} = 0$ has intersection with the branch M_{eq}, so there's equilibrium state at given x_{rest}. It is located around the intersection point. The phase point moves slowly to the direction where $\dot{z} > 0$ while turning as long as it's above the z_{null} plane, and goes toward $\dot{z} < 0$ once it's lower than the z_{null} plane.
For a three cell network modeled using Hindmash-Rose model, the phase lag is hard to define.