Data Structures and Algorithms

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1 Introduction

1.1 Introduction

Definition 1.1.1 (Algorithm) A well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values, as output. An algorithm is thus a sequence of computational steps that transform the input into the output.

Definition 1.1.2 (Data Structure) A way to store and organize data in order to facilitate access and modifications.

Definition 1.1.3 (Abstract Data Type (ADT)) A data type that is defined by the operations that may be performed on it.

1.2 Describing algorithms

A complete description of an algorithm consists of three parts:

1. **The algorithm**
   - A description of the algorithm in a high-level language (English, pseudocode, etc).
   - Include clear specification of used data structures.
2. **Proof of correctness**
3. **Derivation of running time**

1.3 Sorting

Insertion sort

Input: A sequence of n numbers $A = \langle a_1, \ldots, a_n \rangle$
Output: A permutation of the input such that $\langle f(a_1) \leq f(a_2) \leq \cdots \leq f(a_n) \rangle$
initialize: sort $A[1]$ **for** $j = 2$ *to* $A.length$ **do**
    $key = A[j]$
    $i = j - 1$
    while $i > 0$ *and* $A[i] > key$ **do**
        $A[i + 1] = A[i]$
        $i = i - 1$
    end
    $A[i + 1] = key$
end

Loop invariants and the correctness of insertion sort

2 Algorithms

2.1 Analyzing algorithms

Definition 2.1.1 (Running time) The running time of an algorithm on a particular input is the number of primitive operations or "steps" executed.

Definition 2.1.2