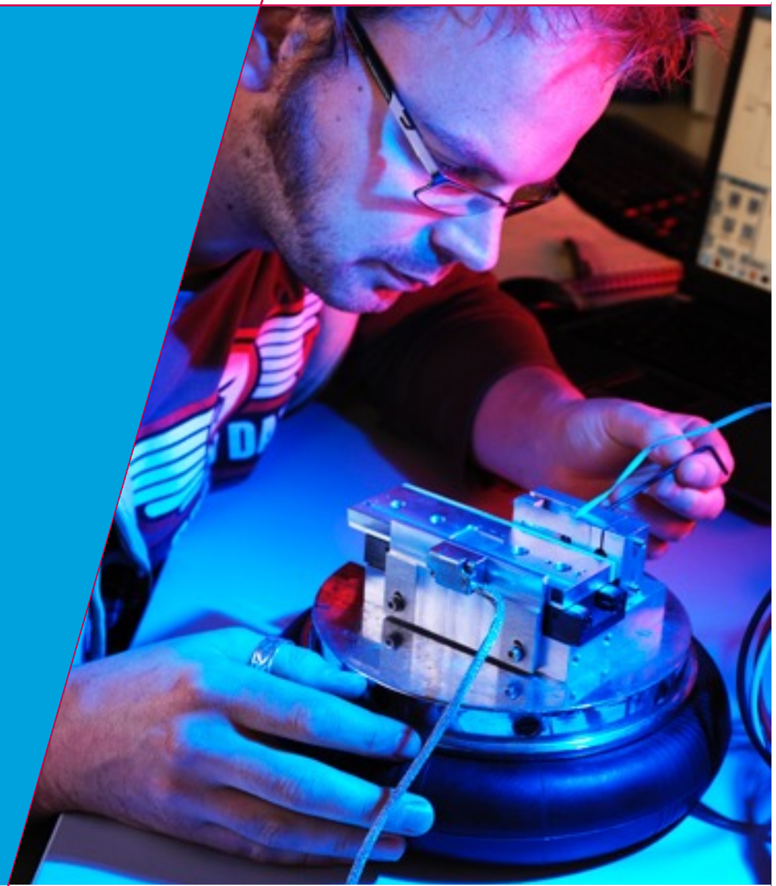
2IC30: Essential computer architecture. Number systems.



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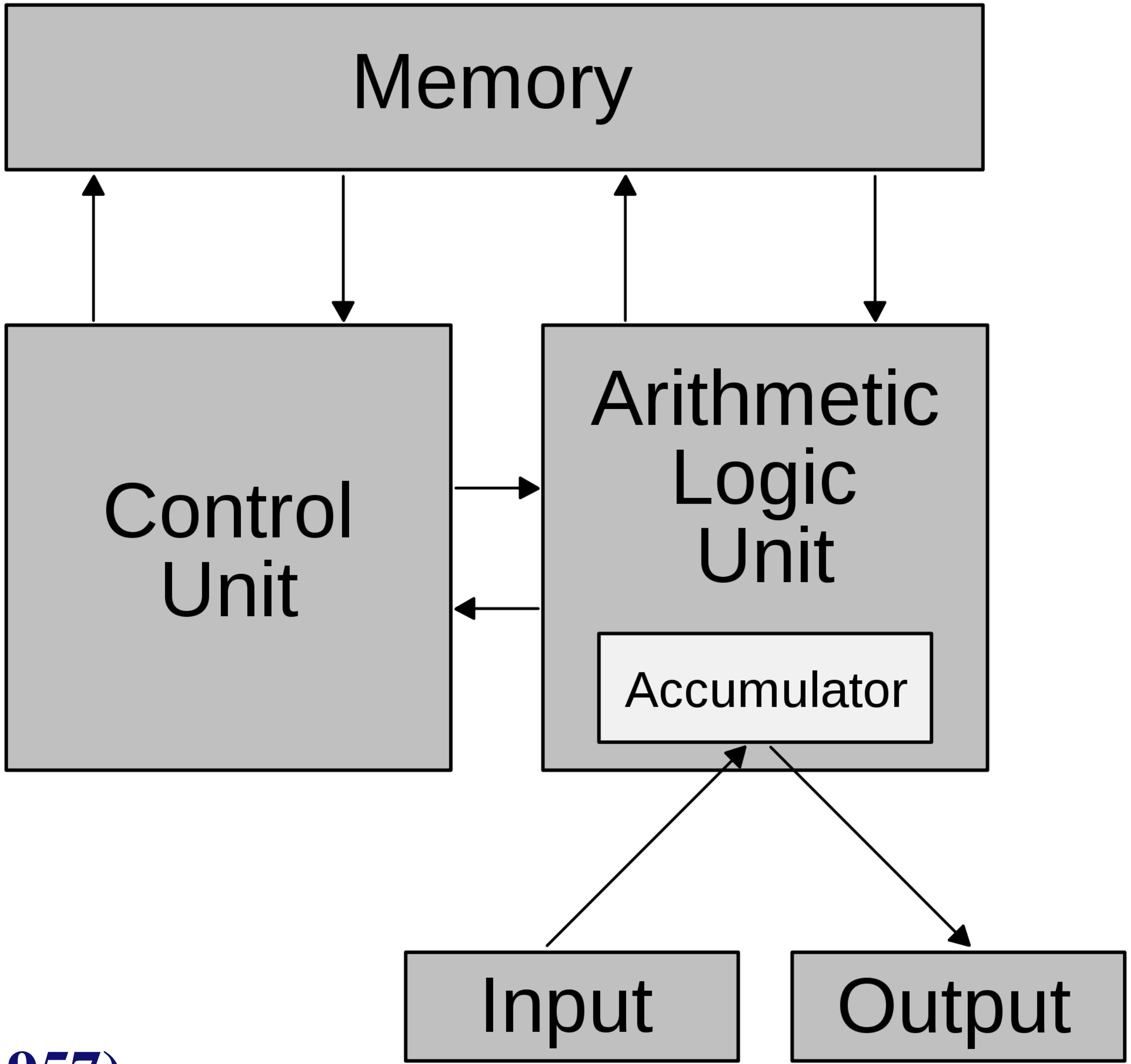
Where innovation starts

How does a computer work?

Von Neumann architecture.

Program is stored in memory.



(1903-1957)

How does a computer work?

Components

Arithmetical logical unit (ALU).

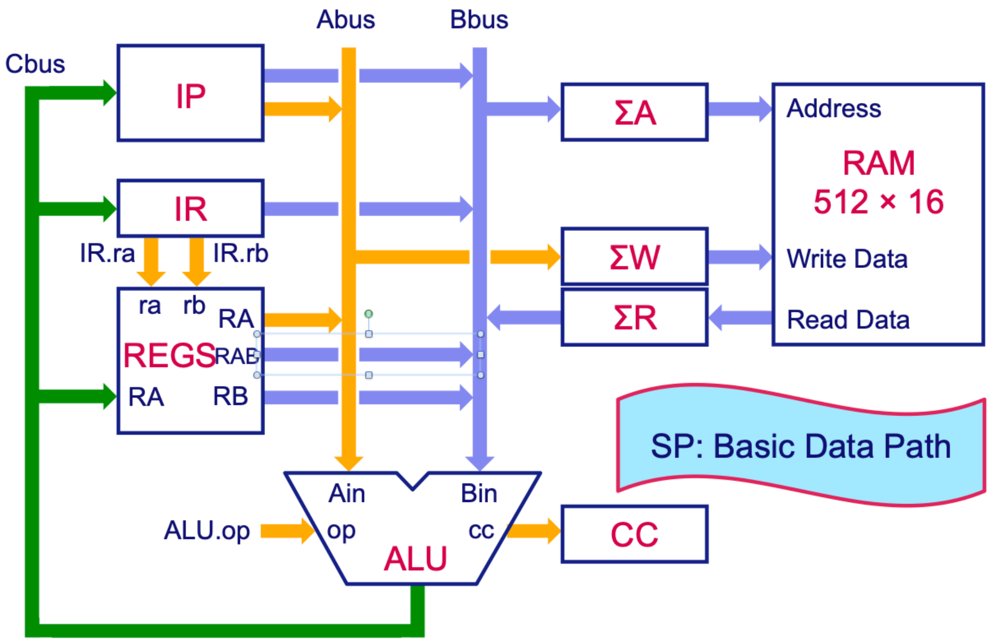
Registers (IP, REGS).

Multiplexers.

Memory address register (ΣA)

Memory Registers (ΣW, ΣR)

Instruction Register (IR)



ALU processes ‘numbers’.

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How do we represent and manipulate numbers?

- Number systems:
  - Represention of natural numbers
  - Represention of negative numbers
  - Addition, subtraction, multiplication, division, …
  - Overflow conditions
- Implementation:
  - Arithmetic circuits
  - Arithmetic Logical Units

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Positional number systems

1) base 10 (decimal; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

$$\langle 154\rangle_{10} = 1 \text{ x } 100 + 5 \text{ x } 10 + 4 \text{ x } 1$$
$$= 1 \text{ x } 10^2 + 5 \text{ x } 10^1 + 4 \text{ x } 10^0$$

2) base 2 (binary; 0, 1)

$$\langle 10011010\rangle_2 = 1 \text{ x } 2^7 + 0 \text{ x } 2^6 + 0 \text{ x } 2^5 + 1 \text{ x } 2^4 + 1 \text{ x } 2^3 + 0 \text{ x } 2^2 + 1 \text{ x } 2^1 + 0 \text{ x } 2^0$$
$$= 128 + 16 + 8 + 2 = 154$$

3) base 8 (octal; 0, 1, 2, 3, 4, 5, 6, 7)

$$\langle 232\rangle_8 = 2 \text{ x } 8^2 + 3 \text{ x } 8^1 + 2 \text{ x } 8^0$$
$$= 2 \text{ x } 64 + 3 \text{ x } 8 + 2 \text{ x } 1 = 128 + 24 + 2 = 154$$

4) base 16 (hexadecimal; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F)

$$\langle 9A\rangle_{16} = 9 \text{ x } 16^1 + 10 \text{ x } 16^0$$
$$= 144 + 10 = 154$$

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General formula

The value of a number $a_{n\text{-}1}\ a_{n\text{-}2}\ \ldots\ a_0$ in base b is:

$$\langle a_{n\text{-}1}a_{n\text{-}2}\ldots a_0\rangle_b =$$
$$a_{n\text{-}1}b^{n\text{-}1} + a_{n\text{-}2}b^{n\text{-}2} + a_{n\text{-}3}b^{n\text{-}3} + a_{n\text{-}4}b^{n\text{-}4}$$
$$+ \ldots + a_3b^3 + a_2b^2 + a_1b^1 + a_0b^0$$

$$\langle a_{n\text{-}1}a_{n\text{-}2}\ldots a_0\rangle_b = \sum_{i=0}^{n\text{-}1} a_i b^i$$

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Numbers in a table.

b_3	b_2	b_1	b_0	decimal	hexadecimal	octal
0	0	0	0	0	0	00
0	0	0	1	1	1	01
0	0	1	0	2	2	02
0	0	1	1	3	3	03
0	1	0	0	4	4	04
0	1	0	1	5	5	05
0	1	1	0	6	6	06
0	1	1	1	7	7	07
1	0	0	0	8	8	10
1	0	0	1	9	9	11
1	0	1	0	10	A	12
1	0	1	1	11	B	13
1	1	0	0	12	C	14
1	1	0	1	13	D	15
1	1	1	0	14	E	16
1	1	1	1	15	F	17

Octal machine.

Binary → octal → hexadecimal

recall: $\langle 154\rangle_{10} = \langle 10011010\rangle_2 = \langle 232\rangle_8 = \langle 9A\rangle_{16}$

grouping bits

$$\underbrace{10}_{2}\underbrace{011}_{3}\underbrace{010}_{2}\;{}_8 \qquad \underbrace{1001}_{9}\underbrace{1010}_{A}\;{}_{16}$$

why does this work?

Consider a n=$4k$ bits binary number → k digit hexadecimal number

$$a_{n-1}2^{n-1} + a_{n-2}2^{n-2} + a_{n-3}2^{n-3} + a_{n-4}\boxed{2^{n-4}} = 2^{4(k-1)}$$

$$+ \ldots + a_3 2^3 + a_2 2^2 + a_1 2^1 + a_0 2^0$$

$$= (a_{n-1}2^3 + a_{n-2}2^2 + a_{n-3}2^1 + a_{n-4}2^0)\boxed{2^{4(k-1)}}$$

$$+ \ldots + (a_3 2^3 + a_2 2^2 + a_1 2^1 + a_0 2^0)\boxed{2^{4(0)}}$$

$$= b_{k-1}16^{k-1} + \ldots + b_0 16^0$$ with b_i as digits

also works the other way round!

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Octal → hexadecimal

$$\langle 232 \rangle_8$$

$$\langle 10\ 011\ 010 \rangle_2$$

$$\langle 9\mathrm{A} \rangle_{16}$$

Base 10 → base X: division with remainder

$\langle 154 \rangle_{10}$

$154 / 3 =$	51	remains 1
$51 / 3 =$	17	remains 0
$17 / 3 =$	5	remains 2
$5 / 3 =$	1	remains 2
$1 / 3 =$	0	remains 1

$\langle 1\,2\,2\,0\,1 \rangle_3$

$= 1 + 18 + 54 + 81 = 154$

why does this work?

let $N = \langle a_{n-1}a_{n-2}\ldots a_1 a_0 \rangle_b$

$= a_{n-1}b^{n-1} + a_{n-2}b^{n-2} + \ldots a_1 b^1 + a_0 b^0$

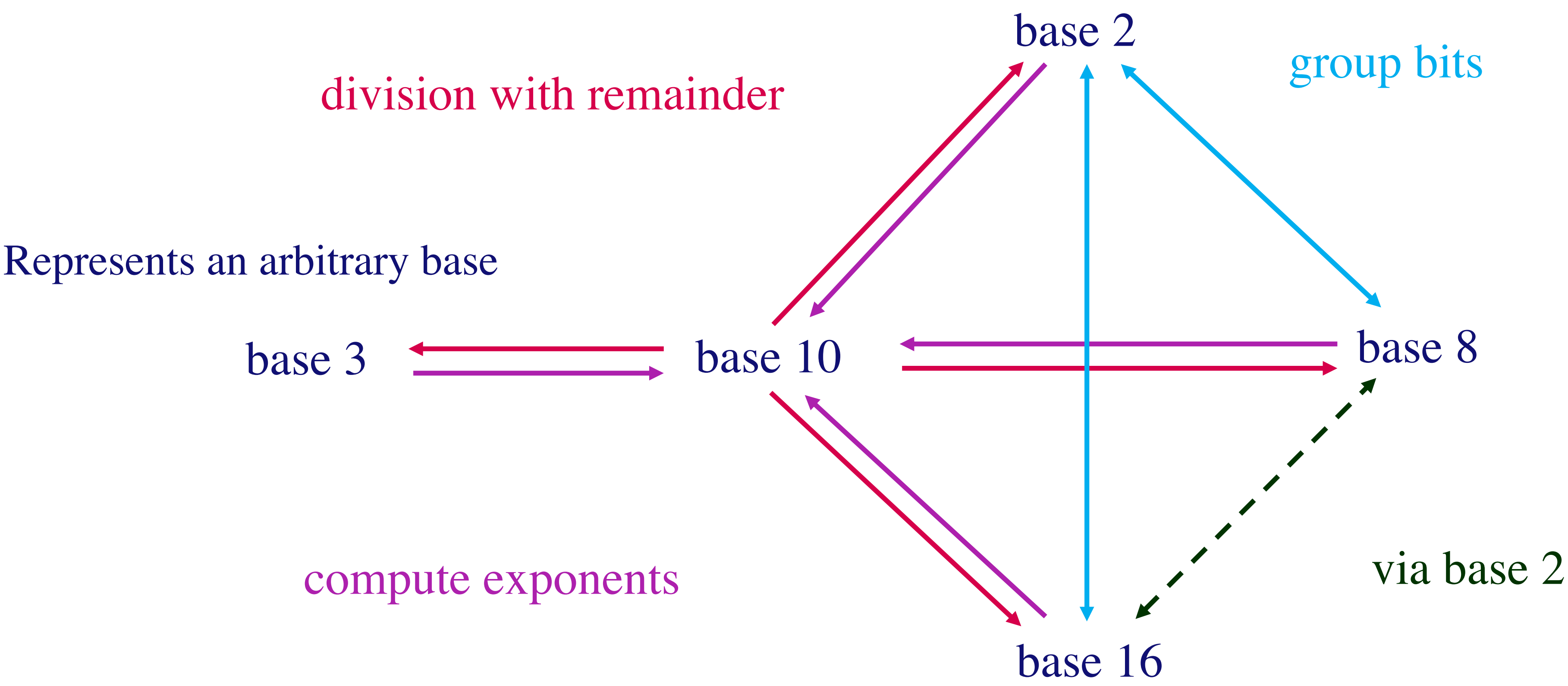
$Q_0 = N/b = \quad a_{n-1}b^{n-2} + a_{n-2}b^{n-3} + \ldots + a_1 b^0 \quad$ remains $\boxed{a_0}$

$Q_1 = Q_0/b = \quad a_{n-1}b^{n-3} + a_{n-2}b^{n-4} + \ldots + a_2 b^0 \quad$ remains $\boxed{a_1}$

$\vdots$

$Q_{n-1} = Q_{n-2}/b = \quad 0 \quad$ remains $\boxed{a_{n-1}}$

Conversions overview



Horner's rule to evaluate polynomials

$$a \times y^3 + b \times y^2 + c \times y^1 + d \times y^0$$

$=$ { isolate d and factor out y }

$$(a \times y^2 + b \times y^1 + c \times y^0) \times y + d$$

$=$ { isolate c and factor out y }

$$((a \times y^1 + b \times y^0) \times y + c) \times y + d$$

$=$ { isolate b and factor out y }

$$((a \times y + b) \times y + c) \times y + d$$

Hence: only 3 multiplications needed instead of 6.

Generally: evaluation of a polynomial with n+1 terms requires n multiplications, instead of ½n(n+1) if exponentiation is calculated by iterative multiplication.

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Horner's rule: conversion of binary to decimal

$\langle 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0 \rangle_2$

$=$ { definition of binary value }

$1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$=$ { Horner's rule }

$(((((((1 \times 2 + 0) \times 2 + 0) \times 2 + 1) \times 2 + 1) \times 2 + 0) \times 2 + 1) \times 2 + 0$

$=$ { arithmetic: inside out }

$((((((2 \times 2 + 0) \times 2 + 1) \times 2 + 1) \times 2 + 0) \times 2 + 1) \times 2 + 0$

$=$ { arithmetic }

$(((((4 \times 2 + 1) \times 2 + 1) \times 2 + 0) \times 2 + 1) \times 2 + 0$

$=$ { and so on ... }

$77 \times 2 + 0$

$=$ { arithmetic }

154 .

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Questions?

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Addition and subtraction

addition base 3

$$
\begin{array}{rrrrrl}
 & 1 & 1 & & 1 & \text{carry (remember)} \\
1 & 2 & 2 & 0 & 1 & \\
 & & 2 & 1 & 2 & + \\
\hline
2 & 0 & 1 & 2 & 0 &
\end{array}
$$

$(154 + 23 = 177)$

$(= 6 + 9 + 162 = 177)$

subtraction base 3

$$
\begin{array}{rrrrrl}
 & & 3 & 3 & 3 & \text{borrow} \\
1 & 2 & 2 & 0 & 1 & \\
 & & -2 & -1 & -2 & \\
 & -1 & -1 & -1 & & + \\
\hline
1 & 1 & 2 & 1 & 2 &
\end{array}
$$

$(154 - 23 = 131)$

$(= 2 + 3 + 18 + 27 + 81 = 131)$

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Arithmetic operations: Addition

Decimal:

$$\begin{array}{rccc} & 1 & 1 & \\ & & 9 & 7 \\ + & & 1 & 6 \\ \hline & 1 & 1 & 3 \end{array}$$

carries

Binary:

$$\begin{array}{rcccc} & 1 & 1 & 1 & \\ & & 1 & 1 & 1 \\ + & & 0 & 1 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

Arithmetic operations: Subtraction

Decimal:

$$
\begin{array}{rccc}
 & -1 & -1 & \\
 & 1 & 1 & 3 \\
- & & 1 & 6 \\
\hline
 & 0 & 9 & 7
\end{array}
$$

borrows

Binary:

$$
\begin{array}{rcccc}
 & -1 & -1 & -1 & \\
 & 1 & 0 & 1 & 0 \\
- & & 0 & 1 & 1 \\
\hline
 & 0 & 1 & 1 & 1
\end{array}
$$

Binary addition of 2 bits: Half Adder

$(c, s) \;=\;$ "binary representation of $a + b$"

half adder

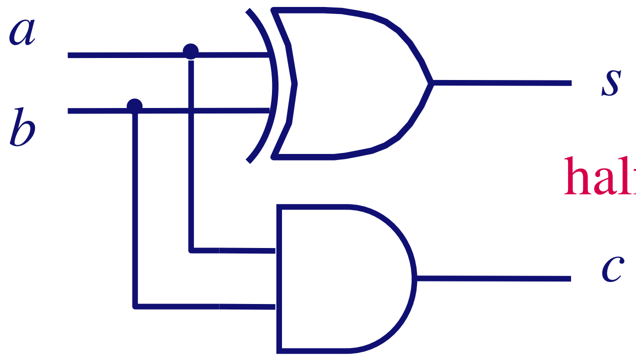
a	b	c	s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$b \backslash a$	0	1
0	0	$\boxed{1}$
1	$\boxed{1}$	0

$b \backslash a$	0	1
0	0	0
1	0	$\boxed{1}$

$$s \;=\; (\neg a \wedge b) \vee (a \wedge \neg b) \qquad\qquad c \;=\; a \wedge b$$

$$= \; a \oplus b \qquad \text{(exclusive OR)}$$



half adder

Binary addition of 3 bits: Full Adder

$$(cout, s) \quad = \quad \text{“binary representation of } a + b + cin \text{”}$$

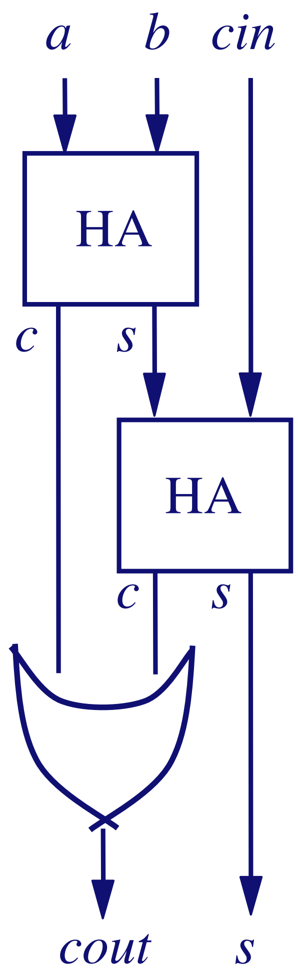
full adder

a	b	cin	$cout$	s
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

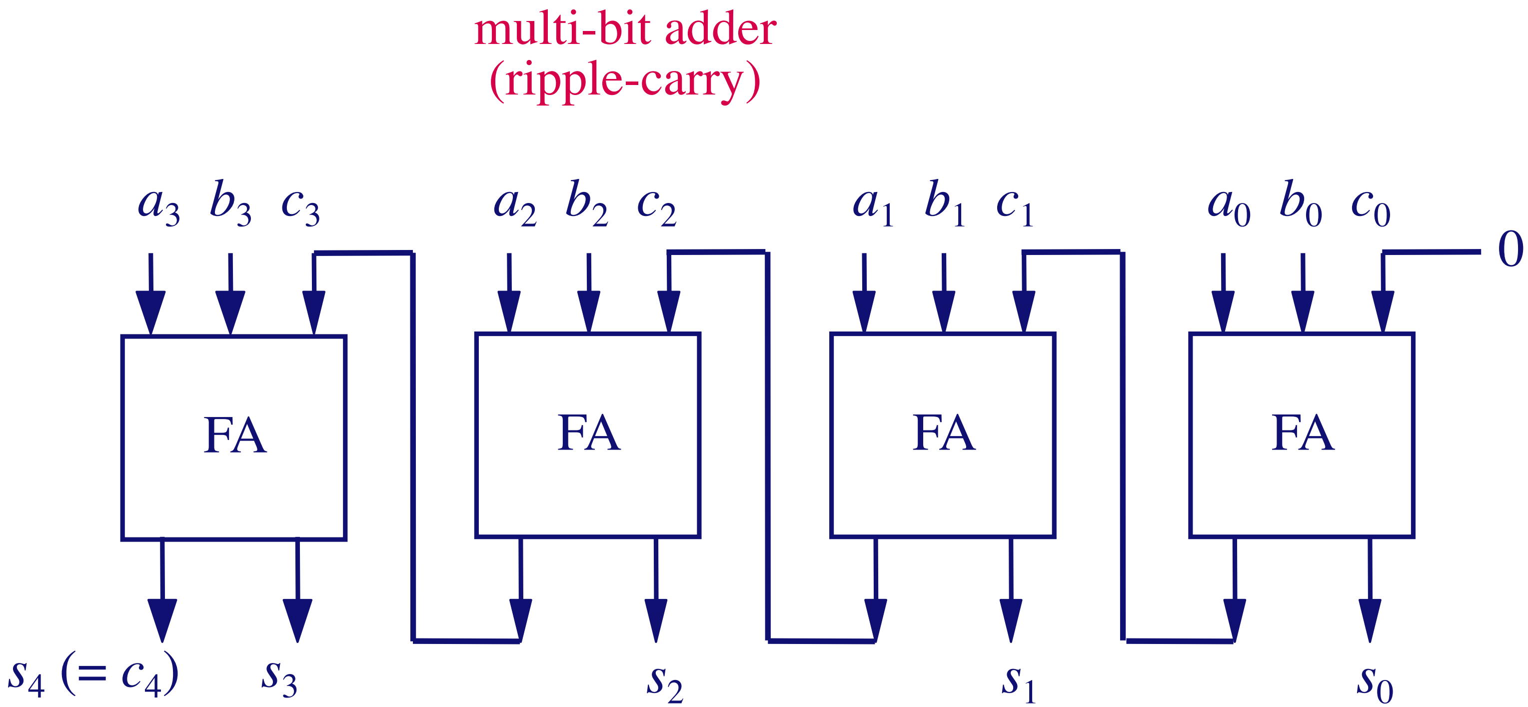
$$s \;=\; a \oplus b \oplus cin$$

$$\begin{aligned} cout &= (a \wedge cin) \vee (b \wedge cin) \vee (a \wedge b) \\ &= ((a \oplus b) \wedge cin) \vee (a \wedge b) \end{aligned}$$

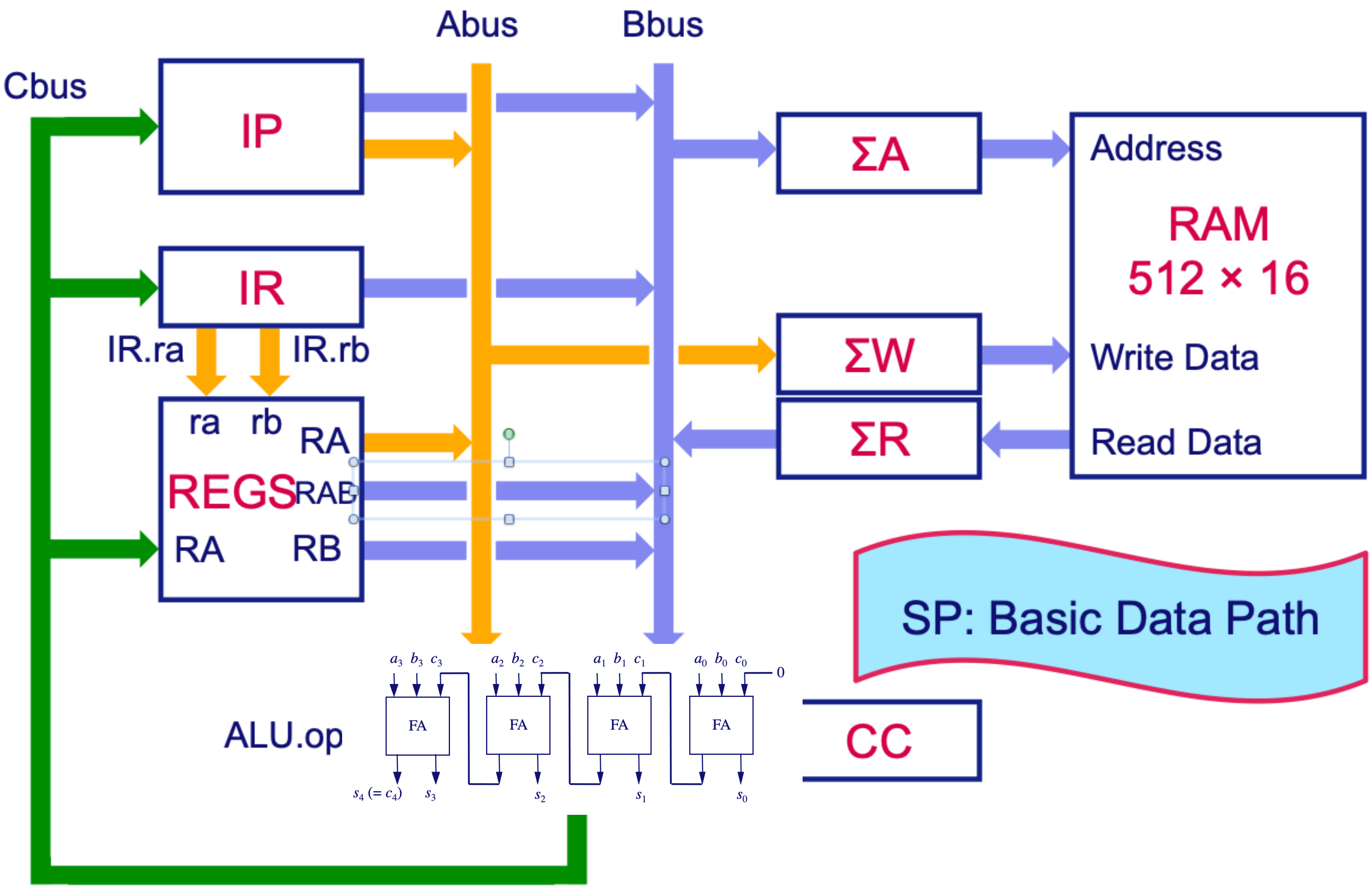
full adder



Binary addition of n-bit numbers



The adder is part of the ALU

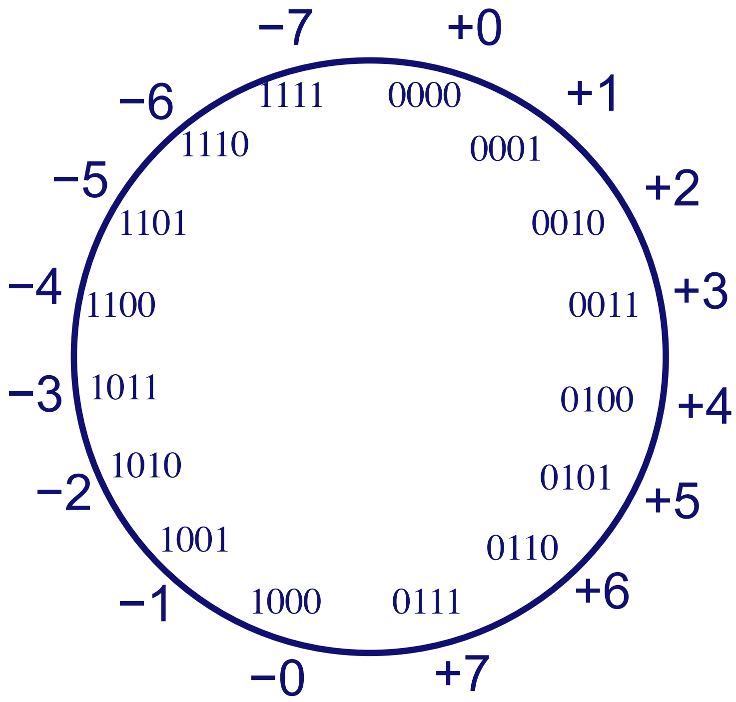


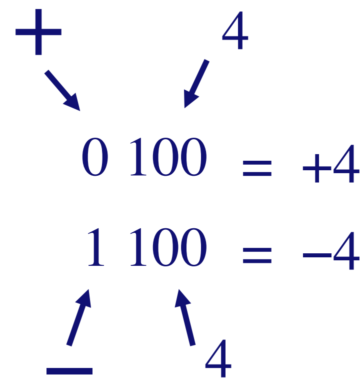
Questions?

Representing negative numbers

- Representation of natural numbers is usually the same for all systems.
- n+1-bits numbers: 2^{n+1} possible combinations, range: $0,\ldots,2^{n+1}-1$.
- Three different ways to represent negative numbers:
  - sign-and-magnitude;
  - 1s-complement;
  - 2s-complement.
- Sign-and-magnitude: used in *floating-point* numbers –*reals*–.
- 1s-complement is inconvenient for integer arithmetic.
- 2s-complement most convenient for integer arithmetic.

Sign-and-Magnitude Representation





Highest order bit is sign bit: 0 = positive (or zero), 1 = negative (or zero).

Three lower order bits are value: 0 (000) t/m 7 (111).

Range for n+1 bits $-(2^n-1) \ldots 2^n-1$, symmetric around 0.

2 representations for 0: +0 and –0.

Complicated addition and subtraction.

Sign-and-Magnitude: Addition and Subtraction

General remark

Subtraction is possible with negation and addition:

Adding number with equal sign bits:

directly add the value

result sign equal to
value signs

$$\begin{array}{r} 1 \\ 3 \\ \hline 4 \end{array}+ \;\Rightarrow\; \begin{array}{r} 0001 \\ 0011 \\ \hline 0100 \end{array} \qquad \begin{array}{r} -1 \\ -3 \\ \hline -4 \end{array}+ \;\Rightarrow\; \begin{array}{r} 1001 \\ 1011 \\ \hline 1100 \end{array}$$

Adding numbers with different sign bits:

how to implement
this?

$$\begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array}+ \;\Rightarrow\; \begin{array}{r} 0100 \\ 1011 \\ \hline 0001 \end{array} \qquad \begin{array}{r} -4 \\ 3 \\ \hline -1 \end{array}+ \;\Rightarrow\; \begin{array}{r} 1100 \\ 0011 \\ \hline 1001 \end{array}$$

subtract smallest from the biggest
and use the sign of the latter

requires subtraction AND comparison

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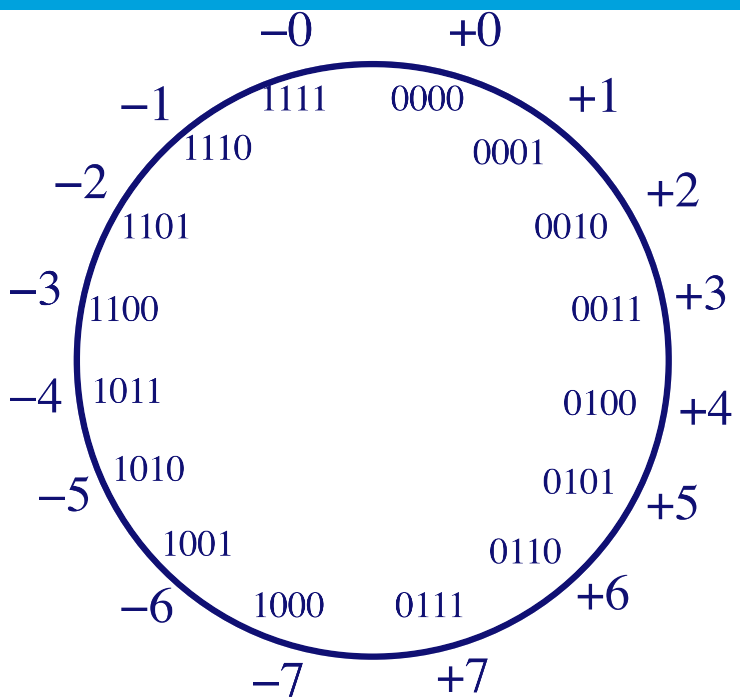
1s-complement



+ 4

$0\ 100_2 = +4$

$1\ 011_2 = -7 + 3 \quad = -4$

–

Highest order bit is sign bit: 0 = positive, 1 = negative.

Range for n+1 bits $-2^n+1,\ldots,2^n-1$, symmetric around 0.

Positive numbers: lower order bits are ''value''.

Negative numbers: lower order bits are: -2^n+1+ ''value'' (with n+1 bits).

A *double* representation for 0.

General formula

The value of a number $a_{n-1}\ a_{n-2}\ \ldots\ a_0$ in 1-complement is:

If a_{n-1}=0 then:

$$a_{n-2}2^{n-2} + a_{n-3}2^{n-3} + \ldots + a_3 2^3 + a_2 2^2 + a_1 2^1 + a_0 2^0$$

If a_{n-1}=1 then:

$$-\ (1-a_{n-2})2^{n-2} + (1-a_{n-3})2^{n-3} + \ldots + (1-a_3)2^3 + (1-a_2)2^2 + (1-a_1)2^1 + (1-a_0)2^0$$

$$\begin{cases} \displaystyle\sum_{i=0}^{n-2} a_i 2^i & \text{if } a_{n-1}=0, \\ \displaystyle -\sum_{i=0}^{n-2} (1-a_i)2^i & \text{if } a_{n-1}=1. \end{cases}$$

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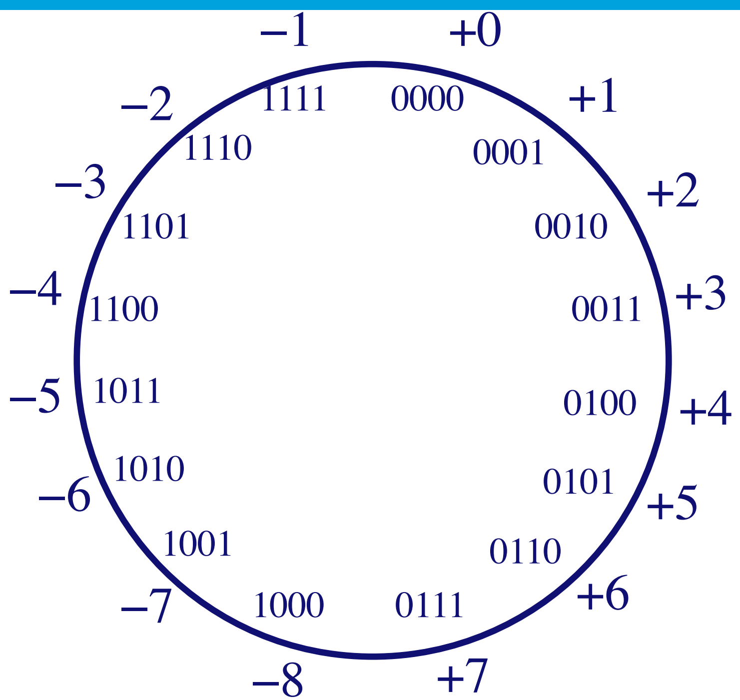
2s-complement (1)



$$\overset{+}{0}\;\overset{4}{100}_2 = +4$$

$$\underset{-}{1}\;\underset{3}{011}_2 = -8 + 3 = -5$$

Highest order bit is sign bit: 0 = positive (or 0), 1 = negative.

Range for n+1 bits $-2^n,\ldots, 2^n-1$, *not* symmetric around 0.

Positive numbers: lower order bits are ''value''.

Negative numbers: lower order bits are: -2^n + ''value'' (with n+1 bits).

A *single* representation for 0 and no complications with addition.

General formula

The value of a number $a_{n-1}\ a_{n-2}\ \ldots\ a_0$ in 2-complement is:

$$[a_{n-1}a_{n-2}\ldots a_0] = -a_{n-1}b^{n-1} + a_{n-2}b^{n-2} + a_{n-3}b^{n-3} + a_{n-4}b^{n-4} + \ldots + a_3b^3 + a_2b^2 + a_1b^1 + a_0b^0$$

$$[a_{n-1}a_{n-2}\ldots a_0] = -a_{n-1}2^{n-1} + \sum_{i=0}^{n-2} a_ib^i$$

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2-s complement (3)

Let m be a 2s complement number in n bits.

What is the 2s-complement n-bit representation of $-m$?

$$-m = -(-a_{n-1} \times 2^{n-1} + a_{n-2} \times 2^{n-2} + \ldots + a_1 \times 2^1 + a_0 \times 2^0)$$

$$= a_{n-1} \times 2^{n-1} - a_{n-2} \times 2^{n-2} - \ldots - a_1 \times 2^1 - a_0 \times 2^0$$

$$= a_{n-1} \times 2^{n-1} - a_{n-2} \times 2^{n-2} - \ldots - a_1 \times 2^1 - a_0 \times 2^0 \underbrace{+ \sum_{i=0}^{n-2} 2^i + 1 - 2^{n-1}}_{0}$$

$$= -(1-a_{n-1}) \times 2^{n-1} + (1-a_{n-2}) \times 2^{n-2} + \ldots + (1-a_1) \times 2^1 + (1-a_0) \times 2^0 + 1$$

This number is representable as an 2s-complement number, except if m is the smallest negative number.

In 4 bits: If m=1000, then $-m$ has value 0111 + 1, which is 8, which cannot be represented.
If m=1111, then $-m$ has value 0000 + 1, which is 1, represented by 0001.

2-s complement (3)

Let m be a 2s-complement number in n bits.

What is the 2s-complement n-bits representation of $-m$?

Calculate 2^n-m where m is interpreted as a positive number and the result is interpreted as an n digit number.

$$2^n - m = 2^n - \sum_{i=0}^{n-1} a_i \times 2^i$$

$$= 2^n - \sum_{i=0}^{n-1} a_i \times 2^i - 2^n + \sum_{i=0}^{n-1} 2^i + 1$$

$$= \sum_{i=0}^{n-1} (1-a_i) \times 2^i + 1$$

$$-m = -(1-a_{n-1})2^{n-1} + \sum_{i=0}^{n-2} (1-a_i) \times 2^i + 1$$

If 1-a_{n-1}=0, they are the same.

If 1-a_{n-1}=1, we interpret the first digit of the result at the left as negative and they are also equal.

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2-s complement (3)

Let m be a positive number.

What is the 2s-complement n+1-bits representation of $-m$?

Example: 2s-complement of –7

$$(2^4)_{10} = (1)0000_2$$
$$(7)_{10} = 0111_{2c} \; -$$
$$1001_{2c} = (-7)_{10}$$

also works with negative numbers!

$$(2^4)_{10} = (1)0000_2$$
$$(-7)_{10} = 1001_{2c} \; -$$
$$0111_{2c} = (7)_{10}$$

Fast method:
invert bits + 1

$$\underset{(7)}{0111} \longrightarrow 1000 + 1 \longrightarrow \underset{(-7)}{1001}$$

as does the fast method

$$\underset{(-7)}{1001} \longrightarrow 0110 + 1 \longrightarrow \underset{(7)}{0111}$$

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2-s complement: sign extension and truncation

- Question: what is –4 represented in 4 bits? Answer: 1 1 0 0 .
- Question: what is –4 represented in 8 bits?
- Answer: just *sign-extend* the 4-bits answer, by replicating the sign bit:

  1 1 1 1 1 1 0 0

- Question: what is –4 represented in 3 bits?
- Answer: just *truncate* the previous 4-bits answer, by deleting the sign bit:

  1 1 1 1 1 1 0 0

- If the sign bit is not changed by truncation the result is correct, but

  if the sign bit is changed the result is not representable: overflow!

2s-complement: Addition and Subtraction

As before,
Subtraction by addition and negation.

adding directly

$$\begin{array}{r} 1 \\ 3 \\ \hline 4 \end{array} + \;\Rightarrow\; \begin{array}{r} 0001 \\ 0011 \\ \hline 0100 \end{array} + \qquad \begin{array}{r} -1 \\ -3 \\ \hline -4 \end{array} + \;\Rightarrow\; \begin{array}{r} 1111 \\ 1101 \\ \hline 1\,1100 \end{array} +$$

ignore carry–out

$4 - 3 =$
$4 + (-3)$:
$-0011=1101$

$$\begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array} + \;\Rightarrow\; \begin{array}{r} 0100 \\ 1101 \\ \hline 1\,0001 \end{array} +$$

$-4 - -3 =$
$-4 + (--3)$

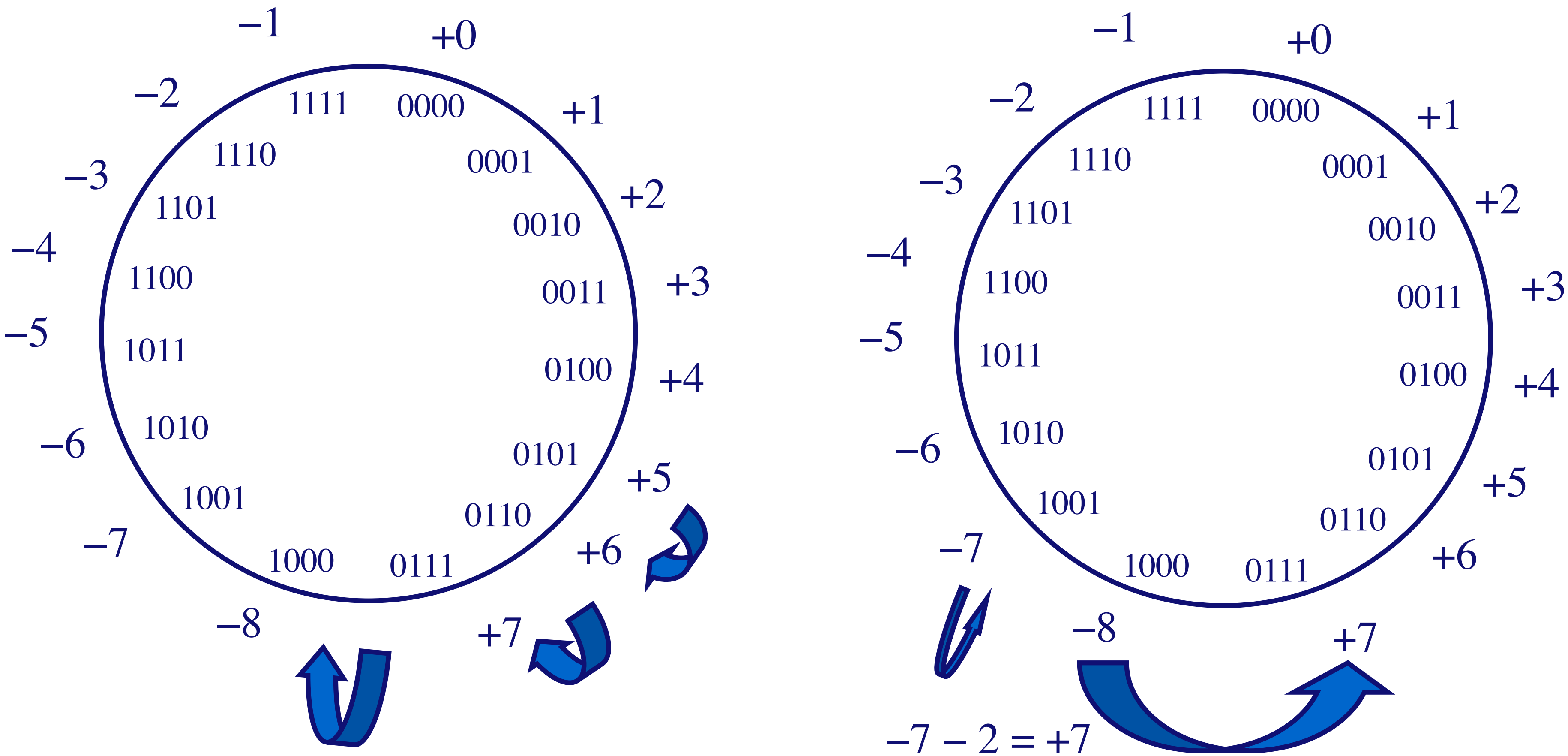
$$\begin{array}{r} -4 \\ 3 \\ \hline -1 \end{array} + \;\Rightarrow\; \begin{array}{r} 1100 \\ 0011 \\ \hline 1111 \end{array} +$$

Adder and bit inverter suffice for implementing addition AND subtraction.

Because of the simpler addition scheme, 2s-complement is the predominant choice for computations with integer number in digital systems.

Result not Representable: Overflow

Adding two positive numbers yields a negative number
or
adding two negative numbers yields a positive number.



−7 − 2 = +7

5 + 3 = −8

with addition, overflows only occur in numbers with equal signs!

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Detecting overflow (1)

carry-in ≠ carry-out

overflow

	0 1 1 1	(carries)
+5	0 1 0 1	
+3	0 0 1 1	
−8	0 1 0 0 0	

overflow

	1 0 0 0
−7	1 0 0 1
−2	1 1 1 0
+7	1 0 1 1 1

no overflow

carry−in = carry−out

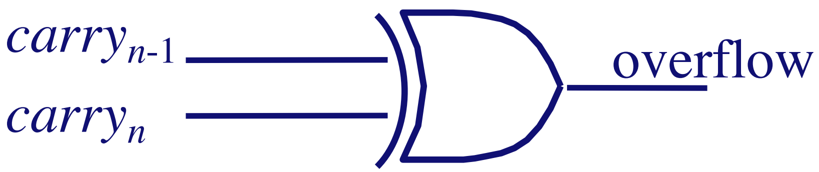
	0 0 0 0
+5	0 1 0 1
+2	0 0 1 0
+7	0 0 1 1 1

	1 1 1 1
−3	1 1 0 1
−5	1 0 1 1
−8	1 1 0 0 0

There is an overflow if and only if most significant carries are different.

Detecting overflow (1)

Overflow occurs when carry-in ≠ carry-out.



Detecting overflow (method for use by hand)

- *Overflow* occurs if the result of the addition/subtraction is *not representable* in the number of bits used.

- If we are working with n-bits 2s complement numbers:

  Sign-extend the two numbers with 1 additional bit, and calculate the result in n+1 bits. The result is representable in n+1 bits, say:

  $a_n \; a_{n-1} \; . \; . \; . \; a_2 \; a_1 \; a_0$ represents the result in n+1 bits; now:

  if $a_n = a_{n\text{-}1}$ the result is *also* representable in n bits: no overflow!

  if $a_{n+1} \neq a_{n\text{-}1}$ the result is *not* representable in n bits: overflow!

- Use?
- Calculations by hand: simple, but *don't forget* the sign-extension!
- Circuit implementations, if the last-but-one carry is not available.

Detecting overflow (method for use by hand)

overflow

	sign extend		sign extend
+5	0 0 1 0 1	−7	1 1 0 0 1
+3	0 0 0 1 1	−2	1 1 1 1 0
−8	0 1 0 0 0	+7	1 0 1 1 1

bits different? overflow!

no overflow

	sign extend		sign extend
+5	0 0 1 0 1	−3	1 1 1 0 1
+2	0 0 0 1 0	−5	1 1 0 1 1
+7	0 0 1 1 1	−8	1 1 0 0 0

bits equal? no overflow!

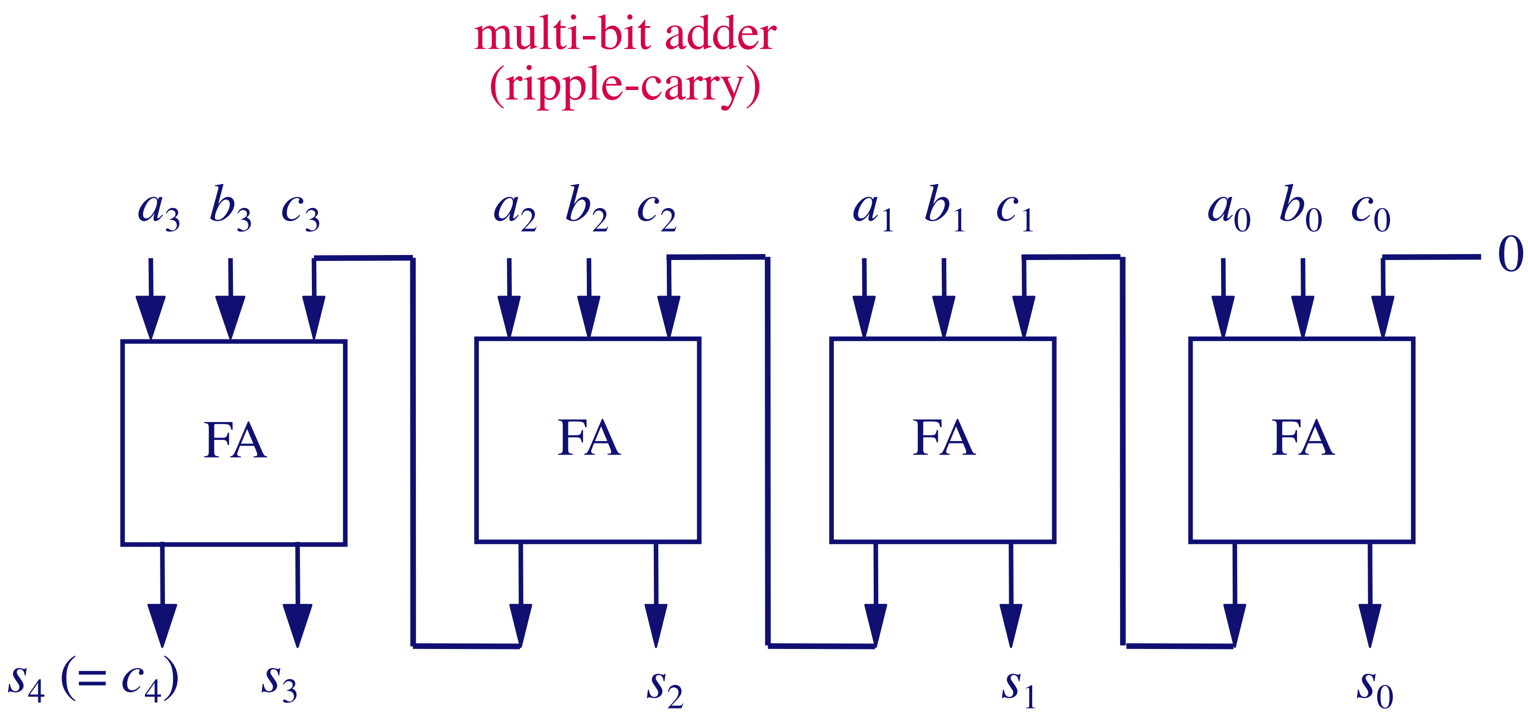
overflow if and only if the left-most 2 bits are *different*

Questions?

Nice theory: now what?

- We will implement all of this with logical gates
- but only for 2s-complement numbers
- and only for addition and subtraction

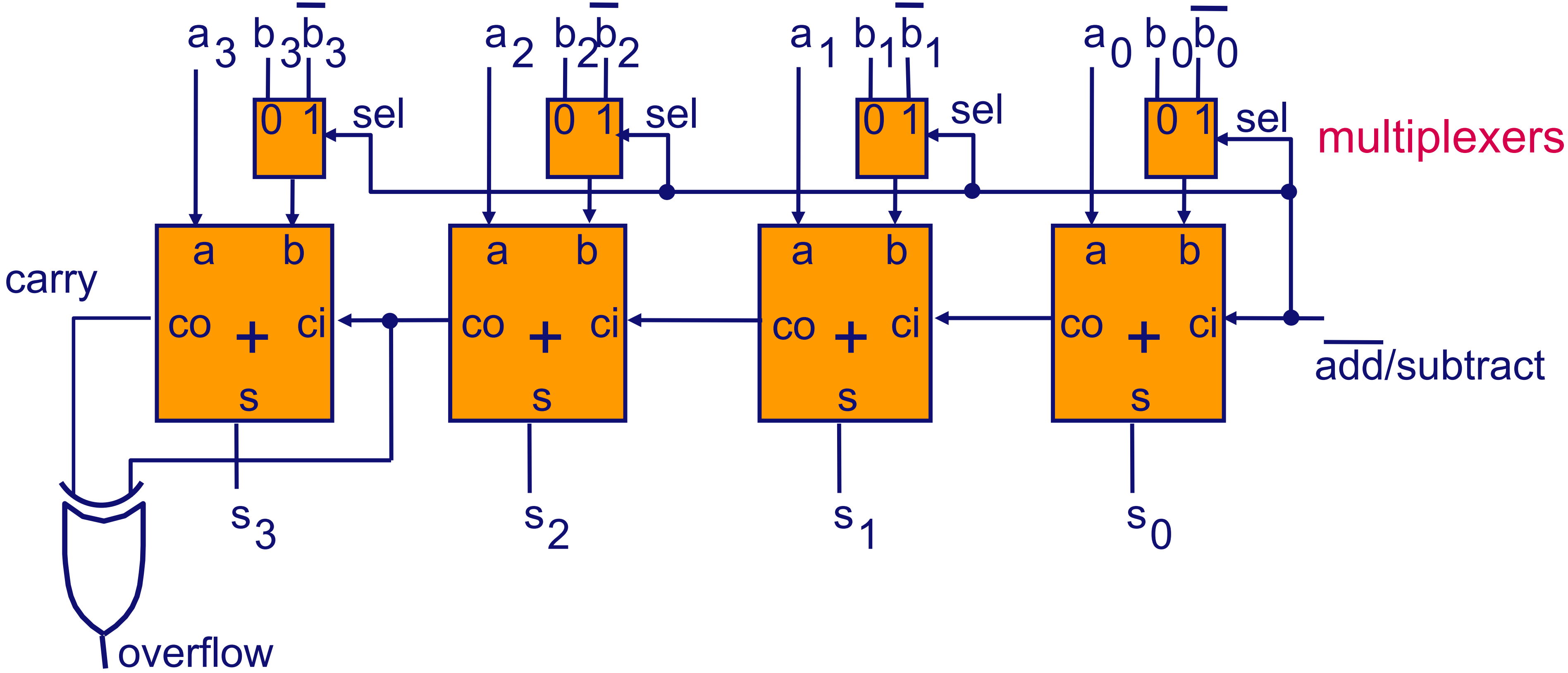
Binary addition of n-bit numbers



Addition and Subtraction in one circuit

$$(a + b)_{10} = (a + b)_{2c}$$

$$(a - b)_{10} = (a + (-b))_{10} = (a)_{2c} + \overline{b}_{2c} + 1$$



Addition and Subtraction in one circuit

$$(a + b)_{10} = (a + b)_{2c}$$

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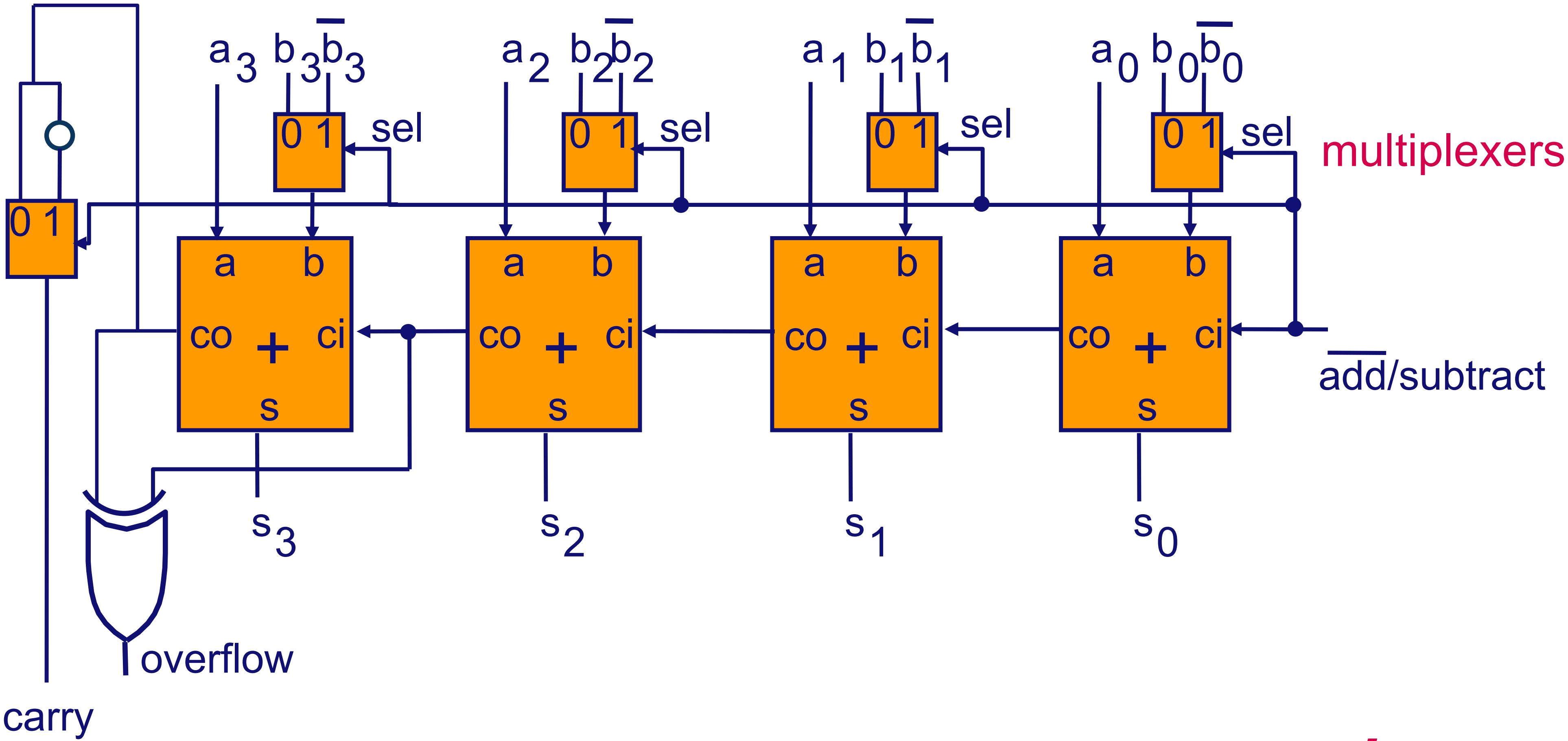
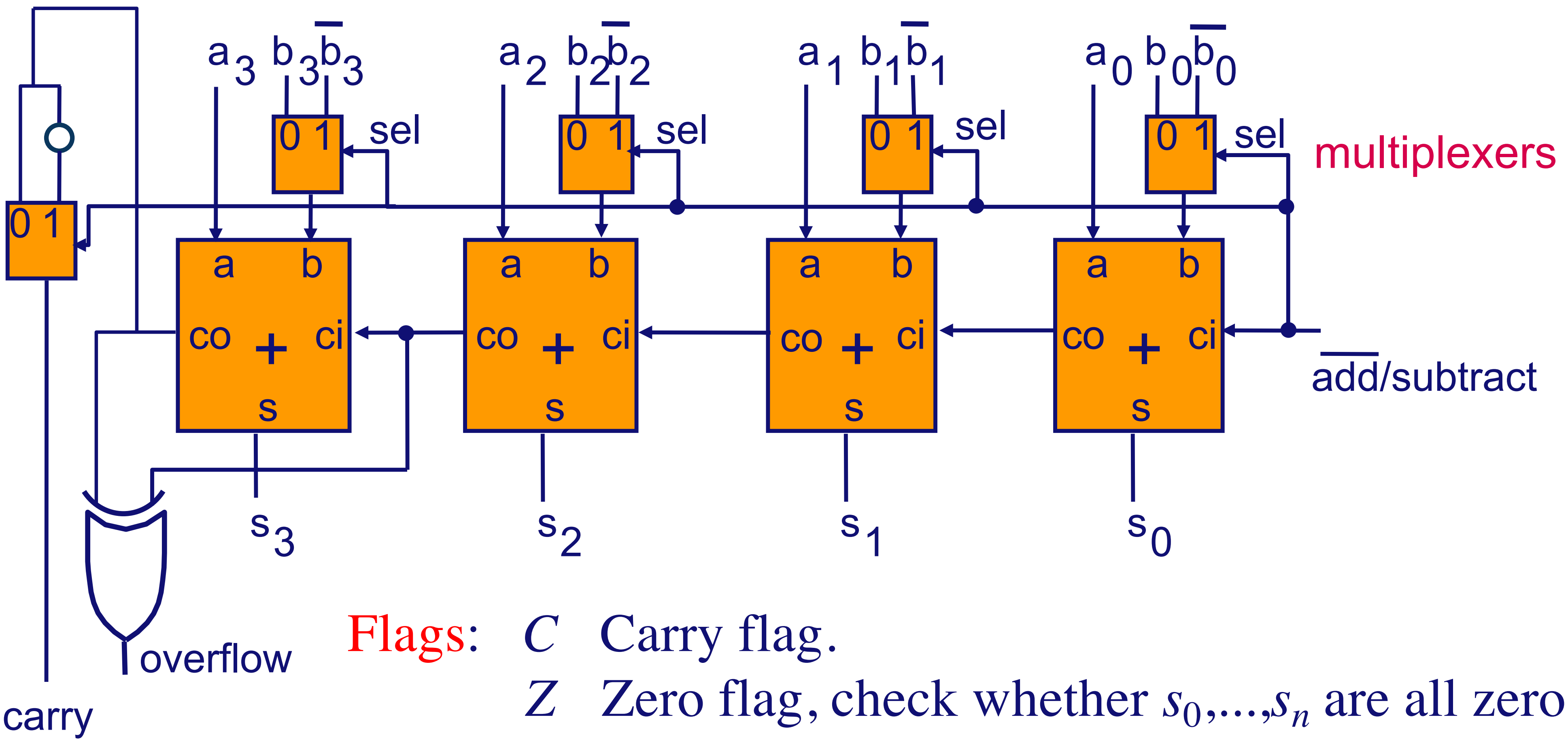


Addition and Subtraction in one circuit



Flags:

- C Carry flag.
- Z Zero flag, check whether $s_0,\ldots,s_n$ are all zero.
- N Negative flag. Equal to s_n.
- V Overflow flag.

Condition Codes: How to be used? (1)

Unsigned and Signed Comparisons

Bit Patterns	Unsigned Values	Two's complement values
0100 1101	+4 < +13 ??	+4 < -3 ??
1100 0011	+12 > +3 ??	-4 > +3 ??

Conclusion: the result of a comparison *depends* on how the bit patterns are interpreted: *unsigned* or *two's-complement*.

Calculate:

α-β

	unsigned	two's complement
$\alpha \leq \beta$	$C=1 \vee Z=1$	$N \neq V \vee Z=1$
$\alpha < \beta$	$C=1$	$N \neq V$
$\alpha = \beta$	$Z=1$	$Z=1$
$\alpha \neq \beta$	$Z=0$	$Z=0$
$\alpha > \beta$	$C=0 \wedge Z=0$	$N=V \wedge Z=0$
$\alpha \geq \beta$	$C=0$	$N=V$

Questions?

Summary

- We know how unsigned numbers are generally represented in other bases and can translate these to each other. Most interesting bases are: decimal, octal, hexadecimal and binary.
- We can design addition on numbers using a ripple adder.
- We know that negative numbers can be represented in (at least) three different ways: sign and magnitude, ones and two’s complement.
- We can add/subtract with negative numbers represented two’s complement, and know how to build a circuit to do that for two’s complement.

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