

# Assignment 9

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## 1 Exercise 12.4.4

**Problem** Let  $(X, \text{dist})$  be a metric space and let  $K \subseteq X$  be a compact subset. Let  $a : \mathbb{N} \rightarrow X$  be a sequence with values in  $X$ , such that

[label=(\*)] for all  $N \in \mathbb{N}$ ,  
there exists  $\ell \geq N$ ,  
 $a_\ell \in K$

1. Use (\*) to inductively define an index sequence  $n : \mathbb{N} \rightarrow \mathbb{N}$  such that for every  $k \in \mathbb{N}$ ,  $a_{n_k} \in K$ .
2. Use the fact that  $K$  is compact to show that there is a point  $p \in K$  and a subsequence of  $a : \mathbb{N} \rightarrow X$  converging to  $p$ .

*Proof.*

□

## 2 Exercise 12.4.5

**Problem**

*Proof.*

□

## 3 Exercise 13.11.1

**Problem**

*Proof.*

□

## 4 Exercise 13.11.2

**Problem**

*Proof.*

□