Assignment 11

Group 1.1

January 14, 2024

14.12.5, 14.12.9, 14.12.10.

1 Exercise 14.12.5

2 Exercise 14.12.9

Problem Let $f : (-\infty, 3) \to \mathbb{R}$ be a continuous function. Assume that

$$\lim_{x \to -\infty} f(x) = \infty$$

and

$$\lim_{x \uparrow 3} f(x) = \infty$$

Show that f attains a minimum on the interval $(-\infty, 3)$.

Proof. Since f is continuous on $(-\infty, 3)$, it holds that for all $a \in (-\infty, 3), \lim_{x \to a} f(x) = f(a)$. And since $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to 3} f(x) = \infty$ it holds that f is bounded below on $(-\infty, 3)$, i.e. $\exists m \in \mathbb{R}, \forall x \in (-\infty, 3), f(x) \geq m$.
Need to show that $\inf f(x) \in f(x)$, then $\min f(x) = \inf f(x)$ □

3 Exercise 14.12.10