Technische Universiteit
Eindhoven
University of Technology

Student name:

Student number:

Examination cover sheet

Course name: Introduction to Discrete Structures — Course code: 2IT80

Date: <Date>

Start time: <Start> — End time : <End>

Number of pages: 13

Number of questions: 9

Maximum number of points/distribution of points over questions:50

Method of determining final grade: divide total of points by 5

Answering style: open questions

Exam inspection: With the instructors at some later to be determined time

Points obtained per question:

1	2	3	4	5	6	7	8	9	Σ

Instructions for students and invigilators

All answers must be written in English and on these sheets. Any additional (scrap) paper used must be handed in with the exam. There is some extra space at the back of the exam if you run out of lines. If you continue an answer on the additional pages, you must clearly indicate this both on these pages and at the original question.

Permitted examination aids (to be supplied by students): absolutely no examination aids allowed!

Important:

- examinees are only permitted to visit the toilets under supervision
- it is not permitted to leave the examination room within 15 minutes of the start and within the final 15 minutes of the examination, unless stated otherwise
- examination scripts (fully completed examination paper, stating name, student number, etc.) must always be handed in
- the house rules must be observed during the examination
- the instructions of examiners and invigilators must be followed
- no pencil cases are permitted on desks
- examinees are not permitted to share examination aids or lend them to each other

During written examinations, the following actions will in any case be deemed to constitute fraud or attempted fraud:

- using another person's proof of identity/campus card (student identity card)
- having a mobile telephone or any other type of media-carrying device on your desk or in your clothes
- using, or attempting to use, unauthorized resources and aids, such as the internet, a mobile telephone, etc.
- using a clicker that does not belong to you
- having any paper at hand other than that provided by TU/e, unless stated otherwise
- visiting the toilet (or going outside) without permission or supervision

Question 1: [4 points] Let $A(n)$ be defined by $A(0) = 0$ and $A(n) = 5A(n-1)+1$ for integer $n \geq 1$. Prove that $A(n) = (5^n - 1)/4$ for integer $n \geq 0$ using induction on n.

Question 2: [2+2+3 points]

(a) Let $f\colon A \to B$ be a function. Show that for every $X, Y \subseteq A$ it holds that $f(X \cap Y) \subseteq f(X) \cap f(Y)$.

(b) Show $f(X \cap Y) = f(X) \cap f(Y)$ does not hold in general by giving an example function and showing that $f(X \cap Y) \neq f(X) \cap f(Y)$ for some choice of X and Y.

(c) Given the relations R_1, R_2, R_3 below on $\mathbb{R} \times \mathbb{R}$, indicate with checkmarks if they are reflexive, irreflexive, symmetric, antisymmetric or transitive in the table below (make sure to check every property that applies).

$(x_1, y_1)R_1(x_2, y_2)$ if and only if $|x_1 - x_2| + |y_1 - y_2| \leq 1$.

$(x_1, y_1)R_2(x_2, y_2)$ if and only if $x_1 < x_2$ and $y_1 < y_2$.

$(x_1, y_1)R_3(x_2, y_2)$ if and only if $x_1^2 + y_1^2 = x_2^2 + y_2^2$.

Relation	**Reflexive**	**Irreflexive**	**Symmetric**	**Antisymmetric**	**Transitive**
R_1	☐	☐	☐	☐	☐
R_2	☐	☐	☐	☐	☐
R_3	☐	☐	☐	☐	☐

Question 3: [2+2 points] Mario needs to make n pizzas each having a unique diameter (they are distinguishable). He has m indistinguishable slices of salami and he needs to decide how the salami is distributed across the pizzas.

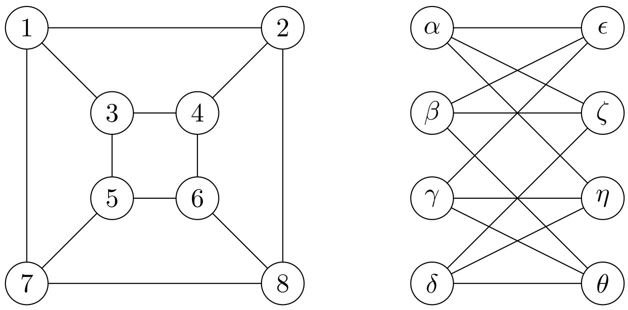
(a) How many ways are there to make the pizzas when each pizza can get arbitrarily many (or even zero) slices of salami. Argue your answer.

(b) Assume $m \leq n$. How many ways are there to make the pizzas when each pizza should get at most one slice of salami. Argue your answer.

Question 4: [5 points] Use the inclusion-exclusion principle to determine the number of permutations of the 26 letters of the English alphabet that *do not* contain any of the strings `fish`, `rat`, `bird`. *Note: You do not need to calculate the actual number without a calculator, but show what calculation would give the solution.*

Question 5: [1+4 points]

(a) Give an isomorphism for the following two graphs. You do not need to argue your answer.



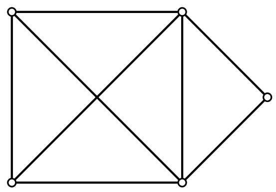
(b) Prove that a bipartite graph does not contain a cycle of odd length.

Question 6: [2+2+4 points]

(a) An Euler path has the same definition as a closed Eulerian tour, with the one notable difference that the Euler path does not need to start and end at the same vertex. Let $G = (V, E)$ be the graph (with 5 vertices), of which a drawing is given below.

1. Does G contain a closed Eulerian tour?
2. Does G contain an Euler path?

Argue your answers.

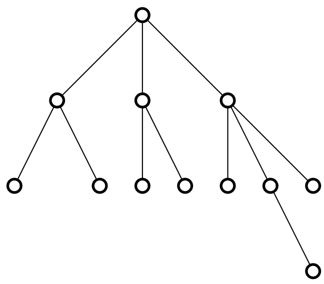


(b) Let $G = (V, E)$ be an undirected graph. For every $v \in V$ there exists a cycle that contains v. Can we be certain that G is 2-connected? Argue your answer.

(c) Recall that a DAG (Directed Acyclic Graph) is a directed graph that does not contain cycles and that a sink is a vertex in a directed graph with no outgoing edges. Prove that in any DAG $G = (V, E)$ (with at least one vertex) every longest path ends at a sink.

Question 7: [1+4 points] This problem is concerned with colorings of graphs, so any coloring discussed should be a valid graph coloring.

(a) Find a graph-coloring of the following tree and color the nodes accordingly using white (leave the node empty) and black (or whichever color your pen is).



(b) Prove that any tree $T = (V, E)$ can be colored using only two colors.

Question 8: [2+2+2 points]

(a) Let $G = (V, E)$ be a connected planar graph on n vertices such that each vertex has degree 3. How many faces are there in a planar drawing of G?

(b) Does there exist a planar graph with the degree sequence $(3, 4, 4, 5, 5, 5, 6)$? Either find one or prove that none exists.

(c) Does there exist a planar graph with $(1, 1, 1, 1, 4, 4, 4)$ as its degree sequence? Either find such a graph or prove that none exists.

Question 9: [2+4 points] In the following questions, start by giving a sample space and define the probability function:

(a) If two fair six-sided dice are tossed, what is the probability that they will show the same number?

(b) If 6 fair dice are tossed, what is the probability that they all show different numbers?