

The Ins and Outs of Barrier Options: Part 1

EMANUEL DERMAN AND IRAJ KANI

Barrier options are extensions of standard stock options. Standard calls and puts have payoffs that depend on one market level: the strike. Barrier options have payoffs that depend on two market levels: the strike and the barrier. Investors can use them to gain exposure to (or enhance returns from) future market scenarios more complex than the simple bullish or bearish expectations embodied in standard options. In addition, their premiums are usually lower than those of standard options with the same strike and expiration.

Part 1 of this article describes many of the most common barrier options available, as well as when to use them in place of standard options. We also describe why their sensitivity to market moves often exceeds that of standard options. This is depicted in several cases with tables and plots, showing the theoretical values and sensitivities for different market levels.

Part 2, which will appear in the Spring 1997 issue of *Derivatives Quarterly*, expands on this presentation. Somewhat more exotic barrier options are explored, and a summary of the mathematical theory behind the valuations is presented.

BARRIER OPTIONS DEFINED

Standard European-style options on stock are characterized by their time to expiration and strike level. At expiration, standard call options pay the owner the difference between the stock

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price and the strike level if the stock is above the strike, and zero otherwise. Similarly, standard put options pay the owner the difference between the strike and the stock price if the stock is below the strike, and zero otherwise. A call owner benefits from an upward stock move; a put owner benefits from a downward stock move.

Barrier options are a modified form of standard options that include both puts and calls. They are characterized by a strike level and a *barrier level*, as well as by a *cash rebate* associated with crossing the barrier. As with standard options, the strike level determines the payoff at expiration. The barrier options contract specifies, however, that the payoff depends on whether the stock price ever crosses the barrier level during the life of the option. In addition, if the barrier is crossed, some barrier option contracts specify a rebate to be paid to the optionholder.

This article discusses only European-style barrier options. Furthermore, in most cases we assume the rebate is zero.

Barriers come in two types. We call a barrier above the current stock level an *up barrier*; if it is ever crossed, it will be from below. We call a barrier below the current stock level a *down barrier*; if it is ever crossed, it will be from above.

Barrier options come in two types: *in options* and *out options*. An *in barrier option*, or *knock-in option*, pays off only if the stock finishes in the money and if the barrier is crossed sometime before expiration. When the stock crosses the barrier, the in barrier option is *knocked in* and becomes a standard option of the same type (call

or put) with the same strike and expiration. If the stock never crosses the barrier, the option expires worthless.

An *out barrier option*, or *knockout option*, pays off only if the stock finishes in the money and the barrier is never crossed before expiration. As long as the stock never crosses the barrier, the out barrier option remains a standard option of the same type (call or put) with the same strike and expiration. If the stock crosses the barrier, the option is *knocked out* and expires worthless.

Therefore, barrier options can be *up-and-out*, *up-and-in*, *down-and-out*, or *down-and-in*. The different barrier options and the effect of crossing the barrier on their payoff are described in Exhibit 1.

In principle, all barrier options can have associated cash rebates that are paid to the optionholder when the barrier is crossed. In practice, however, only out option contracts provide cash rebates, paid to the optionholder as a sort of consolation prize when the option is knocked out.

WHY USE BARRIER OPTIONS?

There are three basic reasons to use barrier options rather than standard options:

BARRIER OPTION PAYOFFS MAY MORE CLOSELY MATCH BELIEFS ABOUT THE FUTURE BEHAVIOR OF THE MARKET

Traders value options based on options theory. In liquid markets, you can value options

EXHIBIT 1

VARIETIES OF BARRIER OPTIONS

Option	Type	Barrier	Effect of Barrier on Payoff	
			Crossed	Not Crossed
Call	Down-and-Out	Below Spot	Worthless	Standard Call
	Down-and-In	Below Spot	Standard Call	Worthless
	Up-and-Out	Above Spot	Worthless	Standard Call
	Up-and-In	Above Spot	Standard Call	Worthless
Put	Down-and-Out	Below Spot	Worthless	Standard Put
	Down-and-In	Below Spot	Standard Put	Worthless
	Up-and-Out	Above Spot	Worthless	Standard Put
	Up-and-In	Above Spot	Standard Put	Worthless

by calculating the expected value of their payoffs, averaging over all stock scenarios whose mean price is the stock's forward price at each future time. According to the theory, you pay for volatility around the forward price.

By buying a barrier option, you can eliminate paying for those scenarios you think are unlikely. Alternatively, you can enhance your return by selling a barrier option that pays off only on scenarios you think are improbable.

Suppose the forward price of the stock a year from today is 105% of spot. You believe that the market is very likely to rise, but that if it somehow drops below a support level of 97%, it will instead decline further. You can buy a down-and-out call struck at 105% that gets knocked out if the stock, at any time, falls below 97%. In this way, you avoid paying for scenarios in which the stock first declines substantially and then rises again. At 20% volatility, this can reduce your premium by half.

Alternatively, you can enhance your return by selling a down-and-in call struck at 105% that gets knocked in only if the market falls below 97%. You can then pick up premium by betting against a scenario you feel is unlikely.

BARRIER OPTIONS MAY MATCH HEDGING NEEDS MORE CLOSELY THAN SIMILAR STANDARD OPTIONS

Suppose you own a stock and have decided that if it ever rises by more than 5% over the next year, you will sell it, but you still want protection against market declines of more than 5%. You can buy a standard put option struck at 95% of spot, but then you get protection even when the market rises more than 5% and you no longer need it. Instead, it would be better to buy an up-and-out put with strike at 95%, barrier at 105%, and no cash rebate. This put will disappear as soon as you sell the stock and have no further need of it.

BARRIER OPTION PREMIUMS ARE GENERALLY LOWER THAN THOSE OF STANDARD OPTIONS

Barrier options are often attractive to investors because their premium is lower than

corresponding standard options. For example, knockout options won't pay off if the stock crosses the knockout barrier. Therefore, they are cheaper than an otherwise identical option without the knockout feature. If you think the chance of knockout is small, you can take advantage of the lower premium and get the same benefits. Or, you can even pay more premium to receive a cash rebate if the option is knocked out.

Similarly, knock-in options have lower premiums than corresponding standard options with the same strike and expiration. You may find these attractively cheap if you feel the probability of getting knocked in is high.

SPECIAL FEATURES OF BARRIER OPTIONS

Managing the risk of an options position is more complicated than managing a stock position alone. You can dynamically hedge options by going short an amount *delta* of stock against a long options position, where delta is the theoretical hedge ratio. If delta is negative, you go long that amount of stock. The value of the option and its delta depend upon both market level and volatility. Standard call options, for example, have delta values between zero and one, and call values increase with increasing volatility.

Barrier options are similar to, but more complicated than, standard options, because they sample payoffs of a smaller and more specialized set of future stock paths. You can dynamically delta hedge them, just as you do with standard options, by using a theoretical model to value the option and calculate its delta. You can also use other options to create static hedges that approximately duplicate the barrier option's payoffs.

The price sensitivities of barrier options can differ markedly from those of standard options. Compare an up-and-out call option with a standard call option. As the stock price moves up, a standard call always gains in value, while an up-and-out call is subject to two opposing effects. As the stock price moves up, the up-and-out call's payoff becomes potentially larger, but the upward move simultaneously threatens to extinguish the total value of the contract by moving it closer to the out barrier. The conflict between these two tendencies makes it extremely sensitive to the

stock's movement near the barrier, and delta can go rapidly from positive to negative values.

We describe the behavior of barrier options in more detail as we examine each type. Meanwhile, be aware of two key ways in which barrier options differ from standard options when the underlying stock or index gets near the barrier level.

First, the delta of the barrier option can differ significantly from the delta of the corresponding standard option. For example, a barrier call can have delta values less than zero or greater than one; an up-and-out deep in-the-money call whose value must vanish at the barrier has a negative delta near the barrier because of the rapid decline in its value there.

Second, barrier option values can actually decrease with increasing volatility. The up-and-out call just described becomes more likely to get knocked out near the barrier as volatility increases. In options parlance, even though you are long volatility or "long gamma" relative to the strike level, you are short volatility or "short gamma" relative to the barrier.

You can often get a good feel for the way a barrier option will behave as the underlying stock price varies by classifying the strike and barrier levels with regard to volatility, and then combining their effects. The owner of a barrier option is long volatility at the strike, long volatility at an in barrier, and short volatility at an out barrier. We make use of this type of analysis later.

In certain cases, like the ones listed next, the strike level of an in option relative to the barrier is such that any non-zero payoff at expiration guarantees that the barrier will be crossed. Such European-style barrier options are identical in payoff and value to standard European-style options with the corresponding strike and expiration. For example:

- Any up-and-in call with strike above the in barrier is worth the same as a standard call, because all future stock paths that lead to payoff also lead to being knocked in.
- For similar reasons, any down-and-in put with strike below the in barrier is worth the same as a standard put.

Similarly, if the strike level of an out barrier

option relative to the barrier is such that any non-zero payoff guarantees the option will be knocked out, then the option is worthless. So, an up-and-out call with strike above the barrier is worthless. Also, a down-and-out put with strike below the barrier has no value.

There is a simple relationship between European-style in options, out options, and standard options. If you own both an in option and an out option of the same type (call or put), with the same expiration, the same respective strike, and the same respective barrier level, you are guaranteed to receive the payoff of the corresponding standard option whether the barrier is crossed or not. Therefore:

- The value of a down-and-in call (put) plus the value of a down-and-out call (put) is equal to the value of a corresponding standard call (put).
- The value of an up-and-in call (put) plus the value of an up-and-out call (put) is equal to the value of a corresponding standard call (put).

SOME EXAMPLES

In this section, we give examples of both out and in European-style options with zero rebate, and compare their theoretical values with those of corresponding European-style standard options. Later, we show how their values and sensitivities vary with market levels for a wide variety of barrier options.

The down-and-out call is close in value to the standard call, because it gets knocked out as the stock moves down to levels where the standard call has little value. The down-and-out put is worth much less than the standard put, because it gets knocked out as the stock price moves down to levels where the standard put becomes deep in the money.

The down-and-in call is worth much less than the standard call. It is knocked in only when the stock has made a large and unlikely downward move. The down-and-in put is close in value to the standard put, because the standard put gets most of its value from downward moves in the stock price, which would also trigger the knock-in of the down-and-in put.

Exhibits 2 and 3 show that the sum of the values of a down-and-in European option and a down-and-out European option equals the value of a corresponding standard European option.

The up-and-out call is worth only a fraction of the standard call's value. It is knocked out for those upward stock moves that contribute most of the value to the standard call. The up-and-out put is similar in value to the standard put. It is knocked out on upward stock moves to levels where the standard put is so far out of the money that it is almost worthless.

The up-and-in call is worth almost the same as the standard call. It is knocked in for those upward stock moves that contribute most of the value to the standard call. The up-and-in put is worth much less than the standard put. It is knocked in on upward moves to levels where the standard put is so far out of the money that it is almost worthless.

Exhibits 4 and 5 show that the sum of the values of an up-and-in European option and an up-and-out European option equals the value of a corresponding standard European option.

VALUES AND SENSITIVITIES OF BARRIER OPTIONS

This section presents plots of the values,

EXHIBIT 3

DOWN-AND-IN OPTIONS

Stock Price:	100
Strike:	100
Barrier:	90
Rebate:	0
Time to Expiration:	1 year
Dividend Yield:	5.0% (annually compounded)
Volatility:	15% per year
Risk-Free Rate:	10.0% (annually compounded)
Standard European	
Call Value:	7.840
Down-and-In	
Call Value:	0.618
Standard European	
Put Value:	3.749
Down-and-In	
Put Value:	3.463

deltas, and gammas of a variety of European-style barrier options with zero rebate. For comparison, we also present the same values and sensitivities for the corresponding standard options. Common parameters used in all plots are shown in Exhibit 6.

In all plots, delta is the number of shares of stock that has the same instantaneous exposure as the option has to infinitesimal changes in the

EXHIBIT 2

DOWN-AND-OUT OPTIONS

Stock Price:	100
Strike:	100
Barrier:	90
Rebate:	0
Time to Expiration:	1 year
Dividend Yield:	5.0% (annually compounded)
Volatility:	15% per year
Risk-Free Rate:	10.0% (annually compounded)
Standard European	
Call Value:	7.840
Down-and-Out	
Call Value:	7.222
Standard European	
Put Value:	3.749
Down-and-Out	
Put Value:	0.286

EXHIBIT 4

UP-AND-OUT OPTIONS

Stock Price:	100
Strike:	100
Barrier:	120
Rebate:	0
Time to Expiration:	1 year
Dividend Yield:	5.0% (annually compounded)
Volatility:	25% per year
Risk-Free Rate:	10.0% (annually compounded)
Standard European	
Call Value:	11.434
Up-and-Out	
Call Value:	0.657
Standard European	
Put Value:	7.344
Up-and-Out	
Put Value:	6.705

EXHIBIT 5**UP-AND-IN OPTIONS**

Stock Price:	100
Strike:	100
Barrier:	120
Rebate:	0
Time to Expiration:	1 year
Dividend Rate:	5.0% (annually compounded)
Volatility:	25% per year
Risk-Free Rate:	10.0% (annually compounded)
Standard European	
Call Value:	11.434
Up-and-In	
Call Value:	10.779
Standard European	
Put Value:	7.344
Up-and-In	
Put Value:	0.639

stock price. You have to short delta shares of stock to dynamically hedge the option. Rapid changes in delta with stock price (high gamma) make the stock difficult to hedge.

Gamma is the sensitivity of delta to changes in stock price. We define it as the ratio of the change in delta for an infinitesimal change in stock price to the corresponding percentage change in stock price.¹ Therefore, gamma is a measure of the degree to which delta hedging becomes more difficult and inaccurate as the stock price changes.

Think of gamma as the risk inherent in trying to trade a hedged options position, or as a measure of the option's sensitivity to volatility. An option with greater gamma must be rehedged more frequently; it takes more premium to pay for greater rebalancing costs.

We plot the value, delta, and gamma for each option as the stock price varies, for one year,

EXHIBIT 6**BARRIER OPTION PARAMETERS**

Strike:	100
Barrier:	80 if down, 120 if up
Rebate:	0
Dividend yield:	5.0% (annually compounded)
Volatility:	20% per year
Risk-free rate:	10.0% (annually compounded)

six months, and one month to expiration. Some of the features we comment on, for example, negative gamma, may be specific to the particular barrier option plotted, and may not apply to similar options with different barrier levels, different market parameters, or different times to expiration.

In the following plots we assume all barrier options have zero rebate. The rebate contribution to the value and sensitivity of barrier options can be separated from the option payoff itself. The rebate associated with an up barrier has the contribution of the *binary up-and-in call*, which we explain in Part 2 of this study (in the Spring 1997 issue of *Derivatives Quarterly*). The rebate associated with a down barrier has the contribution of the *binary down-and-in put*, which will also be explained in Part 2.

INTERPRETING THE PLOTS

Take care in interpreting the plots in Exhibits 7-16. From the point of view of delta-hedging risk, the important regions of stock price are those where delta changes rapidly, and the magnitude of gamma, positive or negative, is correspondingly large. For barrier options, there are often two separate regions where gamma becomes large.

The first is exactly at the barrier. There, a small change in stock price causes a discontinuous change in delta, and gamma becomes infinite. To illustrate, see Exhibit 10, where the call is knocked out at a barrier at 120. Just to the left of this region, the gamma of the one-month up-and-out call is positive. To the right, the call has been knocked out and gamma is zero. At the barrier, gamma is infinite, reflecting the one-time risk that, if the stock price is just below the barrier and moves just above it, continuous delta hedging is impossible.

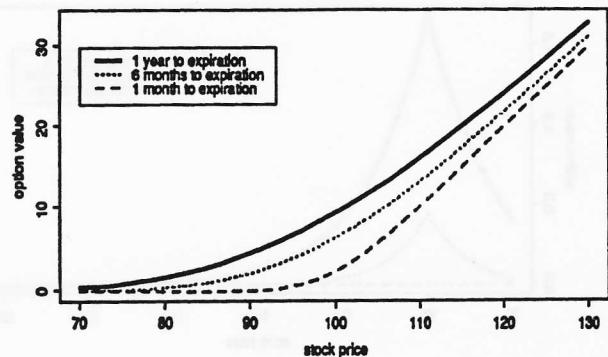
We cannot plot the infinite value of gamma at the barrier. Therefore, for all barrier options, the values of gamma and delta that we display at the barrier are, strictly speaking, the values just left and just right of the barrier. But remember that, even though we don't display it, gamma is infinite at the barrier.

The second region where gamma becomes large is near (but away from) the barrier, where its major effect is felt. It is easy to see this region in

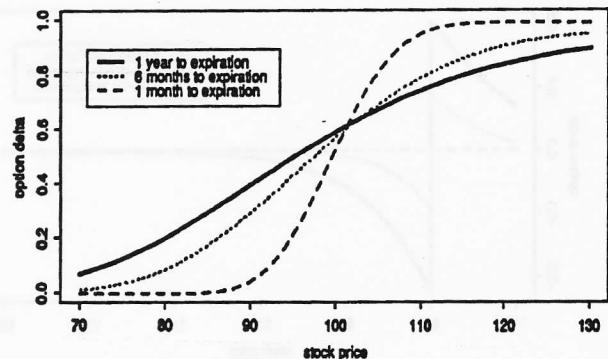
EXHIBIT 7

STANDARD CALL: STRIKE 100

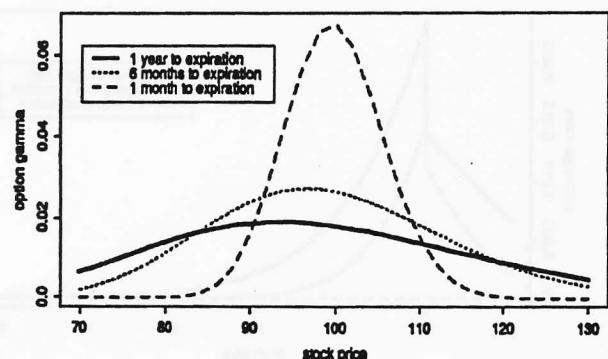
PANEL A. VALUE



PANEL B. DELTA



PANEL C. GAMMA



all of our plots. In Exhibit 10, it is the region with the peak of large negative gamma around a stock price of 112. If the stock price oscillates in this type of high-gamma region, the frequent readjustments of delta can generate large rehedging costs.

In Panel A of Exhibit 7, the value of the standard call increases with increasing stock price

and with time to expiration. As expiration approaches, the call value nears the terminal payoff — zero for stock prices below the strike level of 100, increasing proportionally to the stock price above 100.

At very low stock prices, delta approaches zero (Panel B). You have almost no exposure to the stock. At high stock prices delta approaches one. Owning a deep in-the-money call is like owning the stock itself. The transition from a delta of zero to a delta of one is sharper the closer you are to expiration.

The gamma of the standard call is always positive (Panel C) and is greatest for an at-the-money call where continuous hedging is most difficult. The peak gets more pronounced as the time to expiration nears.

As Panel A of Exhibit 8 shows, the value of the down-and-out call decreases as the stock declines, and must vanish at the barrier where a standard call would still have a small value. The down-and-out call is always worth less than the corresponding standard call, but approaches the same value at very high stock prices.

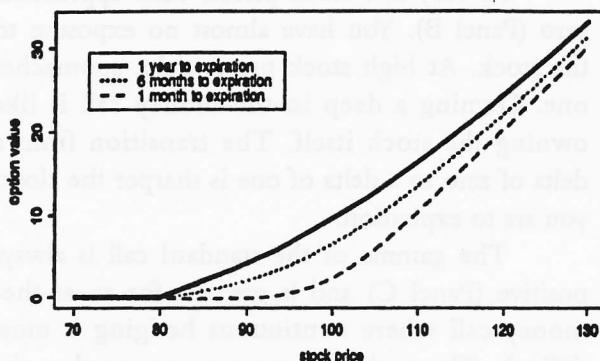
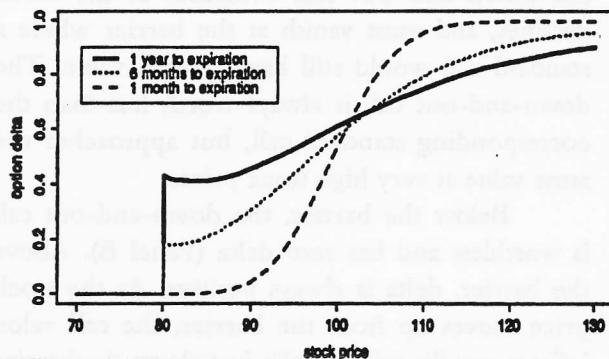
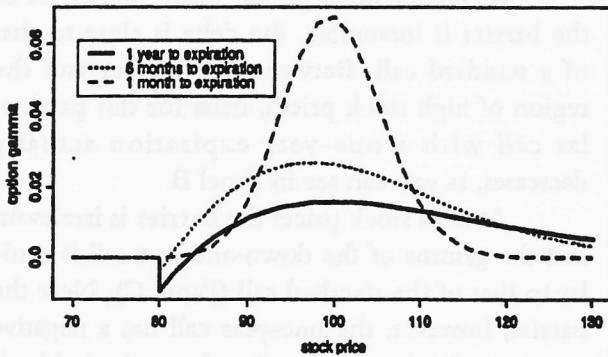
Below the barrier, the down-and-out call is worthless and has zero delta (Panel B). Above the barrier, delta is always positive. As the stock price moves up from the barrier, the call value inflates rapidly, with a delta just above the barrier that can be larger than that of the corresponding standard call.

At higher stock prices, where the effect of the barrier is intangible, the delta is close to that of a standard call. Between the barrier and the region of high stock prices, delta for this particular call with a one-year expiration actually decreases, as you can see in Panel B.

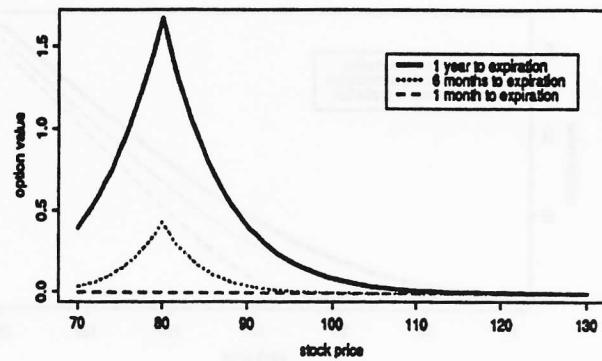
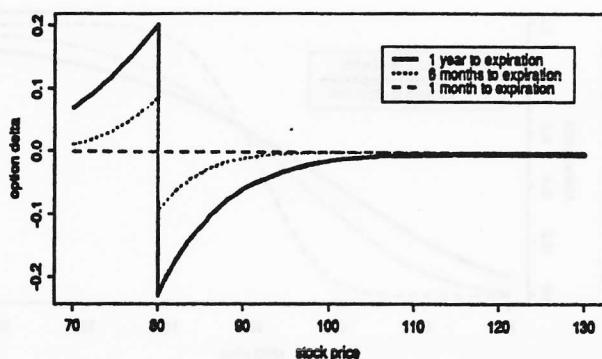
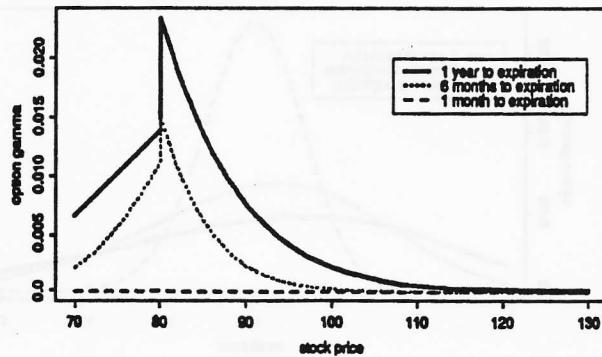
At high stock prices the barrier is irrelevant and the gamma of the down-and-out call is similar to that of the standard call (Panel C). Near the barrier, however, the one-year call has a negative gamma, reflecting the fact that the optionholder is really “short volatility,” or short an option struck at the barrier.

For stock prices near the barrier, an increase in volatility would actually make knockout for this particular option more likely, and so decrease overall option value. Gamma is infinite at the barrier.

As Panel A of Exhibit 9 shows, for stock

EXHIBIT 8**DOWN-AND-OUT CALL:
STRIKE 100, BARRIER 80****PANEL A. VALUE****PANEL B. DELTA****PANEL C. GAMMA**

prices below 80, the call is knocked in and worth the same as a standard call. Its value increases as the stock price rises. Above 80 the call is not yet knocked in. As the stock price increases, the chance of getting knocked in declines, and the call falls in value. Its value is greatest at the barrier. For

EXHIBIT 9**DOWN-AND-IN CALL:
STRIKE 100, BARRIER 80****PANEL A. VALUE****PANEL B. DELTA****PANEL C. GAMMA**

all stock prices, it is always worth less than the corresponding standard call.

Below the barrier, the delta is that of a standard call and rises with stock price (Panel B). Just above the barrier the option owner is effectively short stock (negative delta), because upward

stock moves make knock-in less likely and decrease option value. The sharp change from positive to negative delta across the barrier makes hedging extremely difficult. As the stock moves below 80, you would have to suddenly switch from being long stock to being short.

Gamma is always positive here, as Panel C shows, because the call owner is long volatility at both the strike and the barrier. Gamma is infinite at the barrier where there is a discontinuous change in exposure.

In Panel A of Exhibit 10, above the strike of 100 the call moves in the money, and would increase in value from then on were it not for the up-and-out feature that extinguishes its value at 120. This accounts for the peak in the distribution.

For low stock prices, the exposure increases as the stock moves up toward the strike (Panel B). Nearer the barrier, delta decreases and becomes negative above 110, because of the value-quenching effects of the barrier.

The call owner is long gamma near the strike and short gamma near the barrier (Panel C). The large negative gamma between 110 and 120, corresponding to the rapid change from positive to negative delta near 110 in Panel B, makes hedging inaccurate and difficult, and makes transaction costs large. Gamma is infinite at the barrier.

As Panel A of Exhibit 11 shows, the up-and-in call will knock in only if the stock crosses the barrier before expiration. In that case, the option immediately becomes a deep in-the-money call. That is why the value of the one-month call increases so sharply as the stock approaches 120.

Analogously, as Panel B shows, at 120 the delta of the one-month call spikes far above the delta of a standard call. Above 120, the call has knocked in and has the same positive delta as a deep in-the-money standard call.

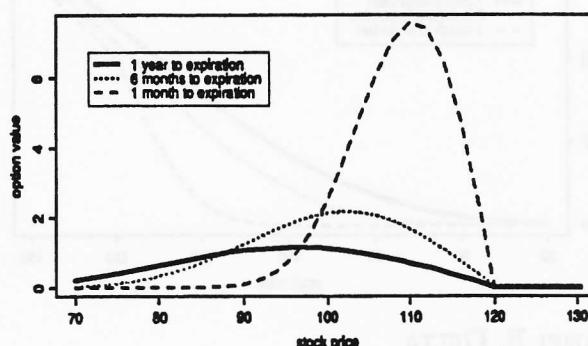
The owner of the call is long considerable volatility at the 120 barrier, whose crossing suddenly brings into existence a valuable deep in-the-money option (Panel C). The call close to expiration has a gamma about three times larger at 110 than that of the corresponding standard call at the strike of 100. Gamma is infinite at the barrier.

In Panel A of Exhibit 12, the value of the standard put increases with decreasing stock price.

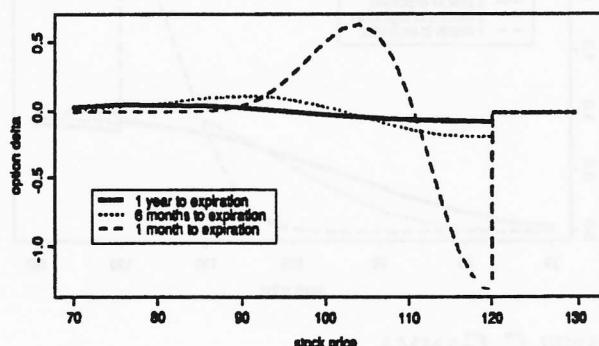
EXHIBIT 10

UP-AND-OUT CALL: STRIKE 100, BARRIER 120

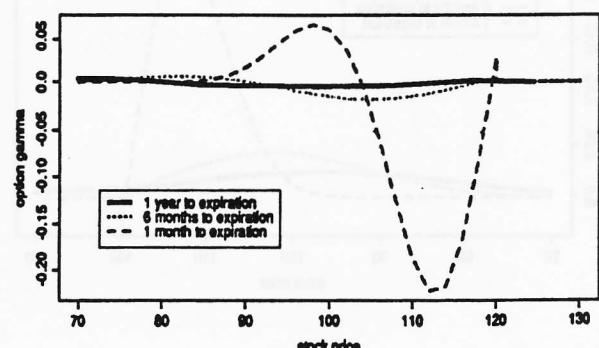
PANEL A. VALUE



PANEL B. DELTA

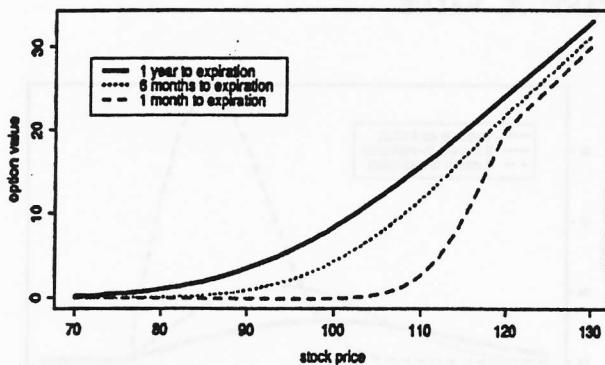
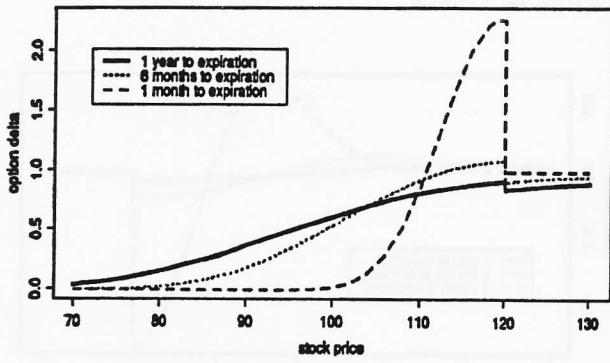
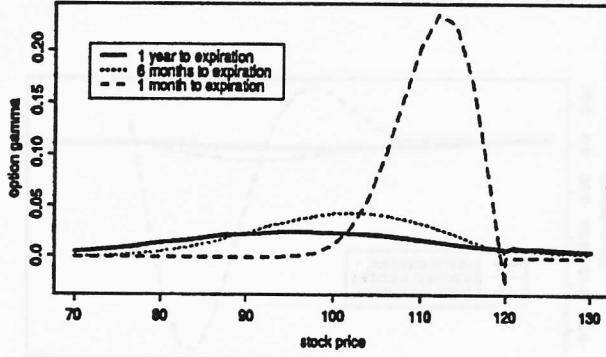


PANEL C. GAMMA



Note that a short-term deep in-the-money put is worth more than a long-term one, because the present value of the strike received is worth more.

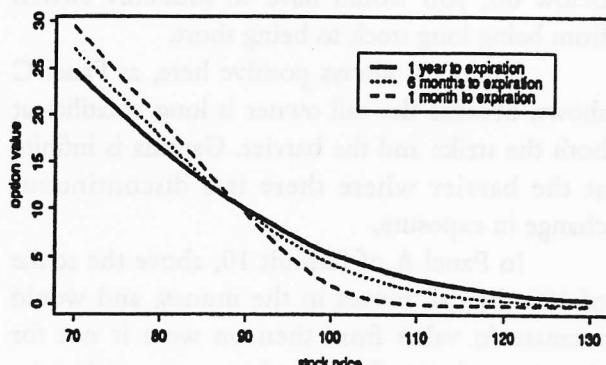
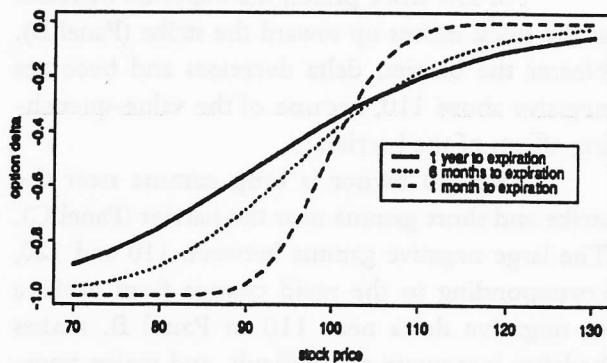
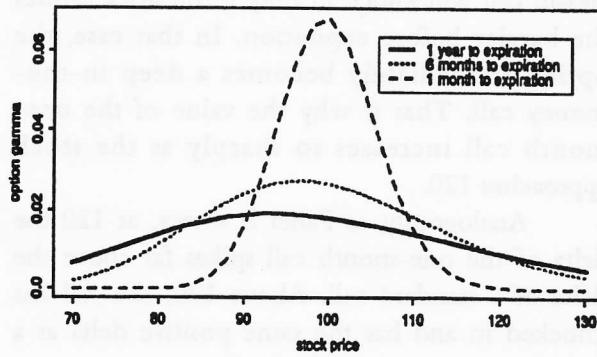
As Panel B shows, at lower stock prices the put owner is effectively short the stock, with a delta value of -1. At high stock prices the delta

EXHIBIT 11
**UP-AND-IN CALL:
STRIKE 100, BARRIER 120**
PANEL A. VALUE**PANEL B. DELTA****PANEL C. GAMMA**

approaches zero.

The gamma of the standard European-style put is identical to the gamma of the corresponding standard call (Panel C). This follows from put-call parity.

As Panel A of Exhibit 13 shows, below the

EXHIBIT 12
**STANDARD PUT:
STRIKE 100**
PANEL A. VALUE**PANEL B. DELTA****PANEL C. GAMMA**

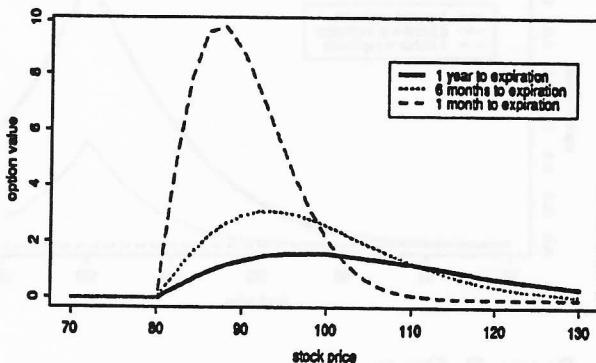
strike of 100, the one-month put moves rapidly into the money. Toward the barrier of 80 its value declines to zero. The peak occurs at about 90.

Just above the barrier, delta is large and positive — the put owner benefits greatly from upward price moves away from the extinguishing

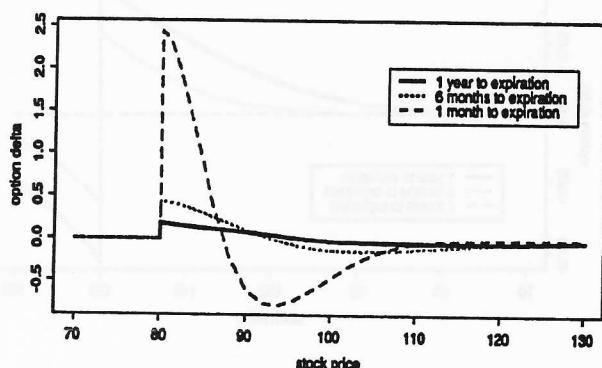
EXHIBIT 13

**DOWN-AND-OUT PUT:
STRIKE 100, BARRIER 80**

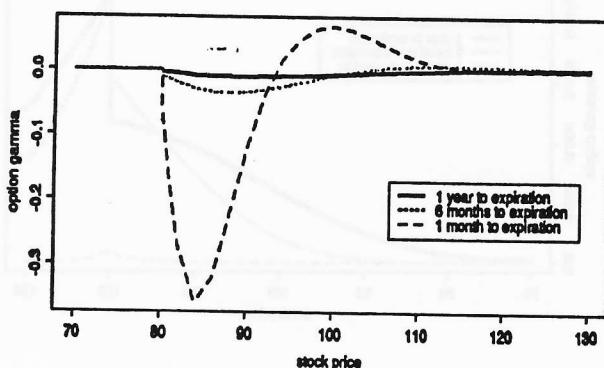
PANEL A. VALUE



PANEL B. DELTA



PANEL C. GAMMA

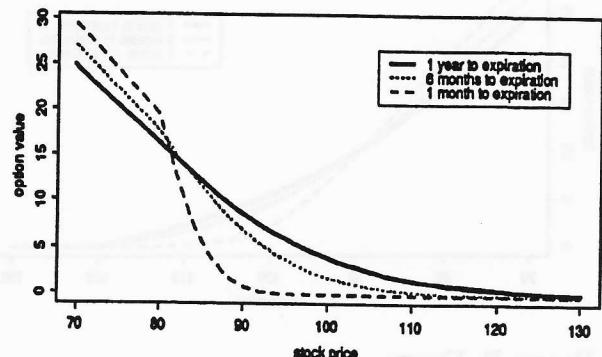


barrier (Panel B). For higher stock prices, the effect of the barrier diminishes, and delta approaches the negative values of a standard put. The rapid change from positive to negative delta makes hedging inaccurate. For large stock prices delta approaches zero.

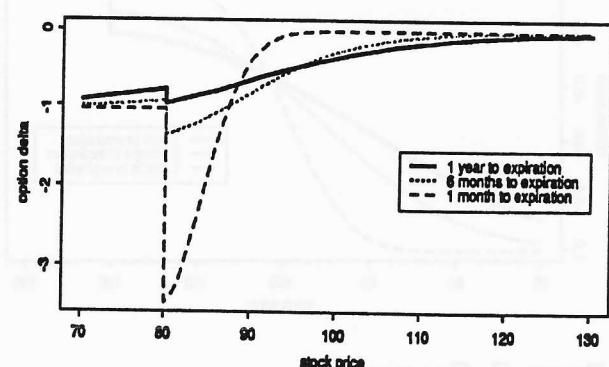
EXHIBIT 14

**DOWN-AND-IN PUT:
STRIKE 100, BARRIER 80**

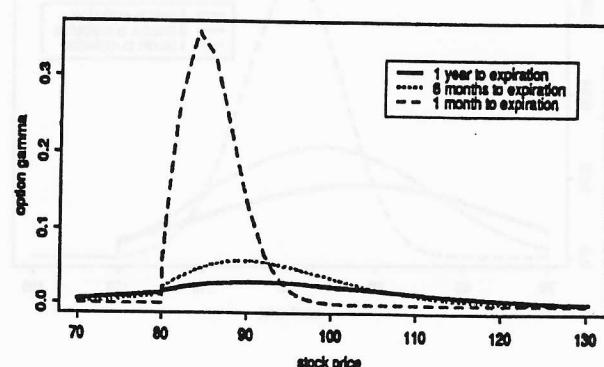
PANEL A. VALUE



PANEL B. DELTA

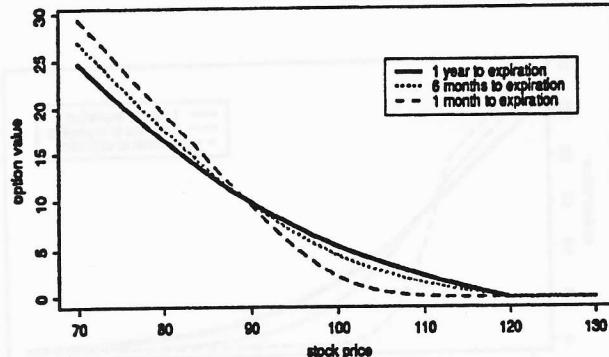
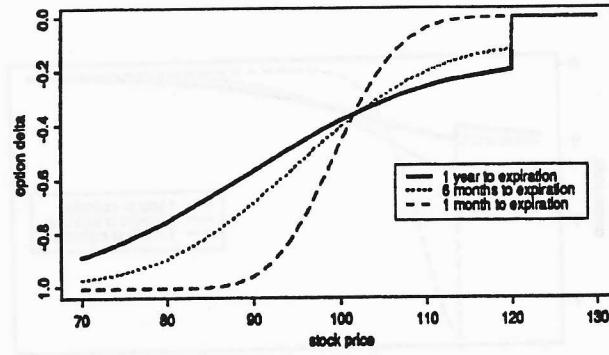
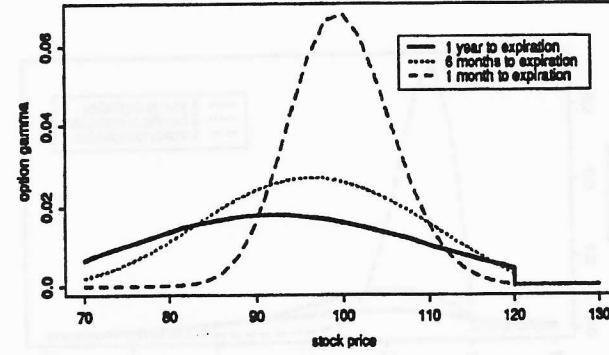


PANEL C. GAMMA



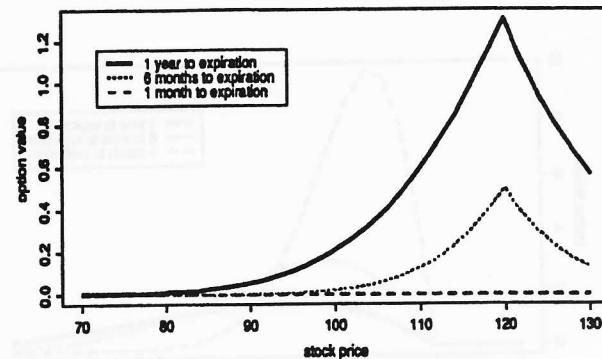
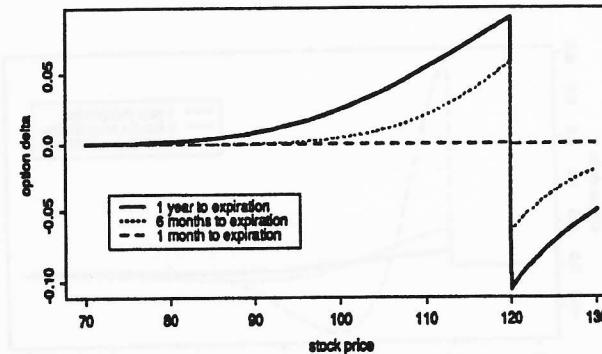
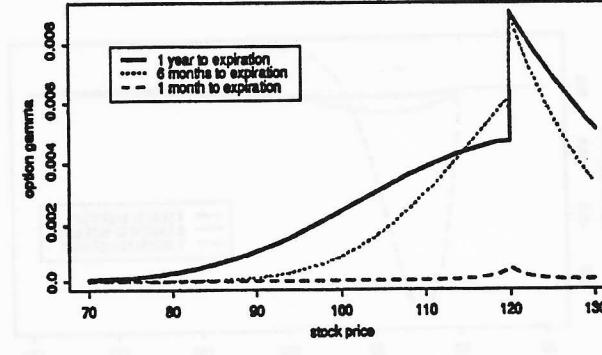
The put owner is long volatility (positive gamma) at the strike of 100 and short volatility (large negative gamma) near the barrier of 80 (Panel C). In addition, gamma is infinite at the barrier.

In Panel A of Exhibit 14, the down-and-in one-month put knocks in deep in the money at

EXHIBIT 15
**UP-AND-OUT PUT:
STRIKE 100, BARRIER 120**
PANEL A. VALUE**PANEL B. DELTA****PANEL C. GAMMA**

80. Approaching the stock price of 80 from above, its value therefore steps up sharply. Below 80, it is knocked in and its value is that of a standard put.

In Panel B, below 80 the delta is that of a standard put, close to -1. Just above 80, the delta of the one-month put drops far below that of a

EXHIBIT 16
**UP-AND-IN PUT:
STRIKE 100, BARRIER 120**
PANEL A. VALUE**PANEL B. DELTA****PANEL C. GAMMA**

standard put because of the rapid decline in its value as the chance of knock-in decreases with increasing stock price. Delta is everywhere negative, and approaches zero for large stock prices where the put is far out of the money.

The put owner benefits from any volatility

that moves the stock into the money or near the knock-in barrier. Therefore, gamma is large and positive near 80 (Panel C). In addition, gamma is infinite at the barrier.

In Panel A of Exhibit 15, the value must vanish at the 120 barrier, where the standard put is worth little anyway. Therefore, the dependence on stock price is similar to that of a standard put. The barrier has little effect in this case, and the option is no riskier than a standard put.

Below the 120 barrier (Panel B), the delta is similar to that of a standard put and is always negative. Above the barrier, the put is extinguished and the delta is zero.

Gamma is similar to that of a standard put below the barrier, and zero above (Panel C). At the barrier, gamma is infinite.

Panel A of Exhibit 16 shows that, as the stock rises to the 120 barrier, the put value increases with its chance of getting knocked in. Above 120, the put is worth the same as a standard put with strike 100, and its value declines as the stock price increases. There is a sharp peak at 120.

The sharp break in delta at 120 makes this put difficult to hedge dynamically (Panel B). As the stock moves above 120, you would have to suddenly switch from shorting stock to going long and buying it.

In Panel C, gamma is everywhere positive, and is very large exactly at the 120 barrier, where sensitivity to volatility is great. Gamma is infinite at the barrier.

SUMMARY

Barrier options on stocks create opportunities for investors that are not available with standard puts and calls, frequently with lower premiums than standard options with similar strike. The payoffs, being a function of both the strike and the barrier, can be complicated. The pricing characteristics, captured by delta and gamma, can change dramatically through time, and exhibit discontinuities.

Options valuation models are theory, and, like all models, are more limited than the real world they attempt to represent. Provided the potential users of barrier options are aware of these limitations, the additional power and flexibility of such options can be important contributors to an overall investment strategy.

ENDNOTES

This article is based on "The Ins and Outs of Barrier Options," a Goldman Sachs Quantitative Strategies Research Notes paper, June 1993.

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*Mathematically, $\gamma = (S/100) \partial\Delta/\partial S$, where S is the stock price.