Multi-Agent Systems

Homework Assignment 3

Martynas Vaznonis (2701013)

m.vaznonis@student.vu.nl

3 Sequential Games with Perfect Information

3.1 Reduced centipede game

Questions:

1. At the final decision node player two can choose between c and s with rewards 3 and 4 respectively, and thus would choose s. Then, player one similarly can choose between c and s with pay-offs 2 and 3, also choosing s. Then player two chooses s because it gives a utility of 2 which is greater than the alternative utility of 1. Finally, player one chooses s as it guarantees a pay-off of 1, which is better than the reward of 0 that he would get if he chose c.

   This strategy makes sense because of its guarantee for player one. But by simply continuing with c actions, both agents could profit far more, and a stochastic strategy is likely much better in the expected utility.

2. The normal form of the game can be seen in the table below. The pure Nash equilibria can be found in the 4 cells where both players start with the s.

2 1	cc	cs	sc	ss
cc	$\underline{5},3$	$2,\underline{4}$	$0,2$	$0,2$
cs	$3,1$	$\underline{3},1$	$0,\underline{2}$	$0,\underline{2}$
sc	$1,\underline{0}$	$1,\underline{0}$	$\underline{1},\underline{0}$	$\underline{1},\underline{0}$
ss	$1,\underline{0}$	$1,\underline{0}$	$\underline{1},\underline{0}$	$\underline{1},\underline{0}$

3. In the smallest subgame, only player 2 has a choice between s and c with the former providing higher reward, making it the NE. This implies that any game with the second action for player 2 as c is not subgame-perfect.

   For the next smallest subgame, player 1 has a choice between s and c with the former providing higher reward, making it the NE regardless of the following action of player 2. This implies that any game with the second action for player 1 as c is not subgame-perfect.

   Finally, in the subgame with just the root node removed, player 2 has a choice between cc, cs, sc, and ss with the two latter ones providing best reward, making them NE if the second action of player one is s. This implies that any game with the first action for player 2 as c is not subgame-perfect.

The table above shows that every NE has to have player 1 choose s as the first action for an NE. With the above conditions, we know that actions one and two for player 2 have to be s, and action two for player 1 also has to be s. Thus, the only subgame-perfect game is the one where all actions are s for both agents.

3.2 Boss and stealing employee

1. Starting backwards in the *No warning* branch, it is better for the boss to ignore and for the employee to continue stealing. Therefore, the value of said branch is (0,1). The backwards induction of the *Warning* branch leads to a better outcome if the boss follows-through and if the employee quits thieving. So, the value of this branch is (1,0). Therefore, the value that is gained in the end is (1,0), and the boss publishes her warning, and the employee stops stealing.

2. The normal form of the game can be seen in the table below.

Emp \ Boss	NIF	NIF_t	NF_iF	NF_iF_t	WIF	WIF_t	WF_iF	WF_iF_t
HQ	0,1	0,1	<u>0,1</u>	<u>0,1</u>	0,1	<u>0,1</u>	0,1	<u>0,1</u>
HT	0,1	0,1	<u>0,1</u>	<u>0,1</u>	1,-2	-1,-1	1,-2	-1,-1
SQ	1,0	1,0	-1,-1	-1,-1	0,1	<u>0,1</u>	0,1	<u>0,1</u>
ST	<u>1,0</u>	<u>1,0</u>	-1,-1	-1,-1	1,-2	-1,-1	1,-2	-1,-1

3. The pure Nash equilibria are denoted in the above table, in the cells with the underlined utilities (not all best responses are underlined for neatness).

4. If the employee plays HQ, the following boss strategies lead to pure NE, NF_iF, NF_iF_t, WIF_t, and WF_iF_t. Also, if the employee plays HT, the following boss strategies lead to pure NE, NF_iF, NF_iF_t. Looking at the subgame where the choice for the employee is between H and S, it is easy to see that the optimal strategy is to choose S which will give a higher pay-off. Therefore, all of these strategies contain this subgame, which has a non-credible threat, and so they are not subgame-perfect NE.

   Similarly, if the employee plays ST, the boss actions for NE are NIF and NIF_t. But in the subgame where the employee has to choose between T and Q as the root node, the employee would always choose the latter. Thus T is a non-credible threat and those NE are not subgame-perfect.

   Finally, if the employee chooses the strategy SQ, the boss can play WIF_t and WF_iF_t. In the subgame where the boss selects between I and F_i, the latter is irrational and so a non-credible threat. Thus, WF_iF_t is not a subgame-perfect NE.

   The game where the boss adopts the WIF_t strategy and the employee chooses the SQ action set is the only subgame-perfect NE.

5. The two solutions are identical as the boss would play W and the employee would respond with Q in either case. The solution found by the subgame-perfect NE also provides information about how the two agents would be have in any possible subgame.