**Multi-Agent Systems**

# Week 2

2-parameter family of social dilemma games:

* Harmony when sucker pay-off and temptation
* Stag hunt when
* Snowdrift volunteer when
* Prisoner's dilemma when both and

**Normal-form game** is a tuple (N, A, u) with:

* N as a set of *n* player agents
* A as strategies, with as the action space for agent
* Element as an action profile
* u as the utility function such that

An action profile is **pareto dominated** by action profile when and

An action is **pareto optimal** if it is not pareto dominated by any other action.

*Strict* vs *weak* strategy domination. A strictly dominated strategy will never be the best response. A dominating strategy can ignore the opponent’s actions.

**Iterated elimination of strictly dominated strategies (IESDS)** – iteratively eliminate strictly dominated actions.

**Best response** with potentially a mixed strategy. Playing best responses sequentially converges to equilibria or cycles.

**Regret minimization** uses regret values to minimize the maximum possible regret.

**Safety strategies** (maximin) set the lower bound of achievable reward:

1. Agent computes the worst possible outcome for all his actions
2. Agent chooses action to maximize the minimal payoff

**Punishment strategies** (minimax) set the upper bound on the utility that an opponent can achieve:

1. Player computes the best responses for player
2. Player takes the action minimizing player ’s utility

**Nash equilibrium:**

* A solution concept based on conditions, not algorithm
* It is the mutual best response
* Strict NE if
* Strict () versus weak () NE
* No-regret/Self-enforcing – no agent has an incentive to unilaterally deviate

A pure Nash equilibrium can be strict or weak, but a mixed Nash equilibrium is necessarily weak.

Determining the mixture parameters in Nash equilibria requires making each agent indifferent to their choice. This is achieved for agent 1 in a two-player game by .

# Week 3

In 2-player 0-sum games NE=Maximin=Minimax.

**Fictitious play (FP)** uses simulations of many game iterations to estimate the Nash equilibria:

* Players play the best response to the empirical distribution of their opponent’s play
* If FP converges, it coincides with Nash strategies
* Works even without knowing the opponent’s utilities
* Fails to detect unstable NE

Iterated elimination of weakly dominated strategies might erase an NE, but IESDS will not.

A game may not have an NE if:

1. State space is not compact
2. Utility function is not continuous

**Backward induction** algorithm:

1. Start at leaf nodes and choose the best node for the current player
2. Propagate decisions and utilities to root
3. The solution path is called the equilibrium path

**Rubinstein’s Model: Infinite Horizon Bargaining**:

* Optimal split: and
* First mover’s advantage for : and

Repeated game strategies:

1. Grim trigger
2. Tit-for-Tat