2.2) Bayes Theorem = $P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

$\tilde{\pi}_1 = P(Y=1|X=1)$, $\tilde{\pi}_2 = P(Y=1|X=2)$

$\nu = P(X=1)$, given $Y=1$

$$P(X=1|Y=1) = \frac{P(Y=1|X=1) \cdot P(X=1)}{P(Y=1)}$$

$$P(Y=1) = P(Y=1 \cap X=1) + P(Y=1 \cap X=2)$$

$$P(Y=1 \cap X=1) = P(Y=1|X=1) \cdot P(X=1)$$

$$P(Y=1 \cap X=2) = P(Y=1|X=2) \cdot P(X=2)$$

$$\frac{P(Y=1|X=1) \cdot P(X=1)}{P(Y=1)} = \frac{P(Y=1|X=1) \cdot P(X=1)}{P(Y=1|X=1)P(X=1) + P(Y=1|X=2)P(X=2)}$$

$$= \frac{\tilde{\pi}_1 \cdot \nu}{[\tilde{\pi}_1 \nu + \tilde{\pi}_2(1-\nu)]} = \frac{0.86 \cdot 0.01}{[0.86 \cdot 0.01 + 0.12(1-0.01)]}$$

$\tilde{\pi}_1 = 0.86$, $\tilde{\pi}_2 = 1 - 0.88 = 0.12$, $\nu = 0.01$

$= 0.0675$