Time Allowed: 1.5 hours

Stu	ident Number
	Desk Number
UNIVERSITY OF WARWICK	EXAMPLE PAPER 2 Solutions
Summer	SEPAI
Business Analytics	EXAMPLE Productions
<u>Instructions</u>	EXAmple Sola
This is a CLOSED book examination.	EXAMIL

One silent calculator, which is not capable of data storage or retrieval is permitted. Electronic devices such as, for example, a mobile phone, tablet, smart watch, fitbit or similar device are not permitted. Graph paper will be provided. Graph paper will be provided.

This examination paper consists of 4 questions. **You should answer <u>ALL</u> questions.** The sum of obtainable marks across all questions is 100.

Answers should be entered on the examination paper in the spaces provided. If you run out of space continue on the back of the page but make sure that you number the answers clearly. Where graph paper is used, make sure that you write your student number on the paper and attach it securely to the examination paper. Do not append any material that you bring with you to the examination as only material written during the examination will be marked.

# Question papers MUST NOT be removed from the Exam Hall.

Add your student number and desk number to the top of this examination paper and make sure that you hand the paper to an invigilator (together with any answer book if required) at the end of the examination.

The next pages (2-4) contains some formulae and statistical tables for your use if you wish. The questions start on page 5.

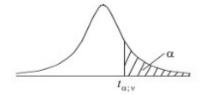
### **Answer all questions**

FOR MARKING PURPOSES ONLY	Q1	Q2	Q3	Q4
Mark				
Marker Initials				

Selected formulae and statistical tables:

# Table of the Student's t-distribution

The table gives the values of  $t_{\alpha;\nu}$  where  $\Pr(T_{\nu} > t_{\alpha;\nu}) = \alpha$ , with  $\nu$  degrees of freedom



va	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7,173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
00	1.282	1.645	1.960	2.326	2.576	3.090	3.291

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Formula sheet continued...\

• Confidence Interval around a regression coefficient:

Regression Coefficient Value 
$$\pm \left[t_{\left(\frac{\alpha}{2},n-k-1\right)} \times SE(regression coefficient)\right]$$

Where k is the number of predictors in the model (not including the intercept).

• T-Test Statistic for regression coefficient:

$$t = \frac{coefficient\ estimate - Assumed\ value\ from\ H_0}{SE(regression\ coefficient)}$$

The degrees of freedom for this test is n - k - 1, where k is the number of predictors in the model (not including the intercept).

- F-Test statistic value =  $MS_{regression}/MS_{residual}$
- Adjusted R-Squared:  $Adj R^2 = R^2 \frac{(1-R^2)k}{(n-k-1)'}$  where k is the number of predictors in the model (not including the intercept).
- The Durbin-Watson (DW) Statistic:

$$DW = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

Where  $e_{t-1} = Y_{t-1} - \hat{Y}_{t-1}$ ,  $e_t = Y_t - \hat{Y}_t$  and n is the number of data points used to fit the model. Note:  $Y_t$  is the observed Y value at time period t and  $\hat{Y}_t$  is the predicted Y value at time period t.

• Predictive Interval (general form):

Forecast<sub>t+p</sub> 
$$\pm t_{\alpha/2,df} \times SE(Y_{t+p} - Forecast_{t+p})$$
  
 $p = 1,2,3,...$  and indicates how far ahead the forecast is,  $df$  represents the degrees of freedom and  $\alpha$  is the significance level.

- The CDF formula for an Exponential distribution with a mean of  $\lambda$ :  $F(x) = 1 e^{-\lambda x}$
- The CDF formula for the continuous Uniform distribution: U(a,b) is  $F(x) = \frac{x-a}{b-a}$

Formula sheet continued over page....\

Formula sheet continued...\

# • Project scheduling formula:

```
EST(i) = earliest start time for activity i, EFT(i) = earliest finish time for activity i  LST(i) = \text{latest start time for activity i}, LFT(i) = \text{latest finish time for activity i}   t_i : \text{duration of activity i}   P_i : \text{ set of immediate predecessors of activity i}, S_i : \text{ set of immediate successors of activity i}   EST(i) = \max\{EFT(j): j \in P_i\}   EFT(i) = EST(i) + t_i   LFT(i) = \min\{LST(j): j \in S_i\}   LST(i) = LFT(i) - t_i
```

Question 1 over the page....\

### Question 1 (33 marks)

The undergraduate program team in the Business School of a British University is interested in what appears to effect student marks in a 2<sup>nd</sup> year core quantitative module called "Business Modelling". The manager collects some data for a randomly selected sample of students who all took this module and sat the same exam. They are courious as to whether the exam score is related to any of the following demographic or academic variables: Gender, A-level/GCSE results, Course of study, class attendance, completion of online quizzes, first language, result in pre-requisite module, first year average. The data set, referred to as the Grades data set from here on, includes 119 students. They decide to analyse this data using multiple regression. Details of the Grades data set are shown in Table 1.1 and data relating to the first 5 students in the data set is shown in Table 1.2.

Variable	Description	Categories
ID	Unique Identifier	N/A
Exam	Mark (out of 100%) attained for the Business  Modelling module.	N/A
Maths	The highest (equivalent) level/grade attained in school age maths exams.	1 = A-level C or above. 2 = A-level D or lower. 3 = AS-level. 4 = GCSE C or above.
Degree	The degree course that the student in registered on.	1 = Management 2 = Finance 3 = Economics
Attendance	Percentage of classes attended by a student for the Business Modelling module. There were 20 hours of classes over the term.	N/A
Quiz	Percentage of online quizzes completed by a student for the Business Modelling module. There were 10 quizzes and each quiz was either completed or not.	N/A
Gender	Whether the student is male or female.	0 = Male 1 = Female
Language	Indicates whether the student's first language is English.	0 = Not English 1 = English
Pre-requisite	The result (out of 100%) obtained in the pre- requisite first year module.	N/A
Year1	The average mark received by a student across all their first year modules (out of 100%).	N/A

Table 1.1: Details of the variables in the Grades data set

Question 1 continued over page....\

ID	Exam	Maths	Degree	Attendance	Quiz	Gender	Language	Pre-requisite	Year1
1	39	1	1	10	40	0	1	38	25
2	39	4	1	65	20	0	0	64	36
3	36	4	1	20	60	0	1	83	92
4	42	2	1	55	20	1	1	60	63
5	50	4	1	45	100	0	0	73	89
:	:	:	:	:	:	:	:	:	:

Table 1.2: Sample of the Grades data set

The manager first graphs each of the explanatory variables against the response variable Exam, as well as each explanatory variable against all other explanatory variables. The following Figures 1.1 to 1.7 are some of these graphs produced by the manager.

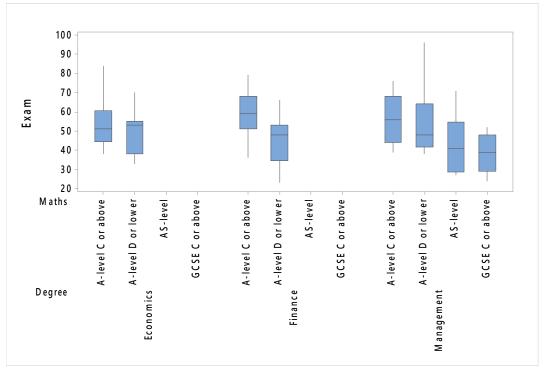


Figure 1.1: box plot of exam mark versus Maths pre-knowledge grouped by Degree course.

Question 1 continued over page....\

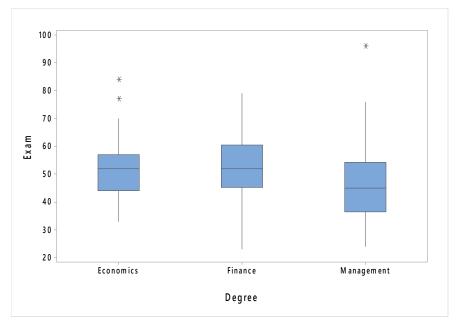


Figure 1.2: box plot of exam mark versus student degree course.

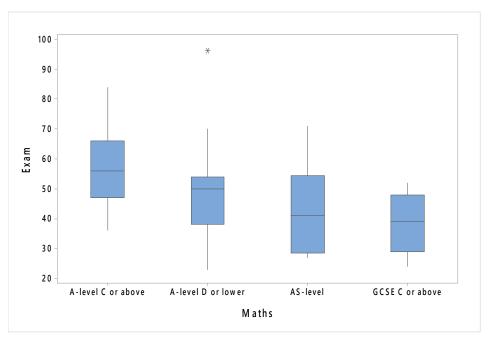


Figure 1.3: box plot of exam mark versus Maths pre-knowledge.

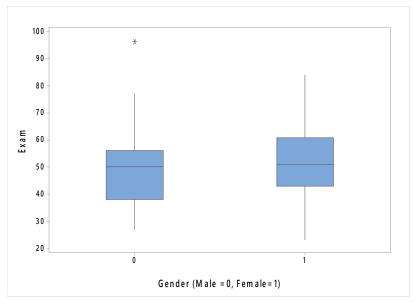


Figure 1.4: box plot of exam mark versus student gender.

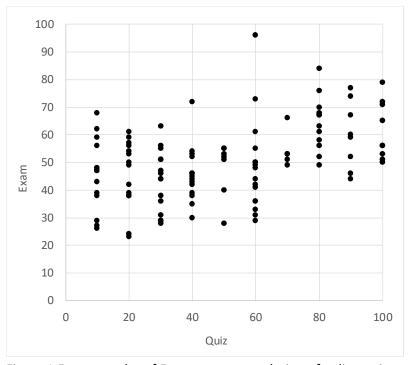


Figure 1.5: scatter plot of Exam versus completion of online quizzes (%).

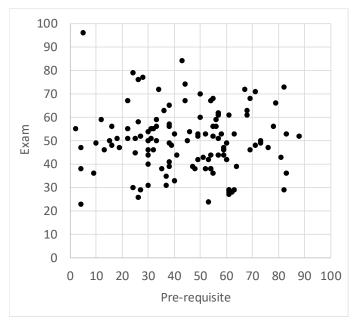


Figure 1.6: scatter plot of Exam versus mark on pre-requisite module (%).

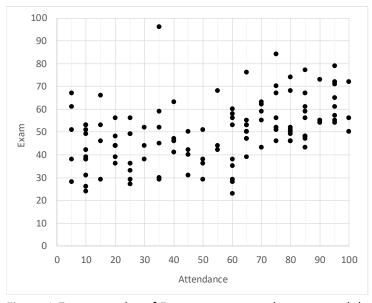


Figure 1.7: scatter plot of Exam versus attendance at module classes (%).

a) Carefully examine each of these graphs (Figures 1.1 to 1.7), fully explain and interpret the information each one gives the manager and hence give recommendations regarding the variables he could use in constructing his multiple regression model of this Grades data.
 (Write your answers in the box over the page) (10 marks)

Question 1a) ANSWER BOX over page....\

#### Question 1a) ANSWER BOX ...\

Sensible and correct full statements that commented on the important characteristics of the given graphs that are connected with constructing the regression model could gain marks:

- Fig 1.1: shows that there is a correlation/correspondence between Maths and Degree, as no students taking Finance or Economics have less than A-level (and/or Only some Management students have GCSE only.)
- Use only one of these variables (Maths, Degree) to avoid multicolinearity issues.
- Choose Maths as more information (and also) Fig 1.2 doesn't show any obvious correlation OR shows only weak correlation between Degree and Exam.
- Fig 1.1 & 1.3 shows correlation between Maths and Exam, so recommend including Maths variable in model construction.
- Figure 1.4 shows maybe very weak correlation between gender and Exam so could try this in model; or calculate correlation or try fitting in simple regression model with response to see further if should try in model.
- Fig 1.5 shows some correlation between Completion and Exam, so recommend including Completion variable in model construction.
- Fig 1.6 shows no obvious correlation between pre-req and exam, so could drop this from model construction or calculate correlation or try fitting in simple regression model with response to see further if should try in model.
- Fig 1.7 shows some correlation between Attendance and Exam, so recommend including Attendance variable in model construction.
- Figs 1.2 to 1.7 all show presence of possible outliers [This statement is only counted once for marking, since most graphs show these same outliers]
- The outliers should be investigated for validity and if valid, recommend fitting model with and without outliers to see what difference is made.
- Variance appears reasonably constant in all graphs [is also considered a valid comment but like outliers, only counted once in marks].

(other valid, logical complete statements could also gain marks)

Question 1a) ANSWER BOX continued over page....\

Question 1a) ANSWER BOX continued\						

Question 1 continued over page....\

The manager decides to work with all the data collected (119 student cases) and selects Exam as the response variable and Maths, Attendance, Quiz, Gender, Language, Pre-requisite and Year1 as explanatory variables. He includes the categorical variables into the model as dummy variables using the coding as stated in Tables 1.3 to 1.5.

MATHS:	$d^{M}_{1}$	$d^{M}_{2}$	$d^{M}{}_{3}$
A-LEVEL C OR ABOVE.	0	0	0
A-LEVEL D OR LOWER.	1	0	0
AS-LEVEL.	0	1	0
GCSE C OR ABOVE	0	0	1

Table 1.3: Coding of variable Maths in the multiple regression model.

GENDER:	$d^G_{1}$
MALE	0
FEMALE	1

Table 1.4: Coding of variable Gender in the multiple regression model.

LANGUAGE:	$d^L{}_1$
NOT ENGLISH	0
ENGLISH	1

Table 1.5: Coding of variable Language in the multiple regression model.

Due to the small size of the data set the manager decides to use a <u>manual backward stepwise</u> <u>regression approach</u>. He fits the regression model using MSExcel. Figure 1.8 below shows selected MSExcel regression output for the first model fitted in the backward stepwise procedure.

Question 1 continued over page....\

SUMMARY OUTPUT						
Regression Sta	tistics					
R Square	?					
Adjusted R Square	0.629545239					
Standard Error	8.32828353					
Observations	119					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	9	14532.88625	1614.76514	23.2808247	1.172E-21	
Residual	109	7560.273415	69.3603066			
Total	118	22093.15966				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	22.85946758	3.463236695	6.60060793	1.5348E-09	15.99544504	29.7234901
Attendance	0.218545593	0.026748433	8.17040728	6.0044E-13	0.165531068	0.27156012
Quiz	0.210239235	0.028711497	7.32247559	4.4309E-11	0.153333982	0.26714449
Gender	0.090992319	1.667351813	0.05457296	0.95657858	-3.21364477	3.39562941
Language	-2.1828386	1.625500964	-1.34287131	0.18210333	-5.40452867	1.03885146
Maths						
A-level D or lower	-7.39444934	1.706296633	-4.33362476	3.2789E-05	-10.7762738	-4.01262489
AS-level	-18.0328069	3.872847183	-4.65621443	9.1356E-06	-25.7086642	-10.3569497
GCSE C or above	-22.4884642	3.162868756	-7.11014776	1.2735E-10	-28.7571672	-16.2197611
Pre-requisite	0.182530813	0.045262353	4.03272922	0.00010252	0.092822301	0.27223932
Year1	0.098510583	0.037477019	2.62855973	0.00981221	0.02423235	0.17278882

Figure 1.8: Regression output from MSExcel for the fitting of the Grades data regression model 1.

b) By examining the results in Figure 1.8 calculate the value of R Square and interpret the results of the (2 tailed) t-test for the coefficients of the variables: Quiz and Gender. (Write your answers in the box over the page)

(5 marks)

Question 1b) ANSWER BOX over page....\

# Question 1b) ANSWER BOX ...\

55	5
14532	.88625
7560.2	273415
22093	.15966

R-square = SS Regression/SS Total = 14532.88625 / 22093.15966 = 0.657800

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	22.85946758					
Attendance	0.218545593	0.026748433	8.17040728	6.0044E-13	0.165531068	0.27156012
Quiz	0.210239235	0.028711497	7.32247559	4.4309E-11	0.153333982	0.26714449
Gender	0.090992319	1.667351813	0.05457296	0.95657858	-3.21364477	3.39562941
Language	-2.1828386	1.625500964	-1.34287131	0.18210333	-5.40452867	1.03885146
Maraba						

#### Quiz

- P-value (of 4.4 x 10<sup>-11</sup> ) <<< 0.05
- Therefore this coefficient is significant at the 5% (or 1%...) level **OR** it is significantly different to zero
- And the quiz variable is therefore explaining some of the variance in the response.

### Gender

- P-value (of 0.957) >> 0.05
- Therefore this coefficient is NOT significant (insignificant) at the 5% (or 10%...) level **OR** it is NOT significantly different to zero
- And the gender variable is therefore NOT significantly explaining the variance in the response.

Question 1b) ANSWER BOX continued over page....\

uestion 1b) ANSWER BOX continued\	

Question 1 continued over page....\

After completing the manual backward stepwise regression, the manager decides upon his final model. The MSExcel regression output for this final model is shown in Figure 1.9 and graphs of the residuals for this model are shown in Figure 1.10.

SUMMARY OUTPUT						
Regression Stat	istics					
Multiple R	0.807444643					
R Square	0.651966852					
Adjusted R Square	0.630018816					
Standard Error	8.322958534					
Observations	119					
ANOVA						
	df	SS	MS	F	Significance F	
Regression	7	14404.00776	2057.71539	29.7050197	9.51868E-23	
Residual	111	7689.151903	69.2716388			
Total	118	22093.15966				
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	22.1925786	3.390593017	6.54533838	1.9024E-09	15.47389219	28.91126501
Attendance	0.213414673	0.026379889	8.0900521	8.2306E-13	0.161141163	0.265688184
Quiz	0.210702285	0.027790794	7.58172957	1.1125E-11	0.155632971	0.265771598
Maths						
A-level D or lower	-7.586654382	1.696713807	-4.4713813	1.8852E-05	-10.9488061	-4.22450271
AS-level	-17.2184202	3.767048385	-4.57079879	1.2683E-05	-24.683078	-9.75376234
GCSE C or above	-21.77280814	2.987987735	-7.28677962	4.931E-11	-27.6937053	-15.851911
Pre-requisite	0.17944275	0.043817831	4.09519924	8.0391E-05	0.092614792	0.266270708
Year1	0.098421579	0.037013073	2.65910317	0.00899358	0.025077704	0.171765453

Figure 1.9: Excel regression output for the final model fitted to the Grades data.

c) Using the output in <u>Figure 1.9</u>, write down the fitted model equation. Fully interpret, in context, the regression coefficients for *Quiz and Maths – 'AS-level'*.

(Write your answers in the boxes over the page).

(8 marks)

Question 1c) ANSWER BOX over page....\

### Question 1c) ANSWER BOX ...\

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	22.1925786	3.390593017	6.54533838	1.9024E-09	15.47389219	28.91126501
Attendance	0.213414673	0.026379889	8.0900521	8.2306E-13	0.161141163	0.265688184
Quiz	0.210702285	0.027790794	7.58172957	1.1125E-11	0.155632971	0.265771598
Maths						
A-level D or lower	-7.586654382	1.696713807	-4.4713813	1.8852E-05	-10.9488061	-4.22450271
AS-level	-17.2184202	3.767048385	-4.57079879	1.2683E-05	-24.683078	-9.75376234
GCSE C or above	-21.77280814	2.987987735	-7.28677962	4.931E-11	-27.6937053	-15.851911
Pre-requisite	0.17944275	0.043817831	4.09519924	8.0391E-05	0.092614792	0.266270708
Year1	0.098421579	0.037013073	2.65910317	0.00899358	0.025077704	0.171765453

Predicted Exam mark = 22.193 + 0.213 Attendance + 0.211 Quiz – 7.587 A-level D or lower (or  $d^{M}_{1}$ ) – 17.218 AS-level ( $d^{M}_{2}$ ) – 21.773 GCSE ( $d^{M}_{3}$ ) + 0.179 Pre-req + 0.098 Yr1

### Quiz:

- The exam mark increases by 0.211 (or by 2.11)
- On average
- For a one *(or 10% if quoted 2.11 above)* percent increase in quiz completion or for one extra completed quiz
- When all other variables remain fixed.

#### Maths: 'AS-level'.

- The exam mark decreases by 17.218
- On average
- If the student has AS-level maths as their highest (equivalent) level/grade attained in school age maths exams.
- Rather than A-level maths grade C or higher
- When all other variables remain fixed.

[other valid ways of stating the above, or different order of statements was fine]

uestion 1c) ANSWER BOX continued\	

Question 1 continued over page....\

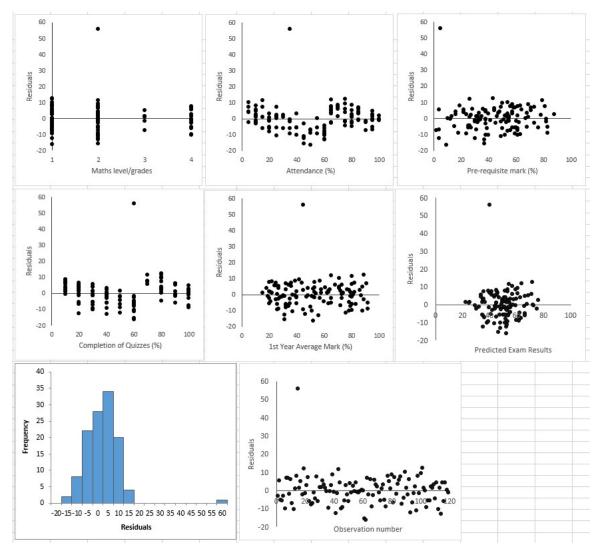


Figure 1.10: Plots of the residuals from the final regression model fitted to the house data.

d) By examining the information in <u>Figures 1.9 and 1.10</u>, comment on the adequacy of the fitted regression model and hence describe any further actions you might want to take as a result. (Write your answers in the box over the page). (10 marks)

Question 1d) ANSWER BOX over page....\

#### Question 1d) ANSWER BOX ...\

Must have at least one statement from each category (A and B) to score full marks:

### A) Comments of adequacy:

- All residual graphs show an obvious outlier
- Graphs of residuals versus Quiz and Attendance show possible strange S (curved, wave...) pattern.
- Graphs of residuals versus pre-req, year1, maths, observation number and pred exams appear reasonable randomly scattered
- Histogram (even if ignore outlier) is a little skewed.
- The residual scatter plots appear to be centred around zero.
- Variance in all scatter plots appears reasonably constant, if ignore outlier.
- Maths level/grades plot may possibly have some decreasing variance
- Adjusted R-squared (R-square) indicates model is explaining around 63% (65%) of the variation in exam marks.
- The Adjusted R-squared value is reasonably high, but around 37% of the variation in exam mark is not explained by the model.
- All explanatory variables are significant in the model (p<0.05)
- F-test is significant (<0.05) so model is significantly explaining variability in the response better than an intercept only model.

#### B) Suggestions of actions to take:

Re. outlier:

- Should check this observation to see if valid value
- If valid value, try fitting model without this outlier to see what difference it makes
- If not valid value, should delete case from sample

Re. pattern in residuals (attendance and quiz):

• Consider transformation of variables

Re. Adj R-square value:

- Could try to find other variables to include in model to explain more of the other 37% of variation in exam mark.
- Increase sample size if possible: since working with 11 potential explanatory variables, 119 observations is quite a small data sample size.

(other valid, relevant and complete statements could gain marks)

### Question 2 (16 marks)

Cedric Achebe works as a data analyst in the United States National Travel and Tourism Office (NTTO). The goal of NTTO is to enhance the international competitiveness of the U.S. travel and tourism industry and increase its exports, thereby creating U.S. employment and economic growth. Cedric's job entails working with data to help the NTTO to make informed decisions regarding the future economic conditions of the country.

Cedric is asked to analyse data (from this point on referred to as the travel data) of the number of visits to the US from Mexico. These observations are given for each quarter of the year from 1<sup>st</sup> quarter (Q1) of 2009 to the 4<sup>th</sup> quarter (Q4) of 2019 (44 data points in total). Table 2.1 below shows the months included within each quarter. Cedric generates a time series plot of the travel data using MSExcel, as shown in Figure 2.1 below.

Quarter 1	Quarter 2	Quarter 3	Quarter 4
January	April	July	October
February	May	August	November
March	June	September	December

Table 2.1: Months of the year included within each Quarter of the year.

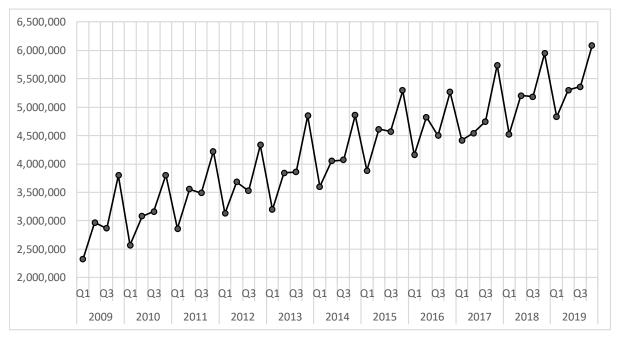


Figure 2.1: Time series plot of the travel data – 2009 Q1 to 2019 Q4 (44 time points)

Question 2 continued over page....\

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a) By looking at the time series plot in Figure 2.1, describe and comment on the main features of this time series. (Put your answer in the box below).

(8 marks)

• Seasonality of period 4 Upward linear trend No obvious outliers by eye, though Q2 of 2017 seems a little unusual No obvious cycles Constant variance Highest visits in 4th Quarter Lowest visits in 1st Quarter Non-stationary data (any other applicable, sensible, complete statement can gain marks)

 b) Cedric decides to fit a multiple regression model to this data to forecast future visitor numbers. Describe and explain in detail the structure of a multiple regression model that ought to fit sufficiently well to the important characteristics in the travel data.
 (Put your answer in the box below).

Predicted visitor numbers = 
$$\beta_0 + \beta_1 t + \beta_2 \mathbf{Q}_1 + \beta_3 \mathbf{Q}_2 + \beta_4 \mathbf{Q}_3 + \varepsilon$$

The response variable is the number of visitors.  $\beta_0$  is the intercept.  $\varepsilon$  is the errors.  $\beta_i$ , for i=1,2,3,4, are the coefficients for each explanatory variable.

The explanatory variables are:

- Time, t, included in the model as a straight forward unit count (t = 1, 2, 3, ..., 27) to model the linear upward trend.
- Seasonality of period 4 included using three dummy variables, coded as shown in table below:

QUARTERS:	$Q_1$	$Q_2$	$Q_3$
1 <sup>ST</sup> QUARTER	0	0	0
2 <sup>ND</sup> QUARTER	1	0	0
3 <sup>RD</sup> QUARTER	0	1	0
4 <sup>™</sup> QUARTER	0	0	1

# Question 3 (17 marks)

Figures 3.1 below shows two excerpts from a spreadsheet model used to model the risk associated with the launch of a new product.

SIMULATION TRIAL   Competitors Sales Price Scott & Co's Sales Price Manket Size Manufacturing Cost Market Size Size Size Size Size Size Size Size	Mobel		A	8		O	Q	ш	ш	Ŋ	I
Market Share   Competitors Sales Price Scott & Co's Sales Price   Market Share   Market Share   Fotal Sales   Fo	Minutation Trial Competitors Sales Price Scott & Cot Sales   13.2   0.303   553902.3     19.25   19.65   19.65   19.65   19.65   19.65   0.317   19.25   19.67   19.65   0.317   19.32   19.67   19.65   0.317   19.32   19.67   19.65   0.317   19.32   19.67   19.65   0.317   19.32   19.67   19.65   0.317   19.65   19.67   19.65   19.67   19.65   19.67   19.			Parameters					MODEL		
1 19.62 21.56 998668.34 13.2 0.039 652390.23  2 2.44 19.67 10.0620.2 16.09 0.379 753427.39  2 2.44 2.16 11.0877.23 16.06 0.4375 9753427.39  2 2.44 2.03 18.23 10.04288.6 13.26 0.5035 97186.22  2 2.56 2.256 11.04 11.0550.13 13.23 0.477 86.05.03  2 2.25 18.24 20.18 988593.7 14.28 0.425 965590.21  2 2.12 2.04 2.047 10.0559.87 14.28 0.425 965590.21  2 2.13 2.244 2.13 10.0559.87 16.29 0.383 825937.48  2 3	10 1962 1156 99866634 112 0.039 6539023 2 19.25 19.67 101068032 16.05 0.379 7542739 3 2.244 2.16 101872.93 13.28 0.547 10640393.73 2 2.24 21.01 1010550.32 13.28 0.547 10640393.73 2 2.25 21.02 20.13 986401.55 13.28 0.547 10640393.73 2 2.25 21.02 20.13 986401.55 13.28 0.547 10640393.73 2 2.25 21.02 20.13 986593.7 16.64 0.482 965590.0.21 2 2.126 2.0.81 986593.7 16.64 0.482 965590.0.21 2 2.126 2.0.81 101055.98 16.64 0.482 955590.0.21 2 2.13 2.0.13 998613.07 16.64 0.482 955590.0.21 2 2.13 2.0.13 998613.07 16.65 0.3895 820440.68 2 2.13 2.244 2.134 998653.07 16.65 0.3895 820440.68 2 2.13 2.244 2.134 998653.01 16.59 0.3895 820440.68 2 2.13 3.13 3.13 0.0459 986633.07 16.68 0.48 957788.98 2 2.14 2.13 2.146 1003574.45 16.29 0.3895 820520.74 2 3 Mean 0.0451.7725 8936508.365 2.176 2 4 St. Deviation 0.082023341 1322468.719 6451.778 2 5 Min 0.02605 5676125.5 589		<b>MULATION TRIAL</b>	Competitors Sa	les Price Sco	tt & Co's Sales Price	Market Size	Manufacturing Cost	t Market Share	Total Sales	<b>Total Profit</b>
2         19.25         19.65         10.0050.32         16.99         0.375         7534237.33           3         20.44         20.3         10.0050.32         16.05         0.375         965788.37           4         20.3         18.23         10.04286         13.26         0.303         348169.22           5         20.01         21.01         10.1801.32         13.38         0.35         748269.23           6         22.56         20.02         18.48         986401.55         13.38         0.547         10640393.73           7         20.02         18.22         20.18         20.04         19.62         99451.26         0.345         90.547         10640393.73           10         20.24         20.05         10.026.35         16.65         0.454         90.455.89         10.0458         96745.88           11         21.79         20.19         998613.07         15.69         0.458         967758.98           12         1         1         4         1         4         0.458         967758.98           1         1         1         1         4         1         4         1         4         1         4         1	2         19.25         19.67         1010660.32         16.05         0.4378         553437739           3         22.44         21.69         101872.93         16.05         0.4378         953437739           4         20.01         20.01         1018501.32         13.26         0.5035         213166-22           5         20.01         21.01         1018501.32         14.97         0.503         7489549.46           6         22.56         19.62         99451.26         13.33         0.547         10540393.73           10         20.64         20.18         998593.7         16.64         0.482         95250402           11         21.78         20.01         1003590.87         14.28         0.4545         970183.46           12         21.26         20.17         1003590.87         14.28         0.4545         970183.46           13         21.74         20.18         1003574.45         16.29         0.485         820440.68           13         22.44         21.34         998655.21         15.54         0.455         966455.33           2         1         0         0         0         11.65         0.455         771784	~	1		19.62	21.56					2529676.4
3         2244         216         11877.93         16.05         0.4375         9667518.37           4         20.3         18.24         10.4288.6         13.26         0.5035         9218469.28           5         22.56         10.26         994451.26         13.23         0.547         10.60393.73           7         20.02         18.48         986401.55         13.23         0.477         869509.21           8         21.82         20.04         989593.7         16.64         0.482         96254.42           10         21.26         20.04         98640.55         13.23         0.477         869500.21           10         21.26         20.04         98640.55         14.28         0.485         96254.42           10         21.26         20.44         20.85         10.0859.34         15.66         0.385         825927.4           11         21.44         21.34         99855.21         15.54         0.485         869455.33           10         1         1         1         1         1         1         1         1         1         1         1         1         1         1         1         1         1         1	3   22.44   21.6   1018772.33   16.05   0.4375   9667518.37     4   20.3   12.3   10042886   13.56   0.4375   9667518.32     5   20.01   21.01   1018501.32   14.35   0.0547   10495844.6     6   22.56   19.62   994451.26   13.38   0.547   1049393.43     7   20.02   18.48   986401.55   13.23   0.477   869509.021     8   21.82   20.01   19.85   13.23   0.477   869509.021     9   21.26   20.05   1010265.35   14.56   0.4842   965500.021     10   20.44   20.05   1010265.35   14.56   0.4845   967758.98     11   21.79   20.13   21.44   10102574.45   15.69   0.48   977758.98     12   22.44   21.34   998552.1   15.69   0.48   977758.98     13   22.44   21.34   998552.1   15.69   0.48   977758.98     14	-	2		19.25	19.67	1010650.32				1026537.74
4         20.3         18.23         1004288.6         13.26         0.5035         918169.22           5         20.01         21.01         101801.32         44.97         0.535         7489549.46           6         20.25         19.64         994451.26         13.38         0.647         805493.73           7         20.02         18.48         986401.55         13.28         0.447         8054900.21           8         21.26         20.24         20.01         1008390.87         14.28         0.482         95250.02           10         20.64         20.64         20.85         1008590.87         14.28         0.485         95250.02           11         20.64         20.64         20.85         100850.87         16.64         0.485         95250.02           12         20.64         20.64         20.04         99861.37         16.65         0.485         97018.46           13         20.44         100350.44         16.64         0.455         960435.33           14         1         70.4         74.4         74.7         74.8         74.4         74.4         74.4         74.4         74.4         74.4         74.4         74.4	4         20.3         18.23         1004288.6         13.26         0.5035         748169.22           5         20.01         21.01         1018201.32         14.97         0.35         748169.24           6         22.56         19.62         91451.26         13.23         0.477         8659020.21           7         20.02         18.48         9895401.55         13.23         0.477         8659020.21           8         21.26         20.18         989593.7         16.64         0.485         965500.21           10         21.26         20.21         100265.35         16.64         0.485         965500.21           11         21.79         20.04         995661.07         14.28         0.445         901834           12         21.73         20.44         100265.35         16.56         0.48         965408.38           13         22.44         21.34         995855.21         15.54         0.455         969455.33           14         0.072         11.67         94444.06         13.24         0.445         969455.33           2         1         0.074         11322468.719         11846428.77         144           2         0.	10	3		22.44	21.69				9667518.37	2513822.2
5         20.001         21.00         1018501.32         14.97         0.35         748954946           6         22.56         19.62         991451.26         13.38         0.547         1040993.73           7         20.02         18.48         985401.55         13.23         0.047         8690021           8         20.02         18.48         985401.55         13.23         0.047         8690021           9         21.28         20.04         986401.55         14.64         0.482         96290221           10         20.04         20.04         98641.55         16.65         0.3895         8204440.68           11         21.79         20.19         998613.07         15.69         0.48         967758.98           12         21.13         22.44         10.0357445         16.29         0.385         8204440.68           13         22.44         21.13         998613.07         16.29         0.385         8204440.68           14         7         7         7         94494.09         16.29         0.385         8204440.8           1         1         0         0         0         16.29         0.385         8204440.8	5         20.01         21.01         1018501.32         1497         0.35         748954946           6         22.56         19.62         991451.36         13.38         0.547         10640333.73           8         21.26         18.48         986401.55         13.38         0.477         1064033.73           9         21.02         20.02         20.18         988593.7         16.64         0.487         965590202           10         20.64         20.08         100265.35         14.28         0.4545         90018346           11         20.79         20.019         98613.07         15.65         0.484         97018346           12         21.13         20.44         20.85         1010265.35         16.65         0.484         97018346           13         22.44         21.34         99585.21         16.29         0.485         969455.33           14         0.074         1.67         4.484         0.454         969455.33         7614678           2         1         0.04PUT         Mean         0.45127725         8936508.365         2176           5         1         Mean         0.045127725         5676125.57         589	10	4		20.3	18.23				9218169.22	2513126.77
6         22.56         19.62         99451.26         13.38         0.547         10640393.73           7         20.02         18.48         986401.55         13.28         0.477         869509.01           8         21.82         20.018         985933.7         16.64         0.487         96590.21           10         20.64         20.85         1002859.87         16.64         0.485         905183.46           11         21.79         20.13         998513.07         15.69         0.48         9677758.98           12         21.13         21.46         103374.45         16.29         0.385         8259327.4           13         22.44         21.34         95855.21         15.56         0.385         8259327.4           14         30.74         31.47         49449.40         15.56         0.385         8259327.4           1         30.74         30.74         34494.60         16.29         0.385         8259327.4           2         4         4         34494.60         132.468.719         6451.87         74147.78           3         5         4         32.468.719         32.468.719         32.468.719         17434 <t< td=""><td>6         22.56         19.62         991451.26         13.38         0.547         10640393.73           7         20.02         18.48         986401.55         13.23         0.477         869509.01           8         21.82         20.01         10.047         986401.55         16.65         0.4545         965500.21           10         20.64         20.01         1010265.35         16.65         0.3895         820440.68           11         21.79         20.19         998613.07         15.69         0.48         967758.98           12         21.13         21.44         21.34         99885.21         15.54         0.455         969455.38           13         22.44         21.34         99885.21         15.54         0.455         969455.33           14         30.74         30.74         40.457         36440.68         7644         7644           2         3         Mean         0.455         14846428.77         6451         276           3         3         3         3         3         3         3         3           4         3         4         3         4         4         4         4         4</td><td></td><td>5</td><td></td><td>20.01</td><td>21.01</td><td></td><td></td><td></td><td></td><td>2153111.79</td></t<>	6         22.56         19.62         991451.26         13.38         0.547         10640393.73           7         20.02         18.48         986401.55         13.23         0.477         869509.01           8         21.82         20.01         10.047         986401.55         16.65         0.4545         965500.21           10         20.64         20.01         1010265.35         16.65         0.3895         820440.68           11         21.79         20.19         998613.07         15.69         0.48         967758.98           12         21.13         21.44         21.34         99885.21         15.54         0.455         969455.38           13         22.44         21.34         99885.21         15.54         0.455         969455.33           14         30.74         30.74         40.457         36440.68         7644         7644           2         3         Mean         0.455         14846428.77         6451         276           3         3         3         3         3         3         3         3           4         3         4         3         4         4         4         4         4		5		20.01	21.01					2153111.79
7         20.02         18.48         986401.55         13.23         0.477         8695090.21           8         21.82         20.18         989593.7         16.64         0.482         9625540.42           9         21.26         20.18         989593.7         16.64         0.482         9625540.42           10         20.64         20.64         20.81         1003590.87         16.65         0.4845         970183.46           11         21.79         20.49         98613.07         15.69         0.488         967387.83           12         21.13         21.34         995855.21         15.69         0.488         967387.83           13         22.44         21.34         995855.21         15.29         0.485         9659455.33           14         70.74         71.67         94444.09         16.84         0.455         9659455.33           2         1         Mean         0.045127725         8936508.365         2176           3         Mean         0.082023341         1322468.719         6451           4         Max         0.06445         11846428.57         388	7         20.02         18.48         986401.55         13.23         0.477         8695090.21           8         21.82         20.18         989593.7         16.64         0.482         965540.42           9         21.26         20.14         100385         14.28         0.4454         90.813.46           10         20.64         20.85         101056.35         16.29         0.485         970440.68           11         21.79         20.19         998613.07         15.69         0.48         97778.88           12         21.13         21.44         100357445         16.29         0.3895         82593274           13         22.44         21.34         99865.21         15.54         0.455         969455.33           14         0UTPUT         71 K7         94404.09         16.29         0.3895         8259327.4           2         Mean         0.45127725         8936508.365         2176           4         \$t. Deviation         0.082023341         1322468.719         6451           5         Max         0.06445         11846428.57         388		9		22.56	19.62					3384100.76
20.182 20.18 989593.7 16.64 0.482 962540.42 14.28 0.4545 920540.42 14.28 0.4545 920183.46 20.12 0.0.17 1003590.87 14.28 0.4545 920183.46 20.17 1003590.87 14.28 0.4545 920183.46 20.18 0.10 20.64 20.85 1010265.35 16.65 0.3895 8204440.68 11.2 21.79 20.13 998613.07 15.69 0.48 9677788.98 204440.68 11.3 22.44 21.34 998853.21 15.54 0.455 969455.33 11.3 22.44 21.34 998853.21 15.54 0.455 969455.33 11.3 NAmber Share Total Sales Total Sales Total Sales Total Sales St. Deviation 0.082023341 1322468.719 64514 1322468.719 64514 28.57 17.5 17.5 17.5 17.5 17.5 17.5 17.5 1	10   1.00   1.		7		20.02	18.48					2470196.08
10   20.64   20.05   10.03590.87   14.28   0.4545   9200183.46   20.64   20.64   20.85   1010265.35   16.65   0.3895   820440.68   20.64   20.24   2	10	0	8		21.82	20.18					1688523.94
10   20.64   20.85   1010265.35   16.65   0.3895   8204440.68   11   21.79   20.19   998613.07   15.69   0.48   967758.98   21.13   22.44   21.34   995855.21   15.54   0.455   9669455.33   14   22.44   21.34   995855.21   15.54   0.455   9669455.33   14   22.44   21.34   995855.21   15.54   0.455   9669455.33   14   22.44   21.34   995855.21   15.54   0.455   9669455.33   14   20UTPUT	10       20.64       20.85       1010265.35       16.65       0.3895       8204440.68       14         11       21.79       20.19       998613.07       15.69       0.48       9677788.98       2         12       21.13       21.44       1003574.45       16.29       0.3835       825937.4       1         13       22.44       21.34       99885.21       15.54       0.485       969455.33       825937.4       1         14       20.07       71.67       94494.09       16.84       0.485       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       7618167.83       761667.83	_	6		21.26	20.17				9200183.46	2686617.78
11 21.79 20.19 998613.07 15.69 0.48 9677758.98 2 12 21.13 21.46 1003574.45 16.29 0.3835 8259374 1 13 22.44 21.34 995855.21 15.54 0.455 9669455.33   14	11 21.79 20.19 998613.07 15.69 0.48 9677788.98 2 12 21.44 10.03574.45 16.29 0.3835 825937.4 1 13 22.44 21.34 99885.21 15.54 0.455 969455.33	2	10		20.64	20.85				8204440.68	1652693.09
12 22.44 21.34 995855.21 16.29 0.3835 825927.4 1  13 22.44 21.34 995855.21 15.54 0.455 9669455.33   14 22.44 21.34 995855.21 15.54 0.455 9669455.33   15 22.44 21.35 992855.21 15.84 0.455 966945.33   16 OUTPUT	12   21.13   21.46   1003574.45   16.29   0.3835   82593274   1	m	11		21.79	20.19					2157004.23
13 22.44 21.34 998855.21 15.54 0.455 9669455.33	13   22.44   21.34   995855.21   15.54   0.455   9669455.33     1	4	12		21.13	21.46				8259327.4	1989782.04
14 I J K L L  1 OUTPUT  2 Market Share Total Sales 3 Mean 0.45127725 8936508.365 4 St.Deviation 0.082023341 1322468.715 5 Min 0.2605 5676125.52 7 Max 0.6445 11846428.57	1	10	13		22.44	21.34					2628061.9
OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	10	14		20 74	71 67					1698004 1
OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57										
OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	OUTPUT         Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57			-		J		×	T		Σ
Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	Market Share         Total Sales           Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57		-	_	OUTPUT						
Mean         0.45127725         8936508.365           St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	Mean       0.45127725       8936508.365         St.Deviation       0.082023341       1322468.719         Min       0.2605       5676125.52         Max       0.6445       11846428.57		14	6			Mark	et Share	Total Sale		al Profit
St.Deviation         0.082023341         1322468.719           Min         0.2605         5676125.52           Max         0.6445         11846428.57	St.Deviation       0.082023341       1322468.719         Min       0.2605       5676125.52         Max       0.6445       11846428.57		(1)			Mean	0.45	127725	8936508.3		3346.255
Min 0.2605 5676125.52 Max 0.6445 11846428.57	Min 0.2605 5676125.52 Max 0.6445 11846428.57		7	54		St.Deviation	0.082	023341	1322468.7		149.9627
Max 0.6445 11846428.57	Max 0.6445 11846428.57		u)	10		Min	0	2605	5676125.5		99.6056
7			v	20		Max	0.0	6445	11846428.		9.68898
				7							
	Figure 3.1: Two excerpts of excel spreadsheet model				Figure 3	3.1: Two excerpts	of excel sp	readsheet model			

Question 3 continued over page....\

In this model there are 2000 simulation trials (i.e. the last simulation trial is in row 2002). In this model there are 4 *inputs* that can be used to estimate the company's sales profit for this new product:

- Competitor's sales price for a competing product (£),
- Scott & Co's sales price (£),
- market size (an absolute count),
- manufacturing cost for the new product (£).

All four inputs have uncertainty / variability in them. The following table shows the distributions that the company have decided best reflect the variability of the four inputs.

NAME	DISTRIBUTION		
Competitor's Sales Price	Uniform(£19, £23)		
Scott & Co's Sales Price	Uniform(£18, £22)		
Market Size	Normal(1,000,000, 10,000)		
Manufacturing Cost	Uniform(£13, £17)		

The model *outputs* (results) of interest to the company are:

- *Market Share* (which depends on Scott & Co's sales price and the competitor's sales price): expressed in proportion rather than percentage.
- *Total Sales* (which depends on the market size, market share and Scott & Co's sales price): expressed in £s.
- *Total Profit* (which depends on the market size, percent market share, Scott & Co's sales price and the manufacturing costs): expressed in £s.

The formulae for calculating these 3 outputs are as follows:

#### Market Share:

0.4 – (0.05\*Scott & Co's Sales Price) + (0.05\*Competitor's Sales Price)

#### **Total Sales:**

Market Size \* Market Share \* Scott & Co's Sales Price

# **Total Profit:**

Market Size\* Market Share\*(Scott & Co's Sales Price – Manufacturing Cost)

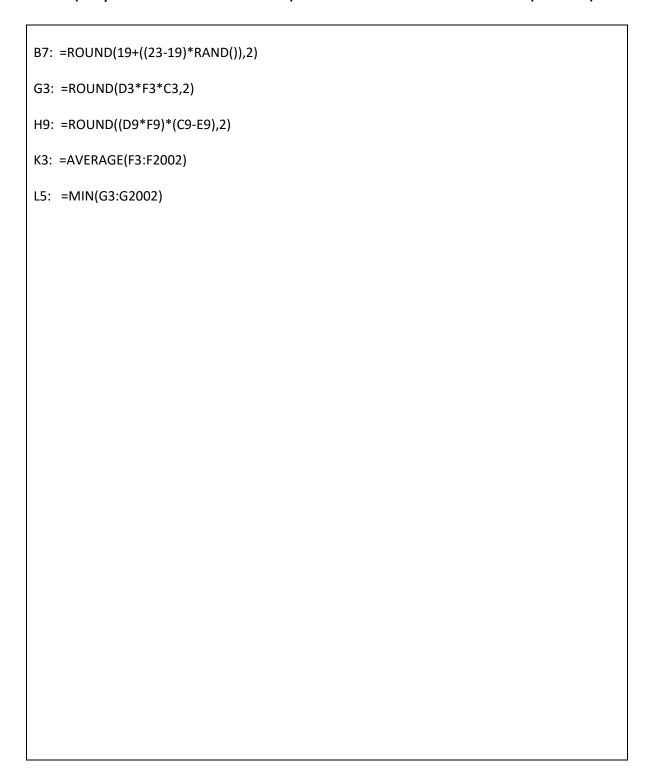
Question 3 continued over page....\

Ougstion	2	continued \
Question	Э	continued\

a) Write the Excel code required to be in cells B7, G3, H9, K3 and L5 for this model to work correctly.

(Put your answers in the box below).

(10 marks)



Question 3 continued over page....\

Question 5 continued \	Question	3	continued	۱	١
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b) Explain in detail what random numbers are.

(Put your answer in the box below).

(7 marks)

Random numbers are a sequence of numbers that appear in a random order.

They can be whole numbers 0-9, 0-99 or real numbers from 0-1 (not including 1).

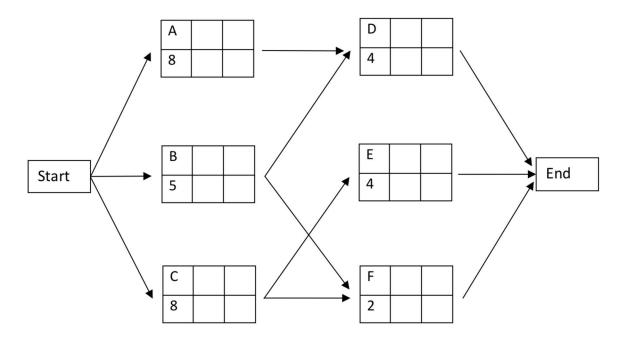
They must have two properties:

- The uniform property: each random number has an equal chance of appearing at any point in the random number sequence.
- The independence property: Once a number has appeared in a sequence, this does not affect its chance of appearing again, or the chance of any other number appearing.

Question 4 over the page....\

# Question 4 (33 marks)

Consider the following project network with activities A to F, and activity durations (in weeks) as indicated in the figure below. The precedence relations between activities are illustrated with arrows.



a) Determine the project completion time and identify critical path(s). For each activity, find the earliest and latest start-times and finishing-times, as well as the slack. You can simply enter the numbers in the graph above.

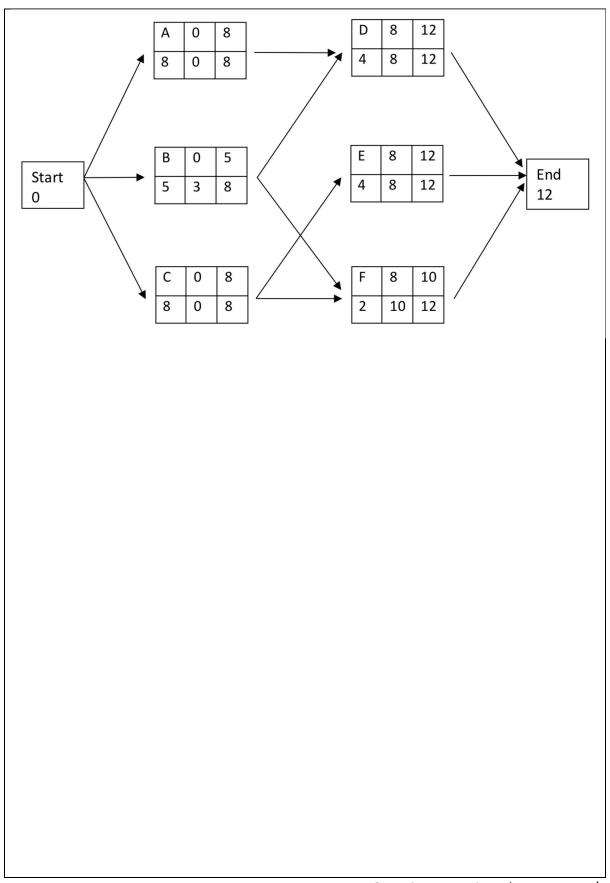
(Please write the additional information in the box below as needed). (13 marks)

Project completion time = 12 weeks

Slack time: S(A) = 0, S(B) = 3, S(C) = 0, S(D) = 0, S(E) = 0, S(F) = 2

Activity B can be delayed by at most three weeks without delaying the project. Similarly, activity F can be delayed by at most 2 weeks without delaying the project. Activities A, C, D and E are critical. Therefore, delaying these activities will increase the project completion time.

Critical paths: A - D and C - E



Question 4 continued over page....\

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b) What happens if we increase the duration of activity B by 3 weeks and decrease the durations of activities D and F by 2 weeks and 1 weeks, respectively? Does the project completion time change, why? Explain your reasoning.

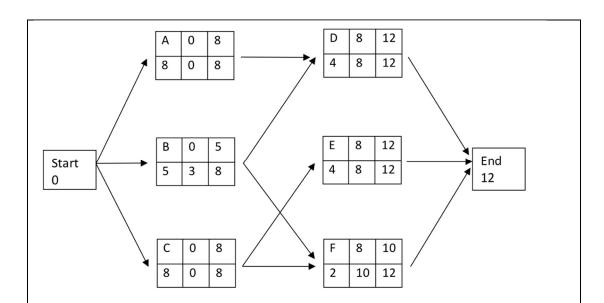
(Write your answer in the box below).	(4 marks)
Activity B has a slack of 3 weeks. Therefore, increasing duration not delay the project.	by 3 weeks does
Activity F has a slack of 2 weeks. Decreasing duration of activity F its slack.	will only increase
Activity D does not have any slack. Therefore, changing its duration project completion time. However, we have two parallel critic problem. Even we complete activity D, we still have to wait for activity project completion time does not change.	cal paths in this

Question 4 continued over page....\

c) The project manager wants to shorten the overall duration of the project found in part (a) by 1 week. Assume that each activity can be crashed by at most 1 week and the associated costs are listed in the table below. Which activity or activities would you choose to crash by 1 week to obtain the minimum total cost? Why? Explain your answer.

(Write your answer in the box below and over the page as needed.) (10 marks)

Activity	A	В	С	D	E	F
Cost per week	15	10	8	12	7	5



Since activities B and F have positive slack, shortening their duration by one week does not change the project completion time.

On the other hand, we have two critical paths A-D and C-E. Crashing just one activity does not change the project completion time. Our options are:

Crash A and C with cost 23

Crash A and E with cost 22

Crash C and D with cost 20

Crash D and E with cost 19 (cheapest) Therefore, we crash D and E.

Question 4 continued over page....\

d) Consider the precedence relation between activities A, B and D. Suppose that  $x_i$  denote the starting time of activity i, for i=A,B,D. Formulate the linear programming constraints that represent the precedence relations between activities A, B and D.

# (Write your answer in the box below).

(6 marks)

 $x_A$ : starting time of activity A

 $x_B$ : starting time of activity B

 $x_D$ : starting time of activity D

$$x_A + 8 \le x_D$$

$$x_B + 5 \le x_D$$

$$x_A \ge 0$$

$$x_B \ge 0$$

# **End of Paper**