기초 과제 2 _ 1171 한은결

#2-2

$$\min_{x \in X} f(x)$$

$$x_{t+1} = x_t - \eta \nabla_x f(x_t)$$

$f(x) = x^4 - 2x^3 - 3x^2 + x$, $x_0 = 1$, $\eta = 0.1$

$\nabla_x f(x) = 4x^3 - 6x^2 - 6x + 1$

$x_1 = x_0 - \eta \nabla_x f(x_0) = 1 - 0.1(4 - 6 - 6 + 1) = 1.8$

$\therefore x_1 = 1.8$

$x_2 = x_1 - \eta \nabla_x f(x_1)$

$= 1.8 - 0.1(4 \cdot 1.8^3 - 6 \cdot 1.8^2 - 6 \cdot 1.8 + 1) = 1.8 + 0.6888 = 3.3888$

$\therefore x_2 = 3.3888$

#3-1.

Entropy : $H(x) = -\sum_{x \in X} P(x) \log_2 P(x)$

X = spam.csv

$P(X = \text{spam}) = \frac{8}{20} = 0.4$

$P(X = \text{Not Spam}) = 0.6$

$H(x) = -\sum_{x \in X} P(x) \log_2 P(x) = -(0.4 \cdot \log_2 0.4 + 0.6 \cdot \log_2 0.6) = 0.97095$

#3-2.

$KL(P||Q) = \sum_{x \in X} P(x) \log\left(\frac{P(x)}{Q(x)}\right)$

P: spam.csv Q = spam-other.csv

$P(X = \text{spam}) = 0.4$ $Q(X = \text{spam}) = 0.6$

$P(X = \text{Not Spam}) = 0.6$ $Q(X = \text{Not Spam}) = 0.4$

$KL(P||Q) = (0.4 \times \log\frac{0.4}{0.6} + 0.6 \times \log\frac{0.6}{0.4}) = 0.08109$

#3-3.

KL-Divergence 는 Asymmetric 하다는 특징이 있음.

$KL(P||Q) = KL(Q||P)$

$KL(Q||P) = \sum_{x \in X} (Q(x) \cdot \log(\frac{Q(x)}{P(x)}))$

$= (0.6 \times \log\frac{0.6}{0.4} + 0.4 \times \log\frac{0.4}{0.6}) = 0.08109$

$\therefore KL(P||Q) = KL(Q||P)$

#4-1.

연세대생이 감기에 걸릴 확률 = DSL인 경우 + DSL이 아닌 경우

DSL 학회원이 아닌 교내생이 감기에 걸릴 확률

$P(\text{감기} | \text{DSL X}) = \frac{P(\text{DSL X} | \text{감기}) \cdot P(\text{감기})}{P(\text{DSL X})} = \frac{99 \cdot 0.1}{0.999} = 9.9$

$P(\text{감기}) = 0.1$

$P(\text{DSL X}) = 1 - 0.001 = 0.999$

$P(\text{DSL X} | \text{감기}) = \frac{P(\text{DSL X} \cap \text{감기})}{P(\text{감기})} = \frac{0.99}{0.1} = 99$

#4-2.

$X_1, \cdots, X_n$은 i.i.d

pdf: $f(x;\theta) = \theta x^{\theta-1}$, $0<x<1$, $0<\theta<\infty$

θ의 MLE인 $\hat{\theta}$를 구하시오.

i) 우도 함수 구하기

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n} \theta x^{\theta-1}$$

$$\log(L(\theta)) = \sum \log f(x_i;\theta) = \sum (\log\theta + (\theta-1)\log x_i)$$

ii) $\frac{d}{d\theta}\log(L(\theta)) = \sum (\frac{1}{\theta} + \log x_i)$

$$\sum (\frac{1}{\theta} + \log x_i) = 0$$

$$\sum (\frac{1}{\theta} + \log x_i) = \frac{n}{\theta} + \sum \log x_i = 0$$

$$\frac{n}{\hat{\theta}} = -\sum \log x_i$$

$$\therefore \hat{\theta} = \frac{n}{-\sum \log x_i}$$