2-1.

2-2 $f(x) = x^4 - 2x^3 - 3x^2 + x$

$\frac{\partial f}{\partial x} = 4x^3 - 6x^2 - 6x + 1$

$x_1 = x_0 - \eta \frac{\partial f}{\partial x}\Big|_{x=0}$

$= 1 - 0.1(4 - 6 - 6 + 1)$

$= 1.7$

$x_2 = x_1 - \eta \frac{\partial f}{\partial x}\Big|_{x=1.7}$

$= 1.7 - 0.1(4(1.7)^3 - 6(1.7)^2 - 6(1.7) + 1)$

$= 2.3888$

3-1 $H(x) = -\sum_{x \in X} p(x) \log_2 p(x)$

$= -0.4 \log_2(0.4) - 0.6 \log_2(0.6)$

$\simeq 0.971$

3-2

$= 0.4 \log\left(\frac{0.4}{0.6}\right) + 0.6 \log\left(\frac{0.6}{0.4}\right)$

$\simeq 0.081$

3-3 For $p(x), Q(x) \geq 0$, i) $p(x) \neq Q(x)$ ii) $p(x) = Q(x)$

Let $KL(P\|Q) - KL(Q\|P) = 0$

$\sum_{x \in X} P(x) \log\left(\frac{P(x)}{Q(x)}\right) - \sum_{x \in X} Q(x) \log\left(\frac{Q(x)}{P(x)}\right) = 0$

$\sum p(x) \log p(x) - p(x) \log Q(x) - Q(x) \log Q(x) + Q(x) \log P(x) = 0$

$\sum ((p(x) + Q(x)) \log p(x) - (p(x) + Q(x)) \log Q(x)) = 0$

$\sum (p(x) + Q(x)) \log \frac{p(x)}{Q(x)} = 0$

$p(x), Q(x) \geq 0$, $p(x) \neq Q(x)$

so, $(p(x) + Q(x)) \geq 0$, $\log \frac{p(x)}{Q(x)} \neq 0$

$KL(P\|Q) - KL(Q\|P) \neq 0$

if ii) $p(x) = Q(x)$ similarly, $KL(P\|Q) - KL(Q\|P) = 0$

$\therefore$ $p(x) \neq Q(x)$ ... Asymmetric

$p(x) = Q(x)$ ... Symmetric

4-1) $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A) =$ 감기 $= 0.1$

$P(B) =$ DSL $= 0.001$) 독립 아니다.

$P(A \cap B) \neq 0.0001, = 0.00001$

$P(A|B) = 0.01 = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.00001}{0.001} = 0.01$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.1 - 0.00001}{1 - 0.001}$$

4-2)

$f(x;\theta) = \theta x^{\theta - 1}, \quad 0 < x < 1, \quad 0 < \theta < \infty$

MLE of θ $(\hat{\theta})$

$$L(n) = \prod_{i=1}^{n} f(x_i;\theta)$$
$$= \prod_{i=1}^{n} \theta x_i^{\theta-1}$$
$$\ln(L(n)) = \ln\theta + \ln\left(\prod_{i=1}^{n} x_i^{\theta-1}\right)$$
$$= \ln\theta + \ln\left(\prod_{i=1}^{n} e^{(\theta-1)\ln x}\right)$$
$$= \ln\theta + \sum_{i=1}^{n} (\theta-1)\ln x$$
$$\frac{\partial \ln(L(n))}{\partial\theta} = \frac{1}{\theta} + \sum_{i=1}^{n} \ln x$$

$$\frac{1}{\hat{\theta}} + \sum_{i=1}^{n} \ln x = 0$$
$$\therefore \hat{\theta} = -\left(\sum_{i=1}^{n} \ln x\right)^{-1}$$