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I-I 모평코 u 와 모반산 p²을 가진 모잡단이 정규분포를 따르지 않아도 표본의 크기 n oi 중분히 크면 , 표본평균 \overline{x} 는 근사적으로 평균이 μ , 부산이 $\frac{\Gamma_{n}^{2}}{n}$ 인 정규분포를 따른다.

즉 x ~ N(U, 뜻)

모평균 4 어디 대한 구간주정을 한다고 하자. 1-2

또는 마음 일고 있으면 정규된 즉 사용한다. H^{ENST} 를 포순하라면 H^{ENST} 를 따라다 F imes F 때문다 F imes F F imes F

일본산 6²을 B2면 표준한다는 대신 사용한다. 이때 亚병생을 표준화하면 t 본포를 따라 : 조-씨 ~ t(N-1)

하시만 표본의 크기가 축분히 크면, 중심국한지리에 의하 정권표를 사용했수 있다.

₹<u>V</u> ~ N(0,1)

► I# 4 : (AI-A)x = 0 의 5H

= $\lambda^2 - 9\lambda + 20$ = (A-5)(A-4) : 1=4,5

 $\lambda = 5: \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} - \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ -2x_1 + 2x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -x_1 + x_2 = 0$

 $\det\left(\begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} - \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}\right) = \det\left(\begin{bmatrix} \Lambda - 6 & 1 \\ -2 & \Lambda - 3 \end{bmatrix}\right)$

 $= (\Lambda - 6)(\Lambda - 3) + 2$ $= \lambda^2 - 9\lambda + 18 + 2$

 $P = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $P^{-1} = \frac{1}{1-2} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ - $D = P^{-1}AP = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} -4 & 4 \\ 10 & -5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$

▷ 고유값 : det (λI-A) = 0 의 5H

2-1 a) $A = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$

 $\Rightarrow \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} S$

⇒ x = [] S

b)
$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \det(\lambda_{1}A) = \det\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 1 \\ 0 & A & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} A-2 & 0 & 0 \\ -1 & A-2 & A-1 \\ -1 & A-2 & A-1 \end{bmatrix}\right)$$

$$= (A-2) \begin{bmatrix} A-2 & -1 \\ 0 & A & 1 \end{bmatrix} = (A-2) (A^{2}3A+2) = (A-2) (A-2)(A-1)$$

$$\therefore A = 1, 2$$

$$\Rightarrow (A1-A) \overrightarrow{X} = 0$$

$$A = 1 : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} \xrightarrow{X} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 \end{bmatrix} \xrightarrow{X_{2}} \begin{bmatrix} X_{1} \\ X_{2} \\ X_{3} \end{bmatrix}$$

$$= \begin{bmatrix} -X_{1} \\ -X_{1} - X_{2} - X_{3} \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{X_{2}} \begin{bmatrix} 0 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{X_{2}} \begin{bmatrix} X_{1} \\ X_{2} \\ X_{3} \end{bmatrix}$$

$$= \begin{bmatrix} -X_{1} \\ -X_{2} - X_{3} \\ X_{1} + X_{3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{X_{1}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{X_{2}} \begin{bmatrix} X_{1} \\ X_{2} \\ X_{3} \end{bmatrix}$$

$$= \begin{bmatrix} -X_{1} - X_{2} \\ -X_{1} - X_{3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{bmatrix} \xrightarrow{X_{1}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{X_{1}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{pmatrix} \lambda - 3 & 0 & 0 \\ 0 & \lambda - 1 & 0 \\ 0 & 0 & \lambda - 2 \end{pmatrix} = (\lambda - 3) \begin{pmatrix} \lambda - 1 & 0 \\ 0 & \lambda - 2 \end{pmatrix}$$
$$= (\lambda - 3) (\lambda - 1) (\lambda - 2) , \lambda_1 = 3, \lambda_2 = 1, \lambda_3 = 2$$

$$\therefore D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$2^{-2} \quad D \quad A^{\mathsf{T}} A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2)
$$A^{T}A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2) $det(\lambda I - A^{T}A) = \begin{bmatrix} \Lambda - 1 & -1 & 0 \\ -1 & \Lambda - 1 & 0 \\ 0 & 0 & \Lambda - 1 \end{bmatrix} = (\Lambda - 1)$

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$$\det(\lambda I - A^T A) = \begin{pmatrix} \Lambda - 1 & -1 & 0 \\ -1 & \Lambda - 1 & 0 \\ 0 & 0 & \Lambda - 1 \end{pmatrix} = (\Lambda - 1) \begin{pmatrix} \Lambda - 1 & 0 \\ 0 & \Lambda - 1 \end{pmatrix} + 1 \begin{pmatrix} -1 & 0 \\ 0 & \Lambda - 1 \end{pmatrix} = (\Lambda - 1)^3 - (\Lambda - 1)$$

$$= (\Lambda - 1) \Lambda (\Lambda - 2) \Rightarrow \Lambda_1 = 2, \Lambda_2 = 1, \Lambda_3 = 0$$

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$$\lambda = 2 : \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ -x_1 + x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = 1 : \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 \\ -x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} C_2 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} O \\ O \\ O \end{bmatrix} \quad \begin{bmatrix} W_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

5)
$$AA^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(\Lambda I - AAT) = \begin{vmatrix} \Lambda - 2 & 0 \\ 0 & \lambda - 1 \end{vmatrix} = (\Lambda - 2)(\Lambda - 1) , \Lambda_1 = 2, \Lambda_2 = 1$$

(XI-AAT)
$$\vec{X} = \vec{0}$$

$$(\lambda \mathbf{I} - \mathbf{A} \mathbf{A}^{\mathsf{T}}) \overrightarrow{\mathbf{X}} = \overrightarrow{\mathbf{0}}$$

$$\cdot \lambda = \mathbf{2} : \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\overrightarrow{\mathbf{w}}_1 = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}$$

$$\lambda = 1: \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \overrightarrow{W_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$A = U \Sigma V^{T} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$3-1$$
 $(n-1)S^2/F^2 \sim \chi^2(n-1)$

$$X_1, \dots, X_n \stackrel{\text{iid}}{\sim} N(\mu, \sigma^2) \implies X \sim N(\mu, \sigma^2)$$

$$V = \sum_{i=1}^{n} \left(\frac{Y_i - \mu}{\sigma}\right)^2 \text{ odd } \left(\frac{X_i - \mu}{\sigma}\right) \sim N(0, 1) \text{ of } 2 \leq V \sim \chi^2(n)$$

$$V = \sum_{i=1}^{n} \frac{(\chi_{i} - \mu)^{2}}{\sigma^{2}} = \sum_{i=1}^{n} \left(\frac{(\chi_{i} - \overline{\chi}) + (\overline{\chi} - \mu)}{\sigma} \right)^{2}$$

$$= \sum_{i=1}^{n} \left(\frac{(\chi_{i} - \overline{\chi})}{\sigma} \right)^{2} + \sum_{i=1}^{n} \left(\frac{\overline{\chi} - \mu}{\sigma} \right)^{2} + 2 \sum_{i=1}^{n} \frac{(\chi_{i} - \overline{\chi})(\overline{\chi} - \mu)}{\sigma^{2}}$$

$$= \sum_{i=1}^{n} \left(\frac{\chi_{i} - \overline{\chi}}{\sigma} \right)^{2} + n \left(\frac{\overline{\chi} - \mu}{\sigma} \right)^{2} + \frac{2}{\sigma^{2}} \sum_{i=1}^{n} \left(\chi_{i} \overline{\chi} - \mu \chi_{i} - \overline{\chi}^{2} + \overline{\chi} \mu \right)$$

$$= \sum_{i=1}^{n} \left(\frac{\chi_{i} - \overline{\chi}}{\sigma} \right)^{2} + n \left(\frac{\overline{\chi} - \mu}{\sigma} \right)^{2}$$

$$= \sum_{i=1}^{n} \left(\frac{\chi_{i} - \overline{\chi}}{\sigma} \right)^{2} + n \left(\frac{\overline{\chi} - \mu}{\sigma} \right)^{2}$$

$$= \sum_{i=1}^{n} \left(\frac{\chi_{i} - \overline{\chi}}{\sigma} \right)^{2} + n \left(\frac{\overline{\chi} - \mu}{\sigma} \right)^{2}$$

$$= \frac{(n-1)}{\Gamma^2} S^2 + \left(\frac{\overline{X} - \mu}{\Gamma \sqrt{\Lambda} n}\right)^2$$

$$V = \frac{(n-1)S^2}{\sigma^2} + \left(\frac{x-\mu}{\sigma (\pi)}\right)^2 + \dots (*)$$
• X 와 $S^2 \in \frac{4}{3}$ 되어 때문에 (성권②) $\frac{(n-1)S^2}{\sigma^2}$ 와 $\left(\frac{x-\mu}{\sigma (\pi)}\right)^2$ 도 독립이다.

·
$$V \sim \chi^2(n)$$
 , $\left(\frac{\sqrt{z-M}}{D\sqrt{m}}\right)^2 \sim \chi^2(1)$ 이므로 , 각각의 mgf 는 $(1-2t)^{-1/2}$ 와 $(1-2t)^{-1/2}$ 이다.

· 식(*) 의 야쫔에
$$mgf 를 구하면,$$

$$(1-2t)^{-n/2} = E \left[exp\left(\frac{(n-1)S^2t}{\Gamma^2} + \left(\frac{\overline{X}-U}{F/M} \right)^2 t \right) \right]$$

$$= E \left[exp\left(\frac{(n-1)S^2}{\sigma^2} t \right) \cdot exp\left(\frac{\overline{y} - u}{\sigma \sqrt{n}} \right)^2 t \right]$$

$$= E \left[exp\left(\frac{(n-1)S^2}{\sigma^2} t \right) \cdot exp\left(\frac{\overline{y} - u}{\sigma \sqrt{n}} \right)^2 t \right]$$

$$= E \left[exp\left(\frac{(n-1)S^2}{\sigma^2} t \right) \right] \cdot E \left[exp\left(\frac{\overline{y} - u}{\sigma \sqrt{n}} \right)^2 t \right]$$

$$= M_{(n-1)S^2}(t) \cdot (|-2t)^{-1/2}$$

$$= M_{\frac{(n-1)S^2}{6^2}}(t) \cdot (|-2t|)^{-1/2}$$

$$\therefore M_{\frac{(n-1)S^2}{6^2}}(t) = (|-2t|)^{-\frac{n}{2} + \frac{1}{2}} = (|-2t|)^{-\frac{(n-1)}{2}} \text{ oles } \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

$$3-2$$
 $T = \frac{\bar{x} - \mu}{S\sqrt{n}} \sim t(n-1)$

〈七분포의 정의〉 확递수 χ 가 표근정뿐표, Y가 자왔 n 인 카이제공 분포를 다드면서 χ 와Y가 독립인마+ : $T = \frac{2}{\sqrt{Y/n}}$ 는 자유도 n 인 + 분포 다른다.

$$T = \frac{\nabla - u}{S / \ln} = \frac{(\nabla - u)}{\sigma / \ln} \cdot \frac{\sigma / \ln}{S / \ln}$$
$$= \frac{(\nabla - u)}{\sigma / \ln} \cdot \sqrt{\frac{\sigma^2}{S^2}}$$

$$= \frac{(\bar{x}-\mu)}{\bar{y}(\bar{y})} \cdot \sqrt{\frac{g^2(n-1)}{g^2(n-1)}}$$

$$=\frac{(\bar{X}-\mu)/(f/n)}{\sqrt{(n-1)s^2/f^2(n-1)}}$$

$$\frac{(\vec{x}-\mu)}{6\pi n} \sim N(0,1) , \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1) \text{ olp3}$$

$$T = \frac{(\vec{y} - \mu)/(\vec{\sigma} \vec{v} \vec{n})}{\sqrt{(\underline{n} + 1)^2}} = \frac{(\vec{y} - \mu)/(\vec{n} \vec{v} \vec{n})}{\sqrt{(\underline{n} +$$

4. 1 X = DSL 사람의 기 , Y = DSL 학회원이 아닌 사랑의 키

 \bar{X} = 173.5, Sx = 7.05 Y = 171.4 SY=7.05

b) 0 = 0.05

. 검정통계2度: $Z = \frac{\overline{X} - \overline{Y}}{S_{P_1} \overline{J}_{P_2} + \overline{J}_{P_2}} \sim N(O, 1)$ because CH至名

4,2

.. निर्श्ति 5 / MIM DSL अधिश 配刊가 DSL संब्रिश अधि। महिला महिला प्राप्त निर्मा चिला नेप निर्मा