Q1 →

$$f(x_i, \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2} \times \sqrt{2\pi}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Likelihood function

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} f(x_i, \theta_1, \theta_2) = \theta_2^{-n/2} (2\pi)^{-n/2} e^{\left[\frac{-1}{2\theta_2} \Sigma (x_i - \theta_1)^2\right]}$$

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log \theta_2 - \frac{n}{2} \log 2\pi - \Sigma \frac{(x_i - \theta_1)^2}{2\theta_2}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = -2 \Sigma \frac{(x_i - \theta_1)(-1)}{2\theta_2} = 0.$$

$$\Sigma x_i - n\theta_1 = 0.$$

$$\hat{\theta_1} = \hat{\mu} = \frac{\Sigma x_i}{n} = \bar{x}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{\Sigma (x_i - \theta_1)^2}{2\theta_2^2} = 0.$$

$$-n\theta_2 + \Sigma (x_i - \theta_1)^2 = 0.$$

$$\hat{\theta_2} = \frac{\Sigma (x_i - \bar{x})^2}{n}.$$

$$\therefore \theta_1 = \frac{\Sigma x_i}{n}, \quad \theta_2 = \frac{\Sigma (x_i - \bar{x})^2}{n}.$$

Q2 →

$y = B(m, \theta)$

$p(y|\theta) = Bin(y; m, \theta)$
$= {}^{m}C_y \times \theta^{y} (1-\theta)^{n-y}$

Log likelihood function = $\log \theta(y|\theta)$
$LL(\theta) = \log {}^{m}C_y + y \log \theta + (m-y) \log(1-\theta)$

Differentiating w.r.t θ

$$\frac{d}{d\theta} LL(\theta) = \frac{y}{\theta} - \frac{(m-y)}{1-\theta} = 0.$$

$$\frac{y}{\hat{\theta}} - \frac{m-y}{1-\hat{\theta}} = 0$$

$$\frac{m-y}{1-\hat{\theta}} = \frac{y}{\hat{\theta}}$$

$$(m-y)(\hat{\theta}) = y(1-\hat{\theta})$$

$$m\hat{\theta} - y\hat{\theta} = y - y\hat{\theta}$$

$$m\hat{\theta} = y$$

$$\boxed{\hat{\theta} = \frac{y}{m}}$$