

0.1 Variables

Phase variables: $N(t)$

Control parameters: $C(t, 1)$, $C(t, 2)$, $\pi(t)$

Exogenous parameters: $p_1(t)$, $p_2(t)$

0.2 Optimality conditions

0.2.1 Euler equations

- $\frac{d}{dt}\alpha_0(t) = 0$

0.2.2 Control optimality conditions

- $$\frac{a_h \lambda_0 r h o_h \left(a_h C^{r h o_h}(t, 1) + \left((1 - a_h) C^{r h o_h}(t, 2) \right)^{\frac{1}{r h o_h}} \right)^{1 - \beta} C^{r h o_h}(t, 1) e^{-d e l t a_h t}}{\left(a_h C^{r h o_h}(t, 1) + \left((1 - a_h) C^{r h o_h}(t, 2) \right)^{\frac{1}{r h o_h}} \right) C(t, 1)} + \alpha_0(t) p_1(t) = 0$$
- $$\frac{\lambda_0 \left((1 - a_h) C^{r h o_h}(t, 2) \right)^{\frac{1}{r h o_h}} \left(a_h C^{r h o_h}(t, 1) + \left((1 - a_h) C^{r h o_h}(t, 2) \right)^{\frac{1}{r h o_h}} \right)^{1 - \beta} e^{-d e l t a_h t}}{\left(a_h C^{r h o_h}(t, 1) + \left((1 - a_h) C^{r h o_h}(t, 2) \right)^{\frac{1}{r h o_h}} \right) C(t, 2)} + \alpha_0(t) p_2(t) = 0$$
- $-\alpha_0(t) = 0$

0.2.3 Transversality conditions

- $\alpha_0(0) = 0$
- $\alpha_0(T) = -\lambda_1$

0.2.4 Dual feasibility and Complementary slackness conditions

- $\lambda_1 \geq 0$
- $\lambda_1 N(T) = 0$