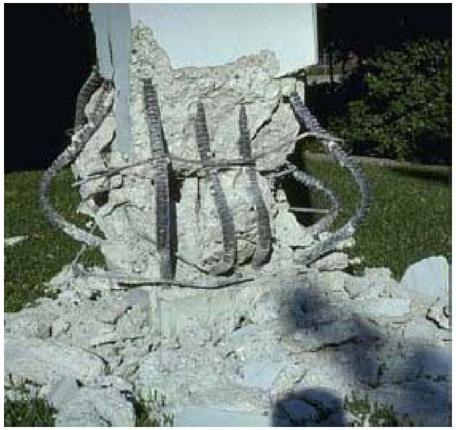
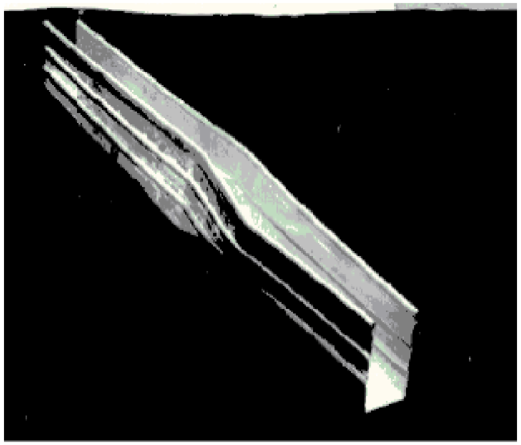
12.1 Introduction to column buckling

- **Buckling:** “Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased.” No failure implied!!!



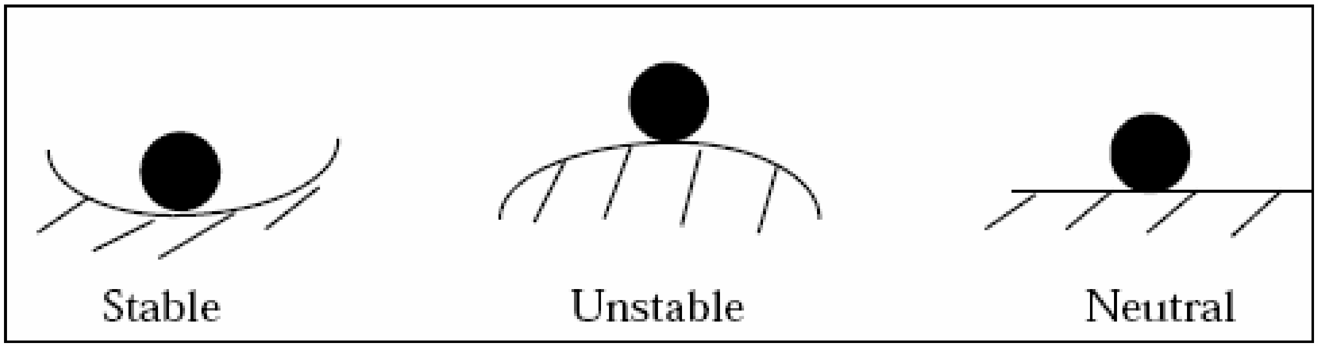
Reinforced concrete



steel

Stability of equilibrium condition

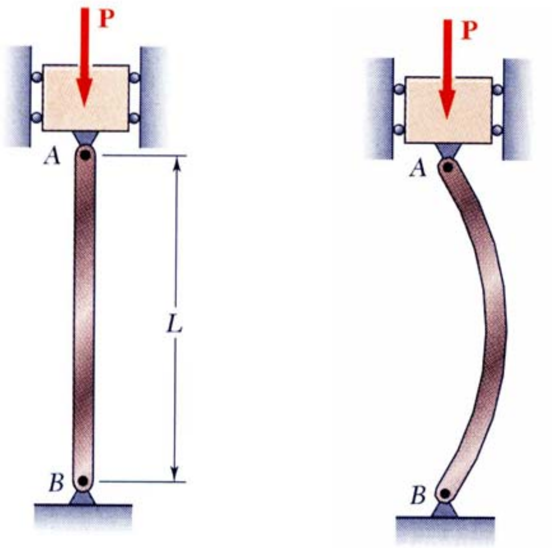
- Easy to visualize for a ball on a surface

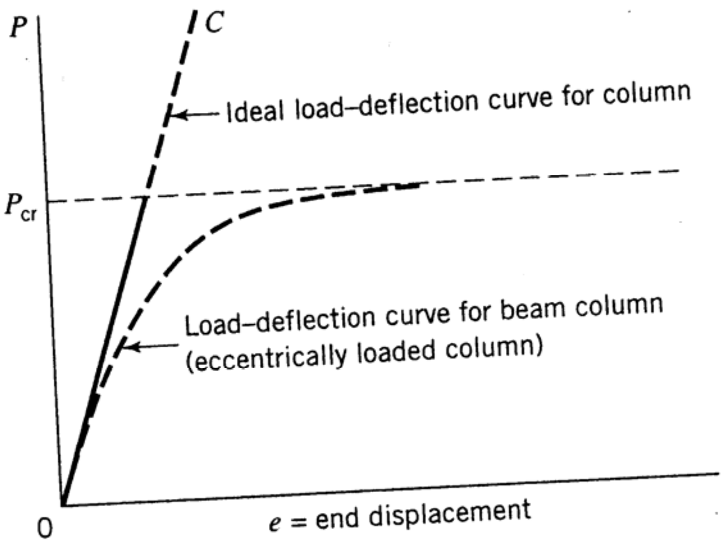


- Note that we use curvature to decide stability, but the surface can curve up with zero curvature

Euler formula

- For simply supported column





$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}} \quad \text{or using } I = Ar^2, \quad \boxed{P_{cr} = \frac{\pi^2 EA}{(L/r)^2}}$$

L/r : Slenderness ratio

Large displacements

- Slenderness ratio and yield stress govern type of post-buckling response

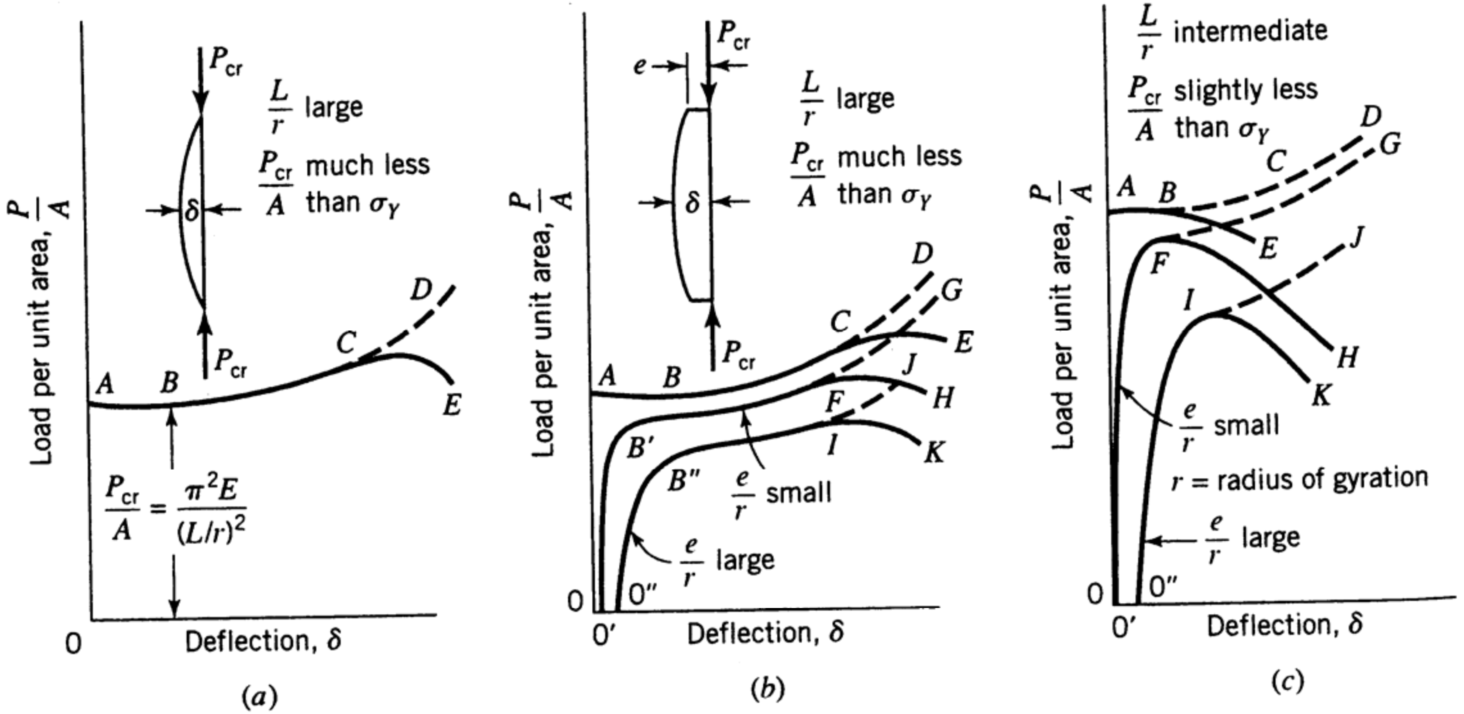
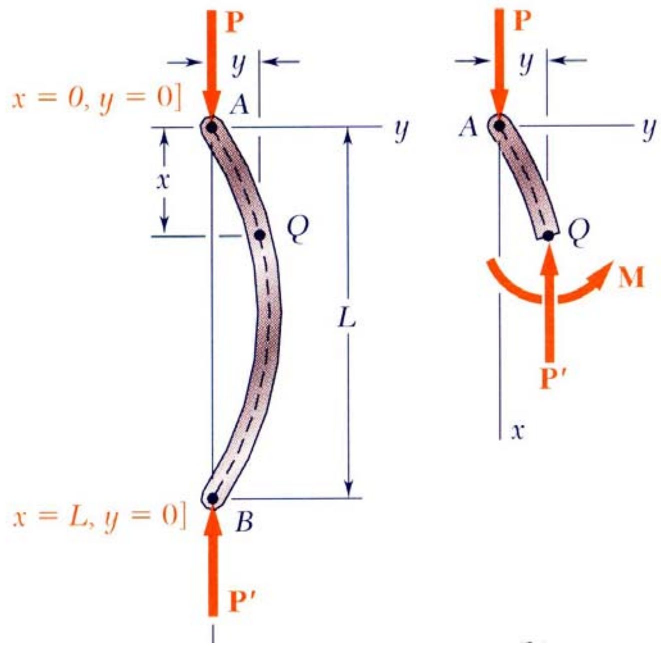


FIGURE 12.3 Relation between load and lateral deflection for columns.

Dashed lines represent behavior without yielding

Governing equation for beam column



$$\frac{d^2y}{dx^2} = \frac{M}{EI} = -\frac{P}{EI}y$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

$$Try: y(x) = A\cos(\omega x) + B\sin(\omega x)$$

$$\frac{d^2y}{dx^2} = -\omega^2 (A\cos(\omega x) + B\sin(\omega x))$$

$$\text{Substitute into } ODE: \omega = \sqrt{\frac{P}{EI}}$$

$$y(x) = A\cos\left(\sqrt{\frac{P}{EI}}x\right) + B\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

Apply boundary conditions

$$y(0)=0 \; and \; y(L)=0$$

$$B\sin\left(\sqrt{\frac{P}{EI}}L\right)=0 \rightarrow \sqrt{\frac{P}{EI}}L=n\pi \quad n=1,2,3,..$$

$$P=\frac{n^2\pi^2 EI}{L^2}$$ n is the integer defining the buckling mode.

$I=Ar^2 \quad where \quad r=\sqrt{\frac{I}{A}}$ is the radius of gyration.

For a column with both ends pinned, n = 1 defines the critical buckling load

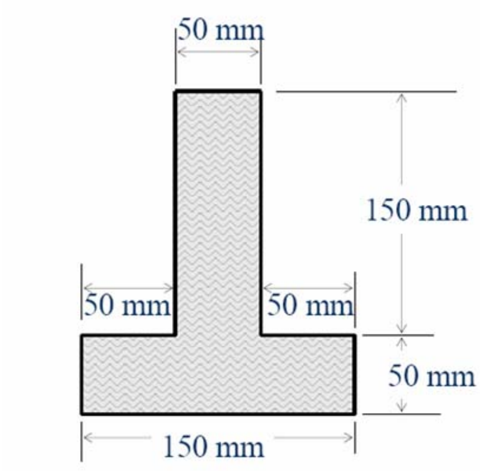
$\boxed{P_{cr}=\frac{\pi^2 EI}{L^2}}$ or using $I=Ar^2$, $\boxed{P_{cr}=\frac{\pi^2 EI}{(L/r)^2}}$

L/r :Slenderness ratio

Compressive (normal) stress at critical buckling load: $\boxed{\sigma_{cr}=\frac{P_{cr}}{A}=\frac{\pi^2 E}{(L/r)^2}}$

Example 1

A 3m column with the cross section shown is constructed from two pieces of timber, that act as a unit. If the modulus of elasticity of timber is E=13 GPa, determine a) The slenderness ratio b) Critical buckling load c) Axial stress in the column when the critical load is applied



- Properties of the cross section

$$A = 2(150)(50) = 15{,}000\,mm^2$$

$$y_c = \frac{25(50)(150) + (50+75)(50)(150)}{15{,}000} = 75\,mm \text{ from bottom}$$

- Moments of inertia about the centroid of the cross section

$$I_x = \frac{150(50)^3}{12} + 150(50)(50)^2 + \frac{50(150)^3}{12} + 150(50)(50)^2 = 53.13\text{x}10^6\,mm^4$$

$$I_y = \frac{50(150)^3}{12} + \frac{150(50)^3}{12} = 15.625\text{x}10^6\,mm^4$$

- Radii of inertia

$$\mathrm{r_x} = \sqrt{\frac{I_x}{A}} = 59.51\,mm \text{ and } \mathrm{r_y} = \sqrt{\frac{I_y}{A}} = 32.3\,mm$$

$$\Rightarrow r = r_{\min} = 32.3\,mm$$

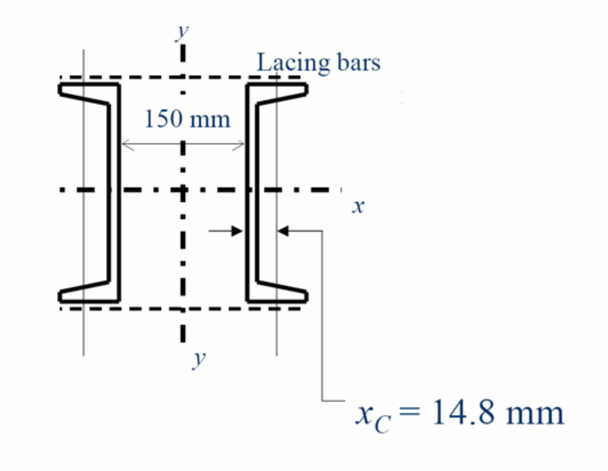
$$a)\text{ Slenderness ratio } \frac{\mathrm{L}}{\mathrm{r}} = \frac{3000}{32.3} = 93$$

$$b)\text{ Critical Buckling Load } P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = 222.75\,kN$$

$$c)\text{ Critical Buckling Stress } \sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2} = 14.85\,MPa$$

Example 2

C229 x 30 structural steel channels with E=200 GPa are used for a 12 m column. Determine the total compressive load required to buckle the column if a) One channel is used b) Two channels are laced 150 mm back to back as shown



Section properties for one channel

$$A = 3795 \, mm^2 \quad x_c = 14.8 mm$$

$$I_{xc} = 25.3 \text{x} 10^6 \; mm^4 \quad I_{yc} = 1.01 \text{x} 10^6 \; mm^4$$

Solution for single channel

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Radii of inertia of a single channel

$$r_x = \sqrt{\frac{I_x}{A}} = 81.1\ mm \text{ and } r_y = \sqrt{\frac{I_y}{A}} = 16.3\ mm$$

$$\Rightarrow r = r_{\min} = 16.3\ mm$$

$$\text{Slenderness ratio}: \frac{L}{r} = \frac{12000}{16.3} = 736.2$$

$$\text{Critical Buckling load}: P_{cr} = \frac{\pi^2 EA}{(L/r)^2} = 13.82\,kN$$

Two laced channels.

$$A = 2(3975) = 7590 mm^3$$

Moments of inertia about the centroid of the cross section:

$$I_x = 2I_{xc} = 50.6 \text{ x} 10^6 \ mm^4$$

$$I_y = 2\left[I_{yc} + A(75+14.8)^2\right] = 63.23 \text{ x} 10^6 \ mm^4$$

Radii of inertia:

$$r_x = \sqrt{\frac{I_x}{A}} = 81.7 \ mm \text{ and } r_y = \sqrt{\frac{I_y}{A}} = 91.3 \ mm \rightarrow r = r_{\min} = 81.7 \ mm$$

Slenderness ratio: $\dfrac{L}{r} = \dfrac{12000}{81.7} = 146.9$

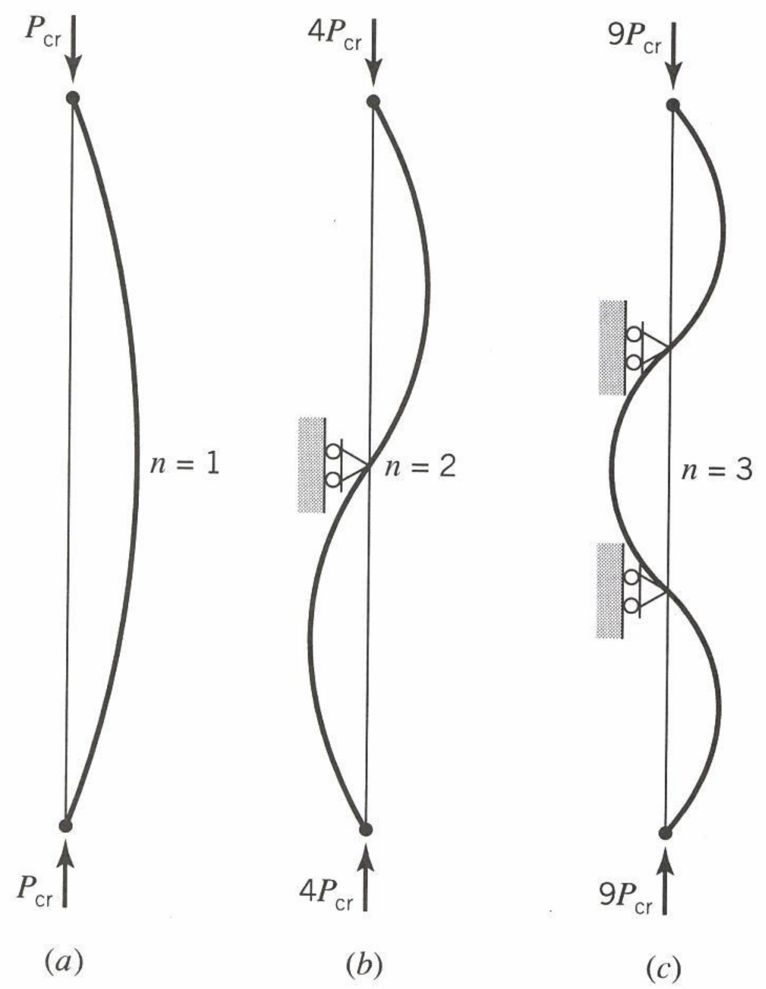
Critical Buckling load: $P_{cr} = \dfrac{\pi^2 EA}{(L/r)^2} = 694.3 \ kN$

Higher buckling modes

- With appropriate boundary conditions can get higher modes

$$P = 4\frac{\pi^2 EI}{L^2} = 4P_{cr}$$

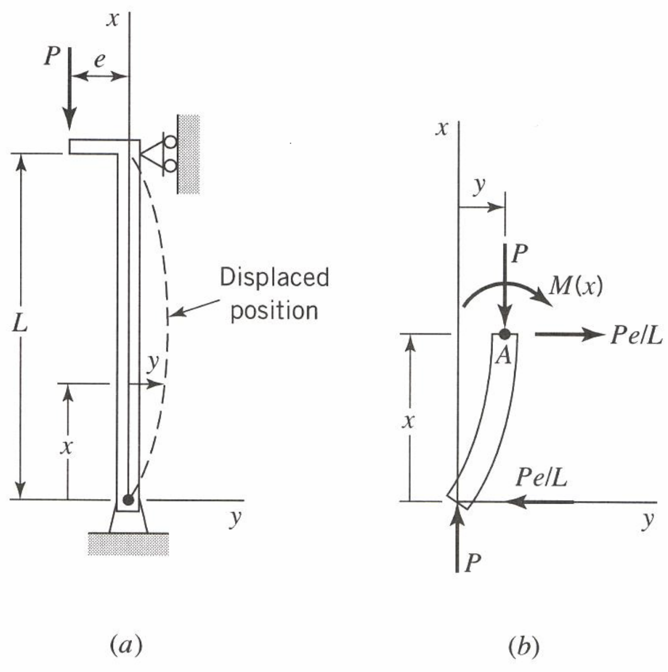
$$\sigma_{cr(2)} = \frac{P}{A} = 4\frac{\pi^2 E}{(L/r)^2}$$



Buckling modes : n = 1,2,3.

Imperfection approach

- Put in imperfection in the form of eccentricity



Eccentrically loaded pinned-end columns

- Moment equation

$$M(x) = -Py - Pe\frac{x}{L}$$

Solution of eccentric load

- Differential equation of equilibrium

$$\frac{d^2y}{dx^2} + k^2 y = -\frac{k^2 ex}{L}, \quad k^2 = \frac{P}{EI}$$

$$y = 0 \quad for \quad x = 0\ ,\ L$$

- General solution

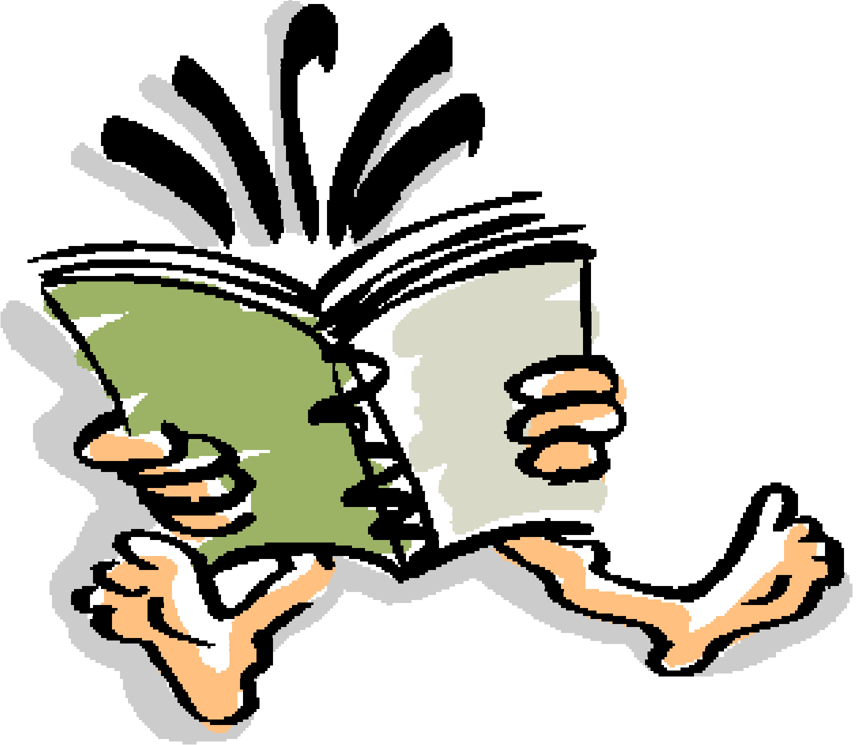
$$y = A\sin kx + B\cos kx - \frac{ex}{L}$$

- With boundary conditions

$$y = e\left(\frac{\sin kx}{\sin kL} - \frac{x}{L}\right)$$

Reading assignment

Sections 12.3-4: Question: Why equation 12.26 does not really guarantee buckling? What does it guarantee?



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