

# Indemnity Payments and Payments for Ecosystem Services: Policy Implications

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## **Abstract**

Payments for Ecosystem Services (PES) are voluntary programs where private or public beneficiaries of ecosystem services (a public good) agree to pay private producers of ecosystem service (ES) inputs. However, when there is private risk to the private provisioning of ES inputs, then there may be gains to offering loss protection (indemnity). This paper characterizes conditions in which it is optimal for a budget constrained regulator to (i) offer indemnity in conjunction with a linear pricing contract and (ii) to pursue both poverty alleviation and the social benefits of the ES inputs.

We find that it is optimal for the regulator to share in the risk of producing ES inputs (or outputs), i.e., offer indemnity if the ES supply function is sufficiently more sensitive to changes in indemnity relative to changes in the linear pay rate. Furthermore, the value from optimally choosing the indemnity, compared to the no-indemnity case, is higher whenever agents are more risk averse. Lastly, we provide a guide to practitioners and empirical researchers on how to evaluate the appeal of indemnity in any particular setting for which PES exists and provision of inputs is risky.

# Introduction

Ecosystem services (sometimes referred to as environmental services) are the aspects of an ecosystem (or environment) that provide benefits to humanity, directly or indirectly (Millennium Ecosystem Assessment, 2005; Fisher, Turner and Morling, 2009).<sup>1</sup> A majority of ecosystem services (ES) fall within the realm of public goods therefore are supplied at suboptimal levels. Worse yet, early assessments indicate that these services are degrading at rapid rates (Millennium Ecosystem Assessment, 2005). One solution for this market failure that has gained popularity among scholars and practitioners is known as payments for ecosystem services (PES) (Salzman et al., 2018). PES programs facilitate the creation of contracts between ES beneficiaries (or their representative such as the government or regulator) and ES suppliers where the beneficiary agrees to pay the supplier for their provision of ES. PES contracts are voluntary programs where agents have to opt into participation which is in contrast to Pigouvian taxation. Around the world, there are now 550 PES programs with a combined annual payment of around \$36 billion USD (Salzman et al., 2018).<sup>2</sup>

PES schemes are essentially an application of the Coase Theorem and can be contracted on either ES outputs (like lower atmospheric CO<sub>2</sub>) or ES inputs (like carbon sequestering land uses) and which one to contract on is still an active area of research (White and Hanley, 2016). PES contracts based on outputs are sometimes referred to as performance-based con-

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<sup>1</sup>Direct benefits could include eco-tourism, pollination for agriculture, carbon sequestration, and general ecosystem health to prevent desertification. Examples for indirect benefits include non-use value from biodiversity, predator populations to prevent deer overpopulation, etc.

<sup>2</sup>In this paper, we abstract away from concerns about the precise definitions of ES (Fisher, Turner and Morling, 2009) and PES (Schomers and Matzdorf, 2013) where there are disagreements on how narrow or broad each definition should be. Throughout this paper, both ES and PES will be used to refer to the broadest definitions of both.

tracts while input-based ones are also known as action-based contracts. One central tradeoff between these two types is that contracting on outputs reduces the uncertainty of benefits from higher ES. However, because of the variability in nature, the ES production function is inherently stochastic which means that contracting on ES outputs could put significant risk on the agent. Further, it is typically much harder to monitor ES outputs compared to ES inputs (Burton and Schwarz, 2013; Derissen and Quaas, 2013).<sup>3</sup> Consequently, many PES programs are contracted on ES inputs rather than outputs (Jack, Kousky and Sims, 2008) with the majority of European PES programs being input-based (Wuepper and Huber, 2022).

The PES literature has acknowledged the important role that risk plays in agents' PES related decisions but rather than formalizing its role, it has chosen to mostly sidestep the issue by narrowing its focus on input-based PES. We argue that risk ought to have a larger consideration in the design of PES programs, regardless of whether it is output or input based. This paper contributes to a small set of papers that analyze the role of risk in PES payments by offering an alternative to increased payments, that is, offering indemnity instead. Our goals with this paper is to (i) define the necessary and sufficient conditions for when it is optimal for the regulator to offer positive indemnity payments and what that optimal indemnity is, (ii) offer an approach to evaluate how large such gains could be, (iii) define further conditions for when it is optimal to pursue the dual objective of additionality and poverty alleviation.<sup>4</sup>

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<sup>3</sup>Though remote sensing is making monitoring environmental factors at large more and more accessible, it is still quite difficult to attribute individual contributions to aggregate environmental quality measures.

<sup>4</sup>Indemnity simply means protection against financial loss/burden. Insurance is a subset of indemnity in that it specifically requires the indemnitee to pay the indemnitor for indemnity.

The model tasks the regulator with representing the beneficiaries of the ES. Their objective is to maximize the gross social benefit given a fixed budget by choosing both payment (or subsidy) level and an indemnity level. The typical PES contract is known as linear pricing contracts which defines a measurable and verifiable ES input and pays a subsidy for each unit of input supplied.<sup>5</sup> The typical linear PES contract chooses the optimal payment level conditional on the indemnity being zero which will henceforth be referred to as the pure payment contract. In contrast, the optimal contract is arrived at by allowing the regulator to freely choose both the payment and indemnity levels.

We find that the pure payment contract always offers full indemnity whenever agents are risk averse but that when agents are risk neutral, the optimal and pure payment contracts provide the same social benefits per government dollar spent. The reason for this has been well understood from the insurance literature which is that the regulator is effectively risk neutral so that the cost of bearing the risk of ES provision is equal to the mean of the cost shock. However, a risk adverse agent would value the cost from risk at the mean cost shock plus some risk premium. Thus, the regulator can always arbitrage this by offering indemnity and “charging” a higher price than it costs to insure. The risk neutrality on the part of the regulator can be justified on the grounds of having a large number of beneficiaries Arrow and Lind (1970). Further, the value gained from implementing the optimal contract, relative to the pure payment contract, increases with the coefficient of relative risk aversion. Following

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<sup>5</sup>Linear pricing contracts are in contrast to optimal contracts from the mechanism design literature. Those contracts are optimal in the sense that it maximizes social welfare by explicitly accounting for the asymmetric information about agent’s counterfactual ES input supply (i.e., additionality). They are a menu of pairs of ES input levels and the corresponding payment, carefully designed to create a separating equilibrium that maximizes expected welfare (Mason and Plantinga, 2013).

Chetty (2006), we show how to estimate a lowerbound for this parameter using estimates of relevant elasticities and moments of the ES cost function. We also conduct a numerical exercise that illustrates the relationship between the magnitude of risk aversion and the value added from implementing the optimal contract. Lastly, we find that the pursuit of the dual objective of maximizing ES benefits with minimizing poverty is optimal only when a particular comparative static is below some threshold. Specifically, the condition states that the business-as-usual level of ES provision must increase with wealth at a sufficiently high rate.

Section will briefly discuss the context and literature review up which this paper hopes to contribute. Section outlines the model detailing both agents' and principal's problems The next section deals with additionality and outlines conditions in which it is optimal for the regulator to pursue the dual objective of additionality and poverty alleviation.

## **Literature on Risk in PES**

This paper relates closely with the literature on risk associated with payments for ecosystem services (PES) which has since established that the required linear payment needed to induce a certain “risky green” action is higher if agents are risk averse compared to being risk neutral (Benítez et al., 2006; De Pinto, Robertson and Obiri, 2013). Benítez et al. (2006) derived the optimal payment needed to induce the adoption of farming practices/technologies that improve soil carbon sequestration and find that such payment increases with risk aversion.

De Pinto, Robertson and Obiri (2013) does something similar for the adoption of farming practices that help with climate change mitigation as well uses Ghana as a case study. The risks faced by agents in each of those papers are related to revenue risk associated with the desirable “green” practices or technologies. However, loss protection, also known as indemnity, is rarely brought up as a potential tool to address risk averse behavior.

There are many other instances in which the participation into PES programs introduces risk to the agent’s bottom line. Another example of risks within input based PES is payments to encourage coexistence with predators where agents are paid to avoid revenge killing of a predator that caused damage to either property (livestock) or human life (Dickman, Macdonald and Macdonald, 2010). Many times, these payments are in conjunction with either payments for or requirements to provision ES inputs such as using guardian dogs to minimize conflicts with predators. It is in the predation context that the literature has explicitly analyzed the role of insurance in conservation and find success in preserving carnivore populations and habitats (see Dickman, Macdonald and Macdonald (2010) for a review). There are also case studies that find that such indemnity schemes lead to moral hazard (Bautista et al., 2019). Other examples of risk associated with PES participation include the risk that payments will not be received due to institutional mistrust (Jack et al., 2022), risks associated with stochasticity of nature within output based PES contracts, and the risk from habitat maintenance which can cause damages to livestock through disease transmission from increased wildlife (Rhyan et al., 2013).

To the best of our knowledge, the literature has not yet addressed what role that “insurance”

or, more precisely, indemnity, can play in achieving ES goals under an economic optimality standpoint.<sup>6</sup> Only Graff-Zivin and Lipper (2008) suggest insurance as a tool to be used in conjunction with the standard linear PES contract. However, those authors mentioned it casually as part of a general discussion rather than giving it a formal treatment. This paper hopes to bring everything together by thinking about the issue that risk poses in PES schemes generally and how a regulator should account for this in its design of linear pricing PES contracts.

## Model

The linear PES contract is characterized by (1) an indemnity rate  $I \in [0, 1]$  that is paid out for each dollar of loss ( $x_i \epsilon_i$ ) that occurs and (2) the ES input payment rate  $p$  which is paid for each unit of ES input  $x_i$  (henceforth referred to as the pay rate). There is a cost shock  $\epsilon_i$  that is distributed  $(\mu_\epsilon, \sigma_\epsilon^2)$  and has support over  $[0, e]$  and when multiplied with  $x_i$  produces the loss that an agent faces for choosing  $x_i$ . I assume that only one ES input  $x_i$  is being contracted and that the input is continuous over  $[0, \infty]$  so that if a cost shock occurs, the PES participant's loss equals  $x_i \epsilon_i$  but receives  $x_i \epsilon_i I$  in indemnity payments. If the agent does not participate in the PES program, then  $x_i = 0$ . Potential enrollees all have heterogeneous initial wealth  $\omega_i$  and are heterogeneous in their known input cost function  $g_i(x_i, \omega_i)$  and for brevity, subscripts  $i$  will be henceforth omitted when risk of confusion is low.<sup>7</sup>

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<sup>6</sup>The main difference between insurance and indemnity is that insurance implies that there is an insurance premium whereas indemnity is a much more general sense of protection against loss and does not require the insuree to pay a premium or cost of insurance.

<sup>7</sup>Small letters will denote individual level values and big letters denoting aggregate values.

Agents have Bernoulli utility  $u(c)$  over consumption  $c$  and a budget constraint defined in (1). Thus, consumption is only stochastic when opting into the PES program by choosing  $x > 0$  and deterministic if  $x = 0$ .<sup>8</sup> For any given policy level  $(I, p)$ , there will be  $X$  units of ES input supplied in the aggregate which then engenders ES output  $ES(X)$ . This ES output then leads to some level of social benefit  $\tilde{B}(ES(X))$  which may be random as well. From here on, denote  $B = \tilde{B} \circ ES$  as the expected social benefit and is a function of the aggregate ES inputs provided,  $X$ .

## Agent's Problem

Agents are expected utility maximizers who take contract  $(I, p)$  as given and chooses ES input level.

$$\begin{aligned} \max_x \mathbb{E}[u(c)] \\ \text{s.t.} \quad c = \omega + xp - g(x, \omega) - x\epsilon(1 - I) \\ x \geq 0 \end{aligned} \tag{1}$$

The ES input cost function is assumed to be increasing and convex ( $g_x(x, \omega) > 0$ ,  $g_{xx}(x, \omega) > 0$ ).<sup>9</sup> The first order condition is given by (2) which implicitly defines the interior solution  $x^*$ .

$$\mathbb{E} \left[ u'(c^*(x^*, \omega)) \frac{\partial c^*}{\partial x} \right] = 0 \tag{2}$$

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<sup>8</sup>There is no loss of generality since one could simply treat  $\omega$  as a random variable as well.

<sup>9</sup>One could allow agents to derive positive benefits from provision of  $x$  and include a second argument into the utility. Then the RHS of (2) would instead be equal to  $\mathbb{E}[u_x(c, x)]$ . Further, one would have to follow more closely to Chetty (2006) in order to account for the role of complementarity between  $x$  and  $c$  in the application stage of this paper.



Using monotone comparative statics, it is possible to sign  $\frac{\partial x^*}{\partial p}$  and  $\frac{\partial x^*}{\partial I}$ .

**Lemma 0.1.** *Given contract  $(I, p)$ , the provision of ES inputs is non-decreasing in indemnity payment rate  $I$  nor is it decreasing in the payment rate  $p$ , i.e.,  $\frac{\partial x_i^*}{\partial I} \geq 0$  and  $\frac{\partial x_i^*}{\partial p} \geq 0$ .*

See proof on page 30.

Lemma 0.1 says that an agent's supply of ES inputs weakly increases with both the pay and indemnity rates. This implies that there must necessarily be a trade off faced by the regulator with a fixed budget  $G$  who maximizes expected social benefit,  $B(X)$ . The natural question then arises, when is it optimal for such a regulator to offer a positive indemnity rate at the expense of offering higher  $p$ ?

## Planner's Problem

The social planner takes government budget  $G$  as fixed and seeks to maximize aggregate supply of ES input  $X = X(x_1^*, \dots, x_n^*) = \sum_i x_i^*$ . Note that the formulation of (3) excludes the welfare of the ES suppliers since the goal is not to achieve a first best outcome, rather the objective is to maximize the benefit of a public good given a fixed budget. If the regulator's budget constraint is not binding then the regulator's solution is isomorphic to the Coasean bargaining outcome and will actually achieve first best.

$$\max_{(I, p)} B(X) \quad \text{s.t.} \quad X(p + \mu_\epsilon I) \leq G \quad (3)$$

With the inclusion of indemnity payments, the government expenditure for any given PES

contract  $(I, p)$  is now stochastic meaning that there are multiple ways to formulate a budget constraint for the regulator. Equation (3) formulates the budget constraint in terms of expectations where  $\mu_\epsilon$  is the expected value of the cost shock which can be justified on the grounds of having a large beneficiary population (Arrow and Lind, 1970). Alternatively, the budget constraint can be formulated in probabilistic or extreme terms, e.g., the probability that the regulator's expenditure exceeds  $G$  is less than some threshold or the budget constraint can be formulated to ensure that the budget will not be exceeded in the event of an extreme shock (where all agents experience shock  $\epsilon_i = e$ ).

## When are indemnity payments optimal?

The formulation of (3) differs slightly from the standard linear pricing PES contracts. The standard linear pricing PES contract takes  $I = 0$  as given, i.e., there is never any indemnity payments and is usually so for exogenous reasons rather than a result of the regulator behaving optimally. As a reminder, this is referred to as the pure payment contract while the optimal contract refers to the case where the regulator optimally chooses both  $I$  and  $p$ . The central question here is whether the regulator can achieve a higher indirect social benefit function when the constraint  $I = 0$  is relaxed. In other words, when is it optimal for the regulator to share the risk of supplying ES inputs with the ES input suppliers?

**Proposition 0.2.** *Given the ES supply function  $X = \sum_i x_i^*$ , where  $x_i^*$  is the solution to (2), it is optimal for the regulator to offer a positive indemnity rate ( $I > 0$ ) if and only if agents are risk averse.*

See proof on page 31.

Under both optimal and pure payment contracts, the regulator's budget will always be exhausted. If the regulator wanted to increase the indemnity rate  $I$  from zero, then their expenditure will increase by  $X\mu_\epsilon$  (assuming no supply responses) but then the pay rate  $p$  must be reduced which lowers their expenditure by  $X$ . However, increasing  $I$  and decreasing  $p$  will lead to countering effects on the supply of ES inputs  $X$  which has further impacts on the budget. Thus, (16) from the proof of Proposition 0.2 simply says that in order for this budget reshuffling to be optimal, the increase in  $X$  in response to an increase in  $I$  must outweigh the decrease  $X$  in response to the required decrease in  $p$ .

This point can be further illustrated by looking at (4) where (4a) shows that increasing  $I$  increases regulator's expected expenditure (i) through an increase in supply  $X$  and (ii) through increasing the amount paid on existing supply  $X\mu_\epsilon$  in expectation. Then to re-balance the budget, a decrease in  $p$  is needed leading to lower spending on existing supply  $X$  and by lowering the total ES offered. Suppose that the condition from Proposition 0.2 holds with equality ( $\partial X/\partial I = \partial X/\partial p \cdot \mu_\epsilon$ ). Then a one unit increase in  $I$  requires  $p$  to decrease by  $\mu_\epsilon$  for budget balance. This would result in zero change in ES input. If the condition from Proposition 0.2 holds with strict inequality, then the pay rate  $p$  would have to decrease by more than  $\mu_\epsilon$  to balance the budget but will still result in a higher  $X$ . For instance, suppose  $\partial X/\partial p \approx 0$  so that the required decrease in  $p$  equals  $\mu + Z$  since it would have to compensate for increasing  $I$  which increases expenditure through two channels. However, the  $X$  would

clearly be higher and so this reshuffling is optimal for the regulator.

$$\frac{\partial G}{\partial I} dI = \left[ \frac{\partial X}{\partial I} (p + I\mu_\epsilon) + X\mu_\epsilon \right] dI \quad (4a)$$

$$\frac{\partial G}{\partial p} dp = \left[ \frac{\partial X}{\partial p} (p + I\mu_\epsilon) + X \right] dp \quad (4b)$$

Proposition 0.2 is quite important as it shows that the regulator can always extract more social value given a fixed budget  $G$  by offering indemnity payments when agents are risk averse. This may leave the reader wondering, when does the condition from Proposition 0.2 hold?

## Optimal Indemnity Under Risk Aversion

After establishing the fact that it is always optimal for the regulator to offer positive indemnity when agents are risk averse, one important question still remains. What is the optimal level of indemnity that the regulator should offer? As it turns out, the intuition in the above section suggests that it is optimal to offer full indemnity ( $I = 1$ ) when the curvature around point  $A$  is sufficiently steeper than around  $B$ .

**Proposition 0.3.** *When agent's are risk averse then it is optimal for the regulator to offer full indemnity, i.e.,  $I^* = 1$ .*

See proof on page 33.

One way to visualize the result from Proposition 0.3 is to look at the iso- $G$  (iso budget) and

iso- $X$  (iso supply) curves in Figure 2. The slope of the iso- $G$  curve can be found by total differentiating the budget equation in (3) and total differentiating  $X(I, p)$ . In Appendix 2 I show that the slope of the iso- $X$  curve is always steeper than that of the iso- $G$  curve when the condition from Proposition 0.2 holds implying that the solution always occurs at  $I = 1$ .

## Graphical Argument

The graphical intuition for Proposition 0.2 can be illustrated with a simple example. Consider the case where the choice variable  $x$  is binary either 0 or 1, the cost shock is also binary either 0 or  $e$ , and agents differ only in their cost of provisioning,  $g$ . Then the pivotal agent's decision problem can be summarized by graphing their indifference curve over state-contingent consumption space  $(c_b, c_g)$  as in Figure 1 where  $c_b$  and  $c_g$  are consumption levels under bad and good states.

When agents are risk neutral ( $R = 0$ ), their indifference curve is a straight line with slope  $\frac{\pi}{1 - \pi}$  where  $\pi = \mathbb{P}(\text{bad state}) = \mathbb{P}(\epsilon = e)$ . Point  $A_0$  would be the pivotal agent's consumption bundle under the PES with no indemnity where  $a_0 = \omega + p_0 - g$  and  $b_0 = \omega + p_0 - g - e$ . By definition, the pivotal agent is indifferent between  $A_0$  and  $C$  which implies that  $p_0 = \mu_e + g = \pi e + g$ . Now imagine the regulator decides to switch from  $I = 0$  to  $I = 1$  while holding  $p = p_0$ . This then pushes the pivotal agent's consumption bundle under PES participation to the 45 degree line (going from  $A_0$  to  $B_0$ ) which is a horizontal shift equal to  $e$ . If the regulator stays at this policy point  $(1, p_0)$ , then the increase in ES supply is proportional to the distance  $CB_0$ . But budget balancing requires a reduction in  $p$  which moves the

consumption bundle towards the origin along the path of the 45° line.<sup>10</sup> Proposition 0.2 says that, at  $B_0$ , if the decrease in  $p_0$  required for budget balancing is higher than  $\mu_\epsilon$ , i.e., need to go from  $B_0$  to  $P_1$ , then it is not optimal to provide any indemnity. This is because the ES supply at  $P_1$  is strictly less than that at  $A_0$ . Whereas if the budget balancing decrease in  $p_0$  is less than  $\mu_\epsilon$ , i.e., need only go from  $B_0$  to  $P_0$ , then it is optimal for the regulator to offer positive indemnity since the regulator will always achieve budget balance and being at  $P_0$  offers higher ES supply than  $A_0$ . It turns out though that the going from  $B_0$  to  $C$  will exactly balance the budget and thus, there is no value added or lost from offering full indemnity which is consistent with Proposition 0.2.

However, when agents are risk averse ( $R > 0$ ), then the pay rate under the pure payment contract  $p_1$  has to be greater than that under the optimal contract  $p_0 = \mu_\epsilon + g$  in order for the same agent to be indifferent between the consumption bundles from not participating ( $C$ ) and from participating ( $A_1$ ). Now if the regulator decides to go from no indemnity to full indemnity (from  $A_1$  to  $B_1$ ), then the ES supply increase is much higher relative to the risk neutral scenario as  $B_1$  is much further up on the 45° line than  $B_0$ . However, the ES supply response to changes in  $p$  is always the same no matter the risk averse behavior. Consequently, the budget balancing required reduction in  $p_1$  is the same as the required reduction in  $p_0$  from the risk neutral case.

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<sup>10</sup>Any change in  $p$  shifts agents' bundles towards the origin on a path parallel to the 45° line. Changes in  $I$  shift bundles horizontally.

## Value Added from Indemnity

One may still be wondering how big is the value added from having optimal indemnity alongside the standard linear pricing PES contracts? In both contracts, the regulator's budget constraint will hold with equality in expectation so that the value added  $V$  is given by (5) where  $X_1 = X(1, p(1))$  represents the aggregate ES input supply under the optimal contract and  $X_0 = X(0, p(0))$  represents the aggregate ES input supply under the pure payment contract.

$$V = B(X_1) - B(X_0) \tag{5}$$

Equation (5) implies that the gain from optimal indemnity PES relative to the pure payment PES is a function of two determinants. First, if the curvature of the expected social benefit function around the neighborhood of  $X_0$  and  $X_1$  is high, then the value added would be higher. Second, if the difference  $X_1 - X_0$  is high then so too would be the value added. What makes the difference between these two ES input supplied great has to do with risk tolerance. This boils down to how much can offering indemnity payments allow the regulator to relax  $p(I)$ , the conditionally optimal pay rate? To see this, note that equation (19) is an increasing function of  $A$  and  $R$ . Not only that, increasing the risk aversion increases the ratio in (19) everywhere over  $I$ . Hence being able to estimate risk aversion allows researchers to estimate the proper policy and the magnitude of the gain.

Table 1 shows simulation results using estimates of risk aversion from Elminejad, Havranek and Irsova (2022) and assuming CRRA utility. Using the parameters from Table 2 and

randomly generated ES supply costs and initial wealth values (distributions shown in Figure 3), we numerically solve the planner's problem in (14) once for  $I = 0$  and another for  $I = 1$  which then allows the calculation of value added. This is then repeated for each risk aversion parameter in Table 1. The results show that the value added from the optimal contract relative to the pure payment contract increase with risk aversion and can range from 5.56% to 42.5% increase in ES supply.

## Estimating $R$

Table 1 is only useful for practitioners if there is a way to estimate risk aversion for any given PES setting with which to compare with the value added figures in the table. In Appendix 3, I show that  $\frac{\bar{A}_\Gamma}{\mu_\Gamma} < \bar{A} < R$  where  $\bar{A}_\Gamma = \frac{-\mu_{u''\Gamma}}{\mu_{u'}}$  is *some* metric for absolute risk aversion and  $\bar{A} = \frac{-\mu_{u''}}{\mu_{u'}}$  is the absolute risk aversion coefficient, loosely speaking. Therefore, estimating  $\bar{A}_\Gamma$  is equivalent to estimating the lower bound for risk aversion  $R$  and thus a lower bound on the value added from implementing the optimal.

To estimate  $\bar{A}_\Gamma$ , we follow a similar procedure outlined in Chetty (2006) to derive a formula for  $\bar{A}_\Gamma$  which I expressed as a function of the price elasticity of Hicksian ES input supply and the income elasticity of ES input supply. The intuition behind this approach is outlined in Chetty (2006) which utilized various labor supply elasticities to compute upperbounds for the coefficient of relative risk aversion with the idea being that labor supply responses to wage changes implies the curvature of the utility function which then gives rise the risk aversion parameter.



First, agents' choices on  $x$  (assuming an interior optimum) must satisfy the first order condition (2). Taking derivatives of their FOC and rearranging algebraically gives (6).

$$\frac{\partial x}{\partial p} = \frac{\mu_{u'} + x\mu_{u''\Gamma}}{\mu_{u'}g_{xx} - \mu_{u''\Gamma^2}} \quad (6a)$$

$$\frac{\partial x}{\partial w} = \frac{(1 - g_\omega)\mu_{u''\Gamma} - \mu_{u'}g_{x\omega}}{\mu_{u'}g_{xx} - \mu_{u''\Gamma^2}} \quad (6b)$$

Using Slutsky's decomposition (shown in Appendix 4) for compensated ES input supply ( $h$ )

$$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} - \frac{\partial x}{\partial \omega}x \quad (7)$$

then the ratio of the substitution effect and income effect is given by (8)

$$\frac{\partial h/\partial p}{\partial x/\partial \omega} = \left( \frac{\partial x/\partial p}{\partial x/\partial \omega} - x \right) \quad (8)$$

Then combining (6) with (8) and some algebraic manipulation gives our equation for the desired risk aversion metric (9).

$$\bar{A}_\Gamma = \frac{1 + g_{x\omega} \left( x + \frac{\varepsilon_{hp}}{\varepsilon_{x\omega}} \frac{\omega}{p} \right)}{(g_\omega - 1) \left( \frac{\varepsilon_{hp}}{\varepsilon_{x\omega}} \frac{\omega}{p} \right) + x(g_\omega - 2)} \quad (9)$$

Where  $\varepsilon_{hp}$  and  $\varepsilon_{x\omega}$  are the price elasticity of Hicksian supply and income elasticity of supply, respectively. Thus to estimate  $R$ , one can simply estimate (9) and treat the estimate of  $\frac{\bar{A}_\Gamma}{\mu_\Gamma}$  as the coefficient of relative risk aversion  $R$ . One downside is that estimate (9) is quite

informationally demanding as it requires the researcher to estimate not only the substitution and income elasticities, but also various aspects of the ES cost function,  $g_\omega$  and  $g_{x\omega}$ . However, if one is able to establish that the cost function does not change with income  $g_\omega \approx 0$ , then by Young's Theorem, the marginal cost will not change either  $g_{x\omega} \approx 0$  which will simplify (9) to (10). Note that  $g_\omega = 0$  implies that  $\varepsilon_{x\omega} < 0$  which makes (10) much easier to estimate.

$$\bar{A}_\Gamma = \left[ -\frac{\varepsilon_{hp}}{\varepsilon_{x\omega}} \frac{\omega}{p} - 2x \right]^{-1} \quad (10)$$

## Additionality and Poverty Alleviation

In practice, private agents may produce strictly positive levels of the ES input in the absence of a PES program and therefore there is considerable interest in the literature and policy arena on the idea of additionality. That is, researchers and policy makers want to make sure that the ES inputs being contracted for are *additional* and hence would not have been procured in the absence of a PES program. It is often very costly to monitor additionality for all program applicants which could render simple linear contracts, like the one proposed above, inefficient. There are many papers that try to create a mechanism to generate a separating equilibrium that is efficient (Mason and Plantinga, 2013). However, this is only necessary when the regulator cannot observe types which is something of the opposite extreme. There are characteristics of households that are cheaply observable such as income or wealth. Then a natural question is when does additionality decrease (or increase) with wealth? Policy makers are often interested in this as this directly answers the question of when is it optimal to have the dual objective of increasing ecosystem services and providing

poverty alleviation. Additionality can be defined as

$$\alpha_i = x_i^* - x_i^{bau} \quad (11)$$

It is clear from (11) that additionality decreases with wealth if and only if (12) holds.

$$\frac{\partial x_i^*}{\partial \omega_i} \leq \frac{\partial x_i^{bau}}{\partial \omega_i} \quad (12)$$

Without functional form assumptions, however, evaluating (12) is quite difficult. Instead, Proposition 0.4 relies on the intuition that in order for additionality to decrease in wealth, the slope of the  $x^{bau}$  curve, when plotted against  $\omega$ , cannot be too flat (shown in Figures 4 and 5). A sufficient condition for (12) to hold is for the  $x^*$  curve to be decreasing in  $\omega$  while the  $x^{bau}$  curve is increasing. However, neither curve is necessarily required to be increasing or decreasing. What is necessary is for the difference between the two to be decreasing in  $\omega$ , hence the restriction the slope of the  $x^{bau}$  curve.

**Proposition 0.4.** *Take PES contract  $(I, p)$  as given. Additionality decreases in wealth (poorer households have higher additionality) if the change in the business-as-usual level of ES input in response to a change in wealth is above some lower bound. Specifically,  $\frac{\partial \alpha}{\partial \omega} < 0$  if and only if*

$$\frac{\partial x^{bau}}{\partial \omega} > \frac{\mu_{A\Gamma}(h_\omega - 1) - h_{\alpha\omega}}{\mu_{A\Gamma}h_\alpha + \mu_{A\Gamma^2}}$$

where  $A = -\frac{u''(c)}{u'(c)}$  is the coefficient of absolute risk aversion,  $\Gamma = p - h_\alpha - \epsilon(1 - I)$  is the random marginal return from chosen additionality  $\alpha$ ,  $h(\alpha, \omega)$  is the deterministic cost function in terms of additionality  $\alpha$  so that  $h_\alpha$ ,  $h_\omega$ , and  $h_{\alpha\omega}$  are all the partials with respect

to the subscripts. The terms  $\mu$  simply denote the expectation of the subscripted variables.

See proof on page 33.

Proposition 0.4 states that in order for additionality to decrease in wealth, i.e., poorer agents having higher additionality, the business-as-usual ES input supply must increase in wealth beyond some threshold. In other words, the ES input supplied for poorer agents, in the absence of PES incentives, must be lower than those of wealthier agents and this difference must exceed a threshold. The reason for why a bound is needed on the slope of  $x^{bau}$  only is because  $\partial x^*/\partial\omega$  has the exact same functional form as  $\partial x^{bau}/\partial\omega$  but the two are evaluated at different values for both  $x$  and  $(I, p)$ . One important take away is that it is not sufficient for the marginal cost of ES input supply to be simply decreasing in wealth in order for additionality to decrease in wealth. That is because the if the marginal cost decreases in wealth, then both the business-as-usual and the PES input supplied will decrease in wealth too leaving the additionality ambiguous.

Lastly, the bound on  $\partial x^{bau}/\partial\omega$  in Proposition 0.4 can be estimated if one could estimate aspects of the cost function and relevant elasticities from (9). However, one would still need to be able to gather sample data on  $x^{bau}$  which is often the difficult part. At the very least, the approach outlined in this paper can be used to motivate such an effort to gather this data. Doing so can go a long way to answer the question of when it is optimal for the regulator to pursue the dual objectives of promoting environmental quality and poverty alleviation.

# 1 Discussion

This paper shows that a regulator should couple indemnity payments with a standard linear pricing PES whenever the ES supply function is sufficiently more responsive to indemnity than it is to pay rate. Further, this condition (that ES supply function is sufficiently more sensitive to  $I$  relative to  $p$ ) always holds when agents are risk averse but not when agents are risk neutral in which case, there are no gains or losses to the regulator from offering indemnity. This result is due to the fact that the risk neutral regulator can supply indemnity at a cost equal to  $\mu_\epsilon$  while risk averse agents are willing to pay more than that to get full indemnity. Lastly, the optimal level of indemnity with risk averse agents is full indemnity when agents exhibit risk aversion and is consistent with the insurance literature. Additionally, the social gain from switching to the optimal contract is increasing in the risk aversion and can range anywhere from 5% to 40% gain in ES supply. Finally, the theoretical results indicate that targeting low wealth households with a PES contract as a means achieving both poverty alleviation and environmental improvement can be optimal if the slope of the business-as-usual ES supply curve (plotted against wealth) is sufficiently steep.

For practitioners, I show how one could estimate a lowerbound on the value added by estimating a lowerbound on the risk aversion parameter  $R$  and certain moments of the ES input cost function,  $g_{x\omega}$  and  $g_\omega$ . The procedure is similar to Chetty (2006) and one important advantage of the approach outlined in this paper is that one can estimate the indemnity response of ES supply without needing to first have indemnity implemented in practice. Although it may still be demanding for researchers to estimate this lowerbound, the infor-

mational requirement is reduced greatly if it is possible to establish that the ES cost function is independent of wealth so that one need only estimate (10) rather than (9).

It is important to note that these results do not depend on the specific functional forms for utility, cost functions, nor the stochastic structure of the cost shock. Furthermore, these insights can easily be applied to output-based PES programs where indemnity can be linked to some environmental index similar to index insurance. However, we leave considerations of moral hazard for future studies but note that moral hazard can manifest itself in numerous ways. One of which is a “scale” response, i.e., the loss protection increases the risky activity which means increasing ES supply. This response works in favor of the regulator but it hurts a third party private insurer. On the other hand, indemnity may discourage activities that reduce the probability and/or the magnitude of a loss but do not affect ES supply. In the predation context, indemnity against depredation may decrease incentives to employ guardian dogs which negatively impact both the regulator and the private insurer’s objectives.

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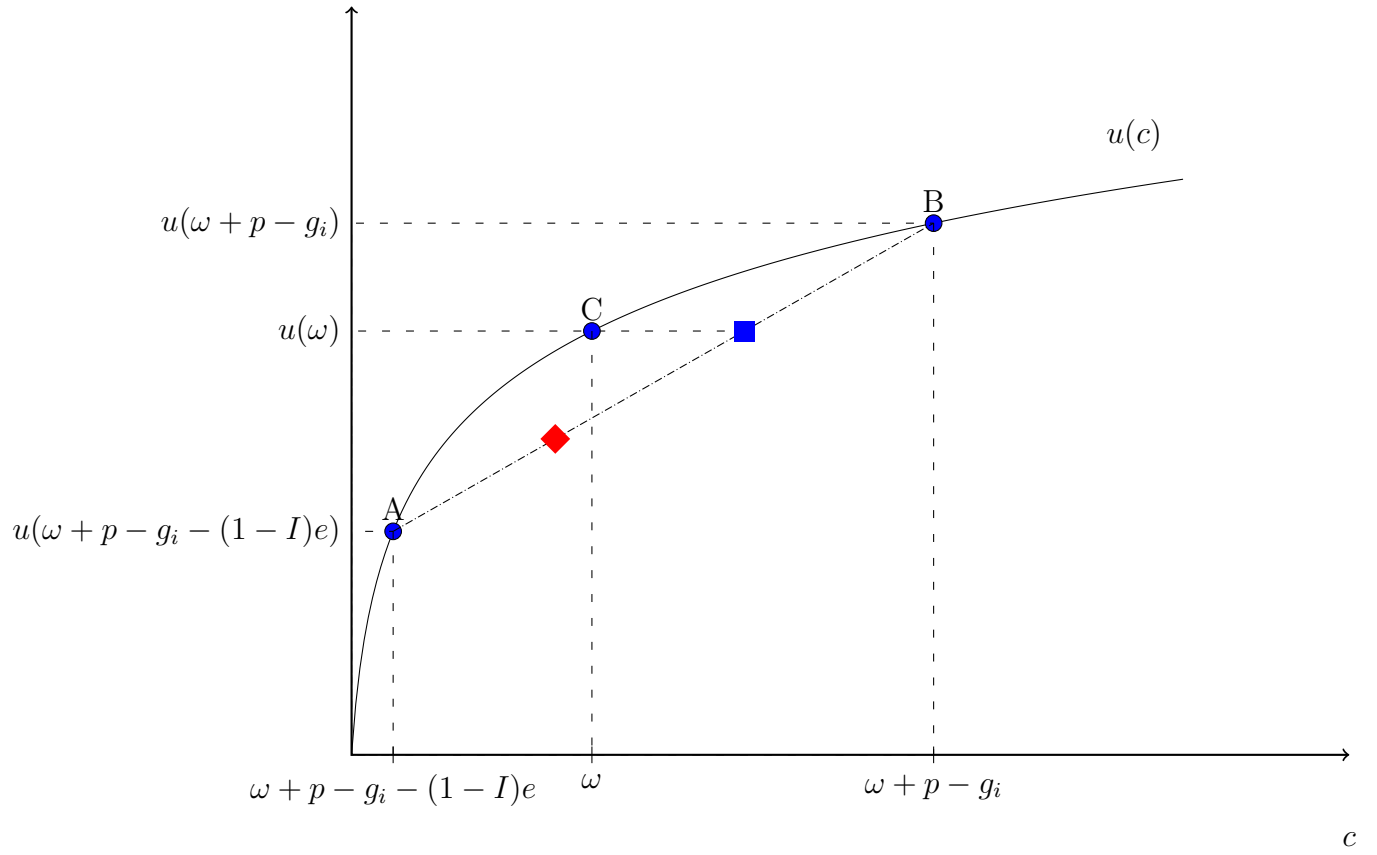
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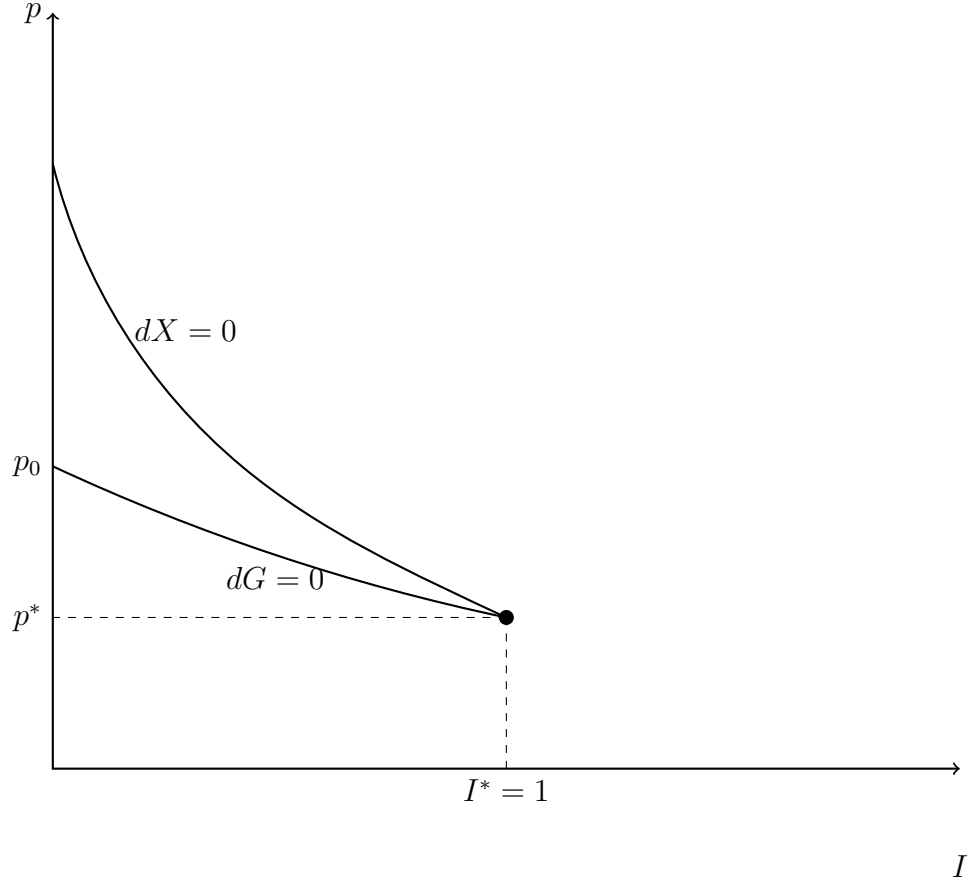
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# Figures

**Figure 1:** Expected Utility of Pivotal Agent: Simple Example



**Figure 2:** Iso- $G$  and Iso- $X$  Curves



**Table 1:** Risk Aversion and Value Added

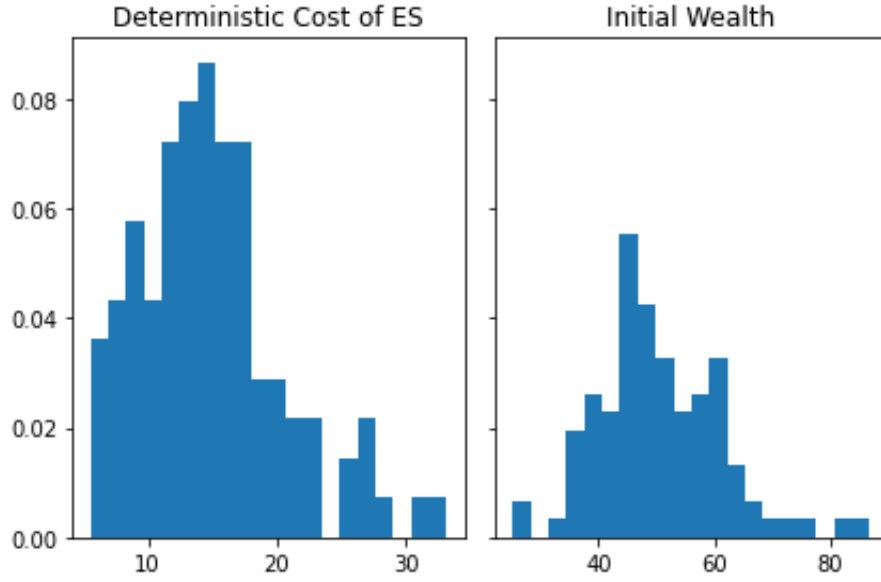
	Estimated risk aversion	Value added (%)
Overall best practice	3.73	18.75
Economics literature	1.24	5.56
Finance literature	7.16	42.5
US	5.81	32.56
EU	1.57	5.56
Stockholder	1.49	5.56
GMM	3.79	18.75
Quarterly data	6.33	35.71

Estimated risk aversion values are taken from Table 6 of Elminejad, Havranek and Irsova (2022).

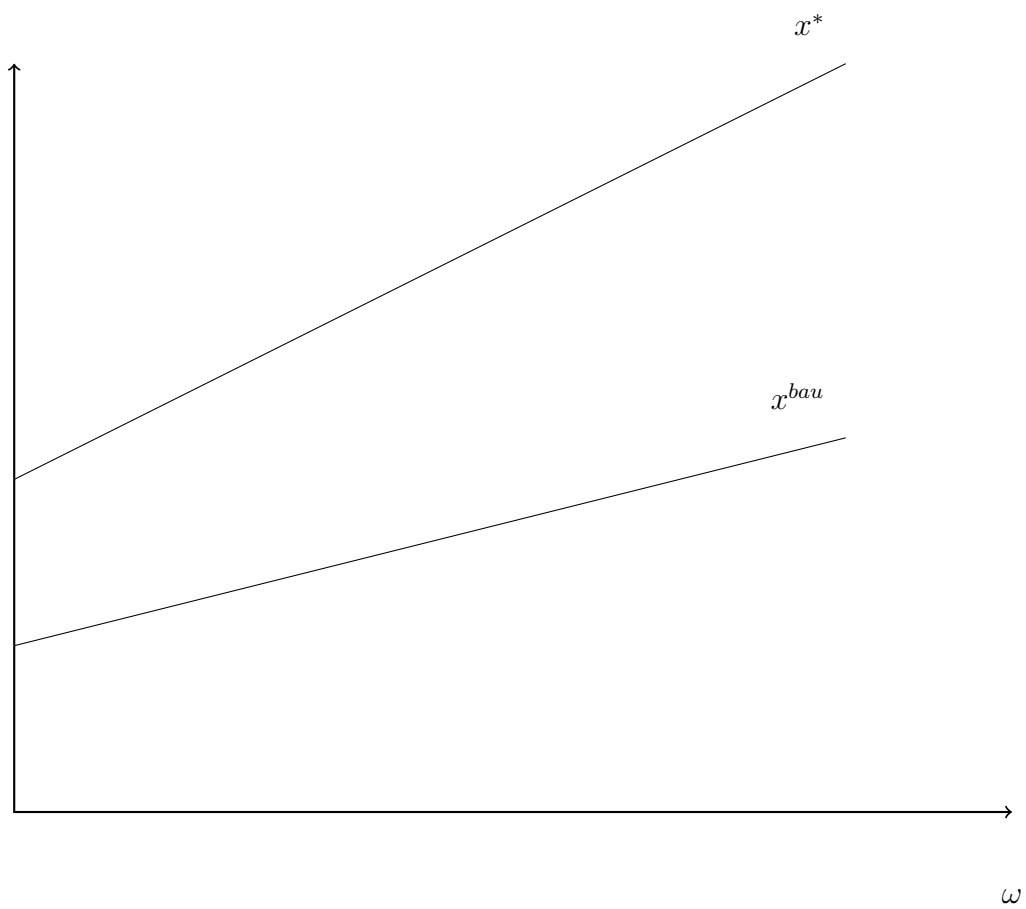
**Table 2:** Simulation Parameters

Parameter	Value
Number of agents	100
Probability of shock	0.10
Cost shock ( $e$ )	25.35
Budget	1000

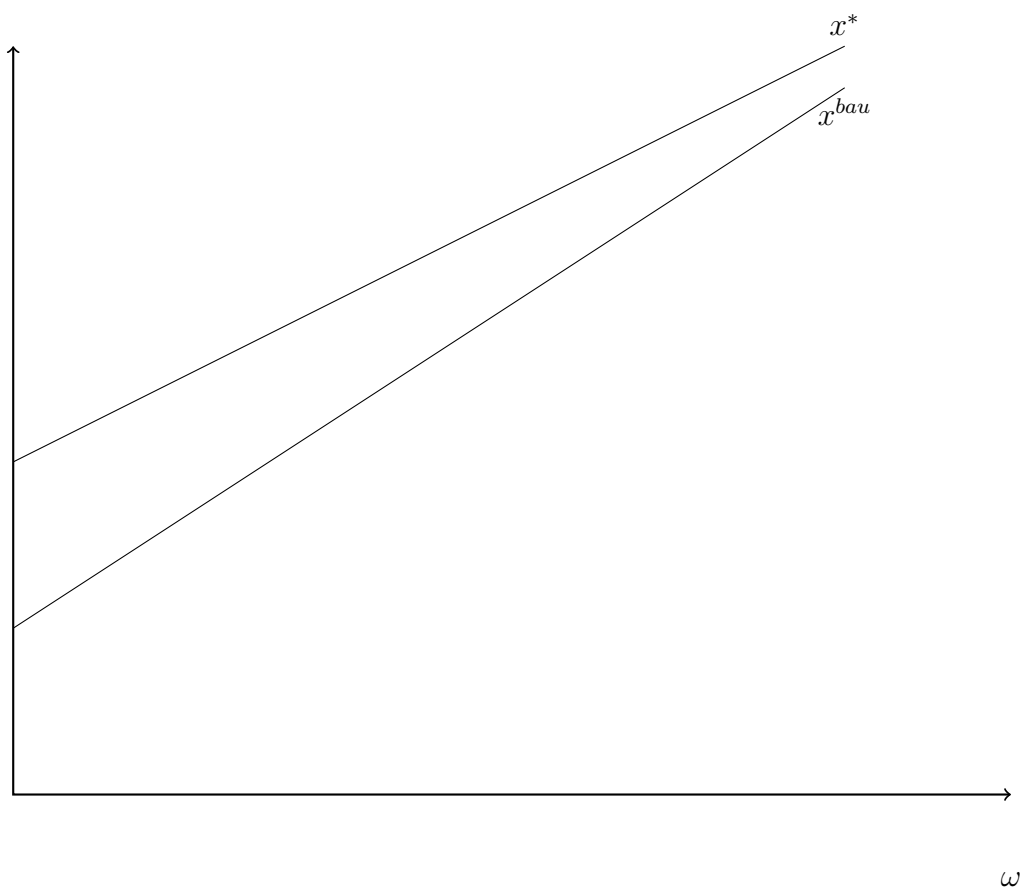
**Figure 3:** Randomly Generated Distribution of  $g$  and  $\omega$



**Figure 4:** Additionality Increasing in Wealth



**Figure 5:** Additionality Decreasing in Wealth



## Appendix: Proofs

*Proof of Lemma 0.1.* To show that  $x^*$  is non-decreasing in  $I$ , we can invoke the Milgrom-Shannon Monotonicity by simply showing that  $\mathbb{E} \left[ u \left( c(x, I) \right) \right]$  is single crossing in  $(x, I)$ . Suppose  $x' > x$  and  $I' > I$ . Want to show that (i)  $\mathbb{E} \left[ u \left( c(x', I) \right) - u \left( c(x, I) \right) \right] \geq 0$  implies  $\mathbb{E} \left[ u \left( c(x', I') \right) - u \left( c(x, I') \right) \right] \geq 0$  and that (ii)  $\mathbb{E} \left[ u \left( c(x', I) \right) - u \left( c(x, I) \right) \right] > 0$  implies  $\mathbb{E} \left[ u \left( c(x', I') \right) - u \left( c(x, I') \right) \right] > 0$ .

Let  $\mathbb{E} \left[ u \left( c(x', I) \right) - u \left( c(x, I) \right) \right] \geq 0$ .

$$\iff \mathbb{E} [c(x', I) - c(x, I)] \geq 0 \quad (13)$$

since  $u(\cdot)$  is a monotonic transformation of  $c(\cdot)$ . Rewriting gives

$$\iff \omega + x'p - g(x') - x'\mu_\epsilon(1 - I) - [\omega + xp - g(x) - x\mu_\epsilon(1 - I)] \geq 0$$

$$\iff p(x' - x) - [g(x') - g(x)] - \mu_\epsilon(1 - I)(x' - x) \geq 0$$

Thus we have

$$\implies p(x' - x) - [g(x') - g(x)] - \mu_\epsilon(1 - I')(x' - x) \geq 0$$

$$\iff \mathbb{E} \left[ u \left( c(x', I') \right) - u \left( c(x, I') \right) \right] \geq 0$$

The proof for part (ii) is almost identical to that of (i) except with strict inequalities.

To show that  $x^*$  is non-decreasing in  $p$  follows an almost exact procedure.  $\square$

*Proof of Proposition 0.2.* The goal is to show that the planner can achieve a higher level of indirect social value when  $I = 0$  is relaxed. First, let  $I$  be a parameter in the planner's problem. The planner then solves the Lagrangian (14) where  $\psi$  is the multiplier (distinct from the multiplier  $\lambda$  in (3)).

$$\mathcal{F} = \max_p B(X) + \psi(G - X(p + \mu_\epsilon I)) \quad (14)$$

The goal can be equivalently stated as wanting to show that  $\frac{\partial \mathcal{F}}{\partial I} > 0$ . We can use the Envelope Theorem to characterize this partial.

$$\begin{aligned} \frac{\partial \mathcal{F}}{\partial I} &= B'(X) \frac{\partial X}{\partial I} - \psi \left( \frac{\partial X}{\partial I} (p + \mu_\epsilon I) + X \mu_\epsilon \right) \\ &= \frac{\partial X / \partial I}{\partial X / \partial p} \left[ B'(X) \frac{\partial X}{\partial p} - \psi \frac{\partial X}{\partial p} (p + \mu_\epsilon I) \right] - \psi X \mu_\epsilon \end{aligned} \quad (15)$$

The first order condition in (14) is given by  $B'(X) \frac{\partial X}{\partial p} - \psi \frac{\partial X}{\partial p} (p + \mu_\epsilon I) = \psi X$  so then we can write

$$\frac{\partial \mathcal{F}}{\partial I} = \left( \frac{\partial X / \partial I}{\partial X / \partial p} - \mu_\epsilon \right) \psi X \quad (16)$$

Since every term is positive, then we have that  $\frac{\partial \mathcal{F}}{\partial I} > 0$  if and only if  $\left( \frac{\partial X / \partial I}{\partial X / \partial p} - \mu_\epsilon \right) > 0$ .

Next, we show that this always holds when agents are risk averse.

We first show that the ES supply response to indemnity ( $\frac{\partial x}{\partial I}$ ) can be expressed as in (17 by



taking derivative of the FOC wrt to  $I$ .<sup>11</sup>

$$\frac{\partial x}{\partial I} = \frac{\mu_{u'\epsilon} + x\mu_{u''\Gamma}\epsilon}{\mu_{u'}g_{xx} - \mu_{u''\Gamma^2}} \quad (17)$$

Where  $\mu_z \equiv \mathbb{E}[z]$  and  $\Gamma = \frac{\partial c}{\partial x} = p - g_x(x) - \epsilon(1 - I)$ . Then using the fact that

$$\mathbb{E}[u'(c)\epsilon(1 - A\Gamma x)] = Cov(u'(c)(1 - A\Gamma x), \epsilon) + \mathbb{E}[u'(c)(1 - A\Gamma x)]\mu_\epsilon$$

and using (18) gives (19).

$$\frac{\partial x}{\partial p} = \frac{\mu_{u'} + x\mu_{u''\Gamma}}{\mu_{u'}g_{xx} - \mu_{u''\Gamma^2}} \quad (18)$$

$$\begin{aligned} \frac{\partial x/\partial I}{\partial x/\partial p} &= \frac{\mathbb{E}[u'\epsilon(1 - A\Gamma x)]}{\mathbb{E}[u'(1 - A\Gamma x)]} \\ &= \frac{\tilde{\sigma}}{\mathbb{E}[u'(c)(1 - R\delta)]} + \mu_\epsilon \end{aligned} \quad (19)$$

Where  $\tilde{\sigma} = Cov(u'(c)(1 - A\Gamma x), \epsilon)$  and  $\delta = \frac{\Gamma x}{c} = \frac{xp - g_x x - x\epsilon(1 - I)}{w + xp - g(x) - x\epsilon(1 - I)}$  and  $A$  and  $R$  are the coefficient of absolute and relative risk aversion, respectively. We can rewrite  $\tilde{\sigma}$  as

$$\tilde{\sigma} = Cov(u'(c)(1 - R\delta), \epsilon)$$

Note that  $\mathbb{E}[u'(c)(1 - A\Gamma x)] = \mathbb{E}[u'(c)(1 - R\delta)]$  is then numerator in (18) which is always positive according to Lemma 0.1. Then (19) implies that (16) is positive if and only if  $\tilde{\sigma} > 0$ .

Since  $u'(c)$  is increasing in  $\epsilon$  and  $\delta$  is decreasing in  $\epsilon$  then  $\tilde{\sigma} > 0$ . However, when agents are

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<sup>11</sup>Note that small  $x$  denotes individual ES input while big  $X$  denotes aggregate ES input.

risk neutral ( $R = 0$ ), then (16) equals zero, i.e.,  $\left(\frac{\partial x}{\partial I} = \frac{\partial x}{\partial p}\mu_\epsilon\right)$ . Then according to (16), risk neutrality implies that there is never any change in regulator's objective function for any change in  $I$ .  $\square$

*Proof of Proposition 0.3.* The proof follows directly from looking at the proof for Proposition 0.2 and noting the fact that  $\frac{\tilde{\sigma}}{\mathbb{E}[u'(c)(1 - R\delta)]} > 0$  holds at all levels of  $I < 1$  and that at  $I^* = 1$ , the FOC from (16) fails to hold with equality meaning that the solution is at a corner one of which is ruled out by Proposition 0.2, hence  $I^* = 1$ .  $\square$

*Proof of Proposition 0.4.* For the comparative statics on additionality  $\alpha$ , it is easier to start with a reframing of the model where the agent's choice variable is  $\alpha = x - x^b$  instead of  $x$  where  $x^b = x^{bau}$  for simplicity. Thus consumption is given by

$$c = w + p(\alpha + x^b) - h(\alpha, \omega) - (\alpha + x^b)\epsilon(1 - I)$$

where  $h(0, \omega) = g(x^b, \omega)$  and  $h(\alpha^*, \omega) = g(x^*, \omega)$ . Then the optimal additionality  $\alpha^*$  is pinned by

$$\mathbb{E}[u'(c)\Gamma] = 0$$

where  $\Gamma \equiv \frac{\partial c}{\partial \alpha}$ . Then using the Implicit function theorem, we can differentiate this FOC WRT  $\omega$  to get

$$\frac{\partial \alpha}{\partial \omega} = \frac{\mu_{A\Gamma}(1 - h_\omega + \frac{\partial x^b}{\partial \omega}h_\alpha) + \mu_{A\Gamma^2}\frac{\partial x^b}{\partial \omega} + h_{\alpha\omega}}{-\mu_{A\Gamma^2} - h_{\alpha\alpha}}$$

and since the denominator is negative, then  $\frac{\partial \alpha}{\partial \omega} < 0$  if and only if the numerator is positive.  $\square$

## 2 Appendix: Slopes of $dG$ and $dX$ Curves

For tractability but without loss of generality, we assume agents are homogeneous so that

$$\frac{\partial X}{\partial I} = n \frac{\partial x}{\partial I}.$$

### $dG$ Curve:

Total differentiation of budget from (3) and setting equal to zero gives

$$0 = \left( \frac{\partial x}{\partial I} dI + \frac{\partial x}{\partial p} dp \right) (p + \mu_\epsilon I) + x(dp + \mu_\epsilon dI)$$

rearranging and solving for  $dp/dI$  gives

$$\frac{dp}{dI} = - \frac{\frac{\partial x}{\partial I}(p + \mu_\epsilon I) + x\mu_\epsilon}{\frac{\partial x}{\partial p}(p + \mu_\epsilon I) + x}$$

Then plugging in (??) gives

$$\frac{dp}{dI} = -\mu_\epsilon + \left( \frac{\tilde{\sigma}x}{1 - x\mu_{A\Gamma}} \right) \left( \frac{\frac{\partial x}{\partial p}(p + \mu_\epsilon I)}{\frac{\partial x}{\partial p}(p + \mu_\epsilon I) + x} \right) \quad (20)$$

### $dX$ Curve:

Total differentiating  $X$  and setting equal to zero and rearranging gives

$$\frac{dp}{dX} = \frac{\partial x / \partial I}{\partial x / \partial p}$$

then plugging in (17) and (18) and rearranging gives

$$\frac{dp}{dX} = -\mu_\epsilon + \frac{x\tilde{\sigma}}{1 - x\mu_{A\Gamma}} \quad (21)$$

Since,  $\tilde{\sigma} < 0$  (from (??)) then it follows from (20) and (21) that the slope of the  $dX$  curve is always steeper than the slope of the  $dG$  curve.

### 3 Appendix: Lower Bound for $\mu_A$

Start with the definition of  $\mu_{A\Gamma}$

$$\mu_{A\Gamma} = Cov(A, \Gamma) + \mu_A \mu_\Gamma$$

Using definition of covariance gives

$$\mu_{A\Gamma} = \rho_{A\Gamma} \sigma_A \sigma_\Gamma + \mu_A \mu_\Gamma$$

$$= -\sigma_A \sigma_\Gamma + \mu_A \mu_\Gamma \quad (A \text{ increases while } \Gamma \text{ decreases in } \epsilon)$$

Hence we have

$$\mu_A > \frac{\mu_{A\Gamma}}{\mu_\Gamma}$$

## 4 Appendix: Application of Slutsky

Let  $e(I, p, u)$  denote the expenditure function defined in (22)

$$\begin{aligned} e &= \min_x g(x, \omega) - xp + x\epsilon(1 - I) \\ \text{s.t. } &u(c) \geq u \end{aligned} \tag{22}$$

The solution to (22) is known as the compensated (Hicksian) ES input supply function  $x_c$ .

Applying the Envelope Theorem gives Sheppard's Lemma.

$$\frac{\partial e}{\partial p} = -x_c$$

Making use of the duality at the optimum produces the equality

$$x_c(I, p, u) = x(I, p, e)$$

Hence

$$\frac{\partial x_c}{\partial p} = \frac{\partial x}{\partial p} - \frac{\partial x}{\partial \omega} x$$

Q.E.D.