Applied Linear Algebra (EE 635): Problem Set-1

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August 1, 2025

1. (a) Consider the system of equations

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1, \\ 2x_1 + 2x_3 &= 1, \\ x_1 - 3x_2 + 4x_3 &= 2. \end{aligned}$$

Does this system have a solution? If so, describe explicitly all solutions.

(b) Find all solutions to the following system of equations by row-reducing the coefficient matrix:

$$\begin{aligned} \frac{1}{3}x_1 + 2x_2 - 6x_3 &= 0, \\ -4x_1 + 5x_3 &= 0, \\ -3x_1 + 6x_2 - 13x_3 &= 0, \\ -\frac{7}{3}x_1 + 2x_2 - \frac{8}{3}x_3 &= 0. \end{aligned}$$

2. Consider the system of linear equations with an unknown parameter k:

$$\begin{aligned} x + 2y + z &= 3, \\ 2x + ky + 3z &= 7, \\ x + 3y + (k-1)z &= 4. \end{aligned}$$

Determine the **condition on** k for which this system is **solvable**.

3. Create examples of $A \in \mathbb{R}^{3\times 3}$, $b \in \mathbb{R}^3$ such that:

(a) $Ax = b$ has no solution.

(b) $Ax = b$ has a unique solution.

(c) $Ax = b$ has infinitely many solutions.

Note: Matrix A must have all entries nonzero.

4. Suppose an economy consists of the Coal, Electric (power), and Steel sectors, and the output of each sector is distributed among the various sectors as shown in Table 1, where the entries in a column represent the fractional parts of a sector's total output. The third column of Table 1, for instance, says that the total output of the Electric sector is divided as follows: 40% to Coal, 50% to Steel, and the remaining 10% to Electric. (Electric treats this 10% as an expense it incurs in order to operate its business.) Since all output must be taken into account, the decimal fractions in each column must sum to 1. Denote the prices (i.e., dollar values) of the total annual outputs of the Coal, Electric, and Steel sectors by P_C, P_E, and P_S, respectively. If possible, find equilibrium prices that make each sector's income match its expenditures.

Table 1: A Simple Economy

Distribution of Output from:	**Coal**	**Electric**	**Steel**
Purchased by:			
Coal	0.0	0.4	0.6
Electric	0.6	0.1	0.2
Steel	0.4	0.5	0.2

5. Propane (C_3H_8) burns in oxygen (O_2) to produce carbon dioxide (CO_2) and water (H_2O) according to the unbalanced chemical equation:

$$(x_1)C_3H_8 + (x_2)O_2 \longrightarrow (x_3)CO_2 + (x_4)H_2O.$$

   (a) Using the conservation of atoms of **carbon (C)**, **hydrogen (H)**, and **oxygen (O)**, write a **system of linear equations** in the variables x_1, x_2, x_3, x_4 that represents the balancing conditions for this chemical reaction.

   (b) Express this system in **matrix form**:

$$A\mathbf{x} = 0,$$

   where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

6. If possible, find some combination of nonfat milk, soy flour, and whey to provide the exact amounts of protein, carbohydrate, and fat supplied by the diet in one day (as shown in Table 2).

Table 2: Amounts (g) Supplied per 100 g of Ingredient and by Cambridge Diet in One Day

Nutrient	**Nonfat milk**	**Soy flour**	**Whey**	**Cambridge Diet (per day)**
Protein	36	51	13	33
Carbohydrate	52	34	74	45
Fat	0	7	1.1	3