

Homework 1: For self-practice

1 Questions

1. Consider two vectors $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \in \mathbb{R}^2$. Define the notation $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + u_2 v_2$.

Now consider the equation of a straight line given by $2x_1 + 3x_2 - 4 = 0$. Show that this equation can be written as $\langle \mathbf{w}, \mathbf{x} \rangle - \theta = 0$, where $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\theta \in \mathbb{R}$.

2. Write Python function `plotline()` which accepts a weight vector $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and a real number θ as arguments. In the function, write code to plot the line $w_1 x_1 + w_2 x_2 - \theta = 0$. Keep x_1 along the vertical axis and x_2 along the horizontal axis. Keep the range of x_1 and x_2 as $[-5, 5]$. Label the axes in the plot and use color `green` for the line.
3. Use `plotline()` to plot the straight line $2x_1 + 3x_2 - 4 = 0$.
4. Write Python function `plotpoint()` which accepts a point vector $\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, along with $\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ and θ as arguments. In this `plotpoint()` function, use `scatterplot` to plot the point. Also check if $\langle \mathbf{w}, \mathbf{z} \rangle - \theta \geq 0$ and color the point with a particular color `red` and if $\langle \mathbf{w}, \mathbf{z} \rangle - \theta < 0$, color the point with a different color `blue`.
5. In the plot obtained in Question 3, plot the points $(4, 3)$ and $(-3, -4)$ using `plotpoint()` function .
6. Generate a set S_1 of ten points from a normal distribution with mean $\mu^1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and covariance matrix $\Sigma^1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. Generate another set S_2 of ten points from a normal distribution with mean $\mu^2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and covariance matrix $\Sigma^2 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.75 \end{bmatrix}$. In the plot obtained in Question 3, plot the points in sets S_1 and S_2 using `plotpoint()` function.
7. For the points in sets S_1 and S_2 in Question 6, find another line of the form $ax_1 + bx_2 - c = 0$ which will separate the `red` points from `blue` points. Explain how you found this new line.