Wye-Delta and Delta-Wye Transformations of Proximally-Coupled Inductor Triads

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***Abstract*—This paper presents a method for Wye-Delta and Delta-Wye transformations of proximally-coupled inductor triads. This is achieved by a two stage process: a topology-preserving 'deproximation' that produces an intermediate coupling-free network which retains the same structure, thus making it possible to apply a straightforward transformation. The developed method is illustrated via numerical examples.**

***Index Terms*—transformation, delta, wye, deproximation, proximal coupling, mutual inductance, equivalent network**

I. Introduction

In networks containing inductors, v-i inter-relationships over and above the topological ones might exist, should some (or all) inductors be placed in magnetic proximity – these latter being collectively referred to as the 'proximal' relationships. Any attempt at analysis of such networks, to be meaningful, would necessitate appropriate consideration of both the aforementioned facets and a comprehensive mathematical treatment thereof.

In the absence of any galvanic interconnections among the elements of a network, this collection of network elements is referred to as the 'primitive' network [1], and connotes a group of unconnected elements, among which relationships that exist, if any, are solely due to magnetic proximity. It is essential to emphasize – this type of relationship is manifest only in inductors [2]. The v-i relationships of a group of unconnected inductors can be collectively quantified by the primitive inductance matrix. Passivity considerations dictate that this matrix be real and positive-definite. The diagonal entries of this matrix are the element inductances (self inductances). The off-diagonal entries, while possessing the dimensions of inductance, quantify the extent of the inter-inductor coupling arising due to magnetic proximity. As with all relationships which owe their origin to proximity, this proximity-originated coupling, succinctly termed as *proximal* coupling, is necessarily mutual. This is the reason to which the inductance matrix owes its symmetry. The term 'mutual inductance' has been widely used to refer to the off-diagonal elements of the primitive inductance matrix.

In the mathematical treatment of a group of proximally coupled inductors, it is sometimes convenient to use the inverse of the primitive inductance matrix and its sub-matrices. This inverse matrix is known as the levitance matrix [3].

In the course of some specific network analyses, one might be faced with the task of performing an alteration of the topological configuration of one specific part of network, from a Wye to a Delta or *vice-versa*. Whereas the means to obtain such a transformation in cases sans proximal coupling are well-known [2], [4]–[14], the same cannot be said of situations involving proximal coupling. This paper furnishes a method wherewith it is possible to perform these transformations even in the face of existing proximal coupling among inductors making up the Wye (or Delta) configurations.

The proposed method is made up of two stages:

1) The given configuration (Wye or Delta) is replaced by a terminal-based topology-preserving equivalent in which the constituent elements are free of proximal coupling. That is to say, a Wye remains a Wye, and a Delta remains a Delta. The so obtained like-topology structure may be said to be the 'deproximated' equivalent of its original. The term 'deproximation' has been coined expressly for this purpose, (for want of a better term) to succintly describe the underlying process in achieving the said task, and shall be referred to as such hereinafter.
   Parameter measurements (in adequate number) are carried out at appropriately constructed ports of access, for both the original and equivalent (deproximated) configurations. By equating these two sets of measurement outcomes, the parameters of the desired deproximated network are obtained.

2) The so obtained deproximated structure may then be subjected to a trivial topological transformation (Wye$\leftrightarrow$Delta), if so desired.

II. A GROUP OF UNCONNECTED INDUCTORS IN MAGNETIC PROXIMITY – THE PRIMITIVE STATE OF EXISTENCE

In the primitive state of existence, the only v-i inter-relationships exhibited by a set of inductors (an inductor *triad*) are those owing to proximal coupling.

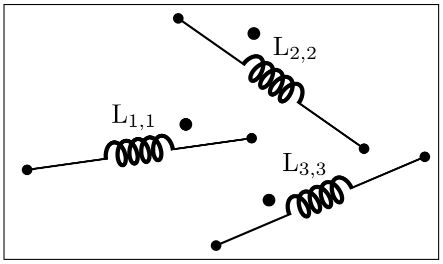

Fig. 1. The Primitive State of Existence

In the presence of proximal coupling, the v-i relationships of a set of three inductors in the primitive state as shown in Fig. 1, are succinctly contained in a (3×3) inductance matrix as shown in (1).

$$\mathbf{L_{primitive}} = \begin{bmatrix} \mathrm{L}_{1,1} & \mathrm{L}_{1,2} & \mathrm{L}_{1,3} \\ \mathrm{L}_{2,1} & \mathrm{L}_{2,2} & \mathrm{L}_{2,3} \\ \mathrm{L}_{3,1} & \mathrm{L}_{3,2} & \mathrm{L}_{3,3} \end{bmatrix} \tag{1}$$

The diagonal elements of $\mathbf{L_{primitive}}$, $\mathrm{L_{i,i}}$ for $i = 1, 2, 3$ represent the element inductance of the inductor i. The off-diagonal elements, $\mathrm{L_{i,j}}$ for all $i \neq j$ and $i, j = 1, 2, 3$ represent the extent (or strength) of proximal coupling between the inductor i and inductor j. Needless to say, the inductance matrix turns out to be diagonal, in the absence of proximal coupling.
Since the inductance matrix is symmetric, $\mathrm{L_{i,j}} = \mathrm{L_{j,i}}$ for all $i, j = 1, 2, 3$. The primitive inductance matrix $\mathbf{L_{primitive}}$ may therefore be rewritten as in (2).

$$\mathbf{L_{primitive}} = \begin{bmatrix} \mathrm{L}_{1,1} & \mathrm{L}_{1,2} & \mathrm{L}_{1,3} \\ \mathrm{L}_{1,2} & \mathrm{L}_{2,2} & \mathrm{L}_{2,3} \\ \mathrm{L}_{1,3} & \mathrm{L}_{2,3} & \mathrm{L}_{3,3} \end{bmatrix} \tag{2}$$

The inverse v-i relationships are analogously contained in the primitive levitance matrix as shown in (4).

$$\mathbf{\Gamma_{primitive}} = [\mathbf{L_{primitive}}]^{\mathbf{-1}} \tag{3}$$

$$= \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & \Gamma_{1,3} \\ \Gamma_{1,2} & \Gamma_{2,2} & \Gamma_{2,3} \\ \Gamma_{1,3} & \Gamma_{2,3} & \Gamma_{3,3} \end{bmatrix} \tag{4}$$

III. WYE-DELTA TRANSFORMATION

In a Wye configuration, one terminal of each inductor is connected to the others to form a node as shown in Fig. 2. The other terminals of each inductor (labelled 1, 2, 3 in Fig. 2) form the terminals on which the transformation is based. It is posited that this set of three inductors does not bear any proximal coupling whatsoever with any other inductors, outside of this set.

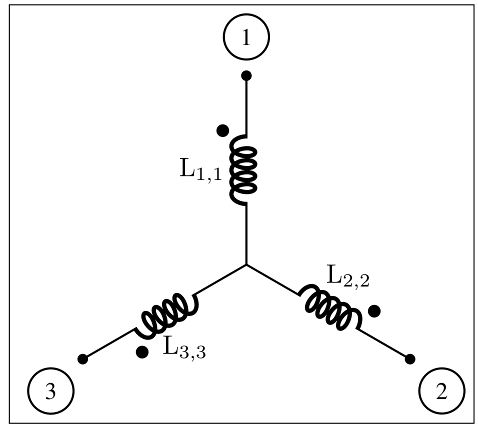

Fig. 2. Original network : Wye with proximal coupling

The symmetric primitive inductance matrix for the original structure in Fig. 2 is given in (5).

$$\mathbf{L}_{\mathbf{Y},\mathbf{original}} = \begin{bmatrix} \mathrm{L}_{1,1} & \mathrm{L}_{1,2} & \mathrm{L}_{1,3} \\ \mathrm{L}_{1,2} & \mathrm{L}_{2,2} & \mathrm{L}_{2,3} \\ \mathrm{L}_{1,3} & \mathrm{L}_{2,3} & \mathrm{L}_{3,3} \end{bmatrix} \tag{5}$$

The network sought retains the Wye topology but the parameter values change such that this network is equivalent to the original network at the three terminals as shown in Fig. 3.

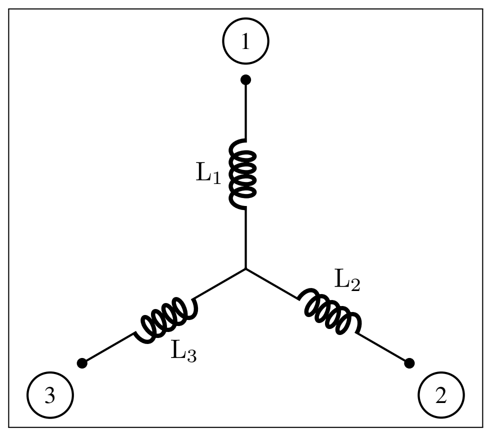

Fig. 3. Equivalent network : Deproximated Wye

The symmetric primitive diagonal inductance matrix for the equivalent network in Fig. 3 is given by (6). The lack of proximal coupling is reflected in the nullity of the off-diagonal element entries.

$$\mathbf{L}_{\mathbf{Y},\mathbf{equivalent}} = \begin{bmatrix} \mathrm{L}_1 & 0 & 0 \\ 0 & \mathrm{L}_2 & 0 \\ 0 & 0 & \mathrm{L}_3 \end{bmatrix} \tag{6}$$

The procedure for testing equivalence at any pair of terminals is as follows: the inductance measured across any given pair of terminals while keeping the remaining terminal open-circuited, should be the same for the original and the equivalent configuration sought. In a Wye configuration, two inductors are always placed in series across any given pair of nodes when the remaining node is open. The well-known expression for the equivalent inductance of a series combination of two coupled inductors is made use of in formulating the following equivalence relations in matrix form.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{L}_1 \\ \mathrm{L}_2 \\ \mathrm{L}_3 \end{bmatrix} = \begin{bmatrix} \mathrm{L}_{1,1} + \mathrm{L}_{2,2} - 2\mathrm{L}_{1,2} \\ \mathrm{L}_{2,2} + \mathrm{L}_{3,3} - 2\mathrm{L}_{2,3} \\ \mathrm{L}_{3,3} + \mathrm{L}_{1,1} - 2\mathrm{L}_{3,1} \end{bmatrix} \tag{7}$$

The solution of the preceding system of equations gives the values of the parameters of the desired network.

$$\mathrm{L}_1 = \mathrm{L}_{1,1} - \mathrm{L}_{1,2} - \mathrm{L}_{1,3} + \mathrm{L}_{2,3} \tag{8a}$$
$$\mathrm{L}_2 = \mathrm{L}_{2,2} - \mathrm{L}_{2,3} - \mathrm{L}_{1,2} + \mathrm{L}_{1,3} \tag{8b}$$
$$\mathrm{L}_3 = \mathrm{L}_{3,3} - \mathrm{L}_{1,3} - \mathrm{L}_{2,3} + \mathrm{L}_{1,2} \tag{8c}$$

The deproximation of the original Wye network is successfully accomplished via equations (8), while preserving the Wye topology. It may be noted that suitably conceived transfer measurements also provide outcomes that are in full agreement with the results shown in (8).

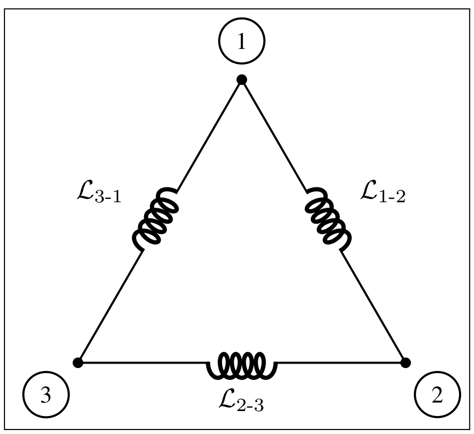

Fig. 4. Equivalent Delta obtained from the deproximated Wye

The deproximated Wye can be further transformed into a Delta, as shown in Fig. 4, using the well-known Wye→Delta transformation.

$$\mathcal{L}_{1\text{-}2} = \mathrm{L}_1 + \mathrm{L}_2 + \frac{\mathrm{L}_1\mathrm{L}_2}{\mathrm{L}_3} \tag{9a}$$
$$\mathcal{L}_{2\text{-}3} = \mathrm{L}_2 + \mathrm{L}_3 + \frac{\mathrm{L}_2\mathrm{L}_3}{\mathrm{L}_1} \tag{9b}$$
$$\mathcal{L}_{3\text{-}1} = \mathrm{L}_3 + \mathrm{L}_1 + \frac{\mathrm{L}_3\mathrm{L}_1}{\mathrm{L}_2} \tag{9c}$$

IV. Delta-Wye Transformation

The Delta configuration considered consists of inductors connected end-to-end as shown in Fig. 5. The procedure to find the deproximated equivalent as seen from the terminals $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$, is facilitated by characterisation in terms of levitance, rather than of inductance. It is posited that this set of three inductors does not bear any proximal coupling whatsoever with any other inductors, outside of this set.

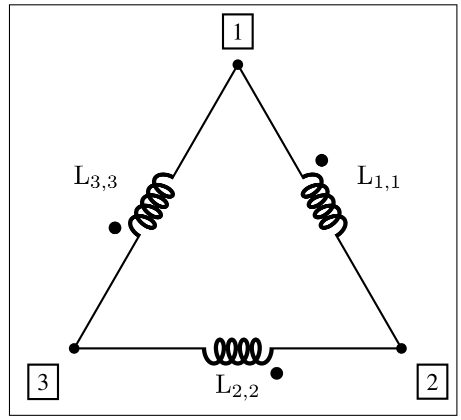

Fig. 5. Original network : Delta with proximal coupling

The primitive inductance matrix for the original Delta configuration in Fig. 5 is written as in (10).

$$\mathbf{L}_{\mathbf{\Delta},\mathbf{original}} = \begin{bmatrix} \mathrm{L}_{1,1} & \mathrm{L}_{1,2} & \mathrm{L}_{1,3} \\ \mathrm{L}_{1,2} & \mathrm{L}_{2,2} & \mathrm{L}_{2,3} \\ \mathrm{L}_{1,3} & \mathrm{L}_{2,3} & \mathrm{L}_{3,3} \end{bmatrix} \tag{10}$$

The levitance matrix of the original network is obtained by taking the inverse of the inductance matrix found in (10). The levitance matrix of the original configuration is obtained as follows:

$$\mathbf{\Gamma}_{\mathbf{\Delta},\mathbf{original}} = [\mathbf{L}_{\mathbf{\Delta},\mathbf{original}}]^{-\mathbf{1}} \tag{11}$$
$$= \begin{bmatrix} \Gamma_{1,1} & \Gamma_{1,2} & \Gamma_{1,3} \\ \Gamma_{1,2} & \Gamma_{2,2} & \Gamma_{2,3} \\ \Gamma_{1,3} & \Gamma_{2,3} & \Gamma_{3,3} \end{bmatrix} \tag{12}$$

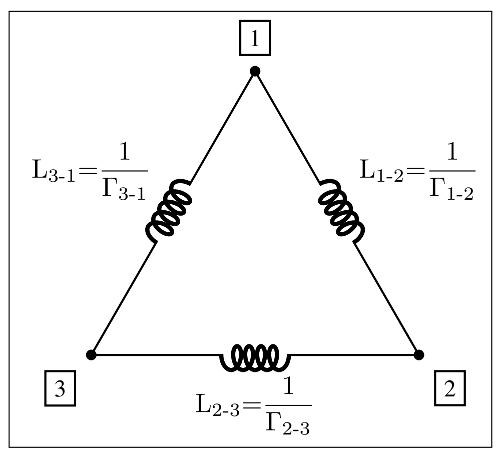

Fig. 6. Equivalent network : Deproximated Delta

The levitance matrix of the equivalent network sought as shown in Fig. 6, can be written as:

$$\mathbf{\Gamma}_{\mathbf{\Delta},\mathbf{equivalent}} = \begin{bmatrix} \Gamma_{1\text{-}2} & 0 & 0 \\ 0 & \Gamma_{2\text{-}3} & 0 \\ 0 & 0 & \Gamma_{3\text{-}1} \end{bmatrix} \tag{13}$$

The equivalence relationships are to be formulated based on the levitance measurements carried out between a given terminal, and a second terminal formed by bunching the remaining two Delta nodes – in cyclic permutation. The levitance manifested across any of the so conceived terminal pairs turns out to be the equivalent levitance of two inductors in parallel.

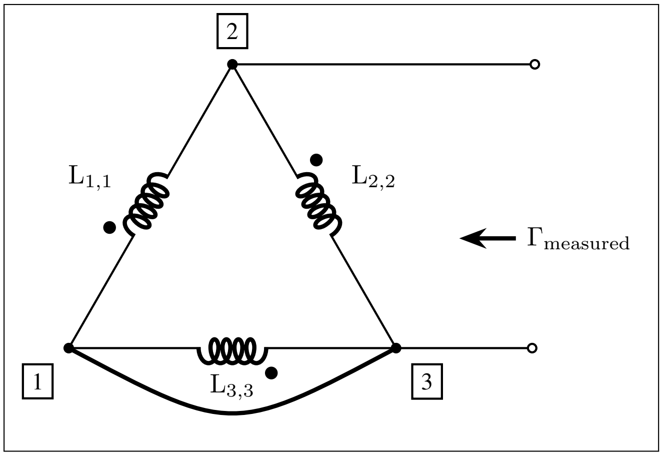

Fig. 7. Levitance measurement between node $\boxed{2}$ and the bunching of nodes $\boxed{3}$ and $\boxed{1}$

Considering the terminal pair defined between node $\boxed{2}$, and the node formed by the bunching of nodes $\boxed{3}$ and $\boxed{1}$, the levitance measured in the original network (as shown in Fig. 7) following the method given in [3] is

$$\Gamma_{\text{measured}} = \Gamma_{1,1} + \Gamma_{2,2} - 2\Gamma_{1,2} \tag{14}$$

whereas, the corresponding measure in the desired equivalent network (which is devoid of coupling) is

$$\Gamma_{\text{measured}} = \Gamma_{\text{1-2}} + \Gamma_{\text{2-3}} \tag{15}$$

Thus, the first of the three equivalence relationships can be written by equating the right hand sides of (14) and (15), as in

$$\Gamma_{\text{1-2}} + \Gamma_{\text{2-3}} = \Gamma_{1,1} + \Gamma_{2,2} - 2\Gamma_{1,2} \tag{16}$$

Two more equations are obtained in similar fashion by measuring the levitance across the other node combinations in cyclic permutation. The three equations thus obtained can be compacted in vector-matrix notation as

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Gamma_{\text{1-2}} \\ \Gamma_{\text{2-3}} \\ \Gamma_{\text{3-1}} \end{bmatrix} = \begin{bmatrix} \Gamma_{1,1} + \Gamma_{2,2} - 2\Gamma_{1,2} \\ \Gamma_{2,2} + \Gamma_{3,3} - 2\Gamma_{2,3} \\ \Gamma_{3,3} + \Gamma_{1,1} - 2\Gamma_{1,3} \end{bmatrix} \tag{17}$$

Upon solution of the system of equations in (17), the following values obtain as the levitances of the desired equivalent network.

$$\Gamma_{\text{1-2}} = \Gamma_{1,1} - \Gamma_{1,2} - \Gamma_{1,3} + \Gamma_{2,3} \tag{18a}$$

$$\Gamma_{\text{2-3}} = \Gamma_{2,2} - \Gamma_{2,3} - \Gamma_{1,2} + \Gamma_{1,3} \tag{18b}$$

$$\Gamma_{\text{3-1}} = \Gamma_{3,3} - \Gamma_{1,3} - \Gamma_{2,3} + \Gamma_{1,2} \tag{18c}$$

The deproximation of the original Delta network is successfully achieved via equations (18). It may be noted that suitably conceived transfer measurements also provide outcomes that corroborate the results shown in (18).

The values of inductances in the deproximated Delta shown in Fig. 6 by taking the reciprocal of the levitances obtained in (18).

$$L_{\text{1-2}} = \frac{1}{\Gamma_{\text{1-2}}}, \quad L_{\text{2-3}} = \frac{1}{\Gamma_{\text{2-3}}}, \quad L_{\text{3-1}} = \frac{1}{\Gamma_{\text{3-1}}} \tag{19}$$

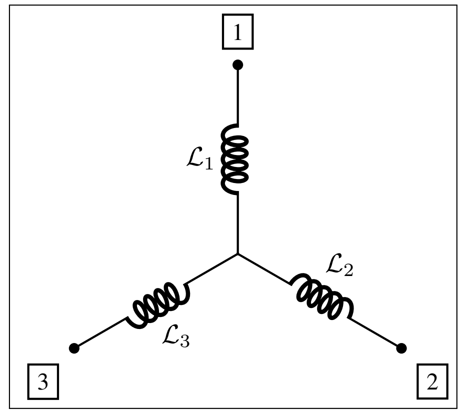

Fig. 8. Equivalent Wye obtained from the deproximated Delta

The deproximated Delta can then be transformed into a Wye, as shown in Fig. 8, using the well-known Delta→Wye tranformation.

$$\mathcal{L}_1 = \frac{L_{\text{1-2}} L_{\text{3-1}}}{L_{\text{1-2}} + L_{\text{2-3}} + L_{\text{3-1}}} \tag{20a}$$

$$\mathcal{L}_2 = \frac{L_{\text{2-3}} L_{\text{1-2}}}{L_{\text{1-2}} + L_{\text{2-3}} + L_{\text{3-1}}} \tag{20b}$$

$$\mathcal{L}_3 = \frac{L_{\text{3-1}} L_{\text{2-3}}}{L_{\text{1-2}} + L_{\text{2-3}} + L_{\text{3-1}}} \tag{20c}$$

V. Numerical Examples Illustrating the Transformations

A. An Example to demonstrate deproximation in Wye and Delta configurations of inductors

The primitive inductance matrix for a set of three inductors is known to be,

$$\mathbf{L_{primitive}} = \begin{bmatrix} 209 & 236 & 258 \\ 236 & 282 & 300 \\ 258 & 300 & 324 \end{bmatrix} \text{mH}$$

The inductors are connected in the Wye configuration as shown in Fig. 9.

In this example, the proximal disposition of the connected inductors is the same as that considered for the derivation of the deproximated equivalent (Fig. 2). Hence,

$$\mathbf{L_{Y,original}} = \mathbf{L_{primitive}}$$

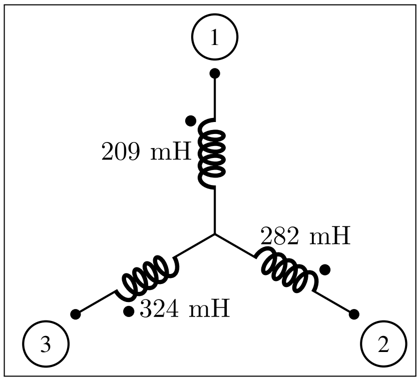

Fig. 9. A Wye configuration of inductors - Example A

The values of the inductances of the equivalent network as shown in Fig. 3 are found using (8).

$$\mathrm{L}_1 = 209 - 236 - 258 + 300 = 15 \text{ mH}$$
$$\mathrm{L}_2 = 282 - 300 - 236 + 258 = 4 \text{ mH}$$
$$\mathrm{L}_3 = 324 - 258 - 300 + 236 = 2 \text{ mH}$$

The inductances of the equivalent Delta post Wye→Delta transformation as shown in Fig. 4 using (9) are:

$$\mathcal{L}_{\text{1-2}} = 49 \text{ mH}$$
$$\mathcal{L}_{\text{2-3}} = \tfrac{98}{15} \text{ mH} = 6.5333 \text{ mH}$$
$$\mathcal{L}_{\text{3-1}} = \tfrac{49}{2} \text{ mH} = 24.5 \text{ mH}$$

The same inductors are connected in the Delta configuration as shown in Fig. 10.

Once again, the proximal disposition of the connected inductors is the same as that considered for the derivation of the deproximated equivalent (Fig. 5). Hence,

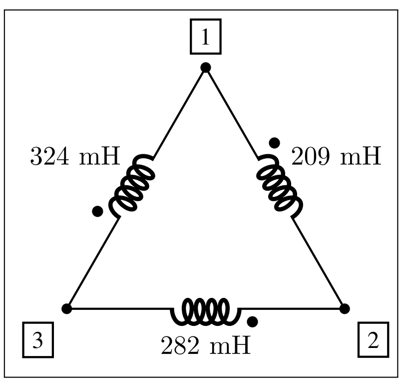

Fig. 10. A Delta configuration of inductors - Example A

$$\mathbf{L}_{\mathbf{\Delta},\mathbf{original}} = \mathbf{L}_{\mathbf{primitive}}$$

The levitance matrix is obtained by inverting the primitive inductance matrix.

$$\begin{aligned}\mathbf{\Gamma}_{\mathbf{\Delta},\mathbf{original}} &= [\mathbf{L}_{\mathbf{\Delta},\mathbf{original}}]^{-1} \\ &= \frac{1}{2160}\begin{bmatrix} 1368 & 936 & -1956 \\ 936 & 1152 & -1812 \\ -1956 & -1812 & 3242 \end{bmatrix}\end{aligned}$$

Equations (18) are used to obtain the values of levitances of the deproximated network.

$$\Gamma_{\text{1-2}} = \tfrac{1368-936-(-1956)+(-1812)}{2160} = \tfrac{4}{15} \ (\text{mH})^{-1}$$
$$\Gamma_{\text{2-3}} = \tfrac{1152-(-1812)-936+(-1956)}{86700} = \tfrac{1}{30} \ (\text{mH})^{-1}$$
$$\Gamma_{\text{3-1}} = \tfrac{3242-(-1956)-(-1812)+936}{86700} = \tfrac{3973}{1080} \ (\text{mH})^{-1}$$

The inductances of the deproximated delta network as shown in Fig. 6 are promptly computed by taking the reciprocal of the calculated levitances.

$$\mathrm{L}_{\text{1-2}} = \tfrac{15}{4} = 3.75 \text{ mH}$$
$$\mathrm{L}_{\text{2-3}} = \tfrac{30}{1} = 30 \text{ mH}$$
$$\mathrm{L}_{\text{3-1}} = \tfrac{1080}{3973} = 0.2718 \text{ mH}$$

The inductances of the equivalent Wye after performing Delta→Wye transformation as shown in Fig. 8 using (20) are:

$$\mathcal{L}_1 = \tfrac{8}{267} = 0.02996 \text{ mH}$$
$$\mathcal{L}_2 = \tfrac{7946}{2403} = 3.3067 \text{ mH}$$
$$\mathcal{L}_3 = \tfrac{64}{267} = 0.2397 \text{ mH}$$

B. An example to demonstrate deproximation in Wye and Delta configurations of inductors with altered proximal dispositions

The same inductors are now connected in the Wye configuration as shown in Fig. 11.

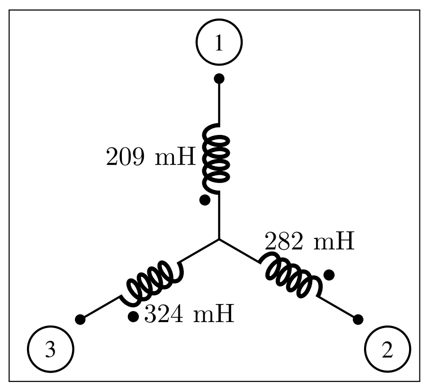

Fig. 11. A Wye configuration of inductors - Example B

Since the proximal dispositions are different from those considered for the derivation, the primitive inductance matrix has to be suitably modified to reflect the changes. The modified matrix is given by,

$$\mathbf{L}_{\mathbf{Y},\mathbf{original}} = \begin{bmatrix} 209 & -236 & -258 \\ -236 & 282 & 300 \\ -258 & 300 & 324 \end{bmatrix} \text{mH}$$

The equivalent inductances of the deproximated network as shown in Fig. 3 are found using (8).

$$\mathrm{L}_1 = 209 - (-236) - (-258) + 300 = 1003 \text{ mH}$$
$$\mathrm{L}_2 = 282 - 300 - (-236) + (-258) = -40 \text{ mH}$$
$$\mathrm{L}_3 = 324 - (-258) - 300 + (-236) = 46 \text{ mH}$$

The equivalent inductances post Wye→Delta transformation as shown in Fig. 4 using (9) are:

$$\mathcal{L}_{1\text{-}2} = \tfrac{2089}{23}\ \text{mH} = 90.826\ \text{mH}$$
$$\mathcal{L}_{2\text{-}3} = \tfrac{4178}{1003}\ \text{mH} = 4.1655\ \text{mH}$$
$$\mathcal{L}_{3\text{-}1} = -\tfrac{2089}{20}\ \text{mH} = -104.45\ \text{mH}$$

The same inductors are connected in the Delta configuration as shown in Fig. 12.

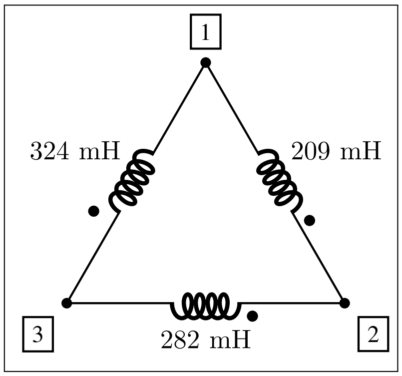

Fig. 12. A Delta configuration of inductors - Example B

Here also, the proximal dispositions are different from those considered for the derivation. Therefore, the primitive inductance matrix has to be suitably modified to reflect the changes. The modified matrix is given by,

$$\mathbf{\Gamma_\Delta} = [\mathbf{L_\Delta}]^{-1} = \frac{1}{2160}\begin{bmatrix} 1368 & -936 & 1956 \\ -936 & 1152 & -1812 \\ 1956 & -1812 & 3242 \end{bmatrix}$$

Equations (18) are used to obtain the values of levitances of the equivalent network.

$$\Gamma_{1\text{-}2} = \tfrac{1368-(-936)-1956+(-1812)}{2160} = -\tfrac{61}{90}\ (\text{mH})^{-1}$$
$$\Gamma_{2\text{-}3} = \tfrac{1152-(-1812)-(-936)+1956}{2160} = \tfrac{122}{45}\ (\text{mH})^{-1}$$
$$\Gamma_{3\text{-}1} = \tfrac{3242-1956-(-1812)+(-936)}{2160} = \tfrac{1081}{1080}\ (\text{mH})^{-1}$$

The inductances of the equivalent delta network as shown in Fig. 6 are calculated by taking the reciprocal of the computed levitances as shown in (19).

$$\text{L}_{1\text{-}2} = -\tfrac{90}{61} = -1.4754\ \text{mH}$$
$$\text{L}_{2\text{-}3} = \tfrac{45}{122} = 0.3688\ \text{mH}$$
$$\text{L}_{3\text{-}1} = \tfrac{1080}{1081} = 0.999\ \text{mH}$$

The inductances of the equivalent Wye after performing Delta→Wye transformation as shown in Fig. 8 using (20) are:

$$\mathcal{L}_1 = \tfrac{96}{7} = 13.714\ \text{mH}$$
$$\mathcal{L}_2 = \tfrac{2162}{427} = 5.0632\ \text{mH}$$
$$\mathcal{L}_3 = -\tfrac{24}{7} = -3.4286\ \text{mH}$$

VI. Conclusion

A means to perform the transformation of a Wye (or Delta) network of proximally coupled inductors into a terminal-based equivalent Delta (or Wye) configuration has been presented. As far as the authors are aware, this paper is the very first instance of identification of such a problem and successful treatment thereof, thereby filling a glaring void.

The developed method has been adequately illustrated via examples. One has to be mindful of the possibility of the inductance values of the obtained equivalent network possessing unreal traits – viz. negativity, nullity, or even infinity. Although this might appear absurd at first sight, it is not an indication of the inapplicability of the transformation scheme; rather a limitation of any method that seeks to obtain a terminal-based equivalent to a given network. Such methods are decidedly restricted in scope of absolute verity. The internal make-up of the equivalent network may even assume forms that defy norms of passivity. This, though seemingly strange, would still give the correct representation of the original network at the terminals.

The procedure presented in this paper can be extended to obtain deproximated equivalents of Wye and Delta configurations of more elaborate network-element combinations in analyses specific to the sinusoidal steady state. It is earnestly hoped that this idea finds due recognition and favourable reception in Electrical Engineering education.

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