भारतीय विज्ञान संस्थान

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IISc QUANTUM TECHNOLOGY INITIATIVE

QT312 - ADVANCED QUANTUM TECHNOLOGY LABORATORY

Project Report

Single Qubit Fidelities

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Contents

1 Rabi Oscillations Without Decoherence

The Schrodinger equation is

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \mathcal{H}|\psi\rangle$$

The aim is to find $|\psi\rangle = \alpha|g\rangle + \beta|e\rangle$ given that $|\psi(0)\rangle = |g\rangle$.
The Hamiltonian for a driven qubit system is given as

$$\mathcal{H} = \hbar\omega\frac{\sigma_z}{2} + \hbar\Omega cos(\omega_d t + \phi_0)\sigma_x$$

This can be written as

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$$

Here, $\mathcal{H}_0 = \hbar\omega\frac{\sigma_z}{2}$ is the bare/uncoupled Hamiltonian and $\mathcal{H}_I = \hbar\Omega cos(\omega_d t+\phi_0)\sigma_x$ is the drive/interaction Hamiltonian, where Ω is the Rabi frequency and $\Omega \propto gV_0$.

To solve the Rabi problem, we go to the interaction picture.
Let

$$|\psi\rangle_I = e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle \tag{1}$$

such that $|\psi\rangle_I = C_g(t)|g\rangle + Ce(t)|e\rangle$.
The Schrodinger equation is

$$i\hbar\frac{\partial|\psi\rangle_I}{\partial t} = \widetilde{\mathcal{H}}|\psi\rangle_I \tag{2}$$

Also at $t = 0$, $|\psi(0)\rangle_I = |\psi(0)\rangle = |g\rangle$ Substituting for $|\psi\rangle_I$ from Eq. 1 in Eq. 2, we get

$$i\hbar\frac{\partial(e^{\frac{i\mathcal{H}_0 t}{\hbar}})|\psi\rangle}{\partial t} = \widetilde{\mathcal{H}}(e^{\frac{i\mathcal{H}_0 t}{\hbar}})|\psi\rangle$$

$$i\hbar\left(\frac{i\mathcal{H}_0}{\hbar}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle + e^{\frac{i\mathcal{H}_0 t}{\hbar}}\frac{\partial|\psi\rangle}{\partial t}\right) = \widetilde{\mathcal{H}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle$$

$$-\mathcal{H}_0 e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle + i\hbar e^{\frac{i\mathcal{H}_0 t}{\hbar}}\frac{\partial|\psi\rangle}{\partial t} = \widetilde{\mathcal{H}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle \tag{3}$$

Pre-multiplying $e^{-\frac{i\mathcal{H}_0 t}{\hbar}}$ to Eq. 3, we get

$$-e^{-\frac{i\mathcal{H}_0 t}{\hbar}}\mathcal{H}_0 e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle + i^{-\frac{i\mathcal{H}_0 t}{\hbar}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}\frac{\partial|\psi\rangle}{\partial t} = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}\widetilde{\mathcal{H}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle$$

But $e^{-\frac{i\mathcal{H}_0 t}{\hbar}}$ and $\mathcal{H}_0$ commute and $e^{-\frac{i\mathcal{H}_0 t}{\hbar}}e^{\frac{i\mathcal{H}_0 t}{\hbar}} = \hat{\mathbb{1}}$

$$\therefore -\mathcal{H}_0|\psi\rangle + i\hbar\frac{\partial|\psi\rangle}{\partial t} = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}\widetilde{\mathcal{H}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle$$

But $i\hbar\frac{\partial|\psi\rangle}{\partial t} = \mathcal{H}|\psi\rangle = (\mathcal{H}_0 + \mathcal{H}_I)|\psi\rangle$

$$-\mathcal{H}_0 + \mathcal{H}_0|\psi\rangle + \mathcal{H}_I|\psi\rangle = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}\widetilde{\mathcal{H}}e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle$$

$$\therefore \mathcal{H}_I = e^{-\frac{i\mathcal{H}_0 t}{\hbar}} \widetilde{\mathcal{H}} e^{\frac{i\mathcal{H}_0 t}{\hbar}}$$

Let $\mathcal{U} = e^{\frac{i\mathcal{H}_0 t}{\hbar}}$ and $\mathcal{U}^\dagger = \mathcal{U}^{-1} = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}$, then

$$\widetilde{\mathcal{H}} = \mathcal{U}\mathcal{H}_I\mathcal{U}^{-1} = \mathcal{U}\mathcal{H}_I\mathcal{U}^\dagger$$

$$\widetilde{\mathcal{H}} = e^{\frac{i\mathcal{H}_0 t}{\hbar}} \mathcal{H}_I e^{-\frac{i\mathcal{H}_0 t}{\hbar}}$$

This is the Hamiltonian in the interaction picture. Since $\mathcal{H}_0 = \hbar\omega_a \frac{\sigma_z}{2}$

$$\mathcal{U} = e^{\frac{i\mathcal{H}_0 t}{\hbar}} = e^{\frac{i\omega_a t\sigma_z}{2}}$$

$$\mathcal{U} = e^{-\frac{i\mathcal{H}_0 t}{\hbar}} = e^{\frac{-i\omega_a t\sigma_z}{2}}$$

We have $\mathcal{H}_I = \hbar\Omega cos(\omega_d t + \phi_0)\sigma_x$

$$\therefore \widetilde{\mathcal{H}} = e^{\frac{i\omega_a t\sigma_z}{2}} \hbar\Omega cos(\omega_d t + \phi_0)\sigma_x e^{\frac{-i\omega_a t\sigma_z}{2}}$$

$$\widetilde{\mathcal{H}} = \hbar\Omega cos(\omega_d t + \phi_0) e^{\frac{i\omega_a t\sigma_z}{2}} \sigma_x e^{\frac{-i\omega_a t\sigma_z}{2}}$$

We know that $\sigma_x = \sigma_+ + \sigma_-$. Using Baker-Campbell-Hausdorf formula, we have

$$e^{\frac{i\omega_a t\sigma_z}{2}} \sigma_+ e^{\frac{-i\omega_a t\sigma_z}{2}} = \sigma_+ e^{i\omega_a t}$$

$$e^{\frac{i\omega_a t\sigma_z}{2}} \sigma_- e^{\frac{-i\omega_a t\sigma_z}{2}} = \sigma_- e^{-i\omega_a t}$$

$$\widetilde{\mathcal{H}} = \hbar\Omega \left(\frac{e^{i(\omega_d t + \phi_0)} + e^{-i(\omega_d t + \phi_0)}}{2} \right) + \phi_0)(\sigma_+ e^{i\omega_a t} + \sigma_- e^{-i\omega_a t})$$

$$\widetilde{\mathcal{H}} = \frac{\hbar\Omega}{2}(\sigma_+ e^{i((\omega_a - \omega_d)t - \phi_0)} + \sigma_i e^{-i((\omega_a - \omega_d)t - \phi_0)} + \sigma_+ e^{i((\omega_a + \omega_d)t - \phi_0)} + \sigma_- e^{-i((\omega_a + \omega_d)t - \phi_0)})$$

If $\omega_a - \omega_d << \omega_a or \omega_d$ (usually $\omega_a \approx \omega_d$), then the first two terms with $(\omega_a - \omega_d)$ rotate slowly and the last two terms with $\omega_a + \omega_d$ rotate rapidly/ So we can ignore the $\omega_a + \omega_d$ terms and thus, we obtain

$$\widetilde{\mathcal{H}} = \frac{\hbar\Omega}{2}(\sigma_+ e^{i((\omega_a - \omega_d)t - \phi_0)} + \sigma_- e^{-i((\omega_a - \omega_d)t - \phi_0)})$$

This is the Rotating Wave Approximation.

The Schrodinger equation becomes

$$i\hbar \frac{\partial |\psi\rangle_I}{\partial t} = \left(\frac{\hbar\Omega}{2}(\sigma_+ e^{i((\omega_a - \omega_d)t - \phi_0)} + \sigma_- e^{-i((\omega_a - \omega_d)t - \phi_0)}) \right) |\psi\rangle_I$$

Now we substitute $|\psi\rangle_I = C_g |g\rangle + C_e |e\rangle$ in this equation.

$$i\hbar \frac{\partial (C_g |g\rangle + C_e |e\rangle)}{\partial t} = \left(\frac{\hbar\Omega}{2}(\sigma_+ e^{i((\omega_a - \omega_d)t - \phi_0)} + \sigma_i e^{-i((\omega_a - \omega_d)t - \phi_0)}) \right) (C_g |g\rangle + C_e |e\rangle)$$

We know that $\sigma_+ |g\rangle = |e\rangle$, $\sigma_+ |e\rangle = |0\rangle$, $\sigma_- |e\rangle = |g\rangle$ and $\sigma_- |g\rangle = |0\rangle$

$$i\hbar(\dot{C}_g |g\rangle + \dot{C}_e |e\rangle) = \frac{\hbar\Omega}{2}(C_g e^{i((\omega_a - \omega_d)t - \phi_0)} |e\rangle + C_e e^{-i((\omega_a - \omega_d)t - \phi_0)}) |g\rangle$$

$|g\rangle$ and $|e\rangle$ are orthogonal so $\langle g\rangle e = 0$. SO we can equate their coefficient terms.
Equating coefficients of $|g\rangle$, we get

$$i\hbar\dot{C}_g = \frac{\hbar\Omega}{2} C_e e^{-i((\omega_a-\omega_d)t-\phi_0)})$$

$$\dot{C}_g = \frac{\Omega}{2i} C_e e^{-i((\omega_a-\omega_d)t-\phi_0)}) \tag{4}$$

Equating coefficients of $|e\rangle$, we get

$$i\hbar\dot{C}_e = \frac{\hbar\Omega}{2}(C_g e^{i((\omega_a-\omega_d)t-\phi_0)})$$

$$\dot{C}_e = \frac{\Omega}{2i}(C_g e^{i((\omega_a-\omega_d)t-\phi_0)}) \tag{5}$$

Differentiate Ee. 5 with respect to time to get

$$\ddot{C}_e = \frac{\Omega}{2i}\left(\dot{C}_g e^{i((\omega_a-\omega_d)t-\phi_0)} + C_g i(\omega_a-\omega_d)e^{i((\omega_a-\omega_d)t-\phi_0)}\right)$$

Substituting values of $\dot{C}_g$ and C_g from Eq. 4 and 5 respectively, we get

$$\ddot{C}_e = \frac{\Omega}{2i}\left(\left(\frac{\Omega}{2i} C_e e^{-i((\omega_a-\omega_d)t-\phi_0)}\right) e^{i((\omega_a-\omega_d)t-\phi_0)} + \frac{2i}{\Omega}\dot{C}_e i(\omega_a-\omega_d)e^{-i((\omega_a-\omega_d)t-\phi_0)} e^{i((\omega_a-\omega_d)t-\phi_0)}\right)$$

$$\ddot{C}_e = -\frac{\Omega^2}{4}C_e + \dot{C}_e i(\omega_a-\omega_d)$$

$$\ddot{C}_e + \dot{C}_e i(\omega_d-\omega_a) + \frac{\Omega^2}{4}C_e = 0$$

The characteristic equation for the above second order differential equation is

$$\lambda^2 + i(\omega_d-\omega_a)\lambda + \frac{\Omega^2}{4} = 0$$

$$\lambda = \frac{i(\omega_d-\omega_a) \pm \sqrt{-(\omega_d-\omega_a)^2 - \Omega^2}}{2}$$

Let $\Delta \equiv (\omega_d - \omega_a)$, then

$$\lambda_\pm = \frac{-i\Delta \pm i\sqrt{\Delta^2-\Omega^2}}{2}$$

$$C_e(t) = Ae^{\lambda_+ t} + Be^{\lambda_- t}$$

At $t = 0$, $C_e(0) = 0$, so we can write,

$$A + B = 0 \implies A = -B$$

At $t = 0$, $\dot{C}_e(0) = \frac{\Omega e^{-i\phi_0}}{2i}$, so we can write,

$$\lambda_+ A + \lambda_- B = \frac{\Omega e^{-i\phi_0}}{2i}$$

$$\lambda_+ A - \lambda_- A = \frac{\Omega e^{-i\phi_0}}{2i}$$

But $(\lambda_+ - \lambda_-) = i\sqrt{\Delta^2 + \Omega^2}$

$$A = -\frac{\Omega e^{-i\phi_0}}{2i\sqrt{\Delta^2+\Omega^2}}$$

$$B = \frac{\Omega e^{-i\phi_0}}{2i\sqrt{\Delta^2+\Omega^2}}$$

$$\therefore C_e = A(e^{i\left(\frac{\Delta+\sqrt{\Delta^2-\Omega^2}}{2}\right)t} - e^{i\left(\frac{\Delta-\sqrt{\Delta^2-\Omega^2}}{2}\right)t})$$

$$C_e = Ae^{-i\Delta}(e^{\frac{-i\sqrt{\Delta^2-\Omega^2}}{2}t} - e^{\frac{i\sqrt{\Delta^2-\Omega^2}}{2}t})$$

$$C_e = \frac{\Omega e^{-i\phi_0}e^{-i\Delta}}{2i\sqrt{\Delta^2+\Omega^2}} \times -2i\sin\left(\frac{\sqrt{\Delta^2-\Omega^2}}{2}\right)$$

$$C_e = -\frac{\Omega e^{-i(\phi_0+\Delta)}}{\sqrt{\Delta^2+\Omega^2}}\sin\left(\frac{\sqrt{\Delta^2-\Omega^2}}{2}\right)$$

At resonance $\omega_a \approx \omega_d$ and $\Delta = 0$ and $\Omega = \Omega_R$, the Rabi frequency. So we have

$$C_e = -e^{-i\phi_0}\sin\left(\frac{\Omega t}{2}\right)$$

Similarly when we solve for C_g, we get

$$C_g = \cos\left(\frac{\Omega t}{2}\right)$$

Let $\theta = \Omega_R t$

$$|\psi\rangle_I = C_g(t)|g\rangle + C_e(t)|e\rangle$$

$$|\psi\rangle_I = \cos\left(\frac{\theta}{2}\right)|g\rangle - e^{-i\phi_0}\sin\left(\frac{\theta}{2}\right)|e\rangle$$

We have $|\psi\rangle_I = e^{\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle$

$$\therefore |\psi\rangle = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}|\psi\rangle_I$$

$$\therefore |\psi\rangle = e^{-\frac{i\mathcal{H}_0 t}{\hbar}}\left(\cos\left(\frac{\theta}{2}\right)|g\rangle - e^{-i\phi_0}\sin\left(\frac{\theta}{2}\right)|e\rangle\right)$$

We also know that

$$e^{-\frac{i\mathcal{H}_0 t}{\hbar}}|g\rangle = e^{\frac{i\omega_a t}{2}}|g\rangle$$

$$e^{-\frac{i\mathcal{H}_0 t}{\hbar}}|e\rangle = e^{-\frac{i\omega_a t}{2}}|e\rangle$$

$$\therefore |\psi\rangle = e^{\frac{i\omega_a t}{2}}\cos\left(\frac{\theta}{2}\right)|g\rangle - e^{-\frac{i\omega_a t}{2}}e^{-i\phi_0}\sin\left(\frac{\theta}{2}\right)|e\rangle$$

Taking out the global phase of $e^{-\frac{i\omega_a t}{2}}$, we get

$$\therefore |\psi\rangle = \cos\left(\frac{\theta}{2}\right)|g\rangle - e^{-i\omega_a t}e^{-i\phi_0}\sin\left(\frac{\theta}{2}\right)|e\rangle$$

The plot for the probability of finding the state to be $|g\rangle$ and $|e\rangle$ versus probability in Rabi Oscillations is shown in Fig 1.

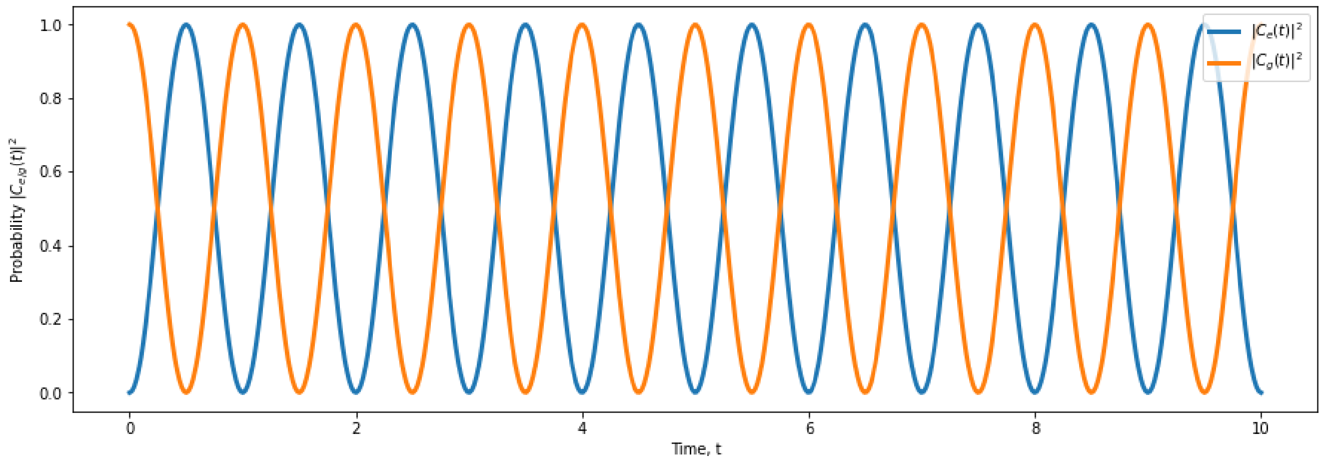

Figure 1: Rabi Oscillations: Probability $|C_{e/g}(t)|^2$ versus Time, t

2 Rabi Oscillations With Decoherence

2.1 Master equation

The master equation in the density matrix formalism is given as

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\mathcal{H}_0 + \mathcal{H}_{d,\rho}] + \frac{\gamma}{2}(N+1)(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a) + \frac{\gamma}{2}(N)(2a^\dagger \rho a - aa^\dagger \rho - \rho a^\dagger a)$$

with $\mathcal{H}_0 = \omega_0(a^\dagger a + \frac{1}{2})$ and $\mathcal{H}_d = -\hbar(a + a^\dagger f(t))$. The parameter γ corresponds to damping.

This equation is often written in the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\mathcal{H}_0 + \mathcal{H}_{d,\rho}] + \gamma_1 \mathcal{D}[a] + \gamma_2 \mathcal{D}[a^\dagger]\rho \tag{6}$$

where the super-operator $\mathcal{D}$ is called the Lindblad super-operator and is defined as

$$\mathcal{D}\rho \equiv A\rho A^\dagger - \frac{1}{2}(A^\dagger A\rho + \rho A^\dagger A)$$

This form of the master equation is called the Lindblas form of the master equation. Here the rate of relaxation and dephasing are given by γ_1 and γ_2 respectively.

2.2 Simulation

The Lindblad master equation is used to simulate a driven damped qubit. The plot for the probability of finding the state to be $|g\rangle$ and $|e\rangle$ versus probability in Rabi Oscillations when is damping/decoherence is shown in Fig 2.

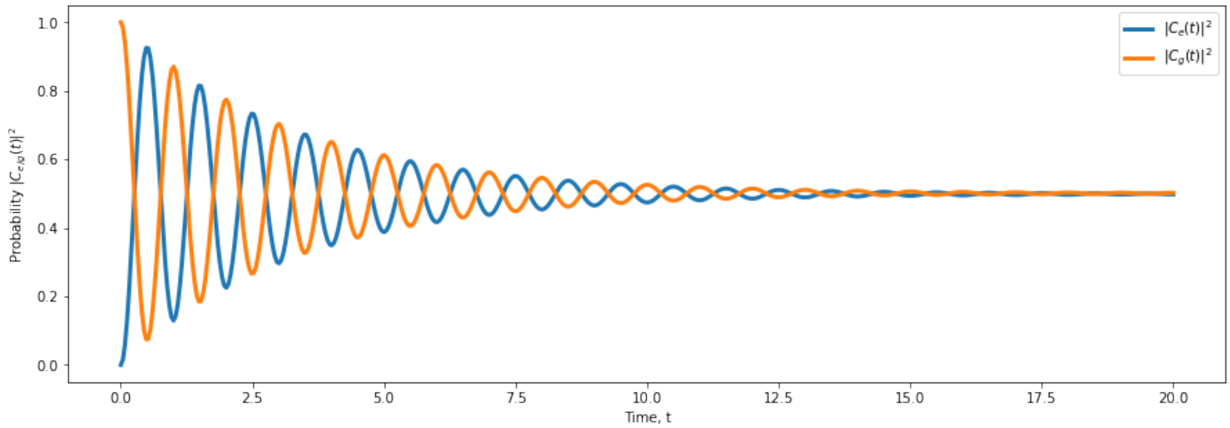

Figure 2: Rabi Oscillations with Decoherence: Probability $|C_{e/g}(t)|^2$ versus Time, t

The rates of relaxation and dephasing were chosen to be 0.2 and 0.15 respectively. It can be seen that the probability for finding both states settles to 0.5 with time after displaying damped oscillations.

2.3 Curve Fitting

The plot in Figure 2 is fitted to obtain the time period of the oscillations T_R. The fitted curve along with the obtained curve is shown in Figure 3 and 4. The equation for the probablities are given by

$$|C_g(\tau)|^2 = \frac{1+\cos(\Omega\tau)}{2}$$
$$|C_e(\tau)|^2 = \frac{1-\cos(\Omega\tau)}{2}$$

The curve fitting for the probability of finding the qubit in the excited state is shown below.

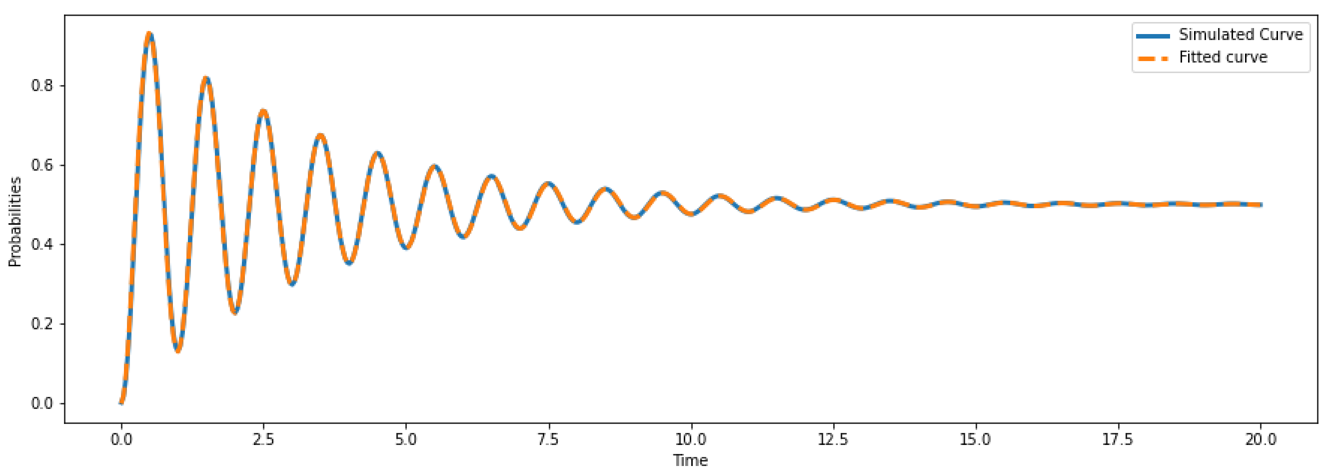

Figure 3: Curve fitting of the Probability $|C_{e/g}(t)|^2$ versus Time plot

The curve fitting for the probability of finding the qubit in the ground state is shown below.

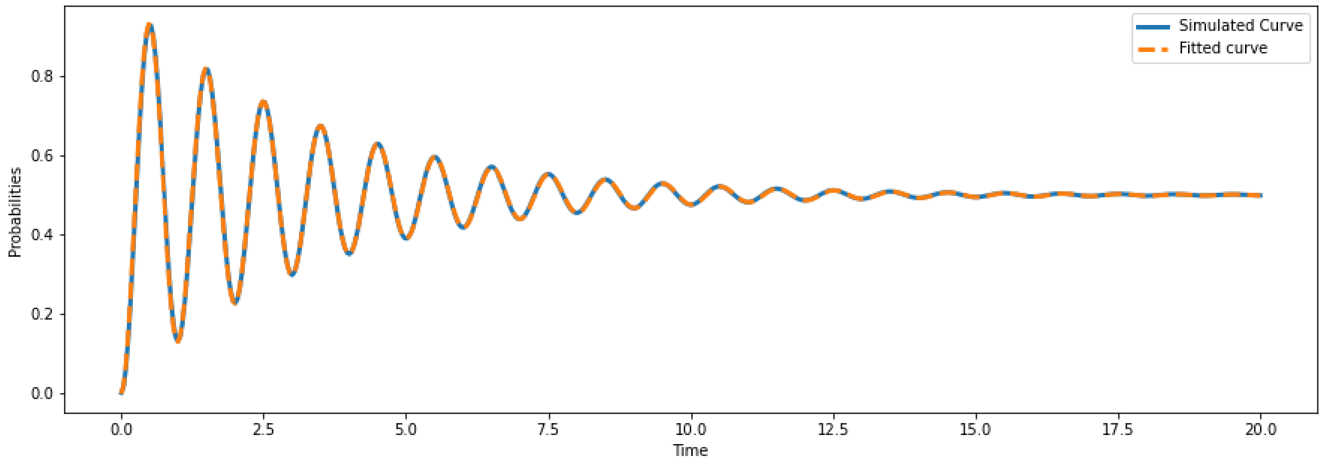

Figure 4: Curve fitting of the Probability $|C_{e/g}(t)|^2$ versus Time plot

The Rabi time T_R is found to be 3.3196 s in both the plots.

2.4 Comparison

We know that $\gamma_1 = \frac{1}{T_1}$ and $\gamma_2 = \frac{1}{T_\phi}$, where T_1 and T_ϕ are the relaxation and dephasing times. And we have

$$\frac{1}{T_R} = \frac{1}{2T_1} + \frac{1}{2T_2}$$

and

$$\frac{1}{T_1} = \frac{1}{T_1} + \frac{1}{T_\phi}$$

$$\therefore \frac{1}{T_R} = \frac{3}{4T_1} + \frac{1}{2T_\phi}$$

$$\gamma_R = \frac{3\gamma_1}{4} + \frac{1\gamma_2}{2}$$

We have $\gamma_1 = 0.2$ and $\gamma_2 = 0.15$.

$$\gamma_R = \frac{3 \times 0.2}{4} + \frac{1 \times 0.15}{2}$$

$$\frac{1}{T_R} = \frac{0.9}{4} = 0.225$$

$$\therefore T_R = 4.444\ s$$

The plot with the fitted and theoretical curbes is shown in Figure 5.

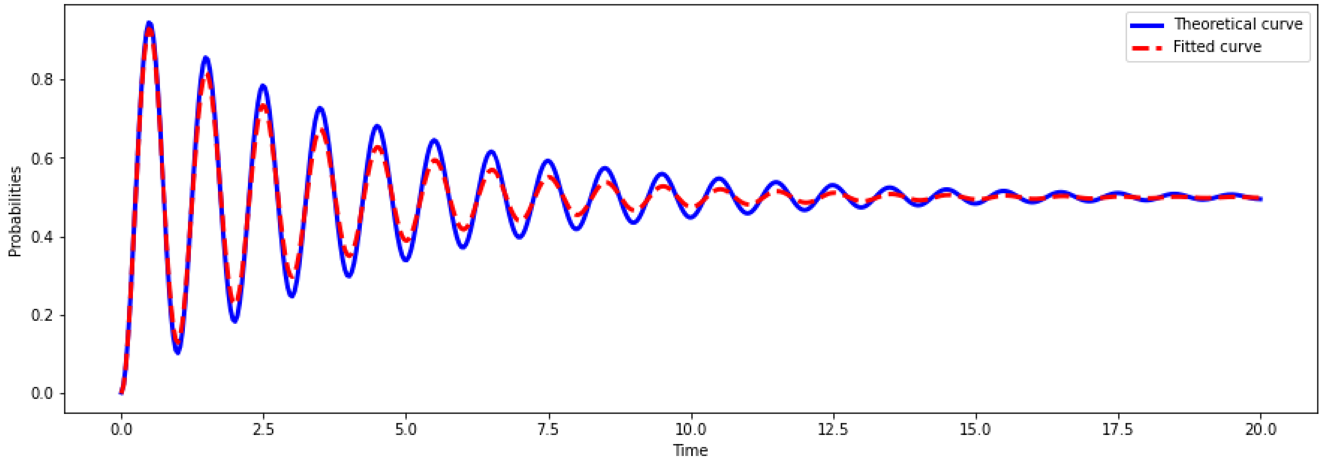

Figure 5: Comparison of the fitted and theoretical curves

3 Qubit fidelity

Fidelity is a measure of the distance between two quantum states. The fidelity of two state $|\phi\rangle$ and $|\psi\rangle$ is their overlap and is given by

$$\mathcal{F}(|\phi\rangle, |\psi\rangle) = |\langle\phi\rangle\,\psi|^2$$

For two density operators ρ and σ, the fidelity is defined by

$$\mathcal{F}(\rho, \sigma) = \left(Tr\left(\sqrt{\rho^{\frac{1}{2}}\sigma\rho^{\frac{1}{2}}}\right)\right)^2$$

The ideal value of fidelity is unity but due to decoherence the qubit states lose fidelity as the rate of dissipation increases.

We can vary relaxation and dephasing separately and study the variation of fidelity of a pi pulse as a function of the relaxation and dephasing rates, γ_1 and γ_2.

The plot for qubit fidelity with varying relaxation rate γ_1 is shown below. The plot for qubit

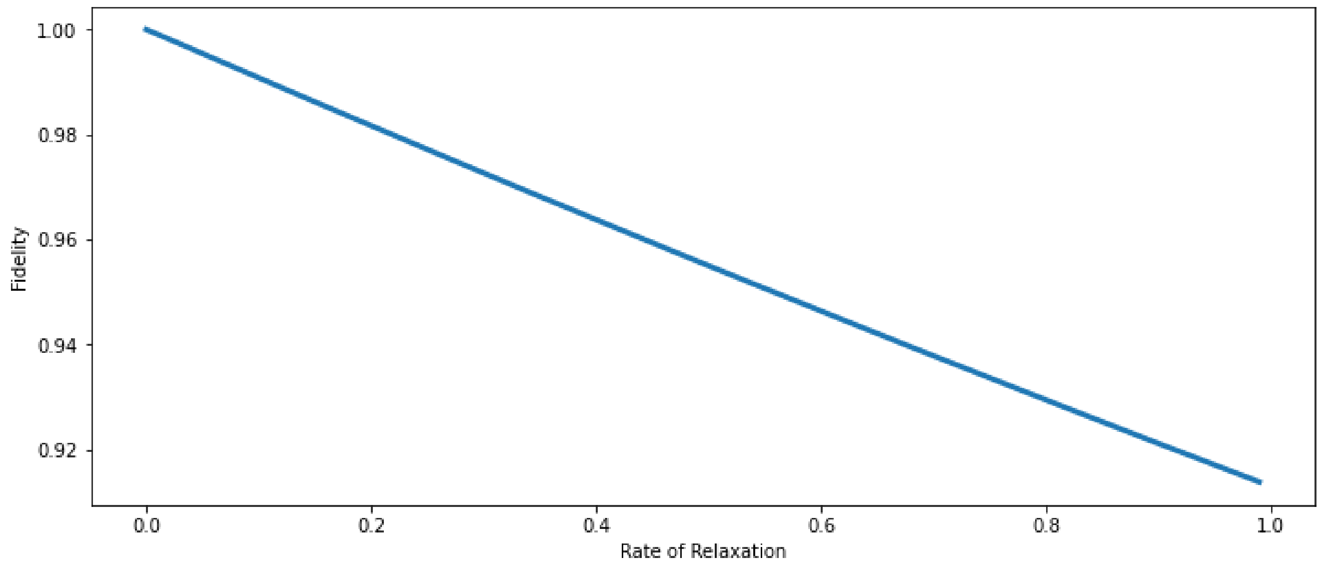

Figure 6: Qubit Fidelity as a function of relaxation rate

fidelity with varying relaxation rate γ_2 is shown below.

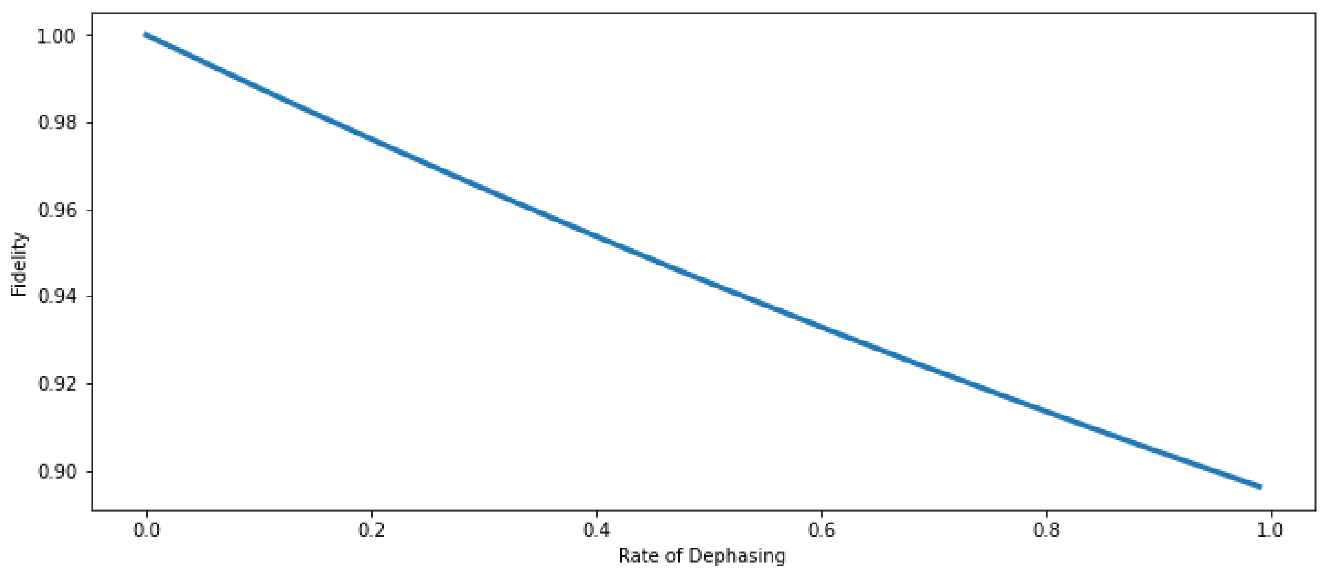

Figure 7: Qubit Fidelity as a function of dephasing rate

4 Code

4.1 Rabi Oscillations

```
#importing required libraries

from qutip import *
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy.optimize import curve_fit
from cmath import *
plt.rcParams['figure.figsize'] = [15, 5]

# Rabi Oscillations

#Parameter values
hbarw_a= 2*np.pi*1
hbarOmega = 2*np.pi*1
w_d= 2*np.pi*1
tlist = np.linspace(0.0, 20, 500)
e = basis(2,0)
g = basis(2,1)
psi0 = g
sz = sigmaz()
sp = sigmap()
sx = sigmax()
sy = sigmay()
sm = sigmam()

# Setting up the Solver
def qubit_integrate(gamma1, gamma2):

    # Setting up the Hamiltonian
    H0 = hbarw_a/2*(1+sz)
    H1 = hbarOmega/2*sp
    H2 = hbarOmega/2*sm

    args = [w_d]

    def H1_coeff(t, args):
        return exp(-1j*w_d*t)

    def H2_coeff(t, args):
        return exp(1j*w_d*t)
```

```
    H = [H0,[H1,H1_coeff],[H2,H2_coeff]]

    # Collapse Operators
    c_ops = []

    n_th = 0

    rate = gamma1 * (1 + n_th)
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sm) # relaxation
    rate = gamma1 * n_th
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sp) # excitation
    rate = gamma2
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sz) #dephasing


    rho_0 = psi0*psi0.dag()
    rho_e = e*e.dag()
    rho_g = g*g.dag()
    e_ops = [rho_e,rho_g]

    output_1 = mesolve(H, rho_0, tlist, c_ops, e_ops)
    output_2 = mesolve(H, rho_0, tlist, c_ops, [])

    return output_1, output_2
# No Decoherence

gamma1 = 0.0
gamma2 = 0.0
result_11, result_12 = qubit_integrate(gamma1, gamma2)

# Probaility Plot for no Decoherence
fig, ax = plt.subplots()
ax.plot(result_11.times, result_11.expect[0],linewidth='3')
ax.plot(result_11.times, result_11.expect[1],linewidth='3')
ax.set_xlabel('Time, t')
ax.set_ylabel('Probability $|C_{e/g}(t)|^2$')
ax.legend(("$|C_e(t)|^2$", "$|C_g(t)|^2$"),loc='upper right')
plt.show()

# Decoherence - both Relaxation and Dephasing
gamma1 = 0.2
gamma2 = 0.15
result_21, result_22 = qubit_integrate(gamma1, gamma2)
```

```
# Probaility Plot for Decoherence
fig, ax = plt.subplots()
ax.plot(result_21.times, result_21.expect[0],linewidth='3')
ax.plot(result_21.times, result_21.expect[1],linewidth='3')
ax.set_xlabel('Time, t')
ax.set_ylabel('Probability $|C_{e/g}(t)|^2$')
ax.legend(("$|C_e(t)|^2$", "$|C_g(t)|^2$"),loc='upper right')
plt.show()

# Curve Fitting
y1= result_21.expect[0]

#Test function with coefficients as parameters
def fit_curve_e(t, a,b, omega, T_Rabi):
    return (a+b*np.cos(omega*t)*np.exp(-t/T_Rabi))
par1,cov = curve_fit(fit_curve_e,result_21.times,y1,p0 = np.array((1,-1,2*np.p
fit1 = fit_curve_e(result_21.times,par1[0],par1[1],par1[2],par1[3])
#Plot the fitted curve with original
fig, ax = plt.subplots()
ax.plot(result_21.times,y1,result_21.times,fit1,'--',linewidth='3')
ax.set_xlabel('Time')
ax.set_ylabel('Probabilities')
plt.legend(['Simulated Curve','Fitted curve'])
plt.show()
print(par1)

# Curve Fitting
y2= result_21.expect[1]

#Test function with coefficients as parameters
def fit_curve_g(t, a,b, omega, T_Rabi):
    return (a+b*np.cos(omega*t)*np.exp(-t/T_Rabi))
par2,cov = curve_fit(fit_curve_g,result_21.times,y2,p0 =
np.array((1,1,2*np.pi,16)))
fit2 = fit_curve_g(result_21.times,par2[0],par2[1],par2[2],par2[3])
#Plot the fitted curve with original
fig, ax = plt.subplots()
ax.plot(result_21.times,y2,result_21.times,fit2,'--',linewidth='3')
ax.set_xlabel('Time')
ax.set_ylabel('Probabilities')
plt.legend(['Simulated curve','Fitted curve'])
plt.show()
print(par2)

#Comparison of fitted curve
```

```
gamma_Rabi = 0.75*gamma1+0.5*gamma2
T_rabi_theo = 1/gamma_Rabi
print(T_rabi_theo)
P_e = 0.5*(1-np.cos(2*np.pi*result_21.times)*
np.exp(-result_21.times/T_rabi_theo))
#Plot the fitted curve with theoretical
fig, ax = plt.subplots()
ax.plot(result_21.times,P_e,'b',result_21.times,
fit1,'r--',linewidth='3')
ax.set_xlabel('Time')
ax.set_ylabel('Probabilities')
plt.legend(['Theoretical curve','Fitted curve'])
plt.show()
```

4.2 Qubit Fidelities

```
 #importing required libraries

from qutip import *
import numpy as np
import matplotlib.pyplot as plt
import scipy as sp
from scipy.optimize import curve_fit
from cmath import *
plt.rcParams['figure.figsize'] = [12, 5]

# Rabi Oscillations

#Parameter values
hbarw_a= 2*np.pi*1
hbarOmega = 2*np.pi*1
w_d= 2*np.pi*1
tlist = np.linspace(0.0, 0.5, 2)
e = basis(2,0)
g = basis(2,1)
psi0 = g
sz = sigmaz()
sp = sigmap()
sx = sigmax()
sy = sigmay()
sm = sigmam()

# Setting up the Solver
def qubit_integrate(gamma1, gamma2):
```

```
    # Setting up the Hamiltonian
    H0 = hbarw_a/2*(1+sz)
    H1 = hbarOmega/2*sp
    H2 = hbarOmega/2*sm

    args = [w_d]

    def H1_coeff(t, args):
        return exp(-1j*w_d*t)

    def H2_coeff(t, args):
        return exp(1j*w_d*t)

    H = [H0,[H1,H1_coeff],[H2,H2_coeff]]

    # Collapse Operators
    c_ops = []

    n_th = 0

    rate = gamma1 * (1 + n_th)
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sm) # relaxation
    rate = gamma1 * n_th
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sp) # excitation
    rate = gamma2
    if rate > 0.0:
        c_ops.append(sqrt(rate) * sz)


    rho_0 = psi0*psi0.dag()
    rho_e = e*e.dag()
    rho_g = g*g.dag()
    e_ops = [rho_e,rho_g]


    output_1 = mesolve(H, rho_0, tlist, c_ops, e_ops)
    output_2 = mesolve(H, rho_0, tlist, c_ops, [])

    return output_1, output_2
# Only Relaxation
gamma1=0.0
gamma2=0.0
ideal_1, ideal_2 = qubit_integrate(gamma1,gamma2)
qubit_fid1=[]
```

```
rate1=[]
for i in range(100):
    gamma1 = 0.01*i
    rate1.append(gamma1)
    result_11, result_12 = qubit_integrate(gamma1, gamma2)
    qubit_fid1.append(fidelity(result_12.states[1],ideal_2.states[1]))

fig, ax = plt.subplots()
ax.plot(rate1,qubit_fid1,linewidth='3')
ax.set_xlabel('Rate of Relaxation')
ax.set_ylabel('Fidelity')
plt.show()

# Only Dephasing
gamma1=0.0
gamma2=0.0
ideal_1, ideal_2 = qubit_integrate(gamma1,gamma2)
qubit_fid2=[]
rate2=[]
for i in range(100):
    gamma2 = 0.01*i
    rate2.append(gamma2)
    result_21, result_22 = qubit_integrate(gamma1, gamma2)
    qubit_fid2.append(fidelity(result_22.states[1],ideal_2.states[1]))

fig, ax = plt.subplots()
ax.plot(rate2,qubit_fid2,linewidth='3')
ax.set_xlabel('Rate of Dephasing')
ax.set_ylabel('Fidelity')
plt.show()
```