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**Term Paper**

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# Optical Squeezed States

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# 1 Introduction

All states of light derive some quantum features from the discreteness of photons and are quantum mechanical. In practice, the non-classical features of light are difficult to observe. The single photon state is the most non-classical of light states but it is also possible to have non-classical states involving a very large number of photons. A criterion for the non-classicality of states is in terms of the quasi-probability distribution known as the Glauber-Sudarshan P function, P($\alpha$).

The coherent state is the most classical of all quantum states while squeezed states are an examples of some the most important non-classical states. These are states having no vanishing expectation values of the field but where fluctuations may be less than those of a coherent state in one part of the field. In other words, a squeezed field has phase-sensitive quantum fluctuations, which, at certain phase angles are less than those of a perfectly coherent field, or of ”no” field at all (the vacuum state).

After the initial discovery of squeezed states in works by Schrodinger, Kennard and Darwin around 1927, they were mathematically rediscovered and discussed extensively in the 1960s and 70s. By mid 1980s, it was understood that quantum fluctuations of light could be lowered below the vacuum-state (shot-noise) limit in various forms of nonlinear optical interactions. The field gained momentum when experimental results were demonstrated by various groups around that time, demonstrating a true quantum effect of light.

# 2 The Basic Principles

A light field may be understood as a collection of harmonic oscillators which is quantized in the standard manner and which fluctuates as a consequence of the basic noncommutability of the canonical field variables. A quantized electromagnetic field can be expressed as a set of quantum harmonic oscillators.

$$\hat{\mathcal{H}} = \sum_k \hbar\omega_k \left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$$

Consider a measure of the uncertainty in the measurement of an observable $\hat{A}$ to be the mean variance $\Delta A$, such that,

$$(\Delta A)^2 = \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2$$

For an eigenstate $|\Psi\rangle$ of the observable $\hat{A}$ (i.e. $\hat{A}|\Psi\rangle = a|\Psi\rangle$) the mean variance is equal to zero. For two non-commuting observables $\hat{A}$ and $\hat{B}$ such that $[\hat{A}, \hat{B}] = i\hat{C}$, there does not exist a common eigenbasis. Therefore, these two observables cannot simultaneously be determined precisely. This is expressed using the uncertainty relations,

$$\langle(\Delta\hat{A})^2\rangle\langle(\Delta\hat{B})^2\rangle \geq \frac{1}{4}|\langle\hat{C}\rangle|^2$$

In the uncertainty relation the lower bound on the product of the two variances explicity depends on the state in which the quantum system is prepared. *Intelligent states* are those states for which the left and right hand side are mutually equal. If the minimum of both left and right hand sides is obtained simultaneously, then the state is called a *minimum uncertainty state* (MUS). The variances $\Delta A$ and $\Delta B$ do not have to be equal. The state $|\Psi\rangle$ is said to be squeezed with respect to the observable $\hat{A}$ if,

$$\langle(\Delta\hat{A})^2\rangle < \frac{1}{2}|\langle\hat{C}\rangle|$$

Also, the state of the system is said to be squeezed if either

$$\langle(\Delta\hat{A})^2\rangle < \frac{1}{2}|\langle\hat{C}\rangle| \quad \text{or} \quad \langle(\Delta\hat{B})^2\rangle < \frac{1}{2}|\langle\hat{C}\rangle|$$

Both the variances cannot be lower than $\frac{1}{2}|\langle\hat{C}\rangle|$ simultaneously because of the uncertainty relation. Squeezed states for which equality holds are referred to as the ideal squeezed states and are an example of intelligent states.

## 2.1 Quadrature Squeezing

For a time dependent electric field operator,

$$\hat{E}_x(t) = \epsilon_0\left(\hat{a}e^{-i\omega t} + \hat{a}^\dagger e^{i\omega t}\right)sin(kz)$$

consider

$$\hat{A} = \hat{X}_1 = \frac{1}{2}(\hat{a} + \hat{a}^\dagger)$$
$$\hat{B} = \hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^\dagger)$$

where $\hat{X}_1$ and $\hat{X}_2$ are called quadrature operators, that are associated with field amplitudes oscillating out of phase with each other by 90° as

$$\hat{E}_x(t) = 2\epsilon_0 sin(kz)[\hat{X}_1 cos(\omega t) + \hat{X}_2 sin(\omega t)]$$

They satisfy the commutation relation

$$[\hat{X}_1, \hat{X}_2] = \frac{i}{2}$$

$$\therefore \langle(\Delta\hat{X}_1)^2\rangle\langle(\Delta\hat{X}_2)^2\rangle \geq \frac{1}{16}$$

Quadrature squeezing occurs whenever

$$\langle(\Delta\hat{X}_1)^2\rangle < \frac{1}{4} \quad \text{or} \quad \langle(\Delta\hat{X}_2)^2\rangle < \frac{1}{4}$$

For a coherent state $|\alpha\rangle$ and for vacuum, the variances of the two quadratures are equal and

$$\langle(\Delta\hat{X}_1)^2\rangle = \langle(\Delta\hat{X}_2)^2\rangle = \frac{1}{4}$$

A squeezed state has less ”noise” in one of the quadratures than for the coherent state or vacuum state. Fluctuations in that quadrature are squeezed but the fluctuations in the other quadrature must be enhanced so as to not violate the uncertainty relation.

Mathematically, squeezed states can be generated using a squeeze operator $\hat{S}(\xi)$,

$$\hat{S}(\xi) = exp\left(\frac{1}{2}(\xi^* a^2 - \xi a^{\dagger 2})\right)$$

where $\xi = re^{i\theta}$, and where $r$ is squeeze parameter and $0 \leq r < \infty$ and $0 \leq \theta \leq 2\pi$. For a special case of $|\psi_S\rangle = \hat{S}(\xi)|\psi\rangle$ where $\psi$ the vacuum state $|0\rangle$, we can write

$$|\xi\rangle = \hat{S}(\xi)|0\rangle$$

On solving for this state at $\theta = 0$ we have

$$\langle(\Delta\hat{X}_1)^2\rangle = \frac{1}{4}e^{-2r}$$

$$\langle(\Delta\hat{X}_2)^2\rangle = \frac{1}{4}e^{2r}$$

There is squeezing in $\hat{X}_1$ quadrature. For $\theta = \pi$, the squeezing occurs in the $\hat{X}_2$ quadrature.

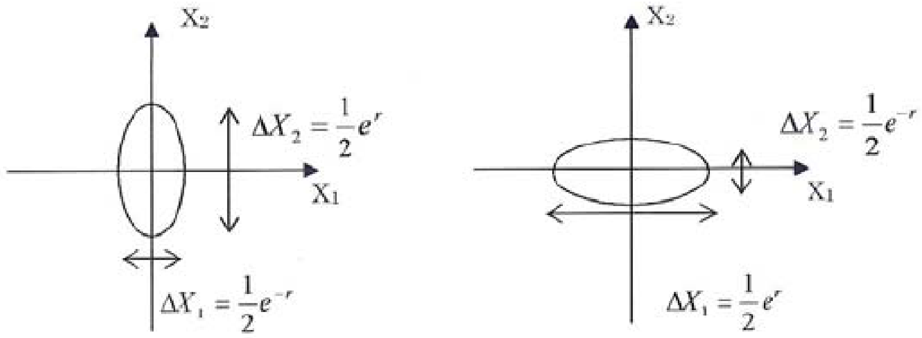



**Figure 1:** Graphical representation: Error ellipse for squeezed state
Reprinted from *Extraction of Thrust from Quantum Vacuum Using Squeezed Light, Yoshinari Minami*

A generalised squeezed state is obtained by adding the displacement operator such that $|\alpha\rangle = \hat{D}(\alpha)|0\rangle$ for a normalized coherent state $|\alpha\rangle$.

$$|\alpha,\xi\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle$$

## 2.2 Amplitude (Number) Squeezing

The number phase commutation relation $[\hat{n},\hat{\phi} = i$ leads to number phase uncertainty relation $\Delta n \Delta\phi \geq \frac{1}{2}$ for large average photon number. The is number squeezing when $\Delta n < \frac{1}{\sqrt{n}}$. Amplitude squeezing is also non-classical. A state exhibiting amplitude squeezing is said to possess sub-Poissonian statistics, the distribution being narrower than for a coherent state of the same average photon number. It is possible for some states of the field to exhibit simultaneously both quadrature and amplitude squeezing. Short and Mandel observed sub-Poissonian statistics for the first time using single-atom resonance fluorescence.

# 3 Methods and Materials

## 3.1 Generation of squeezed light

Squeezed light generation schemes are based on parametric process using nonlinear optical devices, which have an interaction Hamiltonian quadratic in the terms of the creation and annihilation operations $\hat{a}^\dagger$ and $\hat{a}$.

A process called *degenerate parametric down-conversion* a nonlinear medium is pumped by a field of frequency $\omega_p$ and some photons get converted to pairs of identical photons of frequency $\omega = \frac{\omega_p}{2}$ of the signal field. The Hamiltonian is given by

$$\hat{\mathcal{H}} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_p\hat{b}^\dagger\hat{b} + i\hbar\chi^{(2)}(\hat{a}^2\hat{b}^\dagger - \hat{a}^{\dagger 2}\hat{b})$$

where $b$ is the pump mode, $a$ is the signal mode and $\chi^{(2)}$ is a second-order nonlinear susceptibility.

Another nonlinear process is *degenerate four-wave mixing* in which two pump photons are converted into two signal photons of the same frequency. The Hamiltonian is given by

$$\hat{\mathcal{H}} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega\hat{b}^\dagger\hat{b} + i\hbar\chi^{(3)}(\hat{a}^2\hat{b}^{\dagger 2} - \hat{a}^{\dagger 2}\hat{b}^2)$$

where $\chi^{(2)}$ is a third-order nonlinear susceptibility.

The first successful attempt of generating squeezed light states (by Slusher et al) in 1985, used four-wave mixing in an atomic vapour of sodium atoms. The squeezing was measured to be 0.3 dB below the vacuum noise in the squeezed quadrature. The third-order Kerr type non-linearity of $SiO_2$ was exploited and squeezing of 0.6 dB below vacuum noise was measured in a 114 m optical fibre that had been coiled and cooled to 4.2 K in liquid helium in 1986. In the same year, the second-order non-linearity of ferrocrystals were also utilized in the generation of squeezed light states using parametric down-conversion in magnesium doped lithium niobate crystal embedded in a standing-wave cavity to obtain 3.5 dB of squeezing below vacuum noise. Soon after these achievements, current noise suppression was proposed as a method to generate squeezing in the output of a laser diode. Maximum squeezing obtained was 0.3 dB.

## 3.2 Detection of squeezed light

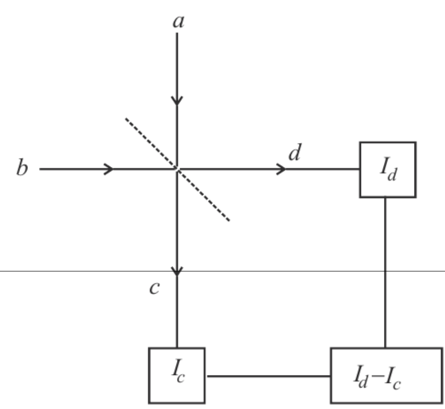
Squeezed light can be detected by using a method called *balanced homodyne detection*. In the Fig. 2, mode $a$ contains the squuezed field and mode $b$ has a strong coherent classical field which is equivalent to a coherent state of amplitude $\beta$.

The input-output relations are

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + i\hat{b}) \quad \hat{d} = \frac{1}{\sqrt{2}}(\hat{b} + i\hat{a})$$

The difference in intensities of detectors is

$$I_c - I_d = \langle \hat{c}^\dagger\hat{c} - \hat{d}^\dagger\hat{d}\rangle = i\langle \hat{a}^\dagger\hat{b} - \hat{a}\hat{b}^\dagger\rangle$$



Let $\langle \hat{n}_{cd} \rangle = \langle \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} \rangle$ and b mode is in state $|\beta e^{-i\omega t}\rangle$ with $\beta = |\beta| e^{-i\psi}$,

$$\langle \hat{n}_{cd} \rangle = |\beta| (\hat{a} e^{i\omega t} e^{-i\theta} + \hat{a}^\dagger e^{-i\omega t} e^{i\theta})$$

where $\theta = \psi + \frac{\pi}{2}$. If a mode light has frequency $\omega$, we have $\hat{a} = \hat{a}_0 e^{-i\omega t}$ so

$$\langle \hat{n}_{cd} \rangle = 2|\beta| \langle \hat{X}(\theta) \rangle$$

with $\hat{X}(\theta) = \frac{1}{2}(\hat{a}_0 e^{-i\theta} + \hat{a}_0^\dagger e^{i\theta})$ as the field quadrature operator. Quadrature of field signal can be measured by changing $\theta$. $\theta$ is chosen to maximise squeezing. For $\langle (\Delta \hat{X}(\theta))^2 \rangle < \frac{1}{4}$ we obtain $\langle (\Delta \hat{n}_{cd})^2 \rangle < |\beta|^2$.

# 4 Applications

There are many applications of optical squeezed states because of their ability to have reduced quantum noise. Some of the common areas in which squeezed states are used are discussed in the following sections.

## 4.1 Interferometry

Squeezed states are used to improve the sensitivity of quantum interferometers for high precision measurements. A Mach-Zehnder interferometer has a small noise which can be hindrance in sensitive measurements. The vacuum noise is reduced by injecting squeezed light rather than allowing the presence of vacuum. With squeezed light the sensitivity can scale as the inverse of circulating power so even with low laser power measurements can be highly precise without redesigning the entire device. LIGO, Virgo and GEO600 are interferometers that are limited by shot noise at higher frequencies and by radiation pressure at lower ones. So phase squeezing is need at high frequencies and amplitude or intensity squeezing is required at lower frequencies.

## 4.2 Quantum Communication

The entangles EPR light can be produces by using squeezed states of light which are an important resource for quantum key distribution. Two initially independent fields interact via non-degenerate parametric amplification before separating. The output beams are strongly correlated so measurements for the output quadrature-phase amplitudes of the idler beam to infer information about the signal beam. Squeezed states allow for measurements with high noise reductions. These states can also be used in teleportation of continuous quantum variables.

## 4.3 Quantum Error Correction

Fault tolerant quantum computing require efficient and robust error-correcting codes. Quantum erasure-correcting codes have been experimentally realised that can simultaneously protect two independent continuous variable quantum systems from loss of photons in transmission channels.

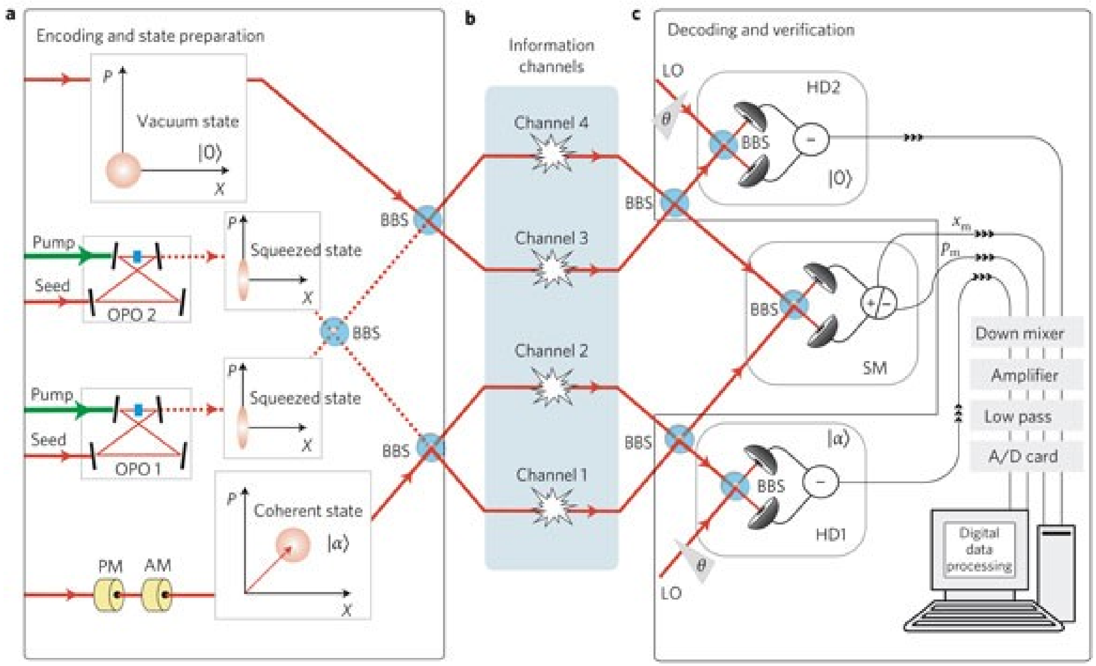



**Figure 3:** Experimental setup for Quantum Error Correction

The four-mode code is prepared using linear interference at three balanced beam-splitters (BBS) between the two input states, $|a\rangle$ and $|0\rangle$, and two ancillary squeezed vacuum states (produced in two optical parametric oscillators, OPO 1 and OPO 2). The coherent state is prepared via a coherent modulation at 5.5 MHz produced by an amplitude (AM) and a phase modulator (PM). The encoded state is injected into four free-space channels that can be independently blocked , thereby mimicking erasures. The corrupted state is decoded, the error is detected by the syndrome measurement (SM) and the state is deterministically corrected or probabilistically selected.
The measurement is an entangled measurement in which the phase and amplitude quadratures of the two emerging states are jointly measured. The error correcting displacement is carried out electronically after the measurement of the transmitted quantum states. These states are measured with two independent homodyne detectors that allow for full quantum state characterization, by scanning the phases (u) of the local oscillators (LOs) with respect to the phases of the signals.

## 4.4 Other applications

Optical squeezed states are also used in phase estimation, quantum imaging of biological samples, clock synchronization and magnetometry. They have also been used in quantum state engineering for generation of non-Gaussian states required for many quantum processing protocols.

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