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33. Noise and Interferometry

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Abstract.
The nature of noise and its varied manifestations in radio and optical interferometry are the subject of this lecture.

1. Preamble

All of the lectures that you have had until now dealt with the theory and practice of how to make images of the radio radiation from the sky, and by now you are probably experts at it. A simplified statement of the principle used is that the similarity between the signals received at spatially separated points contains the information which suitably transformed renders an image of the sky. Although many telescopes are generally used simultaneously in such an exercise, it is the pairwise comparisons that contain the information and that are then put together. It is the similarity between the signals received in two antennas as a function of their vector spacing that is quantified by measuring their complex correlation coefficient. This coefficient will vary from one pair of antennas to the next and measurements on many such pairs go into making the final image. But one pair of telescopes or antennas will suffice for this lecture, whose main purpose is to look a little closer into the meaning of the similarity of signals.

In this last lecture of the school, I shall touch upon a number of topics which have no immediate or everyday relevance to the use of radio interferometers for synthesis imaging, but an appreciation of which could provide greater insight into what your have learnt so far. Towards this end, the following list of items was suggested to me by the organizers of the school:

- The nature of noise.
- Weak and strong signals.
- The two slit paradox.
- Classical versus quantum regimes.
- Relation to intensity interferometry.
- Detection thresholds.
- Amplifiers and amplifier noise.
- Optical versus radio concepts.
- Photon statistics in different regimes.

My talk will attempt to cover all of them, but not necessarily in this sequence.

2. Information and Bandwidth

Noise is unfortunately a bad word for a very good thing like the radiation that emanates from celestial sources bringing us the information we seek. There are also unwanted kinds of noise, like receiver noise that is generated within the receiver itself, or noise from the ground due to spill-over into the antennas. So when I say noise, I just mean a signal with the properties of natural radiation like that which comes to us from radio sources in the sky, or the thermal radiation from a resistor. It is very different for example, from the radiation that you get from a signal generator in the laboratory, whose energy is delivered in such a narrow frequency interval that we can think of it as having no width at all. The latter is very similar to the electric mains where all of the power is in an extremely narrow band (around 60 Hz in this country).

If you take such a source of monochromatic radiation, as it is called, its power and frequency are both fixed for ever. We can learn nothing from it as a function of time, because nothing whatever changes. It is true that the phase of the signal goes round and round, but it does so in a totally predictable fashion and one needs to measure the phase only once to know it precisely at any given future instant. This type of radiation carries no information and as an illustration, let us imagine that a particular radio source in the sky puts out such a signal. If you applied all of the techniques that you have learnt in this synthesis imaging school, all you could obtain, apart from its intensity and polarization, would be its coordinates.

On the other hand, the information needed to make all the lovely images of different radio sources that you have seen is carried in the bandwidth associated with their radiation. It is such broadband signals, as opposed to monochromatic radiation, that we call Noise and that has wonderful properties. Although there are so-called "line" sources that radiate spectral lines like the interstellar hydrogen or OH, as distinct from "continuum" sources, they are all the same for our purpose. If you look at the radiation in a band that is small compared to the actual width of the spectral line, then the characteristics of the signal would be the same as those in a continuum source observed at the same frequency and within the same bandwidth. So what are the characteristics of natural radiation?

3. The Nature of Noise

There are two ways of looking at such a signal which are really equivalent. One is to look at the signal in time and the other in frequency space. If you look at it in time (say on an oscilloscope) you will find that it varies from instant to instant in a way that its average power gets closer and closer to some value the longer the integration time. But the exact value at some future instant remains unpredictable. A signal with a small fractional bandwidth will look like a regular sinewave of approximately the frequency of the center of the band, and in a time of the order of the reciprocal of the bandwidth, its frequency and amplitude change to new values. If the bandwidth is large then these changes happen quickly. If the bandwidth is small then the changes happen more slowly. The data rate is given by the bandwidth, each new piece arriving in a time that is its reciprocal. This is also the rate at which the state of polarization of natural radiation can change to a new state. And finally, an important characteristic of noise is that its distribution in amplitude is Gaussian.

Turning to frequency space, how do we know that the signal is truly broad band? Let us begin by just splitting the band into two halves. We will now find that the average powers are equal, but that the instantaneous value of the signal in the left hand half of the band is quite independent of that in the right hand half, with no correlation whatsoever. We also find that the signals change half as fast as before because the bandwidth has been halved. This suggests that there is signal power at all frequencies and to prove this we could go on subdividing the bands only to find that no matter how narrow we make them, there is always mean power there proportional to the bandwidth.

As noted already we also find that both the amplitude and the phase change more and more slowly as the bands are made narrower and narrower, but the amplitude distribution remains Gaussian however narrow the bands. This leads us to another way of looking at the broad band signal, namely that it consists of an infinite number of monochromatic signals occupying every possible position within this band. Each one of these monochromatic waves is, of course, totally predictable and cannot change for reasons we have already discussed earlier. But when the passband allows a certain range of them to come through, they are all going at different rates and the value of the voltage that we measure at some instant simply happens to be that given by the instantaneous addition of all these infinite monochromatic components. Thanks to the central limit theorem this ensures a Gaussian distribution. At the next instant the voltage will be slightly different and it will go on changing. But the most rapid change possible is that due to the rates associated with the extreme frequencies in the band, and that is why independent amplitudes and phases are separated by time intervals of the order of $(1/\Delta\nu)$. And also why it is enough to sample a band-limited waveform at a finite rate to reconstruct it completely.

4. Interferometers & Coherence

Having described noise, it is time to turn to interferometers again and to see what noise does to them. If a radio source, like the hypothetical one, that I mentioned a little earlier, radiated only a monochromatic wave then the correlation coefficient that would be measured by a pair of antennas receiving the signal would clearly be 100%. We could also say that the signals received by the two antennas were totally coherent.

Now if we did this experiment with real telescopes, for example a pair of antennas of the Very Large Array, the rotation of the earth during our measurement would Doppler shift the apparent frequencies received at the two antennas, offsetting one from the other. This would surprise nobody and the matter would simply be described as saying that we have a fringe rate, meaning thereby that the monochromatic signals received at the two telescopes differed by this amount in frequency. It is this frequency difference and its variation with time which allowed us to measure the source position. One would however still continue to find that the magnitude of the correlation coefficient was 100%, i.e., that the signals received at the two telescopes were fully coherent. This is a simple and convincing way of appreciating that any monochromatic signal is totally coherent with any other monochromatic signal whatever their frequency difference. And it is also a very good way to see that incoherence between two signals requires them to have a finite bandwidth. But broadband signals can also be fully coherent as we shall soon discuss.

Going back to noise, if we look within the time represented by a reciprocal of the bandwidth, then, as I said earlier, each of the antennas will be receiving something that is temporarily monochromatic but which will differ in frequency and amplitude in the two antennas. Nonetheless, for reasons just given, the correlation coefficient for these two should again be unity, and these two signals have to be considered coherent during this small interval. The notion of partial coherence or of a correlation coefficient that is less than 100% can arise only when we combine a large number of samples, each of which can have different values for the phase, finally converging to an average which could be anything between 100% and something close to, but never identically zero.

There is a deep analogy between this and the measurement of the degree of polarization of natural radiation, not least because polarization is a measure of the correlation between orthogonal components of the electric (or magnetic) fields. Any individual sample we measure will be found to be 100% polarized with some particular polarization state even when observing a black body. When we average a large number of samples, and find that the state of polarization of the different samples is distributed uniformly in polarization space, we say that the radiation is randomly polarized. On the other hand, if there is a bias towards any particular state, the radiation is said to be partially polarized. The degree of polarization like the degree of coherence is a notion that can be entertained only when we talk about broad band signals. Thus we see that NOISE is the very stuff of interferometry. Its characteristics provide all the consequences of measuring correlations of the radiation field sampled at locations separated in space and/or time.

5. The Similarity of Signals

We can now look a little closer at examples of cases where there is little, or much, similarity between two signals. If we think of the noise from two different resistors, it is easy to persuade ourselves that their signals will be totally uncorrelated because the microscopic processes that agitate the electrons in these resistors have nothing to do with each other. In the same way, the radiation received from two separated radio sources in the sky or generated in two different receivers will be totally independent and uncorrelated. So also the radiation from separated pieces of ground that different telescopes might see.

A totally different example would be to take a signal that is propagating along a pair of wires and to split it into two. No one would doubt that if you took the output from one of the audio channels of a music system and split it to drive two loud speakers that you would hear the same music from both. Or, that if you split the signal from one resistor into two and put them into a correlator, that you would get an answer of 100% . There is also no difficulty in appreciating why signals which have been derived from a common source, but have different incoherent additions to the two halves (e.g. different receiver noises), will look partially dissimilar. One of today's exercises however is to see how without the addition of extra uncorrelated noise, the division of signals coming from a single source can give unlike samples.

We know that the signal received from a radio source by a telescope has large fluctuations in phase and amplitude because, as already discussed, the signal is broadband or NOISY. Also that it is this noisiness which gives meaning to any measurement of similarity between two signals, and that new information comes only in samples whose rate is given by the bandwidth. But what is it that governs the similarity of the samples received by two telescopes from one source?

Now this is the topic on which, as I said, you must all be experts by now. What you have learnt is that even though a distant radio source might subtend a very small angle in the sky, it is nevertheless composed of different regions, each of which radiates independently. And therefore, when you have separated receiving points, the ways in which the contributions from the different regions combine are not the same at the two telescopes and this results in a correlation coefficient of less than unity.

We can now do a little thought experiment and somehow move the source further away, for example by imagining the expansion of the universe to have gone a lot faster than it does. What we want is the source to retain its characteristics but to simply get more and more distant from the interferometer. Let us see what this does to the (degree of) similarity of the signals received at the two antennas. What is certain is that the angular diameter of the source will decrease as it gets further and further away from us, and it will therefore be less and less resolved by the interferometer. As a consequence, the degree of correlation would definitely increase if this were the only effect. But although it appears more and more like a point source, distance will eventually make it so weak that the division of its signal between the two telescopes will introduce dissimilarity between the two versions. Granularity will appear simply because radiation cannot be divided into pieces smaller than photons. When will this happen?

6. Wave Noise

One way that I like to visualize this is with a long train of boxcars, each piled up with some number of objects which could be bricks or atoms, or in this case photons. At a certain point, the contents of each boxcar has to be divided between two others belonging to two different trains whose pattern of contents we are then going to compare. Let the original train have some enormous number of bricks in each boxcar as in Figure 33–1A. When the contents of each boxcar are divided between two let us assume that the bricks are not counted out but individually thrown randomly into the boxcars of the two trains. The difference between the piles in any corresponding pair of boxcars will be typically of the order of the square root of the number of bricks in each of them; and therefore, the larger the original number, the more exactly similar would be the two portions into which it is divided. Consequently, the pattern of variation of the number of bricks along the trains would be very similar and give us almost perfect correlation of their sizes.

This corresponds to the case of splitting an audio signal into two loudspeakers, and represents the classical limit. What characterises such a classical signal is an enormously high photon occupation number in each sample which permits it to be split into two or more almost identical versions. Even more important is that in the classical limit the voltage (or current) waveform can be measured as a function of time without the measurement process introducing significant error. This implies the ability to record and reproduce a signal with fidelity and is what permits post facto comparison of signals for similarity. VLBI would not be possible without this ability. Its implications, and the price that has to be

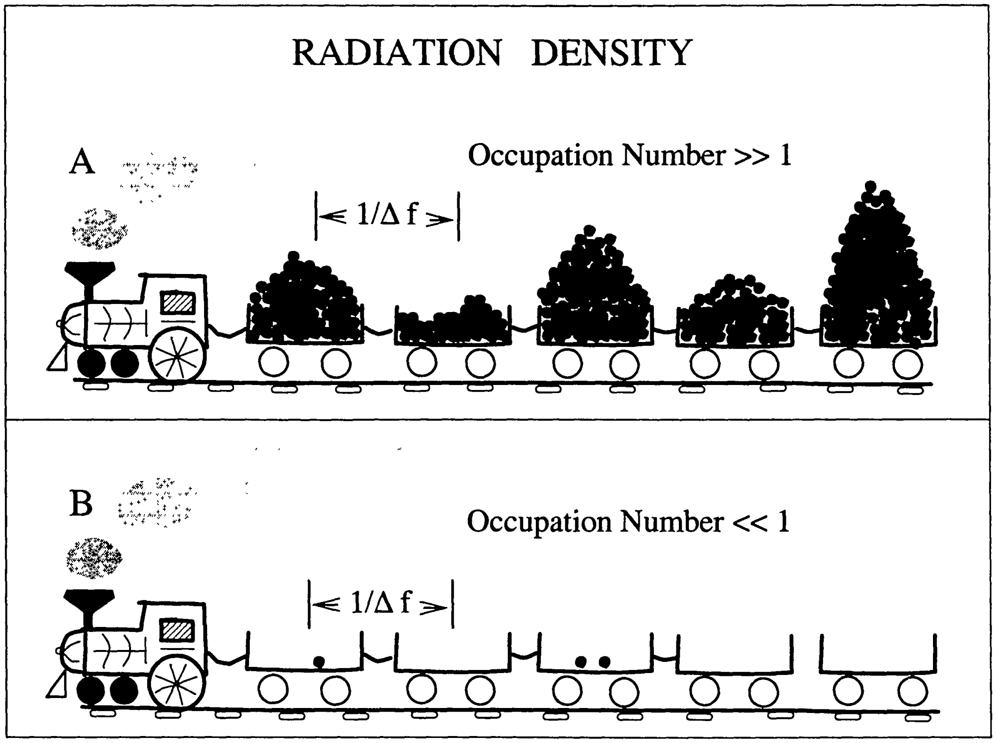

Figure 33–1. Boxcar representation for a stream of radiation. Each boxcar is a sample and corresponds to the reciprocal of the bandwidth, the rate at which new information arrives. A) The high density case where there is an enormous number of photons in each sample and substantial variation from sample to sample. B) The very low density case when the number of photons is minute compared to the number of samples.

paid for it, will become clear when I discuss amplifiers towards the end of my talk.

To avoid confusion with amplitudes, let me point out that the variations from one pile of bricks (photons) to the next as seen in Figure 33–1A represent intensity changes with a time scale of the inverse of the bandwidth. These variations are called wave noise and are associated with high density signals. If the bandwidth is say a hundredth of the frequency, then there are of the order of a hundred cycles of the waveform within one sample. In conventional interferometry, it is the amplitude and phase of this waveform that are correlated with those from other telescopes as you have learnt. The correlation of just the size of one pile with that of another is the correlation of intensities which I shall discuss shortly.

7. Shot Noise

As the other extreme, you could imagine a train in which the number of bricks is minute compared to the number of boxcars, Figure 33–1B. This means that most of them are empty and that occasionally you find a boxcar with a single brick, and even less often a boxcar with more than one. This situation is very different to the picture I have painted above. Firstly, the rate at which the signal brings us new information is not the bandwidth, but rather the mean number of bricks (photons) per second received by the telescope. Then again, if such a sequence consisting mostly of zeros were split into two as we did before, we must surely end up with all the ones leading to a one and a zero in the daughter sequences. This looks like perfect anticorrelation, and suggests that interferometry as I have described it so far, cannot work with such weak signals.

Before discussing how it can and does, let me assure you that although it may seem strange to radio astronomers, this is a faithful description of signals from astronomical sources at optical and all higher frequencies. A square meter of collecting area looking at a zero magnitude star receives on average one optical photon for every ten thousand sample times. The separations get even greater as we go to higher frequencies like X-rays. The arrival of photons at random times with a mean separation that is orders of magnitude greater than the inverse of the filter bandwidth gives rise to the very different, and shot-like noise associated with all such sparse signals. The quantization of the received energy is evident, and it is the associated photon noise, characterized by Poisson statistics, that is responsible for the uncertainty in measurements of such weak intensities.

A telling illustration of the difference between the two cases is a comparison of the accuracy of intensity measurements of radio and optical signals as a function of telescope size. When the size of a radio telescope is big enough to give an antenna temperature on a source that is the main contribution to the total noise, further increase of telescope size makes no improvement even though the source may be totally unresolved by its beam. The uncertainty in a measurement of the intensity depends only on the bandwidth and the integration time, whose product gives you the number of samples, and the square root of which determines the fractional error, Figure 33–2A. In the case of an optical telescope measuring the intensity of light from a star, also unresolved, the square root is of the number of photons detected, which depends on the collecting area and continues to increase with telescope size! (Figure 33–2B).

8. Intensity Interferometry

The possibility of correlating intensities as mentioned a little earlier has played an important part in the development of astronomical interferometry, and also in the launching of a new branch of physics called quantum optics. Two of the most spectacular synthesis images of radio sources that you have surely seen are of Cas A and Cygnus A made here by some of your teachers in this school. The first attempts to resolve these sources at radio wavelengths was in the early fifties by Hanbury Brown and colleagues at Jodrell Bank. It was then believed that these sources might be so small that baselines as large as are now used in the VLBA might be needed to resolve them, and so Hanbury Brown invented, or discovered, a way to do this without a coherent local oscillator at both receivers. It was simply to compare the fluctuations in the demodulated waves which was done by transmitting the detected noise on a radio link. The price paid is a lower signal to noise ratio, which is a small sacrifice if it enables an otherwise impossible observation. The scheme worked perfectly, but to their disappointment it all ended too soon as both sources were resolved with baselines

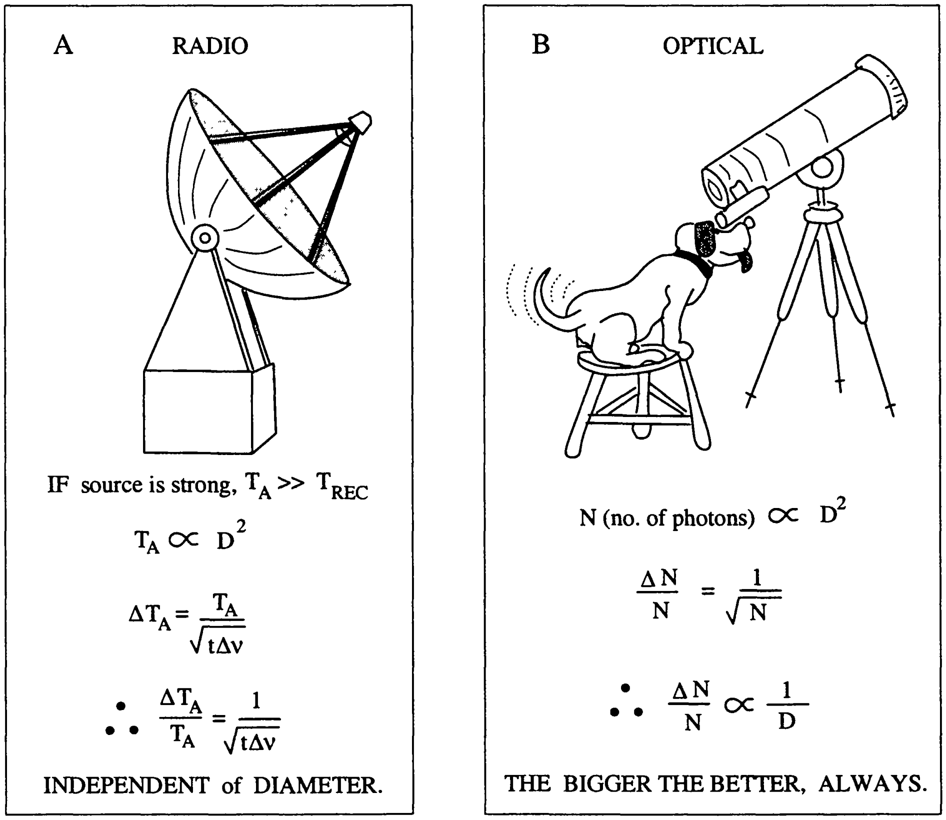

Figure 33–2. The surprising difference in the dependence on telescope size of the accuracy of intensity measurements of radio and optical signals, corresponding typically to the high and low density cases illustrated in Figure 33–1. In the case of a strong radio source, A), the fractional error depends only on the square root of the number of samples and is independent of telescope size. B) In the optical case, where the stellar radiation is the only input, it is the number of photons which matters, as null samples carry no information; the accuracy therefore continues to increase with telescope size. In the general case, the quantity whose square root determines the error in the intensity measurement is the harmonic mean of the number of photons and the number of samples.

of only a few kilometers, and did not really need a scheme which could work across the world.

In Hanbury's words they had used a sledge-hammer to crack a nut, but luckily there were more rewards to come. The fact that the correlations were not affected even when the radio sources were scintillating violently due to the ionosphere revealed that the system could also be made to work through a turbulent medium. And so was born the idea of making an optical intensity interferometer with baselines long enough to resolve main sequence stars. Michelson, the pioneer in this field could do no better than 20 feet, the major difficulty being the turbulence in the earth's atmosphere which blurs the image into a patch which is enormous compared with the true angular size of the star. Apart from being unaffected by this turbulence, another great virtue of comparing intensities was that the mechanical stability needed was only of the order of the reciprocal of the bandwidth of the filter following the detector, about a foot. In the case of a Michelson type interferometer, the stability needed is of the order of a wavelength of light!

A measure of the originality of any idea in science is often the opposition it evokes, and Hanbury Brown and his theoretician-collaborator Twiss got more than their fair share. But they gave as good as they got, and finally managed to obtain funds and to build an instrument in Australia with a maximum baseline of almost two hundred meters. They also achieved their principal scientific objectives. Reasonably precise and reliable measurements were made of 32 single stars; these included the first measurements ever made of a main sequence star and the first measurements of any star earlier than type M. The number of known angular diameters was increased from 6 to 38 and this work stands as a permanent and valuable contribution to stellar astronomy. In addition, the results of their observations on the spectroscopic binary Spica were a striking demonstration of the value of a high resolution interferometer for the study of a close binary star.

From my remarks in the previous section on the sparseness of optical radiation from stars, it might appear that it would be impossible to find correlations in the arrival times of photons at two separated sites. And everybody continued to say so to Hanbury Brown and Twiss till they proved otherwise. To understand why it works you should have heard the Jansky lecture that Hanbury Brown gave here some years ago, or better, read his book on the subject which is also full of his humour. Among the various reasons are that the stars chosen were hot, the reflectors large, and that photons obey Bose statistics. This tends to clump them and in a sense provides a tiny amount of wave noise which is correlated in both detectors as opposed to the shot noises which are uncorrelated.

9. Photons and Interference

I come now to a discussion of those aspects of radiation related to wave-particle duality that have provided a continuing source of confusion ever since quantum theory came into being. From ripples on a pond to Newton's rings, waves have always been the basis of understanding interference phenomena. It is the relative phase of two overlapping waves which decides whether the addition is constructive or destructive and gives rise to all of the effects you have studied at this school. If the picture of radiation as waves with amplitudes and phases is replaced by one in which the energy arrives in discrete bundles, the notion of interference becomes hard to visualize. The principal reason for this difficulty is the misleading concept of photons as particles, and miniscule ones at that. The click of a loudspeaker announcing the arrival of a photon on the tiny area of the cathode of a phototube or the even smaller area of a pixel of a CCD camera, gives an overwhelming but erroneous impression of a photon as a highly localized object in space and time.

It is this false picture of localisation that is responsible for much of the confusion in the discussion of the (in)famous two-slit paradox. Figuring out which of two holes a photon went through is very reminiscent of the preoccupation in an earlier age of assessing how many angels could stand on the head of a pin. Confusion has reigned despite the physicists who created and understood quantum theory telling us that the propagation of radiation, whatever its strength, is always wavelike. And that the discrete nature of the energy manifests itself ONLY in the process of emission or absorption, as for example when it produces a photo-electron. As a test of its correctness we shall try applying this prescription to the functioning of an optical telescope to see if it provides a consistent picture of the behavior of radiation, even if it is not an easily visualizable one.

The diameter of the reflector and the accuracy of its surface determine the collecting area and the resolution of the optical telescope just as for a radio telescope. Ignoring the effect of the atmosphere (which is not relevant to this discussion) the size and shape of the diffraction image will be just the same as would be obtained with intense classical electromagnetic radiation. But I have already mentioned that the signal received by such a telescope from a star is very weak, and consists of photons whose arrival times appear well separated. How does this manifest itself? In several ways. The first is that the exact arrival time of the next photon is totally unpredictable. The next is that the diffraction image is also built up in an unpredictable way, but always mysteriously evolving into the calculated one when enough photons have been accumulated to overcome the granularity. What should be even more surprising is that no matter how weak one makes the radiation, say by moving the source further away as we did before, the image will be the same when we have accumulated the same (adequate) number of photons. How shall we understand this?

10. Uncertainty and Probability

First of all, the notion of intensity has to be replaced by a measure of the probability of detecting a photon within a certain time interval. This applies individually and independently to different parts of the image, and weakening of the radiation would increase proportionately the average time between photons in every part of it. But the fact that the eventual accumulated diffraction pattern is the same, no matter how large this separation in time, is proof that every photon, so to speak, "senses" every part of the telescope aperture. Blocking any small area, by for instance sticking a postage stamp on the mirror, dramatically manifests itself by pushing photons into the nulls of the previous unobstructed aperture diffraction pattern.

This sensing of the whole aperture holds even if there were an opaque strip across its middle separating it in two. The corresponding diffraction pattern is of course different, and has fringes in it, but its building up photon by photon would still be independent of the intensity. It makes no more sense to ask which half the photon went through, than it would have been before to ask where exactly in the undivided aperture any photon went through. Interference can be thought of as the name given to diffraction from a non-contiguous aperture and the photon really only interferes with itself if you want to think of it that way. If you do, you must also think of its size and shape as that of the total aperture in every detail (including separations) that you allowed it to go through before its detection and consequent annihilation.

The replacement of certainty by probability when dealing with the behavior of photons is completely described by Heisenberg's celebrated Uncertainty Principle. It identifies pairs of associated quantities, an accurate determination of one of which automatically implies a large error in the other, the product

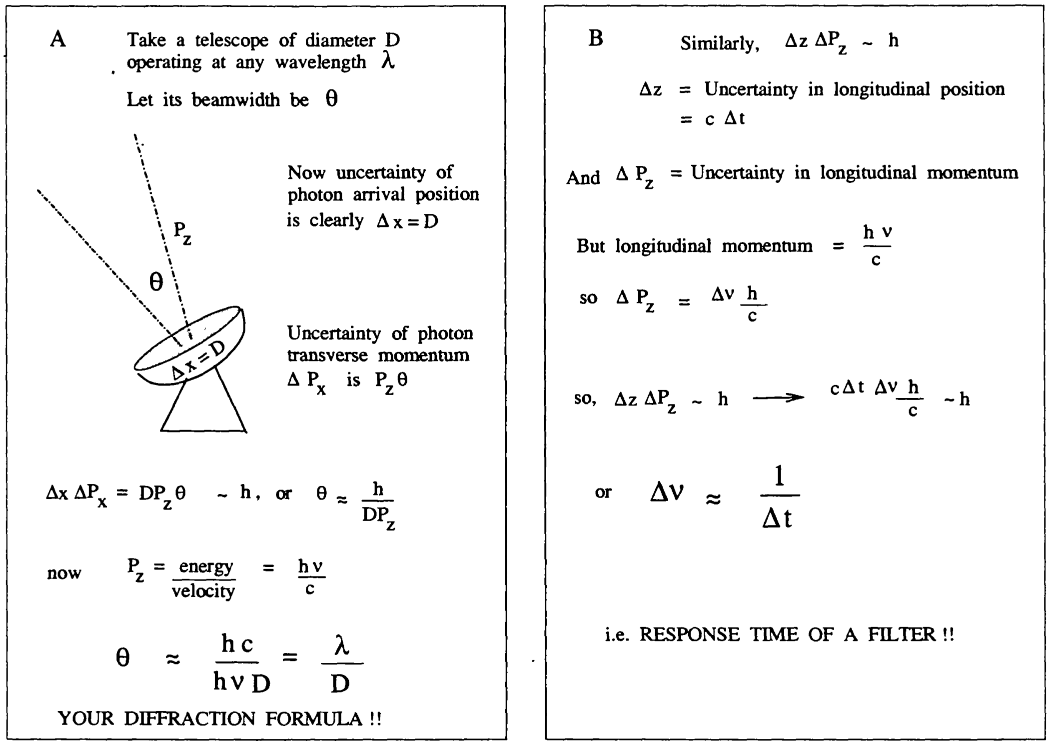

Figure 33–3. Heisenberg's uncertainty principle connects naturally, A) the aperture size and resolving power of telescopes, or B) the bandwidths and response times of filters, even when dealing with single photons.

of the uncertainties not being less than Planck's constant. This quantifies the informational price to be paid for localization in space or time for example, by connecting naturally the aperture size and resolving power of telescopes, or the bandwidth and response times of filters, even when dealing with single photons. The smaller the physical dimensions of a telescope the greater is the accuracy of localization in lateral position of the photon that is collected by it. The price paid, as specified by Heisenberg, is greater uncertainty in determining the lateral component of its momentum, resulting (in our language) in a wider beam, Figure 33–3A. It is the precisely analogous relation between the localization in longitudinal position and longitudinal momentum that connects bandwidth and the time resolution of signals, Figure 33–3B.

The above examples relate to the pair involving position and momentum, and another involves energy and time. A third pair which leads us back to the topic of samples is that relating phase and number (see Lecture 28). A classical signal with a well defined phase is characterized, as seen earlier, by a very large number of photons per sample, and a proportionately large uncertainty in their actual number. At the other extreme is a signal consisting of occasional photons for which the notion of an absolute phase is essentially meaningless. But relative phase differences for different possible paths continues to be meaningful and is what enables us to understand how a photon "interferes with itself."

11. Density in Phase Space

When discussing samples earlier we saw that when subdividing a band into narrower and narrower channels, the mean power available decreases in proportion to the width of the channel. A matched resistor connected to a cable is a good source of natural radiation with all the noise-like properties that I have already discussed, and we know that the power available from it is proportional to $k_B T \Delta\nu$, where k_B is Boltzmann's constant and T is the physical temperature of the resistor in Kelvins. While the power available decreases with bandwidth, we also saw that the rate at which one obtains independent samples goes inversely as the width of the band. Consequently, the energy in each sample is independent of the bandwidth and the average size of this bundle of energy depends only on the temperature characterizing the radiation. And this is equal to $k_B T$ at all frequencies which lie on the lower or Rayleigh-Jeans part of the spectrum.

As the energy of a photon can only be $h\nu$, it is trivial to calculate the number of photons in each sample. And the answer is that when the temperature of the resistor has a value of $h\nu/k_B$, the samples have of the order of one photon in each of them! At temperatures which are very high compared to this value, the signal will appear to be classical and there will be no difficulty in dividing it without introducing dissimilarity. This is the loud speaker case where I will leave the calculation to you as to how many photons of audio frequency per sample are running down the wires to the loudspeaker. But it is the other extreme we are interested in today, and I have some plots to show you precisely at what temperatures for a given frequency, or vice versa, one encounters quantum as opposed to classical behavior.

Let me remind you that by antenna temperature we mean that physical temperature of a matched resistor replacing the antenna that would produce the same intensity of radiation in the same frequency interval. Also that our interest is to see at what antenna temperatures non-classical behavior might manifest itself. The plots show both the black body spectrum at a given temperature and the density of photons per sample as a function of frequency at that temperature. Figure 33–4 shows you the number of photons which you can expect in a sample, as a function of frequency, when observing the cosmic microwave background.

I have chosen to start with this particular radiation to make the point that the temperature it produces in any and every antenna, will be the same. The independence of antenna temperature on the size of a telescope immersed in a black body is analogous to the energy per sample being independent of the width of the passband for broad band radiation. Both are related in a fundamental way to the Uncertainty Principle which says that a sample is a cell in (6-dimensional) phase space whose shape can be changed but whose volume is fixed and equal to Planck's constant cubed. Its occupancy therefore is determined only by the density of radiation in the appropriate part of phase space, which in turn is determined by the temperature characterizing it and the frequency of observation.

The microwave background is the only radiation that can fill the beam of any telescope whatever its size. In the case of sources that are smaller than the antenna beam, we will have so-called beam dilution resulting in an antenna temperature that can be much smaller than their brightness temperatures. Nevertheless, the antenna temperature is the only relevant one determining the density of radiation coming from the feed of the antenna to the receiver. Figure

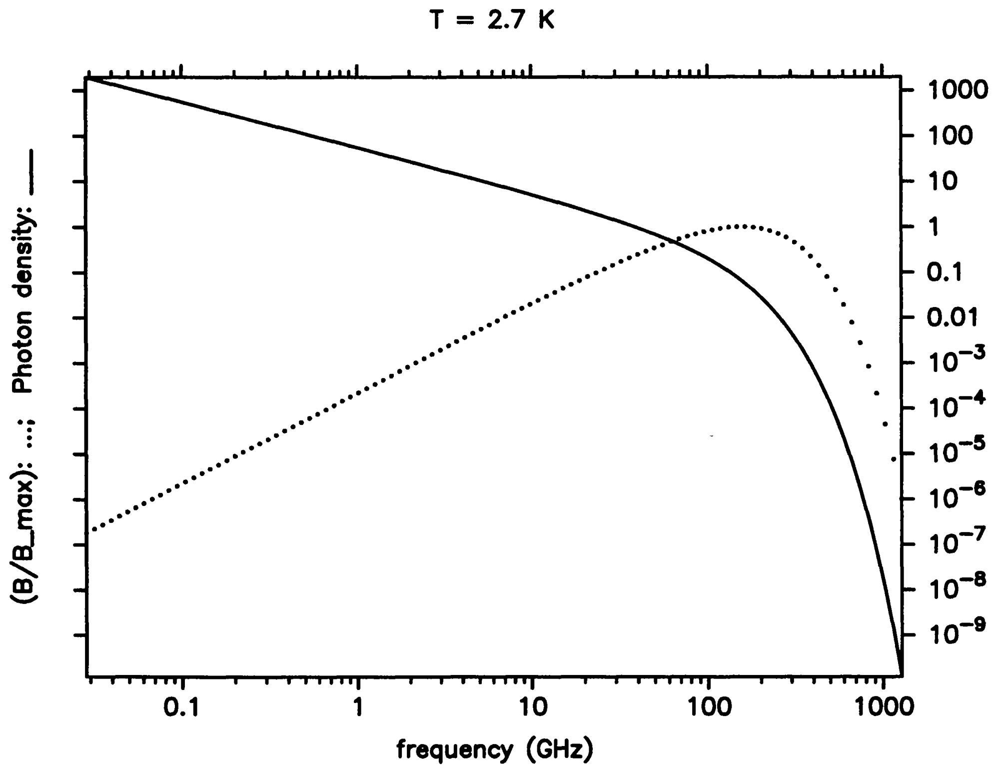

Figure 33–4. The microwave background radiation. The dotted line represents the intensity of a black body of 2.7 K, and the solid line, the number of photons per cell in phase space.

33–5 corresponds to a temperature of 100 K, typically the temperature of neutral interstellar gas. The only change from the previous figure is the relabeling of the frequency axis. The transition region between classical and quantum behavior has moved up as a consequence, and at the frequency of the hydrogen line of 1.4 GHz, a signal becomes sparse at antenna temperatures below a tenth of a degree Kelvin. At higher and lower frequencies the transition will occur at proportionately higher and lower antenna temperatures.

12. The Strength of Astronomical Signals

It is time to see whether all this discussion about weak and strong signals has any relevance to real life radio astronomy, say as practised here. The antennas in both arrays are of 25 meters diameter and would require about 10 Janskys of flux to produce one degree of antenna temperature. Relating this to what I have just said above, the dividing line between weak and strong signals in very very round numbers is at one Jansky per gigaHertz. At the lowest frequencies of operation there will be many situations where the signal can be considered highly classical. But for most of the sensitive observations made with the telescopes, particularly at high frequencies, the antennas collect less than one photon per sample on the average from the source! How come no attention whatever is paid to this circumstance, and no dire consequences result from making all observations in

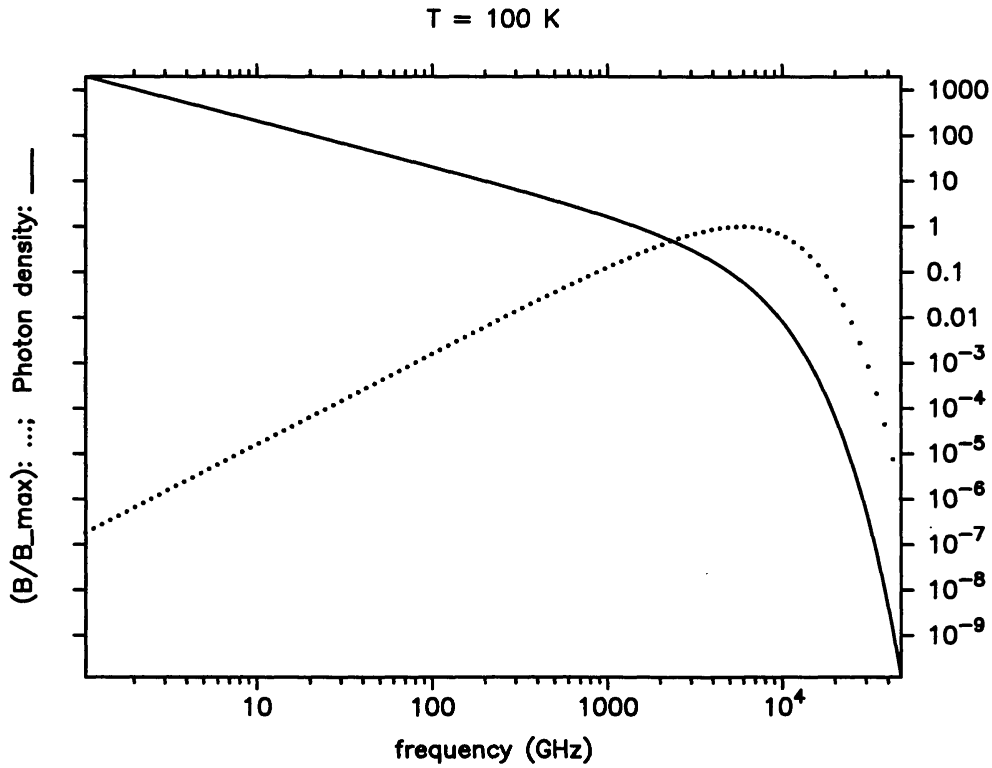

Figure 33–5. The radiation from a black body at 100 K. As in figure 33–4 the dotted line represents the intensity, and the solid line, the number of photons per cell.

the same standard fashion as if they were all of strong classical signals? To understand this we must first look at how astronomical signals are detected.

Barring radiation from our Sun, there are no signals from any astronomical sources that are strong enough to operate a measurement or recording device (other than the eye or a photographic plate) without our providing additional energy. Such devices are called detectors or amplifiers, most or all of which work by accelerating electrons that have either been liberated or set in motion by the astronomical signal. Photons of optical or higher frequencies have adequate energy to overcome the work function in many substances, and a measurement on the electron(s) so liberated is the best way to quantify such signals. In such detectors the photon is generally annihilated and the only information gathered is the energy of the photon, and its direction of arrival to the accuracy the telescope permits. The notion of phase of the signal in these cases is non-existent as already explained.

At usual radio frequencies the photons do not have enough energy to unbind and release electrons, but they can set them in motion in the conductors of which the antennas and feeds are made. But as these currents are orders of magnitude too weak to operate any devices, they have to be amplified first. The devices used for this purpose are called coherent amplifiers and are believed to produce strengthened versions of the input voltage or current waveforms. The amplifier in your hi-fidelity system is the archetypical example of such a device, the very name announcing (at a high decibel level) that the output signal is a faithfully amplified version of the input. We shall discuss in a moment the sustainability of this claim.

I would like to return briefly to the two-slit paradox which was dismissed rather rudely without stating clearly why it would be futile to investigate "which hole the photon went through". It is because any scheme to do so would necessarily introduce attenuation in one or both paths and modify the total diffraction pattern. The interference part of it will be degraded to the same degree as any information is obtained about passage through one of the two holes. Now if there existed truly a way to obtain two or more identical copies of an input signal, then we could have our cake and eat it too. If faithful amplification followed by precise division could provide identical copies, one of the pair could be used to measure what arrives at each hole, and the other to produce the interference pattern. I have already dwelt at length on the errors introduced when dividing a signal into two. To add to that difficulty, Heisenberg has ensured our failure by also prohibiting exact multiplication.

13. Coherent Amplifiers

The uncertainty associated with the energy-time pair of variables manifests itself as a minimum noise of one photon per sample at the input to any coherent amplifier with a large gain factor. Equivalent ways of thinking about this is in terms of the fluctuations of the vacuum field, or spontaneous emission which adds to the emission stimulated by the input signal. The net result is the prediction of a minimum theoretical noise temperature for any high-gain amplifier of order $h\nu/k_B$ degrees Kelvin (equation 28-15). For an input signal with a very large number of photons per sample, the addition of one more would affect neither amplitude nor phase. Coherent amplification therefore works best when the signal is already so strong that division would make no difference, like education being most effective when it is not needed!

On the other hand, compared to typical optical signals, one photon per sample is an enormous amount and is a thousand times as strong as a very bright star seen with a big optical telescope. It is in fact the reason why amplifiers are not used for astronomy at such frequencies and higher. To sum up, the points to remember about coherent amplification are the following. Very low density signals get badly corrupted even with ideal amplifiers. To obtain amplification without damage, the input has already to be a classical signal. No matter how weak the input, even if nothing, the output of a high gain amplifier is always of classical strength, permitting duplication or further amplification (of whatever came out) without further damage.

Historically, radio astronomy was created by radio amateurs and radar engineers who knew nothing about astronomy but all about amplifiers. They knew that their amplifiers added unwanted noise, and a lot of it, and the history of the field is closely tied to the saga of the development of better and lower noise receivers. The excitement of building newer types of receivers and experimenting with them was itself comparable to the astronomical rewards from the higher sensitivities attained. Clever circuits for vacuum tubes were followed by traveling wave amplifiers, parametric amplifiers, masers, field effect transistors and presently higher electron mobility versions of these. At the highest frequencies,

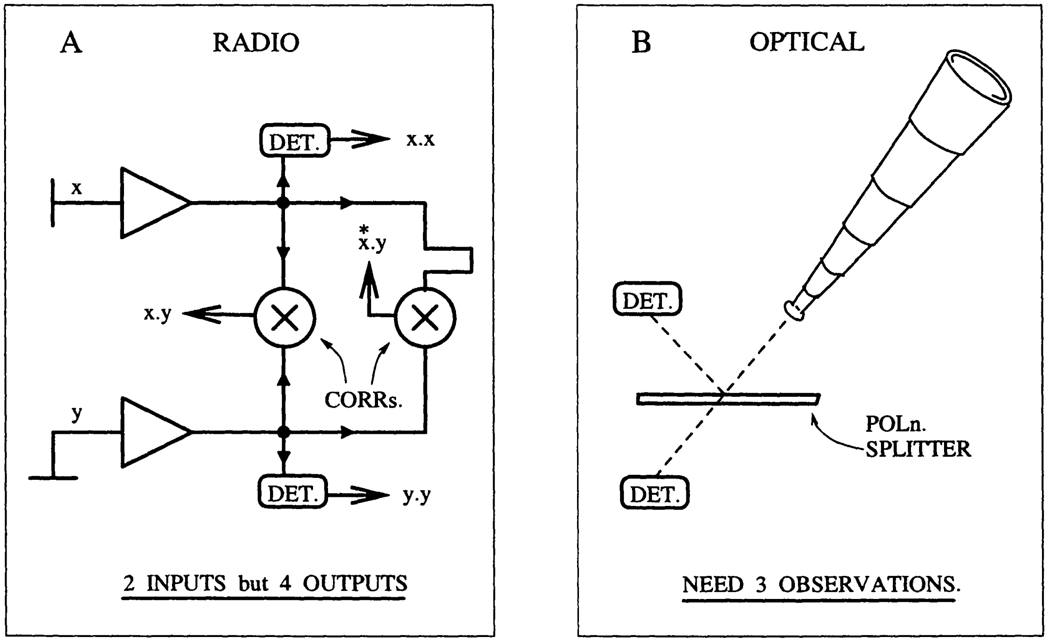

Figure 33–6. The measurement of polarization with and without amplifiers. A) At radio frequencies amplifying and splitting the signal from orthogonally polarized feeds permits the simultaneous determination of all four Stokes parameters. B) An optical telescope would require three observations in sequence for the same determination. Two of the six measurements obtained with different polarization splitters are redundant, but cannot be avoided.

using superconducting devices, one is approaching the achievement of minimum theoretical noise. But if a radio signal is as weak as the optical signals mentioned above, amplification must add an enormous amount of noise that will have to be integrated away before we can see the signal. Why then do we continue to amplify in radio astronomy? There are several reasons as I understand it.

The most serious, as already mentioned, is the work function as compared to the photon energies. Another is the temperature of our physical environment - the night is not dark in the radio. Quite apart from ground radiation which gets into every telescope and can be substantial, even the feeble microwave background of less than 3° K ensures that the occupation number at most radio frequencies is already high. In other words, even though the particular contribution to the signal that we seek is very very weak, it is already in a classical sea of noise and if there are benefits to be derived from retaining the associated aspects, we would be foolish to pass them up. One of them is the ability to measure phase. Even though the contribution of the desired signal to the measured phase is minute, it can be recovered with enough integration. And the most important of these benefits is the ability to duplicate ad nauseum once receiver noise has been added and a high multiplication factor obtained.

14. Epilogue

As a direct consequence of the above, radio astronomers routinely do many things their optical colleagues could not dream of doing. Let me mention some

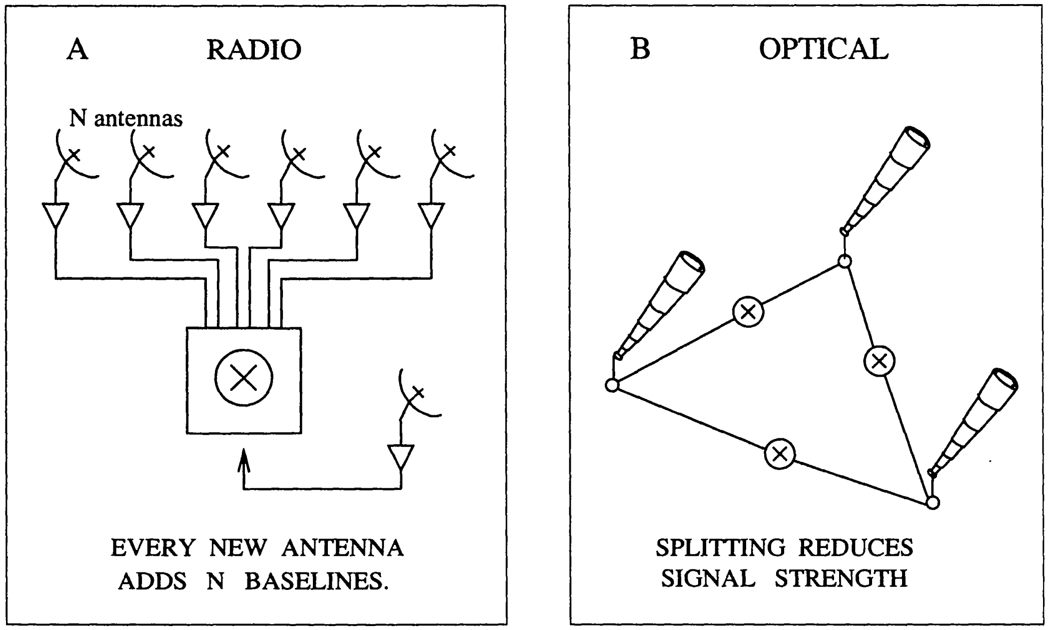

Figure 33–7. Aperture synthesis with and without amplifiers. A) In a radio array, the signal from each antenna is amplified and split N ways to be correlated simultaneously with the signals from all the other antennas. B) In an optical array of Michelson interferometers, splitting the signal reduces its strength and has to be compensated by increased observation to provide the same number of photons per baseline as without splitting.

examples, the first being the measurement of polarization of an astronomical signal. Amplifying and splitting the signal from orthogonally polarized feeds permits the simultaneous measurement of all four Stokes parameters, Figure 33–6A. An optical measurement would require three observations in sequence to obtain the same information because splitting the signal would worsen the signal/noise ratio, Figure 33–6B.

The second example is of a far more important advantage without which this school could not have been held. The signal from each of the twenty seven antennas of the array is amplified and split N ways to perform at the same time all the correlations with the signals from all the other antennas, Figure 33–7A. In the optical Michelson interferometers now being operated in several places, any splitting of the signal for other advantages like obtaining closure phase, Figure 33–7B, has to be compensated by further observations to accumulate the same number of photons as without splitting.

But the most spectacular achievement made possible by coherent amplification is of course VLBI where high fidelity recording and post facto reproduction is its very basis. And this technique has made another giant leap not so long ago with space VLBI in which the NRAO arrays play an important part. Optical technology has made fantastic progress spurred on by the communications industry and it is already many years since phase-locked optical oscillators became commonplace. But I have difficulty even imagining such a thing as optical VLBI. Maybe it will come, but for the moment it is for the radio astronomers to make hay while the sun shines on them as THE experts in synthesis imaging.

15. Acknowledgements

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