Algebraic Structure

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Algebraic Structure

- A non empty set S is called an algebraic structure w.r.t binary operation (*) if it follows the following axioms:
- **Closure:**(a*b) belongs to S for all a,b ? S.

Example :

S = {1,-1} is algebraic structure under *
As 1*1 = 1, 1*-1 = -1, -1*-1 = 1 all results belong to S.

Semi Group

- A non-empty set S, (S,*) is called a semigroup if it follows the following axiom:
  - **Closure:**(a*b) belongs to S for all a, b ? S.
  - **Associativity:** a*(b*c) = (a*b)*c ? a, b ,c belongs to S.
- ***Example:*** *(Set of integers, +), and (Matrix ,*) are examples of semigroup.*

Monoid

- A non-empty set S, (S,*) is called a monoid if it follows the following axiom:
  - **Closure:**(a*b) belongs to S for all a, b ? S.
  - **Associativity:** a*(b*c) = (a*b)*c ? a, b, c belongs to S.
  - **Identity Element:** There exists e ? S such that a*e = e*a = a ? a ? S

EXAMPLE :

- (Set of integers,*) is Monoid as 1 is an integer which is also an identity element.
  (Set of natural numbers, +) is not Monoid as there doesn’t exist any identity element. But this is Semigroup.
  But (Set of whole numbers, +) is Monoid with 0 as identity element.

Group

- A non-empty set G, (G,*) is called a group if it follows the following axiom:
  - **Closure:**(a*b) belongs to G for all a, b ? G.
  - **Associativity:** a*(b*c) = (a*b)*c ? a, b, c belongs to G.
  - **Identity Element:** There exists e ? G such that a*e = e*a = a ? a ? G
  - **Inverses:**? a ? G there exists a^{-1} ? G such that $a*a^{-1} = a^{-1}*a = e$

Abelian Group or Commutative group

- A non-empty set S, (S,*) is called a Abelian group if it follows the following axiom:
  - **Closure:**(a*b) belongs to S for all a, b ? S.
  - **Associativity:** a*(b*c) = (a*b)*c ? a ,b ,c belongs to S.
  - **Identity Element:** There exists e ? S such that a*e = e*a = a ? a ? S
  - **Inverses:**? a ? S there exists a^{-1} ? S such that $a*a^{-1} = a^{-1}*a = e$
  - **Commutative:** a*b = b*a for all a, b ? S

	Must Satisfy Properties
Algebraic Structure	Closure
Semi Group	Closure, Associative
Monoid	Closure, Associative, Identity
Group	Closure, Associative, Identity, Inverse
Abelian Group	Closure, Associative, Identity, Inverse, Commutative


Thank you!!!
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