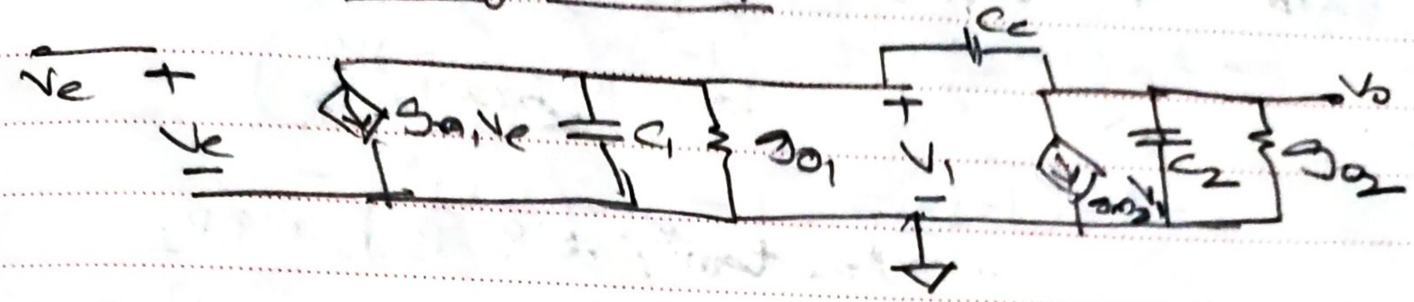
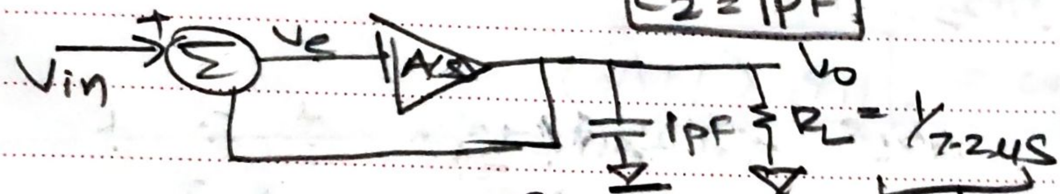
Assignment - 7



$g_{m1} = 20\,\mu S \qquad C_1 = 10\,fF \qquad g_{o1} = 0.2\,\mu S$

$g_{m2} = 80\,\mu S \qquad C_2' = 40\,fF \qquad g_{o2}' = 0.8\,\mu S$

$\boxed{C_2 = 1pF}$



$$\frac{V_o}{V_{in}} = \frac{A(s)}{1 + A(s)}$$

$C_2 = 1pF + 40fF \sim 1pF$

$\rightarrow g_{o2} = g_{o2}' + \frac{1}{R_L}$

$= 7.2\,\mu S + 0.8\,\mu$

$\boxed{g_{o2} = 8\,\mu S}$

And $A(s) = \dfrac{-A_0 (1 - s/z)}{(1 - s/P_1)(1 - s/P_2)}$

$A_0 = \dfrac{g_{m1} g_{m2}}{g_{o1} g_{o2}} \rightarrow 1000 \qquad P_1 \simeq \dfrac{-g_{o1}}{C_1 + C_c\left(1 + \frac{g_{m2}}{g_{o2}}\right)}$

$\underbrace{z = \dfrac{g_{m2}}{C_c}}$ $\qquad P_2 \simeq -\left[\dfrac{g_{o2} + g_{m2}\, C_c/C_1 + C_c}{C_2 + \frac{C_1 C_c}{C_1 + C_c}}\right]$

Want far from $\omega_{u,loop}$ i.e $\dfrac{g_{m2}}{C_c} \geq 10\,\omega_{u,loop}$

$$A(s)\Big|_{s=j\omega} = \frac{-A_0 (1 - j\omega/z)}{\left(1 - \frac{j\omega}{P_1}\right)\left(1 - j\frac{\omega}{P_2}\right)} = \frac{-A_0 (1 - j\omega/z)}{1 - j\omega\left(\frac{1}{P_1} + \frac{1}{P_2}\right) - \frac{\omega^2}{P_1 P_2}}$$

Then, for phase margin, to be 60°,

$$\Rightarrow PM = 180° + \angle LG(s)\Big|_{@\omega_{u,loop}}$$

$$-120° = -\tan^{-1}\left(\frac{\omega}{z}\right) + \tan^{-1}\left(\frac{\omega}{p_1}\right) + \tan^{-1}\left(\frac{\omega}{p_2}\right)$$

As we want $\frac{\omega_{u,loop}}{z} \ll 1$, we can ignore the $\left(\frac{\omega}{z}\right)$ phase part which nearly contributes to zero angle.

$$-120° = +\tan^{-1}\left(\frac{\omega}{-p_1}\right) + \tan^{-1}\left(\frac{\omega}{-p_2}\right)$$

$$\Rightarrow \tan 120° = \tan\left(\tan^{-1}\left(\frac{\omega}{-p_1}\right) + \tan^{-1}\left(\frac{\omega}{-p_2}\right)\right)$$

$$\Rightarrow \boxed{-\sqrt{3} = \frac{\frac{\omega}{-p_1} + \frac{\omega}{-p_2}}{1 - \frac{\omega^2}{-p_1 \times -p_2}}}$$

$$\frac{\omega}{-p_1} + \frac{\omega}{-p_2} = x\sqrt{3}$$

$$1 - \frac{\omega^2}{p_1 p_2} = -x$$

From $\omega_{u,loop}$

$$A_0^2\Big(1 + \underbrace{\frac{\omega^2}{z^2}}_{0}\Big) = \left|1 - j\omega\left(\frac{1}{p_1} + \frac{1}{p_2}\right) - \frac{\omega^2}{p_1 p_2}\right|^2$$

$$= \left|\left(1 - \frac{\omega^2}{p_1 p_2}\right) - j\omega\left(\frac{1}{p_1} + \frac{1}{p_2}\right)\right|^2$$

$$A_0^2 = \left(1 - \frac{\omega^2}{p_1 p_2}\right)^2 + \omega^2\left(\frac{1}{p_1} + \frac{1}{p_2}\right)^2$$

$$\rightarrow A_0^2 = x^2 + 3x^2 = 4x^2$$

$$x^2 = \frac{A_0^2}{4} \Rightarrow \boxed{x = \frac{A_0}{2}} = 500$$

$$\omega_{u,loop}\left(\frac{1}{-P_1} + \frac{1}{-P_2}\right) = 500\sqrt{3} \quad \text{—(1)}$$

$$\omega_{u,loop} = \sqrt{501}\,\sqrt{P_1 P_2} \quad \text{—(2)}$$

$$P_1 = \frac{-g_{o1}}{C_1 + C_c\left(1 + \frac{g_{m2}}{g_{o2}}\right)} = -\frac{0.2\times10^{-6}}{10\times10^{-15} + C_c\left(1 + \frac{80\times10^{-6}}{8\times10^{-6}}\right)}$$

$$P_1 = -\frac{0.2\times10^{-6}}{10\times10^{-15} + 11\times C_c}$$

$$P_2 = -\left[\frac{g_{o2} + g_{m2}\frac{C_c}{C_1+C_c}}{C_2 + \frac{C_1 C_c}{C_1 + C_c}}\right]$$

$$= -\left[\frac{8\times10^{-6} + 80\times10^{-6}\times\frac{C_c}{10\times10^{-15}+C_c}}{1\times10^{-12} + \frac{10\times10^{-15}C_c}{10\times10^{-15}+C_c}}\right]$$

And from (1) and (2),

$$\sqrt{501}\left(\sqrt{\frac{P_2}{P_1}} + \sqrt{\frac{P_1}{P_2}}\right) = 500\sqrt{3}$$

$$\Rightarrow x + \frac{1}{x} = \frac{500\sqrt{3}}{\sqrt{501}}$$

$$\Rightarrow x^2 - \left(\frac{500\sqrt{3}}{\sqrt{501}}\right)x + 1 = 0 \Rightarrow r = 38.66 \text{ or } 0.025$$

$$P_1 = -\frac{0.2\times10^{-6}}{10\times10^{-15}+11\times C_c} \qquad P_2 \simeq -\left[\frac{8\times10^{-6}+80\times10^{-6}\frac{C_c}{10\times10^{-15}+C_c}}{10^{-12}}\right]$$

$$\left(\because 10^{-12} >> \frac{10\times10^{-15}\,C_c}{10\times10^{-15}+C_c}\right)$$

And $P_2 >> P_1$, from ratio, we get

$$\cancel{-}P_2 = (38.66)^2 \times \cancel{-}P_1$$

$$\frac{8\times10^{-6}+80\times10^{-6}\frac{C_c}{10\times10^{-15}+C_c}}{10^{-12}} = \frac{(38.66)^2\times0.2\times10^{-6}}{10\times10^{-15}+11\times C_c}$$

$C_c = c\times10^{-15}$, then

$$\frac{8+80\frac{c}{10+c}}{1000} = \frac{(38.66)^2\,0.2}{10+11c}$$

$$\Rightarrow \frac{80+8c+80c}{1000(10+c)} = \frac{(38.66)^2\,0.2}{10+11c}$$

$$\Rightarrow \frac{\cancel{8}^{4}(11c+10)}{\cancel{1000}_{25}(10+c)} = \frac{(38.66)^2}{11c+10}\,\frac{\cancel{2}}{\cancel{10}}$$

$$\Rightarrow (11c+10)^2 = (5\times38.66)^2(10+c)$$

$$\Rightarrow 121c^2+100+220c = 10\times(5\times38.66)^2 + (5\times38.66)^2 c$$

$121c^2 + (220 - (5\times38.66)^2)c + 100 - 10(5\times38.66)^2$

-371474.89 $\qquad -373548.9$

On solving

$\Rightarrow C = 316.73$

$\Rightarrow \boxed{C_c = 316.73\,fF}$

$\omega_{u,loop} = \sqrt{501} \times \sqrt{P_1P_2}$

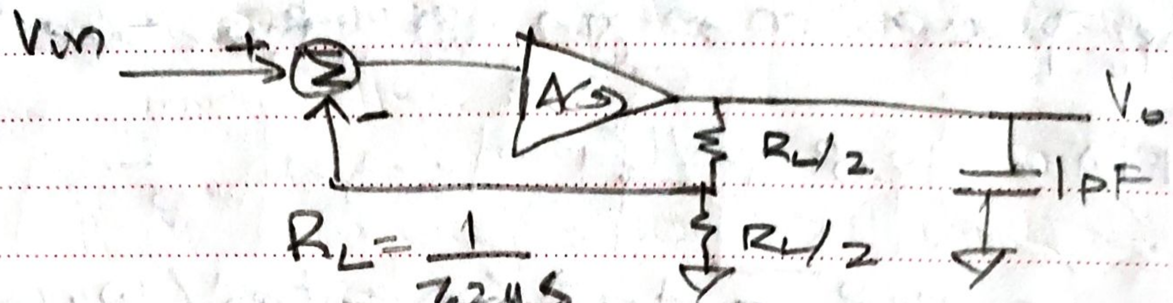
$= \sqrt{501} \times 38.66 \times P_1$

$= \sqrt{501} \times 38.66 \times \dfrac{0.2\times10^{-6}}{10\times10^{-15} + 11\times316.73\times10^{-15}}$

$= 49.53\,M\,rad/s$

$\approx 50\,M\,rad/s$

2)



$R_L = \frac{1}{7.24S}$

Here, $V_o = A(s)\left(V_{in} - \frac{V_o}{2}\right)$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{A(s)}{1 + \frac{A(s)}{2}}$$

This makes the open loop gain,

$$A(s) = \frac{A_0}{2} \cdot \frac{(1 - s/z_1)}{(1 - s/P_1)(1 - s/P_2)}$$

So only $A_0 \rightarrow \frac{A_0}{2}$, then

for $\omega_{u,loop}$:

$$|A(s)| = 1 \Rightarrow \frac{A_0^2}{4}\left(1 + \frac{\omega^2}{z_1^2}\right) = \left(1 + \frac{\omega^2}{P_1^2}\right)\left(1 + \frac{\omega^2}{P_2^2}\right)$$

$$\Rightarrow \frac{A_0^2}{4}\left(1 + \frac{\omega^2}{Z_1^2}\right) = 1 + \omega^2\left(\frac{1}{P_1^2} + \frac{1}{P_2^2}\right) + \frac{\omega^4}{P_1^2 P_2^2}$$

$$\Rightarrow \frac{\omega^4}{P_1^2 P_2^2} + \omega^2\left(\frac{1}{P_1^2} + \frac{1}{P_2^2} - \frac{A_0^2}{4Z_1^2}\right) + 1 - \frac{A_0^2}{4} = 0$$

$$\Rightarrow \omega^4 + \omega^2\left(P_1^2 + P_2^2 - \frac{A_0^2 P_1^2 P_2^2}{4Z_1^2}\right) + P_1^2 P_2^2\left(1 - \frac{A_0^2}{4}\right) = 0$$

$$\Rightarrow \omega^4 + \omega^2(3.776 \times 10^{7}) - 1.6075 \times 10^{34} = 0$$

$$\Rightarrow \omega^2 = 3.860 \times 10^{16} \Rightarrow \boxed{\omega_{u,loop} = 196.49 \text{ Mrad/s}}$$

The phase margin in this case is

$$PM = 180^\circ + \angle LG(s)\big|_{s=j\omega_{u,loop}}$$

$$= 180^\circ - \tan^{-1}\left(\frac{\omega_u}{z}\right) - \tan^{-1}\left(\frac{\omega_{u,loop}}{-P_1}\right) - \tan^{-1}\left(\frac{\omega_{u,loop}}{-P_2}\right)$$

$\omega_{u,loop} = 196.49$ Mrad/s

$-P_1 = 0.057$ M rad/s

$-P_2 = 85.28$ M rad/s

$\frac{g_{m2}}{C_C} \leftarrow z = 252.58$ Mrad/s

On calculating PM turns out to be

$$PM = -14.40^\circ$$

3) For new value of C_c, we should run through the loop of calculation in (1)-9 again with $A_0 \to \frac{A_0}{2}$

Here we have $x = 250$

$$\omega_{u,loop}\left(\frac{1}{-P_1} + \frac{1}{-P_2}\right) = 250\sqrt{3}$$

$$\omega_{u,loop} = \sqrt{251}\,\sqrt{P_1 P_2}$$

$$\Rightarrow \sqrt{\frac{P_2}{P_1}} + \sqrt{\frac{P_1}{P_2}} = \frac{250\sqrt{3}}{\sqrt{251}} \Rightarrow r^2 - \left(\frac{250\sqrt{3}}{\sqrt{251}}\right)r + 1 = 0$$

$$\Rightarrow r = 27.3 \;,\; (or)\; 0.0366$$

Then

$$P_1 = \frac{-0.2\times 10^{-6}}{10\times 10^{-15} + 11C_c} \qquad P_2 = (27.3)^2 P_1$$

Then the quadratic changes to be

$$0 = 121c^2 + (220 - (5\times 27.3)^2) + 100 - 10(5\times 27.3)^2$$

$$-18412.25 \qquad\qquad -186222.5$$

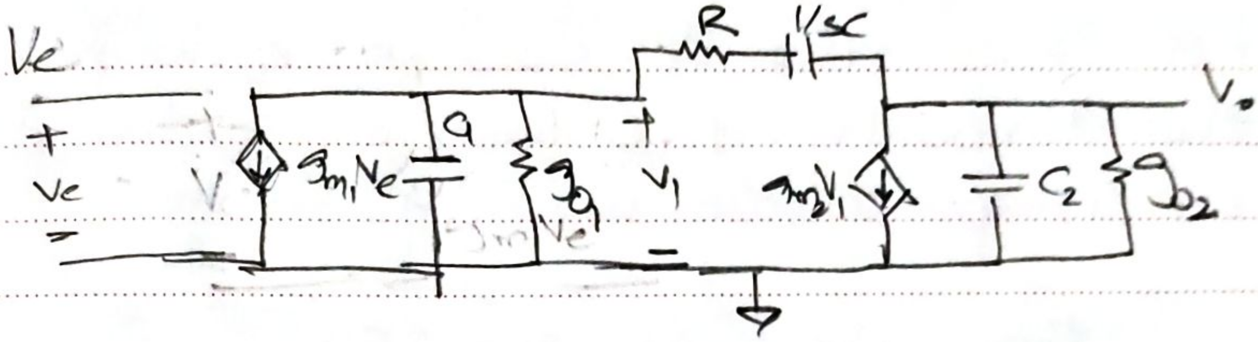
On solving, $c = 161.686$

$$\Rightarrow C_c = 161.686\,fF$$

On Calculating $\Rightarrow$ $\boxed{\omega_{u\,loop} = 48.364\,rad/s}$

$\hookrightarrow \sqrt{251}\times 27.3\times P_1$

4)



(1) → $g_{m_1} V_e - \frac{V_o sC}{1+sRC} = -V_1\left[\frac{sC}{1+sRC} + sC_1 + g_{o_1}\right]$

(2) → $V_o\left[\frac{sC}{1+sRC} + sC_2 + g_{o_2}\right] = V_1\left[\frac{sC}{1+sRC} - g_{m_2}\right]$

By replacing (1) & (2) with V_1,

$$\frac{\left[g_{m_1} V_e - \frac{V_o sC}{1+sRC}\right]}{\left[\frac{sC}{1+sRC} + sC_1 + g_{o_1}\right]} = \frac{-V_o\left[\frac{sC}{1+sRC} + sC_2 + g_{o_2}\right]}{\left[\frac{sC}{1+sRC} - g_{m_2}\right]}$$

$$\Rightarrow \left[\frac{g_{m_1} sC}{1+sRC} - g_{m_1} g_{m_2}\right]V_e - V_o\left[\frac{s^2C^2}{(1+sRC)^2} - \frac{sC g_{m_2}}{1+sRC}\right]$$

$$= -V_o\left[\frac{s^2C^2}{(1+sRC)^2} + \frac{s^2CC_2}{1+sRC} + \frac{s g_{o_2} C}{1+sRC} + \frac{s^2 C_1 C}{1+sRC} + s^2 C_1 C_2 + g_{o_2} sC_1 + \frac{sC g_{o_1}}{1+sRC} + sC_2 g_{o_1} + g_{o_1} g_{o_2}\right]$$

$$\left[\frac{g_{m_1} sC}{1+sRC} - g_{m_1} g_{m_2}\right] V_e = V_o\left[\frac{s^2 C(C_1+C_2)}{1+sRC} + s^2 C_1 C_2 - \frac{sCg_{m_2}}{1+sRC} + \frac{sC(g_{o_1}+g_{o_2})}{1+sRC} + s(C_1 g_{o_2} + C_2 g_{o_1}) + g_{o_1} g_{o_2}\right]$$

$$\Rightarrow \frac{V_o}{V_e} = \frac{g_{m_1} sC - g_{m_1} g_{m_2}(1+sRC)}{s^2 C(C_1+C_2) + s^2 C_1 C_2 (1+sRC) - sCg_{m_2} + sC(g_{o_1}+g_{o_2}) + s(1+sRC)(g_{o_2}C_1 + g_{o_1}C_2) + g_{o_1} g_{o_2}(1+sRC)}$$

$$\Rightarrow \boxed{\frac{V_o}{V_e} = \frac{sC(g_{m_1} - g_{m_1} g_{m_2} R) - g_{m_1} g_{m_2}}{\left[s^3 (RCC_1C_2) + s^2\left(C(C_1+C_2) + C_1C_2 + RC(g_{o_2}C_1 + g_{o_1}C_2)\right) + s\left(C(g_{o_1}+g_{o_2} - g_{m_2}) + g_{o_2}C_1 + g_{o_1}C_2 + g_{o_1}g_{o_2}RC\right) + (g_{o_1} g_{o_2})\right]}}$$

The zero is $s = \dfrac{g_{m_1} g_{m_2}}{C(g_{m_1} - g_{m_1} g_{m_2} R)}$

$$\boxed{Z = \frac{g_{m_2}}{C(1 - g_{m_2} R)}} \rightarrow \text{Sanity Check}$$

$R=0$ $\quad Z = \dfrac{g_{m_2}}{C}$

Here $g_{m_2} = 80\mu S$ And $R = 12.5K$

$$\Rightarrow g_{m_2} R = 80 \times 10^{-6} \times 12.5 \times 10^{-3}$$
$$= 8 \times 125 \times 10^{-3}$$
$$= 10^3 \times 10^{-3} = 1 \quad \text{then } Z \to \infty$$

Then
$$\frac{V_o}{V_e} = \frac{-\dfrac{g_{m_1} g_{m_2}}{g_{o_1} g_{o_2}}}{s^3 \dfrac{RCC_1C_2}{g_{o_1}g_{o_2}} + \dfrac{s^2}{g_{o_1}g_{o_2}}\left(C(C_1+C_2) + C_1C_2 + RC(g_{o_2}C_1 + g_{o_1}C_2)\right) + \dfrac{s}{g_{o_1}g_{o_2}}\left(C(g_{o_1}+g_{o_2}-g_{m_2}) + g_{o_2}C_1 + g_{o_1}C_2 + g_{o_1}g_{o_2}RC\right) + 1}$$

The dc gain remains the same, but the polynomial now is a cubic. Coefficient of $\dfrac{RCC_1C_2}{g_{o_1}g_{o_2}}$

$$\Rightarrow \frac{12.5k \times C \times 10 \times 10^{-15} \times 10^{-12}}{0.2 \times 10^{-6} \times 8 \times 10^{-6}}$$

$$\Rightarrow \frac{125}{16} \times 10 \times 10^{-12} \times C,$$ then $C \sim 1pF - 1\mu F$ s^3 coefficient very very small, hence we can ignore it.

Then

$$\frac{V_o}{V_e} = \frac{\dfrac{-g_{m_1} g_{m_2}}{g_{o_1} g_{o_2}}}{\dfrac{s^2}{g_{o_1}g_{o_2}}\left(C(C_1+C_2)+C_1C_2+RC(g_{o_2}C_1+g_{o_1}C_2)\right)+1+\dfrac{s}{g_{o_1}g_{o_2}}\left(C(g_{o_1}+g_{o_2}-g_{m_2})+g_{o_2}C_1+g_{o_1}C_2+g_{o_1}g_{o_2}RC\right)}$$

$P_1 P_2 \xleftarrow{\text{poles}}$

Considering $P_2 >> P_1$

$$P_2 = -\frac{b}{a} \qquad P_1 = -\frac{c}{b}$$

$$\Rightarrow P_2 = -\frac{C(g_{o_1}+g_{o_2}-g_{m_2})+g_{o_2}C_1+g_{o_1}C_2+g_{o_1}g_{o_2}RC}{C(C_1+C_2)+C_1C_2+RC(g_{o_2}C_1+g_{o_1}C_2)}$$

$$\Rightarrow P_1 = \frac{-g_{o_1}\ g_{o_2}}{C(g_{o_1}+g_{o_2}-g_{m_2})+g_{o_2}C_1+g_{o_1}C_2+g_{o_1}g_{o_2}RC}$$

$$= \frac{-g_{o_1}}{C_1+C\left(1+\dfrac{g_{o_1}}{g_{o_2}}+\dfrac{g_{m_2}}{g_{o_2}}\right)+\underbrace{\dfrac{g_{o_1}C_2}{g_{o_2}}}_{<<1}+g_{o_1}RC}$$

Then approximation gives

$$\boxed{P_1 = \frac{-g_{o_1}}{C_1+C\left(1+\dfrac{g_{m_2}}{g_{o_2}}\right)+g_{o_1}RC}}$$

$g_{o_1}R$

$= 0.2\mu S \times 12.5K$

$= \frac{2}{10} \times \frac{125}{10} \times 10^{-3}$

$= 2.5 \times 10^{-3}$

Similarly for P_2, we get

$g_{o1} R = 2.5\times10^{-3}$

$g_{o2} R = 8\times10^{-3} \times 12.5\times10^{-3}$ (?)

$= 0.1$

$$P_2 = -\left[\frac{g_{o2} + g_{m2}\dfrac{C}{C+C_1+g_{o1}RC}}{C_2 + \dfrac{C_1 C(1+Rg_{o2})}{C+C_1+RCg_{o1}}}\right]$$

Now redoing 1st question

(a) We are good till $P_2 = (38.66)^2 P_1$

$$P_1 = \frac{-0.2\times10^{-16}}{10\times10^{-15} + \underbrace{(1.1+2.5\times10^{-3})}_{\sim 1.1} C} \approx \frac{-0.2\times10^{-16}}{10\times10^{-15} + 1.1C}$$

→ original approximated eqn

$$P_2 = -\left[\frac{8\times10^{-6} + 80\times10^{-6}\dfrac{C}{9+(1+0.25\times10^{-3})C}}{1\times10^{-12} + \dfrac{10\times10^{-15}C\times1.1}{1.1C+10\times10^{-15}}}\right]$$

$$\approx -\left[\frac{8\times10^{-6} + 80\times10^{-6}\dfrac{C}{9+C}}{\underline{1\times10^{-12}} + \dfrac{10\times10^{-15}C}{C+\cancel{10\times10^{-15}}}}\right]$$

→ same as original approx equation

Hence based on the approximations, we can see that the poles position nearly remains unchanged. Hence

a) $C_c \simeq 316.73 fF$ $\omega_{u,loop} \sim 50$ Mrad/s

b) $C_c \simeq 161.686 fF$ $\omega_{u,loop} \sim 48.364$ Mrad/s

These also remain the same.