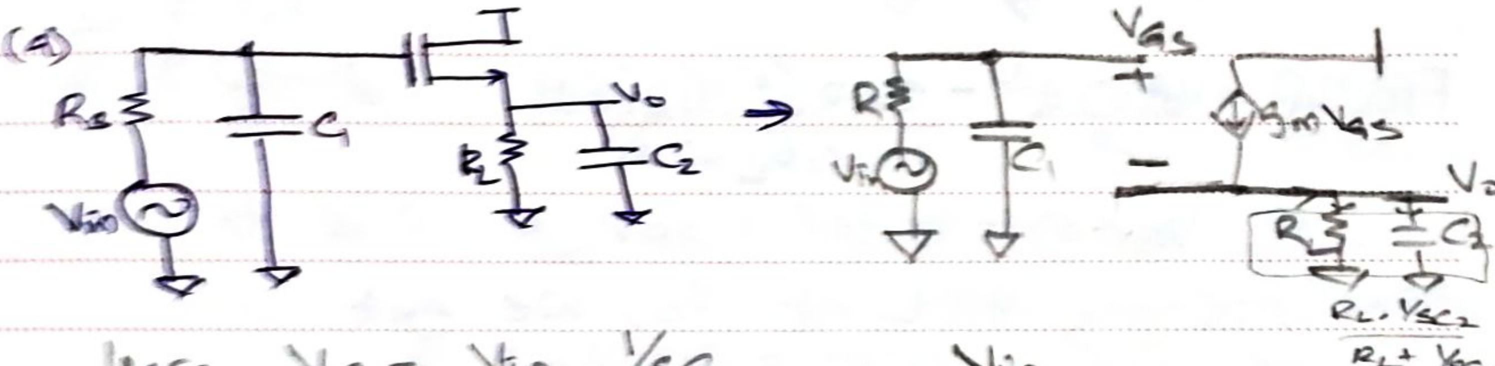
Analog Assignment - 6

1) We need $\frac{V_o(s)}{V_{in}(s)}$

(a)



here, $V_G = V_{in} \frac{1/sC_1}{R + 1/sC_1} = \frac{V_{in}}{1+sRC_1}$

And $V_S = V_o$, hence from the source termin

$\Rightarrow V_o(s) = I(s)\, Z(s) = g_m V_{GS}(s) \frac{R_L \cdot 1/sC_2}{R_L + 1/sC_2}$

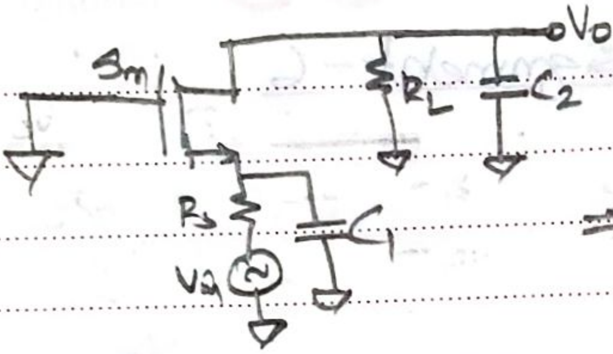
$\Rightarrow V_o(s) = g_m V_{GS}(s) \frac{R_L}{1+sC_2R_L} = g_m (V_G - V_S) \frac{R_L}{1+sC_2R_L}$

$\Rightarrow V_o(s)\left[1 + g_m \frac{R_L}{1+sC_2R_L}\right] = g_m \cdot \frac{V_{in}(s)}{1+sRC_1} \cdot \frac{R_L}{1+sC_2R_L}$

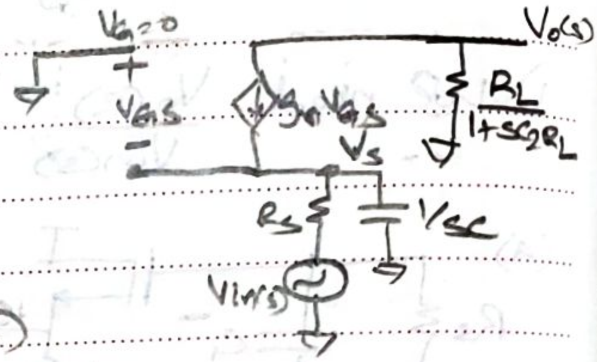
$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{g_m R_L / (1+sRC_1)(1+sR_LC_2)}{1 + g_m R_L/(1+sC_2R_L)}$

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{g_m R_L}{(1+sRC_1)(1+sR_LC_2) + g_m R_L (1+sRC_1)}}$$

1) b)



$\Rightarrow$



Firstly $V_o(s) = -\frac{g_m R_L}{1+sC_2R_L} \cdot V_{GS}(s)$

$V_G(s) = 0$

And applying KCL at V_s, we get

$$g_m V_{GS}(s) = \frac{V_s(s) - V_{in}(s)}{R_s} + \frac{V_s(s)}{1/sC}$$

$$-g_m V_s(s) = V_s(s)\left[\frac{1}{R_s} + sC\right] - \frac{V_{in}(s)}{R_s}$$

$$\Rightarrow \frac{V_{in}(s)}{R_s} = V_s(s)\left[\frac{1}{R_s} + sC + g_m\right]$$

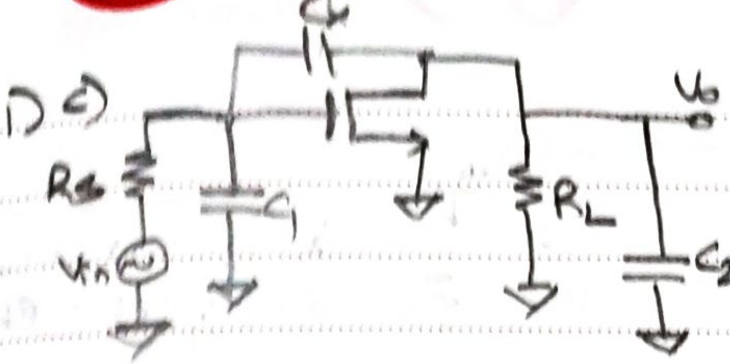
$$\Rightarrow V_s(s) = \frac{V_{in}(s)}{1 + sCR_s + g_m R_s}$$

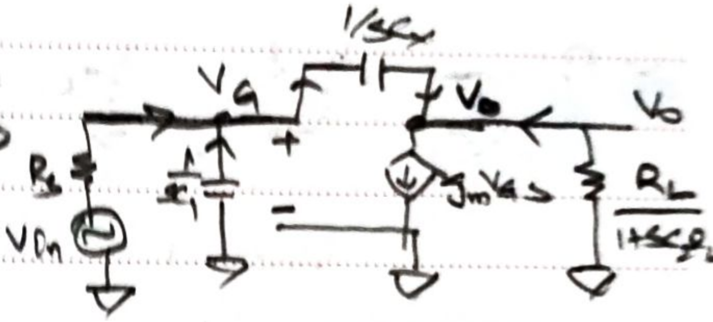
Then, $V_o(s) = \cancel{-}\frac{g_m R_L}{1+sC_2R_L} \cdot \cancel{-} V_s(s)$

$$= \frac{g_m R_L}{1+sC_2R_L} \cdot \frac{V_{in}(s)}{1+sCR_s+g_mR_s}$$

$$\Rightarrow \boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{g_m R_L}{(1+sC_2R_L)(1+sCR_s+g_mR_s)}}$$

1) c)





KCL at V_G: $$\frac{V_G(s)-V_o(s)}{1/sC_x} = \frac{V_{in}-V_G}{R} + \frac{0-V_G}{1/sC_1}$$

KCL at V_o: $$g_m V_{GS} = \frac{V_G(s)-V_o(s)}{1/sC_x} + \frac{V_o}{R_L/1+sC_2R_L}$$

$$\Rightarrow g_m V_G(s) = \frac{V_G(s)}{1/sC_x} - V_o(s)\left[sC_x - \frac{(1+sC_2R_L)}{R_L}\right]$$

$$\Rightarrow V_o(s)\left[sC_x - \frac{(1+sC_2R_L)}{R_L}\right] = V_G(s)\left(sC_x - g_m\right)$$

$$\Rightarrow V_o(s)\left[\frac{sC_xR_L - (1+sC_2R_L)}{sC_xR_L - g_mR_L}\right] = V_G(s)$$

Plug to (1),

$$V_G(s)\left[sC_x + \frac{1}{R} + sC_1\right] = \frac{V_{in}}{R} + V_o(s)\, sC_x$$

$$\Rightarrow V_o(s)\,\frac{(sC_xR_L-(1+sC_2R_L))}{(sC_xR_L - g_mR_L)}\,[1+s(C_1+C_x)R] = V_{in} + V_o(s)sC_xR$$

$$\Rightarrow V_o(s)\left[\frac{(s(C_x-C_2)R_L-1)(1+s(C_1+C_x)R)}{sC_xR_L - g_mR_L} - sC_xR\right] = V_{in}(s)$$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{sC_xR_L - g_mR_L}{s(C_x-C_2)R_L - 1 + s^2(C_x-C_2)(C_x+C_1)RR_L - s(C_1+C_x)R - s^2C_x^2RR_L + sC_xg_mRR_L}$$

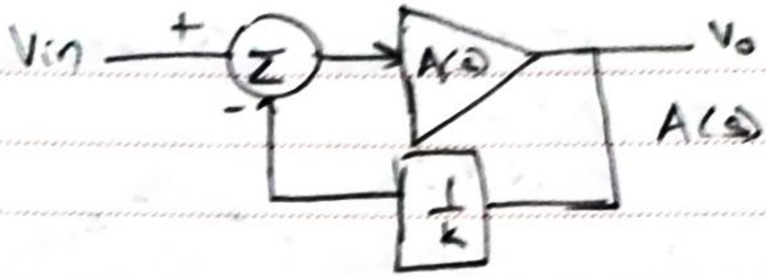
$$= \frac{sC_xR_L - g_mR_L}{-1 + s\left[(C_x-C_2)R_L - (C_1+C_x)R + C_xg_mRR_L\right] + s^2\left[RR_L(C_x-C_2)(C_x+C_1) - C_x^2RR_L\right]}$$

$$= \frac{sC_xR_L - g_mR_L}{-1 + s\left[C_x(R_L - R + g_mRR_L) - C_1R - C_2R\right] + s^2\left[RR_L\cancel{C_x^2} - RR_LC_x(C_1-C_2) - C_1C_2 - \cancel{C_x^2RR_L}\right]}$$

$$= \frac{sC_xR_L - g_mR_L}{-1 + s\left[C_x(R_L-R+g_mRR_L) - C_1R - C_2R\right] - s^2\left[RR_LC_x(C_1-C_2) + C_1C_2\right]}$$

$$\Rightarrow \boxed{\frac{V_o(s)}{V_{in}(s)} = \frac{g_mR_L - sC_xR_L}{1 + s^2\left[RR_LC_x(C_1-C_2) + C_1C_2\right] - s\left(C_x(R_L - R + g_mRR_L) - C_1R - C_2R\right)}}$$

2a)



$$A(s) = \frac{A_0\,(1+s/z)}{(1+\frac{s}{P_1})(1+\frac{s}{P_2})}$$

$A_0 = 10^4 \quad P_1 = 100\ \text{rad/s} \quad K = 10$

a) $z \to \infty$ $\quad A(s) = \dfrac{A_0}{(1+s/P_1)(1+\frac{s}{P_2})}$ $\qquad LG(s) = \dfrac{A_0/K}{(1+\frac{s}{P_1})(1+\frac{s}{P_2})}$

At gain cross over freq, P.M = 60°

$$\Rightarrow 60^\circ = 180^\circ + \angle LG(s)\Big|_{s@j\omega_u}$$

$$\Rightarrow -120^\circ = -\tan^{-1}\left(\frac{\omega_u}{P_1}\right) + \tan^{-1}\left(\frac{\omega_u}{P_2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\omega_u}{P_1} + \frac{\omega_u}{P_2}}{1-\frac{\omega_u^2}{P_1P_2}}\right)$$

$$\Rightarrow \frac{\frac{\omega_u}{P_1} + \frac{\omega_u}{P_2}}{1-\frac{\omega_u^2}{P_1P_2}} = -\sqrt{3}, \quad \text{let } \frac{\omega_u}{P_1} + \frac{\omega_u}{P_2} = \sqrt{3}\,x$$

$$1 - \frac{\omega_u^2}{P_1P_2} = -x$$

$$1 + x = \frac{\omega_u^2}{P_1P_2}$$

At gain crossover frequency, gain = 0

$$\Rightarrow \frac{A_0^2}{k^2} = \left|1 + s\left(\frac{1}{P_1}+\frac{1}{P_2}\right) + \frac{s^2}{P_1P_2}\right|_{s=j\omega_u}$$

$$= \left|1 - \frac{\omega_u^2}{P_1P_2} + j\omega_u\left(\frac{1}{P_1}+\frac{1}{P_2}\right)\right|$$

$$= \left(1-\frac{\omega_u^2}{P_1P_2}\right)^2 + \omega_u^2\left(\frac{1}{P_1}+\frac{1}{P_2}\right)^2$$

$$= x^2 + 3x^2 \Rightarrow x^2 = \frac{A_0^2}{4k^2}$$

Here $A_0 = 10^4$ $k = 10$ $\Rightarrow \boxed{x = \frac{A_0}{2k}}$ (ω_u, P_1, P_2 are +ve)

$$\Rightarrow x = \frac{10^{\cancel{4}3}}{2\times 1\cancel{0}} = 500$$

Hence $\omega_u\left(\frac{1}{P_1}+\frac{1}{P_2}\right) = \sqrt{3}\times 500$

and $\frac{\omega_u^2}{P_1P_2} = 501 \Rightarrow \omega_u^2 = 501\, P_1P_2$

$$\Rightarrow \omega_u = \sqrt{501}\,\sqrt{P_1P_2}$$

Then $\sqrt{501}\,\sqrt{P_1P_2}\left(\frac{1}{P_1}+\frac{1}{P_2}\right) = \sqrt{3}\times 500$

$$\Rightarrow \sqrt{\frac{P_2}{P_1}} + \sqrt{\frac{P_1}{P_2}} = \frac{\sqrt{3}\times 500}{\sqrt{501}}$$

Let $\sqrt{\frac{P_2}{P_1}}$ be some n, then

$$x + \frac{1}{x} = \frac{\sqrt{3} \times 500}{\sqrt{501}}$$

$$x^2 - \frac{\sqrt{3} \times 500}{\sqrt{501}} x + 1 = 0 \quad x^2 - 38.69119x + 1 = 0$$

On solving, $x = 38.6652$ (or) 0.0258

$$\Rightarrow \sqrt{\frac{P_2}{P_1}} = x \Rightarrow P_2 = P_1 x^2$$

$$= 149500.055 \text{ (or) } 0.06689$$

$$\therefore \boxed{P_2 = 149.5k \text{ (or) } 66.89m} \text{ rad/s}$$

Then damping factor would be

$$\xi = \frac{1}{2}\left(\sqrt{\frac{P_1}{P_2}} + \sqrt{\frac{P_2}{P_1}}\right)\sqrt{\frac{K}{A+K}} = \frac{1}{2}\left(x + \frac{1}{x}\right)\sqrt{\frac{10}{10^4+10}}$$

$$= \frac{1}{2} \times \frac{\sqrt{3} \times 500}{\sqrt{501}} \times \sqrt{\frac{1\cancel{0}}{1001\cancel{0}}} = 250 \times \sqrt{\frac{3}{501 \times 1001}}$$

$$= 0.6145$$

$\rightarrow$ Underdamped System (For both cases of P_2)

For the found P_2, the peaking happens as

$\omega_p = \omega_n\sqrt{1-2\xi^2}$ and $\omega_n = \sqrt{\frac{A+K}{K}} \cdot \sqrt{P_1 P_2}$

Hence, for CLG, peaking happens at

i) $P_1 = 100$ $P_2 = 149.5K$ $\xi = 0.61145$

$\omega_n = 122.33K \Rightarrow \boxed{\omega_p = 61.442K}$

ii) $P_1 = 100$ $P_2 = 66.89m$ $\xi = 0.61145$

$\omega_n = 81.827 \rightarrow \boxed{\omega_p = 41.097}$

And the corresponding peaking,

$$CLG(s) = \frac{K}{1 + 1/LG(s)} = \frac{K}{1 + K/A(s)} = \frac{K}{1 + \frac{K(1+s/P_1)(1+s/P_2)}{A_0}}$$

At $s = j\omega$, $$= \frac{K}{1 + \frac{K(1+j\omega/P_1)(1+j\omega/P_2)}{A_0}}$$

By finding the magnitude at the corresponding peaking frequency found, we get

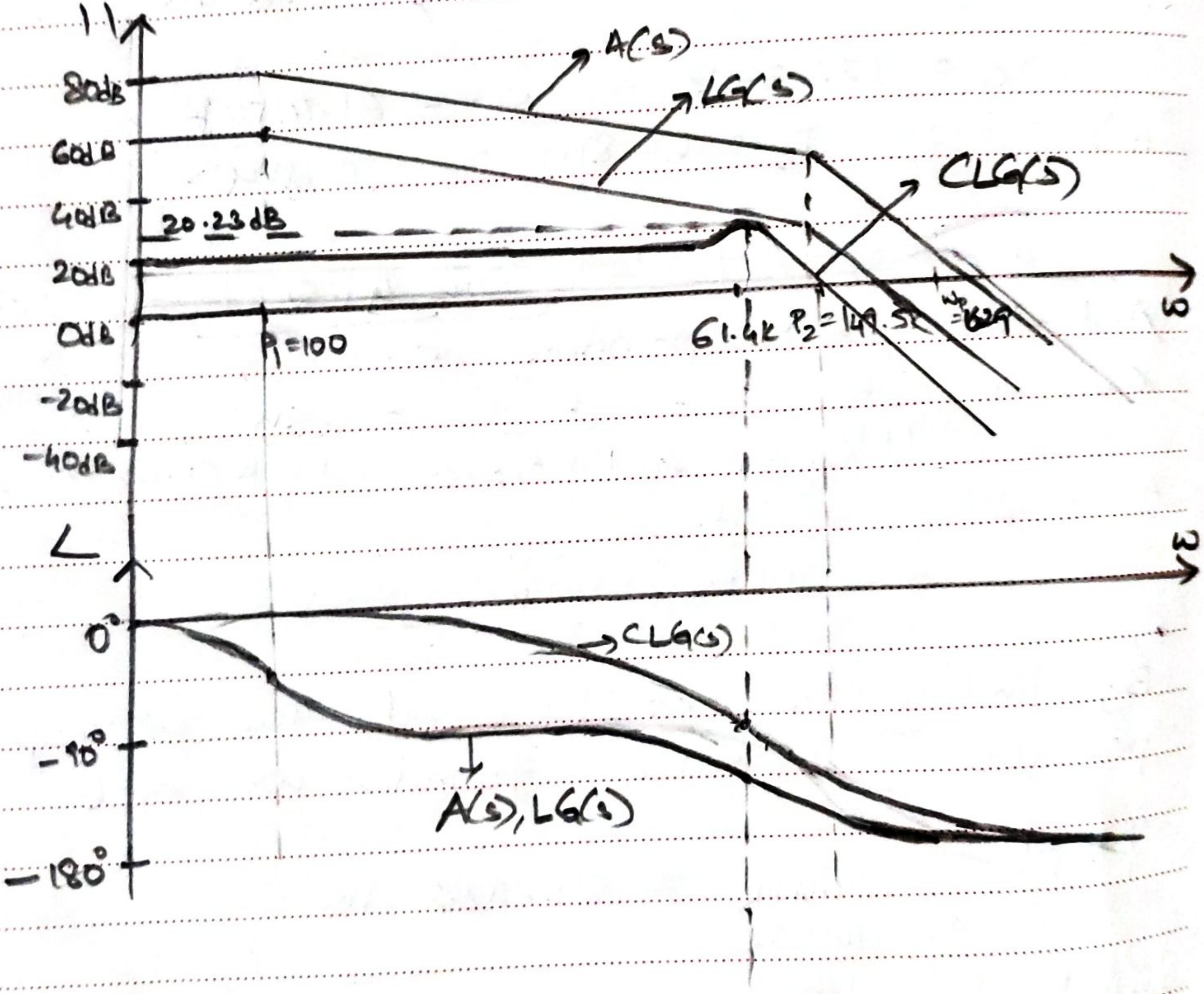
$|T.F|_{@\omega_p = 61.44K,\ P_2 = 149.5K} = 20.276\text{ dB}$

Similarly for

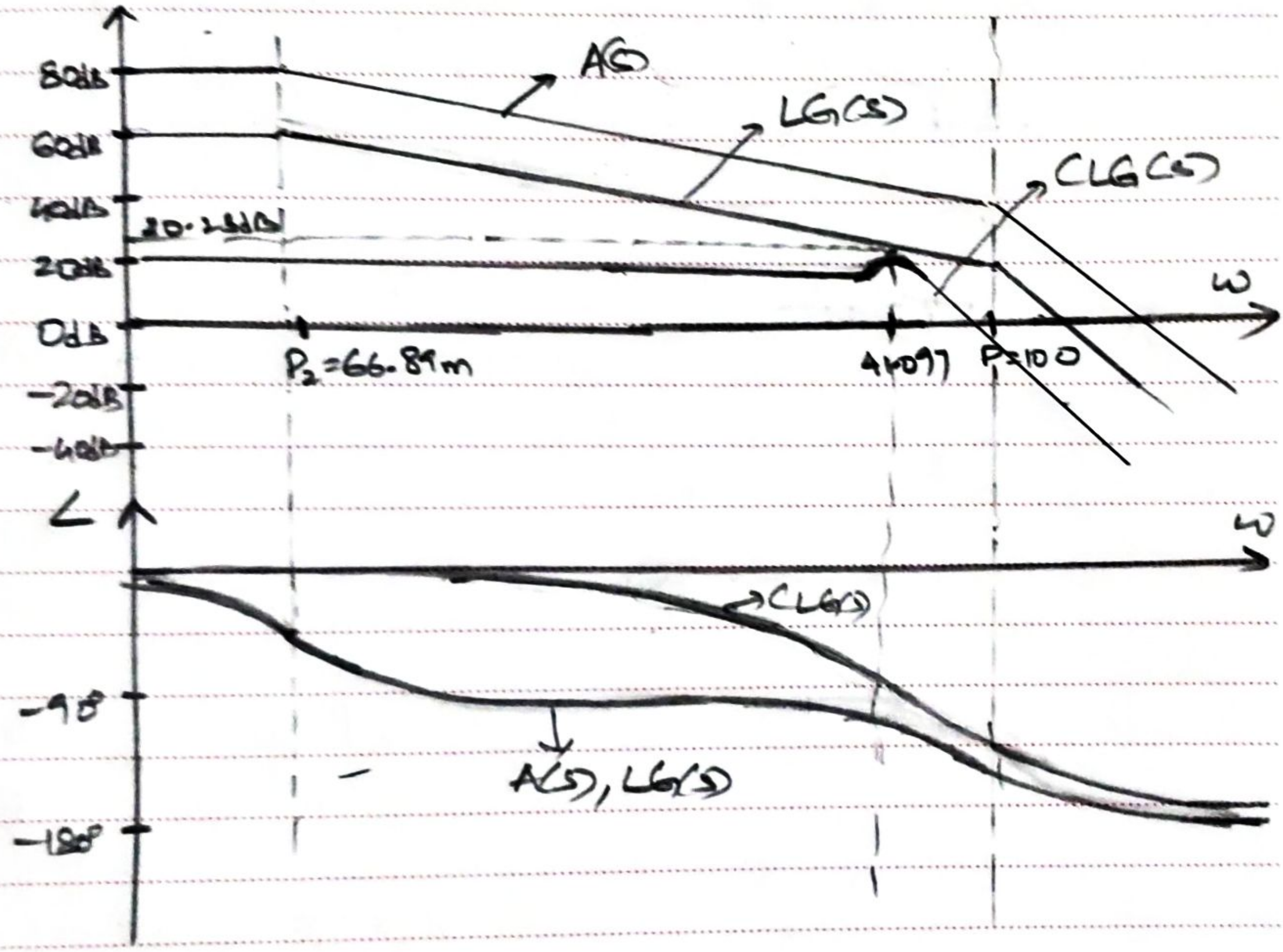
$|T.F|_{@\omega = 81.827,\ P_2 = 66.89m} = 20.276\text{ dB}$

→ Same Peaking

Bode Plot : When $P_2 = 149.5k$ rad/s



Bode Plot : When $P_2 = 66.89m$ rad/s



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2) b) Phase Margin when

i) $Z = P_2$ $\quad A(s) = \dfrac{A_0\cancel{(1+s/P_2)}}{\left(1+\frac{s}{P_1}\right)\cancel{\left(1+\frac{s}{P_2}\right)}} = \dfrac{A_0}{1+\frac{s}{P_1}}$

Gain Crossover frequency is:

$$A_0^2 = 1 + \frac{\omega^2}{P_1^2} \Rightarrow \omega_u = P_1\sqrt{A_0^2-1}$$

Phase Margin: $PM = 180 + \angle LG(s)$

$$= 180 - \tan^{-1}\left(\frac{\omega}{P_1}\right)$$

$$= 180^\circ - \tan^{-1}\left(\sqrt{A_0^2-1}\right)$$

$$= 180^\circ - \tan^{-1}\left(\underbrace{\sqrt{10^8-1}}_{\sim 10^4}\right)$$

$$= 90.005 \sim 90^\circ.$$

ii) $Z = -P_2$ $\quad A(s) = \dfrac{A_0(1-s/P_2)}{\left(1+\frac{s}{P_1}\right)\left(1+\frac{s}{P_2}\right)}$

Gain Crossover frequency:

$$\left(1+\frac{\omega^2}{P_1^2}\right)\cancel{\left(1+\frac{\omega^2}{P_2^2}\right)} = A_0^2\cancel{\left(1+\frac{\omega^2}{P_2^2}\right)}$$

$$\omega_u = P_1 \sqrt{A_0^2 - 1}$$

Phase Margin: $PM = 180 + \angle G(s)$

$$= 180 + -2\tan^{-1}\frac{\omega}{P_2} - \tan^{-1}\frac{\omega}{P_1}$$

$$= 180 - 2\tan^{-1}\frac{P_1}{P_2}\sqrt{A_0^2 - 1} - \tan^{-1}\left(\sqrt{A_0^2-1}\right)$$

$$\simeq 90 - 2\tan^{-1}(10^4)*\frac{P_1}{P_2}$$

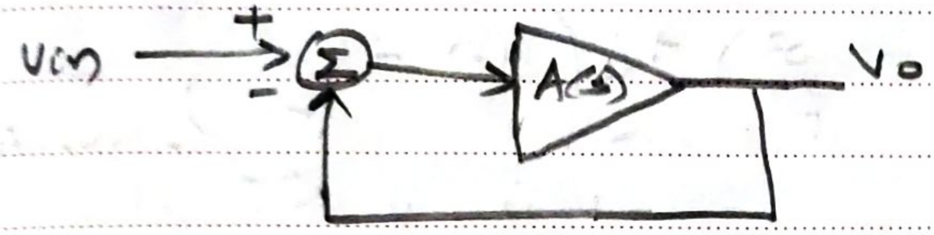
Here there are two cases:

$P_1 = 100$ $\quad$ $P_2 = 66.89m$ (or) $149.505K$

$$\therefore PM_1 = 90 - 2\tan^{-1}\left(10^4 \times \frac{100}{66.898}\right) = -89.99° \approx -90°$$

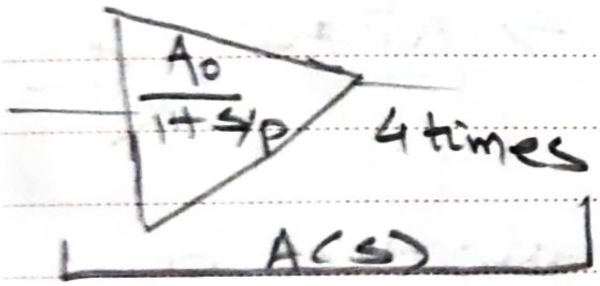
$$PM_2 = 90 - 2\tan^{-1}\left(10^4 \times \frac{100}{149.505 \times 10^3}\right) = -72.99° \approx -73°$$

3)



$$V_o(s) = A(s)\left(V_{in}(s) - V_o(s)\right)$$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{A(s)}{1 + A(s)} = \frac{1}{1 + \frac{1}{A(s)}}$$



$$A(s) = \left(\frac{A_o}{1 + \frac{s}{p}}\right)^4$$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + \left(\frac{1 + s/p}{A_o}\right)^4}$$

Now, for the stability of the system, the denominator $1 + A(s)$ must have no pole in RHP. To find the maximum of such A_o, we consider a marginal case where poles are on imaginary axis. Then

$$1 + A(s) = 0 \,\big|_{s = j\omega}$$

$$\Rightarrow 1 + \left(\frac{A_o}{1 + s/p}\right)^4 = 0 \,\Big|_{s = j\omega}$$

$$\Rightarrow \left(1 + \frac{j\omega}{p}\right)^4 + A_o^4 = 0$$

$$\Rightarrow 1 + 4j\frac{\omega}{p} - \frac{6\omega^2}{p^2} - 4j\frac{\omega^3}{p^3} + \frac{\omega^4}{p^4} + A_o^4 = 0$$

$$\underbrace{\left(1+A_0^4+\frac{\omega^4}{p^4}-\frac{6\omega^2}{p^2}\right)}_{\Downarrow} + j\underbrace{\left(\frac{4\omega}{p}-\frac{4\omega^3}{p^3}\right)}_{\Downarrow} = 0$$

$$\omega = 0,\ \omega = \pm p$$

$$1+A_0^4+\frac{\omega^4}{p^4}-\frac{6\omega^2}{p^2} = 0$$

At $\omega = \pm p$, $1+A_0^4+1-6=0 \Rightarrow A_0^4 = 4$

$$\Rightarrow \boxed{A_0 = \sqrt{2}}$$

Hence for the closed loop system to be stable $A_0 > \sqrt{2}$. (For $A_0 = \sqrt{2}$, system is marginally stable, consider unstable for practical purposes).