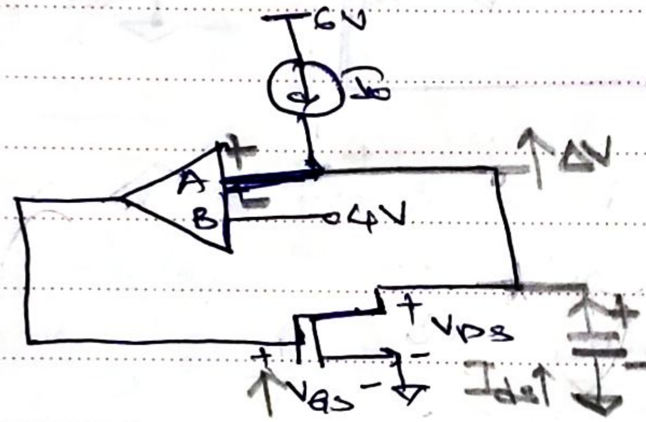
03 FRIDAY MAY 2019

Assignment - 5

1) Given $\mu_n C_{ox} = 100\mu A/V^2$ $\frac{W}{L} = 1$ $V_T = 1V$ $I_o = 200\mu A$

$$\beta = \mu_n C_{ox} \frac{W}{L} = 100\,\mu A/V^2$$

(a)



Sense : Drain

Control : Gate

Now lets assume A(+) and B(−). For finding right feedback, lets slightly increment the sensing voltage by ΔV at V_D. If the feedback is −ve, this V_D has to re-established.

When $V_D \rightarrow V_D + \Delta V$, $V_G \rightarrow V_G + A\Delta V$, when $V_G\uparrow$, $V_{GS}\uparrow$ and so does I_{ds}. Hence $I_{ds} > I_o$, this residual current has to be supplied by the parasthetic Capacitance, there by reducing its voltage, which makes $V_D\downarrow$. Hence, for given sign Configuration, The feedback is −ve i.e A(+ve); B(−ve).

Then,

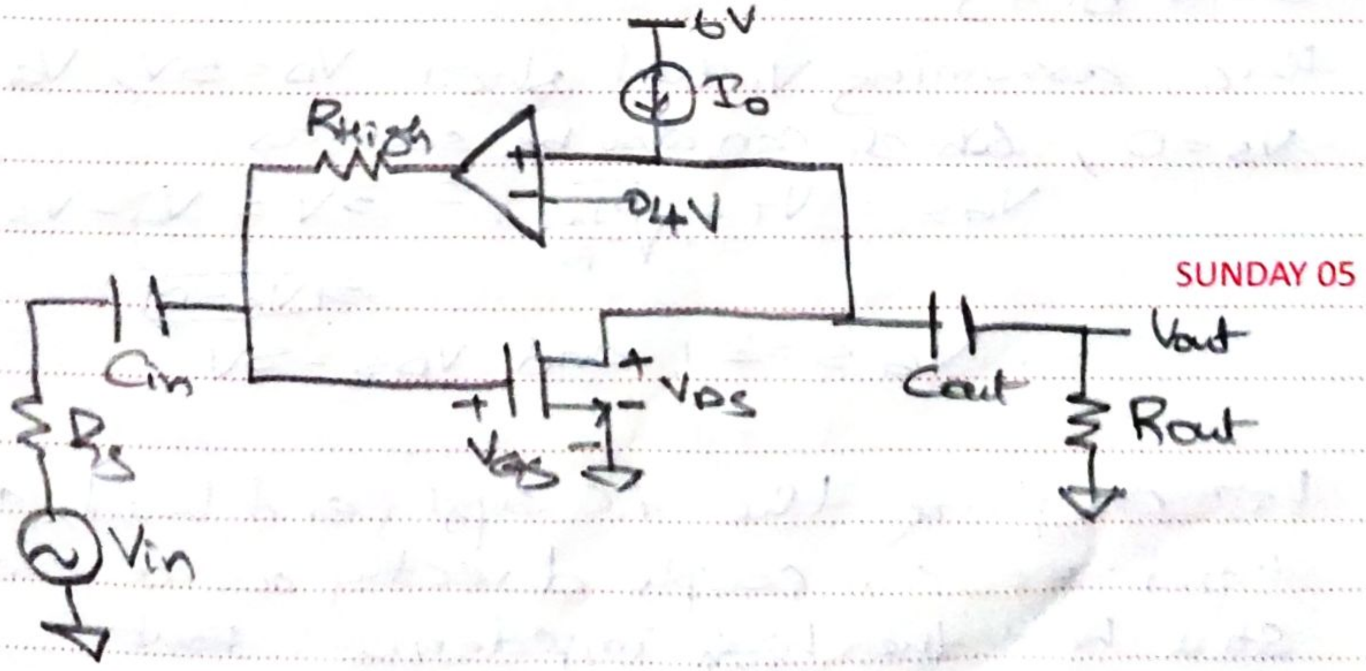
$$V_{DS} = 4V \quad \text{--- (Virtual short)}$$

And $I_{ds} = \frac{\beta}{2} (V_{GS} - V_T)^2$

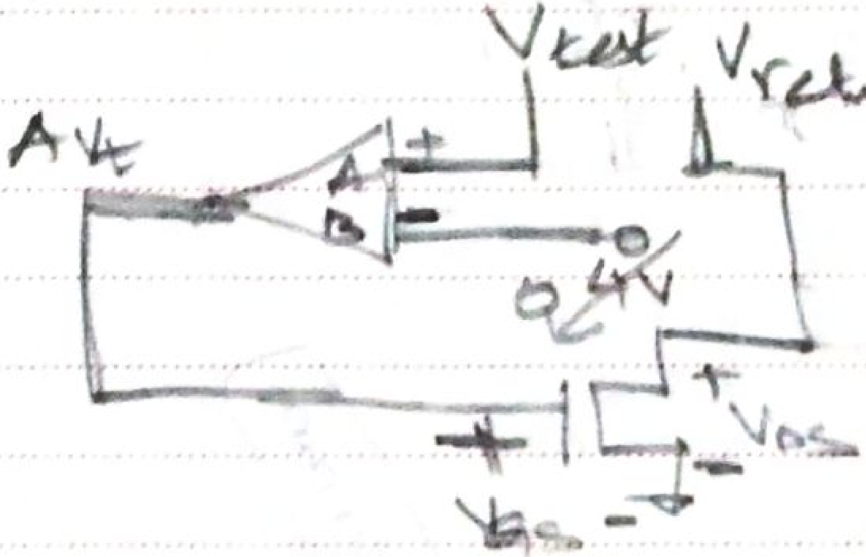
$$\Rightarrow V_{GS} = V_T + \sqrt{\frac{2 I_{ds}}{\beta}} = 1 + \sqrt{\frac{2 \times 200\mu}{100\mu}} = 3V$$

$\therefore$ $V_{GS} = 3V$ and $V_{DS} = 4V$

Now for using it as common source amplifier, the output can be extracted from drain, as op amp has high input impedance, all current flows through the MOS. But for input, a series high resistance is needed to be connected with op amp to prevent input current flow through opamp, not offering high input impedance as shown.



Aliter: Finding Feedback by cutting loop



cut at input of opamp

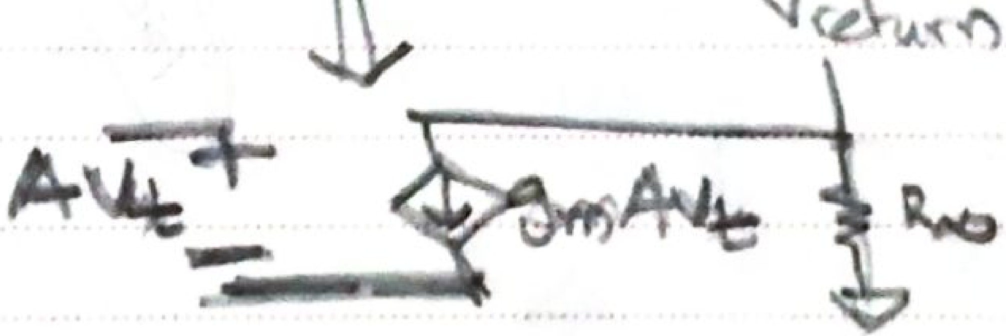
Here,

$V_{return} = -R_o I_d$

$V_{return} = -R_o \cdot g_m A V_t$

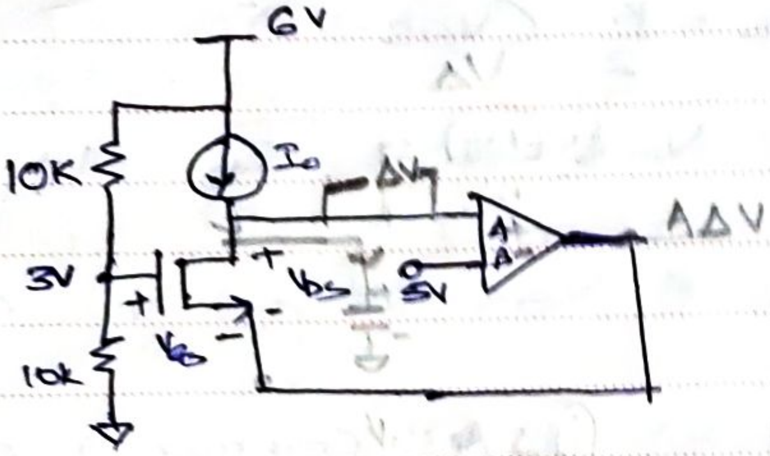
$= -\infty$

$-ve$



∴ This sign of AB gives −ve fb

b)



Sense: Drain
Control: Source

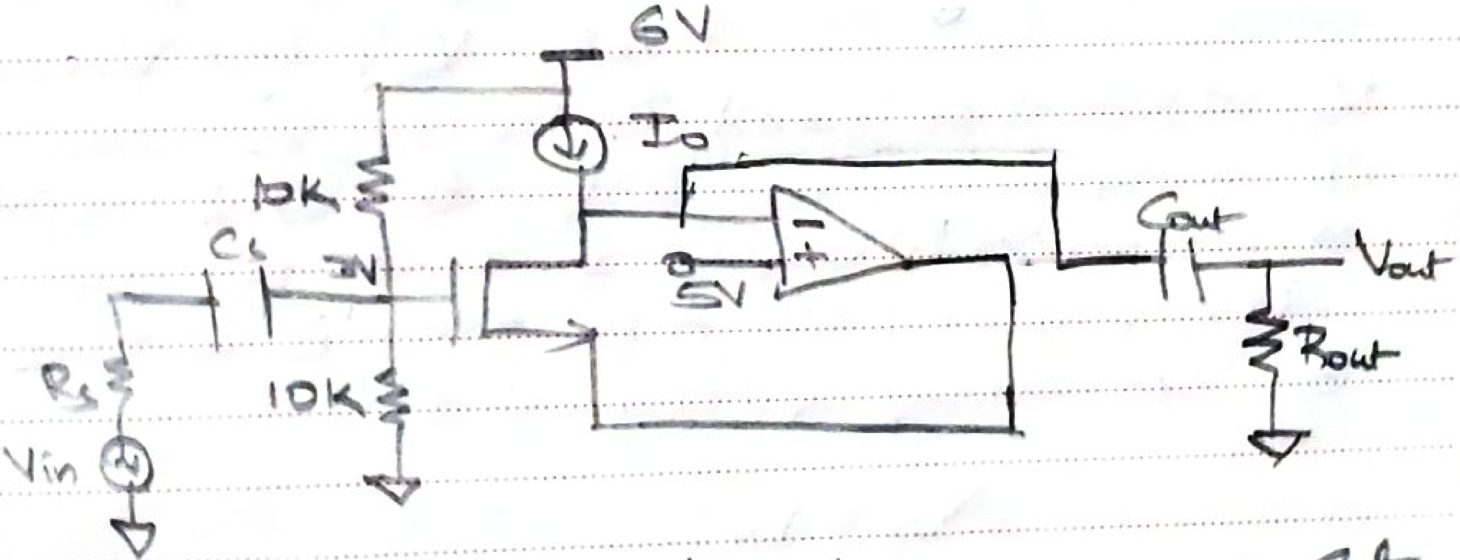
Let assume A(+) and B(−). A small ΔV perturbed at V_D. Then V_S will increase by $A\Delta V$. When $V_S \uparrow$, $V_{GS} \downarrow$ and so does $I_{ds} \downarrow$. Here $I_o > I_{ds}$ so the rest current flows through the cap. thereby even increasing the V_D, hence is serving as +ve feedback. For −ve feedback we have to switch the terminals i.e A(−) and B(+).

Here, assuming virtual short $V_D = 5V$, $V_G = 3V$ $V_S = 0$, ~~which~~ which can also be seen as

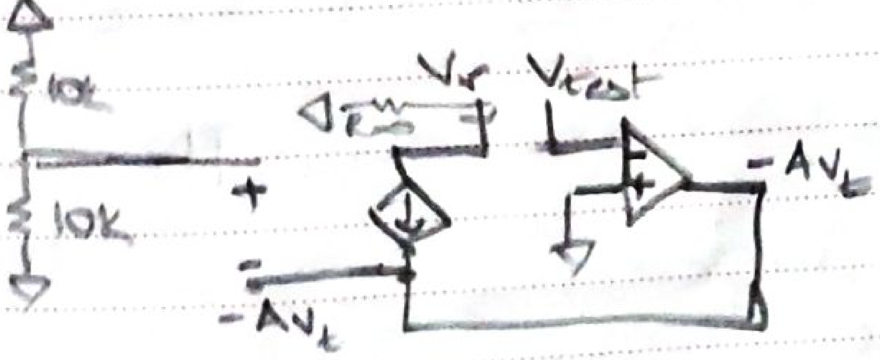
$$V_{GS} = V_T + \sqrt{\frac{2I_{ds}}{\beta}} = 3V = \underset{3}{V_G} - V_s$$
$$\Rightarrow (V_s = 0)$$

$\therefore V_{GS} = 3V$ and $V_{DS} = 5V$

For coupling the AC supply and load, at input we can couple directly as we can still have the high impedence. But output, the current from op-amp interferes with the output's current. To limit the current from op-amp, we can add a high resistance compared to output as shown.



Aliter: For finding the f.b, we can cut loop at input of op-amp.



Hence,

$V_r = -R_{\infty} \cdot AV_t$

$= -\infty$

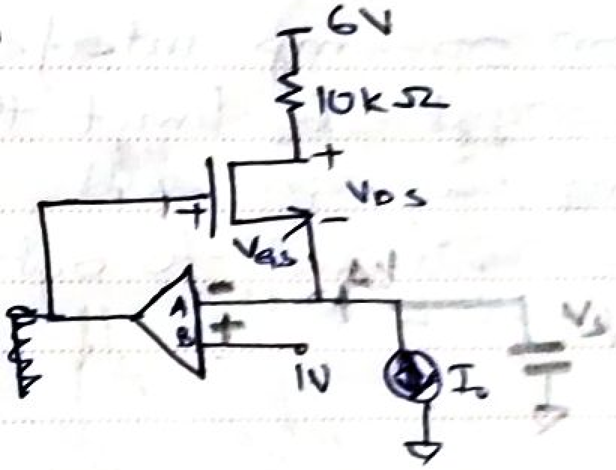
$-ve$

$\therefore$ Feed back is $-ve$

$V_{gs} = AV_t$

$\Rightarrow I_{ds} = g_m AV_t$

c)



Sense : Source

Control : Gate

By giving a small delta increment at sensing voltage, we observe $V_S \uparrow$, $V_G \downarrow$ by $-A\Delta V$. Hence V_{GS} decreases and hence I_{ds} decreases. To maintain I_o, the parasthetic cap at source discharges, decreasing the increment. ∴ A is −ve, B is +ve.

By assuming virtual short,

$$V_S = 1V$$

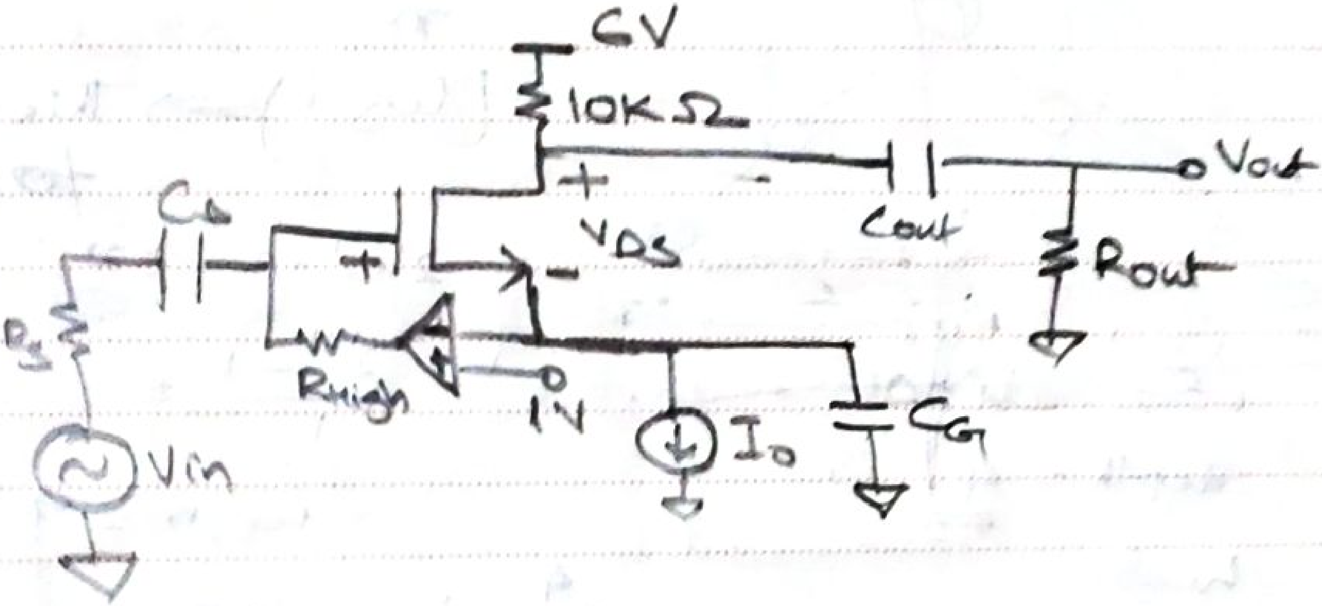
$$V_{DS} = (6V - 10K \times 200\mu A) - 1V$$
$$= 3V$$

And $V_{GS} = V_T + \sqrt{\frac{2I_{ds}}{\beta}} = 1 + 2 = 3V$

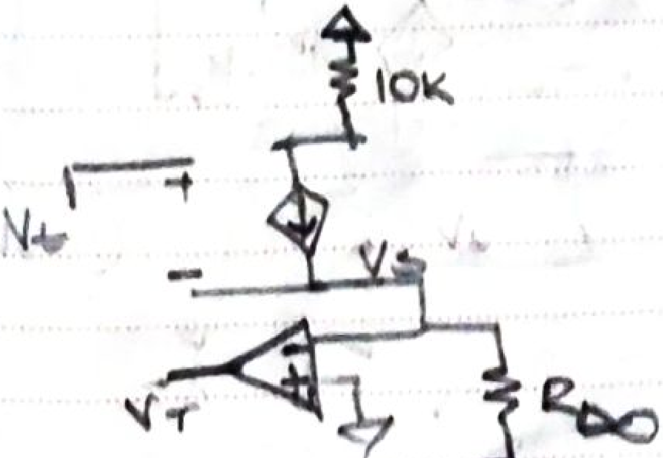
∴ $V_{DS} = 3V$ and $V_{GS} = 3V$

Here the input impedence is not high as the op-amps output terminal can load the input and hence we connect high series resistance.

The output is fine to connect as, the current is flowing completely between MOS and the output resistor.



Altern: For finding feedback, we can also cut the loop and apply test voltage. Suppose we cut it at input of MOS, we get



$$V_s = g_m(V_t - V_s)\cdot R_L$$

$$\Rightarrow V_s = g_m R_{DO}(V_t - V_s)$$

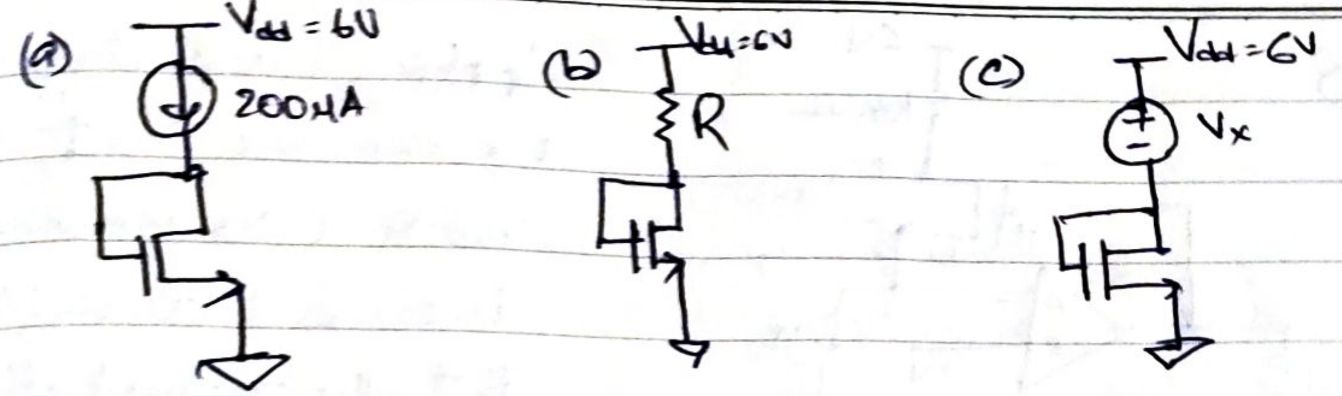
$$V_s = \frac{g_m R_{DO} V_t}{1 + g_m R_{DO}}$$

$$\approx V_t$$

$$\Rightarrow V_r = -V_t \cdot A < 0$$

$\Rightarrow$ -ve f.b for given sign.

7)



$\mu_n C_{ox} = 100 \mu A V^{-2}$ $\frac{W}{L} = 1$ $V_T = 1V$

$\beta = 100 \mu A V^{-2}$

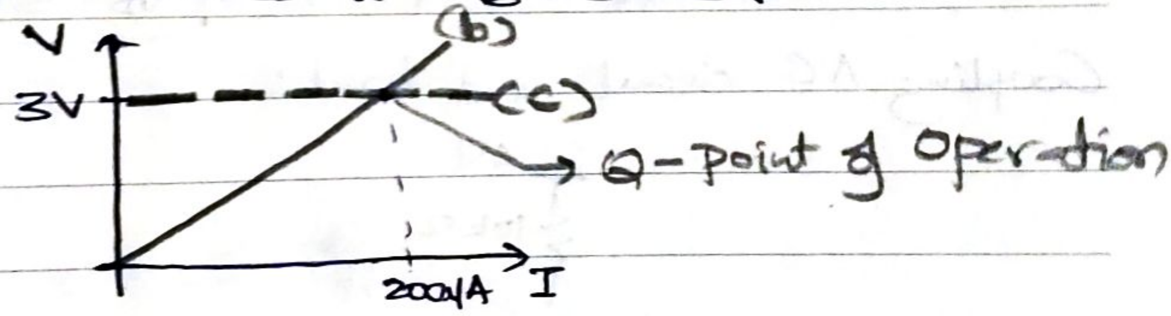
For (a) operating point is

$$V_{GS} = V_T + \sqrt{\frac{2 I_{ds}}{\beta}} = 1 + \sqrt{\frac{2 \times 200\mu}{100\mu}} = 3V$$

$\Rightarrow \boxed{V_{GS} = 3V}$

And Gate drain are shorted $\Rightarrow \boxed{V_{GS} = V_{DS} = 3V}$

As $V_S = 0 \Rightarrow V_D = 3V$. Voltage drop across the current source is $6 - 3 = 3V$.



For the given point of operation, in (b), R should be such that it has same voltage drop across i.e 3V and has to pass same current i.e 200μA. Then

$$R = \frac{V}{I} = \frac{3V}{200 \times 10^{-6} A} = 1.5 \times 10^4 \Omega = 15 k\Omega$$

$\Rightarrow \boxed{R = 15 k\Omega}$

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Similarly, for (c) the drop across the battery has to be 3V and should maintain 200 µA. As a voltage source remains same for all currents we can take,

$$\boxed{V_x = 3V}$$

(ii) Supply change: ΔV_{DD} is given.

Case-a: The I_{ds} has driven by current source and change in supply voltage is taken care by the current source.

$$\Rightarrow \Delta I_{ds} = 0$$

Case-b: Here $I_{ds} = \frac{\beta}{2}(V_{gs} - V_T)^2$

$$= \frac{\beta}{2}(V_{dd} - I_{ds}R - V_T)^2$$

$$\Rightarrow \frac{\partial I_{ds}}{\partial V_{dd}} = \beta(\underbrace{V_{dd} - I_{ds}R}_{V_{GS} = 3} - \underbrace{V_T}_{1}) \cdot \left(\frac{\partial V_{dd}}{\partial V_{dd}} - \frac{R\,\partial I_{ds}}{\partial V_{dd}}\right)$$

$$= 100\times10^{-6} \times 2 \left(1 - 15\times10^{3}\frac{\partial I_{ds}}{\partial V_{dd}}\right)$$

$$\Rightarrow \frac{\partial I_{ds}}{\partial V_{dd}} = \frac{2\times100\times10^{-6}}{1 + 100\times10^{-6}\times2\times15\times10^{3}} = 50\times10^{-6}$$

$$\Rightarrow \Delta I_{ds} = \left.\frac{\partial I_{ds}}{\partial V_{dd}}\right|_{V_{ds}=3V} \cdot \Delta V_{ds} = \boxed{50\times10^{-6}\,\Delta V_{dd}}$$

Case-c: Here $V_{GS} = V_{DS} = V_{dd} - V_x$; $I_{ds} = \frac{\beta}{2}(V_{gs} - V_T)^2$

$$\Rightarrow \frac{\partial I_{ds}}{\partial V_{ds}} = \beta(V_{gs} - V_T)\left[\frac{\partial V_{gs}}{\partial V_{dd}}\right] = 100\times10^{-6}\times2\times\frac{\partial(V_{dd} - V_x)}{\partial V_{dd}}$$

$$= 200\times10^{-6}\times1$$

$$\Rightarrow \boxed{\Delta I_{ds} = 200\times10^{-6}\times\Delta V_{dd}}$$

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iii) Effect of change in V_T.

Case-a: The current I_{ds} is driven by external current source and change V_T results in change in V_{GS} and V_{DS} but I_{ds} remains same.

Case-b: $I_{ds} = \frac{\beta}{2}(V_{GS} - V_T)^2$

$= \frac{\beta}{2}(V_{DD} - I_{DS}R_s - V_T)^2$

$\Rightarrow \frac{\partial I_{ds}}{\partial V_T} = \beta(V_{dd} - I_{ds}R_s - V_T)(-1)$

$= 100\times10^{-6}(2)(-1) = -200\times10^{-6}$

$\Rightarrow \Delta I_{ds} = -200\times10^{-6} \times \Delta V_T$

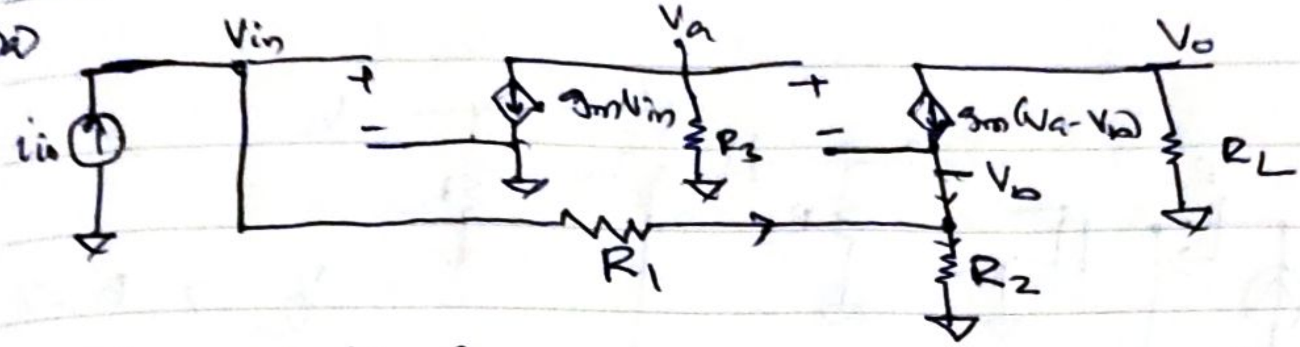
Case-c: $I_{ds} = \frac{\beta}{2}(V_{GS} - V_T)^2 = \frac{\beta}{2}(V_{dd} - V_X - V_T)^2$

$\Rightarrow \frac{\partial I_{ds}}{\partial V_T} = \beta(V_{dd} - V_X - V_T) \cdot -\frac{\partial V_T}{\partial V_T} = -\beta(V_{dd} - V_X - V_T)$

$= -100\times10^{-6}\times(2) = -200\times10^{-6}$

$\Rightarrow \Delta I_{ds} = -200\times10^{-6}\,\Delta V_T$

3)



$$V_a = -g_m V_{in} R_3 \qquad V_o = -g_m R_2 (V_a - V_b)$$

$$\Rightarrow \frac{V_{in} - V_b}{R_1} + g_m (V_a - V_b) = \frac{V_b}{R_2}$$

$$\Rightarrow \frac{V_{in} - V_b}{R_1} + g_m (-g_m V_{in} R_3 - V_b) = \frac{V_b}{R_2}$$

$$\Rightarrow V_{in}\left(\frac{1}{R_1} - g_m^2 R_3\right) = V_b\left[\frac{1}{R_1} + \frac{1}{R_2} + g_m\right]$$

$$\Rightarrow V_b = V_{in} \frac{\left(\frac{1}{R_1} - g_m^2 R_3\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} + g_m\right)}$$

$$i_{in} = \frac{V_{in} - V_b}{R_1} = \frac{V_{in}}{R_1}\left[1 - \frac{\left(\frac{1}{R_1} - g_m^2 R_3\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} + g_m\right)}\right]$$

$$= \frac{V_{in}}{R_1}\left[\frac{\frac{1}{R_2} + g_m + g_m^2 R_3}{\frac{1}{R_1} + \frac{1}{R_2} + g_m}\right]$$

$$= V_{in}\left[\frac{1 + g_m R_2 + g_m^2 R_3 R_2}{R_1 + R_2 + g_m R_1 R_2}\right]$$

$$\Rightarrow R_{in} = \frac{V_{in}}{i_{in}} = \frac{R_1 + R_2 + g_m R_1 R_2}{1 + g_m R_2 + g_m^2 R_2 R_3}$$

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Now, $V_o = -g_m R_L (V_a - V_b)$

$$\Rightarrow V_o = -g_m R_L \left(-g_m V_{in} R_3 - \frac{V_{in}\left(\frac{1}{R_1} - g_m^2 R_3\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2} + g_m\right)}\right)$$

$$= g_m R_L V_{in} \left[g_m R_3 + \left(\frac{\frac{1}{R_1} - g_m^2 R_3}{\frac{1}{R_1} + \frac{1}{R_2} + g_m} \right) \right]$$

$$= g_m R_L V_{in} \left[\frac{\cancel{g_m^2 R_3} + g_m R_3\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_1} - \cancel{g_m^2 R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + g_m} \right]$$

$$= g_m R_L V_{in} \left[\frac{g_m R_3 (R_1 + R_2) + R_2}{R_1 + R_2 + g_m R_1 R_2} \right]$$

$$= g_m R_L \cdot i_{in} R_{in} \left[\frac{g_m (R_3 (R_1 + R_2)) + R_2}{R_1 + R_2 + g_m R_1 R_2} \right]$$

$$\frac{V_o}{i_{in}} = g_m R_L \left[\frac{g_m (R_3 (R_1 + R_2)) + R_2}{1 + g_m R_2 + g_m^2 R_2 R_3} \right]$$

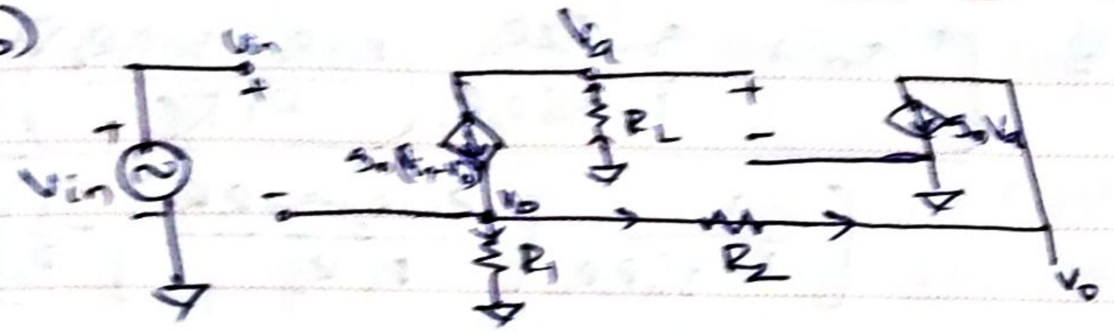
When $g_m \to \infty$,

$$R_{in} = \lim_{g_m \to \infty} \frac{\cancel{g_m^2}}{\cancel{g_m^2}} \frac{((R_1 + R_2)/g_m^2 + R_1 R_2 / g_m)}{1/g_m^2 + R_2/g_m + R_2 R_3} = \frac{0}{R_2 R_3} = \boxed{0 = R_{in}}$$

$$\frac{V_o}{i_n} = \lim_{g_m \to \infty} \frac{\cancel{g_m^2} R_L \left[R_3 (R_1 + R_2) + R_2/g_m \right]}{\cancel{g_m^2} \left[1/g_m^2 + R_2/g_m + R_2 R_3 \right]}$$

$$= \frac{R_L (\cancel{R_3} (R_1 + R_2))}{R_2 \cancel{R_3}} = \boxed{R_L \left(1 + \frac{R_1}{R_2}\right) = \frac{V_o}{i_{in}}}$$

3) b)



KCL ⇒ $V_a = -g_m(V_{in} - V_b) R_L$ ---- (1)

KCL ⇒ $V_b - V_o = g_m V_a R_2$ ----(2)

KCL ⇒ $g_m(V_{in} - V_b) = \frac{V_b}{R_1} + \frac{V_b - V_o}{R_2}$

$$\Rightarrow \boxed{g_m V_{in} + \frac{V_o}{R_2} = V_b\left[\frac{1}{R_1} + \frac{1}{R_2} + g_m\right]} \text{ --- (3)}$$

From (1) and (2),

$$V_b - V_o = g_m R_2 \left(-g_m(V_{in} - V_b)\right) R_L$$

$$= -g_m^2 R_2 R_L V_{in} + g_m^2 R_2 R_L V_b$$

$$V_b(1 - g_m^2 R_2 R_L) = V_o - g_m^2 R_2 R_L V_{in}$$

$$V_b = \frac{V_o - g_m^2 R_2 R_L V_{in}}{1 - g_m^2 R_2 R_L} \longrightarrow \text{Plug in (3)}$$

$$\Rightarrow g_m V_{in} + \frac{V_o}{R_2} = \frac{V_o - g_m^2 R_2 R_L V_{in}}{1 - g_m^2 R_2 R_L}\left[\frac{R_1 + R_2 + g_m R_1 R_2}{R_1 R_2}\right]$$

$$\Rightarrow V_{in}\left[g_m + \frac{g_m^2 R_2 R_L}{1 - g_m^2 R_2 R_L}\left[\frac{R_1 + R_2 + g_m R_1 R_2}{R_1 R_2}\right]\right]$$

$$= V_o\left[\frac{1}{1 - g_m^2 R_2 R_L}\left[\frac{R_1 + R_2 + g_m R_1 R_2}{R_1 R_2}\right] - \frac{1}{R_2}\right]$$

$$\frac{V_o}{V_{in}} = \frac{\left[g_m + \frac{g_m^2 \cancel{R_2} R_L}{1-g_m^2 R_2 R_L}\left(\frac{R_1+R_2+g_m R_1 R_2}{R_1 \cancel{R_2}}\right)\right]}{\left[\frac{1}{1-g_m^2 R_2 R_L}\left[\frac{R_1+R_2+g_m R_1 R_2}{R_1 R_2}\right]-\frac{1}{R_2}\right]}$$

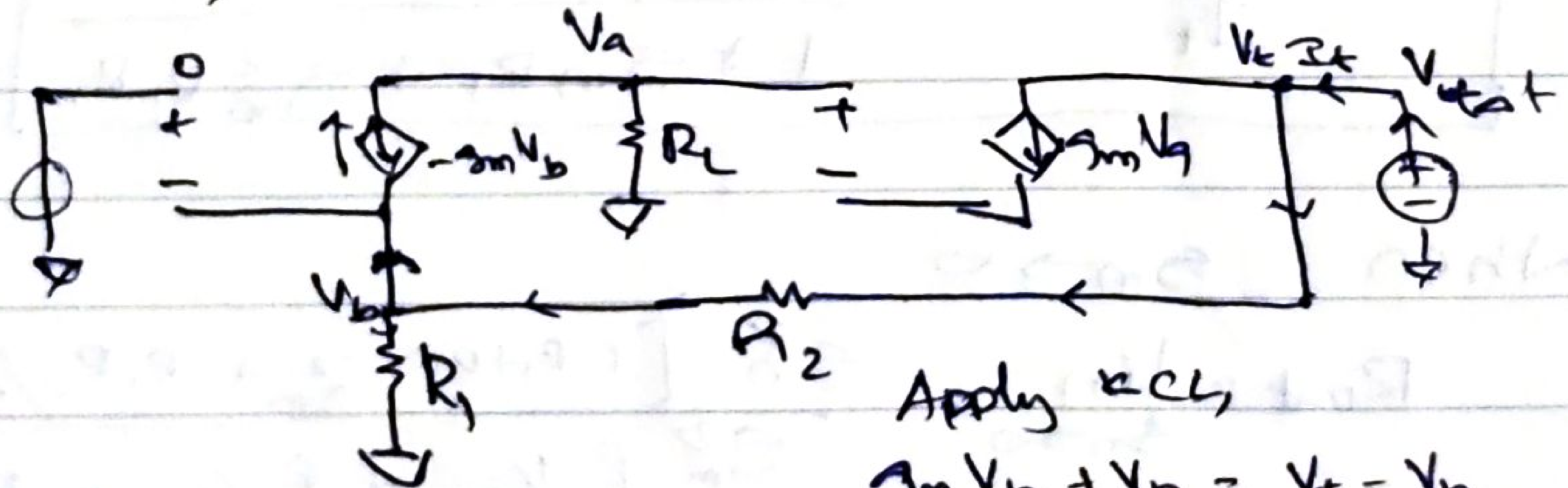
$$= \left[\frac{g_m(1-g_m^2 R_2 R_L)R_1 R_2 + g_m^2 R_2 R_L (R_1+R_2+g_m R_1 R_2)}{R_1+R_2+g_m R_1 R_2 - R_1(1-g_m^2 R_2 R_L)}\right]$$

$$= \frac{g_m R_1 R_2 - \cancel{g_m^3 R_1 R_2^2 R_L} + g_m^2 R_2 R_L (R_1+R_2) + \cancel{g_m^3 R_1 R_2^2 R_L}}{R_2 + g_m R_1 R_2 + g_m^2 R_1 R_2 R_L}$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{g_m R_1 R_2 + g_m^2 R_2 R_L (R_1+R_2)}{R_2 + g_m R_1 R_2 + g_m^2 R_1 R_2 R_L}}$$

As $g_m \to \infty$, $\frac{V_o}{V_{in}} \to \frac{\cancel{R_2}\cancel{R_L}(R_1+R_2)}{R_1 \cancel{R_2}\cancel{R_L}} = \left(1+\frac{R_2}{R_1}\right)$

For Vout,



Apply KCL,

$$g_m V_b + \frac{V_b}{R_1} = \frac{V_t - V_b}{R_2}$$

$$\boxed{V_a = g_m V_b R_L}$$

$$\Rightarrow \boxed{V_b\left[g_m + \frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{V_t}{R_2}}$$

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And, $\dfrac{V_t - V_b}{R_2} = I_{test} - g_m V_a$

$\Rightarrow V_t - V_b = R_2 I_{test} - g_m^2 V_b R_L R_2$

$\Rightarrow V_b = \dfrac{V_t - R_2 I_{test}}{1 - g_m^2 R_L R_2}$

from previous relation of V_b, we get

$$\frac{V_t - R_2 I_t}{1 - g_m^2 R_L R_2}\left[\frac{R_1 + R_2 + g_m R_1 R_2}{R_1 R_2}\right] = \frac{V_t}{R_2}$$

$$\Rightarrow V_t - R_2 I_t = \frac{(1 - g_m^2 R_L R_2) R_1}{R_1 + R_2 + g_m R_1 R_2}$$

$$\Rightarrow V_t\left[\frac{\cancel{R_1} + R_2 + g_m R_1 R_2 + g_m^2 R_L R_1 R_2 - \cancel{R_1}}{R_1 + R_2 + g_m R_1 R_2}\right] = R_2 I_t$$

$$\Rightarrow V_t \cancel{R_2}\left(\frac{1 + g_m R_1 + g_m^2 R_L R_1}{R_1 + R_2 + g_m R_1 R_2}\right) = \cancel{R_2} I_t$$

$$\Rightarrow \boxed{R_{out} = \frac{V_t}{I_t} = \frac{R_1 + R_2 + g_m R_1 R_2}{1 + g_m R_1 + g_m^2 R_L R_1}}$$

When $g_m \to \infty$

$$R_{out} = \lim_{g_m \to \infty} \frac{\cancel{g_m^2}\left[(R_1+R_2)/g_m^2 + R_1 R_2/g_m^2\right]}{\cancel{g_m^2}\left[1/g_m^2 + R_1/g_m + R_1 R_L\right]}$$

$$= \frac{0}{R_1 R_2} \Rightarrow \boxed{R_{out} = 0}$$