

EE2101 : Control Systems

Modeling in the Frequency Domain

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1 Problem

2 Solution

- Finding the constants
- Finding equivalents
- Finding the desired transfer function
- Transfer function in time domain

3 Plot

Problem Statement

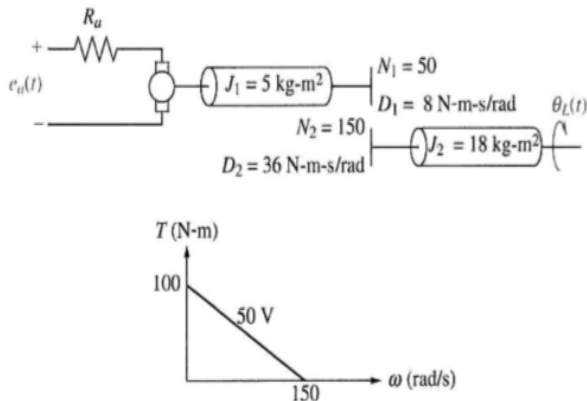


Figure: Problem Figure

For the motor, load, and torque-speed curve shown, find the transfer function, $G(s) = \theta_L/E_a(s)$.

Solution

From the given Torque speed curve, the constants can be found, for motor operating in steady state with d.c. voltage input

$$\frac{R_a}{K_t} T_m + K_b \omega_m = e_a$$

Given $E_a=50\text{V}$, intercepts $T_{stall}=100\text{ N}\cdot\text{m}$ and $\omega_{no-load}=150\text{ rad/s}$.

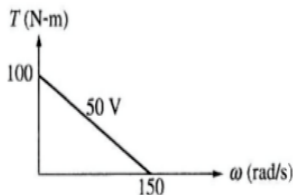


Figure: Torque-speed curve

$$\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} = 2; K_b = \frac{E_a}{\omega_{no-load}} = \frac{1}{3}$$

For finding the equivalent moment of inertia and damping, we translate the load shaft to the source shaft. Hence,

$$J_m = J_1 + J_2 \left(\frac{N_1}{N_2} \right)^2 = 5 + 18(1/3)^2 = 7$$

$$D_m = D_1 + D_2 \left(\frac{N_1}{N_2} \right)^2 = 8 + 36(1/3)^2 = 12.$$

The transfer function for electromechanical system,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s \left[s + \frac{1}{J_m} \left(D_m + \frac{K_t K_b}{R_a} \right) \right]}$$

Substituting the values we get,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2/7}{s \left[s + \frac{1}{7} \left(12 + \frac{2}{3} \right) \right]} = \frac{6}{s(21s + 38)}$$

Finding the desired transfer function

The required transfer function is $G(s) = \frac{\theta_L(s)}{E_a(s)}$.

From the gears: $\theta_L(s) = \frac{1}{3}\theta_m(s)$

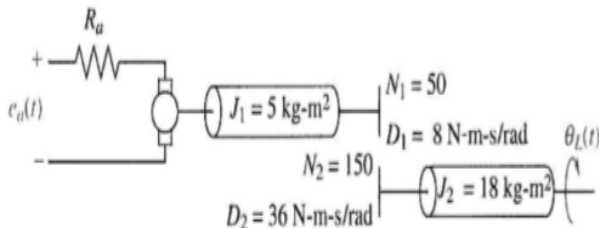


Figure: Electromechanical Systems

Hence

$$G(s) = \frac{2}{s(21s + 38)}$$

Transfer function in time domain

Resolving to partial fractions,

$$G(s) = \frac{2}{s(21s + 38)} = \frac{A}{s} + \frac{B}{21s + 38}$$

On comparing both sides, we get $A=1/19, B=-21/19$, so

$$G(s) = \frac{1}{19s} - \frac{1}{19(s + \frac{38}{21})}$$

In the time domain, i.e. applying Inverse Laplace transform

$$g(t) = \frac{1}{19}(1 - e^{-\frac{38}{21}t})$$

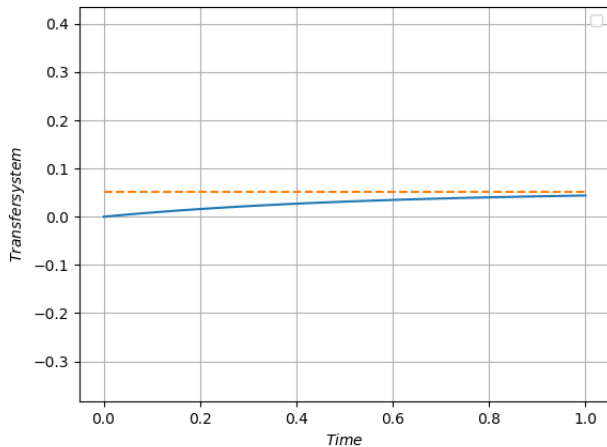


Figure: Transfer system

Thank You

<https://github.com/ee19btech11040/Control-Systems.git>